

Collateral Booms and Information Depletion

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The views expressed in this paper do not reflect those of the ECB or its staff.

Introduction

- ▶ Fluctuations in credit are common (more so in recent years).
Claessens et al. 2011, Mendoza and Terrones 2012, Bakker et al. 2012.

- ▶ Good things happen during credit booms...
 - Asset prices, GDP growth and investment are higher than in normal times.

- ▶ Yet, credit booms are often viewed with suspicion...
 - Fall in lending standards/information quality on borrowers,
Asea and Blomberg 1998; Keys et al. 2010; Becker et al. 2018.
 - Rise in factor misallocation,
Gopinath et al. 2017; Garcia-Santana et al. 2017; Doerr 2018.
 - Often followed by crises and low growth.
Schularick and Taylor 2012.

This paper

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 - Entrepreneurs need credit to undertake long-term projects.
 - ▶ Projects are heterogeneous in “quality,” low or high.
 - ▶ Low quality projects allow entrepreneurs to extract rents.
 - Lenders have two ways of protecting themselves:
 - ▶ *Collateralization*: ask entrepreneurs to put up assets as collateral.
 - ▶ *Screening*: produce costly but durable information about project quality.
 - Collateralization-screening mix depends on aggregate value of collateral.

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 - Is information production efficient during credit booms?
- ▶ We provide new empirical evidence in support of the model’s main predictions using US firm-level data.

The Model

Environment, preferences and endowments

- ▶ Time is infinite, $t = 0, 1, \dots$. Small-open economy.
- ▶ OLG of agents, of constant size and two-period lifetimes.
- ▶ Entrepreneurs and savers, unit mass each, with preferences

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▶ Entrepreneurs:

- Endowed with collateral with value q_t at time t (e.g. land, real estate).
- When young: purchase and invest in capital.
 - ▶ Finance these activities by borrowing from lenders.
- When old: hire labor to produce consumption goods.

Technology

- ▶ **Investment:** one consumption good at $t \rightarrow$ one unit of capital at $t + 1$.
 - Two types of capital, $\theta \in \{L, H\}$, but more on this shortly...
 - A unit's type persists throughout its life.
 - Capital depreciates at rate δ and is reversible.

- ▶ **Production:** Cobb-Douglas technology

$$F_t(k_{it}, l_{it}) = A_t \cdot k_{it}^\alpha \cdot l_{it}^{1-\alpha},$$

where A_t is aggregate productivity, k_{it} are units of *effective* capital and l_{it} are units of labor.

Quality of projects

- ▶ L -type suffers from an agency problem.
 - Entrepreneur can run away with all the resources generated by it.
 - Thus, L -type capital is effectively less pledgeable.
- ▶ **Baseline:** H - and L - types of capital are equally productive.
 - Later, also productivity differences → factor “misallocation.”

Screening and information production

- ▶ Ex-ante, the quality of each unit of investment is uncertain.
 - $\mathbb{P}(\theta = H) = \mu \in (0, 1)$ and quality iid across units.
- ▶ Before investing, each unit can be “screened” at cost ψ_t , in which case its type is publicly revealed.
- ▶ Let s_t denote the units screened in aggregate at t , then we assume that $\psi_t = \psi(s_t)$ with properties $\psi(0) = 0$ and $\psi'(\cdot) > 0$.
 - **Micro-foundation:** screening services and credit provided by competitive intermediary sector that hires experts (e.g., savers) who have heterogeneous screening costs.
- ▶ Past performance of a unit is not publicly observable.

Markets

Notation: θ -type capital k_{it}^θ , unscreened capital k_{it}^μ , and effective capital $k_{it} = k_{it}^H + k_{it}^L + k_{it}^\mu$. The aggregate capital stock is $k_t = \int_i k_{it} di$.

Marginal product of capital: $r_t = A_t \alpha k_t^{\alpha-1}$.

▶ Labor market:

- Old entrepreneurs hire young savers at market wage w_t .

▶ Capital market:

- Old entrepreneurs sell capital to young at prices p_t^j for $j \in \{H, L, \mu\}$.
- Since capital is reversible, the old strictly prefer to sell only if $p_t^j > 1$.

▶ Credit market:

- Young entrepreneur borrows from lenders f_{it} and invests $q_t + f_{it}$.
- Contracts are state-contingent, but there are (endogenous) constraints:

$$R_{it+1} f_{it} \leq \left(r_{t+1} + (1 - \delta) \max\{p_{t+1}^H, 1\} \right) k_{it+1}^H + \left(r_{t+1} + (1 - \delta) \max\{p_{t+1}^\mu, 1\} \right) \mu k_{it+1}^\mu.$$

Equilibrium prices

- ▶ Labor market clearing:

$$w_t = A_t (1 - \alpha) k_t^\alpha.$$

- ▶ Credit market clearing:

$$E_t\{R_{it+1}\} = \rho.$$

- ▶ Capital market clearing:

$$p_t^H = 1 + \frac{\psi(s_t)}{\mu} \geq 1 = p_t^\mu = p_t^L.$$

- **Intuition:** price equals production cost.

Equilibrium dynamics

Given $\{k_0^H, k_0^L, k_0^\mu\}$ and process $\{q_t, A_t\}_{t \geq 0}$, equilibrium is characterized by:

- ▶ Zero expected profits on H -type investment:

$$1 + \frac{\psi(s_t)}{\mu} = \frac{E_t \left\{ r_{t+1} + (1 - \delta) \left(1 + \frac{\psi(s_{t+1})}{\mu} \right) \right\}}{\rho},$$

- ▶ H -type investment: $s_t = \max \left\{ 0, \frac{k_{t+1}^H - (1 - \delta)k_t^H}{\mu} \right\}$,

- ▶ No L -type investment: $k_{t+1}^L = 0$.

- ▶ Unscreened investment constrained by collateral:

$$k_{t+1}^\mu = \min \left\{ \frac{\rho}{\rho - \mu E_t \{ r_{t+1} + 1 - \delta \}} \cdot q_t, k_{t+1}^* \right\},$$

where $r_{t+1} = A_{t+1} \alpha (k_{t+1}^H + k_{t+1}^\mu)^{\alpha-1}$.

Collateral booms and busts

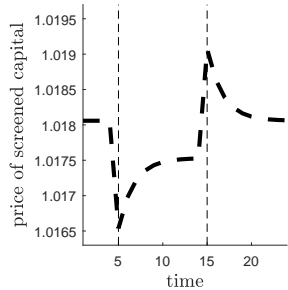
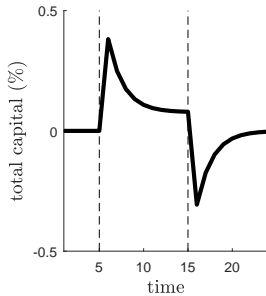
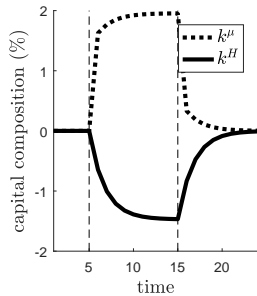
Boom-bust episodes

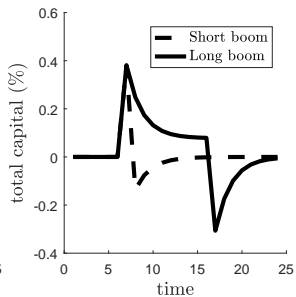
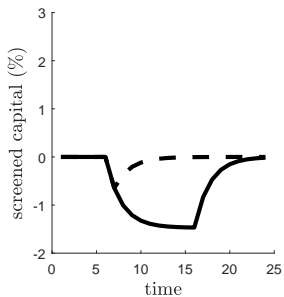
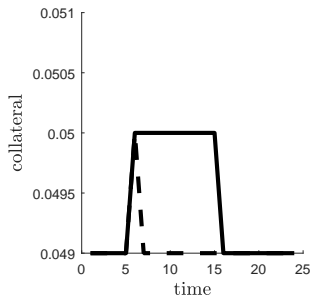
We consider the following experiments:

- ▶ Collateral q takes values in $\{\underline{q}, \bar{q}\}$ with $\mathbb{P}(q_{t+1} = \bar{q} | q_t = \underline{q}) \in (0, \frac{1}{2})$ and $\mathbb{P}(q_{t+1} = \underline{q} | q_t = \bar{q}) \in (0, \frac{1}{2})$.
- ▶ For comparison, productivity A takes values in $\{\underline{A}, \bar{A}\}$ with $\mathbb{P}(A_{t+1} = \bar{A} | A_t = \underline{A}) \in (0, \frac{1}{2})$ and $\mathbb{P}(A_{t+1} = \underline{A} | A_t = \bar{A}) \in (0, \frac{1}{2})$.

Suppose throughout that parameters are such that borrowing constraints bind $\forall t$.

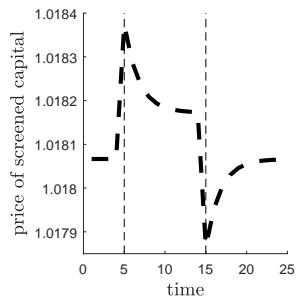
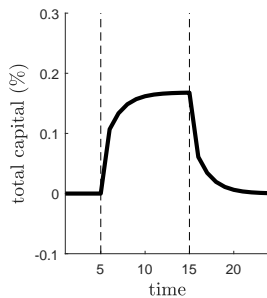
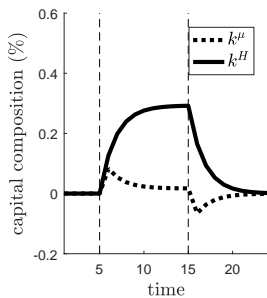
Collateral boom-bust episode



Longer booms \rightarrow larger busts

Source of the boom matters

Productivity boom-bust episode



Normative properties of equilibrium

- ▶ Too little information production?

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- ▶ Consider planner who maximizes discounted consumption subject to same information friction/borrowing constraint as market.

Normative properties of equilibrium

- ▶ Too little information production? No...
- ▶ Consider planner who maximizes discounted consumption subject to same information friction/borrowing constraint as market.
- ▶ Planner optimality condition:

$$1 + \frac{\psi(s_t)}{\mu} = \frac{E_t \left\{ A_{t+1} \alpha k_{t+1}^{\alpha-1} + (1 - \delta) \left(1 + \frac{\psi(s_{t+1})}{\mu} \right) \right\}}{\rho} + \underbrace{\left(\frac{E_t \left\{ A_{t+1} \alpha k_{t+1}^{\alpha-1} + 1 - \delta \right\}}{\rho} - 1 \right)}_{\text{Distortion}} \cdot \frac{\partial k^\mu(k_{t+1}^H, q_t, A_t)}{\partial k_{t+1}^H}$$

- ▶ **Source of inefficiency:** by screening more, entrepreneurs depress MPK, tightening borrowing constraints and crowding out unscreened investment.
- ▶ **Implementation:** can correct inefficiency through Pigouvian taxes on screened investment, with revenues rebated lump sum to savers.

Additional considerations

1. Interpretation through project creation and screening:
 - Each project employs labor and at most \bar{k} units of capital.
 - Projects become obsolete at rate γ ; capital depreciates at rate δ .
2. Credit booms and factor “misallocation”:
 - Assume that H -type projects are also more productive.
 - TFP dispersion across projects increases in q_t if starting with small q_t .
3. Interpretation of fluctuations in q_t :
 - Activity organized within firms = collection of projects.
 - Rational bubbles on firms randomly appear and burst.
4. Irreversibilities and “fire-sales”:
 - Assume that capital can be converted to $\chi \in (0, 1)$ units of consumption.
 - During the bust, some of the effect is absorbed by project prices.

Conclusions

- ▶ Model of **Collateral Booms and Information Depletion**.
 - Rising collateral values boost investment and economic activity,
 - But reallocate investment towards less information-intensive activities:
 - ▶ Lower incentives to produce information.
 - ▶ Information depletion over time...
 - Longer booms → more info depletion → larger busts.
- ▶ Source of the credit boom matters.
 - Productivity-driven booms do not deplete information.
- ▶ Normative aspects of credit booms:
 - “Misallocation” may increase during booms, but save on screening costs.
 - If anything, due to pecuniary externalities, there is too much information!
- ▶ We provide new empirical evidence in support of the model’s main predictions using US firm-level data.

Related literature

- ▶ **Credit booms and lending standards:** Manove et al. (2011), Ruckes (2004), Martin (2005), Dell'Ariccia and Marquez (2006), Gorton and He (2008), Favara (2012), Petriconi (2015), Krishnamurthy and Muir (2017).
- ▶ **Information production in macro:** Van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), Gorton and Ordoñez (2014, 2016), Fajgelbaum et al. (2017), Straub and Ulbricht (2017).
- ▶ **Collateral and investment:** Peek and Rosengreen (2000), Gan (2007), Chaney et al. (2012).
- ▶ **Financial frictions and investment composition:** Matsuyama (2007), Diamond et al. (2018).
- ▶ **Pecuniary externalities:** Caballero and Krishnamurthy (2003), Lorenzoni (2008), Dávila and Korinek (2017).

Appendix

Testable implications

1. Investment is increasing in collateral values.
2. Share of unscreened investment is increasing in collateral values.
3. Collateral bust is followed by fall in investment, and more so the larger is the share of unscreened investment in the boom.

Empirical strategy

Two challenges:

1. Identify shocks to outside collateral:
 - Build on Chaney et al. (2012): effect of real estate prices on investment.
 - Extend sample: COMPUSTAT firms 1993-2012.
 - Real estate assets in 1993: infer market value using local real estate inflation.
2. Measure unscreened investment: proxy firm-level information with
 - Bid-ask spread on firm stock,
 - Ratio of intangible to tangible fixed assets, and
 - Analyst coverage.

Implicit mapping: high-info firms \approx screened investment; low-info firms \approx unscreened investment.

Firm-level regressions

1. How does the value of real estate affect investment by firm i in location k ?

$$I_{it} = \alpha_i + \delta_t + \beta \cdot RE_{it} + \gamma \cdot P_{kt} + controls_{it} + \varepsilon_{it}$$

2. How is this effect correlated with firm level information, e.g. spread?

$$I_{it} = \alpha_i + \delta_t + \beta_1 \cdot Spread_{it} + \beta_2 \cdot RE_{it} + \\ + \beta_3 \cdot RE_{it} \cdot Spread_{it} + \gamma \cdot P_{kt} + controls_{it} + \varepsilon_{it}$$

Empirical findings

Dependent variable I_{it}

Table 3: Investment and collateral

VARIABLES	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	OLS	OLS	IV
RE Value (State Prices)	0.0622***	0.0563***	0.0478***		
	(0.00345)	(0.00361)	(0.00349)		
State Prices	-0.0999*	-0.367	-0.142		
	(0.0529)	(0.305)	(0.347)		
Cash			0.0253***	0.0262***	0.0269***
			(0.00241)	(0.00276)	(0.00293)
Market/Book			0.0577***	0.0604***	0.0605***
			(0.00282)	(0.00295)	(0.00318)
RE Value (MSA Prices)				0.0461***	0.0506***
				(0.00395)	(0.00752)
MSA Prices				-0.465	-0.447
				(1.061)	(0.375)
Initial Controls x State Prices	No	Yes	Yes	No	No
Initial Controls x MSA Prices	No	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	34 986	34 746	31 351	26 596	22 901
Adjusted R-squared	0.270	0.281	0.311	0.320	0.322

Empirical findings

Dependent variable I_{it}

Table 4: Investment, collateral and information

VARIABLES	(1) OLS	(2) OLS	(3) OLS	(4) OLS	(5) IV
RE Value (State Prices)	0.0617*** (0.00373)	0.0474*** (0.00403)	0.0414*** (0.00405)		
Spread	-0.0286*** (0.00182)	-0.0321*** (0.00183)	-0.0209*** (0.00173)	-0.0214*** (0.00204)	-0.0216*** (0.00227)
RE Value (State Prices) x Spread	0.00303*** (0.000712)	0.00438*** (0.000649)	0.00280*** (0.000708)		
State Prices	-0.204*** (0.0605)	-1.052** (0.452)	-0.802* (0.453)		
Cash			0.0256*** (0.00282)	0.0262*** (0.00324)	0.0268*** (0.00343)
Market/Book			0.0641*** (0.00293)	0.0645*** (0.00319)	0.0638*** (0.00346)
RE Value (MSA Prices)				0.0414*** (0.00438)	0.0462*** (0.00525)
RE Value (MSA Prices) x Spread				0.00280*** (0.000926)	0.00280*** (0.00104)
MSA Prices				-1.644 (1.066)	0.294 (0.351)
Initial Controls x State Prices	No	Yes	Yes	No	No
Initial Controls x MSA Prices	No	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	28 370	28 256	26 535	22 568	19 448
Adjusted R-squared	0.349	0.363	0.391	0.396	0.392

Aggregate regressions

1. Does high-spread investment in region k increase when the value of real estate increases?

$$\left(\frac{I^{HS}}{I}\right)_{kt} = \alpha_k + \delta_t + \beta \cdot RE_{kt} + \gamma \cdot P_{kt} + \nu_{kt}$$

2. Does the allocation of investment during boom years (2001-2006) affect investment in the bust (2007-2012)?

$$I_{kt} = \alpha_k + \delta_t + \beta_1 \cdot RE_{kt} + \beta_2 \cdot RE_{kt} \cdot \left(\Delta \frac{I^{HS}}{I}\right)_k^{boom} + \gamma \cdot P_{kt} + \nu_{kt}$$

Empirical findings

Dependent variable $\left(\frac{IHS}{I}\right)_{kt}$

Table 6: Investment, collateral and information: aggregate results

VARIABLES	(1) Aggregate	(2) Boom years (2001-2006)
RE Value (State Prices) at State Level	0.0308** (0.0134)	0.0623** (0.0301)
State Prices	-4.454 (2.801)	-8.724 (5.443)
Year FE	Yes	Yes
State FE	Yes	Yes
Observations	900	190
R-squared	0.110	0.213

Empirical findings

Dependent variable $I_{k,t}$

Table 7: Investment, collateral and information: aggregate results during busts

VARIABLES	(1) Bust years (2007-2012)
Δ Investment Ratio during Boom at State Level x RE Value (State Prices) at State Level	0.293*** (0.0863)
RE Value (State Prices) at State Level	0.0649*** (0.0199)
State Prices	0.00679 (0.0942)
Year FE	Yes
State FE	Yes
Observations	216
R-squared	0.707

Additional findings

Table 1: Summary statistics

	Mean	Median	SD	25th percentile	75th percentile	Obs.
Firm-level data						
Investment	0.33	0.20	0.38	0.11	0.38	34 986
Cash	0.04	0.26	1.78	-0.09	0.63	35 204
Market / Book	2.16	1.52	1.76	1.10	2.42	32 512
Spread	2.26	1.36	3.07	0.77	2.56	28 643
Analysts	7.93	5.00	7.46	2.00	11.00	19 921
Intangibility	0.51	0.35	0.62	0.16	0.64	31 167
RE Value (State Prices)	0.89	0.26	1.44	0.00	1.14	35 430
RE Value (MSA Prices)	0.88	0.26	1.42	0.00	1.13	34 892
State Prices	0.29	0.24	0.12	0.20	0.33	35 430
MSA Prices	0.14	0.12	0.05	0.10	0.17	34 907
Housing Supply Elasticity	1.17	0.90	0.67	0.65	1.42	30 753
Initial firm level data (1993)						
Age	8.09	8.00	4.66	3.00	13.00	2 855
ROA	-0.01	0.07	0.25	-0.04	0.12	2 844
Log(Asset)	4.05	3.96	2.19	2.58	5.46	2 852

Additional findings

Dependent variable P_{kt}

Table 2: First-stage regression: the impact of local housing supply elasticity on housing prices

VARIABLES	(1) MSA Prices	(2) MSA Prices
Housing supply elasticity	0.00990*** (0.00274)	
First quartile of elasticity		-0.0225*** (0.00682)
Second quartile of elasticity		-0.00548 (0.00751)
Third quartile of elasticity		0.00141 (0.00744)
Year FE	Yes	Yes
MSA FE	Yes	Yes
Observations	2 232	2 232
R-squared	0.892	0.893

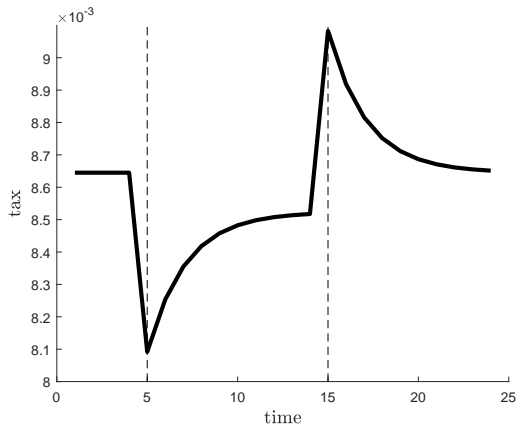
Additional findings

Dependent variable I_{it}

Table 5: Investment, collateral and information: alternative proxies

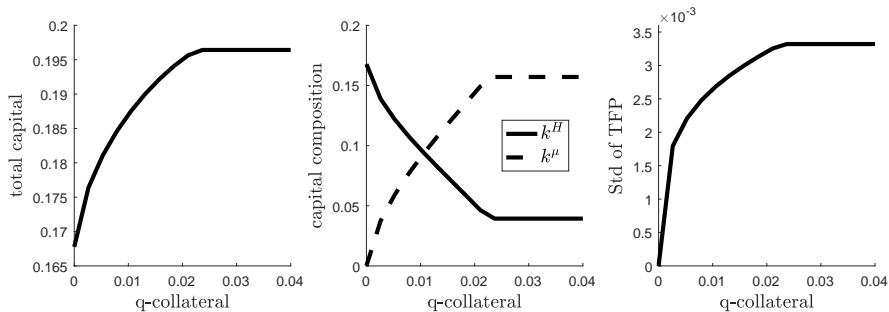
VARIABLES	(1) OLS	(2) OLS	(3) IV	(4) OLS	(5) OLS	(6) IV
RE Value (State Prices)	0.0526*** (0.00779)			0.0365*** (0.00432)		
RE Value (MSA Prices)		0.0510*** (0.00835)	0.0598*** (0.00982)		0.0346*** (0.0048)	0.0400*** (0.00569)
Analysts	0.0238*** (0.00818)	0.0294*** (0.00846)	0.0300*** (0.00941)			
RE Value (State Prices) x Analysts	-0.00704** (0.00349)					
RE Value (MSA Prices) x Analysts		-0.00721** (0.00364)	-0.00949** (0.00396)			
Intangibility				-0.00359 (0.00491)	-0.00419 (0.00531)	-0.00604 (0.00551)
RE Value (State Prices) x Intangibility				0.00546** (0.00249)		
RE Value (MSA Prices) x Intangibility					0.00609** (0.00263)	0.00673** (0.0028)
State Prices	-2.071*** (0.785)			-1.075 (0.983)		
MSA Prices		-2.573 (2.359)	0.41 (0.532)		-0.648 (2.646)	0.854 (0.541)
Cash	0.0313*** (0.00442)	0.0308*** (0.00486)	0.0303*** (0.00521)	0.0277*** (0.00304)	0.0287*** (0.00337)	0.0296*** (0.00355)
Market/Book	0.0649*** (0.00388)	0.0666*** (0.00395)	0.0665*** (0.00426)	0.0621*** (0.00311)	0.0644*** (0.00325)	0.0644*** (0.00351)
Initial Controls x State Prices	Yes	No	No	Yes	No	No
Initial Controls x MSA Prices	No	Yes	Yes	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	17 051	14 517	12 432	22 436	19 134	16 510
Adjusted R-squared	0.452	0.469	0.471	0.371	0.379	0.380

Pigouvian tax over a collateral-driven boom-bust cycle



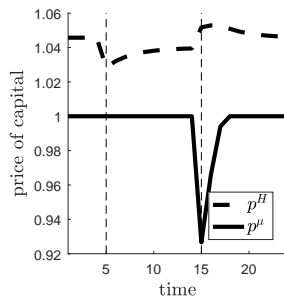
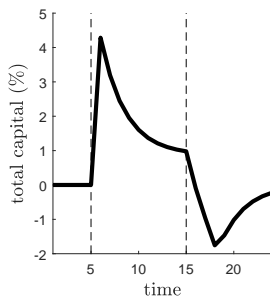
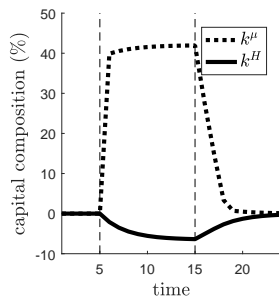
Collateral booms and misallocation

Here, $\delta = 1$ and $k_t = k_t^H + \lambda k_t^L + (\mu + (1 - \mu)\lambda)k_t^\mu$ for $\lambda \in (0, 1)$



Irreversibilities and “fire-sales”

Each unit of capital can be converted into $\chi = 0.9$ units of consumption



Micro-foundation for screening cost

Each period:

- ▶ Many banks, provide loans and screening services to entrepreneurs.
- ▶ Hire experts (e.g. savers) in a competitive market to do the screening.
 - Each expert can screen at most n projects at cost c per project.
 - $F(\cdot)$ is the distribution of costs in the population of experts, which is continuous with support $[0, \infty)$.
- ▶ Let s be the measure of projects screened, then the market clearing expert wage is given by $\psi \geq 0$ such that $s = n \cdot F(\psi)$.
 - Defines map $\psi(\cdot)$ with properties $\psi(0) = 0$ and $\psi'(\cdot) > 0$.
- ▶ $\psi(s)$ is the marginal cost and $\int_0^s \psi(x)dx$ is the total cost of screening.

Normative properties of equilibrium

- ▶ The social planner's objective is to maximize:

$$E_0 \sum_{t=0}^{\infty} \rho^{-t} C_t,$$

which is equivalent to p.v. of social welfare with relative weight ρ .

- ▶ Set $\rho > 1$ so that the economy is dynamically efficient.
- ▶ Information friction: needs to screen to invest in H -type capital.
- ▶ Financial friction: unscreened investment must be collateralized by q .
- ▶ Assume parameters are such that borrowing constraints bind for the planner.

Normative properties of equilibrium

- Formally, the planner's problem is:

$$V(k_t^H, k_t^\mu, q_t, A_t) = \max_{s_t} \left\{ Ak_t^\alpha + (1 - \delta)k_t - k_{t+1} - \int_0^{s_t} \psi(x) dx + q_t \right. \\ \left. + \rho^{-1} E_t V(k_{t+1}^H, k_{t+1}^\mu, q_{t+1}, A_{t+1}) \right\}$$

where $k_t = k_t^H + k_t^\mu$, subject to:

$$s_t = \max \left\{ 0, \frac{k_{t+1}^H - (1 - \delta)k_t^H}{\mu} \right\}, \\ k_{t+1}^\mu = \frac{\rho}{\rho - \mu E_t \{ A_{t+1} \alpha (k_{t+1}^H + k_{t+1}^\mu)^{\alpha-1} + 1 - \delta \}} \cdot q_t.$$

- From borrowing constraint, $k_{t+1}^\mu = k^\mu(k_{t+1}^H, q_t, A_t)$ is decreasing in k_{t+1}^H .