Mirrlees meets Diamond-Mirrlees: Simplifying Nonlinear Income Taxation

by Florian Scheuer and Ivan Werning

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- This connection with associated conclusion is important as it unifies and tell us the D-M formula is the one we should expect to hold in general
- They also make a serious effort to show how it is useful 'in practice'

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$$\begin{aligned} U(q;\theta) \quad &:= \quad \max_{c,\phi} \; u(c,\phi;\theta) = c - \sum_{i=1}^{I} \phi_i v(y_i;\theta) \\ s.t. \quad & \quad c \leq \sum_i q_i \phi_i + q_0, \quad \sum_i \phi_i = 1. \end{aligned}$$

where $q_i = y_i - t(y_i)$ and ϕ_i is a probability measure.

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Suppose we can focus on $\phi_i \in \{0, 1\}$ (not needed).

$$
\begin{aligned}
A_i(q) \quad &:= \quad \{\theta \mid U(q,\theta) = q_i + q_0 - v(y_i,\theta)\} \\
\text{Aggregate Demand}: \; \mu_i(q) \quad &:= \quad \mu(A_i(q)) = \int_{A^i(q)} dF(\theta).
\end{aligned}
$$

General Optimal Tax Program

$$\max_{q} \quad \sum_{i=1}^{I} \int_{A_i(q)} W\left(q_i + q_0 - v(y_i, \theta); \theta\right) dF(\theta)$$

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- Let the average Pareto associated to agents who have positive 'demand' in income i be:

$$P(A_i) := \frac{1}{\mu_i} \frac{\int_{A_i} W'\left(q_i + q_0 - v(y_i,\theta);\theta\right) dF(\theta)}{\gamma}$$

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- The FOC w.r.t. to $q_s, s > 0$ reads

$$\mu_s(q)\beta_s(q) = -\sum_{i=1}^{I} \frac{\partial \mu_i(q)}{\partial q_s} t_i,$$

where $\beta_s(q) := P(A_s(q)) - 1$ and $t_i := y_i - q_i$.

DM Formula: Lesson 1 (Slutsky symmetry)

Compare our 'naive' formula (FOC)

$$\mu_s(q)\beta_s(q) = -\sum_{i=1}^{I} \frac{\partial \mu_i(q)}{\partial q_s} t_i,$$

and DM with no wealth effects

$$\mu_s(q)\beta_s(q) = -\sum_{i=1}^{I} \frac{\partial \mu_s(q)}{\partial q_i} t_i.$$

DM is better since it refers to the same good. And as such it delivers nice interpretations ... With no wealth effects we indeed have:

$$\frac{\partial \mu_s(q)}{\partial q_i} = \frac{\partial \mu_i(q)}{\partial q_s}$$

DM Formula: Lesson 2 (redefine Pareto weights)

What if we have income effects? Use 'compensated' demands

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Hence DM allowing for wealth effects

$$\mu_s(q)\hat{\beta}_s(q) = -\sum_{i=1}^{I} \frac{\partial \mu_s^c(q)}{\partial q_i} t_i.$$

where

$$\hat{\beta}_s(q) = \beta_s(q) + \sum_{i=1}^{I} \frac{\mu_i^c(q)}{\partial q_0} t_i$$

because

$$\frac{\partial \mu_s^c(q)}{\partial q_i} := \frac{\partial \mu_s(q)}{\partial q_i} + \frac{\partial \mu_s(q)}{\partial q_0} \mu_i$$

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And FOC w.r.t. q_0 delivers (if no wealth effects always true)

$$\sum_s \mu_s(q)\hat{\beta}_s(q) = 0.$$

What is special to Mirrlees framework?

1. As in our model, there is a fixed supply of workers

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- Rearranging FOCs one can isolate $t_i - t_{i-1}$ for all $i > 1$
- This amounts to increase jointly $q_i, q_{i+1}, \ldots, q_I$ (well known)
- Equivalently, the set $\cup_{s=1}^{I} A_i$ has only one border: with A_{i-1}

Redefining goods instead of re-arranging FOCs

- The authors show that if we re-define goods as the bundles $(y_i, y_{i+1}, \dots, y_I)$ then the associated tax is $t_i - t_{i-1}$ (integrations by parts), and there are no cross-price effects, i.e. DM delivers directly Mirrlees formula:

$$
\begin{aligned}
(t_i - t_{i-1})\frac{\partial \mu_i^c(q)}{\partial q_i} &= \sum_{s=i}^{I} \mu_s(q)\hat{\beta}_s(q), \\
\Longleftrightarrow \quad \frac{t_i - t_{i-1}}{q_i - q_{i-1}} &= \frac{1}{\varepsilon_i^c(q)}\frac{1}{\mu_i(q)}\sum_{s=i}^{I} \mu_s(q)\hat{\beta}_s(q), \\
\Longleftrightarrow \quad \frac{\tau_{i,i-1}}{1-\tau_{i,i-1}} &= \frac{1}{\varepsilon_i^c(q)}\frac{1}{\mu_i(q)}\sum_{s=i}^{I} \mu_s(q)\hat{\beta}_s(q),
\end{aligned}
$$

where $\tau_{i,i-1} := \frac{t_i - t_{i-1}}{y_i - y_{i-1}}$ and $\varepsilon_i^c(q) := \frac{\partial \mu_i^c(q)}{\partial (q_i - q_{i-1})}\frac{q_i - q_{i-1}}{\mu_i(q)}$

Lessons

1. What matters are aggregate demands.
2. Cross-price effects can be seen as 'active borders' (or relevant IC constraints)
3. Slutsky symmetry holds for 'compensated demands' (at the aggregate), this suggests a redefinition of Pareto weights (DM suggest a way to deal with income effects)
4. M has two main peculiarities: a. specific structure of cross-price effects; b. fixed supply of workers
5. The authors' reading of DM recall us that with a proper redefintion of the 'good' any perturbation can become another perturbation

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Q4: Advantages of the continuum of goods?

Q5: Do we really have to focus on the no-bunching case?