Cournot Fire Sales

Thomas M. Eisenbach and Gregory Phelan*

February 2018

Abstract

In standard Walrasian macro-finance models, pecuniary externalities such as fire sales lead to overinvestment in illiquid assets or underprovision of liquidity. We investigate whether imperfect competition (Cournot) improves welfare through internalizing the externality and find that this is far from guaranteed. In a standard model of liquidity shocks, when liquidity is sufficiently scarce, Cournot competition leads to even less liquidity than the Walrasian equilibrium. In a standard model of productivity shocks, the Cournot equilibrium over-corrects for the fire-sale externality and holds less capital than socially efficient. Implications for welfare and regulation therefore depend highly on the nature of the shocks and the competitiveness of the industry considered.

JEL: D43, D62, E44, G18, G21.

Keywords: liquidity; fire sales; overinvestment; financial regulation; macroprudential regulation.

*Eisenbach: Research Group, Federal Reserve Bank of New York, thomas.eisenbach@ny.frb.org; Phelan: Department of Economics, Williams College, gp4@williams.edu. The views expressed in the paper are those of the authors and are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System. For valuable comments, we thank Gara Afonso, Nina Boyarchenko, Adam Copeland, Keshav Dogra, Michael Lee, as well as audience members at the New York Fed. Any errors are our own.

1 Introduction

The macro-finance literature has taken great interest in fire-sale externalities. Such pecuniary externalities can lead to underprovision of liquidity and overinvestment, because price-taking agents do not internalize how their portfolio choices will affect prices after adverse shocks. The standard macro-finance externalities in the literature would be partially mitigated if agents internalized their effects on prices: agents would hold more liquidity, or borrow less, so that asset prices are supported when bad aggregate shocks occur. In this paper, we show that a very simple and natural modification of standard macro-finance models can lead to very different, nonintuitive, conclusions about how less than perfect competition (internalizing price effects) mitigates, exacerbates, or reverses pecuniary externalities. Critically, how internalizing price impacts affects pecuniary externalities depends on whether fire sales are driven by liquidity or productivity shocks and whether the shocks are purely aggregate or have an idiosyncratic component.

We consider two standard macro-finance models—a model of liquidity shocks with illiquid assets, and a model of productivity shocks with borrowing constraints—with the crucial modifications that (i) the economy features both aggregate and idiosyncratic risk and (ii) firms internalize how their initial portfolio choices will affect future prices à la Cournot competition. Because not all firms receive the same shocks, when fire sales occur because some agents receive bad shocks, other agents receive good shocks and are therefore in a favorable position to buy fire-sale assets cheap. Agents strategically consider how their portfolios will affect prices, both when they receive bad shocks and contribute to fire sales, and when they receive good shocks and benefit from fire sales.

Crucially, we show that this strategic consideration as buyer and seller has important consequences for how imperfect competition affects investment and liquidity. A social planner considers how initial decisions will *in the aggregate* affect fire-sale prices, and then weighs the social value of those price impacts by averaging over the marginal consequences for buyers and sellers. In particular, buyers will benefit from low prices, and sellers will benefit from high prices, and the social value of affecting prices depends on the marginal utilities of buyers and sellers. In contracts, Cournot agents consider separately how their initial decisions will affect prices *when they are a buyer* and *when they are a seller*. The private value of those price impacts will then be weighed according to the marginal utility when a buyer and the marginal utility when a seller, but a Cournot firm does not average across these states to consider the aggregate price impact of its decision. Cournot firms do not care about the aggregate price impacts but rather the price impact *when* they will buy or sell. Importantly, the implications of this strategic consideration depend critically on whether uncertainty originates from liquidity shocks (liabilities side) or productivity shocks (asset side).

Standard models of liquidity risk typically consider idiosyncratic liquidity shocks that incomplete markets fail to provide adequate insurance for. If investors held more liquidity, the interim (illiquid) asset price would be higher, which implicitly provides insurance to investors receiving liquidity shocks (and selling assets). Hence, the standard model features *under-provision* of liquidity in Walrasian equilibrium. In our model, investors know that holding more liquidity will push up the asset price, which is good when they are sellers but bad when they are buyers. The relative weight given to the two states depends on the marginal effect of holding more liquidity on the prices in the two states. Importantly, whether the price is more responsive to the buyers' liquid asset holdings (demand) or the sellers' illiquid asset holdings (supply) depends on the level of the asset price. When the price is sufficiently low, investors have a greater strategic incentive to push down the price (to buy at cheap prices when they are buyers). This is precisely what would occur in the presence of aggregate liquidity risk: we suppose there is a good aggregate state in which no investors have liquidity shocks, and as a result investors hold less liquidity ex ante. As a result, fire sales are more extreme, and Cournot competition leads to even *lower* asset prices. Thus, Cournot competition exacerbates the pecuniary externality rather than mitigating it, and Cournot equilibrium has *under-under-provision* of liquidity.

Standard models of productivity risk with borrowing constraints typically consider "pure aggregate risk" so that all agents receive a bad shock at the same time, forcing sales of capital to repay debts, pushing down asset prices, and requiring even more sales in order to raise funds. If firms invested less initially (i.e., borrowed less), then fire sales would be smaller, and less capital would be reallocated to inefficient users of capital. Hence, the standard model features *over-investment* in Walrasian equilibrium. To this standard setup we introduce idiosyncratic production risk in the bad state, so that some firms have good production and can buy up capital at cheap prices. With Cournot competition, firms receiving bad shocks know that they will sell capital, and they strategically would like to hold less capital to minimize the price impact. Firms receiving good shocks will buy capital, and they would like to purchase capital at lower prices, which they would do by having fewer funds available to buy capital—which occurs by having less capital. So whether a buyer or a seller, firms strategically would like to hold less capital *in either case*. As a result, the Cournot equilibrium features *under-investment* relative to the efficient level because shocks to capital determine the funds available to repay debts or buy new capital.

In summary, while Cournot competition can mitigate the externality in both models, in the liquidity risk model Cournot can also *exacerbate* the externality and in the productivity risk model Cournot can instead *reverse* the externality. Given these vastly different results, it is worth highlighting why the nature of the fire sales in the models are so different. First, it may seem trivial to point out that in either model, the price impact of additional funds used to buy assets, or of additional assets for sale, depends on the total supply of assets and funds in the market. But in the liquidity risk model with liabilities-side shocks, the supply of funds and of assets is completely determined by the initial liquidity decision. The value of liquid assets is not affected by whether or not an agent receives a liquidity shock. Hence, what matters for price impacts of buying and selling is initial liquidity holdings, which are primarily determined by the aggregate risk of agents receiving liquidity shocks. In contrast, in the production model with asset-side shocks, the supply of funds to buy capital does depend on the productivity shock, and the supply of capital for sale also depends on the productivity shock. A firm with high productivity has more funds to buy capital—and the more capital initially invested, the more a high productivity shock increases funds available to buy capital. In the same way, firms with low productivity sell more capital to repay debts because productivity is low—and the more capital initially invested, the more a low productivity shock requires selling capital to repay debts. The source of the shock—asset-side or liabilities-side—has very different implications for the supply of assets for sale and of funds available to buy those assets.

The theoretical contribution of our paper is itself of important interest to the macro-finance literature, given the importance of pecuniary externalities for understanding macro-finance frictions. Furthermore, our analysis potentially sheds light on recent trends in market power amongst firms and financial institutions. First, many markets are increasingly dominated by a small number of very large firms. At the same time, many of these large firms have enormous cash holdings and by some measures investment has been low among these firms. Our analysis provides an explanation, that firms with market power will tend to under-invest when the primary shocks they face are productivity (asset-side) shocks. Additionally, financial markets have become increasingly concentrated, and our results suggest that liquidity holdings, in the absence of regulation, would be inefficiently low. Increased market power in the banking sector could lead to decline in liquidity holdings below the Walrasian level, making effects of a moderate or severe financial crisis more damaging than would occur otherwise (in the absence of regulation). Our results therefore suggest that regulation addressing fire sales must carefully consider the source of shocks driving fire sales and the degree of price-taking behavior among market participants.

Related literature. The literature on generic inefficiency arising from pecuniary externalities dates to Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986), which provide justifications for policy interventions when private agents do not internalize their effects on prices. Dávila (2015) and Dávila and Korinek (2017) provide recent analysis of pecuniary externalities in macro-finance models with borrowing constraints, showing that terms of trade and collateral externalities are distinct, as are the issues of efficiency and amplifications. Stein (2013) is an example of policy thinking based on academic insights.

Closely related to the literature on pecuniary externalities are the papers on fire sales and limits to arbitrage: Shleifer and Vishny (1992), Gromb and Vayanos (2002), Shleifer and Vishny (2011). All of these papers on pecuniary externalities share the feature that inefficiencies arise because price-taking agents do not internalize how their portfolio decisions affects prices, affecting risk sharing and borrowing capacities.

The literature on liquidity provision includes Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Jacklin (1987), and Allen and Gale (2004). Recently, Farhi et al. (2009) and Geanakoplos and Walsh (2017) study inefficient liquidity provision with private trades in financial markets. These papers study how incomplete markets lead to under-provision of liquidity (typically, though different specifications of shocks can lead to over-provision).

Diamond and Rajan (2011) argue that limited liability constraints make banks "seekers of liquidity." Gale and Yorulmazer (2013) argue that costly bankruptcy and incomplete markets cause inefficient liquidity hoarding. Malherbe (2014) argues that liquidity provision can exacerbate adverse selection. Perotti and Suarez (2002) highlight the incentive to be the "last bank standing."

Our paper relates to the literature on overinvestment, which includes Caballero and Krishnamurthy (2001), He and Kondor (2016), and Lorenzoni (2008).

A few macro-finance papers consider the implications of firms interalizing their affect on prices. Corsetti et al. (2004) consider how the presence of a large traders effects the likelihood of currency crises, as small traders take into account strategically the behavior of the large trader (small traders are more aggressive). Dávila and Walther (2017) consider the leverage decisions of large and small banks when banks internalize how their leverage and size affect bailout probabilities (small firms use more leverage in the presence of large firms).

2 Cournot in a model of liquidity risk

We first consider a standard model of liquidity risk à la Diamond and Dybvig (1983) and show that internalizing the pecuniary externality through Cournot behavior can exacerbate the inefficiency by leading to underprovision of liquidity even compared to the Walrasian equilibrium.

There are three periods $t = 0, 1, 2$, and $2N$ investors that have two investment opportunities at $t = 0$: (i) *liquid* assets, which deliver 1 in $t = 1$ or $t = 2$ for each unit invested at $t = 0$ (think of liquidity as cash); (ii) *illiquid* assets, which deliver $R > 1$ at $t = 2$ for each unit invested at $t = 0$ but nothing in $t = 1$. In $t = 1$ illiquid assets can be traded in the market for an endogenous price p. We suppose that the interim asset price p is determined by market clearing (cash-in-the-market pricing) and that investors behave as price takers in this period. In Appendix A we consider strategic behavior at $t = 1$ and show that, except for a knife-edge case, price-taking behavior at $t = 1$ is without loss of generality. It is at $t = 0$ where strategic vs. price-taking behavior crucially affects the results.

Investors start with one unit to invest and have utility following Diamond and Dybvig (1983): investors can be either early or late consumers, and late period utility is discounted by $\beta \leq 1$ and with $\beta R > 1$. We will throughout our analysis suppose that investors have relative risk aversion greater than 1. Together with $\beta \leq 1$, our assumptions on preferences imply that investors have demand for liquid claims. Accordingly, we will say that investors are subject to *liquidity shocks* forcing them to liquidate their holdings, e.g. to meet liquidity needs from the liability side of the balance sheet.

We suppose that there are two aggregate states in the economy at $t = 1$, a good state and a bad state. In the good state, no liquidity shocks occur, and so all investors consume late. In the bad state, half the investors, randomly selected, receive liquidity shocks. These investors sell their illiquid assets at a price p to the the other half that did not receive liquidity shocks. The good aggregate state occurs with probability α.[1]

Denote the fraction invested in liquidity by ℓ_i (hence, $1 - \ell_i$ invested in illiquid assets). When no liquidity shocks occur in the aggregate, consumption is $\bar{c} = \ell_i + (1 - \ell_i) R$, which is the return from investing in liquid and illiquid assets. If an investors receives a liquidity shock in the bad state, it sells its illiquid assets and hence receives $c_L = \ell_i + (1 - \ell_i) p$ (L for low). When an investors does not receive a liquidity shock in the bad state, the investors can use liquid investments to buy illiquid assets (potentially cheap), and hence receives $c_H = \ell_i \frac{R}{p} + (1 - \ell_i) R$ (H for high). It is clear that $p \leq R$ in equilibrium since H types would not be willing to pay more than R for illiquid assets. For now we suppose that the only buyers of illiquid assets are other investors. Figure 1 summarizes the states.

[1] Without changing the analysis, one can also consider a third "very bad" or "crisis" aggregate state in which *all* investors receive liquidity shocks. We think of the crisis state as an extreme financial crisis, a very rare event occurring once or twice a century. Adding this state would increase the equilibrium price in the bad state (in accordance with the discussion below), which could possibly lead to an over-provision of liquidity. However, we think this is not an empirically relevant case to consider.

Aggregate state	Probability	Liquidity shock	Consumption
Good state	α	Nobody hit	$\bar{c} = \ell_i + (1 - \ell_i) R$
Bad state	$1 - \alpha$	Hit ($\Pr = \frac{1}{2}$)	$c_L = \ell_i + (1 - \ell_i) p$
		Not hit ($\Pr = \frac{1}{2}$)	$c_H = \ell_i \frac{R}{p} + (1 - \ell_i) R$

Figure 1: Summary of aggregate states and investor consumption.

Accordingly, the expected utility of investors is given by

$$\alpha \beta u(\bar{c}) + (1 - \alpha) \left(\tfrac{1}{2} u(c_L) + \tfrac{1}{2} \beta u(c_H) \right)$$

2.1 Walrasian Equilibrium

Taking p as exogenous, the first order condition of an investor in the Walrasian equilibrium (WE) is

$$\underbrace{\frac{(1-\alpha)}{2} (1 - p) \left(u'(c_L) + \frac{1}{p} \beta R u'(c_H) \right)}_{\text{benefit in bad state}} = \underbrace{\alpha \beta (R - 1) u'(\bar{c})}_{\text{cost in good state}} \tag{1}$$

Since the RHS is positive, it must be that the liquidation price satisfies $p \leq 1$, with strict equality whenever the good state occurs with positive probability. In a symmetric equilibrium in which all $2N$ investors hold $\ell_i = \ell$ in liquid assets, the asset price thus satisfies

$$N (1 - \ell) p = N \ell \quad \Rightarrow \quad p = \frac{\ell}{1 - \ell}.$$

Critically, if there is no aggregate risk so that only the bad state can occur, then $p = 1$ in equilibrium. This case provides a nice intuition. In the Walrasian equilibrium, additional liquidity affects both states L and H the same way. If $p < 1$ then assets are traded below cost so no-one wants to invest in them; sellers (state L) would rather hold liquidity and buyers (state H) would rather buy assets cheaply. Analogously for $p > 1$. Equilibrium is pinned down by the no-arbitrage condition $p = 1$ which leads to $c_L = 1$ and $c_H = R$. Furthermore, since the model with only liquidity shocks corresponds to the standard macro-finance liquidity risk model, we now consider the Social Planner and Cournot allocations without aggregate risk.

2.2 No aggregate risk

We first focus on the bad state alone (and set $\alpha = 0$) which corresponds to the standard macro-finance liquidity risk model. Since the model is standard, we find the standard (and intuitive) result that the Walrasian equilibrium provides too little liquidity, and Cournot partially corrects the inefficiency. The section therefore serves as a benchmark for Section 2.3, where the introduction of aggregate risk can easily overturn this intuition. We first consider the Social Planner's liquidity decision and then the equilibrium outcome with Cournot firms.

Social planner. Taking into account that $p = \ell/(1-\ell)$, the social planner (SP) has an additional term in the first order condition, which considers how liquidity holdings will affect the asset price:

$$(1-p)\left(u'(c_L) + \frac{1}{p}\beta R u'(c_H)\right) + \underbrace{(1-\ell)\left(u'(c_L) - \frac{1}{p}\beta R u'(c_H)\right)\frac{dp}{d\ell}}_{\text{Social planner price effect}} = 0$$

The social planner takes into account that p is increasing in ℓ,

$$\frac{dp}{d\ell} = \frac{1}{(1-\ell)^2},$$

and that a higher price benefits sellers L at the expense of buyers H.

The social planner chooses higher liquidity than the Walrasian equilibrium if the Walrasian equilibrium allocation results in $u'(c_L) > \beta R u'(c_H)$. Substituting $dp/d\ell$ into the first order condition yields the standard risk sharing condition of Diamond and Dybvig (1983) for the social planner allocation:

$$u'(c_L) = \beta R u'(c_H)$$

The social planner chooses a higher price and improves liquidity insurance so that $c_L > 1$ and $c_H < R$.

The condition for insufficient liquidity in the Walrasian equilibrium, $u'(1) > \beta R u'(R)$, is the standard condition of Diamond and Dybvig (1983) which is satisfied for $\beta \leq 1$ and relative risk aversion greater than 1. The Walrasian equilibrium is inefficient because the no-arbitrage condition (1) applies between periods 0 and 1 while the insurance problem applies between periods 1 and 2. Our setup with trading essentially corresponds to the

Jacklin (1987) model, and our result that liquidity under-provision can be corrected by increasing the asset price is found also in Farhi et al. (2009) and Geanakoplos and Walsh (2017).

Cournot equilibrium. In the Cournot equilibrium (CE), investors taking into account the effect of their own liquidity choice also have an additional term in the first order condition:

$$(1-p)\left(u'(c_L)+\frac{1}{p}\beta R u'(c_H)\right)+\underbrace{(1-\ell)\left(\frac{dp_L}{d\ell}u'(c_L)-\frac{dp_H}{d\ell}\frac{1}{p}\beta R u'(c_H)\right)}_{\text{Cournot price effect}}=0$$

Similar to the social planner, the Cournot agent also takes into account that liquidity affects price and that a higher price benefits herself as a sellers but hurts herself as a buyer. However, the Cournot agent takes into account only a "partial" price impact (compared to the SP) since she is only considering her own choice: the effect of holding more liquidity when buying (state L) or the effect of holding fewer assets when selling (state H).

With $2N$ investors, we have

$$\frac{dp_L}{d\ell}=\frac{1}{N}\frac{\ell}{(1-\ell)^2} \quad \text{and} \quad \frac{dp_H}{d\ell}=\frac{1}{N}\frac{1}{1-\ell}$$

Compared to the social planner's price impact, $dp/d\ell = 1/\left(1-\ell\right)^2$, the Cournot agent's price impacts are uniformly lower, biasing downward the Cournot liquidity choice.[2] However, the price impacts also differ depending whether the Cournot agent is a buyer or a seller and the relative size of these two price impacts depends crucially on the level of p. If $p > 1$, the effect on the seller price, where the agent prefers a higher price, is greater than the effect on the buyer price, where the agent prefers a lower price, as illustrated in Figure 2. This effect biases upward the Cournot liquidity choice if $p > 1$ and further downward if $p < 1$. Accordingly, Cournot agents do not weigh their effects on the price in the same way the social planner does. While the social planner averages the price effect over all agents in all states (buyers and sellers), Cournot agents weigh their price impact separately depending on whether they will be a buyer or seller.

Substituting into the first order condition yields

$$u'(c_L)-\beta R u'(c_H)=\left(1-\frac{1}{N}\right)\left(pu'(c_L)-\frac{1}{p}\beta R u'(c_H)\right)$$

[2] Notice that $N\frac{dp_L}{d\ell}+N\frac{dp_H}{d\ell}=\frac{dp}{d\ell}$, so that the price impacts of all agents indeed sum to the price impact calculated by the social planner.

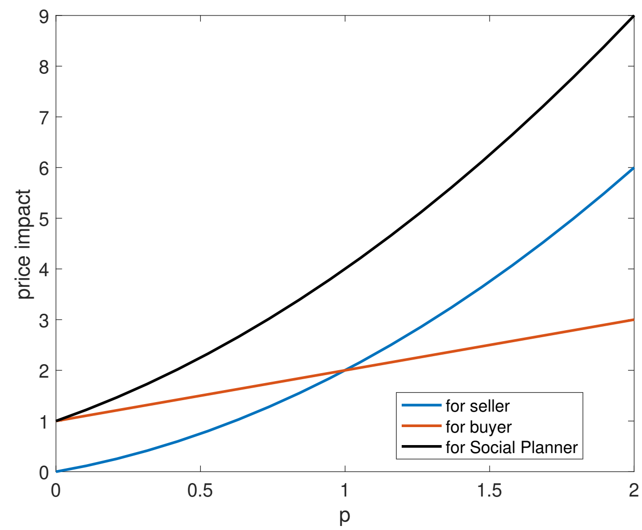

Figure 2: Effects of liquidity on Cournot price ($N = 1$)

For $N = 1$, the two biasing effects exactly offset and Cournot yields the efficient level of liquidity. For $N > 1$, however, the downward bias prevails and Cournot yields inefficiently low liquidity, albeit more than the Walrasian equilibrium. For $N \to \infty$, Cournot approaches the Walrasian equilibrium with $p = 1$.

In sum, under the standard condition $u'(1) > \beta R u'(R)$, the Walrasian equilibrium provides too little liquidity, and Cournot partially corrects the inefficiency. This is the natural intuition implicit in standard macro-finance models with pecuniary externalities. As we show next, this intuition is not robust to the introduction of aggregate risk.

2.3 Aggregate risk

Aggregate risk adds an additional term to the first order condition, the cost of holding liquidity in the good state where it is not needed since no investor receives a liquidity shock. However, since assets are not traded in the good state, this term doesn't involve p and therefore does not differ across the first order conditions of Walrasian equilibrium, social planner, and Cournot. Through this additional cost of holding liquidity, aggregate risk pushes equilibrium liquidity holdings — and therefore the price in the bad state — lower than in the case without aggregate risk.

Social planner. The social planner has the same additional term on the LHS of the first-order condition as before, and the new RHS due to aggregate risk:

$$\frac{(1-\alpha)}{2}(1-p)\left(u'(c_L)+\frac{1}{p}\beta R u'(c_H)\right)+\overbrace{\frac{(1-\alpha)}{2}(1-\ell)\left(u'(c_L)-\frac{1}{p}\beta R u'(c_H)\right)\frac{dp}{d\ell}}^{\text{price effect same as w/o agg. risk}}$$
$$=\underbrace{\alpha\beta(R-1)\,u'(\bar{c})}_{\text{cost same as WE}}$$

The condition for extra liquidity is unchanged, $u'(c_L) > \frac{1}{p}\beta R u'(c_H)$, which still holds for $\beta \leq 1$ and relative risk aversion greater than 1 (even for $p \neq 1$) so the Walrasian equilibrium still chooses inefficiently low liquidity. However, in the presence of aggregate risk, the social planner may find it optimal to set $p < 1$. This occurs if the crisis state is not too severe (if α is high and/or R high).

Cournot equilibrium. The Cournot first-order condition extends analogously to

$$\frac{(1-\alpha)}{2}(1-p)\left(u'(c_L)+\frac{1}{p}\beta R u'(c_H)\right)+\overbrace{\frac{(1-\alpha)}{2}(1-\ell)\left(\frac{dp_L}{d\ell}u'(c_L)-\frac{dp_H}{d\ell}\frac{1}{p}\beta R u'(c_H)\right)}^{\text{price effect same as w/o agg. risk}}$$
$$=\underbrace{\alpha\beta(R-1)\,u'(\bar{c})}_{\text{cost same as WE}}$$

As before, Cournot accounts for the price effect only in a partial way and differentially for being a buyer and a seller.

Whether Cournot leads to inefficiently low or high liquidity, depends on whether, at the efficient allocation,

$$\frac{dp_L}{d\ell}u'(c_L)-\frac{dp_H}{d\ell}\frac{1}{p}\beta R u'(c_H) \lesseqgtr \left(u'(c_L)-\frac{1}{p}\beta R u'(c_H)\right)\frac{dp}{d\ell}$$

Substituting in for the price effects and setting $N=1$ yields

$$\ell u'(c_L)-(1-\ell)\frac{1}{p}\beta R u'(c_H) \lesseqgtr u'(c_L)-\frac{1}{p}\beta R u'(c_H) \tag{2}$$

While the RHS is decreasing in ℓ, the LHS is always *less* decreasing (and can be increasing in ℓ). Further, the LHS and RHS are equal at the efficient allocation without aggregate risk. Therefore, Cournot leads to inefficiently low liquidity when aggregate risk lowers the effi-

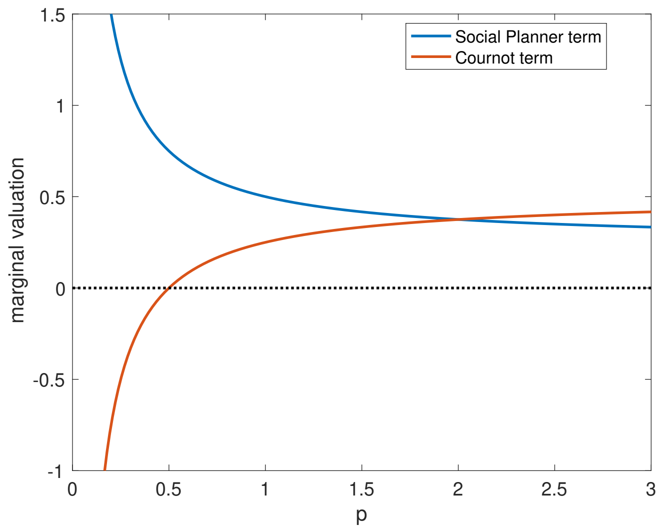

Figure 3: Condition for Cournot overprovision of liquidity with log utility and $\beta = 0.5$.

cient level but inefficiently high liquidity when aggregate risk increases the efficient level. Figure 3 illustrates the LHS and RHS of condition (2) for log utility, where overprovision occurs for $p > 1/\beta$.[3]

Comparing to the Walrasian equilibrium, Cournot yields less liquidity if, at the Walrasian allocation,

$$\frac{dp_L}{d\ell}u'(c_L) - \frac{dp_H}{d\ell}\frac{1}{p}\beta R u'(c_H) < 0$$

$$\Leftrightarrow \quad \ell u'(c_L) - (1-\ell)\frac{1}{p}\beta R u'(c_H) < 0$$

If the LHS is increasing in ℓ, then Cournot yield less liquidity than both the Walrasian equilibrium and the social planner for sufficiently low ℓ, i.e. if the crisis state is sufficiently unlikely. Figure 3 illustrates this for log utility, where the condition simplifies to $p < \beta$.

What is the intuition for the underprovision of liquidity by Cournot, even compared to the Walrasian equilibrium? As $\ell \to 0$ and $p \to 0$ (driven by a decreased likelihood receiving liquidity shocks, as determined by the aggregate risk), we have that seller price impact $dp_L/d\ell$ goes to zero while buyer price impact $dp_H/d\ell$ does not. This implies that $\frac{dp_L}{d\ell}u'(c_L)$ goes to zero (as long as risk aversion is not too high). Cournot then holds very little liquidity because more liquidity would have negligible benefit in the low state but non-zero cost in the high state.

[3]Note that, although not the most natural case, a high price can occur if a bad crisis state in which all investors receive liquidity shocks is sufficiently likely.

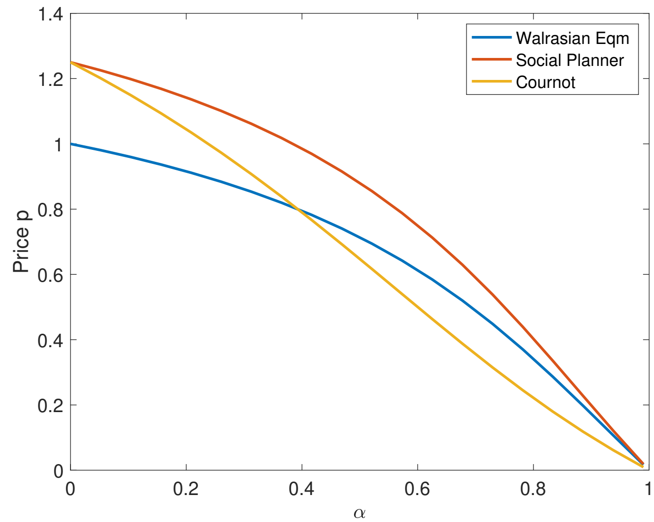

Figure 4: Comparison of liquidity provision of Walrasian and Cournot equilibrium to the efficient level with log utility, $\beta = 0.8$, and $R = 1.5$.

In sum, with aggregate risk, Cournot can provide potentially even less liquidity than the Walrasian equilibrium, in violation of the intuition that internalizing the pecuniary externality should lead to an allocation closer to the social planner's . Figure 4 compares the levels of liquidity provision in the Walrasian and Cournot equilibria to the efficient level varying the probability of the good aggregate state. As the good state without liquidity shocks becomes more likely (α increases), the efficient level of liquidity declines but is always higher than the one provided by the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency as long as liquidity risk is sufficiently high (α sufficiently low). Once the good state is sufficiently likely and liquidity risk therefore sufficiently low, the Cournot equilibrium exacerbates the underprovision of liquidity in the Walrasian equilibrium.

3 Cournot in a model of productivity risk

We now consider a standard model of fire sales with production and show that internalizing the pecuniary externality through Cournot behavior can overcorrect the inefficiency by leading to underinvestment even compared to the social planner. This is the polar opposite of the result from the liquidity risk model of Section (2), highlighting just how incomplete the intuition of Cournot behavior correcting pecuniary externalities is.

Firms have a production technology with uncertain output. When output is low firms must sell capital to repay debts. The main result of this analysis is that, when firms receiving good shocks will buy capital at fire-sale prices, then the Walrasian equilibrium features over-investment (standard), whereas the Cournot equilibrium features *under*-investment relative to the efficient level, overcorrecting the pecuniary externality. This key difference with the liquidity model arises because capital production is the source of funds to repay debts and the source of funds to buy new capital at fire-sale prices. Accordingly, holding less capital always moves prices in the direction that a buyer or seller would prefer to move, even though in the aggregate less capital always leads to higher prices.

There are three periods, $t = 0, 1, 2$. There are two types of agents, $2N$ firms and $2N$ households. Households are risk neutral with deep pockets and do not discount consumption. Firms consume at $t = 2$ and have risk-averse utility $u(c)$ with $\lim_{c \to 0} u'(c) = \infty$. Firms are each endowed with n units of consumption goods at $t = 0$. Consumption goods are perfectly durable. As we did in the liquidity risk model, we suppose that agents behave strategically in the initial period but are price takers in the interim date. In Appendix A we consider strategic behavior at $t = 1$ and show that price-taking behavior is always without loss of generality in the production model.

Firms have access to a linear production technology using capital in each period. Production at $t = 1$ is risky: Capital k invested at $t = 0$ produces Ak consumption goods at $t = 1$, where A is uncertain, with $\mathbb{E}[A] = 1$. Production at $t = 2$ is risk-free with every unit of period-1 capital producing one unit of consumption at $t = 2$.[4]

Households have access to a production technology that yields $F(k) = a \log(1 + k)$ at $t = 2$ for capital holdings k at $t = 1$. Households will buy capital to produce if the capital price at $t = 1$ is below a. To ensure that households only buy capital following a fire sale, we suppose that $a \leq 1$.

To simplify the analysis, we suppose that at $t = 0$ capital can be produced from consumption goods at a linear rate $q_0 \leq 1$ so that the capital price at $t = 0$ is $q_0 \leq 1$. Capital is fully durable and cannot be produced at $t = 1$.

In the absence of constraints, the capital price at $t = 1$ would equal 1. We suppose that, at $t = 1$, there is no borrowing, and so if firms have to sell capital to repay debts the capital price can be less than 1. However at $t = 0$ firms can borrow from households at a rate of 0 (risk free). Since $q_0 < 1$, firms will leverage to buy capital, and since firms are sufficiently risk averse (and we do not model repayment frictions) all borrowing is risk free. Denoting

[4]Modeling firms as risk-averse with linear production is a tractable way to generate a motive for insurance. We could also model firms as risk-neutral with curvature in their production technology (see Holmström and Tirole, 1998).

Aggregate state	Probability	Productivity shock	Consumption
Good state	α	$\overline{A}$	$\bar{c} = n + \left(\overline{A} + 1 - q_0\right) k$
Bad state	$1-\alpha$	A_L	$c_L = \frac{n}{\tilde{q}} + \frac{\tilde{q}+A_L-q_0}{\tilde{q}} k$
		A_H	$c_H = \frac{n}{\tilde{q}} + \frac{\tilde{q}+A_H-q_0}{\tilde{q}} k$

Figure 5: Summary of aggregate states and firm consumption.

borrowing by d, the firm's balance sheet at $t = 0$ satisfies

$$q_0 k = n + d.$$

In this setup, allowing firms to hold liquid assets in addition to capital is equivalent to having firms simply hold less debt. Accordingly, we will consider firms' investment and borrowing decisions and discuss over-borrowing or over-investment, though the reader should understand that over-borrowing is equivalent to under-provision of liquidity (i.e., holding too few liquid assets).[5]

As in the liquidity risk model in Section 2, there are two aggregate states in the economy at $t = 1$, a good state and a bad state. In the good state, all firms have productivity $\overline{A} > 1$ and are able to repay their debt without selling capital. In the bad state, half the firms, randomly selected, have low productivity A_L (they must sell capital to repay debts) and the other half A_H with $A_L \leq A_H$. Additionally, this bad state has low average productivity

$$\widetilde{A} = \tfrac{1}{2}A_L + \tfrac{1}{2}A_H < q_0,$$

which ensures that households, in addition to firms with good shocks, will buy capital in the bad state (i.e., the aggregate production from capital is not sufficiently high that firms with good production can buy up all the capital sold by those with low production). Figure 5 summarizes the states.

3.1 Pure aggregate risk

We first consider a setup corresponding to the standard macro-finance model of fire sales due to purely aggregate risk. This generates the standard result of overinvestment in the Walrasian equilibrium and the intuitive result that Cournot behavior partially corrects the

[5] In this case, letting ℓ denote investments in liquid assets (e.g., cash), the budget constraint would be $q_0 k + \ell = n + d$. It is easy to verify that consumption in each state, as well as quantities of assets sold/purchased, are just a function of $d - \ell$, and so ignoring liquidity holdings is equivalent to folding liquidity holdings into the debt in our baseline analysis.

inefficiency. Similar to our analysis of the liquidity risk model, this served as a benchmark for Section 3.2, where the introduction of idiosyncratic risk can easily overturn this intuition.

We restrict the model to purely aggregate risk by supposing that in the bad state the high and low productivities are the same $A_H = A_L = \underline{A}$. (Since all firms in the bad state receive the same shock, we will denote variables in this state by an underline to distinguish from when there is idiosyncratic risk.) In the good production state, the capital price is $\overline{q} = 1$ and firms are able to repay their debts. In the bad state, we suppose firms have to sell capital to repay their debt, and so households buy capital. To ensure that firms must be sellers in the low state, we require that

$$d > \underline{A}k \quad \Leftrightarrow \quad (q_0 - \underline{A})\, k > n,$$

which will occur in equilibrium so long as q_0 is sufficiently low (the returns to holding capital are high). Substituting in for d from the budget constraint, the units of capital sold by a firm in the bad state is

$$z = \frac{d - \underline{A}k}{\underline{q}} = \frac{(q_0 - \underline{A})\, k - n}{\underline{q}}.$$

Firm consumption in the good and bad states are given by

$$\begin{aligned} \overline{c} &= \left(\overline{A} + 1\right) k - d = n + \left(\overline{A} + 1 - q_0\right) k, \\ \underline{c} &= k - \frac{d - \underline{A}k}{\underline{q}} = \frac{n}{\underline{q}} - \frac{q_0 - \underline{A} - \underline{q}}{\underline{q}} k. \end{aligned}$$

A household's demand for capital, coming from the first order condition, is

$$x^h = \frac{a}{\underline{q}} - 1,$$

and so market clearing ($2Nz = 2Nx^h$) implies that the price of capital in the bad state is

$$\underline{q} = a + n - (q_0 - \underline{A})\, k.$$

It is clear that a higher initial capital stock k pushes down the price in the bad aggregate state. With more capital and low production there is more forced selling to repay debts (more fire sales), whereas less borrowing (equivalently, more liquidity) would increase the price since less capital would be sold to repay debts.

At a higher price, firms would have more capital at $t = 1$ to produce at $t = 2$, which would lead to higher output and higher welfare, but in Walrasian equilibrium firms do not internalize how their initial investment/borrowing decisions affect the capital price at $t = 1$. This is a classic externality that has been studied extensively in the literature. A social planer would internalize the effect on $\underline{q}$ and have firms hold lower levels of capital.

Walrasian equilibrium. Taking $\underline{q}$ as exogenous, the first order condition of a firm in the Walrasian equilibrium is

$$\overline{\alpha}\left(\overline{A}+1-q_0\right)u'(\overline{c})-\underline{\alpha}\,\frac{q_0-\underline{A}-\underline{q}}{\underline{q}}\,u'(\underline{c})=0.$$

Social planner. When considering the value of capital, the social planner considers an additional term that captures the effect of capital holdings on the fire-sale price:[6]

$$\overline{\alpha}\left(\overline{A}+1-q_0\right)u'(\overline{c})-\underline{\alpha}\,\frac{q_0-\underline{A}-\underline{q}}{\underline{q}}\,u'(\underline{c})+\underbrace{\underline{\alpha}\,\frac{(q_0-\underline{A})\,k-n}{\underline{q}^2}\,u'(\underline{c})\,\frac{d\underline{q}}{dk}}_{\text{Social planner price effect}}=0. \tag{3}$$

Since the social planner takes into account that $\underline{q}$ is decreasing in k,

$$\frac{d\underline{q}}{dk}=-\left(q_0-\underline{A}\right)<0,$$

the additional term in the first order condition (3) is negative. Thus, the social planner holds less capital, which reduces fire sales, increasing the asset price in the low state and increasing production.[7] This is the standard fire-sale result.

Cournot equilibrium. A firm taking into account the effect of its capital choice has the same additional term in the first order condition as the social planner, except that the price impact is only partial. With $2N$ firms, we have

$$\frac{d\underline{q}}{dk}=-\frac{1}{2N}\left(q_0-\underline{A}\right)$$

[6] We suppose the social planner only considers the firms (since households have deep pockets and are risk neutral, this is without loss of generality so long as the social planner can also implement ex-ante transfers, see Dávila, 2015).

[7] Allowing for ex-ante transfers would improve welfare for households as well as firms.

Cournot firms partly internalize the price effect and, therefore, choose a level of capital between the social planner and the Walrasian equilibrium.

3.2 Aggregate and idiosyncratic production risk

We now suppose that in the bad state there is idiosyncratic productivity risk where half the firms have high productivity A_H and half have low productivity $A_L < A_H$. We first suppose that $A_H > q_0$ so that firms with good productivity shocks are buyers of capital. Later we consider the case when idiosyncratic risk is smaller (i.e., $A_H < q_0$).

Large idiosyncratic risk

In the bad state with idiosyncratic risk, similar to the pure aggregate risk case just considered, a firm with low productivity fire-sells capital to repay debts and supplies

$$z_L = \frac{d - A_L k}{\widetilde{q}} = \frac{k(q_0 - A_L) - n}{\widetilde{q}}.$$

A firm with high productivity, however, has the means to purchase fire-sold capital and demands[8]

$$x_H = \frac{A_H k - d}{\widetilde{q}} = \frac{n + (A_H - q_0)k}{\widetilde{q}}.$$

In addition, household demand for capital is, as before, $x^h = a/\widetilde{q} - 1$. Thus, market clearing for capital, $Nz_L = Nx_H + 2Nx^h$, implies that the price of capital in the bad state is

$$\widetilde{q} = a + n - \left(q_0 - \widetilde{A}\right) k. \tag{4}$$

As in the bad aggregate state without idiosyncratic risk, a higher capital stock leads to a lower price since the aggregate production is low. In this aggregate state, consumption for the high and low types are given by

$$c_H = k + \frac{A_H k - d}{\widetilde{q}} = \frac{n}{\widetilde{q}} + \frac{\widetilde{q} + A_H - q_0}{\widetilde{q}} k,$$
$$c_L = k - \frac{d - A_L k}{\widetilde{q}} = \frac{n}{\widetilde{q}} - \frac{q_0 - A_L - \widetilde{q}}{\widetilde{q}} k.$$

[8] High types are guaranteed to be buying capital since we assume $A_H > q_0$. Even if this condition does not hold, if initial wealth n is sufficiently high then high types may buy (endogenously) even if $q_0 > A_H$ (note that in this case firms are not borrowing very much). Our results continue to hold in this case, though the reasoning is slightly more subtle.

Walrasian equilibrium. The bad state with idiosyncratic risk adds a new term to the Walrasian first order condition

$$\alpha\left(\overline{A}+1-q_0\right)u'(\bar{c})+\underbrace{\frac{(1-\alpha)}{2}\left(\frac{\widetilde{q}+A_H-q_0}{\widetilde{q}}\,u'(c_H)-\frac{q_0-A_L-\widetilde{q}}{\widetilde{q}}\,u'(c_L)\right)}_{\text{Idiosyncratic risk}}=0. \qquad (5)$$

Intuitively, holding more capital benefits a firm in the bad state if it has high productivity since it allows for more profitable purchases of fire-sold capital, but hurts a firm with low productivity since it forces more costly sales of capital.

Social planner. Compared to the Walrasian equilibrium first order condition (5), the social planner's first order condition again considers the impact of capital holdings on the price:

$$\alpha\left(\overline{A}+1-q_0\right)u'(\bar{c})+\overbrace{\frac{(1-\alpha)}{2}\left(\frac{\widetilde{q}+A_H-q_0}{\widetilde{q}}\,u'(c_H)-\frac{q_0-A_L-\widetilde{q}}{\widetilde{q}}\,u'(c_L)\right)}^{\text{Idiosyncratic risk same as WE}}$$
$$\underbrace{+\frac{(1-\alpha)}{2}\left(-\frac{n+(A_H-q_0)\,k}{\widetilde{q}^2}\,u'(c_H)+\frac{(q_0-A_L)\,k-n}{\widetilde{q}^2}\,u'(c_L)\right)\frac{d\widetilde{q}}{dk}}_{\text{Social planner price effect}}=0.$$

A higher level of capital decreases the fire-sale price,

$$\frac{d\widetilde{q}}{dk}=-\left(q_0-\widetilde{A}\right)<0,$$

which is good for high types who buy capital, but bad for low types who sell capital. Thus, the social planner trades off the relative gain to high types against the relative loss to low types (marginal-utility weighted). We can simplify the term in parentheses,

$$-\frac{n+(A_H-q_0)\,k}{\widetilde{q}^2}\,u'(c_H)+\frac{(q_0-A_L)\,k-n}{\widetilde{q}^2}\,u'(c_L)=\frac{1}{\widetilde{q}}\left(-x_H\,u'(c_H)+z_L\,u'(c_L)\right),$$

so the social planner chooses less capital than the Walrasian equilibrium if

$$x_H u'(c_H)<z_L u'(c_L).$$

By assumption, high types are not able to purchase all the assets sold by the low types, $x_H<z_L$. Furthermore, high consumption exceeds low consumption and so marginal util-

ity of high types is less than that of low types. Thus, it is always the case that this condition holds.[9]

Cournot equilibrium. In considering their effect on the price, Cournot firms' first order condition considers separately the value of changing the capital price when they are high or low types.

$$\alpha\left(\overline{A}+1-q_0\right)u'(\overline{c})+\overbrace{\frac{(1-\alpha)}{2}\left(\frac{\widetilde{q}+A_H-q_0}{\widetilde{q}}\,u'(c_H)-\frac{q_0-A_L-\widetilde{q}}{\widetilde{q}}\,u'(c_L)\right)}^{\text{Idiosyncratic risk same as WE}}$$
$$+\underbrace{\frac{(1-\alpha)}{2}\left(-\frac{n+(A_H-q_0)\,k}{\widetilde{q}^2}\,u'(c_H)\,\frac{d\widetilde{q}_H}{dk}+\frac{(q_0-A_L)\,k-n}{\widetilde{q}^2}\,u'(c_L)\,\frac{d\widetilde{q}_L}{dk}\right)}_{\text{Cournot price effect}}=0.$$

Rewriting the equilibrium capital price (4) to explicitly account for the roles of capital held by high and low types,

$$\widetilde{q}=a+n+\underbrace{\tfrac{1}{2}\left(A_H-q_0\right)k}_{\text{high types}}-\underbrace{\tfrac{1}{2}\left(q_0-A_L\right)k}_{\text{low types}},$$

we see that, with $2N$ firms,

$$\frac{d\widetilde{q}_H}{dk}=\frac{1}{2N}\left(A_H-q_0\right)>0 \qquad \text{and} \qquad \frac{d\widetilde{q}_L}{dk}=-\frac{1}{2N}\left(q_0-A_L\right)<0.$$

While the relationship between the capital price and the aggregate level of capital is negative, the effect on the capital price as a high or low type differs. A high type has a positive effect on the price since its available cash scales with its initial investment; a low type has a negative effect since its forced sales to repay debt also scale with its initial investment.

Comparing the first-order conditions, Cournot firms hold *even less* capital than the social planner if, at the efficient allocation,

$$-x_H u'(c_H)\,\frac{d\widetilde{q}}{dk_H}+z_L u'(c_L)\,\frac{d\widetilde{q}}{dk_L}<\left(-x_H u'(c_H)+z_L u'(c_L)\right)\frac{d\widetilde{q}}{dk}$$
$$\Leftrightarrow \quad x_H u'(c_H)\left(\frac{d\widetilde{q}}{dk}-\frac{d\widetilde{q}}{dk_H}\right)<z_L u'(c_L)\left(\frac{d\widetilde{q}}{dk}-\frac{d\widetilde{q}}{dk_L}\right)$$

[9] Furthermore, comparing the extra social planner term in this case to when there is pure aggregate risk reveals that the social value of capital is higher with idiosyncratic risk. It is immediately clear that the difference between these terms is the high-type term in the social planner term.

which holds for sufficiently low N since for $N = 1$

$$\frac{d\widetilde{q}}{dk_L} < \frac{d\widetilde{q}}{dk} < \frac{d\widetilde{q}}{dk_H}.$$

A higher price always hurts buyers and benefits sellers. For the social planner choosing aggregate capital, less capital increases the price and there is a trade off between the effects on buyers and sellers. In contrast, Cournot firms reason as follows: holding *more* capital will push *down* the price when they are a *seller*, but that is when they want the capital price to be higher; holding more capital will push *up* the capital price when they are a *buyer*, but that is when they want the capital price to be lower. Thus, whether they will be a buyer or a seller, strategically it makes sense to hold less capital, and so Cournot firms will want to hold less capital to move the capital price in the appropriate direction *given* the shock they receive. However, the social planner knows that *improving* the price for the buyer (seller), simultaneously *hurts* the price for the seller (buyer). The social planner takes into account that ex-ante less aggregate capital is not *actually* better for buyers of capital because *in the aggregate* less capital will push up the price.

Low idiosyncratic risk

Suppose now instead that $q_0 > A_H$, which implies production is below what high types paid for capital. In this case, even firms with high productivity shocks may be forced to sell capital to repay debts. In this case, higher capital holdings by high types decrease the asset price, which is good for high types since they are net buyers (remember that when $A_H > q_0$ buying firms prefer lower levels of capital, not higher). Note that we can write the Cournot term as

$$\underbrace{(q_0 - A_H)}_{<2(q_0 - \widetilde{A})} \left(\underbrace{u'(c_H)\frac{x_H}{\widetilde{q}}}_{\text{high types}} - \underbrace{\frac{q_0 - A_L}{q_0 - A_H} u'(c_L)\frac{z_L}{\widetilde{q}}}_{\text{low types}} \right).$$

Overall, Cournot firms internalize a lower overall effect on the price (since they take the other firms' capital as given), but they also place a higher relative weighting on the low state (when they are sellers). Cournot firms consider that they have a greater effect on the price when they are sellers, and so they have a greater incentive to hold less capital in order to increase the price they get when they sell. Remember that the social planner considers the price effect for both high and low types, effectively averaging across both types. Cournot firms, however, consider their effect when high and low separately, ignoring the

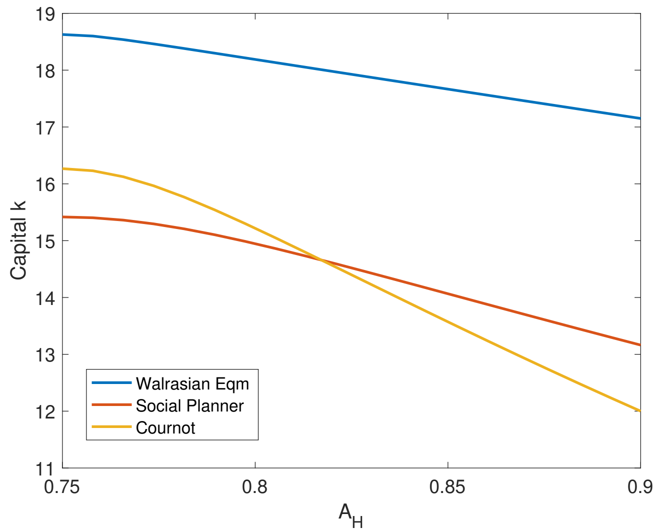

Figure 6: Comparison of capital investment of Walrasian and Cournot equilibrium to the efficient level with relative risk aversion 2, $\alpha = 0.5$, $a = 0.95$, $n = 1$, $q_0 = 0.85$, and $\widetilde{A} = 0.75$.

effect on the other firm type.

In this case we can rearrange the Cournot condition to

$$x_H u'(c_H) < \left(\frac{q_0 - A_L}{q_0 - A_H} \right) z_L u'(c_L).$$

As previously discussed, it is always true that $x_H u'(c_H) < z_L u'(c_L)$, and so Cournot firms will hold less capital than the Walrasian equilibrium. By assumption on productivities we have

$$q_0 - A_L > q_0 - A_H \quad \Rightarrow \quad \frac{q_0 - A_L}{q_0 - A_H} > 1.$$

Comparing to earlier, Cournot firms are putting a relatively greater weight on the low-type marginal utility than the Social Planner does. Thus, Cournot may lead to under-investment relative to the Social Planner so long as A_H and A_L are sufficiently different (i.e., when idiosyncratic risk is high). We already know that if $A_H = A_L$ then Cournot partially mitigates the externality, holding less capital than the Walrasian level but still more than the Social Planner. Hence, when idiosyncratic risk is low and productivities in the bad state are close, Cournot mitigates the externality, but when idiosyncratic risk is sufficiently large Cournot reverses the externality.

Figure 6 compares the levels of investment in capital in the Walrasian and Cournot equilibria to the efficient level in a setting with only two aggregate states, the good state with high productivity and the bad state with idiosyncratic productivity shocks. The figure varies the degree of productivity risk by varying A_H (and A_L) and keeping average productivity constant, $\widetilde{A} = 0.75$. As the degree of productivity risk increases (higher A_H and lower A_L), the efficient level of investment declines and is always lower than the level of investment in the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency as long as productivity risk is sufficiently low. Once productivity risk is sufficiently high, the Cournot equilibrium overcorrects the overinvestment of the Walrasian equilibrium, leading to inefficiently *low* investment.

4 Conclusion

We have considered standard macro-finance models of fire sales and have incorporated market power with both aggregate and idiosyncratic risks. With liquidity shocks, Cournot competition can lead to under-provision of liquidity even below what occurs in Walrasian equilibrium, thus exacerbating the standard pecuniary externality rather than mitigating it. With productivity shocks and borrowing constraints, Cournot competition leads to *under-investment* and under-borrowing rather than the standard over-investment and over-borrowing result. Our results highlight that the simple intuition of internalizing pecuniary externalities is flawed and points to the type of shocks considered as an important determinant of the welfare effects.

References

Allen, F. and D. Gale (2004). Financial intermediaries and markets. *Econometrica*, 1023–1061.

Bhattacharya, S. and D. Gale (1987). *Preference shocks, liquidity, and central bank policy*.

Caballero, R. J. and A. Krishnamurthy (2001). International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics 48*(3), 513–548.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2004). Does one soros make a difference? a theory of currency crises with large and small traders. *The Review of Economic Studies 71*(1), 87–113.

Dávila, E. (2015). Dissecting fire sales externalities. Working Paper, New York University.

Dávila, E. and A. Korinek (2017). Pecuniary externalities in economies with financial frictions. *The Review of Economic Studies*, rdx010.

Dávila, E. and A. Walther (2017). Does size matter? bailouts with large and small banks. Technical report, NYU.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Diamond, D. W. and R. G. Rajan (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics 126*(2), 557–591.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A theory of liquidity and regulation of financial intermediation. *The Review of Economic Studies 76*(3), 973–992.

Gale, D. and T. Yorulmazer (2013). Liquidity hoarding. *Theoretical Economics 8*(2), 291–324.

Geanakoplos, J. and H. M. Polemarchakis (1986). Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete. *Uncertainty, information and communication: essays in honor of KJ Arrow 3*, 65–96.

Geanakoplos, J. and K. J. Walsh (2017). Inefficient liquidity provision. *Economic Theory*, 1–21.

Greenwald, B. C. and J. E. Stiglitz (1986). Externalities in economies with imperfect information and incomplete markets. *The quarterly journal of economics 101*(2), 229–264.

Gromb, D. and D. Vayanos (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of financial Economics 66*(2), 361–407.

He, Z. and P. Kondor (2016). Inefficient investment waves. *Econometrica 84*(2), 735–780.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Jacklin, C. J. (1987). Demand deposits, trading restrictions and risk sharing. In E. C. Prescott and N. Wallace (Eds.), *Contractual Arrangements for Intertemporal Trade*. University of Minnesota Press.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*, 809–833.

Malherbe, F. (2014). Self-fulfilling liquidity dry-ups. *The Journal of Finance 69*(2), 947–970.

Perotti, E. C. and J. Suarez (2002). Last bank standing: What do i gain if you fail? *European economic review 46*(9), 1599–1622.

Shleifer, A. and R. Vishny (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives 25*(1), 29–48.

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–1366.

Stein, J. (2013). The fire-sales problem and securities financing transactions. In *Speech by Governor Jeremy C. Stein at the Federal Reserve Bank of New York during the Workshop on Fire Sales as a Driver of Systemic Risk in Triparty Repo and other Secured Funding Markets, New York*.

Appendices

A Strategic behavior at $t = 1$

In the main analysis, we suppose that agents strategically choose portfolios at $t = 0$ (understanding that their portfolios will affect prices at $t = 1$), but at $t = 1$ agents act as price takers, selling all assets when required to and using all funds to purchase assets when they can. In this section we extend the previous analysis to allow agents to also act strategically at $t = 1$. Buyers may choose to use only a portion of their available funds to buy assets, in order to maintain lower prices on the asset they purchase. And sellers may choose to sell only a portion of their assets, thus receiving higher prices. We show in this section that our assumption of price-taking behavior at $t = 1$ is without loss of generality, except for a knife-edge case in the liquidity model. Indeed, agents will choose to behave as discussed in the main analysis.

To incorporate strategic behavior in the interim period, we suppose that agents trade at Shapley-Shubik trading posts. Buyers choose a value of funds V with which they will purchase assets. Sellers choose the quantity of assets z to sell. The price is determined given the funds supplied to purchase assets and the quantity of assets supplied, which is essentially cash-in-the-market pricing, with the cash and assets in the market determined strategically.

For expositional purposes, we first consider the production model, and then we consider the liquidity model.

A.1 Strategic interim behavior in the production model

Consider the production model of fire sales and now suppose that at $t = 1$ firms (but not households) compete via Shapley-Shubik trading posts. First, firms with bad shocks choose to sell z units of capital, and they must raise at least $d - A_L k$ in value to repay debts, thus they need $zq \geq d - A_L k$, where these firms strategically consider how they affect q. Second, firms with good shocks choose the value V of wealth they'd like to use to purchase capital. They may use fewer funds than available after repaying debts, i.e., $V \leq A_H k - d$.

Low type firms want to sell as little capital as required. High type firms want to use as few funds as possible to get as much capital as possible. Since household demand is $a/q - 1$, market clearing implies

$$\frac{2a}{q} - 2 + \frac{V}{q} = z,$$

where we have scaled to have 2 households, which implies

$$q = \frac{2a + V}{2 + z}.$$

Thus, a high firm will get $x = \frac{V}{2a+V}(2 + z)$ units of capital with V worth of funds. Differentiating we have

$$\frac{dx}{dV} = (2 + z)\frac{2a}{(2a + V)^2} > 0.$$

Thus, high types will get more capital if they use more funds. Note that if we did not have households ($a = 0$) then x would be independent of V (unit price elasticity) and then firms would use $V = 0$ in funds to buy all the capital at zero price (this case arises as a knife-edge case in the liquidity model). But critically, we do have households, which breaks the unit elasticity, and so high firms will use all their funds even when they have the strategic choice. Hence, our initial modeling decision is appropriate for buyers.

A low firm will raise $zq = (2a + V)\frac{z}{2+z}$ funds from selling z units of capital. Setting this equal to the funds needed to repay debts we have

$$d - A_L k = (2a + V)\frac{z}{2 + z},$$

which becomes

$$\begin{aligned}(2 + z)(k(q_0 - A_L) - n) =& (2a + n + (A_H - q_0)k)z,\\ 2k(q_0 - A_L) - 2n =& (2a + 2n + (A_H - q_0)k + (A_L - q_0)k)z,\end{aligned}$$

and so

$$z = \frac{k(q_0 - A_L) - n}{a + n + (\tilde{A} - q_0)k}.$$

Notice that this implies that the capital price satisfies

$$q = a + n + (\tilde{A} - q_0)k,$$

which is what we had *without* the strategic decision at $t = 1$. Hence, the results in the production model taking as given the behavior at $t = 1$ are without loss of generality. Indeed, the strategic consideration predicts doing exactly this. Furthermore, it is easy to show that extending the model to $N > 1$ does not change the analysis.

A.2 Strategic interim behavior in the liquidity model

We now consider the liquidity model, which has a few slight differences. First, sellers now have no alternative use of illiquid assets if they do not sell, which was not true in the production model. Thus, sellers would withhold selling illiquid assets only if doing so increased the value of the assets they do sell, requiring the price elasticity is greater than one. Buyers may withhold using liquid assets to purchase illiquid assets, because liquid assets do have alternative value. As we will see, by the same logic as in the production model, except for a knife-edge case, this strategic decision will not change the previous analysis.

We make the following slight generalization to the model. In addition to investors not receiving liquidity shocks, we suppose there are outside investors (households) who provide ϵ worth of funds to purchase illiquid assets at the interim date. Consider a buyer choosing V funds to use to purchase illiquid assets, and let V' be the funds used by the other $N-1$ buyers. Consider a seller choosing to sell z assets while the other investors sell z'. Then the asset price satisfies

$$p = \frac{\epsilon + (N-1)V' + V}{(N-1)z' + z}.$$

By the exact same analysis done for the production model, if either $N > 1$ or $\epsilon > 0$, then buyers will use all their liquid funds to buy illiquid assets (doing so gets them the most assets with the least money), and sellers will sell all their illiquid assets (doing so raises the most funds). Thus, robustly, the analysis in the main text carries through when we allow strategic decisions at $t = 1$.

However, the analysis is changed only when $N = 1$ and $\epsilon = 0$, a knife-edge case which has theoretical interest but almost surely has little empirical relevance. In this case, the price is $p = V/z$, implying the price has a unit elasticity. In this case, the selling investor will earn V regardless of how many assets sold (though this investor has no incentive to sell fewer), and the buyer investor will receive z units of illiquid assets for any quantity of funds supplied. This investor clearly has an incentive to use as few funds as possible and would thus deliver $V = 0$ funds and keep all liquid assets for consumption next period. Thus, the asset price would be $p = 0$, and there would be no risk-sharing through markets. The initial liquidity holdings would thus reflect that consumption after receiving a bad shock comes exclusively from liquid investments, thus providing a higher incentive for initial liquidity holdings. It is worth pointing out that a Social Planner would choose a high V in order to provide insurance to low types, who have higher marginal utility.

In this knife-edge case, it is sensible to consider when buyers are committed, perhaps partially, to use liquidity to buy distressed assets. Since many investment firms and corporations carefully report their uses of credit lines, for example, this is a reasonable assumption. consider when buyers must use a fraction ϕ of their liquidity to buy assets. Then we have

$$p = \frac{\phi \ell}{1 - \ell} = \phi p.$$

Thus, all the price impacts in the base analysis are decreased by a fraction ϕ. This has the equivalent effect of increasing the number of competing investors. The initial liquidity holding would also change to reflect this effect on interim prices, so comparing the Cournot and Walrasian equilibrium requires slightly more care.