Mirrlees meets Diamond-Mirrlees

Simplifying Nonlinear Income Taxation

Florian Scheuer
Iván Werning

Motivation

- Two workhorse models of Public Finance
  - Linear commodity taxation
    Diamond-Mirrlees (1971)
  - Nonlinear income taxation
    Mirrlees (1971)
- Widely used formulas

  Optimum ⟷ Empirical counterparts

- Very different approaches

Motivation

- Two workhorse models of Public Finance
  - Linear commodity taxation
    Diamond-Mirrlees (1971)
  - Nonlinear income taxation
    Mirrlees (1971)
- Widely used formulas

  Optimum ⟷ Empirical counterparts

- Very different approaches

→ Connection?

This Paper

- Show that Mirrlees is an application of DM
  - DM: only linear taxation, but…
  - … can treat each income level as different good

This Paper

- Show that Mirrlees is an application of DM
  - DM: only linear taxation, but…
  - … can treat each income level as different good
- Derive Mirrlees formula from DM formula
  - Simpler than standard approach, once…
  - … extension to continuum of goods

This Paper

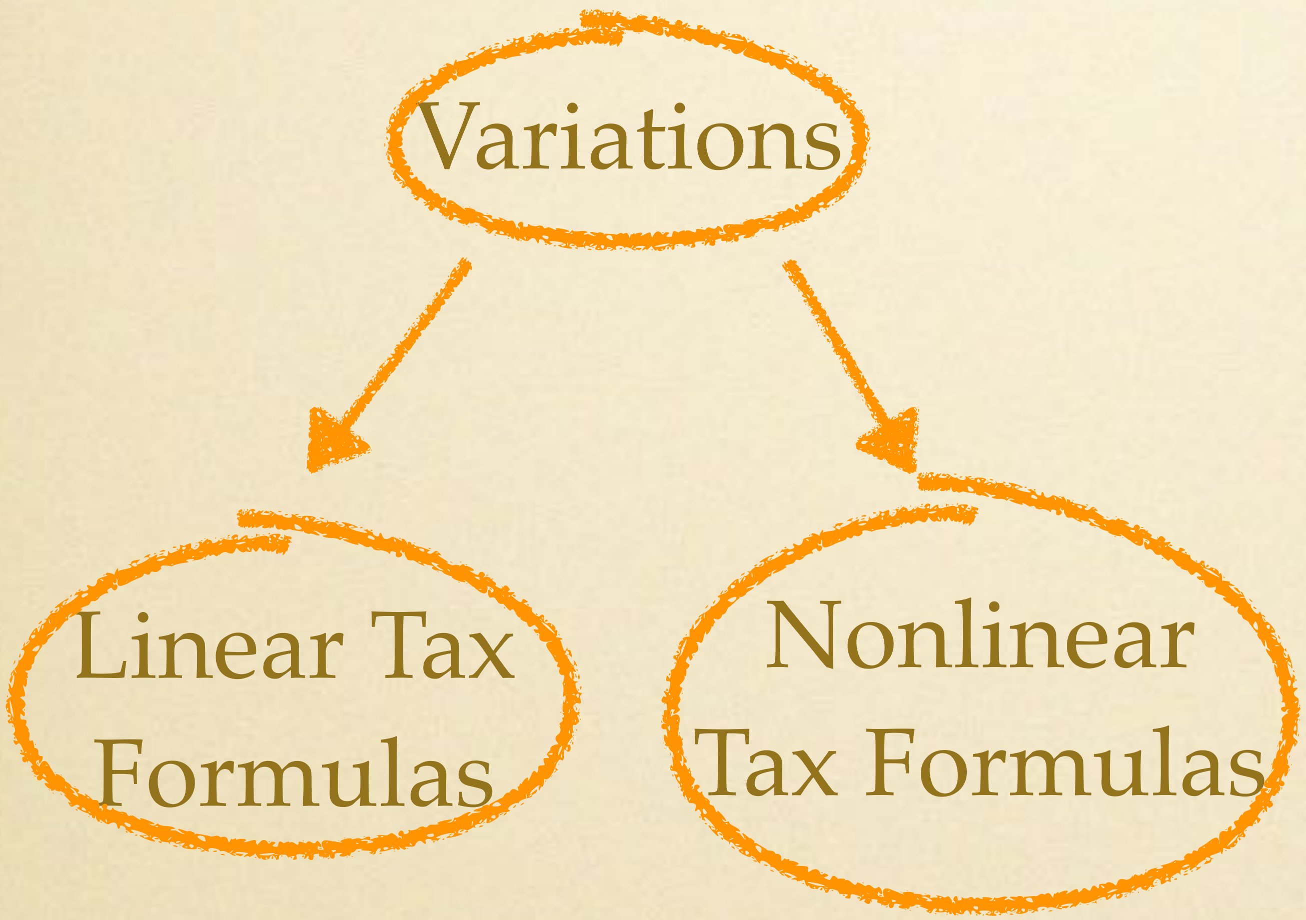
- Show that Mirrlees is an application of DM
  - DM: only linear taxation, but…
  - … can treat each income level as different good

- Derive Mirrlees formula from DM formula
  - Simpler than standard approach, once…
  - … extension to continuum of goods

- Benefits
  - Greater generality Technology, Heterogeneity
  - Common economic interpretation "Inverse Elasticity Rule"
  - Can easily tackle richer problems Dynamics, Behavioral Margins

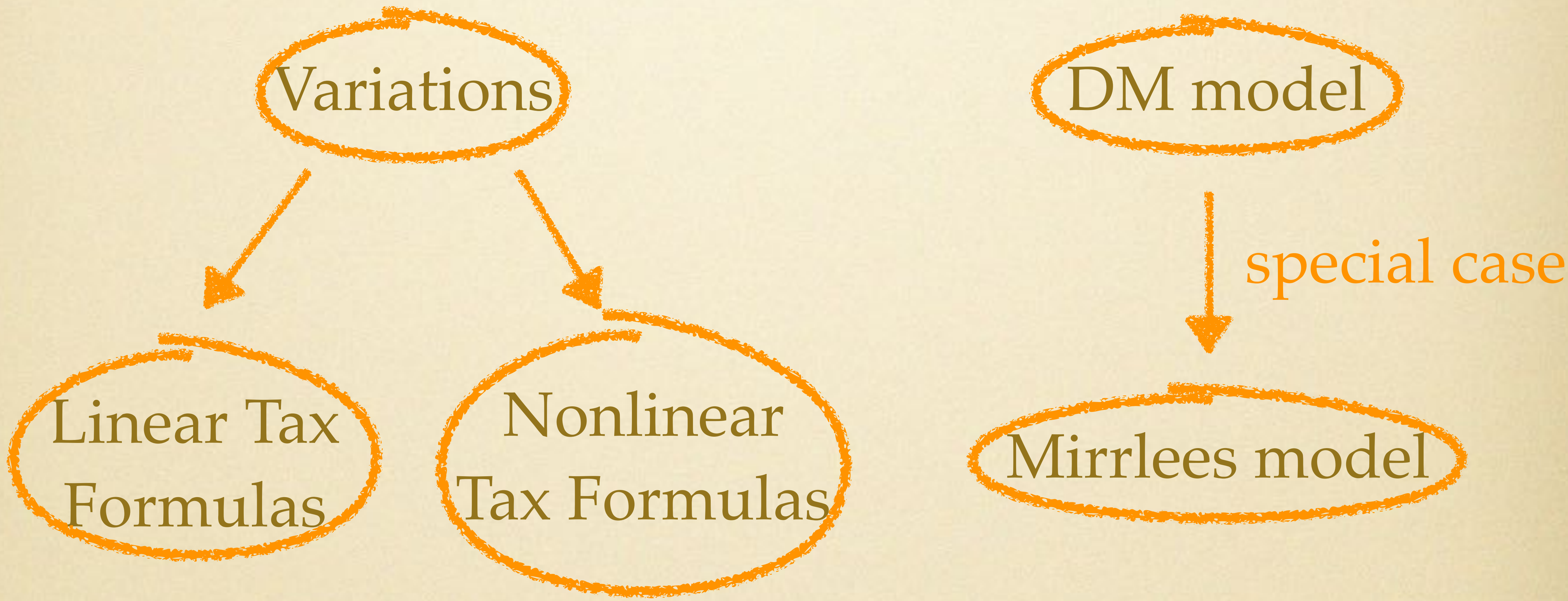
Related Literature

- Variational approach
  Piketty (1997), Saez (2001), Golosov et al. (2015)



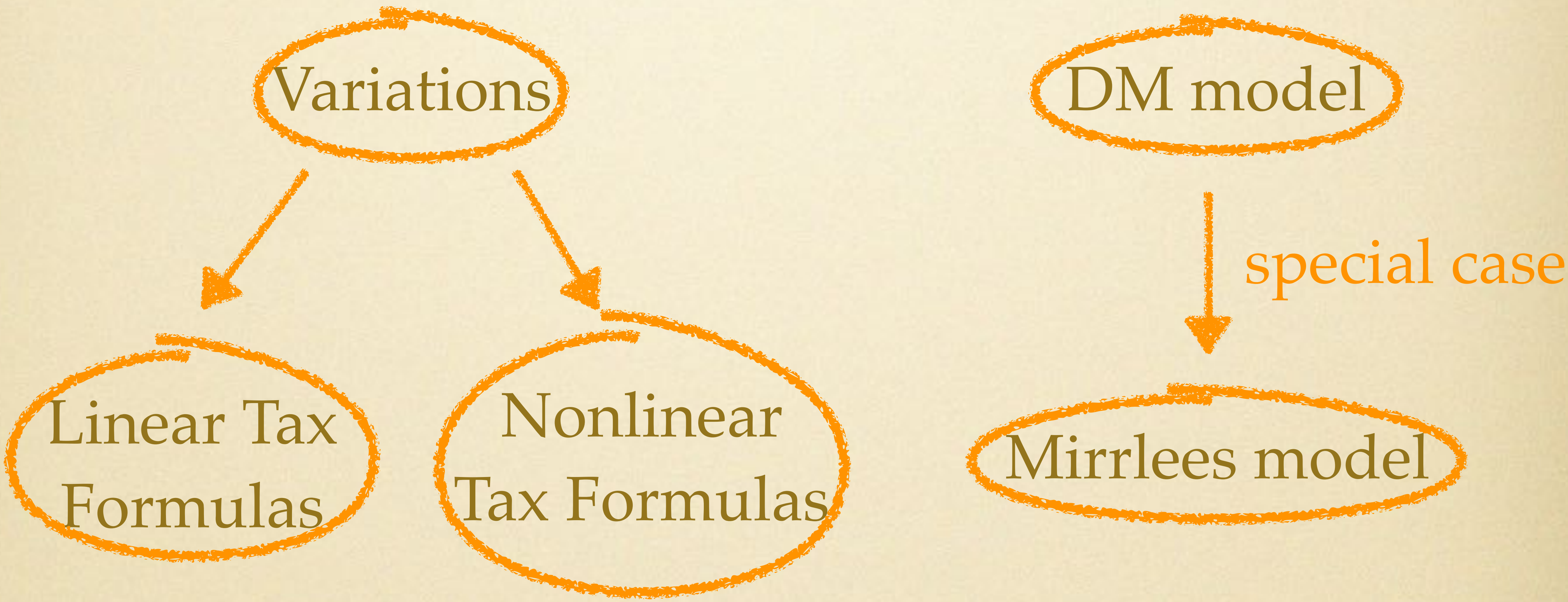
Related Literature

- Variational approach
  Piketty (1997), Saez (2001), Golosov et al. (2015)



Related Literature

- Variational approach
  Piketty (1997), Saez (2001), Golosov et al. (2015)



- Mirrleesian taxation with GE effects
  Rothschild-Scheuer (2013, 2016), Ales et al. (2015), Scheuer-Werning (2017), Sachs et al. (2017), Costinot-Werning (2018)

Outline

1. DM and Mirrlees models
2. Mirrlees as special case of DM
3. Optimal tax formulas
4. Mirrlees formula derived from DM formula
5. Applications and extensions

DM and Mirrlees Models

DM Model

- Agents $h \in H$
  - utility $u^h(x^h)$
  - net demands $x \in X$
- Technology $G(\overline{x}) \leq 0$
  - aggregate net demands $\overline{x}$
- Budget constraints $B(x^h, q) = I$
  - lump-sum tax I
  - consumer prices q
- Social welfare function $W\left(\{u^h\}\right)$

DM Model

- Agents $h \in H$
  - utility $u^h(x^h)$
  - net demands $x \in X$

- Technology $G(\overline{x}) \leq 0$
  - aggregate net demands $\overline{x}$

- Budget constraints $B(x^h, q) = I$
  - lump-sum tax I
  - consumer prices q

DM: $I = 0$

- Social welfare function $W\left(\{u^h\}\right)$

DM Model

- Policy problem

$$\max_{q,\{x^h\}} W\left(\{u^h(x^h)\}\right)$$

s.t. $$G(\bar{x}) \leq 0$$

and $$x^h \in \arg\max_x u^h(x) \text{ s.t. } B(x,q) = 0$$

DM Model

- Policy problem

$$\max_{q,\{x^h\}} W\left(\{u^h(x^h)\}\right)$$

s.t. $$G(\overline{x}) \leq 0$$

and $$x^h \in \arg\max_x u^h(x) \text{ s.t. } B(x,q) = 0$$

- If G is convex and has CRS, can decentralize
  - firms max profits at prices $p \neq q$
  - taxes $t = q - p$

DM Model

- Finite agents and goods
  - finite population
  - aggregate demands
  - finite set of goods
  - individual demands
  - budget constraints

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands
  - finite set of goods
  - individual demands
  - budget constraints

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands $\bar{x} = \sum_{h=1}^{M} x^h$
  - finite set of goods
  - individual demands
  - budget constraints

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands $\bar{x} = \sum_{h=1}^{M} x^h$
  - finite set of goods $i \in \{1, 2, ..., N\}$
  - individual demands
  - budget constraints

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands $\bar{x} = \sum_{h=1}^{M} x^h$
  - finite set of goods $i \in \{1, 2, ..., N\}$
  - individual demands $x^h = (x_1^h, x_2^h, ..., x_N^h)$
  - budget constraints

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands $\bar{x} = \sum_{h=1}^{M} x^h$
  - finite set of goods $i \in \{1, 2, ..., N\}$
  - individual demands $x^h = (x_1^h, x_2^h, ..., x_N^h)$
  - budget constraints $q \cdot x^h = \sum_{i=1}^{N} q_i x_i^h = 0$

DM Model

- Finite agents and goods
  - finite population $H = \{1, 2, ..., M\}$
  - aggregate demands $\bar{x} = \sum_{h=1}^{M} x^h$
  - finite set of goods $i \in \{1, 2, ..., N\}$
  - individual demands $x^h = (x_1^h, x_2^h, ..., x_N^h)$
  - budget constraints $q \cdot x^h = \sum_{i=1}^{N} q_i x_i^h = 0$
- Continuum of agents and goods
  - measure of agents μ_h over set $\mathcal{H}$
  - agents consume signed measure χ^h over X
  - linear budget constraints $B(\chi^h, q) = 0$

Mirrlees Model

- Agents
  - skills $\theta \sim F(\theta)$
  - utility $U(c, y; \theta)$ e.g. $u(c, y/\theta)$
- Technology

$$\int c(\theta) dF(\theta) \leq \int y(\theta) dF(\theta)$$

- Budget constraints

$$c(\theta) \leq y(\theta) - T(y(\theta)) \equiv R(y(\theta))$$

- Social welfare function $W\Big(\{U(c(\theta), y(\theta); \theta)\}\Big)$

Mirrlees Model

- Policy problem

$$\max_{\{c(\theta),y(\theta)\},R(.)} W\Big(\{U(c(\theta),y(\theta);\theta)\}\Big)$$

s.t. $$\int c(\theta)dF(\theta) \leq \int y(\theta)dF(\theta)$$

and $c(\theta), y(\theta) \in \arg\max_{c,y} U(c,y;\theta)$ s.t. $c \leq R(y)$

Mirrlees Model

- Policy problem

$$\max_{\{c(\theta),y(\theta)\},R(.)} W\Big(\{U(c(\theta),y(\theta);\theta)\}\Big)$$

s.t. $$\int c(\theta)dF(\theta) \leq \int y(\theta)dF(\theta)$$

and $c(\theta), y(\theta) \in \arg\max_{c,y} U(c,y;\theta)$ s.t. $c \leq R(y)$

- Solve using mechanism design
  - incentive compatible allocations $c(\theta), y(\theta)$
  - optimal control problem

Mirrlees as Special Case of DM

Mirrlees as Special Case of DM

- Commodity space
  - single consumption good
  - continuum of labor varieties $y \geq 0$
  - agent θ chooses $c \geq 0$ and measure $H_\theta(y)$

Mirrlees as Special Case of DM

- Commodity space
  - single consumption good
  - continuum of labor varieties $y \geq 0$
  - agent θ chooses $c \geq 0$ and measure $H_\theta(y)$
- Budget constraint $c \leq \int_0^\infty q(y) dH_\theta(y)$

Mirrlees as Special Case of DM

- Commodity space
  - single consumption good
  - continuum of labor varieties $y \geq 0$
  - agent θ chooses $c \geq 0$ and measure $H_\theta(y)$
- Budget constraint $c \leq \int_0^\infty q(y) dH_\theta(y)$
- Taxes
  - $q(y) - p(y)$ on variety y (zero tax on c)
  - $p(y) = y$ with linear technology

Mirrlees as Special Case of DM

- Commodity space
  - single consumption good
  - continuum of labor varieties $y \geq 0$
  - agent θ chooses $c \geq 0$ and measure $H_\theta(y)$
- Budget constraint $c \leq \int_0^\infty q(y) dH_\theta(y)$
- Taxes
  - $q(y) - p(y)$ on variety y (zero tax on c)
  - $p(y) = y$ with linear technology
- Preferences
  - must put full mass on one value of y
  - $H_\theta(y)$ is a step function $\rightarrow$ $c \leq q(y)$

Issues

- Natural quantities are densities
  - budget constraint

$$c \leq \int_0^\infty q(y) h_\theta(y) dy$$

  - but $H_\theta(y)$ does not admit density

Issues

- Natural quantities are densities
  - budget constraint

$$c \leq \int_0^\infty q(y) h_\theta(y) dy$$

  - but $H_\theta(y)$ does not admit density

- Discontinuous effects on demand
  - suppose $q(y) = p(y) = y$
    → density over y in aggregate
  - raise $q(y_0)$ at a point y_0
    → mass of agents shift to y_0
  - reduce $q(y_0)$ at y_0
    → density drops discontinuously to 0

A Reformulation

- Integrate budget constraint by parts

$$c \leq \int_0^\infty q'(y)(1 - H_\theta(y))dy + I$$

  - price $q'(y)$
  - quantity $1 - H_\theta(y)$

A Reformulation

- Integrate budget constraint by parts

$$c \leq \int_0^\infty q'(y)(1 - H_\theta(y))dy + I$$

→ price $q'(y)$

→ quantity $1 - H_\theta(y)$

- Overcomes previous problems
  - quantities always well-defined

$$1 - H_\theta(y) = \mathbb{I}(y \leq y(\theta))$$

  - Aggregate demand no longer discontinuous w.r.t. small changes in $q'(y)$

$$1 - H(y) = \int_0^\infty (1 - H_\theta(y))dF(\theta)$$

Tax Formulas

DM Formula

Diamond (1975) "Many-Person Ramsey Tax Rule"

$$\frac{\partial}{\partial \tau}\left(\sum_{h=1}^{M} x_i^{c,h}(q+\tau t)\right)\Bigg|_{\tau=0} = \sum_{h=1}^{M} \hat{\beta}^h x_i^h$$

with

$$\hat{\beta}^h = \beta^h - 1 + \frac{\partial}{\partial I}\left(\sum_{j=1}^{N} t_j x_j^h(q, I)\right)$$

DM Formula

Diamond (1975) "Many-Person Ramsey Tax Rule"

$$\frac{\partial}{\partial \tau}\left(\sum_{h=1}^{M} x_i^{c,h}(q+\tau t)\right)\Bigg|_{\tau=0} = \sum_{h=1}^{M} \hat{\beta}^h x_i^h$$

with

$$\hat{\beta}^h = \beta^h - 1 + \frac{\partial}{\partial I}\left(\sum_{j=1}^{N} t_j x_j^h(q, I)\right)$$

→ Encourage goods consumed by those with high $\hat{\beta}^h$

DM Formula

- In terms of elasticities

$$\sum_{j=1}^{N} \frac{t_j}{q_j} \varepsilon_{ij}^{c} = \frac{\sum_h \hat{\beta}^h x_i^h}{X_i}$$

DM Formula

- In terms of elasticities

$$\sum_{j=1}^{N} \frac{t_j}{q_j} \varepsilon_{ij}^c = \frac{\sum_h \hat{\beta}^h x_i^h}{X_i}$$

- Special case: no cross-price effects

$$\frac{t_i}{q_i} = \frac{1}{\varepsilon_{ii}^c} \frac{\sum_h \hat{\beta}^h x_i^h}{X_i}$$

→ "Inverse elasticity rule"

Mirrlees Formula

$$\frac{T'(y)}{1-T'(y)}\varepsilon^c(y)yh(y) = \int_y^\infty (1-\beta_{\tilde{y}})dH(\tilde{y}) + \int_y^\infty \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\eta(\tilde{y})dH(\tilde{y})$$

with elasticities from

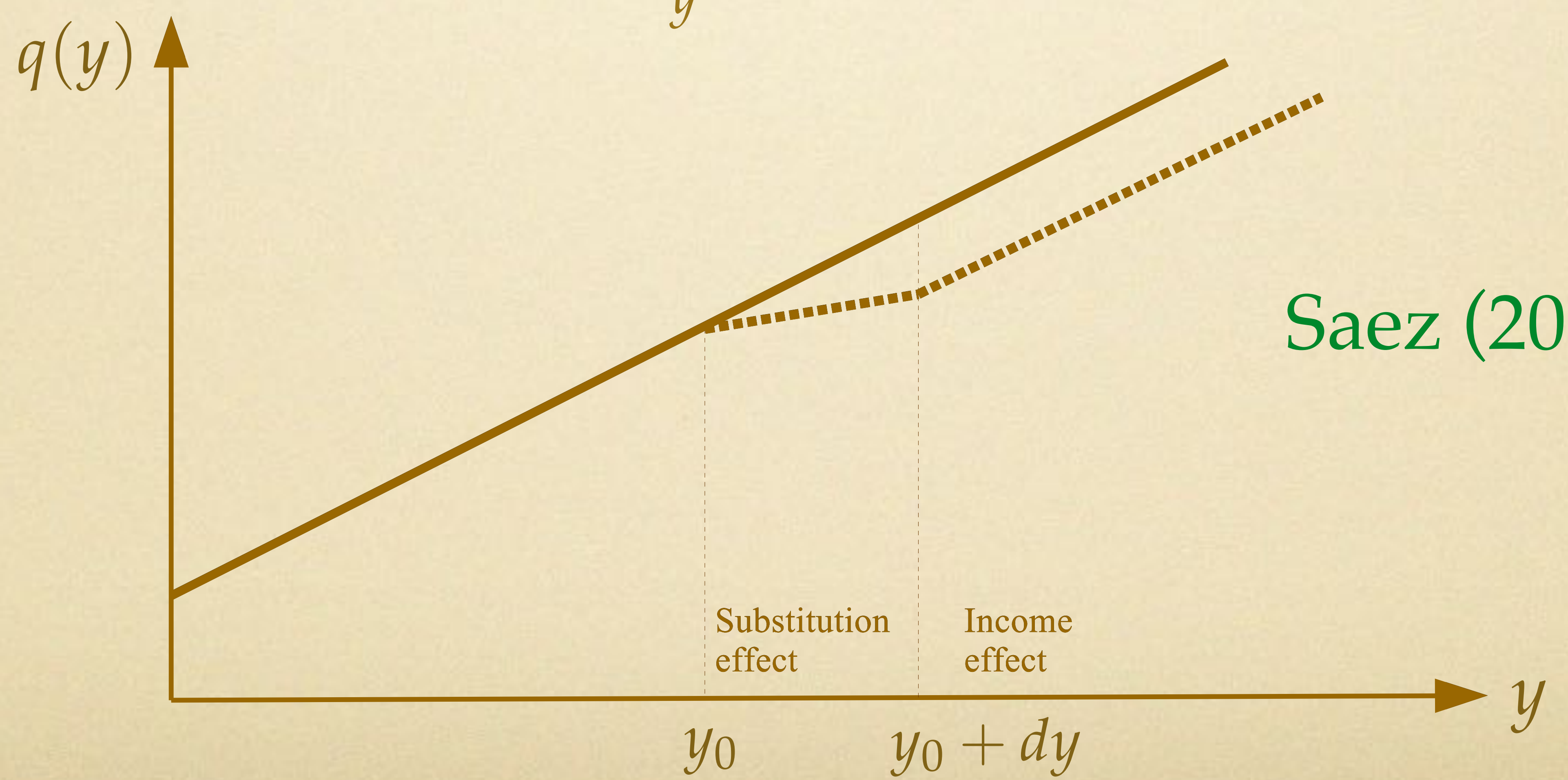
$$y(\tau, I) \in \arg\max_y U(q(y) - \tau y + I, y; \theta)$$

Mirrlees Formula

$$\frac{T'(y)}{1-T'(y)}\varepsilon^c(y)yh(y) = \int_y^\infty (1-\beta_{\tilde{y}})dH(\tilde{y}) + \int_y^\infty \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\eta(\tilde{y})dH(\tilde{y})$$

with elasticities from

$$y(\tau, I) \in \arg\max_y U(q(y) - \tau y + I, y; \theta)$$



Saez (2001)

From DM to Mirrlees

LHS of DM

Recall $$\frac{\partial}{\partial \tau}\left(\sum_h x_i^{c,h}(q+\tau t)\right)\Bigg|_{\tau=0}$$

LHS of DM

Recall $$\frac{\partial}{\partial \tau}\left(\sum_h x_i^{c,h}(q+\tau t)\right)\Bigg|_{\tau=0}$$

- corresponds to $q'(y) \quad \to \quad q'(y;\tau) = q'(y) + \tau t'(y)$

LHS of DM

Recall $$\frac{\partial}{\partial \tau}\left(\sum_h x_i^{c,h}(q+\tau t)\right)\Big|_{\tau=0}$$

- corresponds to $q'(y) \quad \to \quad q'(y;\tau) = q'(y) + \tau t'(y)$
- $\frac{\partial}{\partial \tau}\left(1 - H^c(y;\tau)\right)\Big|_{\tau=0}$

LHS of DM

Recall $$\frac{\partial}{\partial \tau}\left(\sum_h x_i^{c,h}(q+\tau t)\right)\Bigg|_{\tau=0}$$

→ corresponds to $q'(y) \quad \rightarrow \quad q'(y;\tau) = q'(y) + \tau t'(y)$

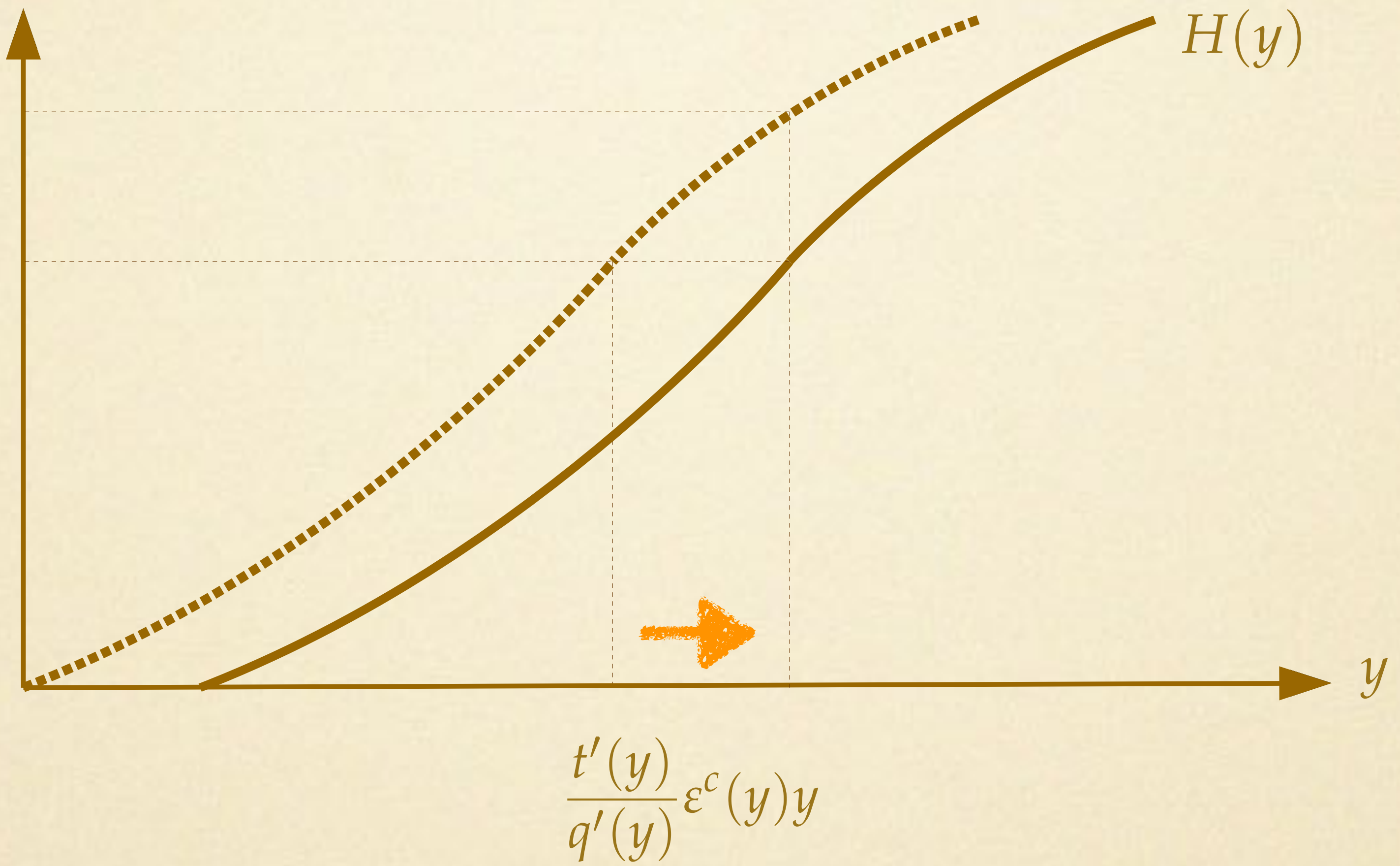
→ $$\frac{\partial}{\partial \tau}\left(1 - H^c(y;\tau)\right)\Bigg|_{\tau=0}$$

Agents raise y in response to small τ by

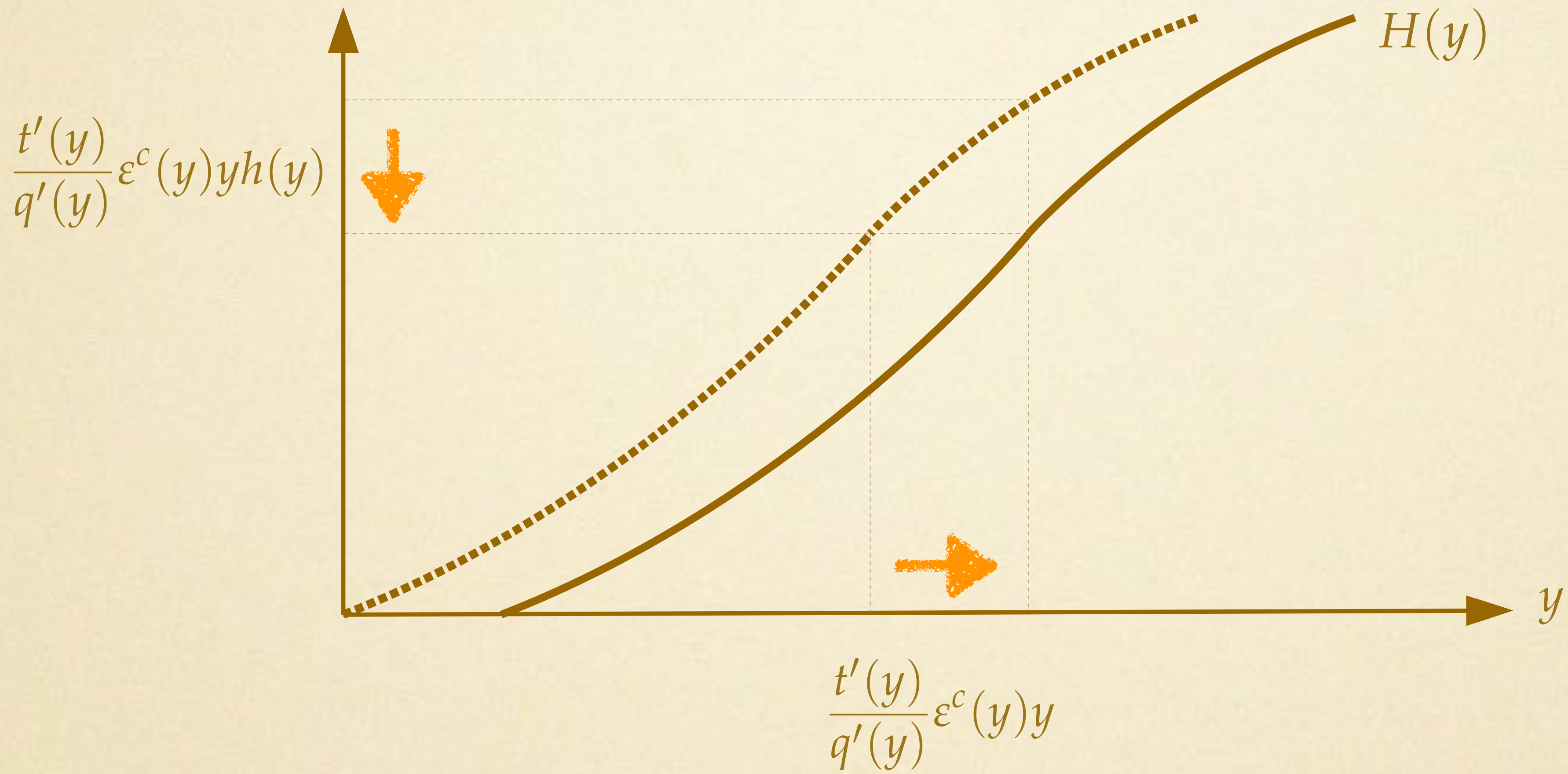
$$t'(y)\frac{\varepsilon^c(y)y}{q'(y)}$$

→ rightward shift in H(y)

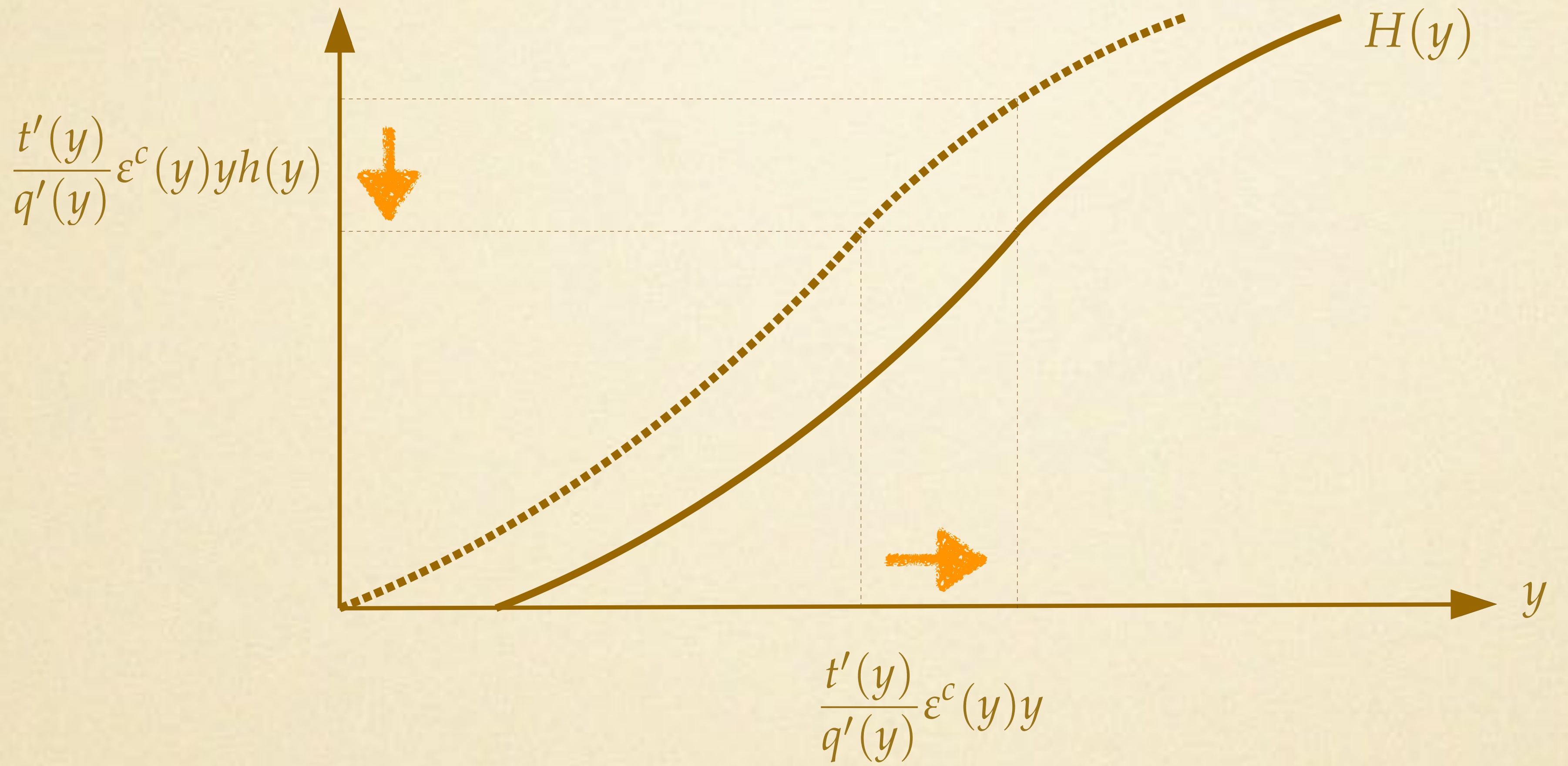
LHS of DM



LHS of DM



LHS of DM



$$\frac{\partial}{\partial\tau}\left(1 - H^c(y;\tau)\right)\Big|_{\tau=0} = \frac{t'(y)}{q'(y)}\varepsilon^c(y)yh(y)$$

Inverse elasticity rule!

RHS of DM

Recall $$\sum_h x_i^h \left(\beta_h - 1 + \frac{\partial}{\partial I} \left(\sum_j t_j x_j^h(q, I) \right) \right)$$

RHS of DM

Recall $$\sum_h x_i^h \left(\beta_h - 1 + \frac{\partial}{\partial I} \left(\sum_j t_j x_j^h(q, I) \right) \right)$$

- corresponds to

$$\int_0^\infty (1 - H_\theta(y)) \left(\beta_\theta - 1 - \frac{\partial}{\partial I} \int_0^\infty t'(z)(1 - H_\theta(z; I))dz \right) dF(\theta)$$

RHS of DM

Recall $$\sum_h x_i^h \left(\beta_h - 1 + \frac{\partial}{\partial I} \left(\sum_j t_j x_j^h(q, I) \right) \right)$$

- corresponds to

$$\int_0^\infty (1 - H_\theta(y)) \left(\beta_\theta - 1 - \frac{\partial}{\partial I} \int_0^\infty t'(z)(1 - H_\theta(z; I))dz \right) dF(\theta)$$

- use step-function and change variables

$$\int_y^\infty (\beta_{\tilde{y}} - 1) dH(\tilde{y}) + \int_y^\infty \frac{t'(\tilde{y})}{q'(\tilde{y})} \eta(\tilde{y}) dH(\tilde{y})$$

RHS of DM

Recall $$\sum_h x_i^h \left(\beta_h - 1 + \frac{\partial}{\partial I} \left(\sum_j t_j x_j^h(q, I) \right) \right)$$

- corresponds to

$$\int_0^\infty (1 - H_\theta(y)) \left(\beta_\theta - 1 - \frac{\partial}{\partial I} \int_0^\infty t'(z)(1 - H_\theta(z; I))dz \right) dF(\theta)$$

- use step-function and change variables

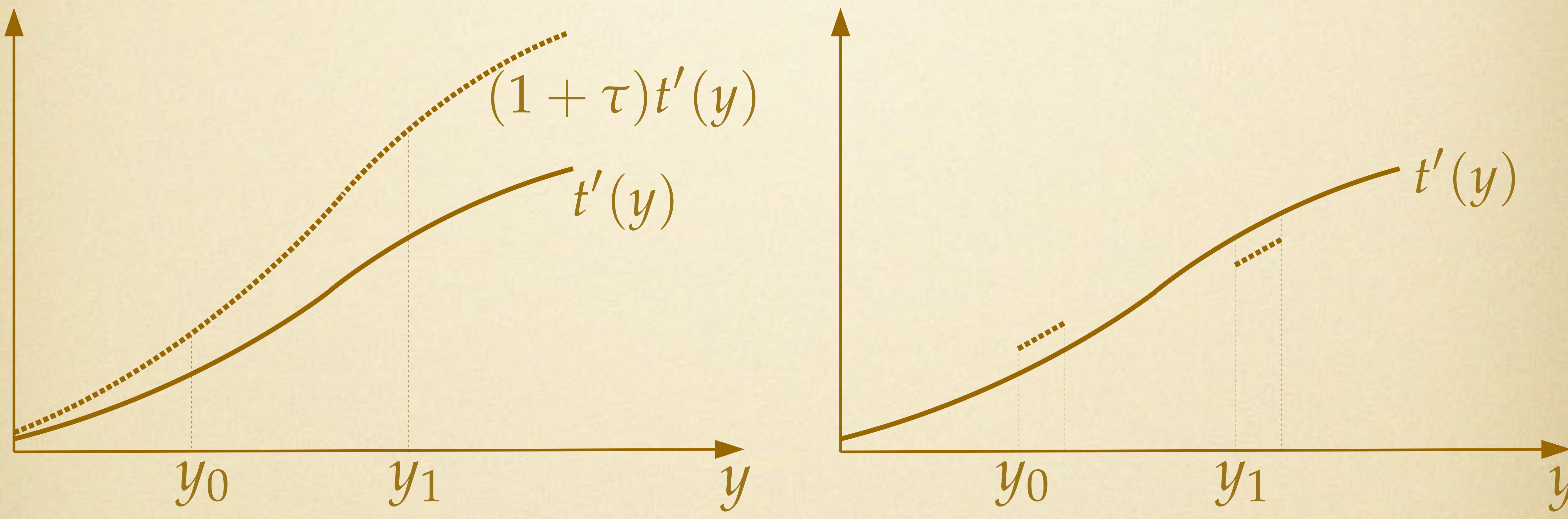
$$\int_y^\infty (\beta_{\tilde{y}} - 1) dH(\tilde{y}) + \int_y^\infty \frac{t'(\tilde{y})}{q'(\tilde{y})} \eta(\tilde{y}) dH(\tilde{y})$$

Combining with LHS immediately gives Mirrlees formula!

Variational Approach

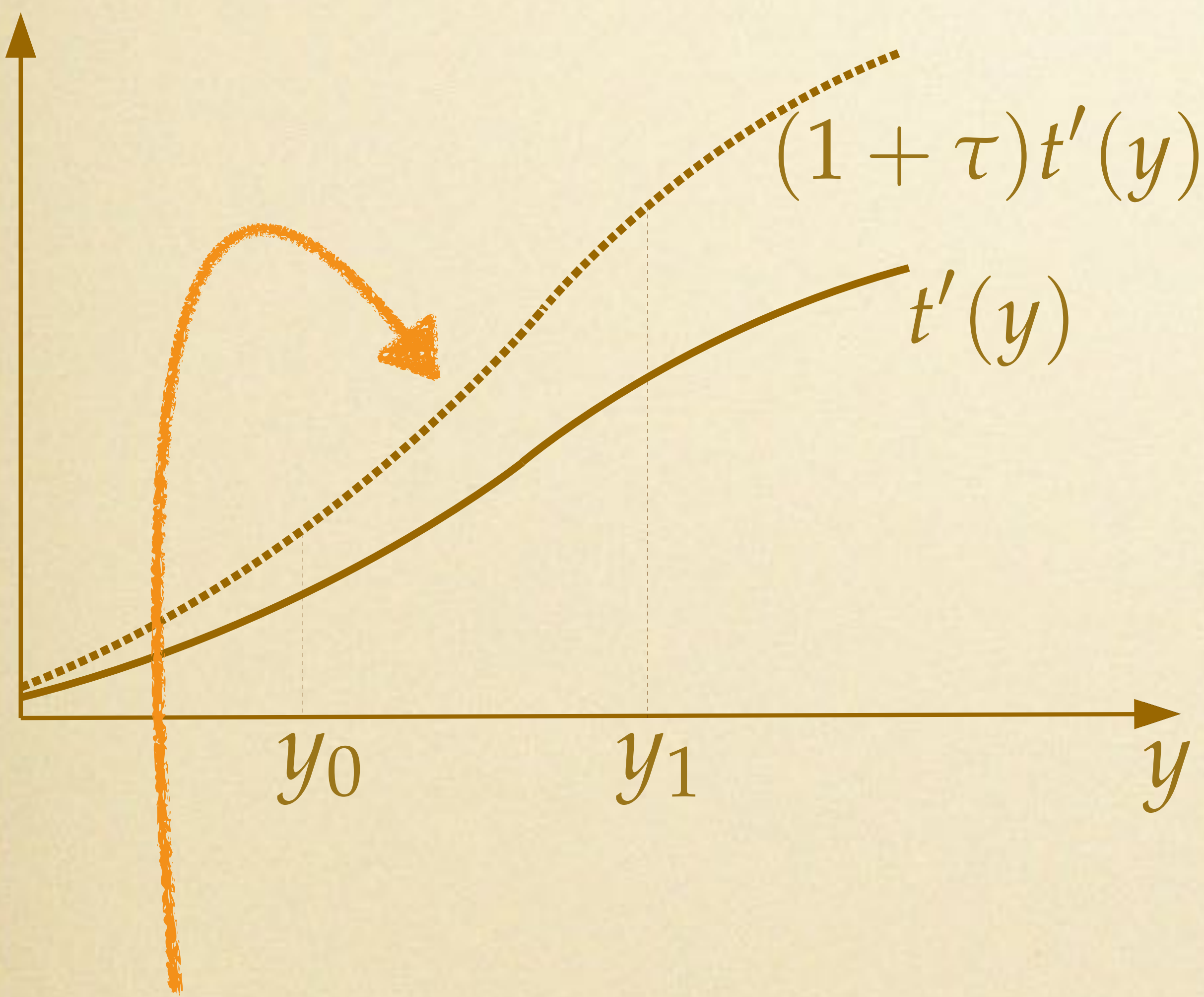
Diamond-Mirrlees (1971)

Saez (2001)



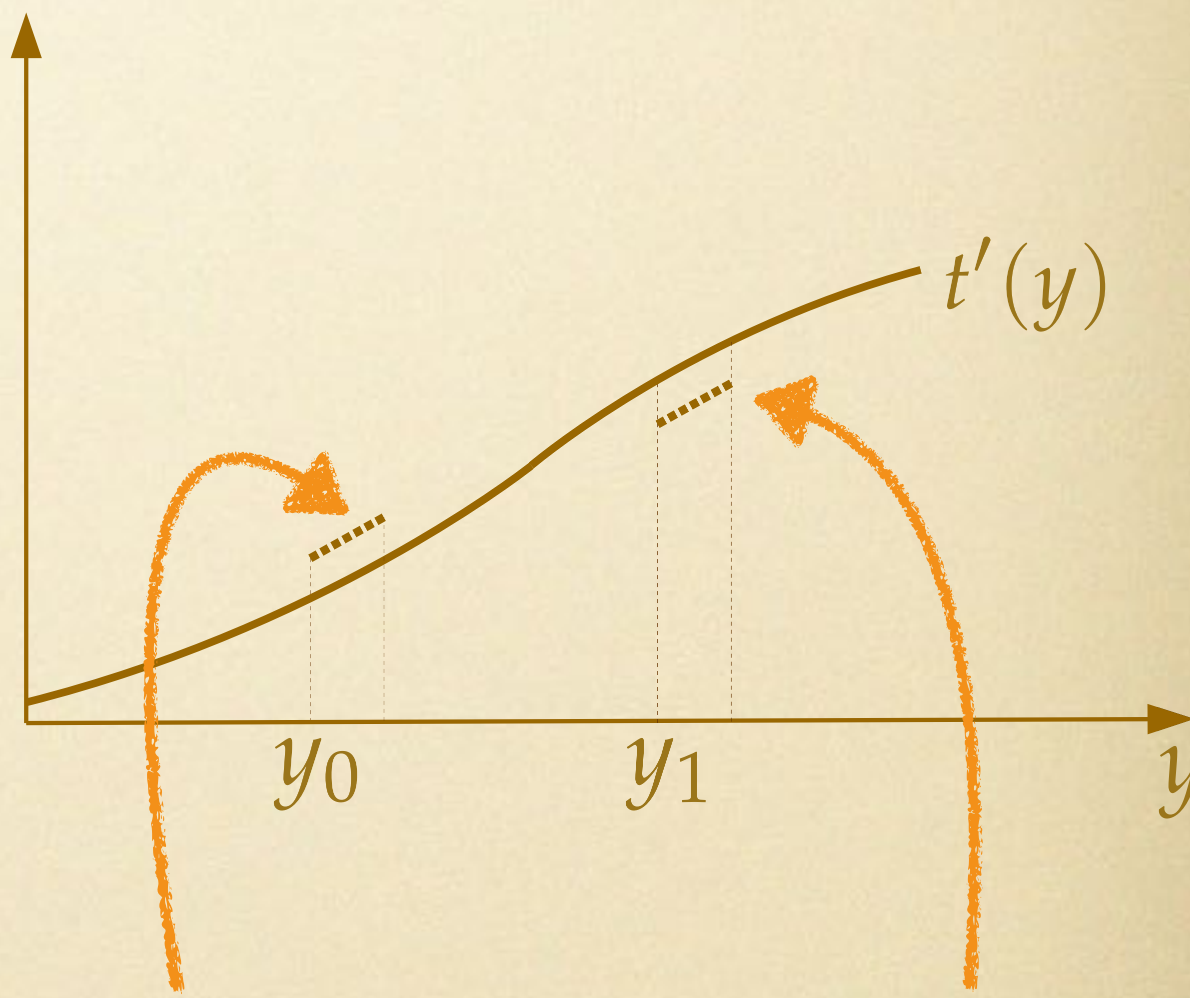
Variational Approach

Diamond-Mirrlees (1971)



Single global variation

Saez (2001)



Different local variations

Discussion

- Technology
  - DM holds for general $G(H)$ → extends to Mirrlees!
  - $G(H^1, \ldots H^S)$ → neutrality still holds with $t^1(y), \ldots, t^S(y)$
  - production efficiency is optimal
  - restricted tax instruments → modifications
    tax formula: Rothschild-Scheuer (2013), Ales et al. (2015), Sachs et al. (2017), production efficiency: Costinot-Werning (2018)

Discussion

- Technology
  - DM holds for general $G(H)$ → extends to Mirrlees!
  - $G(H^1, ...H^S)$ → neutrality still holds with $t^1(y), ..., t^S(y)$
  - production efficiency is optimal
  - restricted tax instruments → modifications
    tax formula: Rothschild-Scheuer (2013), Ales et al. (2015), Sachs et al. (2017), production efficiency: Costinot-Werning (2018)

- DWL interpretation

$$\frac{\partial D(t)/\partial t_i}{\sum_h x_i^h} = 1 - \frac{\sum_h \beta^h x_i^h}{\sum_h x_i^h} \forall i$$

DM

Discussion

- Technology
  - DM holds for general $G(H)$ → extends to Mirrlees!
  - $G(H^1, ...H^S)$ → neutrality still holds with $t^1(y), ..., t^S(y)$
  - production efficiency is optimal
  - restricted tax instruments → modifications
    tax formula: Rothschild-Scheuer (2013), Ales et al. (2015), Sachs et al. (2017), production efficiency: Costinot-Werning (2018)

- DWL interpretation

$$\frac{\partial D(t)/\partial t_i}{\sum_h x_i^h} = 1 - \frac{\sum_h \beta^h x_i^h}{\sum_h x_i^h} \forall i \rightarrow \frac{\frac{\partial}{\partial \tau} D(T' + \tau \delta_y)\Big|_{\tau=0}}{1 - H(y)} = 1 - \frac{\int_y^\infty \beta_{\tilde{y}} dH(\tilde{y})}{1 - H(y)} \forall y$$

DM Mirrlees

Extensions

Lifecycle and Annual Taxation

- Dynamics
  - ex ante $\theta \sim F(\theta)$
  - lifecycle $\delta \sim P(\delta|\theta)$
- "Annual" income tax $q(y)$
- Complete markets

$$c = \int_0^\infty q(y(\delta;\theta))dP(\delta|\theta)$$

Lifecycle and Annual Taxation

- Dynamics
  - ex ante $\theta \sim F(\theta)$
  - lifecycle $\delta \sim P(\delta|\theta)$
- "Annual" income tax $q(y)$
- Complete markets

$$c = \int_0^\infty q(y(\delta;\theta))dP(\delta|\theta)$$

- Preferences $U(c, Y; \theta)$

with

$$Y = \int_0^\infty v(y(\delta;\theta),\delta)dP(\delta|\theta)$$

Lifecycle and Annual Taxation

$$\left(\frac{T'(y)}{1-T'(y)}+\Lambda(y)\right)\varepsilon^F(y)yh(y)=\int_y^\infty(1-\bar{\beta}_{\tilde{y}})dH(\tilde{y})+\int_y^\infty\frac{T'(\tilde{y})}{1-T'(\tilde{y})}\bar{\eta}(\tilde{y})dH(\tilde{y})$$

Lifecycle and Annual Taxation

$$\left(\frac{T'(y)}{1-T'(y)} + \Lambda(y)\right)\varepsilon^F(y)yh(y) = \int_y^\infty (1-\bar{\beta}_{\tilde{y}})dH(\tilde{y}) + \int_y^\infty \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\bar{\eta}(\tilde{y})dH(\tilde{y})$$

Frisch elasticity

- Frisch elasticity holds $\lambda = -U_c/U_Y$ fixed

Lifecycle and Annual Taxation

$$\left(\frac{T'(y)}{1-T'(y)} + \Lambda(y)\right)\varepsilon^F(y)yh(y) = \int_y^\infty (1-\bar{\beta}_{\tilde{y}})dH(\tilde{y}) + \int_y^\infty \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\bar{\eta}(\tilde{y})dH(\tilde{y})$$

lifetime effects → $\Lambda(y)$

Frisch elasticity → $\varepsilon^F(y)$

- Frisch elasticity holds $\lambda = -U_c/U_Y$ fixed
- Lifetime effects on λ

→ vanish with $U\left(c - \int_0^\infty v(y(\delta;\theta),\delta)dP(\delta|\theta);\theta\right)$

Lifecycle and Annual Taxation

$$\left(\frac{T'(y)}{1-T'(y)} + \Lambda(y)\right)\varepsilon^F(y)yh(y) = \int_y^\infty (1-\bar{\beta}_{\tilde{y}})dH(\tilde{y}) + \int_y^\infty \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\bar{\eta}(\tilde{y})dH(\tilde{y})$$

lifetime effects — Frisch elasticity — averages

- Frisch elasticity holds $\lambda = -U_c/U_Y$ fixed
- Lifetime effects on λ
  → vanish with $U\left(c - \int_0^\infty v(y(\delta;\theta),\delta)dP(\delta|\theta);\theta\right)$
- Fundamental welfare weights β_θ only vary with θ
  → $\bar{\beta}_y$ varies less than in static framework
  → T=0 if inequality from δ only

Human Capital

- Education e
  - affects productivity profile $P(\delta|\theta, e)$
  - lifetime utility $U(c, Y; \theta, e)$

Human Capital

- Education e
  - affects productivity profile $P(\delta|\theta, e)$
  - lifetime utility $U(c, Y; \theta, e)$
- Annual tax $q(y)$

Human Capital

- Education e
  - affects productivity profile $P(\delta|\theta, e)$
  - lifetime utility $U(c, Y; \theta, e)$
- Annual tax $q(y)$

→ Same tax formula goes through!

→ Λ now also captures effects of taxes on e

General Heterogeneity

- DM: arbitrary heterogeneity
- Mirrlees: only one dimension, single-crossing

General Heterogeneity

- DM: arbitrary heterogeneity
- Mirrlees: only one dimension, single-crossing

- Groups indexed by $\phi \in \Phi$
  - preferences $U(c, y; \theta, \phi)$
  - single-crossing only in terms of θ within ϕ
  - Φ can be arbitrary

General Heterogeneity

- DM: arbitrary heterogeneity
- Mirrlees: only one dimension, single-crossing

- Groups indexed by $\phi \in \Phi$
  - preferences $U(c, y; \theta, \phi)$
  - single-crossing only in terms of θ within ϕ
  - Φ can be arbitrary

→ Standard formula goes through

→ Replace elasticities/welfare weights by averages at y

$$\bar{\varepsilon}^c(y) = \mathbb{E}[\varepsilon^c(y, \phi)|y] = \int_\Phi \varepsilon^c(y, \phi) dP(\phi|y)$$

Extensive Margin

- Preferences

$$V(c,y;\theta,\varphi) = \begin{cases} U(c,y;\theta) & \text{if } y > 0 \\ u(c;\theta,\varphi) & \text{if } y = 0 \end{cases}$$

Extensive Margin

- Preferences

$$V(c,y;\theta,\varphi)=\begin{cases} U(c,y;\theta) & \text{if } y>0 \\ u(c;\theta,\varphi) & \text{if } y=0 \end{cases}$$

$$\frac{T'(y)}{1-T'(y)}\varepsilon^c(y)yh(y)=\int_y^\infty\left(1-\bar{\beta}_{\tilde{y}}+\frac{T'(\tilde{y})}{1-T'(\tilde{y})}\eta(\tilde{y})-\frac{T(\tilde{y})-T(0)}{q(\tilde{y})-q(0)}\rho(\tilde{y})\right)dH(\tilde{y})$$

Extensive Margin

- Preferences

$$V(c,y;\theta,\varphi) = \begin{cases} U(c,y;\theta) & \text{if } y > 0 \\ u(c;\theta,\varphi) & \text{if } y = 0 \end{cases}$$

$$\frac{T'(y)}{1-T'(y)}\varepsilon^c(y)yh(y) = \int_y^\infty \left(1 - \bar{\beta}_{\tilde{y}} + \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\eta(\tilde{y}) - \frac{T(\tilde{y}) - T(0)}{q(\tilde{y}) - q(0)}\rho(\tilde{y})\right) dH(\tilde{y})$$

Participation elasticity $\rho(y) = \left.\frac{\partial h(y)}{\partial(q(y) - q(0))}\frac{q(y) - q(0)}{h(y)}\right|_{\{y(\theta)\}}$

Extensive Margin

- Preferences

$$V(c,y;\theta,\varphi) = \begin{cases} U(c,y;\theta) & \text{if } y > 0 \\ u(c;\theta,\varphi) & \text{if } y = 0 \end{cases}$$

$$\frac{T'(y)}{1-T'(y)}\varepsilon^c(y)yh(y) = \int_y^\infty \left(1 - \bar{\beta}_{\tilde{y}} + \frac{T'(\tilde{y})}{1-T'(\tilde{y})}\eta(\tilde{y}) - \frac{T(\tilde{y}) - T(0)}{q(\tilde{y}) - q(0)}\rho(\tilde{y})\right) dH(\tilde{y})$$

Participation elasticity $\rho(y) = \frac{\partial h(y)}{\partial(q(y) - q(0))} \frac{q(y) - q(0)}{h(y)}\Big|_{\{y(\theta)\}}$

Pure extensive margin $\frac{T(y) - T(0)}{q(y) - q(0)} = \frac{1 - \bar{\beta}_y}{\rho(y)}$

Conclusion

- Connection between canonical models in public finance
  - Link between widely used tax formulas
  - Integrated approach to linear and nonlinear taxation
- Simpler treatment of nonlinear taxation
- Allows for…
  - … weaker conditions for known results
  - … novel extensions that would not be tractable otherwise