Uncertainty and Economic Activity: A Multi-Country Perspective

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*The views expressed in this paper do not necessarily reflect the position of the Bank of England.
Strong and robust association between measures of uncertainty and economic activity

Data for 32 countries, covering about 90 percent of world GDP

- **Uncertainty proxy**: Quarterly realized equity market volatility computed from daily returns.
- **Economic activity proxy**: Quarterly real GDP growth.
Theory is ambiguous about the direction of causation.

- Uncertainty dampens activity (precautionary savings, irreversible investments, financial frictions, ...).
- Recessions can also increase uncertainty (financial and information frictions).

Question of whether uncertainty *causes* GDP to contract (and by how much) is important for policy makers.
But difficult to interpret

- Theory is ambiguous about the direction of causation.
  - Uncertainty dampens activity (precautionary savings, irreversible investments, financial frictions,...).
  - Recessions can also increase uncertainty (financial and information frictions).

- Question of whether uncertainty causes GDP to contract (and by how much) is important for policy makers.

- Identification problem typically tackled in a single-country framework.
This paper

- Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.
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**Figure:** Average pairwise correlation of volatility (yellow bars)
This paper proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.

**Figure**: Average pairwise correlation of volatility (yellow bars) and GDP growth (blue bars).
This paper

- Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.

- Empirically, there are two identification problems:
  1. Identification of the common factors.
  2. Identification of the country-specific shocks.
This paper

- Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.

- Empirically, there are two identification problems:
  - [1] Identification of the common factors.

- This paper contributes to [1]
  - Identification of a **growth** and a **financial** factors exploiting different patterns of cross-country correlations of volatility and GDP growth rates.
The growth factor accounts for the bulk of the (unconditional) negative correlation between volatility and GDP growth for most countries.
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Shocks to the financial factor (common to volatility only) explain a non-trivial ($\sim 10\%$) share of GDP growth forecast error variance, and can have strong and persistent contractionary effects.
This paper: Main Findings

- The **growth factor** accounts for the bulk of the (unconditional) negative correlation between volatility and GDP growth for most countries.

- Shocks to the **financial factor** (common to volatility only) explain a non-trivial (∼10%) share of GDP growth forecast error variance, and can have strong and persistent contractionary effects.

- **Country-specific volatility shocks** play a negligible role, i.e. explain ∼2% of GDP growth forecast error variance (irrespective of within-country identification scheme used).
Related literature

- **Uncertainty can respond to the business cycle** [Ludvigson, Ma, Ng (2017), Berger, Dew-Becker and Giglio (2017), Berger and Vavra (2017)]
  - We impose restrictions on the cross-section rather than on individual countries.
  - Restrictions consistent with observable properties of the data.
  - We obtain similar results with very different approach.
Related literature

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  - We impose restrictions on the cross-section rather than on individual countries.
  - Restrictions consistent with observable properties of the data.
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- **International dimension** [Carriere-Swallow and Cespedes (2013), Baker and Bloom (2013), Hirata, Kose, Otrok and Terrones (2012), Carriero, Clark and Marcellino (2017), Mumtaz and Musso (2018)]
  - Multi-country framework, as opposed to a set of countries considered in isolation.
  - We do not assume volatility is exogenous.
[1] **Theoretical model**  
Equity returns and volatility in a multi-country business cycle model.

[2] **Empirical framework**  
A multi-country factor model for volatility and the business cycle.

[3] **Data & Empirical results**

[4] **Conclusions**
Theoretical model: Multi-country Lucas tree model with stochastic volatility

- Multi-country version of the Lucas (1978) tree model where country-specific output growth is driven by two processes:
  - Persistent global growth factor with stochastic volatility and heterogeneous loadings.
  - Country-specific business cycle component with stochastic volatility.
Theoretical model: Multi-country Lucas tree model with stochastic volatility

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  - Persistent global growth factor with stochastic volatility and heterogeneous loadings.
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- Two steps:
  - [1] Solve for the risk free rate as a function of global growth shocks and their volatility.
Theoretical model: Multi-country Lucas tree model with stochastic volatility

- Multi-country version of the Lucas (1978) tree model where country-specific output growth is driven by two processes:
  - Persistent global growth factor with stochastic volatility and heterogeneous loadings.
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- Main insights:
  - Global growth shock is sufficient to explain cross-country differences in output growth.
  - At least one additional common shock (a composite of second and higher-order moment shocks) is required to explain the cross-country differences of realized volatility.
Empirical Framework
A multi-country econometric framework

- First-order panel vector autoregressive (PVAR) model in $\Delta y_{it}$ and $v_{it}$ for $i = 1, 2, ..., N$:

\[
\begin{align*}
\Delta y_{it} &= a_{iy} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + e_{iy,t}, \\
v_{it} &= a_{iv} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + e_{iv,t}.
\end{align*}
\]
A multi-country econometric framework

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  \[ v_{it} = a_{iv} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + e_{iv,t}. \]

- Consistent with the theoretical model and the stylized facts, we posit the following unobservable common-factor representation for $e_{iy,t}$ and $e_{iv,t}$:

  \[ e_{iy,t} = \gamma_i \zeta_t + \varepsilon_{it}, \]
  \[ e_{iv,t} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}. \]

- **Objective** Identification of factors $(\zeta_t, \xi_t)$ and loadings $(\lambda_i, \gamma_i, \theta_i)$. 
A multi-country econometric framework

- First-order panel vector autoregressive (PVAR) model in $\Delta y_{it}$ and $v_{it}$ for $i = 1, 2, ..., N$:

$$\Delta y_{it} = a_i y + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + e_{iy,t},$$

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- **Our approach** Identification is achieved by placing restrictions on the cross-country correlations of the country-specific innovations, $Corr(\varepsilon_{it}, \varepsilon_{jt})$ and $Corr(\eta_{it}, \eta_{jt}) \forall i, j$. 

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Introduction: Theoretical model Empirical framework Data & Empirical Results Conclusions
A multi-country econometric framework

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  $$

  $$
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  $$

- **Note #1** Within-country correlation $Corr(\varepsilon_{it}, \eta_{it})$ for $i = 1, 2, ..., N$ is left unrestricted. To identify the country-specific shocks we will use auxiliary assumptions typically used in the literature.
A multi-country econometric framework

- First-order panel vector autoregressive (PVAR) model in $\Delta y_{it}$ and $v_{it}$ for $i = 1, 2, ..., N$:

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  $e_{iy,t} = \gamma_i \zeta_t + \varepsilon_{it}$,
  
  $e_{iv,t} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}$.

- **Note #2** In a single-country model, the common factors $(\zeta_t, \xi_t)$ or their loadings $(\lambda_i, \gamma_i, \theta_i)$ cannot be identified even if it is assumed that $\text{Corr}(\varepsilon_{it}, \eta_{it}) = 0$. 

Definitions & Identifying assumptions

Definitions

- Define global GDP growth ($\Delta \bar{y}_\omega, t$) and global volatility ($\bar{v}_\omega, t$) as weighted ($w_i$) averages over a large number of countries:

$$\Delta \bar{y}_\omega, t = \sum_{i=1}^{N} w_i \Delta y_{it}, \quad \bar{v}_\omega, t = \sum_{i=1}^{N} w_i v_{it}$$
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Identifying assumptions

1. Common shocks & Loadings: pervasive factors ($\xi_t$ for both volatility and activity, $\zeta_t$ for volatility only).
2. Weights: granularity (weights $w_i$ are not dominated by a few cross-section units).
3. Cross-sectional correlations: weak dependence of country-specific innovations (pairwise correlations of $\varepsilon_{it}$ and $\eta_{it}$ tend to zero).
The common growth shock ($\zeta_t$): Identification by aggregation

- For simplicity, and to focus on our novel approach, consider first a ‘static’ case:

$$\Delta y_{it} = \gamma_i \zeta_t + \varepsilon_{it},$$
$$v_{it} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}.$$
The common growth shock ($\zeta_t$): Identification by aggregation

- For simplicity, and to focus on our novel approach, consider first a ‘static’ case:

$$
\Delta y_{it} = \gamma_i \zeta_t + \varepsilon_{it},
$$
$$
u_{it} = \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}.
$$

- In this simple case, the factor $\zeta_t$ can be identified by $\bar{y}_{\omega,t}$ (up to a constant):

$$
\zeta_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p \left( N^{-1/2} \right).
$$

- **Intuition** The GDP growth innovations $\varepsilon_{it}$ only include idiosyncratic risk that wash out when $N$ is large.
The common growth shock ($\zeta_t$): Identification by aggregation

**Proof** Consider the weighted average of the country-specific systems:

\[
\begin{align*}
\Delta \bar{y}_{\omega,t} &= \gamma \zeta_t + \bar{\varepsilon}_{\omega,t}, \\
\bar{v}_{\omega,t} &= \lambda \zeta_t + \theta \xi_t + \bar{\eta}_{\omega,t}.
\end{align*}
\]

where $\bar{\eta}_{\omega,t} = w' \eta_t$ and $\bar{\varepsilon}_{\omega,t} = w' \varepsilon_t$. Under the above assumptions it now readily follows that:

\[
\zeta_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} + \frac{\bar{\varepsilon}_{\omega,t}}{\gamma} = O_p\left( N^{-1/2} \right)
\]

and for $N$ sufficiently large $\zeta_t$ can be approximated by $\Delta \bar{y}_{\omega,t}$ (up to a constant), as

\[
\text{Var} \left( \bar{\varepsilon}_{\omega,t} \right) = w' \Sigma_{\varepsilon} w \leq (w' w) \rho_{\text{max}} \left( \Sigma_{\varepsilon} \right)
\]

and so $\text{Var} \left( \bar{\varepsilon}_{\omega,t} \right) = O \left( w' w \right) = O \left( N^{-1} \right)$. 

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**Introduction**

Theoretical model

Empirical framework

Data & Empirical Results

Conclusions
Remarks on the Common Growth Shock ($\zeta_t$): Remarks

Remark: Interpretation of $\zeta_t$

Because $\zeta_t$ is the same as world growth rescaled, we label it a common “growth” shock, in line with our theoretical derivations.
The common growth shock ($\zeta_t$): Remarks

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**Remark** Cross-sectional average of volatility

The cross-sectional average of the volatilities series $v_{it}$ does not identify $\zeta_t$, because we would not be able to disentangle $\zeta_t$ from $\xi_t$. 
The common growth shock ($\zeta_t$): Remarks

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The cross-sectional average of the volatilities series $v_{it}$ does not identify $\zeta_t$, because we would not be able to disentangle $\zeta_t$ from $\xi_t$.

**Remark** The role of $N$
The triangular factor structure alone does not identify $\zeta_t$. To get identification we need large $N$, otherwise we would not be able to disentangle $\zeta_t$ from $\bar{\varepsilon}_{\omega,t}$. 
The common financial shock ($\xi_t$)

- The main empirical result of the paper does not require identification of the financial factor $\xi_t$.

- But doing so permits exploring other properties of the data.
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- But doing so permits exploring other properties of the data.

- Conditional on $\zeta_t$, for $N$ large enough, $\xi_t$ is determined by:

\[
\xi_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta \gamma} \Delta \bar{y}_{\omega,t} + \frac{\bar{\eta}_{\omega,t}}{\theta} + O_p\left(\frac{N^{-1/2}}{\sqrt{\theta}}\right).
\]

- **Proof** Same logic as above applied to $\eta_{it}$. Factor $\xi_t$ can be identified from the data as a linear combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ (up to an orthonormal transformation).
The common financial shock ($\xi_t$): Remarks

**Remark** We label $\xi_t$ a global ‘financial’ shock to highlight its role in capturing all higher-order terms present in our theoretical derivations once we account for the common growth factor shock, $\zeta_t$, as well as any bubble component, financial friction, or time-varying risk preference component that might be present in the volatility data.
A dynamic multi-country heterogeneous model

Consider the following first-order dynamic version of the above static model:

\[
\Delta y_{it} = a_{iy} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \gamma_i \zeta_t + \varepsilon_{it}
\]

\[
v_{it} = a_{iv} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}
\]

Identification of the common factors in this dynamic and heterogeneous setting is sensibly more challenging.

Stronger assumptions on PVAR coefficients \((\phi_{i,rs})\) and higher order lags of \((\bar{v}_\omega,t, \Delta \bar{y}_\omega,t)\) are needed.

Skip derivations
Dynamic model: Assumptions

- **Factor loadings** The factor loadings $\lambda_i$, $\theta_i$, and $\gamma_i$ (i.e., the non-zero elements of $\Gamma_i$) are independently distributed across $i$, and of the common factors, $f_t$, for all $i$ and $t$, with non-zero means $\lambda$, $\theta$, and $\gamma$, and second-order moments. Furthermore:

$$\Gamma = \mathbb{E} (\Gamma_i) = \begin{pmatrix} \gamma & 0 \\ \lambda & \theta \end{pmatrix}$$

- **Coefficients** The constants $a_i$ are bounded, $\Phi_i$ and $\Gamma_i$ are independently distributed for all $i$, the support of $\varrho (\Phi_i)$ lies strictly inside the unit circle, for $i = 1, 2, ..., N$, and the inverse of the polynomial $\Lambda (L) = \sum_{\ell=0}^{\infty} \Lambda_\ell L^\ell$, where $\Lambda_\ell = \mathbb{E} (\Phi_i^\ell)$ exists and has exponentially decaying coefficients, namely $\|\Lambda_\ell\| \leq K \rho^\ell$, where $K$ is a fixed constant and $0 < \rho < 1$. 

Back
Proposition Common factors $\zeta_t$ and $\xi_t$ are given by:

$$
\begin{align*}
\zeta_t &= b_\zeta + \gamma^{-1} \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty} c_{1,\ell}' \bar{z}_{\omega,t-\ell} + O_p \left( N^{-1/2} \right), \\
\xi_t &= b_\xi + \theta^{-1} \left( \bar{v}_{\omega,t} - \frac{\lambda}{\gamma} \Delta \bar{y}_{\omega,t} \right) + \sum_{\ell=1}^{\infty} c_{2,\ell}' \bar{z}_{\omega,t-\ell} + O_p \left( N^{-1/2} \right),
\end{align*}
$$

where $z_{it} = (\Delta y_{it}, v_{it})'$.
Dynamic model: Consistent estimation of orthogonal factors

**Proposition** Consistent estimators of the common shocks, denoted by $\hat{\zeta}$ and $\hat{\xi}$, can be obtained as residuals from the following OLS regressions:

$$
\hat{\zeta} = \Delta \tilde{y}_\omega - \bar{Z}_\omega \hat{C}_1, \\
\hat{\xi} = \bar{v}_\omega - \hat{\lambda} \hat{\zeta} - \bar{Z}_\omega \hat{C}_2,
$$

where $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_T)'$, $\xi = (\xi_1, \xi_2, \ldots, \xi_T)'$, 
$\bar{Z}_\omega = (\tau_T, \bar{z}_\omega,-1, \bar{z}_\omega,-2, \ldots, \bar{z}_\omega,-p)$, $\bar{z}_\omega,-l = (\Delta \tilde{y}_\omega,-l \bar{v}_\omega,-l)$, 
$\Delta \tilde{y}_\omega,-l = (\Delta \tilde{y}_\omega,1-l, \Delta \tilde{y}_\omega,2-l, \ldots, \Delta \tilde{y}_\omega,T-l)'$, $\Delta \tilde{y}_\omega = \Delta \tilde{y}_\omega,0$, 
$\bar{v}_\omega,-l = (\bar{v}_\omega,1-l, \bar{v}_\omega,2-l, \ldots, \bar{v}_\omega,T-l)'$, $\bar{v}_\omega = \bar{v}_\omega,0$, and $p$ denotes a suitable number of lags (or truncation order).
Dynamic model: Factor-augmented large VAR

- Theoretical results carry through a fully heterogeneous dynamic version of the model.

- Country-specific model with orthonormal factors:

\[
\Delta y_{it} = \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \beta_{i,11} \hat{\xi}_t + \sum_{\ell=1}^{p} \psi'_{\Delta y,i\ell} \bar{z}_{\omega,t-\ell} + \varepsilon_{it}
\]

\[
v_{it} = \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \beta_{i,21} \hat{\xi}_t + \beta_{i,22} \hat{\xi}_t + \sum_{\ell=1}^{p} \psi'_{v,i\ell} \bar{z}_{\omega,t-\ell} + \eta_{it}
\]

where \(\bar{z}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta \bar{y}_{\omega,t})\).

- Country-specific models can be combined in a large model of the global economy.
Volatility measurement

- We compute the realized volatility for country $i$ in quarter $t$ as:

\[ \sigma_{it}^2 = \sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2 \]

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter $t$, and $D_t$ is the number of trading days in quarter $t$.

- We work with log of $\sigma_{it}$.

- In recent literature focus has shifted to implied volatility measures from option prices (e.g., VIX). But:
  - Not available for a large number of countries over a long period of time.
  - Berger, Dew-Becker, Giglio (2017): conditional on realized volatility, VIX is not associated with indicators of economic activity.
Data & Empirical Results
Data & Empirical results

Data


Empirical results

- Factors estimates.
- Within-country *conditional* correlation of volatility and GDP growth.
- IRFs and FEVDs to factors and country-specific shocks.
- Evidence in support of identifying assumptions (cross-country correlations).
The common growth shock ($\hat{\zeta}$)

- Common growth shock:
The common growth shock ($\hat{\zeta}$)

- **Common growth shock:**

> ![Common Growth Shock and Natural Rate]

- **Correlated with:**

  * Proxy for the world natural rate (0.5) from Holston, Laubach, Williams (2017).
The common growth shock ($\hat{\zeta}$)

- Common growth shock:

- Correlated with:
  - Proxy for the world natural rate (0.5) from Holston, Laubach, Williams (2017).
  - Proxy for global TFP (0.65) and global utilization adjusted TFP (0.33) from Huo, Levchenko, Pandalai-Nayar (2018).
  - Proxy for global long-run risk (0.29) from Colacito, Croce, Gavazzoni, Ready (2018).
The common financial shock ($\hat{\xi}$)

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- Common financial shock:

- Correlated with:
  - Financial uncertainty measure (0.43) from Ludvigson, Ma and Ng (2017).
The common financial shock ($\hat{\xi}$)

- Common financial shock:

- Correlated with:
  - Financial uncertainty measure (0.43) from Ludvigson, Ma and Ng (2017).
  - Excess Bond Premium (0.35) Gilchrist and Zakrajsek (2012).
Within-country correlation of volatility and growth: unconditional

- Unconditional correlation between $\Delta y_{it}$ and $v_{it}$ (same as Figure shown in introduction).
Within-country correlation of volatility and growth: conditioning on common growth shock only

- Correlation between $\varepsilon_{it}$ and $u_{it}$ conditional on growth common shock ($\hat{\zeta}_t$) only.

$$\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$

$$v_{it} = \beta_{i,11} \hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it}$$
Within-country correlation of volatility and growth: conditioning on both common shocks

- Correlation between $\varepsilon_{it}$ and $\eta_{it}$ conditional on growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) common shocks

\[
\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}
\]

\[
v_{it} = \beta_{i,11} \hat{\zeta}_t + \beta_{i,12} \hat{\xi}_t + \text{cross-section averages and lagged values} + \eta_{it}
\]
Growth factor \( \hat{\zeta}_t \), purple areas) and country-specific growth innovations (\( \varepsilon_{it} \), green areas) jointly explain less than \(< 5\%\) of volatility forecast error variance.
Financial factor ($\hat{\xi}_t$, dark blue areas) explains a significant share of growth forecast error variance (about $\sim 10\%$).
Financial factor ($\hat{\xi}_t$, dark blue areas) explains a significant share of growth forecast error variance (about $\sim 10\%$).

But country-specific volatility shocks ($\eta_{it}$, red areas) diversified away.
Growth common shocks ($\hat{\zeta}_t$) lead to endogenous movements in volatility

- Positive growth shock ($\hat{\zeta}_t$) leads to an increase in world GDP growth...
- ... and to a countercyclical fall in volatility.
Financial common shocks \( (\xi_t) \) lead to sharp recessions

- Adverse financial shock \( (\hat{\xi}_t) \) leads to an increase in world volatility...
- ... and to a sharp and persistent contraction of world GDP growth.
Country-specific volatility shocks ($\hat{\eta}_t$): Omitted variable bias

- Compare IRFs from a single-country VAR (with recursive identification) with those obtained from our multi-country model.

- Single-country model largely overestimates the impact of domestic shocks. growth.
Are the identifying assumptions on cross-sectional dependence consistent with the data?

- Estimate country models with growth shock \( \hat{\zeta}_t \) only:

\[
\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}
\]

\[
v_{it} = \beta_{i,11} \hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it}
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$$

$$
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$$

---

**NOTE.** Average pairwise correlation of the $u_{it}$ (yellow bars) and the $\varepsilon_{it}$ (blue bars).
Are the identifying assumptions on cross-sectional dependence consistent with the data?

- Estimate country models with growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) shocks:

$$
\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \epsilon_{it}
$$

$$
v_{it} = \beta_{i,11} \hat{\zeta}_t + \beta_{i,12} \hat{\xi}_t + \text{cross-section averages and lagged values} + \eta_{it}
$$

**NOTE.** Average pairwise correlation of the $\eta_{it}$ (yellow bars) and the $\epsilon_{it}$ (blue bars).
Tests of cross-sectional dependence don’t reject identifying assumptions

- CD and Exponent of cross-sectional dependence tests [Pesaran (2015), Bailey et al. (2016)].
- Results in accordance with assumptions of:
  - Weak/strong cross-sectional dependence of $\varepsilon_{it}/u_{it}$, respectively.
  - Weak cross-sectional dependence of both $\varepsilon_{it}$ and $\eta_{it}$.

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<th></th>
<th>$CD$</th>
<th>Lower 5%</th>
<th>$\hat{\alpha}$</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{it}$</td>
<td>104.57</td>
<td>0.94</td>
<td>0.99</td>
<td>1.05</td>
</tr>
<tr>
<td>$\Delta y_{it}$</td>
<td>55.73</td>
<td>0.87</td>
<td>1.00</td>
<td>1.14</td>
</tr>
<tr>
<td><strong>Innovations (conditional on $\hat{\zeta}_t$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{it}$</td>
<td>110.89</td>
<td>0.96</td>
<td>1.00</td>
<td>1.04</td>
</tr>
<tr>
<td>$\varepsilon_{it}$</td>
<td>-2.90</td>
<td>0.56</td>
<td>0.62</td>
<td>0.67</td>
</tr>
<tr>
<td><strong>Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_{it}$</td>
<td>-5.12</td>
<td>0.58</td>
<td>0.64</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Robustness

- Granularity assumption:
  - Results are robust when dropping (i) the US, (ii) China, and (iii) both the US and China.
  - Results are not robust when replacing the global factors with US GDP growth and US realized volatility.
Robustness

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- Sample period:
  * Results are robust to dropping the period of the global financial crisis.
  * Results are robust to a longer (unbalanced) sample period.
Robustness

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- **Realized versus implied volatility:**
  - Results are robust when using the VIX Index as a measure of volatility for the United States.
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▶ Realized versus implied volatility:
  * Results are robust when using the VIX Index as a measure of volatility for the United States.

▶ Alternative assumptions for identification of country-specific shocks:
  * Results are robust to block-diagonal covariance matrix with recursive (within-country) identification.
  * Unrestricted covariance matrix.
Paper takes a new common-factor approach in a multi-country model to the relation between volatility and growth without imposing restrictions on the direction of causation at the level of individual country.
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- Paper exploits the different cross-country correlation structure of volatility and growth innovations to identify a ‘growth’ factor and a ‘financial’ factor.
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* Common factors are important to understand relation between volatility and economic activity.
Paper takes a new common-factor approach in a multi-country model to the relation between volatility and growth without imposing restrictions on the direction of causation at the level of individual country.

- Paper exploits the different cross-country correlation structure of volatility and growth innovations to identify a ‘growth’ factor and a ‘financial’ factor.
- Common factors are important to understand relation between volatility and economic activity.

Key implication is that part of explanatory power typically attributed to country-specific uncertainty shocks is due to omitted common factors.
Uncertainty and Economic Activity: A Multi-Country Perspective

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\textsuperscript{2}University of Southern California and Trinity College, Cambridge
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2nd Annual CEBRA Intl. Finance and Macroeconomics Meeting
November 29-30, 2018

*The views expressed in this paper do not necessarily reflect the position of the Bank of England.
Appendix
A1::Model
A2::Assumptions
Assumptions

Assumption 1: Common factors and their loadings  The common unobservable factors, $\zeta_t$ and $\xi_t$, have zero means and unit variances, and are serially uncorrelated. The factor loadings, $\lambda_i$, $\gamma_i$, and $\theta_i$, are distributed independently across $i$ and from the common factors $f_t$ and $g_t$ for all $i$ and $t$, with non-zero means $\lambda$, $\gamma$, and $\theta$ ($\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0$), and satisfy the following conditions, for a finite $N$ and as $N \to \infty$:

\[
N^{-1} \sum_{i=1}^{N} \lambda_i^2 = O(1) \quad \lambda = \sum_{i=1}^{N} w_i \lambda_i \neq 0
\]

\[
N^{-1} \sum_{i=1}^{N} \gamma_i^2 = O(1) \quad \gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0
\]

\[
N^{-1} \sum_{i=1}^{N} \theta_i^2 = O(1) \quad \theta = \sum_{i=1}^{N} w_i \theta_i \neq 0
\]
Assumptions (Cont.)

▶ Assumption 2: Aggregation weights  Let $\mathbf{w} = (w_1, w_2, \ldots, w_N)'$ and $\mathbf{\hat{w}} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_N)'$ be the $N \times 1$ vectors of non-stochastic weights with $w_i, \hat{w}_i > 0$, $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} \hat{w}_i = 1$, such that the following “granularity” conditions are met:

$$
||\mathbf{w}|| = O(N^{-1}), \quad \frac{w_i}{||\mathbf{w}||} = O(N^{-1/2})
$$

and

$$
||\mathbf{\hat{w}}|| = O(N^{-1}), \quad \frac{\hat{w}_i}{||\mathbf{\hat{w}}||} = O(N^{-1/2})
$$

for all $i$. 
Assumptions (Cont.)

- **Assumption 3: Cross-section correlations** The country-specific innovations, $\eta_{it}$ and $\varepsilon_{it}$, have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. Furthermore, denoting the covariance matrices of the $N \times 1$ innovation vectors $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt})'$ and $\eta_t = (\eta_{1t}, \eta_{2t}, \ldots, \eta_{Nt})'$ by $\Sigma_{\varepsilon\varepsilon} = \text{Var} (\varepsilon_t)$ and $\Sigma_{\eta\eta} = \text{Var} (\eta_t)$, respectively, it is assumed that:

\[ \rho_{\text{max}} (\Sigma_{\varepsilon\varepsilon}) = O(1) \]
\[ \rho_{\text{max}} (\Sigma_{\eta\eta}) = O(1) \]
A3::Econometric model
Estimating observable and orthogonal factors

- **Issue** Factors $f_t$ and $g_t$ are unobservable, and even if known, would be correlated with each other.

- For ease of interpretation it is standard to work with the orthogonalized version of the factors:
  - This task is simplified due to the triangular way the factors affect the global variables, $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$.

- Proceed recursively:
  - Factor $f_t$ can be identified up to a constant
    \[
    f_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} \Rightarrow \hat{\zeta}_t = \Delta \bar{y}_{\omega,t}
    \]

  - Factor $g_t$ can then be approximated by the residuals of a regression of world volatility $\bar{v}_{\omega,t}$ on world growth
    \[
    g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta \gamma} \Delta \bar{y}_{\omega,t} \Rightarrow \bar{v}_{\omega,t} = \hat{\beta} \Delta \bar{y}_{\omega,t} + \xi_t
    \]
Consistent estimation of orthogonal factors

**Proposition 3** Let $\hat{\zeta}_t$ and $\hat{\xi}_t$ be consistent, orthonormalized estimators of $f_t$ and $g_t$, respectively, where $f_t$ and $g_t$ are defined by (??) and (??). Then, $\hat{\zeta}_t$ can be obtained by re-scaling $\Delta \bar{y}_{\omega,t}$ so that its variance is 1, while $\hat{\xi}_t$ can be obtained as the standardized residual of a least squares regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$.

**Proof** Consider equation (??) and (??) and set the coefficients $\alpha_g = (\alpha_{1g}, \alpha_{2g})'$, such that $T^{-1} \sum_{t=1}^{T} \hat{\zeta}_t \hat{\xi}_t = 0$. This yields:

$$\frac{\hat{\alpha}_{2g}}{\hat{\alpha}_{1g}} = \frac{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t} \bar{v}_{\omega,t}}{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t}^2},$$

which is the OLS estimate of the coefficient on $\Delta \bar{y}_{\omega,t}$ in a regression of $\bar{v}_t$ on $\Delta \bar{y}_{\omega,t}$. Next, set $\alpha_f$ and $\alpha_{1g}$ so that $\zeta_t$ and $\xi_t$ have unit in-sample standard deviations.
A3::Additional results
Granularity: No US

Correlation

Argentina  Australia  Austria  Belgium  Brazil  Canada  Chile  China  Finland  France  Germany  India  Indonesia  Italy  Japan  Korea  Malaysia  Mexico  Netherlands  New Zealand  Norway  Philippines  Singapore  South Africa  Spain  Sweden  Switzerland  Thailand  Turkey  United Kingdom

-0.2  0  0.2  0.4  0.6

Back
Granularity: No China
Granularity: No US & No China

A1::Model
A2::Assumptions
A3::Econometric model
A3::Additional results
Granularity: US as the global factor

![Correlation Chart]

- Argentina
- Australia
- Austria
- Belgium
- Brazil
- Canada
- Chile
- China
- Colombia
- Denmark
- Estonia
- Finland
- France
- Germany
- India
- Indonesia
- Ireland
- Italy
- Japan
- Korea
- Latvia
- Lithuania
- Luxembourg
- Malaysia
- Mexico
- Netherlands
- New Zealand
- Norway
- Philippines
- Poland
- Portugal
- Singapore
- South Africa
- Spain
- Sweden
- Switzerland
- Thailand
- Turkey
- United Kingdom

Correlation values range from -0.2 to 0.6.
Comparison between VIX and US realized volatility: Data

Realized Volatility vs VIX

Correlation: 0.93

RV
VIX
Comparison between VIX and US realized volatility: IRF

VOL response

GDP response

RV
VIX

Back
Excluding the global financial crisis: FEVDs

Volatility ($v_{it}$), average

Real GDP Growth ($\Delta y_{it}$), average

A1::Model  A2::Assumptions  A3::Econometric model  A3::Additional results
Excluding the global financial crisis: IRFs

(A) $v_{it}$ to a $\zeta_t$ shock

(B) $\Delta y_{it}$ to a $\zeta_t$ shock

(C) $v_{it}$ to a $\xi_t$ shock

(D) $\Delta y_{it}$ to a $\xi_t$ shock
Longer sample period: Common Shocks

Panel A: Common real shock ($\hat{\zeta}_t$)

Panel B: Common financial shock ($\hat{\xi}_t$)
Average FEVD: Block-diagonal covariance matrix

- Allow for within-country correlation of the innovations ($\varepsilon_{it}$ and $\eta_{it}$).
- Orthogonalize with Cholesky decomposition, with volatility ordered first (block-diagonal covariance matrix)
Allow for within- and across-country correlation of the innovations ($\varepsilon_{it}$ and $\eta_{it}$).

Compute Generalized FEVD with threshold covariance matrix.
Country-specific response to the factors

(A) Volatility response to $\zeta_t$

(B) GDP growth response to $\zeta_t$

(C) Volatility response to $\xi_t$

(D) GDP growth response to $\xi_t$