# Uncertainty and Economic Activity: A Multi-Country Perspective

A. Cesa-Bianchi[1] M.H. Pesaran[2] A. Rebucci[3]

[1] Bank of England
[2] University of Southern California and Trinity College, Cambridge
[3] Johns Hopkins University Carey Business School, CEPR and NBER

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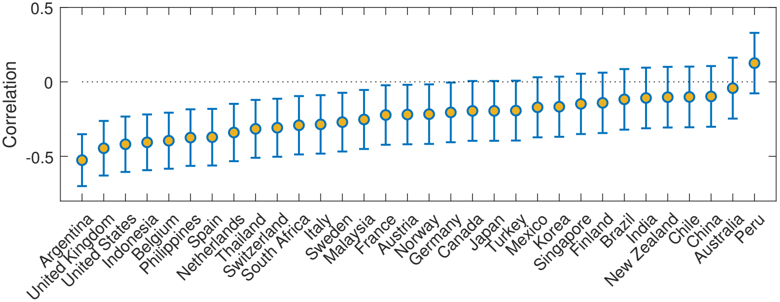
*The views expressed in this paper do not necessarily reflect the position of the Bank of England.

# Strong and robust association between measures of uncertainty and economic activity




- Data for 32 countries, covering about 90 percent of world GDP
  - **Uncertainty proxy**: Quarterly realized equity market volatility computed from daily returns.
  - **Economic activity proxy**: Quarterly real GDP growth.

# But difficult to interpret

- Theory is ambiguous about the direction of causation.
  - Uncertainty dampens activity (precautionary savings, irreversible investments, financial frictions,...).
  - Recessions can also increase uncertainty (financial and information frictions).

- Question of whether uncertainty *causes* GDP to contract (and by how much) is important for policy makers.

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  - Recessions can also increase uncertainty (financial and information frictions).
- Question of whether uncertainty *causes* GDP to contract (and by how much) is important for policy makers.
- Identification problem typically tackled in a single-country framework.

# This paper

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**FIGURE**: Average pairwise correlation of volatility (yellow bars)

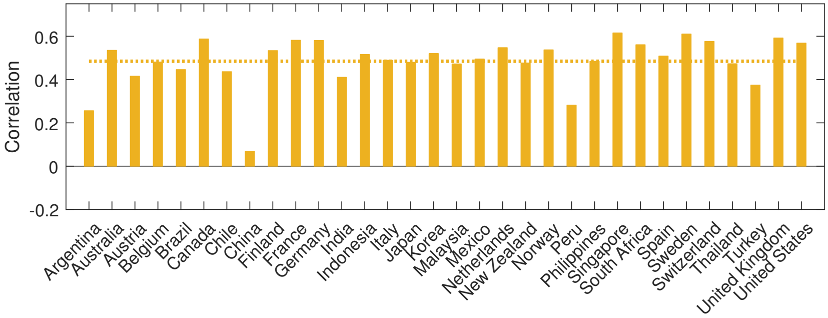

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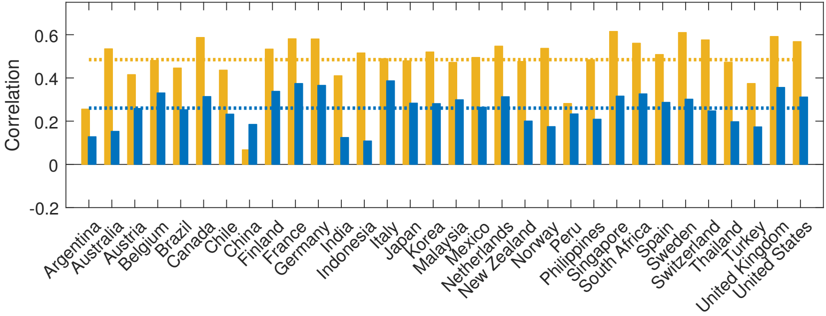
**FIGURE:** Average pairwise correlation of volatility (yellow bars) and GDP growth (blue bars).



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- Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.

- Empirically, there are two identification problems:
  [1] Identification of the common factors.
  [2] Identification of the country-specific shocks.

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- Empirically, there are two identification problems:
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  [2] Identification of the country-specific shocks.

- This paper contributes to [1]
  * Identification of a **growth** and a **financial** factors exploiting different patterns of cross-country correlations of volatility and GDP growth rates.

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# This paper: Main Findings

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- Shocks to the **financial factor** (common to volatility only) explain a non-trivial ($\sim 10\%$) share of GDP growth forecast error variance, and can have strong and persistent contractionary effects.
- **Country-specific volatility shocks** play a negligible role, i.e. explain $\sim 2\%$ of GDP growth forecast error variance (irrespective of within-country identification scheme used).

# Related literature

- **Uncertainty can respond to the business cycle** [Ludvigson, Ma, Ng (2017), Berger, Dew-Becker and Giglio (2017), Berger and Vavra (2017)]
  - We impose restrictions on the cross-section rather than on individual countries.
  - Restrictions consistent with observable properties of the data.
  - We obtain similar results with very different approach.

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- **International dimension** [Carriere-Swallow and Cespedes (2013), Baker and Bloom (2013), Hirata, Kose, Otrok and Terrones (2012), Carriero, Clark and Marcellino (2017), Mumtaz and Musso (2018)]
    - Multi-country framework, as opposed to a set of countries considered in isolation.
    - We do not assume volatility is exogenous.

# Outline

[1] **Theoretical model**
Equity returns and volatility in a multi-country business cycle model.

[2] **Empirical framework**
A multi-country factor model for volatility and the business cycle.

[3] **Data & Empirical results**

[4] **Conclusions**

# Theoretical model: Multi-country Lucas tree model with stochastic volatility

- Multi-country version of the Lucas (1978) tree model where country-specific output growth is driven by two processes:
  - Persistent global growth factor with stochastic volatility and heterogeneous loadings.
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- Two steps:
  [1] Solve for the risk free rate as a function of global growth shocks and their volatility.

  [2] Solve for the country-specific equity returns using Campbell and Shiller (1989) approximation and Bansal and Yaron (2004) approach.

Model details

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- Main insights:
  - Global growth shock is sufficient to explain cross-country differences in output growth.
  - At least one additional common shock (a composite of second and higher-order moment shocks) is required to explain the cross-country differences of realized volatility.

# Empirical Framework

# A multi-country econometric framework

- First-order panel vector autoregressive (PVAR) model in $\Delta y_{it}$ and $v_{it}$ for $i = 1, 2, ..., N$:

$$\begin{aligned} \Delta y_{it} &= a_{iy} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + e_{iy,t}, \\ v_{it} &= a_{iv} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + e_{iv,t}. \end{aligned}$$

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- Consistent with the theoretical model and the stylized facts, we posit the following unobservable common-factor representation for $e_{iy,t}$ and $e_{iv,t}$:

$$\begin{aligned} e_{iy,t} &= \gamma_i \zeta_t + \varepsilon_{it}, \\ e_{iv,t} &= \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}. \end{aligned}$$

- **Objective** Identification of factors ($\zeta_t$, $\xi_t$) and loadings ($\lambda_i$, $\gamma_i$, $\theta_i$).

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- **Our approach** Identification is achieved by placing restrictions on the *cross-country* correlations of the country-specific innovations, $Corr(\varepsilon_{it}, \varepsilon_{jt})$ and $Corr(\eta_{it}, \eta_{jt})\ \forall i, j$

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- **Note #1** *Within-country* correlation $Corr(\varepsilon_{it}, \eta_{it})$ for $i = 1, 2, ..., N$ is left unrestricted. To identify the country-specific shocks we will use auxiliary assumptions typically used in the literature.

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- **Note #2** In a single-country model, the common factors ($\zeta_t$, $\xi_t$) or their loadings ($\lambda_i$, $\gamma_i$, $\theta_i$) cannot be identified even if it is assumed that $Corr(\varepsilon_{it}, \eta_{it}) = 0$.

# Definitions & Identifying assumptions

## Definitions

- Define global GDP growth ($\Delta\bar{y}_{\omega,t}$) and global volatility ($\bar{v}_{\omega,t}$) as weighted ($w_i$) averages over a large number of countries:

$$\Delta\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}, \qquad \bar{v}_{\omega,t} = \sum_{i=1}^{N} w_i v_{it}$$

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## Identifying assumptions

1. Common shocks & Loadings: pervasive factors ($\zeta_t$ for both volatility and activity, $\xi_t$ for volatility only).
2. Weights: granularity (weights $w_i$ are not dominated by a few cross-section units).
3. Cross-sectional correlations: weak dependence of country-specific innovations (pairwise correlations of $\varepsilon_{it}$ and $\eta_{it}$ tend to zero).

Details

# The common growth shock ($\zeta_t$): Identification by aggregation

- For simplicity, and to focus on our novel approach, consider first a 'static' case:

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\Delta y_{it} &= \gamma_i \zeta_t + \varepsilon_{it}, \\
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- In this simple case, the factor $\zeta_t$ can be identified by $\bar{y}_{\omega,t}$ (up to a constant):

$$\zeta_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right).$$

- **Intuition** The GDP growth innovations $\varepsilon_{it}$ only include idiosyncratic risk that wash out when $N$ is large.

# The common growth shock ($\zeta_t$): Identification by aggregation

- **Proof** Consider the weighted average of the country-specific systems:

$$\begin{aligned} \Delta \bar{y}_{\omega,t} &= \gamma \zeta_t + \bar{\varepsilon}_{\omega,t}, \\ \bar{v}_{\omega,t} &= \lambda \zeta_t + \theta \xi_t + \bar{\eta}_{\omega,t}. \end{aligned}$$

where $\bar{\eta}_{\omega,t} = \mathbf{w}'\boldsymbol{\eta}_t$ and $\bar{\varepsilon}_{\omega,t} = \mathbf{w}'\boldsymbol{\varepsilon}_t$. Under the above assumptions it now readily follows that:

$$\zeta_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} + \underbrace{\frac{\bar{\varepsilon}_{\omega,t}}{\gamma}}_{O_p\left(N^{-1/2}\right)}.$$

and for $N$ sufficiently large $\zeta_t$ can be approximated by $\Delta \bar{y}_{\omega,t}$ (up to a constant), as

$$Var\left(\bar{\varepsilon}_{\omega,t}\right) = \mathbf{w}'\boldsymbol{\Sigma}_{\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right)\varrho_{\max}\left(\boldsymbol{\Sigma}_{\varepsilon}\right)$$

and so $Var\left(\bar{\varepsilon}_{\omega,t}\right) = O\left(\mathbf{w}'\mathbf{w}\right) = O\left(N^{-1}\right)$.

# The common growth shock ($\zeta_t$): Remarks

**Remark Interpretation of $\zeta_t$**

Because $\zeta_t$ is the same as world growth rescaled, we label it a common "growth" shock, in line with our theoretical derivations.

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**Remark The role of $N$**

The triangular factor structure alone does not identify $\zeta_t$. To get identification we need large $N$, otherwise we would not be able to disentangle $\zeta_t$ from $\bar{\varepsilon}_{\omega,t}$.

# The common financial shock ($\xi_t$)

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- But doing so permits exploring other properties of the data.

- Conditional on $\zeta_t$, for $N$ large enough, $\xi_t$ is determined by:

$$\xi_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma}\Delta\bar{y}_{\omega,t} + \underbrace{\frac{\bar{\eta}_{\omega,t}}{\theta}}_{O_p\left(N^{-1/2}\right)}.$$

- **Proof** Same logic as above applied to $\eta_{it}$. Factor $\xi_t$ can be identified from the data as a liner combination of $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ (up to an orthonormal transformation).

# The common financial shock ($\xi_t$): Remarks

**Remark** We label $\xi_t$ a global 'financial' shock to highlight its role in capturing all higher-order terms present in our theoretical derivations once we account for the common growth factor shock, $\zeta_t$, as well as any bubble component, financial friction, or time-varying risk preference component that might be present in the volatility data.

# A dynamic multi-country heterogeneous model

- Consider the following first-order dynamic version of the above static model:

$$\begin{aligned}\Delta y_{it} &= a_{iy} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \gamma_i \zeta_t + \varepsilon_{it} \\ v_{it} &= a_{iv} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}\end{aligned}$$

- Identification of the common factors in this dynamic and heterogeneous setting is sensibly more challenging.
- Stronger assumptions on PVAR coefficients ($\phi_{i,rs}$) and higher order lags of $(\bar{v}_{\omega,t}, \Delta \bar{y}_{\omega,t})$ are needed.

Skip derivations

# Dynamic model: Assumptions

- **Factor loadings** The factor loadings $\lambda_i$, $\theta_i$, and $\gamma_i$ (i.e., the non-zero elements of $\mathbf{\Gamma}_i$) are independently distributed across $i$, and of the common factors, $\mathbf{f}_t$, for all $i$ and $t$, with non-zero means $\lambda$, $\theta$, and $\gamma$, and second-order moments. Furthermore:

$$\mathbf{\Gamma} = \mathbb{E}\left(\mathbf{\Gamma}_i\right) = \begin{pmatrix} \gamma & 0 \\ \lambda & \theta \end{pmatrix}$$

- **Coefficients** The constants $\mathbf{a}_i$ are bounded, $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are independently distributed for all $i$, the support of $\varrho\left(\mathbf{\Phi}_i\right)$ lies strictly inside the unit circle, for $i = 1, 2, ..., N$, and the inverse of the polynomial $\mathbf{\Lambda}\left(L\right) = \sum_{\ell=0}^{\infty} \mathbf{\Lambda}_\ell L^\ell$, where $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients, namely $\|\mathbf{\Lambda}_\ell\| \leq K\rho^\ell$, where $K$ is a fixed constant and $0 < \rho < 1$.

Back

# Dynamic model: Identification of the factors

- **Proposition** Common factors $\zeta_t$ and $\xi_t$ are given by:

$$\begin{aligned} \zeta_t &= b_\zeta + \gamma^{-1}\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty}\mathbf{c}'_{1,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \\ \xi_t &= b_\xi + \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right) + \sum_{\ell=1}^{\infty}\mathbf{c}'_{2,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \end{aligned}$$

where $\mathbf{z}_{it} = (\Delta y_{it}, v_{it})'$.

Proof

# Dynamic model: Consistent estimation of orthogonal factors

- **Proposition** Consistent estimators of the common shocks, denoted by $\hat{\zeta}$ and $\hat{\xi}$, can be obtained as residuals from the following OLS regressions:

$$\begin{aligned}\hat{\zeta} &= \Delta\bar{\mathbf{y}}_{\omega} - \bar{\mathbf{Z}}_{\omega}\hat{\mathbf{C}}_1, \\ \hat{\xi} &= \bar{\mathbf{v}}_{\omega} - \hat{\lambda}\hat{\zeta} - \bar{\mathbf{Z}}_{\omega}\hat{\mathbf{C}}_2,\end{aligned}$$

where $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ..., \zeta_T)'$, $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_T)'$, $\bar{\mathbf{Z}}_{\omega} = (\boldsymbol{\tau}_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-2}, ..., \bar{\mathbf{z}}_{\omega,-p})$, $\bar{\mathbf{z}}_{\omega,-l} = (\Delta\bar{\mathbf{y}}_{\omega,-l}\bar{v}_{\omega,-l})$, $\Delta\bar{\mathbf{y}}_{\omega,-l} = (\Delta\bar{y}_{\omega,1-l}, \Delta\bar{y}_{\omega,2-l}, ..., \Delta\bar{y}_{\omega,T-l})'$, $\Delta\bar{\mathbf{y}}_{\omega} = \Delta\bar{\mathbf{y}}_{\omega,0}$, $\bar{\mathbf{v}}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, ..., \bar{v}_{\omega,T-l})'$, $\bar{\mathbf{v}}_{\omega} = \bar{\mathbf{v}}_{\omega,0}$, and $p$ denotes a suitable number of lags (or truncation order).

Proof

# Dynamic model: Factor-augmented large VAR

- Theoretical results carry through a fully heterogeneous dynamic version of the model.
- Country-specific model with orthonormal factors:

$$\Delta y_{it} = \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \beta_{i,11} \hat{\zeta}_t + \sum_{\ell=1}^{p} \psi'_{\Delta y, i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \varepsilon_{it}$$

$$v_{it} = \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \beta_{i,21} \hat{\zeta}_t + \beta_{i,22} \hat{\xi}_t + \sum_{\ell=1}^{p} \psi'_{v, i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \eta_{it}$$

where $\bar{\mathbf{z}}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta \bar{y}_{\omega,t})$.

- Country-specific models can be combined in a large model of the global economy.

# Volatility measurement

- We compute the realized volatility for country $i$ in quarter $t$ as:

$$\sigma_{it}^2 = \sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter $t$, and $D_t$ is the number of trading days in quarter $t$.

- We work with log of $\sigma_{it}$.

- In recent literature focus has shifted to implied volatility measures from option prices (e.g., VIX). But:
  - Not available for a large number of countries over a long period of time.
  - Berger, Dew-Becker, Giglio (2017): conditional on realized volatility, VIX is not associated with indicators of economic activity.

# Data & Empirical Results

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## Data

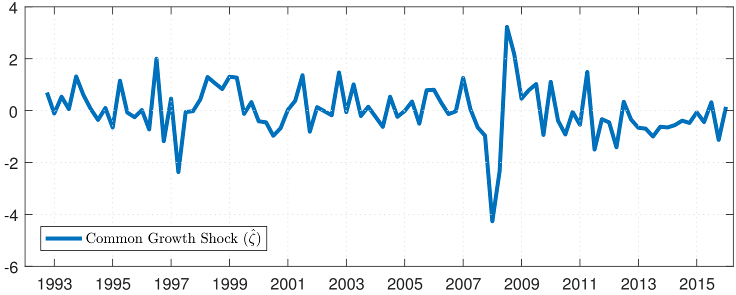
- Balanced panel data for 32 countries from 1993:Q1 to 2016:Q4.

## Empirical results

- Factors estimates.
- Within-country *conditional* correlation of volatility and GDP growth.
- IRFs and FEVDs to factors and country-specific shocks.
- Evidence in support of identifying assumptions (cross-country correlations).

# The common growth shock ($\hat{\zeta}$)

- Common growth shock:



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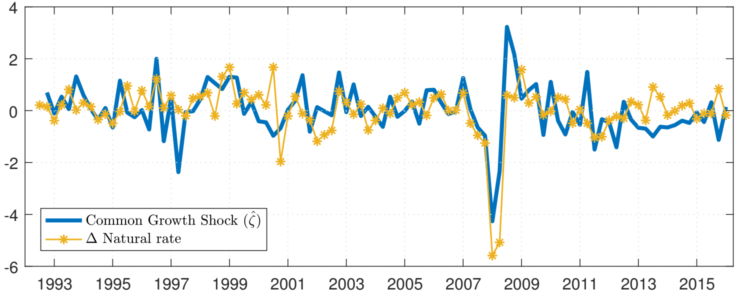


- Correlated with:
  - Proxy for the world natural rate ($0.5$) from Holston, Laubach, Williams (2017).

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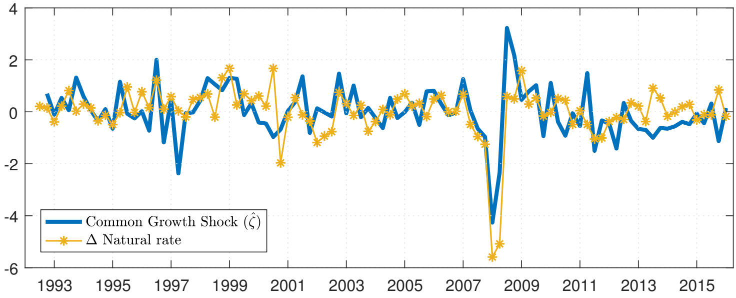


- Correlated with:
  - Proxy for the world natural rate ($0.5$) from Holston, Laubach, Williams (2017).
  - Proxy for global TFP ($0.65$) and global utilization adjusted TFP ($0.33$) from Huo, Levchenko, Pandalai-Nayar (2018).
  - Proxy for global long-run risk ($0.29$) from Colacito, Croce, Gavazzoni, Ready (2018).

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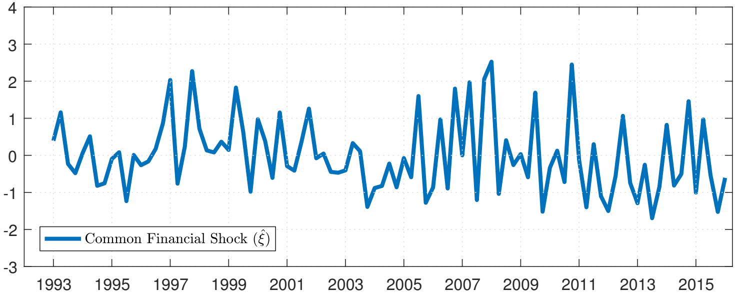

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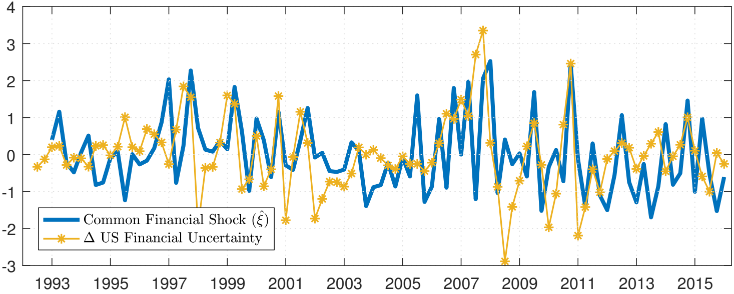


- Correlated with:
  - Financial uncertainty measure ($0.43$) from Ludvigson, Ma and Ng (2017).

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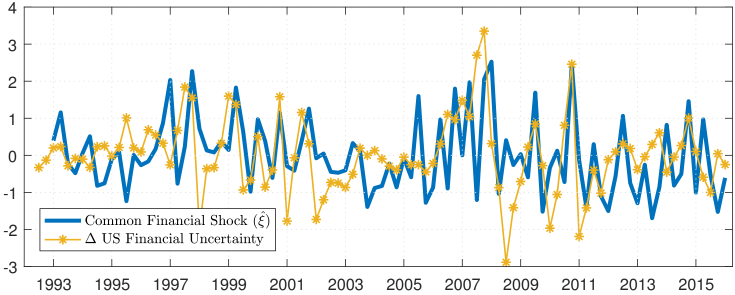
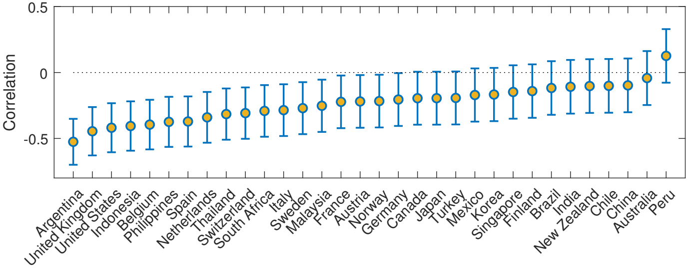


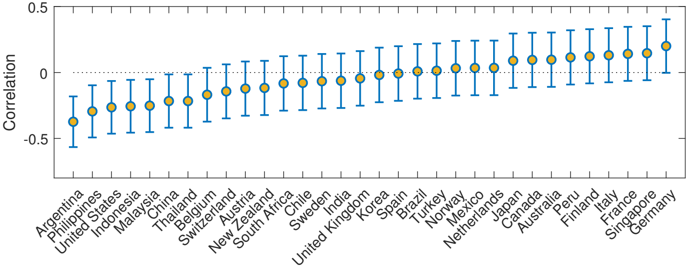

- Correlated with:
  - Financial uncertainty measure ($0.43$) from Ludvigson, Ma and Ng (2017).
  - Excess Bond Premium ($0.35$) Gilchrist and Zakrajsek (2012).

# Within-country correlation of volatility and growth: unconditional




- Unconditional correlation between $\Delta y_{it}$ and $v_{it}$ (same as Figure shown in introduction).

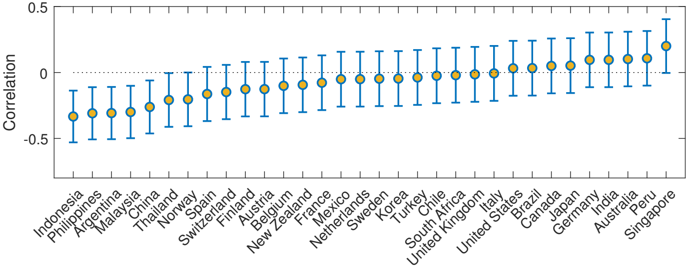
# Within-country correlation of volatility and growth: conditioning on common growth shock only




- Correlation between $\varepsilon_{it}$ and $u_{it}$ conditional on growth common shock ($\hat{\zeta}_t$) only.

$$\Delta y_{it} = \beta_{i,21}\hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$
$$v_{it} = \beta_{i,11}\hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it}$$

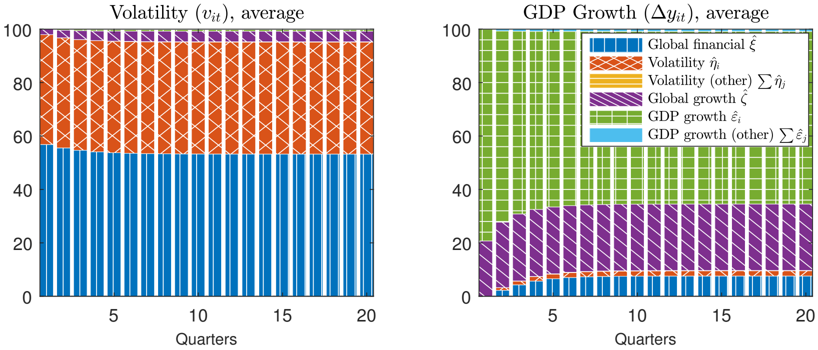
# Within-country correlation of volatility and growth: conditioning on both common shocks




- Correlation between $\varepsilon_{it}$ and $\eta_{it}$ conditional on growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) common shocks

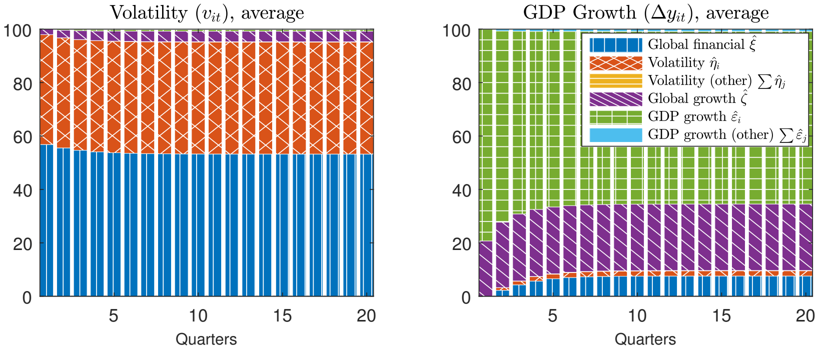
$$
\begin{aligned}
\Delta y_{it} &= \beta_{i,21}\hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it} \\
v_{it} &= \beta_{i,11}\hat{\zeta}_t + \beta_{i,12}\hat{\xi}_t + \text{cross-section averages and lagged values} + \eta_{it}
\end{aligned}
$$

# Forecast error variance decomposition



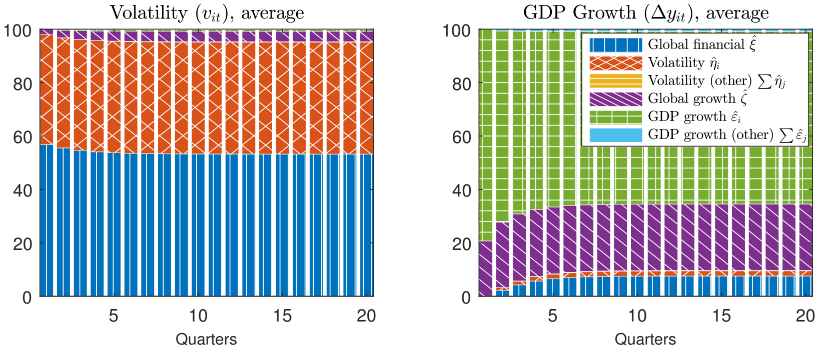

- Growth factor ($\hat{\zeta}_t$, **purple areas**) and country-specific growth innovations ($\varepsilon_{it}$, **green areas**) jointly explain less than $< 5\%$ of volatility forecast error variance .

# Forecast error variance decomposition



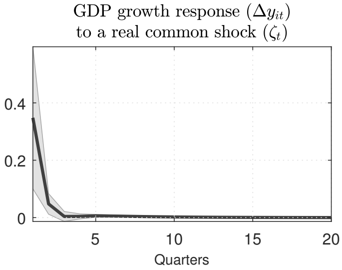

- Financial factor ($\hat{\xi}_t$, **dark blue areas**) explains a significant share of growth forecast error variance (about $\sim 10\%$).

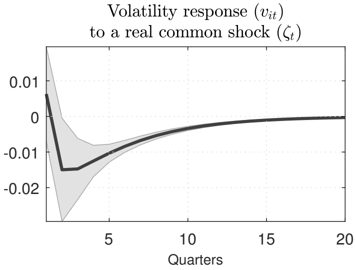
# Forecast error variance decomposition




- Financial factor ($\hat{\xi}_t$, **dark blue areas**) explains a significant share of growth forecast error variance (about $\sim 10\%$).
- But country-specific volatility shocks ($\eta_{it}$, **red areas**) diversified away.

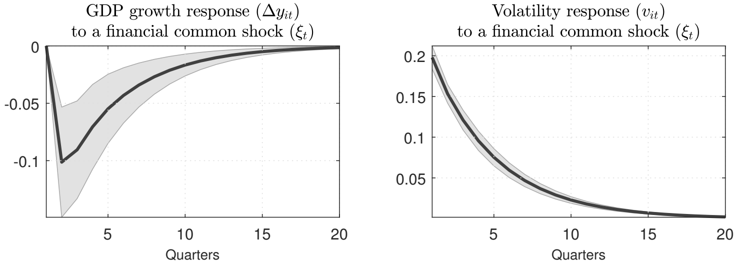
# Growth common shocks ($\hat{\zeta}_t$) lead to endogenous movements in volatility




- Positive growth shock ($\hat{\zeta}_t$) leads to an increase in world GDP growth...
- ... and to a countercyclical fall in volatility.

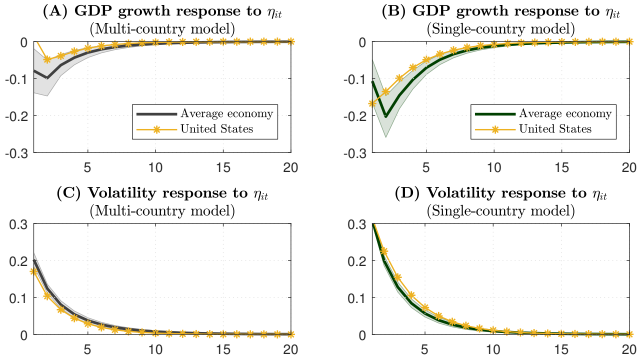
# Financial common shocks ($\hat{\xi}_t$) lead to sharp recessions




- Adverse financial shock ($\hat{\xi}_t$) leads to an increase in world volatility...
- ... and to a sharp and persistent contraction of world GDP growth.

# Country-specific volatility shocks ($\hat{\eta}_t$): Omitted variable bias

- Compare IRFs from a single-country VAR (with recursive identification) with those obtained from our multi-country model.




- Single-country model largely overestimates the impact of domestic shocks. growth.

# Are the identifying assumptions on cross-sectional dependence consistent with the data?

- Estimate country models with growth shock ($\hat{\zeta}_t$) only:

$$\begin{aligned} \Delta y_{it} &= \beta_{i,21}\hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it} \\ v_{it} &= \beta_{i,11}\hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it} \end{aligned}$$

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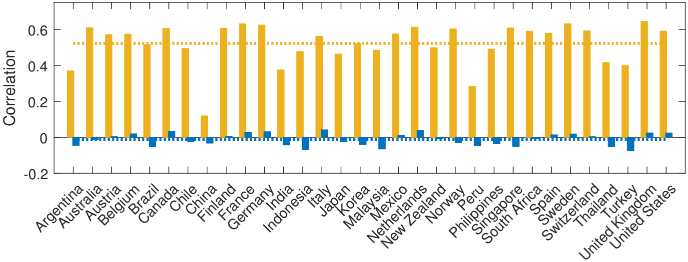


NOTE. Average pairwise correlation of the $u_{it}$ (yellow bars) and the $\varepsilon_{it}$ (blue bars).

# Are the identifying assumptions on cross-sectional dependence consistent with the data?

- Estimate country models with growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) shocks:

$$\Delta y_{it} = \beta_{i,21}\hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$
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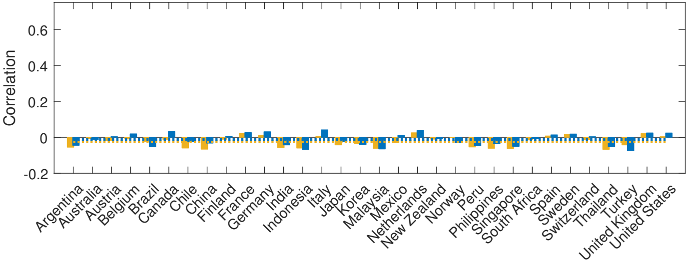


NOTE. Average pairwise correlation of the $\eta_{it}$ (yellow bars) and the $\varepsilon_{it}$ (blue bars).

# Tests of cross-sectional dependence don't reject identifying assumptions

- CD and Exponent of cross-sectional dependence tests [Pesaran (2015), Bailey et al. (2016)].
- Results in accordance with assumptions of:
  - Weak/strong cross-sectional dependence of $\varepsilon_{it}/u_{it}$, respectively.
  - Weak cross-sectional dependence of both $\varepsilon_{it}$ and $\eta_{it}$.

| | $CD$ | Lower $5\%$ | $\hat{\alpha}$ | Upper $95\%$ |
|---|---|---|---|---|
| *Data* | | | | |
| $v_{it}$ | 104.57 | 0.94 | 0.99 | 1.05 |
| $\Delta y_{it}$ | 55.73 | 0.87 | 1.00 | 1.14 |
| Innovations (conditional on $\hat{\zeta}_t$) | | | | |
| $u_{it}$ | 110.89 | 0.96 | 1.00 | 1.04 |
| $\varepsilon_{it}$ | -2.90 | 0.56 | 0.62 | 0.67 |
| Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$) | | | | |
| $\eta_{it}$ | -5.12 | 0.58 | 0.64 | 0.70 |

# Robustness

- Granularity assumption:
  - Results are robust when dropping (i) the US, (ii) China, and (iii) both the US and China. Go
  - Results are not robust when replacing the global factors with US GDP growth and US realized volatility. Go

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- Realized versus implied volatility:
  - Results are robust when using the VIX Index as a measure of volatility for the United States. Go
- Alternative assumptions for identification of country-specific shocks:
  - Results are robust to block-diagonal covariance matrix with recursive (within-country) identification. Go
  - Unrestricted covariance matrix. Go

# Conclusions

- Paper takes a new common-factor approach in a multi-country model to the relation between volatility and growth without imposing restrictions on the direction of causation at the level of individual country.

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  - Common factors are important to understand relation between volatility and economic activity.

# Conclusions

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  - Paper exploits the different cross-country correlation structure of volatility and growth innovations to identify a 'growth' factor and a 'financial' factor.
  - Common factors are important to understand relation between volatility and economic activity.
- Key implication is that part of explanatory power typically attributed to country-specific uncertainty shocks is due to omitted common factors.

# Uncertainty and Economic Activity: A Multi-Country Perspective

A. Cesa-Bianchi[1] M.H. Pesaran[2] A. Rebucci[3]

[1] Bank of England
[2] University of Southern California and Trinity College, Cambridge
[3] Johns Hopkins University Carey Business School, CEPR and NBER

2nd Annual CEBRA Intl. Finance and Macroeconomics Meeting
November 29-30, 2018

*The views expressed in this paper do not necessarily reflect the position of the Bank of England.

# Appendix

# A1::Model

# A2::Assumptions

# Assumptions

- **Assumption 1: Common factors and their loadings** The common unobservable factors, $\zeta_t$ and $\xi_t$, have zero means and unit variances, and are serially uncorrelated. The factor loadings, $\lambda_i$, $\gamma_i$, and $\theta_i$, are distributed independently across $i$ and from the common factors $f_t$ and $g_t$ for all $i$ and $t$, with non-zero means $\lambda$, $\gamma$, and $\theta$ $(\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0)$, and satisfy the following conditions, for a finite $N$ and as $N \to \infty$:

$$N^{-1} \sum_{i=1}^{N} \lambda_i^2 = \mathcal{O}(1) \qquad \lambda = \sum_{i=1}^{N} \mathring{w}_i \lambda_i \neq 0$$

$$N^{-1} \sum_{i=1}^{N} \gamma_i^2 = \mathcal{O}(1) \qquad \gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0$$

$$N^{-1} \sum_{i=1}^{N} \theta_i^2 = \mathcal{O}(1) \qquad \theta = \sum_{i=1}^{N} w_i \theta_i \neq 0$$

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# Assumptions (Cont.)

- **Assumption 2: Aggregation weights** Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\mathring{\mathbf{w}} = (\mathring{w}_1, \mathring{w}_2, ..., \mathring{w}_N)'$ be the $N \times 1$ vectors of non-stochastic weights with $w_i, \mathring{w}_i > 0$, $\sum_{i=1}^N w_i = 1$ and $\sum_{i=1}^N \mathring{w}_i = 1$, such that the following "granularity" conditions are met:

$$||\mathbf{w}|| = O(N^{-1}), \ \frac{w_i}{||\mathbf{w}||} = O(N^{-1/2})$$

and

$$||\mathring{\mathbf{w}}|| = O(N^{-1}), \ \frac{\mathring{w}_i}{||\mathring{\mathbf{w}}||} = O(N^{-1/2})$$

for all $i$.

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# Assumptions (Cont.)

- **Assumption 3: Cross-section correlations** The country-specific innovations, $\eta_{it}$ and $\varepsilon_{it}$, have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. Furthermore, denoting the covariance matrices of the $N \times 1$ innovation vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t}, ..., \eta_{Nt})'$ by $\Sigma_{\varepsilon\varepsilon} = Var(\boldsymbol{\varepsilon}_t)$ and $\Sigma_{\eta\eta} = Var(\boldsymbol{\eta}_t)$, respectively, it is assumed that:

$$\varrho_{\max}(\Sigma_{\varepsilon\varepsilon}) = O(1)$$

$$\varrho_{\max}(\Sigma_{\eta\eta}) = O(1)$$

Back

# A3::Econometric model

# Estimating observable and orthogonal factors

- **Issue** Factors $f_t$ and $g_t$ are unobservable, and even if known, would be correlated with each other
- For ease of interpretation it is standard to work with the orthogonalized version of the factors
  * This task is simplified due to the triangular way the factors affect the global variables, $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$
- Proceed recursively
  * Factor $f_t$ can be identified up to a constant

$$f_t = \frac{\Delta\bar{y}_{\omega,t}}{\gamma} \Rightarrow \hat{\zeta}_t = \Delta\bar{y}_{\omega,t}$$

  * Factor $g_t$ can then be approximated by the residuals of a regression of world volatility $\bar{v}_{\omega,t}$ on world growth

$$g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma}\Delta\bar{y}_{\omega,t} \Rightarrow \bar{v}_{\omega,t} = \hat{\beta}\Delta\bar{y}_{\omega,t} + \hat{\xi}_t$$

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# Consistent estimation of orthogonal factors

- **Proposition 3** Let $\hat{\zeta}_t$ and $\hat{\xi}_t$ be consistent, orthonormalized estimators of $f_t$ and $g_t$, respectively, where $f_t$ and $g_t$ are defined by (**??**) and (**??**). Then, $\hat{\zeta}_t$ can be obtained by re-scaling $\Delta \bar{y}_{\omega,t}$ so that its variance is $1$, while $\hat{\xi}_t$ can be obtained as the standardized residual of a least squares regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$.

- **Proof** Consider equation (**??**) and (**??**) and set the coefficients $\boldsymbol{\alpha}_g = (\alpha_{1g}, \alpha_{2g})'$ , such that $T^{-1} \sum_{t=1}^{T} \hat{\zeta}_t \hat{\xi}_t = 0$. This yields:

$$\frac{\hat{\alpha}_{2g}}{\hat{\alpha}_{1g}} = \frac{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t} \bar{v}_{\omega,t}}{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t}^2},$$

which is the OLS estimate of the coefficient on $\Delta \bar{y}_{\omega,t}$ in a regression of $\bar{v}_t$ on $\Delta \bar{y}_{\omega,t}$. Next, set $\alpha_f$ and $\alpha_{1g}$ so that $\zeta_t$ and $\xi_t$ have unit in-sample standard deviations.

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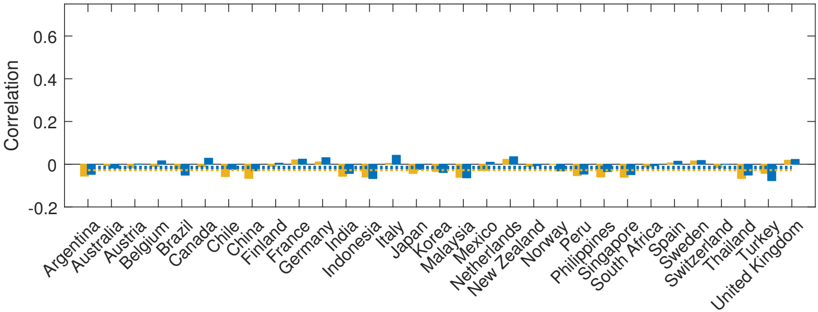
# A3::Additional results

# Granularity: No US

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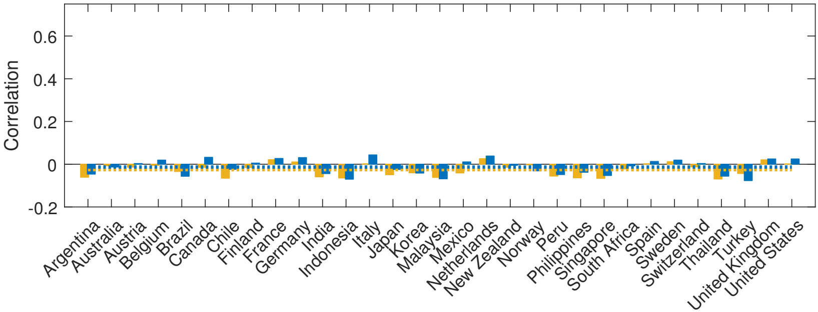
# Granularity: No China

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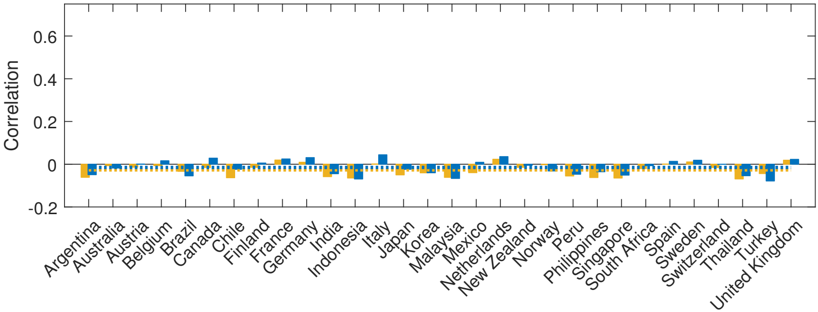
# Granularity: No US & No China

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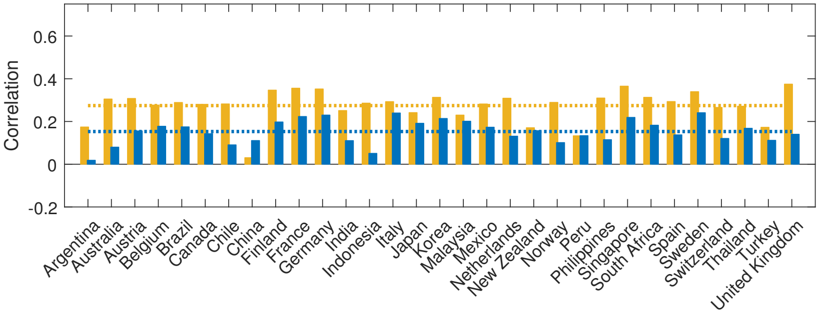
# Granularity: US as the global factor

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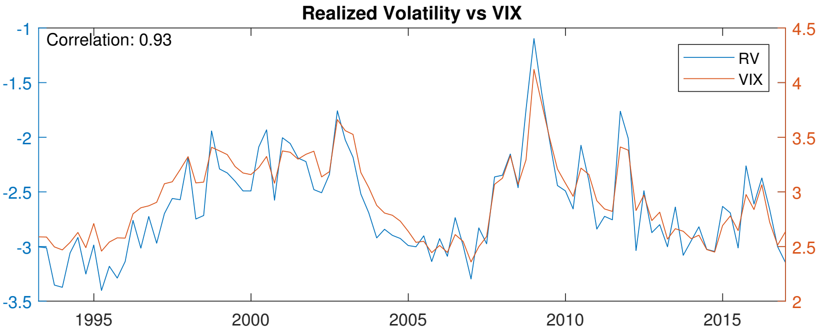
# Comparison between VIX and US realized volatility:Data

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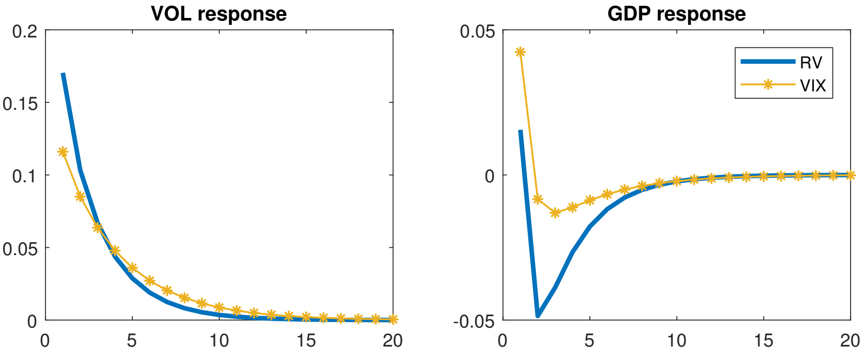
# Comparison between VIX and US realized volatility: IRF

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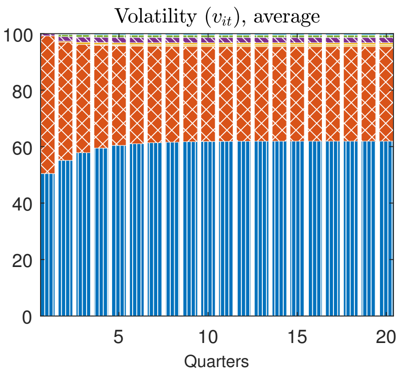
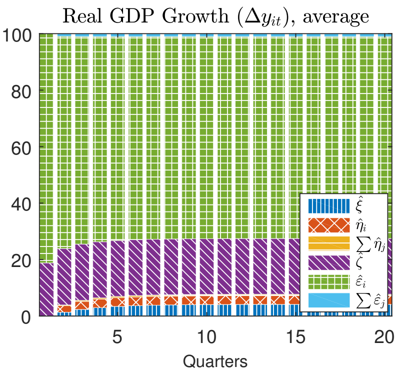
# Excluding the global financial crisis: FEVDs

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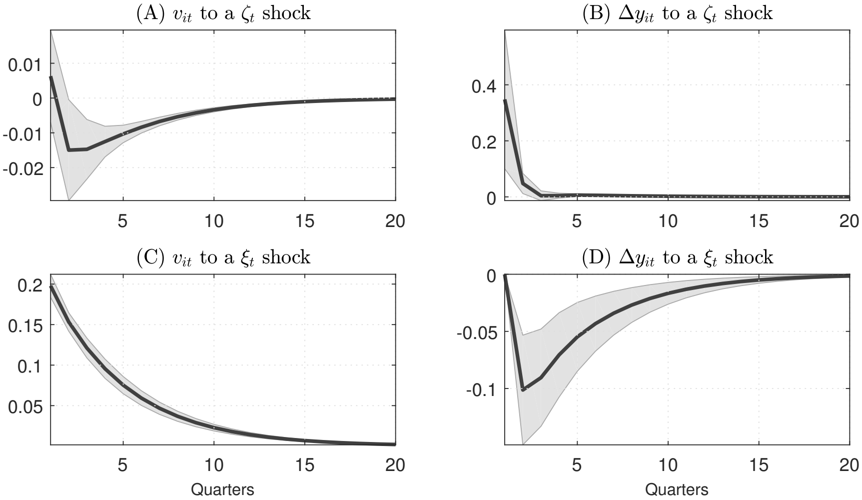
# Excluding the global financial crisis: IRFs

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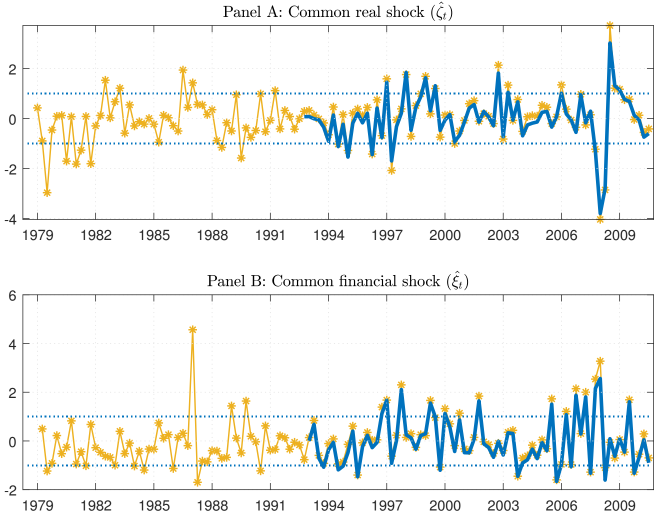
# Longer sample period: Common Shocks

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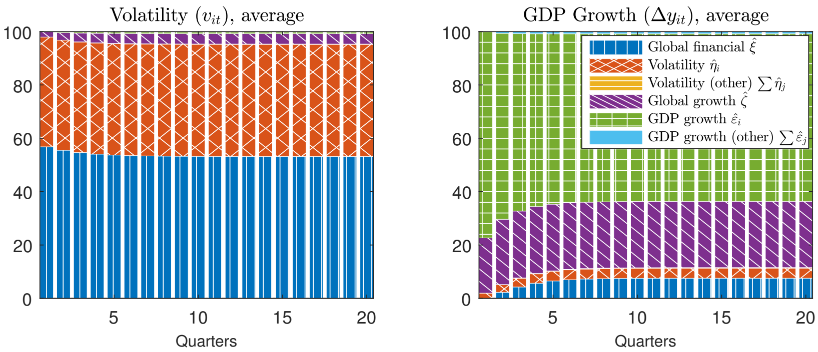
# Average FEVD: Block-diagonal covariance matrix




- Allow for within-country correlation of the innovations ($\varepsilon_{it}$ and $\eta_{it}$).
- Orthogonalize with Cholesky decomposition, with volatility ordered first (block-diagonal covariance matrix)

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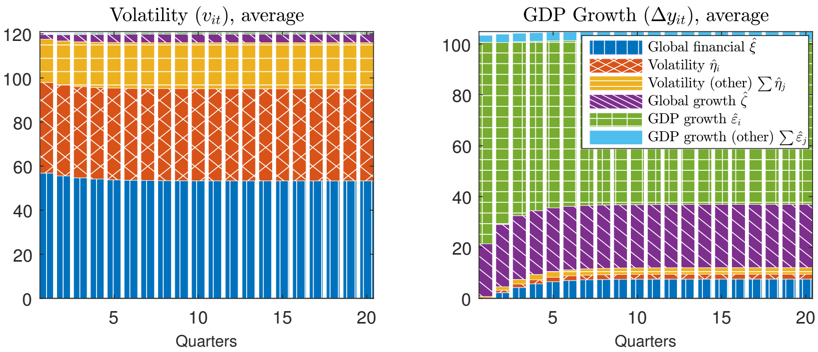
# Average GFEVD: Threshold covariance matrix




- Allow for within- and across-country correlation of the innovations ($\varepsilon_{it}$ and $\eta_{it}$).
- Compute Generalized FEVD with threshold covariance matrix.

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# Country-specific response to the factors