Fiscal policy and forward guidance with preferences over wealth

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Abstract

I examine how the effects of fiscal policy and forward guidance are shaped by preferences over wealth calibrated to reflect microeconomic evidence on household saving behavior and individual discount rates. This assumption effectively limits the horizon of unconstrained households and introduces real wealth into their Euler equation. Therefore, the effect of permanent and temporary cuts of government expenditure when monetary policy is constrained by the zero lower bound become much more alike due to smaller crowding in of unconstrained household consumption in response to permanent cuts. Furthermore, the inflationary and GDP effects of forward guidance policies become much smaller.

JEL Codes: E52; E62; E32. Keywords: Forward guidance, Fiscal consolidation, finite horizon, zero lower bound.

1 Introduction

The manner in which households and firms respond to the future stance of fiscal and monetary policy is at the heart of recent macroeconomic debates. More specifically, in standard models, a fiscal contraction enacted during a period when monetary policy is constrained by the zero lower bound has very different effects on private consumption depending whether it is anticipated to be temporary or permanent (e.g. Woodford (2011), Denes et al. (2013)). A temporary fiscal contraction lasting as long as the period of constrained monetary policy reduces consumption of forward looking households by lowering inflation and thus increasing the real interest rate. By contrast, a permanent fiscal contraction tends to crowd in the consumption of these households, as it entails the promise of a period of looser-than-otherwise monetary policy after the exit from the zero lower bound and a higher private consumption

[†]The opinions expressed here are those of the author and do not necessarily represent those of the Central Bank of Ireland or the European System of Central Banks.

level in the new steady state the economy will ultimately converge to, which via consumption smoothing immediately increases consumption. Another topical example is the effect of forward guidance by the central bank regarding the path of the future policy interest rate, which can in principle overcome the zero lower bound constraint (e.g. Eggertsson and Woodford (2003)). As shown by Del Negro et al. (2015) and Carlstrom et al. (2015), the effects of such policies in estimated DSGE models are very strong and bigger than suggested by the available empirical evidence. A related finding is that the impact of forward guidance increases exponentially in the length of period for which the interest rate is expected to be fixed below what it would be in the absence of the policy. A key factor behind these results is the consumption response of forward looking households.

However, the standard infinite horizon model of the consumer is at odds with the micro evidence on the inter-temporal choices of high income households, who would seem to be the natural real world counterpart of "Riccardian" households with their unconstrained consumption smoothing opportunities. Firstly, as has been shown by Carroll (2000), the standard model underpredicts the saving of high income households relative to their permanent income. More specifically, the marginal propensity to save out of an increase in their permanent income is zero in the model, but appears be large in the data (Dynant et al. (2004) and Kumhof et al. (2014)). Secondly, the micro evidence on individual discount rates typically estimates them to substantially exceed market interest rates relevant for the inter temporal choice under examination by the researcher, even for income rich and highly educated individuals (e.g. Harrison et al. (2002) and Warner and Pleeter (2001)). This discrepancy cannot be accommodated by the standard model, where the individual discount rate applied to future nominal income streams always equals the inverse of the (gross) nominal interest rate, implying that financial market exactly compensate the household's impatience.

I examine the macroeconomic effects of fiscal policy and forward guidance at the zero lower bound in a framework where consumption smoothing households derive utility from accumulating wealth in the form of safe assets. Carroll (2000), Francis (2009) and Kumhof et al. (2014) show that such a "capitalist spirit" assumption allows the infinite horizon model to replicate the saving behavior of rich households, while Fisher (2015) appeals to liquidity preference as an additional motivation. By generating a marginal benefit of saving over and above increasing future consumption, this assumptions allows the equilibrium market interest rate to fall short of the individual discount rate households apply to future income streams, thus introducing discounting into the linearized Euler equation. It also implies a positive effect of real wealth on unconstrained household consumption.

I embed these preferences in the small New Keynesian model used by Woodford (2011) to examine the effects of fiscal policy at the zero lower bound. It turns out that the effect of temporary and permanent fiscal contractions in the presence of the zero lower bound become much more alike than in without preferences over wealth. A permanent contraction in government consumption causes a smaller crowding in of private consumption as households respond less to the associated future monetary easing and their own higher consumption level in the new steady state. Furthermore, the decline in their real wealth associated with the decline in government debt caused by the fiscal contraction increases the marginal utility of wealth, inducing them to save more. The impact of preferences over wealth on the fiscal multiplier becomes even bigger in the presence of a share of credit constrained households who simply consume their disposable income.

I then examine the macroeconomic effect of forward guidance. I assume that the Central Bank is able to affect the private sector's expectations regarding the deviation of the policy interest rate after the exit from the ZLB. Without preferences over wealth, GDP and inflation increase exponentially in the expected length for which the interest rate is expected to deviate from it's path in the absence of the forward guidance announcement, and more so with a share of credit constrained households. With preferences over wealth, the effect is substantially muted, and for reasonable calibrations increases linearly in the length of the forward guidance episode. The reason is that the low interest rates promised by forward guidance for the recovery period have a lower effect on current consumption than with an infinite horizon. Furthermore, the decline in the government budget deficit caused by the increase in GDP and the increase in inflation both lower real government debt and thus the real wealth of unctnstrained households, which with curvature in preferences over wealth motivates households to save more. Hence it seems that adding preferences over wealth is a possible solution to the "Forward Guidance puzzle".

Finally, in the Appendix, I embed these preferences into a medium scale DSGE model with capital accumulation, sticky prices and wages and credit constrained households and firms, and show that the results arising from the simple model persist in this richer environment.

To my knowledge, this paper is the first to analyze the consequences of discounting the Euler equation for the effect of fiscal policy changes at the zero lower bound. It also proposes a partially novel approach to addressing the forward guidance puzzle. While Campbell et al. (2016) also use preferences over wealth, they find that this assumption makes only a minor difference for their simulated effect of forward guidance. Part of the explanation might be that their calibration of preferences over wealth is not based on micro evidence on individual discount rates, implying less discounting in the Euler equation than my calibration. Furthermore, in their model there is only a representative agent in the model and the government runs a balanced budget, implying real financial wealth is constant and thus the wealth effect on consumption is absent and thus cannot attenuate the effect of forward guidance. At the same time, their estimated wage and price markup coefficients are very small, the estimated degree of habit formation is high and the wealth effect on labor supply is largely absent, all of which would tend to reduce the effect of forward guidance even in the absence of preferences over wealth.

By contrast, Del Negro et al. (2015) assume a Blanchard-Yaari type perpetual youth structure to limit the horizon of unconstrained households. However, in order to achieve a quantitatively important attenuation of the effect of forward guidance, they also have to assume counterfactually high death rates (see McKay et al. (2017)). Finally, there is an ongoing debate on whether incomplete market models with heterogenous agents can overcome the forward guidance puzzle. McKay et al. (2016) develop a New Keynesian incomplete markets model where agents respond less to future real interest rates for fear of running up against their borrowing constraint in the future. However, Verning (2015) argues that this lower interest rate is not a consquence of the incomplete market assumption per se, but relate to other, auxiallary assumptions of McKay et al. (2016) which render idiosyncratic income risk procyclical and liquidity risk counterycyclical.[1] Verning (2015) shows that with either countercyclical income risk or procyclical liquidity risk, consumption becomes more sensitive to current and future interest rates than in a representative agent model.

Although the degree of heterogeneity I allow for is much simpler than in Verning (2015)'s framework, one might interpret the version of my model with constrained households as a shortcut to a heterogenous agents model where the cyclicality of income and borrowing constraints differ across agents. The model captures the flavour of Verning (2015)'s mechanism by implying a procyclical constrained household income share, and therefore a greater sensitivity of aggregate demand with respect to the unconstrained household consumption and in turn the the path of future interest rates. In this setup, preferences over wealth lower the effect of forward guidance firstly by reducing the sensitivity of unconstrained household consumption to future interest rates. Crucial for this effect is that given the amount of real wealth of the unconstrained household, the marginal benefit from an additional is unrealted to the business cycle, which could reflect acyclical income risk. Secondly, preferences over wealth create a consumption wealth effect for unconstrained households, in line with a positive effect of wealth on the ability to self insure, implying that the forward guidance policy related decline in real government debt which constitutes unconstrained wealth limits their overall consumption increase in response to the policy. To the extent that constrained agents are likely to be poorer and less well educated than unconstrained households, the greater procyclicality of of constrained household than unconstrained household income and acyclical income risk of unconstrained households are in line with the microevidence of Heathcote et al. (2010) and Murphy and Topel (1987).

Finally, Lemoine and Linde (2016) examine the effect of permanent government spending cuts in a monetary union under imperfect credibility regarding the spending cut's duration. This approach represents an alternative mechanism limiting the crowding in of unconstrained household consumption and thus increasing the cost of permanent spending cuts if monetary policy is constrained.

In the next section, I develop a simple New Keynesian model with preferences over wealth and credit constrained households, section three discusses the calibration, while section four and five present the main results of the paper. Section six summarizes the results from the medium scale model, which are discussed in more detail in the Appendix.

2 A stylized model with preferences over wealth and constrained households

2.1 Households

The economy features a fraction $(1-\omega)$ of households who participate in credit markets and are thus able to smooth consumption inter-temporally by trading a safe

[1]Income risk becomes procyclical because McKay et al. (2016) assume that firm profits, which relative to labour income are countercyclical, are distributed equally across agents, implying that the share of an agents income subject to idiosyncratic uncertainty increases during a boom.

bond B_t, and a fraction ω which consumes only their disposable income. Household j derives utility from consumption and the possession of safe government bonds, and disutility from supplying labor. Her objective is given by

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{j,t+i}^{1-1/\sigma_H}}{1-1/\sigma_H} - \chi N_{j,t+i}^{1+\eta} + \phi_B \left(\frac{B_{j,t+i}}{P_{t+i}} \right)^{1-\sigma_B} \right] \right\} \tag{1}$$

Her budget constraint is given by

$$B_t + P_t C_{O,t} = R_{t-1} B_{t-1} + W_t N_{O,t} - P_t T_{O,t} + P_t \Xi_t \tag{2}$$

where $P_{t,}$, $C_{O,t}$, R_t, W_t,$N_{O,t}$, $T_{O,t}$ and Ξ_t denote the price level, the consumption basket, the nominal interest rate on bonds (which is also the policy rate set by the central bank), the nominal wage, labor supply, real lump sum taxes and real profits of firms, respectively, while the subscript O refers to unconstrained ("optimizing") households. Throughout the paper I adopt the convention that only period t decision variables are indexed with t, implying that B_t denotes the stock of safe bonds at the end of period t. Preferences over wealth have been found useful, or indeed necessary, to explain a range of phenomena, the most conventional example being liquidity preference used to explain the presence of money in agents portfolios. Krishnamurthy and Vissing Jorgenson (2012) argue that liquidity preference may extend to assets with a positive yield if they have money-like qualities, and argue that preferences over such assets explain the demand curve for US treasuries. Following KVJ, Fisher (2015) argues that the "risk premium shock" in the estimated DSGE model of Smets and Wouters (2007) can be interpreted as shocks to the demand for safe and liquid assets (i.e. shocks to ϕ_B). Carroll (2000) argues that "capitalist spirit" type preferences over wealth are necessary to explain the saving behavior of rich households in US data. Specifically, the standard life-cycle model substantially under predicts the amount of wealth rich households hold relative to (estimates of) their permanent income and their propensity to save out of permanent income changes, as is also argued by Dynant et al. (2004). Francis (2009) shows that preferences over wealth can replicate this phenomenon. Kumhof et al. (2014) build on this literature in order to link the increase in US inequality, household leverage and financial fragility in the decades before the 2008-2009 financial crisis.

The FOCs of unconstrained households with respect to consumption and bonds are given by

$$\Lambda_{O,t} = \beta \frac{R_t}{\hat{\Pi}_{t+1}} E_t \Lambda_{O,t+1} + \phi_B \left(\frac{B_{O,t}}{P_t} \right)^{-\sigma_B} \tag{3}$$

$$\Lambda_{O,t} = C_{O,t}^{-1/\sigma_H} \tag{4}$$

where $\Lambda_{O,t}$ denotes the real marginal utility of consumption. Linearizing and combining equations (3) and (4) yields

$$\hat{C}_{O,t} = -\theta \sigma_H \left[\hat{R}_t - E_t \hat{\Pi}_{t+1} \right] + \theta E_t \hat{C}_{O,t+1} + (1-\theta) \sigma_H \sigma_B \hat{b}_{O,t} \tag{5}$$

where $b_{O,t} = \frac{B_{O,t}}{P_t}$, a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with $\theta = \beta \frac{R}{\Pi}$, i.e. the product of the steady-state household discount factor and the real interest rate. Iterating (5) until period t+n yields

$$\hat{C}_{O,t} = E_t \left\{ \sum_{i=0}^{n} -\theta^n \sigma_H \left[\theta \left(\hat{R}_{t+i} - \hat{\Pi}_{t+1+i} \right) + (1-\theta)\sigma_B \hat{b}_{O,t+i} \right] \right\} + \theta^{n+1} E_t \hat{C}_{O,t+1+n} \tag{6}$$

θ may be interpreted as the equilibrium weight the household attaches to period $t+1$ consumption, i.e. the net effect of utility discounting and the (steady state) market real interest rate. In the absence of preferences over wealth ($\theta = 1$), the equilibrium weight of an additional unit of future consumption -no matter how far removed from the present- equals one. Hence a permanent increase in government consumption will reduce (increase) current consumption (the current marginal utility of consumption) of optimizing households to the same extend that it reduces (increases) their future (marginal utility of) consumption, no matter how distant the point in time when the new steady state will be reached. Future changes in the real interest rate are equally powerful no matter how far away from period t they are located. By contrast, with preferences over wealth ($\theta < 1$), the equilibrium weight the the household attaches to future periods declines in their distance from the present. Thus future events will have a smaller effect on the choices of the household in period t, the more so the further away in time they are located. Even though the household maximizes over an infinite number of periods, her effective horizon is thus finite. Furthermore, for $\sigma_B > 0$, current and future real household wealth has a positive effect on consumption, as the marginal utility from wealth is declining relative to the marginal utility from consumption.

Constrained households are assumed to have identical preferences over consumption and labor but preference over wealth, and are unable to save or borrow, implying that their consumption is given by

$$C_{ROT,t} = w_t N_{ROT,t} - T_{ROT,t} \tag{7}$$

where we use the subscript ROT (=Rule of Thumb) to denote constrained households. Their real marginal utility of consumption is given by

$$\Lambda_{ROT,t} = C_{ROT,t}^{-1/\sigma_H} \tag{8}$$

2.2 Wage setting

I assume that there is a continuum of unions which recruit the labor of both household types according to their share in the population, and transforms their labor into a specific variety i, which forms part of a CES labor basket employed by firms. The union operates under monopolistic competition and sets a joint wage for both household types. The demand curve for labor variety i is given by

$$N(i)_t = \left(\frac{w_t(i)}{w_t} \right)^{-e_w} N_t$$

where $w_t = \frac{W_t}{P_t}$ and $e_w > 1$ denotes the demand elasticity for type i labor. Due to the assumption of identical preferences over consumption and labor across household types, we can write the union's problem as maximizing

$$w_t(i)\,\Lambda_t N(i)_t + \chi\frac{N(i)_t^{1+\eta}}{1+\eta}$$

subject to

$$N(i)_t = \left(\frac{w_t(i)}{w_t}\right)^{-e_w} N_t$$

where

$$\Lambda_t = \omega\Lambda_{ROT,t} + (1-\omega)\,\Lambda_{O,t} \tag{9}$$

The union's FOC is thus given by

$$w_t\Lambda_t = \chi N_t^{\eta}\mu_w \tag{10}$$

where $\mu_w > 1$ denotes the markup of the real wage over the aggregate marginal rate of substitution of both household types.

2.3 Retailers

There is a continuum of monopolistically competitive firms owned by unconstrained households which each produce a variety j from a CES basked of goods. They set prices subject to nominal rigidities in the form of Calvo (1983) pricing, i.e. a fraction ξ_p of firms is unable to reset their price in a given period. The production technology of firm j is given by

$$Y_t(j) = N_t^{\alpha}(j) \tag{11}$$

with $\alpha \leq 1$, implying that their real marginal cost $mc_t$is denoted as

$$mc_t = \frac{w_t}{N_t^{\alpha-1}} \tag{12}$$

These assumptions imply that up to first order, inflation evolves according to the familiar New Keynesian Phillips curve

$$\hat{\Pi}_t = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\hat{mc}_t + \beta E\hat{\Pi}_{t+1} \tag{13}$$

2.4 Government

The government levies taxes and buys goods from retailers. Its budget constraint is given by

$$B_t = (1+R_t)\,B_{t-1} + P_tG_t - P_t\left((1-\omega)\,T_{O,t} + \omega T_{ROT,t}\right) \tag{14}$$

For simplicity, I assume that in the steady state, government debt is zero, taxes on unconstrained households T_O exactly equal the profits of firms plus the interest earned on government bonds, and taxes on constrained households are zero. This assumption requires that the share of government expenditure in GDP equals the profit share.[2] As a consequence, the steady state consumption level is identical across household types. Taxes outside the steady state are determined by the following fiscal rule

$$\hat{T}_t = \tau_y \hat{Y}_t + \tau_b \hat{b}_{t-1} \tag{15}$$

where a hat above a fiscal variable such as taxes, debt or government consumption denotes the deviation of that variable from its steady as a percentage of steady state GDP, with $\tau_y, \tau_b > 0$. $\hat{T}_t$ is split across household types according to their population share.

Monetary policy is described by a simple rule where the Central Bank responds to inflation and the deviation of output from its flexible price level $\Gamma\hat{G}_t$, which for a given actual output level depends positively on government consumption as a consequence of the wealth effect on labor supply.

$$\hat{R}_t = max\left(\phi_\pi \hat{\Pi}_t + \phi_y\left(\hat{Y}_t - \Gamma\hat{G}_t\right), \hat{R}_L\right) \tag{16}$$

where $\hat{R}_L < 0$ denotes a lower bound on the (percentage deviation from its steady state of the) nominal interest rate $\hat{R}_t$. If the lower bound on level of the policy interest rate is zero, $\hat{R}_L = -\frac{R-1}{R}$. Below I will refer to the expression $\phi_\pi \hat{\Pi}_t + \phi_y\left(\hat{Y}_t - \Gamma\hat{G}_t\right)$ as the off-ZLB interest feedback rule of the central bank.

2.5 Equilibrium

Total consumption is the sum of constrained and unconstrained household consumption:

$$C_t = (1-\omega)C_{O,t} + \omega C_{ROT,t} \tag{17}$$

GDP is the sum of household and government consumption

$$Y_t = C_t + G_t \tag{18}$$

2.6 Linearized equations

Linearizing and combining the above equations allows to express the model in five equations (for details see Appendix 9):

$$\begin{aligned}\hat{Y}_t - \left(\sigma_g \hat{G}_t - \sigma_T \hat{T}_t\right) = &-\tilde{\sigma}\left[\theta\left[\hat{R}_t - E_t\hat{\Pi}_{t+1} - r_t^{net}\right] - (1-\theta)\tilde{\sigma}_B \hat{b}_t\right] \\ &+ \theta\left(E_t\hat{Y}_{t+1} - \left(\sigma_g E_t\hat{G}_{t+1} - \sigma_T E_t\hat{T}_{t+1}\right)\right)\end{aligned} \tag{19}$$

$$\hat{\Pi}_t = \kappa(\hat{Y}_t - \Gamma\hat{G}_t) + \beta E_t\hat{\Pi}_{t+1} \tag{20}$$

$$\hat{R}_t = max\left(\phi_\pi \hat{\Pi}_t + \phi_y\left(\hat{Y}_t - \Gamma\hat{G}_t\right), \hat{R}_L\right) \tag{21}$$

[2] Hence $\frac{G}{Y} = \left(1 - \frac{1}{\mu_P}\right)$, where μ_P denotes the price markup.

$$\hat{b}_t = \frac{R}{\Pi}\left(\hat{b}_{t-1} + \frac{b}{Y}\left(\hat{R}_{t-1} - \hat{\Pi}_t\right)\right) + \hat{G}_t - \hat{T}_t \tag{22}$$

$$\hat{T}_t \;=\; \tau_y \hat{Y}_t + \tau_b \hat{b}_{t-1} \tag{23}$$

The meaning of the reduced form coefficients can be obtained from Table 1. In the absence of constrained households ($\omega = 0$), and without preferences over wealth ($\theta = 1$), the model collapses to the familiar New Keynesian model (i.e. $\tilde{\sigma} = \sigma$and $\sigma_g = 1$). By contrast, for $\omega > 0$, the model features a "Keynesian multiplier effect" of the type observed in the simple Keynesian income-expenditure model ($\sigma_g > 1$). An increase in government spending $\hat{G}_t$ increases the consumption of constrained households due to higher employment and wages, implying that for a constant real interest rate and constant t+1 expectations, the overall increase in GDP exceeds the increase in government consumption.

Furthermore, for $\omega > 0$, GDP also becomes more sensitive to the real interest rate. The reason is that the increase in GDP caused by the consumption increase of unconstrained households in response to a lower real interest rate disproportionately benefits constrained households, because the real wage increases in employment, which benefits constrained households at the expense of unconstrained households, who own the firms.[3] To the extent that constrained households are likely to poorer and less well educated than unconstrained households, the model thus captures in a crude fashion the empirical finding that the earnings of lower income households are more cyclical than the earnings of median and higher income households (see Heathcote et al. (2010), Figures 9 and 7, and Murphy and Topel (1987)).[4]

Finally, r_t^{net} denotes a consumption preference shock affecting unconstrained households (sometimes also interpreted as a "risk premium shock), i.e. a shock to the natural rate of interest.

Table 1: Reduced form coefficients of the stylized model

σ	$\tilde{\sigma}$	σ_g	σ_T	β	Γ
$\sigma_H \frac{C}{Y}$	$\frac{\sigma(1-\omega)}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$	$\frac{\left(1-\omega\frac{C}{Y}\frac{1}{\sigma}\right)}{\left(1-\omega\frac{C}{Y}\frac{1}{\sigma}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$	$\frac{\omega}{\left(1-\omega\frac{C}{Y}\frac{1}{\sigma}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$	$\frac{\theta}{R/\Pi}$	$\frac{\frac{1}{\sigma}}{\frac{1}{\sigma}+\eta_v}$

κ	η_v
$\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\left(\frac{1}{\sigma}+\eta_v\right)$	$\frac{1-\alpha+\eta}{\alpha}$

3 Calibration

Regarding the standard parameters (Table 2), I follow Woodford (2011). The assumed target for $\frac{R}{\Pi}$ implies that for $\theta = 1$, β is as in Woodford (2011). For the

[3]If firm profits were split equally between constrained and unconstrained households, the interest rate sensitivity of GDP would equal σ and would thus be independent of the share of credit constrained households.

[4]Admittedly, as pointed out by Heathcote et al. (2010), the greater increase in lower income household earnings during expansions is more related to a decline in unemployment and an increase in hours worked, than an increase in the hourly wage. We abstract from this aspect in order to keep the analysis simple.

share of constrained households, I consider two values, namely 0 and 0.15. I assume that monetary policy outside the ZLB is described by the standard Taylor rule coefficients. I set $\tau_y = 0.34$, in line with evidence on the estimated semi-elasticity of the primary deficit with respect to GDP by Girouard and Andre (2005) for the US, while τ_b is set to small value sufficient to guarantee debt stationarity.

Table 2: Parameters in the stylized model

σ	η	$\frac{R}{\Pi}$	α	Γ	ω	$\frac{C}{Y}$	ϕ_y	ϕ_π	τ_y	τ_b
0.8621	1.6	1/0.997	1	0.4203	0; 0.15	0.8	0.5/4	1.5	0.34	0.0530

μ	θ	$\tilde{\sigma}_B$
0.875	$0.96 - 1.0$	0; 0.2

For θ, I consider values between 0.96 and 1. This calibration is based on a large literature attempting to estimate the personal discount rate from micro data. Note that the bond Euler equation may be rearranged as

$$1 - \frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_{O,t}} = R_t \beta E_t \left\{ \frac{\Lambda_{O,t+1}}{\hat{\Pi}_{t+1}\Lambda_{O,t}} \right\}$$

or

$$1 - \frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_{O,t}} = \frac{1+i_t}{1+d_t} \tag{24}$$

where $i_t = R_t - 1$ and $1 + d_t = \beta \frac{\Lambda_{O,t+1}}{\hat{\Pi}_{t+1}\Lambda_{O,t}}$ denotes the -time varying- stochastic discount factor the household applies to nominal $t+1$ income. In the steady state, we have $1 - \frac{\phi_B \left(\frac{B_O}{P}\right)^{-\sigma_B}}{\Lambda_O} = \beta \frac{R}{\Pi} = \theta$. However, for θ close 1, $\frac{1+i_t}{1+d_t}$ represents a good approximation of θ even outside the steady state.[5] Therefore, even for large deviation of consumption or bonds from their respective steady states, we have

$$\theta \approx \frac{1+i_t}{1+d_t} \tag{25}$$

Hence θ may be estimated using an estimate of the personal discount rate and an appropriate market interest rate. Economists have attempted to estimate the personal discount rate at least since Friedman's (1957) seminal tests of the permanent income hypotheses by studying economic agents behavior when faced with a variety of inter temporal trade-offs (see Table 3). These range from trading off the energy efficiency and price price of household appliances (Ruderman et al. (1984)) to the effects of paying bonuses (Cylke et al. (1982)) or severance packages (Warner and Pleeter (2001)) as a lump sump sums instead of installments, as well as field experiments where probants choose between a payment and a higher deferred payment (Harrison

[5] A linear approximation of the left hand side of equation (24) yields $\frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_O} \left(\hat{\Lambda}_{O,t} + \sigma_B \hat{b}_{O,t}\right) = (1-\theta)\left(\sigma_B \hat{b}_{O,t} - \frac{1}{\sigma_H}\sigma_H \hat{C}_{o,t}\right)$. Hence for $1 - \theta$ close to zero and reasonable calibrations of σ_H and σ_B even large deviations of $\hat{C}_{O,t}$and $\hat{b}_{O,t}$ would only slightly bias the estimate of θ.

Table 3: Empirical evidence on θ

Sample period	d (APR)	i_t (APR)	Implied θ	Source of D; R used for comparison	Estimate of D based on
1929-1948	33.0*	0.8*	0.82	Friedman (1962,1957); real treasury maturity$\geq$ 10 years	Tests of PIH
1960	19.6*	2.0*	0.96	Heckman (1976); real 10 year treasury	Estimated life cycle earnings model
1979	27.4	9.5	0.96	Cylke et al. (1982); 5 year treasury	US Military reenlistment decisions
1972; 1978; 1980	54.7; 64.0; 72.1*	2.9;1.9;2.8*	0.9; 0.89; 0.88	Ruderman et al. (1984), Median; 10 year real treasury	Price of household appliances
1982-1989	18.3	8.6	0.98	Ausubel (1991); one month certificate of deposit	US credit card interest rates
1992-1993	18.7	6.3	0.97	Warner and Pleter (2001), 20 year treasury	US officers severance package choices
1996	22.5	4.2	0.96	Harrison et al. (2002); 1 year money market rate	Experiment, income rich households
2008	28.2/19.0	1.82/3.7	0.9/0.97	Wang et al. (2016); see note.	Experiment, US economics students, hyp. discounting

Notes: If information on the horizon of the choice of the agent under observation was available, i_t is the the safe (e.g. government) interest rate with a maturity as close as possible to this horizon during the year the decision was made. In most other cases, I use the 10 year government bond yield. Numbers marked with a * are estimates of the *real* personal discount rate. The corresponding R I use to compute θ is therefore a measure of the real interest rate, where expected inflation is assumed to equal the average CPI inflation rate over the preceding 10 years. In case of Friedman (1962, 1957), I calculated the relevant i_t as the difference between the average interest rate on long term government bonds (maturity 10 yeas or more, the only long term government bond series for this period I am aware of) over this period, and the average PCE deflator inflation rate.
Ausubel's (1991) investigation of the US market for credit cards is frequently cited as evidence in favor of high personal discount rates. In his sample, more than three quarters of customers holding credit credit cards incur finance charges on substantial outstanding balances in spite of credit card interest rates ranging between 18 and 19%, and he cites industry publications saying that about 90% of an issuers outstanding balance accrue interest.
Wang et al. (2016) allow for hyperbolical discounting and therefore allow the discount rate applied to a payment received one year ahead to exceed the discount rate between any future period. I report both rates and thus two values of theta. The interest rates use to compute θ are the one year treasury bond rate, and the 9 year forward rate one year hence implied by the one and 10 year treasury bond rate.

et al. (2002)). As can be obtained from Table 3, the elicited discount rates are quite high, although typically below the estimate of 33% of Friedman (1962,1957). What is more, they also typically exceed safe market interest rates on safe investments with a comparable maturity observed at the time the discount rates were elicited, implying that the implied value of θ is smaller than one, sometimes substantially so. Since I interpret the optimizing households as rich and educated households, the contributions of Harrison et al. (2002) and Warner and Pleeter (2001) are of particular relevance. Harrison et al. (2002) report estimates for (income-) rich households, while Warner and Pleeter's (2002) elicit discount rate of officers of the United States armed forces choosing between two severance packages during the 1992-1995 military draw-down.[6] My calibration of θ is thus at the upper end of what is implied by the available evidence.

Following Kumhof et al. (2014), I calibrate σ_B such that the unconstrained households marginal propensity to save (MPS) out of an increase in permanent income in a partial equilibrium exercise matches the micro evidence on the saving behavior of high income households provided by Dynant et al. (2004) and Kumhof et al. (2014). For the top 5% income group, Kumhof et al. (2014) report an MPS of 0.4 based on their own estimates and an upper bound of 0.5 based on the estimates of Dynant (2004). I use an MPS of 0.4-0.5 as a target for the calibration σ_B because wealth and consumption are likely to be concentrated among a subset of those households forming the empirical counterpart of my unconstrained household group. However, my main results become even stronger if I target a lower value of the MPS, which would imply a bigger value of σ_B. I provide details on this partial equilibrium calibration exercise in Appendix 8.

4 Fiscal multipliers

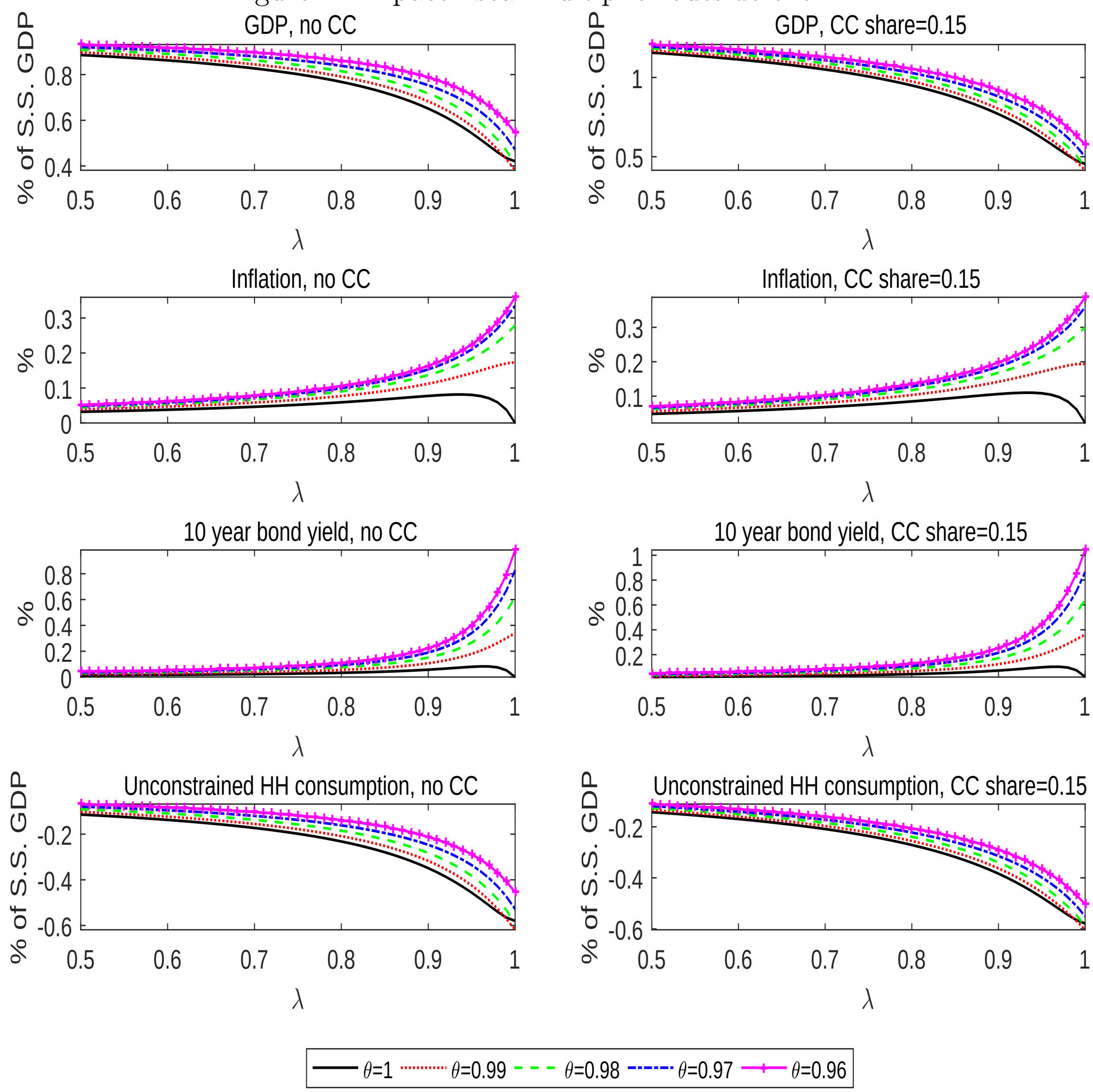
I first examine the impact effect of an increase in government spending outside the zero lower bound. Government spending follows an AR(1) process with persistence λ. As can be obtained from Figure 1, the more persistent the government spending increase the stronger the crowding out of unconstrained household consumption. Adding constrained households (right panel) increases the fiscal multiplier, as the consumption of theses households increases in response to the fiscal expansion, and the crowding out of unconstrained household consumption.

With preferences over wealth ($\theta < 1$) the decrease of the multiplier in the persistence of the government spending shock decreases somewhat, as forward looking households become less responsive to the rise in the monetary policy interest rate triggered by the the increase in GDP and inflation, and respond positively to the increase in their real wealth associated with the increase in government debt. Therefore the policy interest rate has to increase by more in order to achieve the a similar crowding out as in the absence of preferences over wealth. The increase is larger and more persistent the more persistent the government spending increase, as reflected by the rise of the 10 year yield implied by agent's expectations of future short

[6] The authors report that virtually all of the officers in their sample have a college degree, while according to the Current Population survey the same was true for only 24.5% of individuals in the same age group.

term rates. Without curvature in preferences over wealth and thus no wealth in the unconstrained household Euler equation ($\tilde{\sigma}_B = 0$, not shown), the macroeconomic effects are very similar, while required increase in the monetary policy interest rate is smaller.

Figure 1: Impact fiscal multiplier outside the ZLB



Note: Effects of increasing government spending by 1% of GDP outside the zero lower bound. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. Government spending follows an AR(1) process with persistence λ. The calibration is as in Table 2, with $\tilde{\sigma}_B = 0.2$.

I now examine the effect of an increase in government spending in the presence of the zero lower bound. For that purpose, I assume that an exogenous decline in the natural rate of interest hits the economy, i.e. r_t^{net} takes a negative value r^L, with the superscript L denoting the "low state" of the economy, following Eggertsson (2008) and Woodford (2011). This decline in the natural rate is sufficiently big to reduce the policy rate to its lower bound . Furthermore, with probability μ, r_t^{net}will continue to equal r^L in the following quarter, while with probability $(1-\mu)$, it will return to zero, and is expected to remain there forever. Throughout this section and the next, I will restrict the analysis to values of $\mu$for which the equilibrium is stable and unique, given the values of the other parameters. I then assume that government spending increases for the duration of the low state by $\hat{G}_L$. The increase in government expenditure is assumed to be too small to achieve and exit of the economy from the ZLB. After the exit from the low state, in each quarter there is a probability $1-\lambda$ that government spending returns to its steady state, and in this event is expected to remain there forever. The effect of this path for government expenditure on the economy is described by the following set of equations:

$$\begin{aligned}\hat{Y}_{L,t} - \left(\sigma_g \hat{G}_L - \sigma_T \hat{T}_{L,t}\right) = \tilde{\sigma}\left[\theta\left[\left(\mu_L \hat{\Pi}_{L,t+1} + (1-\mu_L)\,\hat{\Pi}_{S,t+1}\right) + r_L\right] + (1-\theta)\tilde{\sigma}_B \hat{b}_{t,L}\right] \quad (26)\\ + \theta\left(\mu_L \hat{Y}_{L,t+1} + (1-\mu_L)\,\hat{Y}_{S,t+1}\right)\\ - \theta\left(\left(\mu_L + (1-\mu_L)\,\lambda\right)\sigma_g \hat{G}_L - \sigma_T\left[\mu_L \hat{T}_{L,t+1} + (1-\mu_L)\hat{T}_{S,t+1}\right]\right)\end{aligned}$$

$$\hat{\Pi}_{L,t} = \kappa(\hat{Y}_{L,t} - \Gamma\hat{G}_L) + \beta\left[\mu_L \hat{\Pi}_{L,t+1} + (1-\mu_L)\,\hat{\Pi}_{S,t+1}\right] \quad (27)$$

$$\hat{T}_{L,t} = \tau_y \hat{Y}_{L,t} + \tau_b \hat{b}_{L,t-1} \quad (28)$$

$$\hat{b}_{L,t} = \frac{R}{\Pi}\left(\hat{b}_{L,t-1} - \frac{b}{Y}\left(\hat{\Pi}_{L,t}\right)\right) + \hat{G}_L - \hat{T}_{L,t} \quad (29)$$

where L refers to the low state and the subscript $S, t+1$ refers to the value of a variable in the first quarter outside the low state, assuming that the economy has been in the low state in quarter t, where monetary policy follows its off-ZLB interest feedback rule. Furthermore, a hat now refers to the effect of setting government expenditure to $\hat{G}_L$on the deviation of that variable from its steady state.[7]

A useful special case is a version of the model without credit constrained households and linear preferences over wealth ($\omega = 0$ and thus σ_T=0, $\sigma_g = 1$, and $\tilde{\sigma} = \sigma$), as under these assumption government debt does not matter for the equilibrium values of GDP and inflation, and equations equations (26) and (27) simplify to

$$\hat{Y}_L - \hat{G}_L = \frac{\sigma\theta\left(\mu_L \hat{\Pi}_L + \lambda(1-\mu_L)\,\gamma_{\Pi G}\hat{G}_L\right) + \lambda\theta(1-\mu_L)(\gamma_{YG} - 1)\,\hat{G}_L}{(1-\theta\mu_L)} \quad (30)$$

$$\hat{\Pi}_L = \frac{\kappa(\hat{Y}_L - \Gamma\hat{G}_L) + \lambda\beta(1-\mu_L)\,\gamma_{\Pi G}\hat{G}_L}{1-\beta\mu_L} \quad (31)$$

where $\gamma_{\Pi G}$ and γ_{YG} denote the effects of government expenditure on inflation and GDP outside the low state, as plotted in the left column of Figure 1.

[7] This reinterpretation of the hat notation allows me to drop $\hat{R}_{L,t}$ and r_L from the exposition, as they are by assumption unaffected by government expenditure.

We first consider the case of a stimulus expected to last only as long as the economy's low state, i.e. $\lambda = 0$, and the special case of the model described by the aggregated demand and aggregate supply equation (30) and (31) (see the left column of Figure 2)). For $\theta = 1$, as shown by Woodford (2011), the multiplier is larger or equal than one and increases exponentially the expected duration of the low state D_L, for the following reasons. With a zero probability of the low state persisting into the next period ($\mu_L = 0 \Longleftrightarrow D_L = 1$) increasing government expenditure leaves all t+1 variables unchanged. Hence the expected sum of future inflation and (since the nominal interest rate is fixed) real interest rates $\frac{\mu_L \hat{\Pi}_L}{(1-\theta\mu_L)}$ in the aggregate demand equation increase (30) remains at zero. By contrast, if the low state and the fiscal expansion are expected to persist with some probability ($\mu_L > 0 \Longleftrightarrow D_L > 1$), the expected sum of future real interest rates declines and unconstrained household consumption is crowded in. The associated higher GDP increases the expected sum of future output gaps $\frac{\kappa(\hat{Y}_L - \Gamma\hat{G}_L)}{1-\beta\mu_L}$ in the aggregate supply relation (31) and thus $\hat{\Pi}_L$, which feeds back into aggregated demand (31), thus accelerating the increase in $\frac{\mu_L \hat{\Pi}_L}{(1-\theta\mu_L)}$ and thus GDP. The interaction between these two infinite sums gives rise to the exponential relationship between the multiplier and D_L displayed in Figure 1.

However, the increase in the multiplier is smaller for lower values of θ (left panel of Figure 2). For instance, for $D_L = 10$, the multiplier equals 2.1 without preferences over wealth, but only 1.5 for $\theta = 0.96$. With $\theta < 1$, households attach an exponentially declining weight to future periods, implying that importance of future real interest rates for current consumption of forward looking households declines exponentially the further away from the current quarter they are located in time (see equation (30))). Lowering θ also lowers the effect of the future output gap on current inflation by lowering β (see equation 31), though the attenuation of this mechanism is quantitatively less important.

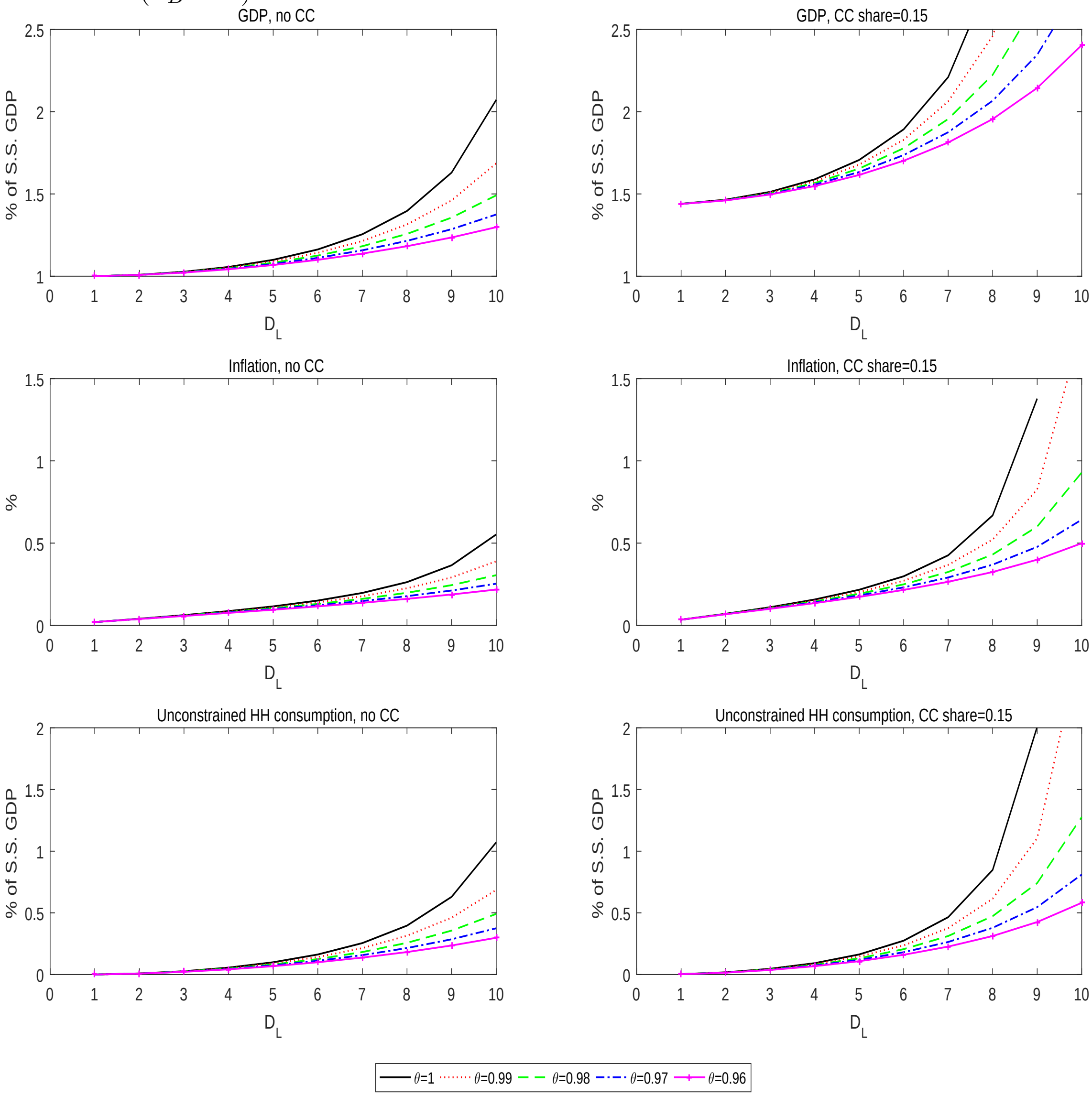
Adding credit constraint households to the model ($\omega = 0.15$, right panel of Figure 2) increases the fiscal multiplier for all values of μ_L and θ, and also raises the effect of increasing D_L (i.e. the slope of the plotted line). A (debt-financed) fiscal stimulus directly increases the consumption of unconstrained households by increasing employment and thus disposable income (i.e. $\sigma_g > 1$ for $\omega > 0$ in equation 26), implying that the multiplier exceeds one even for $D_L = 1$. Furthermore, any decline in the real interest rate now has a bigger impact on aggregate demand (as $\tilde{\sigma} > \sigma$).[8] As a result, the attenuation of the relationship between D_L and the multiplier achieved by lowering θ below one is also much bigger.

Allowing for curvature ($\tilde{\sigma}_B = 0.2$) further reduces the fiscal multiplier in the presence of constrained households (see Figure 3) further for values of D_L>7. With the high fiscal multipliers and the associated strong increase in inflation, the government deficit and real government debt $\hat{b}_t$ decline in response to the government spending increase. The decline in their real wealth motivates unconstrained households to save more, thus limiting their consumption crowding in. Thus for $\theta \leq 0.97$, the crowding of unconstrained household consumption and thus the GDP increase become

[8] Under the assumption that government debt is repaid by taxes on unconstrained households alone, adding constrained households would imply that equation 30 would become $\hat{Y}_L - \sigma_g \hat{G}_L = \frac{\tilde{\sigma}\theta\left(\mu_L \hat{\Pi}_L + r_L - \hat{R}_L\right)}{(1-\theta\mu_L)}$.

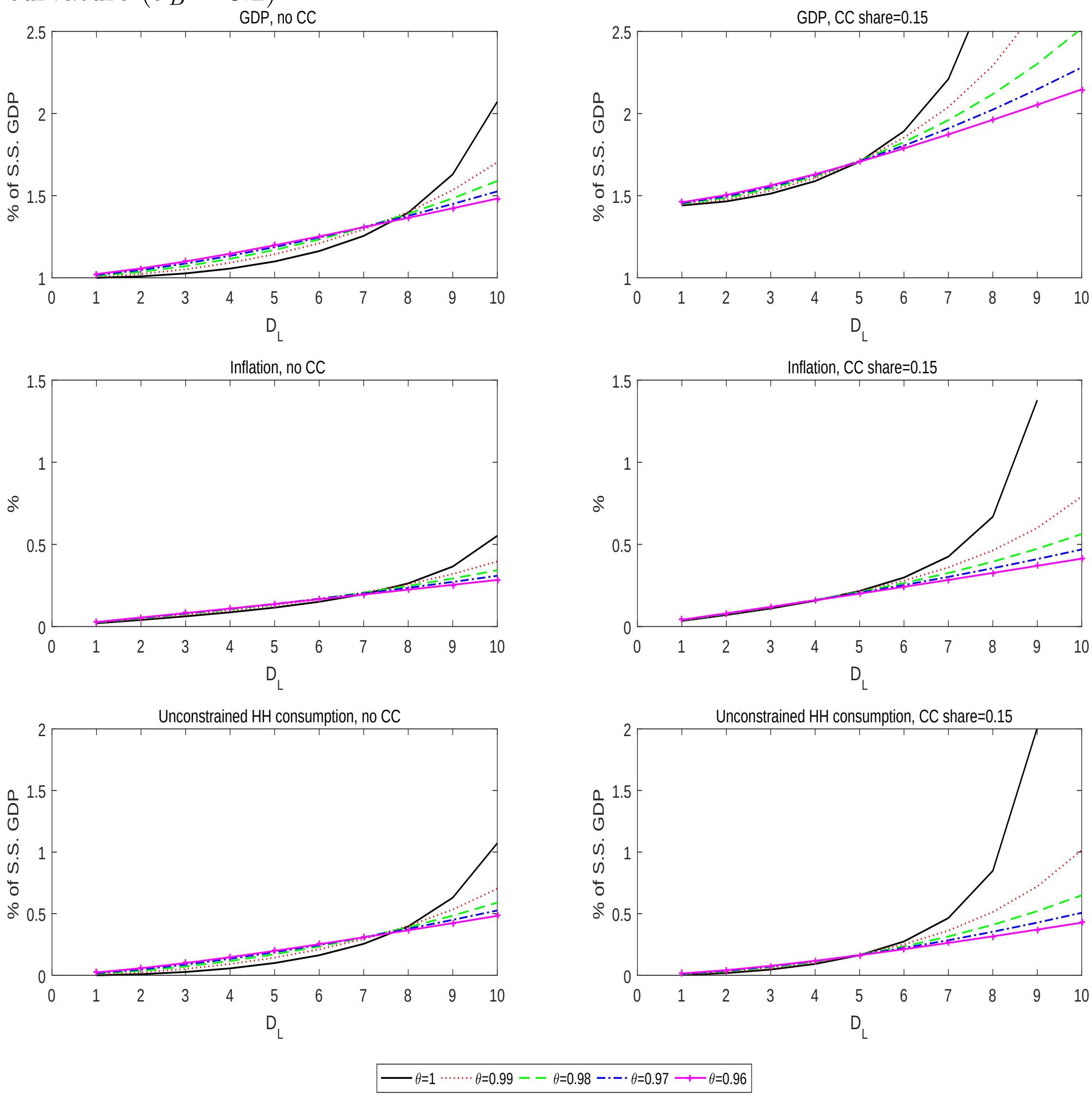
approximately linear in the expected duration of the low state D_L.

Figure 2: Impact fiscal multiplier during low state, short term stimulus($\lambda = 0$), no curvature ($\tilde{\sigma}_B = 0$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP during the low state only ($\lambda = 0$). GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis depicts the expected duration of the low state $D_L = \frac{1}{1-\mu_L}$. All other parameters are as in Table 2. For details on the computation of the displayed values, see Appendix 11.

Figure 3: Impact fiscal multiplier during low state, short term stimulus($\lambda = 0$), with curvature ($\tilde{\sigma}_B = 0.2$)



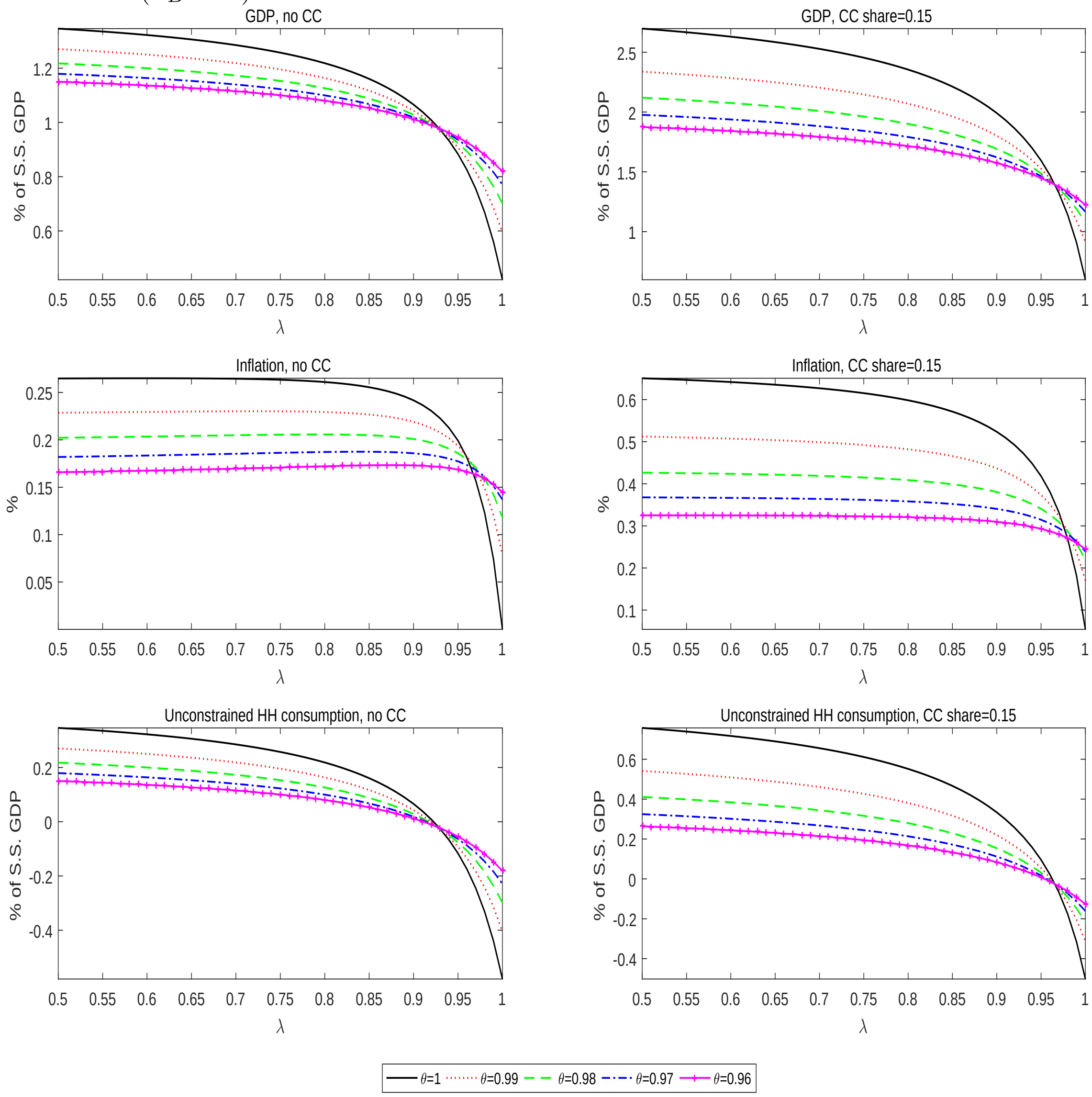
Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP during the low state only ($\lambda = 0$). GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis depicts the expected duration of the low state $D_L = \frac{1}{1-\mu_L}$. All other parameters are as in Table 2. For details on the computation of the displayed values, see Appendix 11.

I now examine the case where the increase in government expenditure is expected to persist beyond the duration of the low state (i.e. $\lambda > 0$), abstracting again from constrained households and assuming linear preferences over wealth (see Figure 4, left panel). This reason is that outside the zero lower bound, an increase in government expenditure crowds out private consumption (Figure 1), the more so the more persistent it is expected to be, which via consumption smoothing tends to lower consumption during the low state. In equation (30), the expression $\lambda\theta\,(1-\mu_L)\,(\gamma_{YG}-1) < 0$ captures this mechanism. The expression is strictly negative as the fiscal multiplier outside the zero lower bound γ_{YG} is strictly smaller than one and declines in λ. Therefore, without preferences over wealth, and $\lambda = 0$, 0.8 and 1 (i.e. a permanent increase), the multiplier equals 1.4, 1.2 and 0.4, respectively.

By contrast, for $\theta < 1$, the multiplier is much less sensitive to increasing λ. For instance, for $\theta = 0.96$, values of $\lambda = 0$, 0.8 and 1 correspond to multipliers of 1.2, 1.1 and 0.8. The reason is that for $\theta < 1$, the household attach a smaller weight to the low consumption/ high marginal utility state they will enter upon the exit from the low state and thus smooth their consumption less. Furthermore, there is also less consumption crowding out outside the low state with $\theta < 1$ (Figure 1). As a result of theses mechanisms, the magnitude of the increase in inflation and the associated fall in the real interest rate during the low state also declines less in λ than for $\theta = 1$, which tends to increase consumption as well.

In the presence of constrained households ($\omega = 0.15$), the impact of λ on the multiplier in the low state is even bigger, as the real interest rate sensitivity of GDP is larger, and thus the decline in the inflationary effect during the low state is associated with a much bigger decline of the aggregate demand effect through the real interest rate channel. For $\theta = 1$, the multiplier decreases from 2.7 to 0.6 asλ moves from zero to one. However, the decline of the multiplier is smaller for $\theta < 1$, as the aforementioned decline in the magnitude of the real interest rate drop is smaller as well. For instance, for $\theta = 0.96$, the multiplier decreases merely from 2.0 to 1.2 (see Figure 4, right panel).

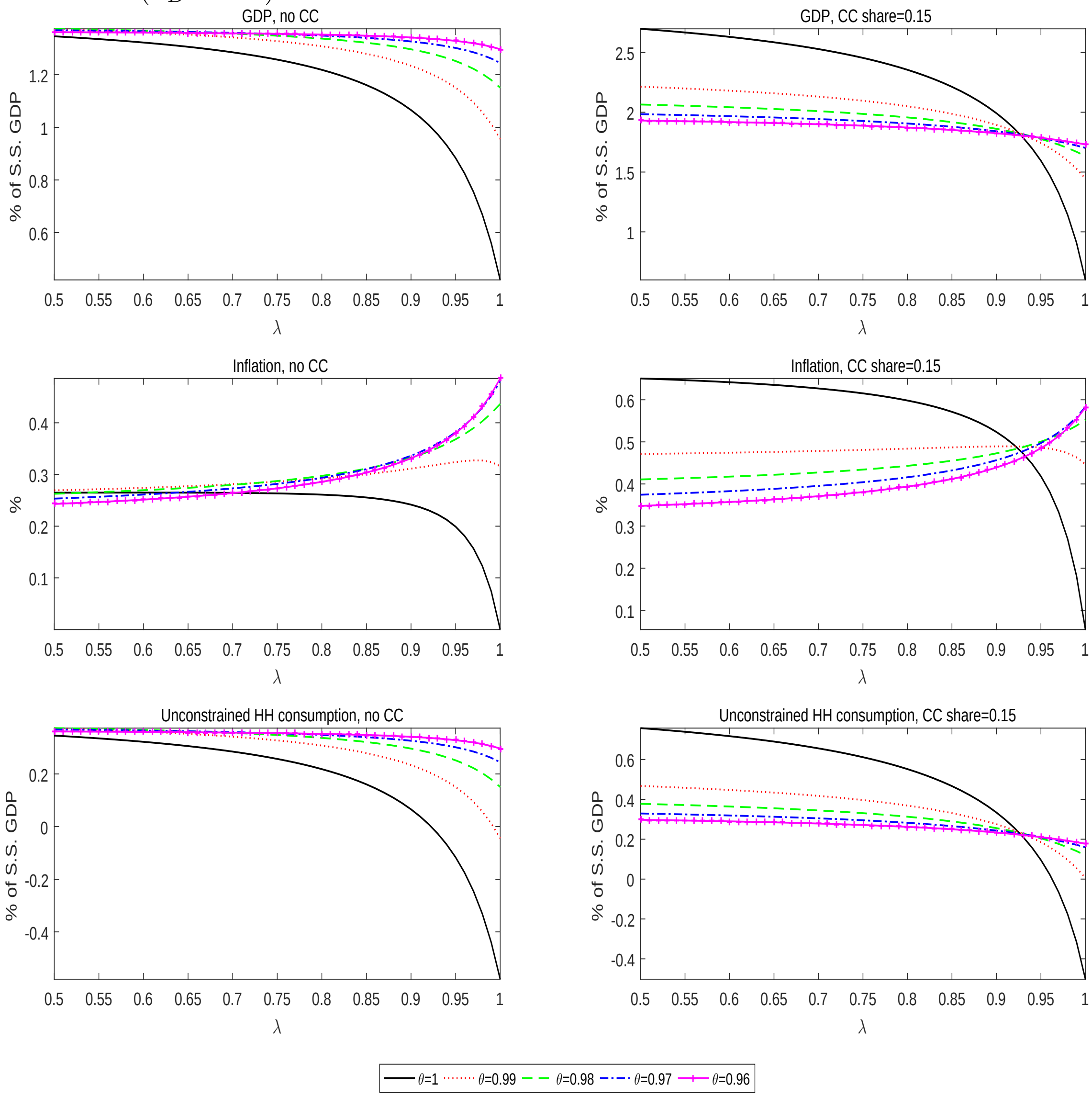
Figure 4: Impact fiscal multiplier during low state, persistent increase($\lambda \geq 0$), no curvature ($\tilde{\sigma}_B = 0$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis displays the probability λ that the stimulus persists after the economy's exit from the low state and thus the ZLB. $D_L = 8$ (i.e.$\mu_L = 0.875$). All other parameters are as in Table 2.

With curvature in preferences over wealth (see Figure 5), the dependence of the multiplier on the persistence of the government spending increase is reduced further. For $\theta = 0.96$, and no unconstrained households, values of $\lambda = 0$, 0.8 and 1 yield multipliers of 1.4, 1.4 and and 1.3, while with credit constrained households, the corresponding multipliers are 1.9, 1.9 and 1.7. The increase in government debt associated with a persistent increase in government consumption increases the real wealth of unconstrained households, which motivates them to spend more both inside the low state (equation 26) and after the exit from the low state . As a result, for persistent spending increases, their consumption is higher than without curvature.

Figure 5: Impact fiscal multiplier during low state, persistent increase($\lambda \geq 0$), with curvature ($\tilde{\sigma}_B = 0.2$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis displays the probability λ that the stimulus persists after the economy's exit from the low state and thus the ZLB. $D_L = 8$ (i.e.$\mu_L = 0.875$). All other parameters are as in Table 2.

With preferences over wealth, the multipliers of permanent and temporary increases in government expenditure thus become much more alike if the zero lower bound is binding. Table 4 summarizes this result by displaying the multipliers of both a perfectly timed and a permanent change in government consumption and an expected length of the zero lower bound state of 8 quarters ($D_L = 8$). Furthermore, for a permanent fiscal expansion, adding credit constrained households makes

Table 4: Impact fiscal multiplier, $D_L = 8$

	No constrained HH	
	Perfectly timed ($\lambda = 0$)	Permanent($\lambda = 1$)
$\theta = 1$	1.4	0.4
$\theta = 0.96, \tilde{\sigma}_B = 0$	1.2	0.8
$\theta = 0.96, \tilde{\sigma}_B = 0.2$	1.4	1.3
	Constrained HH=0.15	
	Perfectly timed($\lambda = 0$)	Permanent($\lambda = 1$)
$\theta = 1$	2.8	0.6
$\theta = 0.96, \tilde{\sigma}_B = 0$	2.0	1.2
$\theta = 0.96, \tilde{\sigma}_B = 0.2$	2.0	1.7

a bigger difference for the multiplier with preferences over wealth than without. The main reason is that with preferences over wealth, the increase in the total wage bill is bigger than without even in the absence of credit constrained households, implying that adding constrained households whose consumption equals their labour income implies a bigger demand increase. Furthermore, adding credit constrained households increases the crowding out of unconstrained household consumption after the exit from the low state, which via consumption smoothing implies lower consumption than without credit constrained households during the low state as well, but less so with preferences over wealth since the future is discounted more heavily.

While the model is sufficiently simple to illustrate the key mechanisms underlying my results -discounting in the Euler equation and the direct effect of wealth on unconstrained household consumption-, constrained households and/ or declining marginal utility from wealth imply that the model has an endogenous state variable, namely government debt. In Appendix 12, I show that in an analytically solvable model which assumes linear preferences over wealth and a fiscal policy where government debt accumulated during the low state is funded by taxes on unconstrained agents alone, the results are very similar to the $\tilde{\sigma}_B = 0$ case just discussed.

5 Forward guidance

I now examine the effects of forward guidance regarding the path of the short term interest rate. Specifically, I assume that during the low state, the central bank via its announcements creates the expectation that once the economy exits the low state, it will move the policy interest rate by an amount $\hat{R}_f < 0$ away from its trajectory in the absence of the policy. Furthermore, it will keep this policy in place in subsequent periods with probability μ_f, while with probability $1-\mu_f$ it will revert to its off-ZLB interest feedback rule, with $D_f = \frac{1}{1-\mu_f}$ denotes the expected length of the forward guidance episode. I restrict attention to values of μ and μ_f for a unique and stable equilibrium exists given the other values of the parameters. For future reference, it is again useful to describe the special case without credit constrained households and linear preferences over wealth ($\omega = 0$ and thus σ_T=0, $\sigma_g = 1$, and $\tilde{\sigma} = \sigma$). Under these assumptions the effect of forward guidance on GDP and inflation during the low state are determined by

$$\hat{Y}_L = \frac{\sigma\theta\mu_L\hat{\Pi}_L + \theta(1-\mu_L)\hat{Y}_f}{1-\theta\mu_L} \tag{32}$$

$$\hat{\Pi}_L = \frac{\kappa\hat{Y}_L + \beta(1-\mu_L)\hat{\Pi}_f}{1-\beta\mu_L} \tag{33}$$

$$\hat{Y}_f = \frac{\sigma\theta\left(\mu_f\hat{\Pi}_f - \hat{R}_f\right)}{1-\theta\mu_f} \tag{34}$$

$$\hat{\Pi}_f = \frac{\kappa\hat{Y}_f}{1-\beta\mu_f} \tag{35}$$

Equations (32) and (33) are the aggregate demand and aggregate supply equations during the low state, which determine $\hat{Y}_L$ and $\hat{\Pi}_L$as a function of $\hat{Y}_f$ and $\hat{\Pi}_f$, the level of GDP and inflation during the state where the announced policy is actually implemented. For a given $\hat{R}_f$, the larger D_f and thus μ_f, the higher$\hat{Y}_f$ and $\hat{\Pi}_f$.

To investigate the effect of increasing the expected length of the forward guidance episode D_f (where $D_f = \frac{1}{1-\mu_f}$), I assume an expected length of the low state $D_L = 6$. This assumption is consistent with the evidence provided by Del Negro et al. (2015) on financial market expectations regarding the timing of the exit of the Federal Funds rate from the ZLB prior to the forward guidance announcements of the US Federal Reserve in September 2011, January 2012 and September 2013, as well as their evidence on the announcement's effect on private sector forecasts of three month and 10 year treasury bonds. Furthermore, I assume that the central bank sets $\hat{R}_f$ to an annualized value of -0.2%, which is also in line with the evidence of Del Negro et al. (2015).

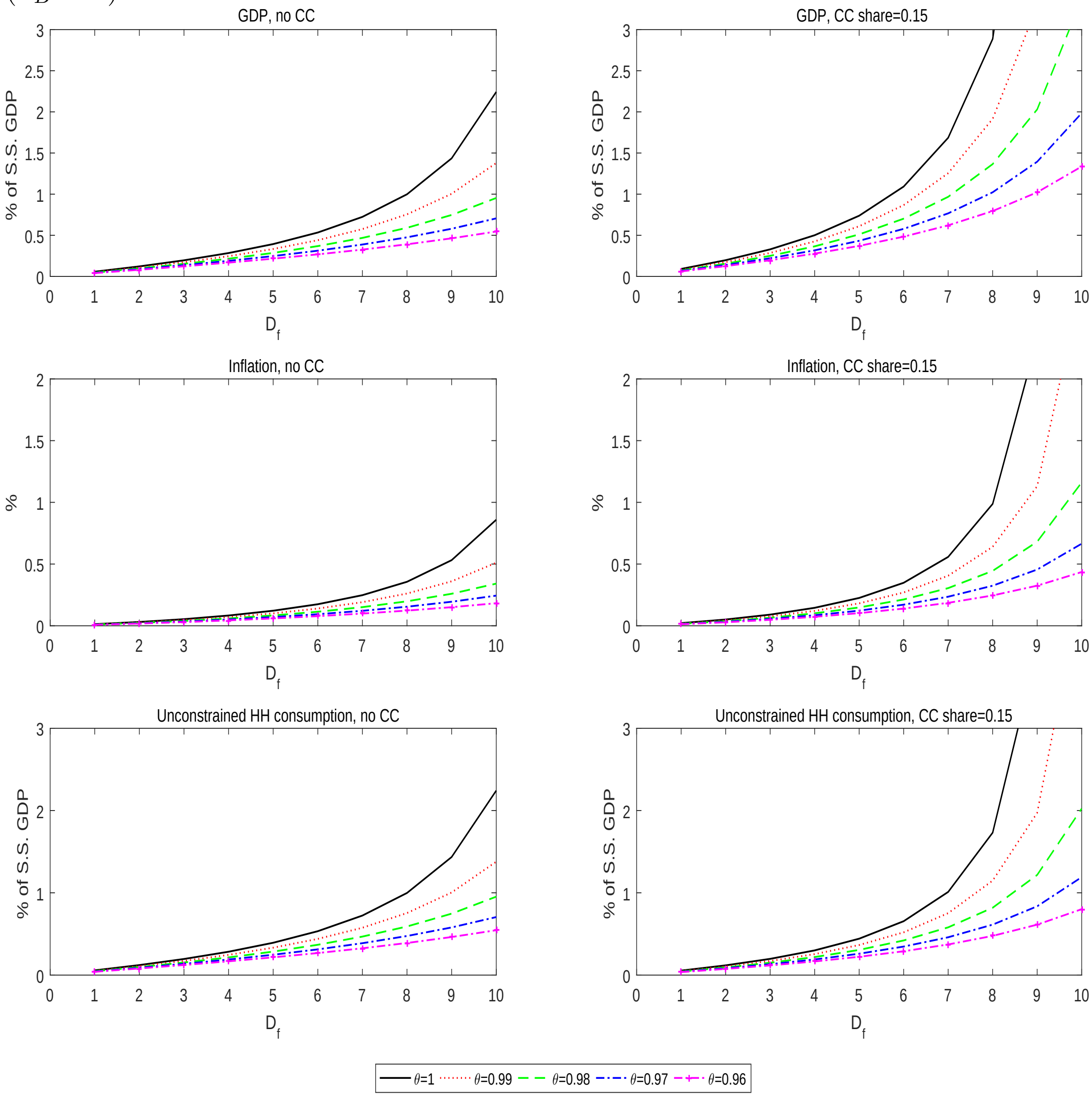
As can be obtained from Figure 6, with no preferences over wealth and no credit constrained households ($\omega = 0$), the impact effect of the policy on GDP and inflation in the low state increase exponentially in the expected length of the forward guidance episode D_f. As in the analysis of a perfectly timed fiscal stimulus, the reason for this exponential relationship is the interaction between the expected sum of real interest rates and the expected sum of future output gaps in the aggregate demand and supply relationships. With credit constrained households and the associated greater interest rate sensitivity of aggregate demand, the effect of forward guidance on $\hat{Y}_L$ and $\hat{\Pi}_L$ is substantially magnified. The result mirrors the finding of Carlstrom et al. (2015), who analyze a deterministic forward guidance policy.

However, in the presence of linear preferences over wealth ($\theta < 1$ and $\tilde{\sigma}_B = 0$) and the associated lower weight households and firms attach to future periods, the increase in GDP is much lower, especially for $\theta = 0.96$. The attenuation is even bigger in the presence of credit constrained households, both in absolute and relative terms.

Allowing for curvature in preferences over wealth ($\tilde{\sigma}_B > 0$), further reduces the effects of forward guidance (see Figure 7). The reason is that the forward guidance policy erodes unconstrained household real wealth by increasing the governments tax revenue and inflation, allowing the government to delever. With curvature, this decline in real wealth increases the marginal utility unconstrained households derive from saving, thus dampening the amount by which they increase their consumption in response to the policy and thus the increase in aggregate demand (see equations 19 and 6). Preferences over wealth thus seem to have the potential to alleviate the so called "Forward guidance puzzle" documented by Del Negro et al. (2015) and Carlstrom et al. (2015).

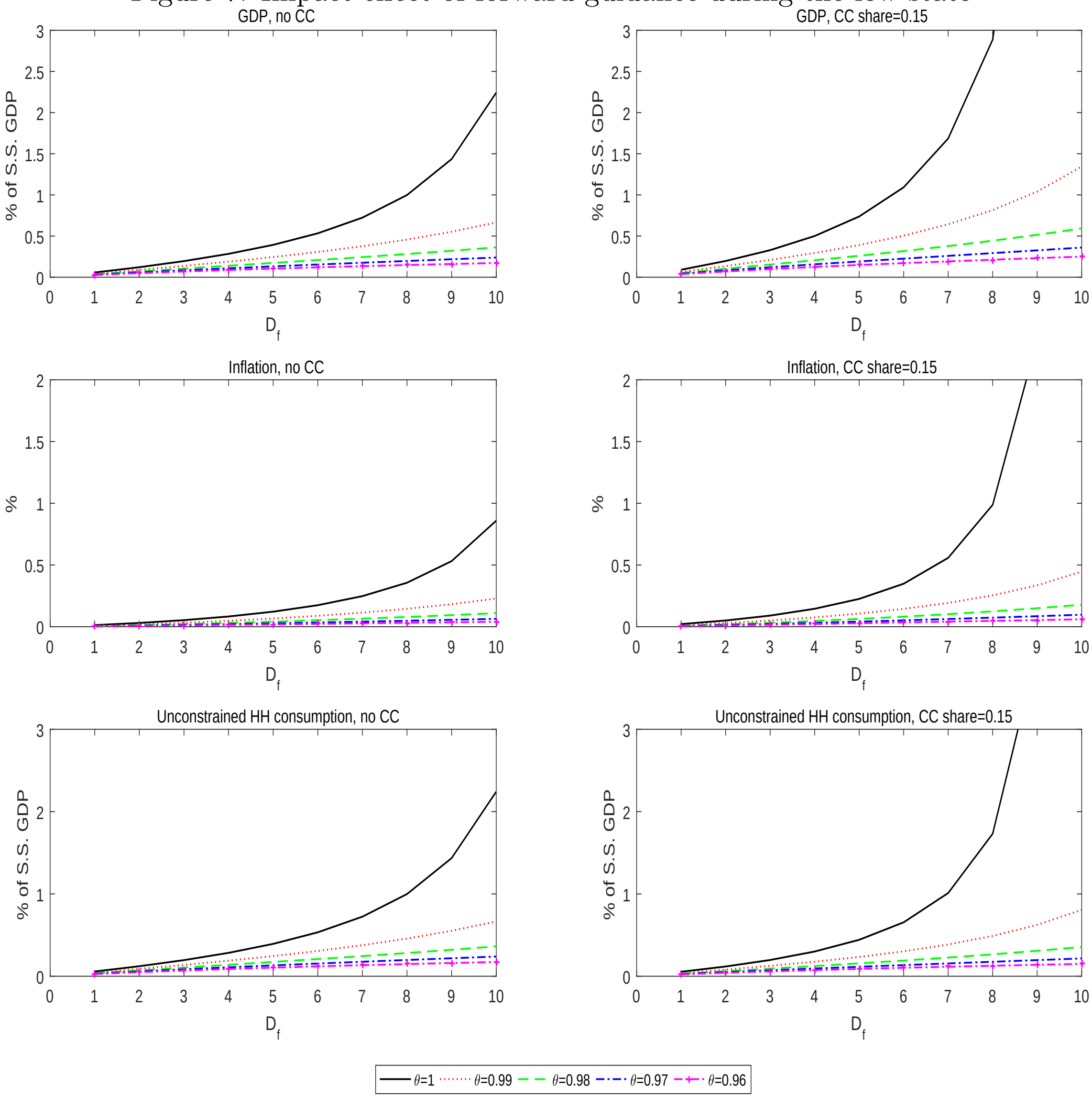
For the aforementioned analytically tractable version of the model with linear preferences over wealth and government debt funded by unconstrained households, results are similar to the model without curvature (see Appendix 12) .

Figure 6: Impact effect of forward guidance during the low state, no curvature ($\tilde{\sigma}_B = 0$)



Note: Effects on output and annualized inflation in the low state of fixing the annualized interest rate at 0.2% below its steady state level once the economy has exited the low state. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis depicts the expected length of the fixed interest rate policy D_f (where $D_f = \frac{1}{1-\mu_f}$). The expected length of the low state is 6 quarters (i.e. $\mu_L = 0.83$). All other parameters are as detailed in Table 2.

Figure 7: Impact effect of forward guidance during the low state



Note: Effects on output and annualized inflation in the low state of fixing the annualized interest rate at 0.2% below its steady state level once the economy has exited the low state. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis depicts the expected length of the fixed interest rate policy D_f (where $D_f{=}\frac{1}{1-\mu_f}$). The expected length of the low state is 6 quarters (i.e. $\mu_L = 0.83$). All other parameters are as detailed in Table 2.

6 The impact of preferences over wealth in a medium scale model

In the appendix I show the impact of preferences over wealth on the effects of fiscal policy and forward guidance carry over to a richer, quantitative model, with both credit constrained households and firms. Like the simple model of the previous section, it features constrained and rule of thumb (or credit constrained) households.

Unconstrained households save in the form of government bonds and safe deposits issued be financial intermediaries, and have preferences over wealth. Both household types supply labor to retailers via a labor union, which sets a common wage for both.

On the firms side, an entrepreneurial sector owns and accumulates the capital stock, and obtains external funds in the form of one period loans from a financial intermediary. The cost of external finance of entrepreneurs increases in their leverage due to a costly state verification (CSV) problem as in the Bernanke et al. (1999) financial accelerator model, implying that the investment response to any shock which increases the present discounted value of a unit of capital is amplified compared to a model without a financial accelerator. Since entrepreneurs consume their net worth once they die, the financial accelerator also creates a link between asset prices and consumption.

Entrepreneurs rent their capital stock to retailers. Retailers produce the homogeneous output good employing physical capital and labor in a Cobb Douglas technology. Investment good producers owned by unconstrained households convert the output good into new capital goods and sell them to entrepreneurs. Investment goods producers are subject to investment adjustment costs, while retailers and unions are subject to nominal rigidities in the form of Calvo contracts. The full set of linearized equations can be obtained from Appendix 14. I estimate key parameters by matching the Impulse response functions to a monetary policy and a fiscal policy shock from the identified VAR model estimated by Blanchard et al. (2015) on Euro Area data.

In the simulations of the effects of fiscal policy and forward guidance, I proxy the zero lower bound by switching off the central banks interest feedback rule for a specific period, and conduct the simulations under perfect foresight. Regarding fiscal policy, I find that the effects of a government spending change lasting as long as the period during which monetary policy is constrained is similar both in the presence and in the absence of preferences over wealth. For a permanent change, as in the simple model, the fiscal multiplier is substantially lower without preferences over wealth due to crowding out of unconstrained household consumption. Therefore, with a permanent change, adding credit constrained households and firms to the model without preferences over wealth increases the multiplier only slightly, as the employment increase which determines the increase in constrained household consumption is low, and the low overall GDP effect implies a small decline in Tobin's Q. By contrast, with preferences over wealth, the multipliers of temporary and permanent changes in government expenditure become very similar, and the increase in the multiplier associated with adding credit constrained households and firms is substantially bigger.

Regarding the effect of forward guidance, I confirm the results from the stylized model that preferences over wealth strongly attenuate the effect of forward guidance, especially so with declining marginal utility from wealth. Via higher higher inflation and tax revenues, the forward guidance policy redistributes real wealth from unconstrained households to the government and entrepreneurs, which the increase in unconstrained household consumption.

7 Conclusion

This paper examines how the effects of fiscal policy and forward guidance are shaped by preferences over wealth calibrated based on microeconomic evidence on household saving behavior and individual discount rates in a stylized and a medium scale DSGE model. This assumption effectively limits the horizon of unconstrained households, as the intrinsic benefit of wealth over and above allowing more future consumption implies that the equilibrium real interest rate is smaller than the individual discount factor of the household. Furthermore, real wealth enters the unconstrained household Euler equation. Therefore, the contractionary effect of a permanent cut in government expenditure implemented during a period when monetary policy is constrained by the zero lower bound becomes larger due to smaller crowding in of unconstrained household consumption.

Furthermore, preferences over wealth much reduce the effect of forward guidance on the future policy interest rate. Once I allow for curvature in wealth, the attenuation of the effect of forward guidance becomes even bigger as forward guidance policies imply a decline in unconstrained household real wealth, which motivates them to save more.

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8 Calibration of the curvature of household preferences over wealth

Following Kumhof et al. (2014), I calibrate the curvature of preferences over wealth σ_B by targeting an empirical estimate of the marginal propensity to save out of an increase in permanent income. I assume that constrained household disposable income$Y_{O,t}$ is exogenous, implying that its budget constraint is given by

$$B_{O,t} + P_t C_{O,t} = R_{t-1} B_{O,t-1} + Y_{O,t} \tag{36}$$

The linearized partial equilibrium economy is described by the following equations:

$$\hat{\Lambda}_{O,t} = \frac{-\sigma_H \left(\hat{C}_{O,t} - h\hat{C}_{O,t-1}\right)}{1-h} \tag{37}$$

$$\hat{\Lambda}_{O,t} = \theta \left(\hat{\Lambda}_{O,t} + \hat{R}_t - \hat{\Pi}_{t+1}\right) - (1-\theta)\, \sigma_B \hat{b}_{O,t} \tag{38}$$

$$\hat{b}_{O,t} = \frac{R}{\Pi}\left(\hat{b}_{O,t-1} + \hat{R}_t - \hat{\Pi}_t\right) + \frac{Y\hat{Y}_{o,t} - C_O \hat{C}_{O,t}}{b_O} \tag{39}$$

Following Kumhof et al. (2014). I then assume that $\hat{Y}_{O,t}$ increases permanently in period one to a level $\hat{Y}_{O,P}$ and calculate the MPS over a six year horizon, holding inflation and the nominal interest rate constant. The reason for the six year horizon is that the empirical estimates of the MPS of Kumhof et al. (2014) and Dynant et al. (2004) uses data on saving rates which is six years apart. Hence the model counterpart of their empirical estimate of the MPS is given by

$$MPS = \frac{b_O \hat{b}_{O,24}}{24\hat{Y}_{O,P} Y_O} \tag{40}$$

9 Derivation of the stylized model

9.1 Households

Households seek to maximize the objective

constrained households do not receive profit income

$$\sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-1/\sigma_H}}{1-1/\sigma_H} - \chi N_{t+i}^{1+\eta} + \phi_B \left(\frac{B_{j,t+i}}{P_{t+i}} \right)^{1-\sigma_B} \right] \tag{41}$$

A fraction of households $(1-\omega)$has unconstrained access to financial markets and thus can vary its holding of safe bonds B_t. The remaining fraction ω of households is restricted to $B_t = 0$. A common wage is set for both households groups. Wage setting is discussed in the next section.

The budget constrained of optimizing households is (denoted with the subscript O) is given by

$$B_t + P_t C_{O,t} = R_{t-1} B_{t-1} + W_t N_{O,t} - P_t T_{O,t} \tag{42}$$

Maximizing (41) subject to (42) yields

$$\Lambda_{O,t} = \beta \frac{R_t}{\hat{\Pi}_{t+1}} \Lambda_{O,t+1} + \phi_B \left(\frac{B_{j,t+i}}{P_{t+i}} \right)^{1-\sigma_B} \tag{43}$$

$$\Lambda_{O,t} = C_{O,t}^{-1/\sigma_H} \tag{44}$$

while for ROT households we have

$$C_{ROT,t} = w_t N_{ROT,t} - T_{ROT,t} \tag{45}$$

$$\Lambda_{ROT,t} = C_{ROT,t}^{-1/\sigma_H} \tag{46}$$

Aggregate consumption is given by

$$C_t = (1-\omega) C_{O,t} + \omega C_{ROT,t} \tag{47}$$

10 Firms

There is a continuum of monopolistically competitive firms who set price subject to nominal rigidities in the form of Calvo pricing. The production technology of firm j is given by

$\mu_1 = \mu_L$ [The initial state is now the low state.]

$$Y_t(j) = N_t^{\alpha}(j) \tag{48}$$

Real marginal cost $mc_t$is given by

$$mc_t = \frac{w_t}{N_t^{\alpha-1}} \tag{49}$$

$$\pi_t = \kappa \hat{mc}_t + \beta \hat{\Pi}_{t+1} \tag{50}$$

10.1 Wage setting

Assume that there is a union maximizing the weighted utilities of ROT and optimizing households. Assume identical preferences. Union supplies labor variety i to labor packer, using labor from both households types. objective is given by

$$w_t(i)\left(\omega\Lambda_{ROT,t}N(i)_{ROT,t}+(1-\omega)\Lambda_{O,t}N(i)_{O,t}\right)+\omega\chi\frac{N(i)_{ROT,t}^{1+\eta}}{1+\eta}+(1-\omega)\chi\frac{N(i)_{O,t}^{1+\eta}}{1+\eta}$$

where$w_t(i)=\frac{W(i)_t}{P_t}$ Assumption: Firms higher household types on proportion to their population share, hence $N_{O,t}=N_{ROT,t}=N_t$. Hence

$$w_t(i)\Lambda_t N(i)_t+\chi\frac{N(i)_t^{1+\eta}}{1+\eta}$$

with

$$\Lambda_t=\omega\Lambda_{ROT,t}+(1-\omega)\Lambda_{O,t} \tag{51}$$

and

$$N(i)_t=\left(\frac{w_t(i)}{w_t}\right)^{-e_w}$$

The FOC is then given by (taking into account that all unions set the same wage)

$$w_t\Lambda_t=\chi N_t^{\eta}\mu_w \tag{52}$$

with $\mu_w>1$denoting the wage markup.

10.2 Log-linearization: Aggregate demand

Linearizing (43) yields

$$\hat{\Lambda}_{O,t}=\theta\left[\hat{R}_t-\hat{\Pi}_{t+1}+\hat{\Lambda}_{O,t}\right]+\sigma_B(1-\theta)\hat{b}_{O,t}$$

$$\hat{C}_{O,t}=-\theta\sigma_H\left[\hat{R}_t-\hat{\Pi}_{t+1}\right]+\theta\hat{C}_{O,t+1}+\sigma_H\sigma_B(1-\theta)\hat{b}_{O,t}$$

or

$$\hat{C}_{O,t}=-\theta\sigma_H\left[\hat{R}_t-\hat{\Pi}_{t+1}\right]+\theta\hat{C}_{O,t+1}+\sigma_H\tilde{\sigma}_B(1-\theta)\hat{b}_t \tag{53}$$

with $\tilde{\sigma}_B=\sigma_B\frac{Y}{b}$ and $\hat{b}_t=\frac{db_t}{Y}$

where a hat denotes the percentage deviation of a variable from its steady state, with the exception of government spending, where it is expressed as a percentage of steady state GDP.

GDP is given by

$$Y_t=C_t+G_t$$

Substituting (47) and linearizing yields

$$\hat{Y}_t Y = (1-\omega) C_O \hat{C}_{O,t} + \omega C_{ROT} \hat{C}_{ROT,t} + Y \hat{G}_t \tag{54}$$

Linearizing (45) and (48) as well as using $N_{O,t} = N_{ROT,t} = N_t$ yields

$$\hat{C}_{ROT,t} = \hat{w}_t + \frac{1}{\alpha} \hat{Y}_t - \hat{T}_{ROT,t} \tag{55}$$

Linearizing (51), (44) and (46) and using $C_O = C_{ROT}$ and $\Lambda_{ROT} = \Lambda_O$ yields yields

$$\hat{\Lambda}_t = -\frac{1}{\sigma_H} \hat{C}_t \tag{56}$$

Linearizing (52) and (48) and combining the result with (56) yields $\hat{w}_t - \frac{1}{\sigma_H} \hat{C}_t = \eta \hat{N}_t$

$$\hat{w}_t = \frac{\eta}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \hat{C}_t$$

or, using $\hat{C}_t = \frac{Y}{C} (\hat{Y}_t - \hat{G}_t)$

$$\hat{w}_t = \frac{\eta}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) \tag{57}$$

Substituting this equation into (55) yields

$$\hat{C}_{ROT,t} = \frac{\eta+1}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) - \hat{T}_{ROT,t} \tag{58}$$

Combining (54) and (58) yields $(1-\omega)\hat{C}_{O,t} = \left(\hat{Y}_t - \hat{G}_t\right) \frac{Y}{C} - \omega \left(\frac{\eta+1}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) - \hat{T}_{ROT,t} \right)$, or

$$(1-\omega)\hat{C}_{O,t} = \left(\hat{Y}_t - \hat{G}_t\right) \frac{Y}{C} \left(1 - \omega \frac{1}{\sigma_H}\right) - \omega \frac{\eta+1}{\alpha} \hat{Y}_t + \omega \hat{T}_{ROT,t} \tag{59}$$

Combining (59) and (53) yields

$$\begin{aligned} \left(\hat{Y}_t - \hat{G}_t\right) \left(1 - \omega \frac{1}{\sigma_H}\right) - \frac{C}{Y} \omega \frac{\eta+1}{\alpha} \hat{Y}_t + \omega \frac{C}{Y} \hat{T}_{ROT,t} \\ = -(1-\omega)\sigma_H \frac{C}{Y} \left[\theta \left(\hat{R}_t - \hat{\Pi}_{t+1} \right) + (1-\theta) \tilde{\sigma}_B \hat{b}_t \right] \\ +\theta \left(\left(\hat{Y}_{t+1} - \hat{G}_{t+1}\right) \left(1 - \omega \frac{1}{\sigma_H}\right) - \frac{C}{Y} \omega \frac{\eta+1}{\alpha} \hat{Y}_{t+1} + \omega \frac{C}{Y} \hat{T}_{ROT,t+1} \right) \end{aligned}$$

or

$$\begin{aligned} \hat{Y}_t \left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha}\right) - \hat{G}_t \left(1 - \omega \frac{1}{\sigma_H}\right) + \omega \frac{C}{Y} \hat{T}_{ROT,t} \\ = -(1-\omega)\sigma_H \frac{C}{Y} + i_\pi e_{i,S} \left[\theta \left(\hat{R}_t - \hat{\Pi}_{t+1} \right) + (1-\theta) \tilde{\sigma}_B \hat{b}_t \right] \\ +\theta \left(\hat{Y}_{t+1} \left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha}\right) - \left(1 - \omega \frac{1}{\sigma_H}\right) \hat{G}_{t+1} + \omega \frac{C}{Y} \hat{T}_{ROT,t+1} \right) \end{aligned}$$

or

$$\hat{Y}_t - \frac{\left(1-\omega\frac{1}{\sigma_H}\right)\hat{G}_t - \omega\frac{C}{Y}\hat{T}_{ROT,t}}{\left(1-\omega\frac{1}{\sigma_H}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)} = -\frac{(1-\omega)\sigma_H\frac{C}{Y}}{\left(1-\omega\frac{1}{\sigma_H}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}\left[\theta\left(\hat{R}_t-\hat{\Pi}_{t+1}\right)+(1-\theta)\tilde{\sigma}_B\hat{b}_t\right]$$
$$+\theta\left(\hat{Y}_{t+1} - \frac{\left(1-\omega\frac{1}{\sigma_H}\right)\hat{G}_{t+1} - \omega\frac{C}{Y}\hat{T}_{ROT,t+1}}{\left(1-\omega\frac{1}{\sigma_H}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}\right)$$

10.3 Phillips Curve

Linearizing and combining equations (49) and (48) yields $\hat{mc}_t = \hat{w}_t + \frac{1-\alpha}{\alpha}\hat{Y}_t$. Substituting (57) yields

$$\hat{mc}_t = \frac{1}{\sigma}(\hat{Y}_t - \hat{G}_t) + \frac{1-\alpha+\eta}{\alpha}\hat{Y}_t = \left(\eta_v + \frac{1}{\sigma}\right)\hat{Y}_t - \frac{1}{\sigma}\hat{G}_t = \left(\eta_v + \frac{1}{\sigma}\right)\left(\hat{Y}_t - \Gamma\hat{G}_t\right)$$

This equation is identical to the marginal cost schedule in Woodford (2011), equation. (3.7), implying that the Phillips curve is identical as well, with $\sigma = \frac{u'(C_O)}{u''(C_O)Y}$, $\eta_v = \frac{1-\alpha+\eta}{\alpha}$and $\Gamma = \frac{\frac{1}{\sigma}}{\frac{1}{\sigma}+\eta_v}$.

Furthermore, note that for $\hat{mc}_t = 0$, we have $\hat{Y}_t = \hat{Y}_t^*$, implying that

$$\hat{Y}_t^* = \Gamma\hat{G}_t \tag{60}$$

11 Note on the solution of the model used in the main text

To solve the simple model, I cast it in a form displayed as equations (61) to (73) below, where

- A hat now indicates the effect of a policy on the percentage deviation of the respective variable from its steady state (allowing us to disregard the risk premium shock which induces the ZLB situation).
- $e_{F,t}$ and $e_{G,t}$ denote one-off shocks with $E_t e_{F,t} = E_t e_{G,t} = 0$, and a standard deviation of 1.
- Subscript 1 (2) indicates the initial (subsequent) state.
- Variables with subscript F are relevant for the forward guidance policy only. They represent the effect of pegging the policy rate at $e_{F,t}$ during the first period of the peg.
- $\mu_1$denotes the probability of remaining in the initial state in period t+1, while $(1-\mu_1)$ denotes the probability of moving to the second state. No return to the first state is expected.

- D_{ZLB} is a dummy variable (in Dynare: a parameter) with $D_{ZLB} = 1$ indicating the ZLB state, or -with forward guidance- the state where the central bank pegs the interest rate.

- The$\gamma_{..}$ coefficients denote either the effect of an an exogenous (i.e. $\hat{G}_{1,t}$) or endogenous state variable (e.g. $\hat{b}_{1,t-1}$ and $\hat{R}_{1,t-1}$) on $\hat{Y}_{2,t}$, $\hat{\Pi}_{2,t}$ and $\hat{T}_{2,t}$. They affect the solution during state 1 iff $\mu_1 < 1$.

$$\hat{Y}_{1,t} - \left(\sigma_g \hat{G}_1 - \sigma_T \hat{T}_{1,t}\right) = -\tilde{\sigma}\left[\theta\left[\hat{R}_t - \left(\mu_1 \hat{\Pi}_{1,t+1} + (1-\mu_1)\,\hat{\Pi}_{2,t+1}\right)\right] - (1-\theta)\tilde{\sigma}_B \hat{b}_t\right] + \theta\left(\mu_1 \hat{Y}_{1,t+1} + (1 - \right. \tag{61}$$

with $\sigma_g = \frac{\left(1-\omega\frac{1}{\sigma_H}\right)}{\left(1-\omega\frac{1}{\sigma_H}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$, $\sigma_T = \frac{\omega\phi_{ROT}}{\left(1-\omega\frac{1}{\sigma_H}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$, $\sigma = \sigma_H \frac{C}{Y}$, $\tilde{\sigma} = \frac{\sigma(1-\omega)}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$

$$\hat{\Pi}_{1,t} = \kappa(\hat{Y}_{1,t} - \Gamma\hat{G}_{1,t}) + \beta\left[\mu_1 \hat{\Pi}_{1,t+1} + (1-\mu_1)\,\hat{\Pi}_{2,t+1}\right] \tag{62}$$

$$\hat{G}_{1,t} = \rho\hat{G}_{1,t-1} + e_{G,t} \tag{63}$$

$$\hat{T}_{1,t} \;\; = \;\; \tau_y \hat{Y}_{1,t} + \tau_b \hat{b}_{1,t-1} \tag{64}$$

$$\hat{b}_{1,t} = \frac{R}{\Pi}\left(\hat{b}_{1,t-1} + \frac{b}{Y}\left(\hat{R}_{1,t-1} - \hat{\Pi}_{1,t}\right)\right) + \hat{G}_{1,t} - \hat{T_{1,t}} \tag{65}$$

$$\hat{R}_{1,t} = (1 - D_{ZLB})\left[\phi_\pi \hat{\Pi}_{1,t} + \phi_y(\hat{Y}_{1,t} - \Gamma\hat{G}_{1,t})\right] + D_{ZLB}\left[\hat{R}_{1,t-1} + R_F e_{F,t}\right] \tag{66}$$

$$(1-\omega)\hat{C}_{O,1,t} = \left(\hat{Y}_{1,t} - \hat{G}_{1,t}\right)\frac{Y}{C}\left(1 - \omega\frac{1}{\sigma_H}\right) - \omega\frac{\eta+1}{\alpha}\hat{Y}_{1,t} + \omega\frac{Y}{C}\phi_{ROT}\hat{T}_{1,t} \tag{67}$$

Condition on the t-1 being the initial state, quarter t GDP, inflation and taxes in the second state are determined by

$$\hat{Y}_{2,t} = \lambda\gamma_{YG}\hat{G}_{1,t} + \gamma_{Yb}\hat{b}_{1,t-1} + \gamma_{YR}\hat{R}_{1,t-1} + \hat{Y}_{2,F,t-1} \tag{68}$$

$$\hat{\Pi}_{2,t} = \lambda\gamma_{\Pi G}\hat{G}_{1,t} + \gamma_{\Pi b}\hat{b}_{1,t-1} + \gamma_{\Pi R}\hat{R}_{1,t-1} + \hat{\Pi}_{2,F,t-1} \tag{69}$$

$$\hat{T}_{2,t} = \lambda\gamma_{\Pi G}\hat{G}_{1,t} + \gamma_{\Pi b}\hat{T}_{1,t-1} + \gamma_{\Pi R}\hat{R}_{1,t-1} + \hat{T}_{2,F,t-1} \tag{70}$$

The $\gamma_{..}$ coefficients will be determined below.
The following equations are relevant for the forward guidance simulation only

$$\hat{T}_F = \hat{T}_{F,t-1} + \gamma_{Te_F} e_{F,t} \tag{71}$$

$$\hat{Y}_F = \hat{Y}_{F,t-1} + \gamma_{Ye_F} e_{F,t} \tag{72}$$

$$\hat{\Pi}_F = \hat{\Pi}_{F,t-1} + \gamma_{\Pi e_F} e_{F,t} \tag{73}$$

Equations (61) to (73) appear in the Dynare code. The MSV solution of the model for the variables of interest as a function of exogenous and endogenous states and shocks can be written as

$$\hat{Y}_{1,t} = \Gamma_{YG}\hat{G}_{1,t-1}+\Gamma_{Yb}\hat{b}_{1,t-1}+\Gamma_{YR}\hat{R}_{1,t-1}+\Gamma_{YT_F}\hat{T}_{F,t-1}+\Gamma_{YY_F}\hat{Y}_{F,t-1}+\Gamma_{e,YG}e_{G,t}+\Gamma_{e,YF}e_{F,t} \tag{74}$$

$$\hat{\Pi}_{1,t} = \Gamma_{\Pi G}\hat{G}_{1,t-1}+\Gamma_{\Pi b}\hat{b}_{1,t-1}+\Gamma_{\Pi R}\hat{R}_{1,t-1}+\Gamma_{\Pi T_F}\hat{T}_{F,t-1}+\Gamma_{\Pi Y_F}\hat{Y}_{F,t-1}+\Gamma_{e,\Pi G}e_{G,t}+\Gamma_{e,\Pi F}e_{F,t} \tag{75}$$

$$\hat{T}_{1,t} = \Gamma_{TG}\hat{G}_{1,t-1}+\Gamma_{Tb}\hat{b}_{1,t-1}+\Gamma_{TR'}\hat{R}_{1,t-1}+\Gamma_{TT_F}\hat{T}_{F,t-1}+\Gamma_{TY_F}\hat{Y}_{F,t-1}+\Gamma_{e,TG}e_{G,t}+\Gamma_{e,TF}e_{F,t} \tag{76}$$

$$\hat{C}_{O,1,t} = \Gamma_{CG}\hat{G}_{1,t-1}+\Gamma_{Cb}\hat{b}_{1,t-1}+\Gamma_{CR'}\hat{R}_{1,t-1}+\Gamma_{CT_F}\hat{T}_{F,t-1}+\Gamma_{CY}\hat{Y}_{F,t-1}+\Gamma_{e,CG}e_{G,t}+\Gamma_{e,CF}e_{F,t} \tag{77}$$

and is produced by Dynare.

11.1 Government spending multipliers

To compute the government spending effects reported in the paper, the following steps need to be followed sequentially.

11.1.1 Step I: Model solution with active monetary policy (Figure 1)

1. Set
   (a) $D_{ZLB} = \gamma_{..} = 0$ [no ZLB]
   (b) $\mu_1 = 1$ [Economy remains outside ZLB with probability 1]
   (c) $\rho = \lambda$
   (d) $R_F = 0$
2. Solve model with Dynare. This step yields values for the $\Gamma_{..}$ coefficients.
3. **The impact effect of a government spending increase outside the ZLB is given by** $\Gamma_{e,YG}$, $\Gamma_{e,\Pi F}$, $\Gamma_{e,TG}$, $\Gamma_{e,CG}$.

Figure 1 is obtained by performing Step I for the respective values of λ, ω and θ.

11.1.2 Step II: Government spending multiplier during ZLB (Figures 2-5)

1. Set
   (a) $D_{ZLB} = 1$ [During the ZLB, the interest rate does not respond to GDP and inflation]
   (b) $\gamma_{Yb} = \Gamma_{Yb}$[Use solution of model in Step I in order to determine the effect of endogenous state variables on the economy in period t+1 in the event the economy exits the ZLB state in t+1.]

(c) $\gamma_{\Pi b} = \Gamma_{\Pi b}$

(d) $\gamma_{Tb} = \Gamma_{Tb}$

(e) $\gamma_{YR} = \Gamma_{YR}$

(f) $\gamma_{\Pi R}$=$\Gamma_{\Pi R}$

(g) $\gamma_{TR} = \Gamma_{TR}$

2. Set

   (a) $\mu_1 = \mu_L$ [The initial state is now the low state.]

   (b) $\rho = 1$ [Gov. spending remains at $\hat{G}_L$ for at least the duration of the low state.]

   (c) $\gamma_{YG} = \Gamma_{e,YG}$[Use the solution obtained in Step I to determine the effect of a government spending increase on GDP outside the ZLB.]

   (d) $\gamma_{\Pi G} = \Gamma_{e,\Pi G}$

   (e) $\gamma_{TG} = \Gamma_{e,TG}$

3. Solve model with Dynare, obtain new values for $\Gamma_{..}$.

4. **The impact effect of an increase in government spending during the ZLB is then given by** $\Gamma_{e,YG}$, $\Gamma_{e,\Pi F}$, $\Gamma_{e,TG}$, $\Gamma_{e,CG}$.

Figures 2 to 5 are obtained by conducting step I and II for the respective values of λ, $\mu_L = 1 - \frac{1}{D_L}$ and .

11.2 Effect of forward guidance (Figures 6 and 7)

The effect of forward guidance is computed by following steps $I_F - III_F$:

- Step I_F: Same as step I.1 and I.2.
- Step II_F (Compute model solution during the "voluntary pegging state")):
  - Perform step II.1
  - Then (instead of step II.2) set
    * $\hat{R}_f$ to the desired level.
    * $\mu_1 = \mu_F = 1 - \frac{1}{D_F}$ [The initial state is now the state during which the interest rate cut is actually implemented, i.e. the "voluntary pegging state"]
  - Solve model with Dynare, obtain new values for the $\Gamma_{..}$ coefficients.
- Step III_F (Compute impact of forward guidance during the ZLB state).
  - Perform step II.1
  - Then (instead of step II.2) set
    * $\mu_1 = \mu_L = 1 - \frac{1}{D_L}$ [Initial state is now the ZLB state, state 2 is the "voluntary pegging state"]

    * $\gamma_{Ye_F} = \Gamma_{e,YF}$ [Effect of the forward guidance policy on GD].
    * $\gamma_{\Pi e_F} = \Gamma_{e,\Pi F}$
    * $\gamma_{Te_F} = \Gamma_{e,TF}$
  - Solve model with Dynare to obtain new values of $\Gamma_{..}$
  - **The impact of forward guidance during the ZLB is then given by** $\Gamma_{e,YF}$, $\Gamma_{e,\Pi F}$, $\Gamma_{e,TF}$, $\Gamma_{e,CF}$.

- Figures 6 and 7) are obtained by repeating steps $I_F - III_F$ for the respective values of D_F and ω.

12 Simple Model without endogenous state variable

In this section I analyze a simplified case of the model treated in the main text. Specifically, I assume that there is no curvature over wealth ($\tilde{\sigma}_B = 0$) and that whenever the government accumulates debt, it is repaid by taxes on unconstrained households alone. Using $dT_{ROT,t} = \phi_{ROT} dG_t$ and $C_{ROT} = C$, we can write $\hat{T}_{ROT,t} = \frac{Y}{C}\phi_{ROT}\hat{G}_t$, with $\phi_{ROT} = 1$ if credit constrained households contribute to the increase in government expenditure according to their population share. I can then write$\sigma = \sigma_H \frac{C}{Y}$, $\tilde{\sigma} = \frac{\sigma(1-\omega)}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$, $\sigma_{g,t} = \frac{\left(1-\frac{\omega C}{\sigma Y}\right)-\omega\phi_{ROT,t}}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$.

$$\hat{Y}_t = \theta\hat{Y}_{t+1} - \theta\tilde{\sigma}\left(i_t - \hat{\Pi}_{t+1} - \bar{r}\right) + \sigma_{g,t}\left(\hat{G}_t - \theta\hat{G}_{t+1}\right)$$

In the absence of ROT households ($\omega = 0$),$\tilde{\sigma} = \sigma$and $\sigma_g = 1$, and thus the equation reduces to equation (3.12) in Woodford (2011).

The simple model is then written as

$$\hat{Y}_t = \theta\hat{Y}_{t+1} - \theta\tilde{\sigma}\left(\hat{R}_t - \hat{\Pi}_{t+1} - r_t^{net}\right) + \sigma_{g,t}\left(\hat{G}_t - \theta\hat{G}_{t+1}\right) \tag{78}$$

$$\hat{\Pi}_t = \kappa(\hat{Y}_t - \Gamma\hat{G}_t) + \beta\hat{\Pi}_{t+1} \tag{79}$$

$$\hat{R}_t = \left(\bar{r} + \phi_\pi\hat{\Pi}_t + \phi_y(\hat{Y}_t - \Gamma\hat{G}_t), -i\right) \tag{80}$$

with $\sigma_{g,t} = \frac{\left(1-\frac{\omega C}{\sigma Y}\right)-\omega\phi_{ROT,t}}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$.

12.1 Determinacy and MSV solution outside the ZLB

12.1.1 Determinacy

Proposition 1: Consider an economy described by equations (78) to (80). Then the necessary and sufficient condition for a unique and stable equilibrium is given by $\frac{\phi_y(1-\beta)}{\kappa} + \phi_\pi > 1 - \frac{(1-\beta)(1-\theta)}{\theta\kappa\tilde{\sigma}}$.

For $\theta = 1$, the condition collapses to the familiar requirement that to ruling out sunspot equilibria requires an active monetary policy, i.e. the real interest rate has to rise if inflation increases (e.g. Woodford (2003)). For $\theta < 1$, the economy becomes less prone to expectation driven fluctuations, as the right hand side of the equation becomes smaller as θ and thus β decline, as expectations of future real interest rates now have an exponentially decreasing effect on aggregate demand (see equation 78). However, I restrict the discussion below to calibrations of the monetary policy rule which yield a unique and stable equilibrium for all values of θconsidered.

Proof:

We first eliminate all exogenous variables and bring the system into canonical form:

From (78)

$$\hat{\Pi}_t = \kappa \hat{Y}_t + \beta E_t \hat{\Pi}_{t+1}$$

$$\Leftrightarrow E_t \hat{\Pi}_{t+1} = -\frac{\kappa}{\beta}\hat{Y}_t + \frac{\hat{\Pi}_t}{\beta}$$

From (79)

$$\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right)\hat{Y}_t = \theta\left[\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right)\hat{Y}_{t+1}\right] - \sigma\theta\left(1-\omega\right)\left(i_t - \hat{\Pi}_{t+1} - \bar{r}\right)$$

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \frac{\sigma\theta\left(1-\omega\right)}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right)}\left(i_t - \hat{\Pi}_{t+1} - \bar{r}\right)$$

$$\hat{Y}_t = \theta(E_t \hat{Y}_{t+1}) + \tilde{\sigma}\theta\left(E_t\hat{\Pi}_{t+1} - \left(\phi_\pi \hat{\Pi}_t + \phi_y \hat{Y}_t\right)\right)$$

$$\Longleftrightarrow \hat{Y}_t\left(1 + \tilde{\sigma}\theta\phi_y\right) + \tilde{\sigma}\theta\phi_\pi\hat{\Pi}_t = \theta(E_t\hat{Y}_{t+1}) + \tilde{\sigma}\theta\left(\frac{\hat{\Pi}_t}{\beta} - \frac{\kappa}{\beta}\hat{Y}_t\right)$$

$$\Longleftrightarrow \hat{Y}_t\left(1 + \frac{\tilde{\sigma}\kappa\theta}{\beta} + \tilde{\sigma}\theta\phi_y\right) + \left(\tilde{\sigma}\theta\phi_\pi - \frac{\tilde{\sigma}\theta}{\beta}\right)\hat{\Pi}_t = \theta E_t \hat{Y}_{t+1}$$

$$\Longleftrightarrow E_t\hat{Y}_{t+1} = \hat{Y}_t\left(\frac{1}{\theta} + \frac{\tilde{\sigma}\kappa\theta}{\theta\beta} + \tilde{\sigma}\phi_y\right) + \left(\tilde{\sigma}\phi_\pi - \frac{\tilde{\sigma}}{\beta}\right)\hat{\Pi}_t$$

$$E_t\hat{Y}_{t+1} = \hat{Y}_t\left(\frac{1}{\theta} + \frac{\tilde{\sigma}\kappa}{\beta} + \tilde{\sigma}\phi_y\right) + \left(\tilde{\sigma}\phi_\pi - \frac{\tilde{\sigma}}{\beta}\right)\hat{\Pi}_t$$

$$E_t \hat{\Pi}_{t+1} = -\frac{\kappa}{\beta}\hat{Y}_t + \frac{\hat{\Pi}_t}{\beta}$$

System in Matrix form:

$$\begin{matrix} E_t\hat{Y}_{t+1} \\ E_t\hat{\Pi}_{t+1} \end{matrix} = A\begin{pmatrix} \hat{Y}_t \\ \hat{\Pi}_t \end{pmatrix} \tag{81}$$

with $A = \begin{pmatrix} \frac{1}{\theta} + \frac{\tilde{\sigma}\kappa}{\beta} + \frac{\tilde{\sigma}\phi_y}{1} & \frac{\tilde{\sigma}\phi_\pi}{1} - \frac{\tilde{\sigma}}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}$

Calculate determinant and trace:

$$|A| = \left(\frac{1}{\theta} + \frac{\tilde{\sigma}\kappa}{\beta} + \tilde{\sigma}\phi_y\right)\frac{1}{\beta} - \frac{\kappa}{\beta}\left(\frac{\tilde{\sigma}}{\beta} - \tilde{\sigma}\phi_\pi\right) = \frac{1}{\theta\beta} + \frac{\tilde{\sigma}\phi_y}{\beta} + \frac{\kappa\tilde{\sigma}\phi_\pi}{\beta}$$

,

$$tr(A) = \frac{1}{\theta} + \frac{\tilde{\sigma}\kappa}{\beta} + \frac{\tilde{\sigma}\phi_y}{1} + \frac{1}{\beta}$$

According to Woodford (2003)'s Case I, a unique and stable equilibrium exists of the following three conditions are jointly met:

$$\begin{aligned} det\left(A\right) &> 1 \\ det\left(A\right) - tr\left(A\right) &> -1 \\ det\left(A\right) + tr\left(A\right) &> -1 \end{aligned}$$

$$|A| = \frac{1}{\theta\beta} + \frac{\tilde{\sigma}\phi_y}{\beta} + \frac{\kappa\tilde{\sigma}\phi_\pi}{\beta} > 1$$

This condition is always fulfilled since $\theta\beta < 1$. Regarding the second condition, we have

$$|A| - tr\left(A\right) \quad > \quad -1$$

$$\Leftrightarrow \frac{1}{\theta\beta} + \frac{\tilde{\sigma}\phi_y}{\beta} + \frac{\kappa\tilde{\sigma}\phi_\pi}{\beta} - \frac{1}{\theta} - \frac{\tilde{\sigma}\kappa}{\beta} - \tilde{\sigma}\phi_y - \frac{1}{\beta} > -1$$

$$\Leftrightarrow 1 + \tilde{\sigma}\theta\phi_y + \theta\kappa\tilde{\sigma}\phi_\pi - \frac{\beta}{1} - \frac{\theta\tilde{\sigma}\kappa}{1} - \theta\beta\tilde{\sigma}\phi_y - \theta > -\theta\beta$$

$$\Leftrightarrow \tilde{\sigma}\theta\phi_y\left(1-\beta\right) + \theta\kappa\tilde{\sigma}\phi_\pi - \frac{\theta\tilde{\sigma}\kappa}{1} > \theta - 1 + \beta\left(1-\theta\right) = -\left(1-\beta\right)\left(1-\theta\right)$$

$$\Leftrightarrow \frac{\theta\phi_y\left(1-\beta\right)}{\kappa} + \theta\phi_\pi > \theta - \frac{\left(1-\beta\right)\left(1-\theta\right)}{\kappa\tilde{\sigma}}$$

$$\Leftrightarrow \frac{\phi_y\left(1-\beta\right)}{\kappa} + \phi_\pi > 1 - \frac{\left(1-\beta\right)\left(1-\theta\right)}{\theta\kappa\tilde{\sigma}}$$

For $\theta = 1$, condition collapses to Woodford (2003). For $\theta < 1$, the condition for determinacy becomes less strict than in Woodford (2003): Determinacy may be achieved even if LHS<1.

12.1.2 MSV solution

Proposition 2: Consider an economy described by equations (78) to (80), with government spending described by $\hat{G}_t = \rho \hat{G}_{t-1}$, with $\rho < 1$. Assume also that the percapita tax burden of ROT households per unit of dG_t remains fixed at 1. Assume also that the determinacy condition given of Proposition 1 holds. The minimum state variable solutions for output and inflation are given by

$$\hat{Y}_t = \gamma_y \hat{G}_t$$

$$\hat{\Pi}_t = \gamma_\pi \hat{G}_t$$

with

$$\gamma_y = \frac{\sigma_g(1-\rho\theta) + \Gamma\tilde{\sigma}\theta(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)})}{(1-\rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)}$$

$$\gamma_\pi = \frac{\kappa(1-\rho\theta)\left(\sigma_g - \Gamma\right)}{(1-\rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)}$$

In the absence of constrained households (σ_g=1), $\gamma_y < 1$ as an increase in government spending increases flexible price output by less than one for one ($\Gamma < 1$). Furthermore, the fiscal multiplier decreases in the persistence of the government spending shock ρ, the slope of the Phillips curve κ, the hawkishness of monetary policy (ϕ_y and ϕ_π) and θ. For a permanent government spending increase and no preferences over wealth ($\rho = 1$ and $\theta = 1$), $\gamma_y = \Gamma$, i.e. the increase in GDP corresponds exactly to the increase in flexible price output caused by the wealth effect on labor supply, implying that inflation remains is ($\gamma_\pi = 0$). Intuitively, for a given household income, the increase in government consumption reduces private consumption one-for one, which however expands labor supply and GDP, implying that in general equilibrium private consumption declines by less. With $\theta < 1$, $\gamma_y > \Gamma$ and $\gamma_\pi > 0$, households discount the higher marginal utility associated with a unit of future consumption more heavily compared to the market interest rate, implying that even for a given expected path of their income, their consumption declines less than one for one. As a result, the real interest rate increases permanently.

Proof:

The MSV solution has the form

$$\begin{aligned}\hat{Y}_t &= \gamma_y \hat{G}_t \\ \hat{\Pi}_t &= \gamma_\pi \hat{G}_t \\ \hat{R}_t &= \gamma_i \hat{G}_t\end{aligned}$$

Euler equation:

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \theta\tilde{\sigma}\left(\hat{R}_t - \hat{\Pi}_{t+1}\right) + \sigma_g\left(\hat{G}_t - \theta\hat{G}_{t+1}\right)$$

$$\gamma_y \hat{G}_t + i_y e_{i,t} = \theta\left(\rho\gamma_y \hat{G}_t\right) - \theta\tilde{\sigma}\left(\gamma_i \hat{G}_t - \rho\gamma_\pi \hat{G}_t\right) + \sigma_g\left(\hat{G}_t - \theta\rho\hat{G}_t\right)$$

$$\left[\gamma_y\left(1-\rho\theta\right) + \tilde{\sigma}\theta\gamma_i - \tilde{\sigma}\theta\rho\gamma_\pi\right]\hat{G}_t = \sigma_g\left(1-\theta\rho\right)\hat{G}_t \tag{82}$$

Phillips curve:

$$\gamma_\pi \hat{G}_t + i_\pi e_{i,t} = \kappa(\gamma_y \hat{G}_t - \Gamma\hat{G}_t + i_y e_{i,t}) + \beta\gamma_\pi\rho\hat{G}_t$$

$$(-\kappa\gamma_y + \gamma_\pi(1-\beta\rho))\hat{G}_t = -\kappa\Gamma\hat{G}_t \tag{83}$$

MP rule:

$$\gamma_i \hat{G}_t = \phi_\pi\gamma_\pi\hat{G}_t + \phi_y(\gamma_y\hat{G}_t - \Gamma\hat{G}_t)$$

$$(-\gamma_y\phi_y + \gamma_i - \phi_\pi\gamma_\pi)\hat{G}_t = -\phi_y\Gamma\hat{G}_t \tag{84}$$

Equation system to solve for $\gamma_y, \gamma_\pi, \gamma_i$ using Kramer's rule (collecting equations (82) to (84))

$$A_G \begin{pmatrix} \gamma_y \\ \gamma_i \\ \gamma_\pi \end{pmatrix} = \begin{matrix} \sigma_g(1-\rho\theta) \\ -\kappa\Gamma \\ -\phi_y\Gamma \end{matrix} \tag{85}$$

with

$$A_G = \begin{pmatrix} (1-\rho\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho \\ -\kappa & 0 & (1-\beta\rho) \\ -\phi_y & 1 & -\phi_\pi \end{pmatrix}$$

Solution

$$|A_G| = (1-\rho\theta)\begin{vmatrix} 0 & (1-\beta\rho) \\ 1 & -\phi_\pi \end{vmatrix} - \tilde{\sigma}\theta\begin{vmatrix} -\kappa & (1-\beta\rho) \\ -\phi_y & -\phi_\pi \end{vmatrix} - \tilde{\sigma}\theta\rho\begin{vmatrix} -\kappa & 0 \\ -\gamma_y & 1 \end{vmatrix}$$

$$= -(1-\rho\theta)(1-\beta\rho) - \tilde{\sigma}\theta\left[\kappa\phi_\pi + \phi_y(1-\beta\rho)\right] + \tilde{\sigma}\theta\rho\kappa$$

$$= -(1-\beta\rho)\left[(1-\rho\theta) + \tilde{\sigma}\theta\frac{\kappa\phi_\pi}{(1-\beta\rho)} - \frac{\tilde{\sigma}\theta\rho\kappa}{(1-\beta\rho)} + \tilde{\sigma}\theta\phi_y\right]$$

$$= -(1-\beta\rho)\left[(1-\rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi-\rho)}{(1-\beta\rho)}\right)\right]$$

$$|A_{G,Y}| = \begin{vmatrix} \sigma_g(1-\rho\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho \\ -\kappa\Gamma & 0 & (1-\beta\rho) \\ -\phi_y\Gamma & 1 & -\phi_\pi \end{vmatrix}$$

$$= -\sigma_g(1-\rho\theta)(1-\beta\rho) - \tilde{\sigma}\theta \begin{vmatrix} -\kappa\Gamma & (1-\beta\rho) \\ -\phi_y\Gamma & -\phi_\pi \end{vmatrix} - \tilde{\sigma}\theta\rho \begin{vmatrix} -\kappa\Gamma & 0 \\ -\phi_y\Gamma & 1 \end{vmatrix}$$

$$= -\sigma_g(1-\rho\theta)(1-\beta\rho) - \tilde{\sigma}\theta(\kappa\Gamma\phi_\pi + \phi_y\Gamma(1-\beta\rho)) + \tilde{\sigma}\theta\rho\kappa\Gamma$$

$$= -(1-\beta\rho)\left[\sigma_g(1-\rho\theta) + \tilde{\sigma}\theta\phi_y\Gamma + \frac{\Gamma\tilde{\sigma}\kappa\phi_\pi}{(1-\beta\rho)} - \frac{\Gamma\kappa\tilde{\sigma}\theta\rho}{(1-\beta\rho)}\right]$$

$$= -(1-\beta\rho)\left[\sigma_g(1-\rho\theta) + \Gamma\tilde{\sigma}\theta(\phi_y + \frac{\kappa(\phi_\pi-\rho)}{(1-\beta\rho)})\right]$$

$$|A_{G,\pi}| = \begin{vmatrix} (1-\rho\theta) & \tilde{\sigma}\theta & \sigma_g(1-\rho\theta) \\ -\kappa & 0 & -\kappa\Gamma \\ -\phi_y & 1 & -\phi_y\Gamma \end{vmatrix}$$

$$= \kappa\left(-\tilde{\sigma}\theta\phi_y\Gamma - \sigma_g(1-\rho\theta)\right) + \kappa\Gamma\left((1-\rho\theta) + \tilde{\sigma}\theta\phi_y\right) = -\kappa\sigma_g(1-\rho\theta) + \kappa\Gamma(1-\rho\theta) = -\kappa(1-\rho\theta)\left(\sigma_g - \Gamma\right)$$

$$\gamma_y = \frac{|A_{G,Y}|}{|A|} = \frac{\sigma_g(1-\rho\theta) + \Gamma\tilde{\sigma}\theta(\phi_y + \frac{\kappa(\phi_\pi-\rho)}{(1-\beta\rho)})}{(1-\rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi-\rho)}{(1-\beta\rho)}\right)}$$

$$\gamma_\pi = \frac{|A_{G,\pi}|}{|A|} = \frac{\kappa(1-\rho\theta)\left(\sigma_g - \Gamma\right)}{(1-\rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi-\rho)}{(1-\beta\rho)}\right)}$$

12.2 Determinacy and MSV solution inside the ZLB

12.2.1 Determinacy

Proposition 3: Consider an economy described by equations (78) to (80), and assume that r_t^{net} *takes a value* r^L *sufficiently negative for* $\hat{R}_L = -i$. *Assume further that* $r_{t+1}^{net} = r^L$ *with probability* μ_L, *and* $r_{t+1}^{net} = 0$ *with probability* $(1-\mu_L)$, *and that once* $r_{t+i}^{net} = 0$, *it is expected to remain there forever after. Then the necessary and sufficient condition for a unique and stable equilibrium is given by* $(1-\mu_L\beta)(1-\theta\mu) > \theta\tilde{\sigma}\kappa\mu_L$.

From (79) we get (again ignoring all exogenous variables)

$$\hat{\Pi}_t = \kappa\hat{Y}_t + \mu_L\beta E_t\hat{\Pi}_{t+1} \Leftrightarrow E_t\hat{\Pi}_{t+1} = -\frac{\kappa}{\mu_L\beta}\hat{Y}_t + \frac{\hat{\Pi}_t}{\mu_L\beta}$$

From (78)

$$\hat{Y}_t = \theta(\mu_L E_t \hat{Y}_{t+1}) - \tilde{\sigma}\theta \left(-\mu_L E_t \hat{\Pi}_{t+1} \right)$$

$$\hat{Y}_t = \theta(\mu_L E_t \hat{Y}_{t+1}) + \tilde{\sigma}\theta \left(\mu_L E_t \hat{\Pi}_{t+1} \right)$$

$$\Longleftrightarrow \hat{Y}_t \left(1 + \tilde{\sigma}\theta\phi_y\right) + \tilde{\sigma}\theta\phi_\pi \hat{\Pi}_t = \theta(\mu_L E_t \hat{Y}_{t+1}) + \tilde{\sigma}\theta \left(\frac{\hat{\Pi}_t}{\beta} - \frac{\kappa}{\beta}\hat{Y}_t \right)$$

$$\Longleftrightarrow \hat{Y}_t \left(1 + \frac{\tilde{\sigma}\kappa\theta}{\beta} + \tilde{\sigma}\theta\phi_y \right) + \left(\tilde{\sigma}\theta\phi_\pi - \frac{\tilde{\sigma}\theta}{\beta} \right) \hat{\Pi}_t = \theta\mu_L E_t \hat{Y}_{t+1}$$

$$\Longleftrightarrow E_t \hat{Y}_{t+1} = \hat{Y}_t \left(\frac{1}{\theta\mu_L} + \frac{\tilde{\sigma}\kappa}{\mu_L\beta} + \frac{\tilde{\sigma}\phi_y}{\mu_L} \right) + \left(\frac{\tilde{\sigma}\phi_\pi}{\mu_L} - \frac{\tilde{\sigma}}{\mu_L\beta} \right) \hat{\Pi}_t$$

Hence in canonical form, the system reads:

$$\begin{pmatrix} E_t \hat{Y}_{t+1} \\ E_t \hat{\Pi}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{Y}_t \\ \hat{\Pi}_t \end{pmatrix}$$

with

$$A = \begin{pmatrix} \frac{1}{\theta\mu_L} + \frac{\tilde{\sigma}\kappa}{\mu_L\beta} & -\frac{\sigma}{\mu_L\beta} \\ -\frac{\kappa}{\mu_L\beta} & \frac{1}{\mu_L\beta} \end{pmatrix}$$

$$|A| = \left(\frac{1}{\theta\mu_L} + \frac{\sigma\kappa}{\beta\mu_L} \right) \frac{1}{\mu_L\beta} - \frac{\kappa}{\mu_L\beta} \frac{\tilde{\sigma}}{\mu_L\beta} = \frac{1}{\theta\mu_L^2\beta}$$

$$tr\left(A\right) = \frac{1}{\theta\mu_L} + \frac{\tilde{\sigma}\kappa}{\beta\mu_L} + \frac{1}{\mu_L\beta}$$

According to Woodford (2003)'s Case I, a unique and stable equilibrium exists of the following three conditions are jointly met:

$$\begin{aligned} det\left(A\right) &> 1 \\ det\left(A\right) - tr\left(A\right) &> -1 \\ det\left(A\right) + tr\left(A\right) &> -1 \end{aligned}$$

Condition I and III are always met.

Condition II:

$\beta = \frac{\theta}{\frac{R}{\Pi}}$

$$|A| - tr\left(A\right) > -1 \Leftrightarrow \frac{1}{\theta\mu_L^2\beta} - \frac{1}{\theta\mu_L} - \frac{\tilde{\sigma}\kappa}{\beta\mu_L} - \frac{1}{\mu_L\beta} > -1 \Leftrightarrow 1 - \frac{\mu_L\beta}{1} - \frac{\theta\tilde{\sigma}\kappa\mu_L}{1} + -\theta\mu_L > -\theta\mu_L^2\beta$$

$$\Leftrightarrow 1 - \mu_L\beta - \theta\mu_L + \theta\mu_L^2\beta > \theta\tilde{\sigma}\kappa\mu_L$$

$$\Leftrightarrow \left(1 - \mu_L\beta\right)\left(1 - \theta\mu_L\right) > \theta\tilde{\sigma}\kappa\mu_L$$

12.2.2 Fiscal multiplier

Proposition 4: Take the assumptions from proposition 3 and ad that during the low state L $\hat{G}_t = \hat{G}_L$*, while after the end of state L,* $\hat{G}_t = \hat{G}_L$ *with probability* λ *and* $\hat{G}_t = 0$ *with probability* $(1-\lambda)$*. Furthermore, during state L* $\phi_{ROT,t} = \phi_{ROT,L} = 0$*, and during state S,* $\phi_{ROT,t} = \phi_{ROT,S} = 1$*, respectively. Then GDP and inflation during the low state are described by*

$$\hat{Y}_L = \gamma_{G_L Y_L}\hat{G}_L + \gamma_{r_L Y_L}\left(r_L - \hat{R}_L\right)$$

$$\hat{\Pi}_L = \gamma_{G_L \Pi_L}\hat{G}_L + \gamma_{r_L \Pi_L}\left(r_L - \hat{R}_L\right)$$

with

$$\gamma_{G_L Y_L} = \frac{(1-\theta\mu_L)(1-\beta\mu_L)\sigma_{g,L} - \kappa\theta\tilde{\sigma}\mu_L\Gamma + \tilde{\sigma}\theta(1-\mu_L)\lambda\gamma_\pi + \theta\lambda(1-\mu_L)(1-\beta\mu_L)(\gamma_y - \sigma_{g,S})}{(1-\theta\mu_L)(1-\beta\mu_L) - \kappa\theta\tilde{\sigma}\mu_L}$$

$$\gamma_{\Pi_L} = \kappa\frac{(1-\theta\mu_L)(\sigma_{g,L} - \Gamma) + \frac{\tilde{\sigma}\theta(1-\mu_L)\lambda\gamma_\pi}{1-\beta\mu_L} + \theta\lambda(1-\mu_L)(\gamma_y - \sigma_{g,S})}{(1-\theta\mu_L)(1-\beta\mu_L) - \kappa\theta\tilde{\sigma}\mu_L} + \frac{\beta(1-\mu_L)\lambda\gamma_\pi}{1-\beta\mu_L}$$

with

$$\gamma_{r_L Y_L} = \frac{\tilde{\sigma}\theta(1-\beta\mu_L)}{(1-\theta\mu_L)(1-\beta\mu_L) - \kappa\theta\tilde{\sigma}\mu_L}$$

$$\gamma_{r_L \Pi_L} = \frac{\kappa\tilde{\sigma}\theta}{(1-\theta\mu_L)(1-\beta\mu_L) - \kappa\theta\tilde{\sigma}\mu_L}$$

$$\sigma_{g,S} = \frac{\left(1 - \frac{\omega C}{\sigma Y}\right) - \omega\phi_{ROT}}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$$

$$\sigma_{g,L} = \frac{\left(1 - \frac{\omega C}{\sigma Y}\right)}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$$

with γ_y *and* γ_π *as determined in proposition 2 and* $\rho = \lambda$*.*

Proof:

From (78) and using the assumptions stated in the proposition yields

$$\hat{Y}_L - \sigma_{g,L}\hat{G}_L = \theta(\mu_L\hat{Y}_L - \mu_L\sigma_{g,L}\hat{G}_L + \lambda(1-\mu_L)\hat{Y}_S - \lambda(1-\mu_L)\sigma_{g,S}\hat{G}_S) - \tilde{\sigma}\theta\left(\hat{R}_L - \mu_L\hat{\Pi}_L - (1-\mu_L)\lambda\hat{\Pi}_S - r_L\right.$$

$$\left(\hat{Y}_L - \sigma_{g,L}\hat{G}_L\right)(1-\theta\mu_L) = \tilde{\sigma}\theta\left(\mu_L\hat{\Pi}_L + (1-\mu_L)\lambda\hat{\Pi}_S + r_L - \hat{R}_L\right) + \theta\lambda(1-\mu_L)\left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S\right) \quad (86)$$

From (79), we have

$$\hat{\Pi}_L = \kappa(\hat{Y}_L - \Gamma\hat{G}_L) + \beta\left(\mu_L\hat{\Pi}_L + (1-\mu_L)\lambda\hat{\Pi}_S\right)$$

$$\hat{\Pi}_L = \frac{\kappa}{1-\beta\mu_L}(\hat{Y}_L - \Gamma\hat{G}_L) + \frac{\beta\left(1-\mu_L\right)\lambda\hat{\Pi}_S}{1-\beta\mu_L} \tag{87}$$

Combining (86) and (87):

$$\left(\hat{Y}_L - \sigma_{g,L}\hat{G}_L\right)(1-\theta\mu_L) = \tilde{\sigma}\theta\left(-\hat{R}_L + \mu_L\frac{\kappa}{1-\beta\mu_L}(\hat{Y}_L - \Gamma\hat{G}_L) + r_L + (1-\mu_L)\lambda\hat{\Pi}_S + \frac{\mu_L\beta\left(1-\mu_L\right)}{1-\beta\mu_L}\right.$$

$$+\theta\lambda(1-\mu_L)\left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S\right)$$

$$\hat{Y}_L\left((1-\theta\mu_L) - \mu_L\frac{\kappa\tilde{\sigma}\theta}{1-\beta\mu_L}\right) = \left((1-\theta\mu_L)\,\sigma_{g,L} - \tilde{\sigma}\theta\left(\mu_L\frac{\kappa}{1-\beta\mu_L}\Gamma\right)\right)\hat{G}_L + \tilde{\sigma}\theta\left(r_L - \hat{R}_L + \frac{1-\beta\mu_L}{1-\beta}\right.$$

$$\hat{Y}_L = \frac{\left((1-\theta\mu_L)\,\sigma_g - \tilde{\sigma}\theta\left(\mu_L\frac{\kappa}{1-\beta\mu_L}\Gamma\right)\right)\hat{G}_L + \tilde{\sigma}\theta\left(r_L - \hat{R}_L + \frac{1}{1-\beta\mu_L}\left(1-\mu_L\right)\lambda\hat{\Pi}_S\right)}{\left((1-\theta\mu_L) - \mu_L\frac{\kappa\theta\tilde{\sigma}}{1-\beta\mu_L}\right)}$$

$$+\frac{\theta\lambda(1-\mu_L)\left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S\right)}{\left((1-\theta\mu_L) - \mu_L\frac{\kappa\theta\tilde{\sigma}}{1-\beta\mu_L}\right)}$$

$$\hat{Y}_L = \frac{\left((1-\beta\mu_L)\left(1-\theta\mu_L\right)\sigma_g - \tilde{\sigma}\theta\mu_L\kappa\Gamma\right)\hat{G}_L + \tilde{\sigma}\theta\left(1-\beta\mu_L\right)\left(r_L - \hat{R}_L + \frac{1}{1-\beta\mu_L}\lambda\left(1-\mu_L\right)\hat{\Pi}_S\right)}{\left((1-\theta\mu_L)\left(1-\beta\mu_L\right) - \mu_L\kappa\theta\tilde{\sigma}\right)} \tag{88}$$

$$+\frac{\theta(1-\mu_L)\lambda\left(1-\beta\mu_L\right)\left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S\right)}{\left((1-\theta\mu_L)\left(1-\beta\mu_L\right) - \mu_L\kappa\theta\tilde{\sigma}\right)}$$

With $\hat{G}_S = \hat{G}_L$, we have $\hat{Y}_S = \gamma_y\hat{G}_S$ and $\hat{\Pi}_S = \gamma_\pi\hat{G}_S$

Solution for

$$\hat{Y}_L = \frac{(1-\theta\mu_L)\left(1-\beta\mu_L\right)\sigma_g - \kappa\theta\tilde{\sigma}\mu_L\Gamma + \tilde{\sigma}\theta(1-\mu_L)\lambda\gamma_\pi + \theta\lambda(1-\mu_L)\left(1-\beta\mu_L\right)\left(\gamma_y - \sigma_{g,S}\right)}{(1-\theta\mu_L)\left(1-\beta\mu_L\right) - \kappa\theta\tilde{\sigma}\mu_L}\hat{G}_L \tag{89}$$

$$+\frac{\tilde{\sigma}\theta\left(1-\beta\mu_L\right)}{(1-\theta\mu_L)\left(1-\beta\mu_L\right) - \kappa\theta\tilde{\sigma}\mu_L}\left(r_L - \hat{R}_L\right)$$

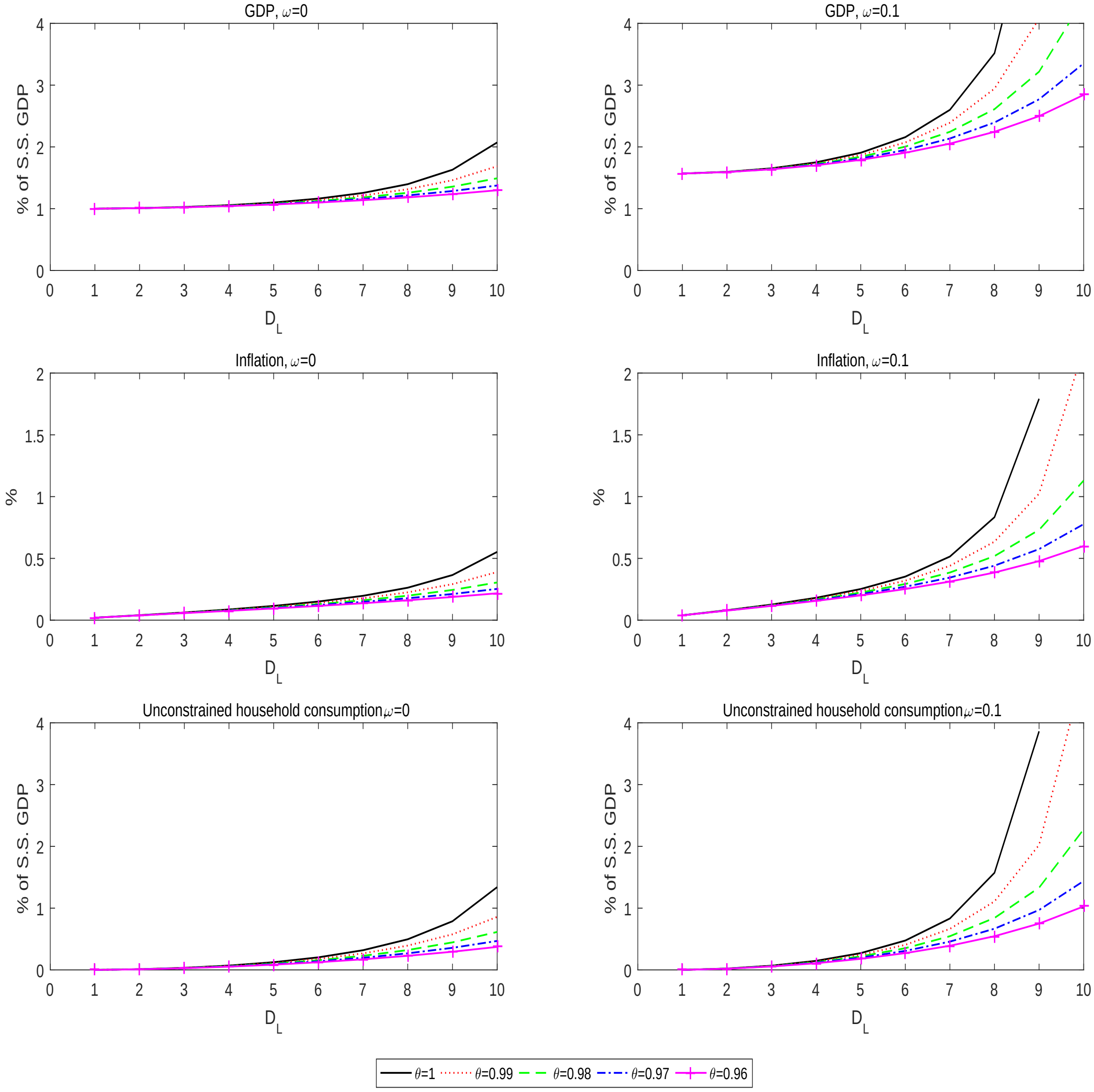
$\frac{(1-\theta\mu_L)(1-\beta\mu_L)-\kappa\theta\tilde{\sigma}\mu_L\Gamma}{(1-\theta\mu_L)(1-\beta\mu_L)-\kappa\theta\tilde{\sigma}\mu_L} > 1$ since $\Gamma < 1$

Solution for $\hat{\Pi}_L$

$$\hat{\Pi}_L = \left(\kappa\frac{(1-\theta\mu_L)(\sigma_g-\Gamma)+\frac{\tilde{\sigma}\theta(1-\mu_L)\lambda\gamma_\pi}{1-\beta\mu_L}+\theta\lambda(1-\mu_L)(\gamma_y-\sigma_{g,S})}{(1-\theta\mu_L)(1-\beta\mu_L)-\kappa\theta\tilde{\sigma}\mu_L}+\frac{\beta(1-\mu_L)\lambda\gamma_\pi}{1-\beta\mu_L}\right)\hat{G}_L+\frac{}{(1-\theta\mu_L)}$$

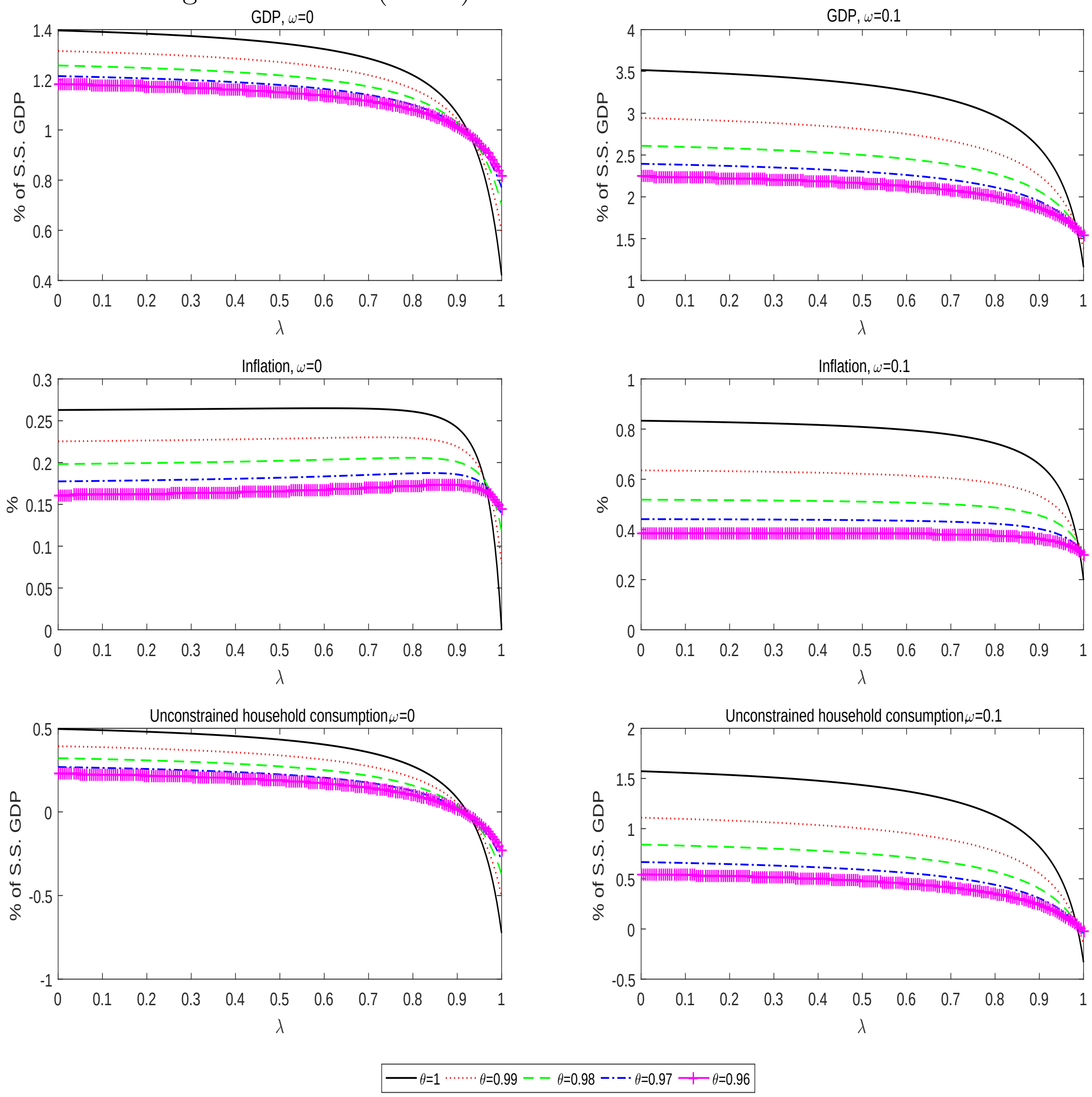
As can be seen from Figures 8 and 9, the effect of lowering θ below one is qualitatively the same as in the model with curvature and the more realistic fiscal rule and remains quantitatively significant.

Figure 8: Fiscal multiplier during low state, short term stimulus($\lambda = 0$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP during the low state only ($\lambda = 0$). GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. Taxes on credit constrained households are fixed at zero ($\phi_{ROT,L} = 0$). The horizontal axis depicts the expected duration of the low state $D_L = \frac{1}{1-\mu_L}$. All other parameters are as in Table 2.

Figure 9: Fiscal multiplier during low state with the change in government expenditure outlasting the low state($\lambda > 0$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis displays the probability λ that the stimulus persists after the economy's exit from the low state and thus the ZLB. The expected length of the low state is 8 quarters (i.e.$\mu_L = 0.875$). During the low state, taxes on credit constrained households are fixed at zero ($\phi_{ROT,L} = 0$). During the transitional state, the share of the government consumption increase funded via taxes on credit constrained households corresponds to their share in the population ($\phi_{ROT,S} = 1$). All other parameters are as in Table 2.

12.2.3 Forward guidance

Proposition 5: Take the assumptions from proposition 3 and ad that once the economy exits the low state, the central bank, with probability fixes the interest rate at a level

$\hat{R}_f$. Conditional on fixing the interest rate in the first post-ZLB quarter, it will keep the quarterly interest rate at $\hat{R}_f$ in subsequent periods with probability μ_f. With probability $1 - \mu_f$ it will revert to its interest feedback rule (Equation 21), and is expected to stick to this rule forever after. Output and inflation in the low state are then given by

$$\hat{Y}_L = \gamma_{R_f Y_L} \hat{R}_f + \gamma_{r_L Y_L} \left(r_L - \hat{R}_L \right)$$

$$\hat{\Pi}_L = \gamma_{R_f \Pi_L} \hat{R}_f + \gamma_{r_L \Pi_L} \left(r_L - \hat{R}_L \right)$$

with

$$\gamma_{R_f Y_L} = -\frac{\tilde{\sigma}\theta^2 \lambda_f (1 - \mu_L)(\kappa\tilde{\sigma} + (1 - \beta\mu_L)(1 - \beta\mu_f))}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})((1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f)}$$

$$\gamma_{R_f \Pi_L} = -\frac{(1 - \mu_L)\lambda_f \kappa\tilde{\sigma}\theta \left[\theta \left(\frac{\kappa\tilde{\sigma}}{1 - \beta\mu_L} + 1 - \beta\mu_f \right) + \beta \left(1 - \theta\mu_L - \frac{\mu_L \kappa\theta\tilde{\sigma}}{1 - \beta\mu_L} \right) \right]}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})((1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f)}$$

Proof:

GDP and Inflation in the low state are determined by (88) and (87), where we set $\hat{G}_L = 0$ and replace the subscript S with f, where f refers to the state during which the announced policy is actually implemented:

$$\hat{Y}_L = \frac{\tilde{\sigma}\theta (1 - \beta\mu_L) \left(r_L - \hat{R}_L + \frac{1}{1 - \beta\mu_L} \lambda_f (1 - \mu_L) \hat{\Pi}_f \right)}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})} + \frac{\theta(1 - \mu_L)(1 - \beta\mu_L) \hat{Y}_f}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})}$$

where $\hat{\Pi}_f$ and $\hat{Y}_f$ refer to inflation and GDP once r is at zero but the policy rate is still fixed. $\hat{\Pi}_f$ and $\hat{Y}_f$ are given by

$$\hat{Y}_f = -\frac{\tilde{\sigma}\theta (1 - \beta\mu_f)}{(1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f} \left(\hat{R}_f \right) \tag{91}$$

$$\hat{\Pi}_f = -\frac{\kappa\tilde{\sigma}\theta}{(1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f} \left(\hat{R}_f \right)$$

Hence

$$\hat{Y}_L = -\left[\frac{\tilde{\sigma}\theta (1 - \mu_L) \kappa\tilde{\sigma}\theta}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})((1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f)} + \frac{\theta(1 - \mu_L)(1 -}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa}\right.$$

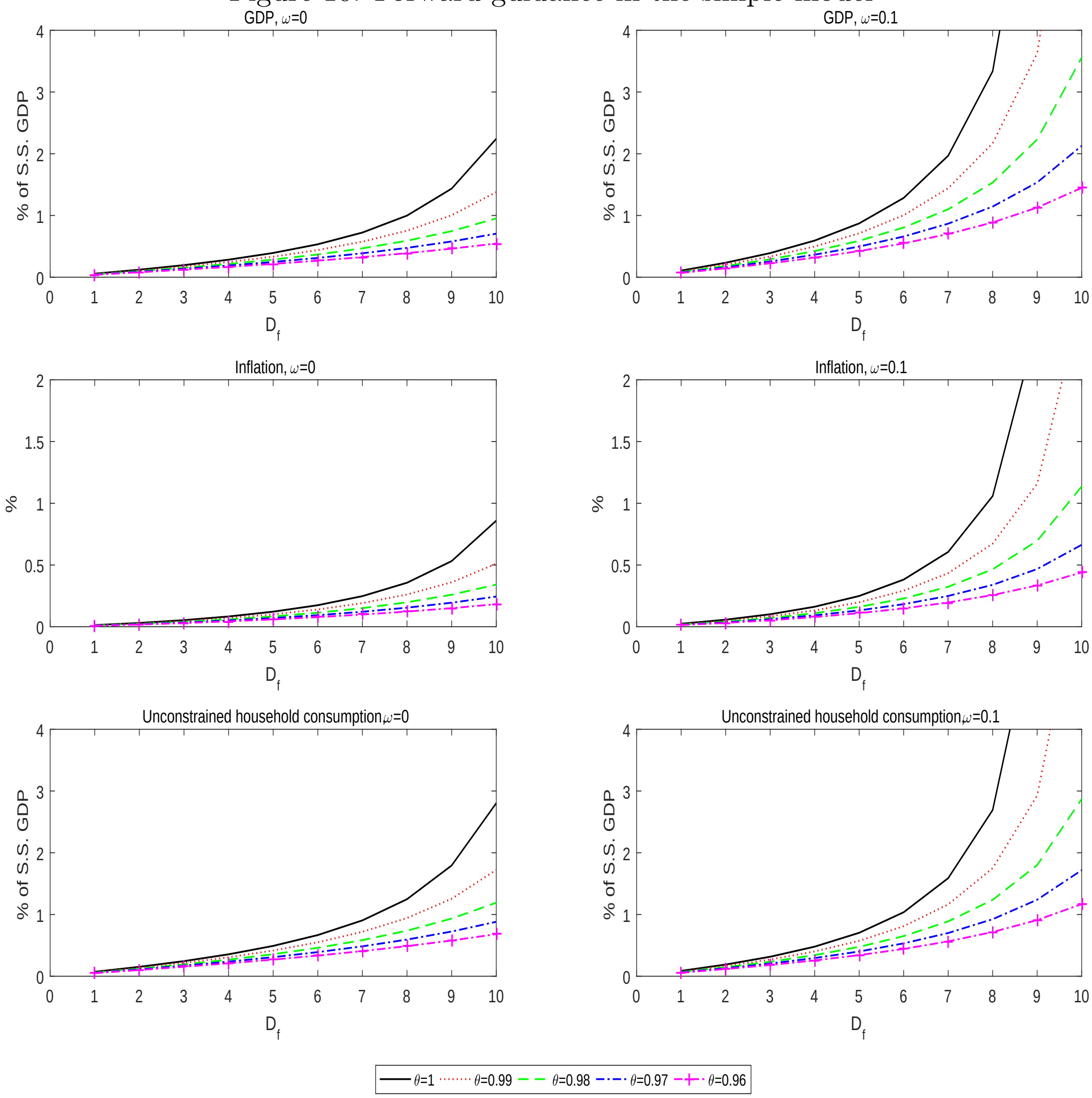
$$= -\left[\frac{\tilde{\sigma}\theta^2 (1 - \mu_L)(\kappa\tilde{\sigma} + (1 - \beta\mu_L)(1 - \beta\mu_f))}{((1 - \theta\mu_L)(1 - \beta\mu_L) - \mu_L \kappa\theta\tilde{\sigma})((1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f)} \right] \hat{R}_f + \frac{\tilde{\sigma}\theta (1 - \beta\mu_L)}{(1 - \theta\mu_L)(1 - \beta\mu_L) - \kappa\theta}$$

and

$$
\begin{aligned}
\hat{\Pi}_L &= \frac{\kappa}{1-\beta\mu_L}\hat{Y}_L + \frac{\beta(1-\mu_L)\hat{\Pi}_f}{1-\beta\mu_L} \\
&= -\left[\frac{\kappa\tilde{\sigma}\theta^2(1-\mu_L)\left(\frac{\kappa\tilde{\sigma}}{1-\beta\mu_L}+(1-\beta\mu_f)\right)}{((1-\theta\mu_L)(1-\beta\mu_L)-\mu_L\kappa\theta\tilde{\sigma})((1-\theta\mu_f)(1-\beta\mu_f)-\kappa\theta\tilde{\sigma}\mu_f)}\right]\hat{R}_f + \frac{\tilde{\sigma}\theta(1-\beta\mu_L)}{(1-\theta\mu_L)(1-\beta\mu_L)-\kappa\theta \cdots} \\
&= -\left[\frac{\kappa\tilde{\sigma}\theta^2(1-\mu_L)\left(\frac{\kappa\tilde{\sigma}}{1-\beta\mu_L}+(1-\beta\mu_f)\right)}{((1-\theta\mu_L)(1-\beta\mu_L)-\mu_L\kappa\theta\tilde{\sigma})((1-\theta\mu_f)(1-\beta\mu_f)-\kappa\theta\tilde{\sigma}\mu_f)}\right]\hat{R}_f + \frac{\tilde{\sigma}\theta(1-\beta\mu_L)}{(1-\theta\mu_L)(1-\beta\mu_L)-\kappa\theta \cdots} \\
&= -(1-\mu_L)\kappa\tilde{\sigma}\theta\left[\frac{\theta\left(\frac{\kappa\tilde{\sigma}}{1-\beta\mu_L}+1-\beta\mu_f\right)+\beta\left(1-\theta\mu_L-\frac{\mu_L\kappa\theta\tilde{\sigma}}{1-\beta\mu_L}\right)}{((1-\theta\mu_L)(1-\beta\mu_L)-\mu_L\kappa\theta\tilde{\sigma})((1-\theta\mu_f)(1-\beta\mu_f)-\kappa\theta\tilde{\sigma}\mu_f)}\right]\hat{R}_f + \frac{\tilde{\sigma}\theta(\cdots}{(1-\theta\mu_L)(1\cdots}
\end{aligned}
$$

As can be obtained from Figure 10, the effect of lowering θ below one is qualitatively the same as in the model with curvature and the more realistic fiscal rule and remains quantitatively significant. It is however smaller than in the model discussed in the main text due to the absence of a direct effect of the decline in real unconstrained household wealth on their consumption.

Figure 10: Forward guidance in the simple model



Note: Effects on output and annualized inflation in the low state of fixing the annualized interest rate at 0.2% below its steady state level once the economy has exited the low state. The horizontal axis depicts the expected length of the fixed interest rate policy D_f (where $D_f = \frac{1}{1-\mu_f}$). The expected length of the low state is 6 quarters (i.e. $\mu_L = 0.83$), and the Central Bank embarks on its fixed interest rate policy after the end of the low state with certainty ($\lambda_f = 1$). All other parameters are as detailed in Table 2.

13 The impact of preferences over wealth in a medium scale model

I now investigate whether the impact of preferences over wealth on the effects of fiscal policy and forward guidance carry over to a richer, quantitative model. Like the simple model of the previous section, it features constrained and rule of thumb (or credit constrained) households. Unconstrained households save in the form of government bonds and safe deposits, and have preferences over wealth, where I examine both linear preferences and preferences with curvature. Both household types supply labor to retailers via a labor union, which sets a common wage for both. An entrepreneurial sector owns and accumulates the capital stock, who obtains external funds in the form of one period loans from the financial intermediary. Thus the wealth of unconstrained households now consists of government debt and -via the financial intermediary- the debt of entrepreneurs.

The cost of external finance of entrepreneurs increases in their leverage due to a costly state verification (CSV) problem as in Bernanke et al. (1999). Entrepreneurs rent their capital stock to retailers. Retailers produce the homogeneous output good employing physical capital and labor in a Cobb Douglas technology. Perfectly competitive investment good producers owned by unconstrained households convert the output good into new capital goods and sell them to entrepreneurs. Investment goods producers are subject to investment adjustment costs, while retailers and unions are subject to nominal rigidities in the form of Calvo contracts. The full set of linearized equations can be obtained from Appendix 14. I estimate key parameters of the model by matching the impulse response functions of a VAR to a monetary policy and a government spending shock.

13.1 Financial accelerator

We will consider a version of the model with a financial accelerator along the lines of Bernanke et al. (1999). Risk neutral entrepreneurs accumulate the physical capital stock $\widehat{K}_t$ and rent it to retailers. After the collection of rental income, the liquidate their capital stock. Their period t return on capital $\widehat{R}_t^K$ is thus given by

$$\widehat{R}_t^K = \widehat{\Pi}_t + \frac{\Pi\left(dr_t^k\left(1-\tau_K\right)+\widehat{Q}_t\left(1-\delta\right)\right)}{R^k} - \widehat{Q}_{t-1} \tag{92}$$

where dr_t^k and $\hat{Q}_t$denote the deviation of the rental rate on physical capital and the price of a unit of capital from their respective steady states, respectively. Entrepreneurs fund their capital stock using their own net worth $\hat{N}_t$ and a loan from a financial intermediary. As a consequence of idiosyncratic shocks to the return on capital, some entrepreneurs default on their debt in period $t+1$. In case of default, the bank seizes a fraction $(1-\mu)$of the assets of the entrepreneur, while the remainder represents a monitoring costs. Banks pass the costs of bankruptcy to entrepreneurs in the form a of a state contingent debt contract, implying that the always earn the save interest rate R_t. As a result, the entrepreneurs first order conditions require a positive relationship between the spread of the entrepreneurs expected return on capital $E_t\widehat{R}_{t+1}^K$ over the risk free rate $\widehat{R}_t$ and entrepreneurial leverage:

$$E_t\widehat{R}_{t+1}^K - \widehat{R}_t = \chi^{\phi^e}\left(\hat{Q}_t + \widehat{K}_t - \hat{N}_t\right) \tag{93}$$

with $\chi^{\phi^e} > 0$ if bankruptcy is costly (i.e. if $\mu > 0$). $E_t\widehat{R}_{t+1}^K - \widehat{R}_t$ is typically referred to as the cost of external finance. Furthermore, each period a small fraction $1-\gamma$ of entrepreneurs dies each and consumes its net worth, and are replaced with a fraction of newly born entrepreneurs. This assumption assures that entrepreneurs never become fully self-financing. Entrepreneurs supply one unit of labor to retailers at wage $w_{e,t}$, which allows newly borne entrepreneurs to start their business. Entrepreneurial net worth thus depends positively on on past net worth and the real return on capital net of bankruptcy costs $(1-\mu G(\overline{\omega}))\left(\widehat{R}_t^K-\widehat{\Pi}_t\right)$ as well as negatively on the real save interest rate $\widehat{R}_{t-1}-\widehat{\Pi}_t$

$$\hat{N}_t=\gamma\left[\begin{array}{c}\frac{R}{\Pi}\widehat{N}_{t-1}+\phi^e\frac{R^K}{\Pi}(1-\mu G(\overline{\omega}))\left(\widehat{R}_t^K-\widehat{\Pi}_t\right)+\phi^e\left(\frac{R^k}{\Pi}(1-\mu G(\overline{\omega}))-\frac{R}{\Pi}\right)\left(\hat{Q}_{t-1}+\widehat{K}_{t-1}\right)\\-\frac{R}{\Pi}(\phi^e-1)\left(\widehat{R}_{t-1}-\widehat{\Pi}_t\right)-\phi^e\frac{R^K}{\Pi}\mu G'(\overline{\omega})\,\varpi\widehat{\varpi}_t\end{array}\right]+(1- \tag{94}$$

where $\phi^e=\frac{K}{N}$, $\widehat{\varpi}_t$, $G(\overline{\omega})$ and γ denote steady state leverage, the bankruptcy threshold, the steady-state probability weighted expected value of the idiosyncratic shock ω(conditional on $\omega<\bar{\omega}$) and the survival probability of entrepreneurs, respectively. The financial accelerator works as follows: Any shock causing a jump in $\widehat{Q}_t$ by say lowering the monetary policy rate $\widehat{R}_t$ and/ or increasing expected future output and thus dr_t^k (see equations (92) and (93)) will increase not only investment, but also $\widehat{R}_t^K$ and thus period t net worth (see equation (94)). The financial accelerator amplifies the jump in $\hat{Q}_t$and thus in investment by letting the associated decline in entrepreneurial leverage $\left(\hat{Q}_t+\widehat{K}_t-\hat{N}_t\right)$ reduce the external finance premium $E_t\widehat{R}_{t+1}^K-\widehat{R}_t$. Furthermore, for a given nominal interest rate, an increase in inflation $\widehat{\Pi}_t$ lowers entrepreneurial leverage and thus the cost of external finance by lowering their real debt burden.

Finally, entrepreneurial consumption $\hat{C}_t^e$is given by

$$\hat{C}_t^e=\frac{(1-\gamma)N}{\gamma V}\hat{N}_t \tag{95}$$

Financial intermediaries fund their loans by collecting deposits from unconstrained households. Hence the safe assets $B_{O,t}$ entering their utility function now consist of both bank deposits and government bonds.

13.2 Government

Government revenue consists of labor taxes borne by the employer and the employee, profit, lump sum and consumption taxes. In order to ensure the stationarity of government debt in the long run, the government adjusts the consumption tax according to the following fiscal rule:

$$\hat{\tau}_{C,t}=(1-d_{\tau_C,t})\left[(1-\rho_\tau)\phi_\tau\hat{b}_t+\rho_\tau\hat{\tau}_{C,t-1}\right] \tag{96}$$

where $\hat{\tau}_{C,t}$ and $\hat{b}_t$ denote the percentage deviation of the employees labor tax and real government debt from their respective steady states, while $d_{\tau_C,t}$ denotes a dummy variable with a value of zero unless otherwise mentioned. Using using consumption taxes as the fiscal instrument in the fiscal rule simplifies the analysis of the permanent cut in government consumption considered below by ensuring that the long run percentage increase of unconstrained households, constrained households and entrepreneurial consumption is almost identical. Furthermore, the long run decline in the consumption tax offsets the decline in labor supply associated with higher household disposable income, implying that hours and GDP change only marginally in the long run.

For the IRF matching exercise to be conducted below, I assume that government spending follows a simple auto-regressive process

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + e_{G,t} \tag{97}$$

where $e_{G,t}$denotes an i.i.d. government spending shock.

Monetary policy is described by a rule relating the policy rate to its own lag as well as inflation and the deviation of real marginal cost from its steady state $\widehat{mc}_t$, which serves a proxy for the output gap

$$\widehat{R}_t R = (1 - d_{R,t})(1 - \rho_R)\left[\psi_\pi \widehat{\Pi}_t + \psi_y (\widehat{mc}_t)\right] + \rho_R \widehat{R}_{t-1} + e_{R,t} \tag{98}$$

where d_R denotes a dummy variable with a value of zero unless otherwise mentioned.

13.3 Calibration

I calibrate the parameters to Euro Area data where possible and divide them into two groups. Parameters in the first group are set to standard values in the literature, empirical estimates (see Table 6) or indirectly calibrated by setting targets for the steady state values of some variables (see Table 5). I set the share of unconstrained households $1-\omega$ to 75%, in line with the Euro Area estimates of Coenen and Straub (2005). Regarding household preferences over consumption, I assume log utility ($\sigma_H = 1$), set the degree of habit formation to the median estimate reported by Havranek et al. (2015) for European countries and assume an inverse Frisch elasticity of labor supply η of 2. I consider three specifications for preferences over wealth, namely no preferences over wealth ($\theta = 1$), linear preferences over wealth ($\sigma_B = 0$) with $\theta = 0.96$, and finally the case of $\theta = 0.96$ and curvature of the utility from wealth of$\sigma_B = 0.5$, where the procedure again follows Kumhof et al. (2014). Details of the partial equilibrium calibration exercise are provided in appendix 8. I assume a wage markup of 1/3 and and an elasticity of production with respect to physical capital of 1/3.

I set the elasticity of the external finance premium with respect to entrepreneurial leverage χ^e and the survival probability of entrepreneursγ equal to the Euro Area estimates of Gelain (2010). Given the choices of χ^e, I set the degree of idiosyncratic uncertainty σ, the bankruptcy cost parameter μ and the transfer to entrepreneurs such that steady state entrepreneurial leverage and the entrepreneurial bankruptcy rate equal the values reported by Christiano et al. (2010) and Gelain (2010). Given the respective value of θ, I calibrated the household discount factor β, the steady state level of government expenditure G, the markup charged by retailers and the depreciation rate such that the steady state real interest rate as well as the shares of government expenditure, private investment and the compensation of employees are close to their respective averages over the 1972Q1-2008Q4 period.

Finally, I assume long run responses of the policy rate to inflation and the output gap of 2.0 and 0.05, respectively, close to the Euro Area estimates of Gaddatsch et al. (2016). I obtain the steady state tax rates from the ECBs New Area Wide Model (Coenen et al. (2008)) and calibrate the fiscal rule such that in the long run such that the consumption tax rate moves very gradually in response to deviations of government debt from its steady state, while the response is still sufficiently strong to guarantee long-run debt stationarity.

Table 5: Values of steady state targets and empirical counterparts/ sources

Variable	Model	AWM database or other source
$\frac{R}{\Pi}$, APR	2.2	2.2
$\frac{I}{GDP}$	22.7	22.3
$\frac{G}{GDP}$	19.5	19.5
$\frac{W(1+\tau_{W_h,t})l}{GDP}$	50.1	52.6
$\frac{K}{N}$	2.1	As in Christiano et al. (2010) and Gelain (2010)
$F(\bar{\omega})$	0.0075	Gelain (2010)
MPS	0.5	MPS out of an increase in permanent income, estimates of Dynant et al. (2004) and Kumhof et al. (2014).

Note: Unless otherwise mentioned, the data is taken from the Area Wide Model database (see Fagan et al. (2001)).
Averages from the AWM where calculated over the 1972Q1-2008Q4.

Table 6: Medium scale model: calibrated Parameters

Parameter	Name	Value
χ^e	Elasticity of EFP w.r.t. leverage	0.038, in line with Gelain (2010)
μ	Bankruptcy cost	0.1, in line with Gelain (2010)
σ	Idiosyncratic uncertainty of capital return	0.25*
γ	Survival probability of entrepreneurs	0.9797 as in Gelain (2010)
τ_{W_h}	Labor tax borne by employee	0.24, as in Coenen et al. (2008)
τ_{W_f}	Labor tax borne by employer	0.219, as in Coenen et al. (2008)
τ_C	Consumption tax	0.183, as in Coenen et al. (2008)
τ_K	Capital income tax	0.184, as in Coenen et al. (2008)
α	Labor share	0.33, as in Coenen et al. (2008)
ϕ_τ	Long run response to debt level	0.2
ρ_τ	Response to lagged tax rate	0.98
σ_H	Intertemporal elasticity of substitution	1.0
h	Degree of habit formation	0.6, as estimated by Havranek et al. (2015)
σ_B	Curvature preferences over wealth	0/0.5, evidence on MPS of Dynant (2004) and Kumhof et al. (2014)
θ	$\beta\frac{R}{\Pi}$	0.96, 1
δ	Depreciation rate physical capital	0.03*
ϵ_p	Goods market, elasticity of substitution	11*
ϵ_W	Labor market, elasticity of substitution	4
ψ_π	Interest feedback rule: Inflation response	2.0, as in Christoffel et al. (2008)
ψ_Y	Interest feedback rule: output gap response	0.05, as estimated by Gaddatsch et al. (2016)

I estimate the second group of parameters by matching the impulse response functions of GDP, private consumption, private Investment, government expenditure, the nominal wage, inflation (measured by the change in the GDP deflator) and the policy rate to a government spending shock and a monetary policy from a structural VAR model estimated for this purpose by Blanchard et al. (2015) over the 1972Q1-2008Q4 period using the Area Wide Model (AWM) database of Fagan et al. (2002). On top of the aforementioned series, the VAR also includes the effective real exchange rate as an additional endogenous variable (which is however absent from the model), as well as US GDP, the Federal Funds rate, commodity prices and quadratic time trend as exogenous variables. The two shocks are identified by assuming (1) that government spending does not respond contemporaneously to any of the endogenous variables (following Blanchard and Perotti (2002)) and (2) that none of the endogenous variables except for the real exchange rate respond contemporaneously to policy rate innovations (following Christiano et al. (1999)).[9] These assumptions are also implemented in the model when computing impulse response functions to the two shocks.[10] I set the standard deviations of the government spending and monetary policy shocks to their VAR estimates and collect the remaining estimated parameters in the vector $\Omega = \{\phi_I, \kappa_p, \kappa_w, \rho_g \rho_R\}$, and choose Ωin order to minimize the criterion function

$$\left(\hat{\Psi} - \Psi\left(\Omega\right)\right)' F^{-1} \left(\hat{\Psi} - \Psi\left(\Omega\right)\right)$$

where $\hat{\Psi}$ and $\Psi\left(\Omega\right)$denote vectors stacking all VAR and model impulse response functions (for a given value of Ω) on top of each other (excluding the contemporaneous response to the monetary policy shock), respectively. F denotes a diagonal weighting matrix, where each element equals the estimated variance of the corresponding IRF element from the VAR. [11] I match the first 20 periods of each impulse response, and estimate a model variant without preferences over wealth ($\theta = 1$), with linear preferences over wealth ($\theta = 0.96$, $\sigma_B = 0$) and with curvature in preferences over wealth ($\theta = 0.96$, $\sigma_B = 0.5$).

As can be obtained from Figure 11, the three models respond in a virtually identical identical fashion to the monetary policy shock (see the dashed and circled lines). They perform well at tracking the empirical impulse response of GDP, as well as the stronger decline of private investment relative to private consumption. All three models understate the persistence of the consumption decline. Furthermore, the models are unable to match the initial increase in inflation (usually referred to as the "price puzzle"). Both models somewhat understate the initial response of GDP to the government spending shock and are unable to generate the observed increase in private investment. The estimated price and wage markup coefficients are low (see table 7), although , Linde et al. (2016) and Brave et al. (2012) estimate wage

[9]I would like to thank Jesper Linde for sharing their code for the estimation of the VAR as well as valuable hints on how to implement the restrictions of the VAR in the model.

[10]Specifically, I assume that financial variables (e.g. Tobin's Q and the net worth of entrepreneurs) respond to the policy rate innovation contemporaneously, but that the information set of households and firms does not include a contemporaneous policy innovation nor its effect on financial variables. For instance, unconstrained households act as if the (expected) value of current and future real interest rates is the same as in the absence of the policy rate innovation, while investment good producers ignore the effect of the innovation on Tobin's Q.

[11]Hence with the length of each IRF denoted as T, $\hat{\Psi}$ and $\Psi\left(\Omega\right)$each comprise n=7X(2T-1) elements, while the dimension of F is nXn.

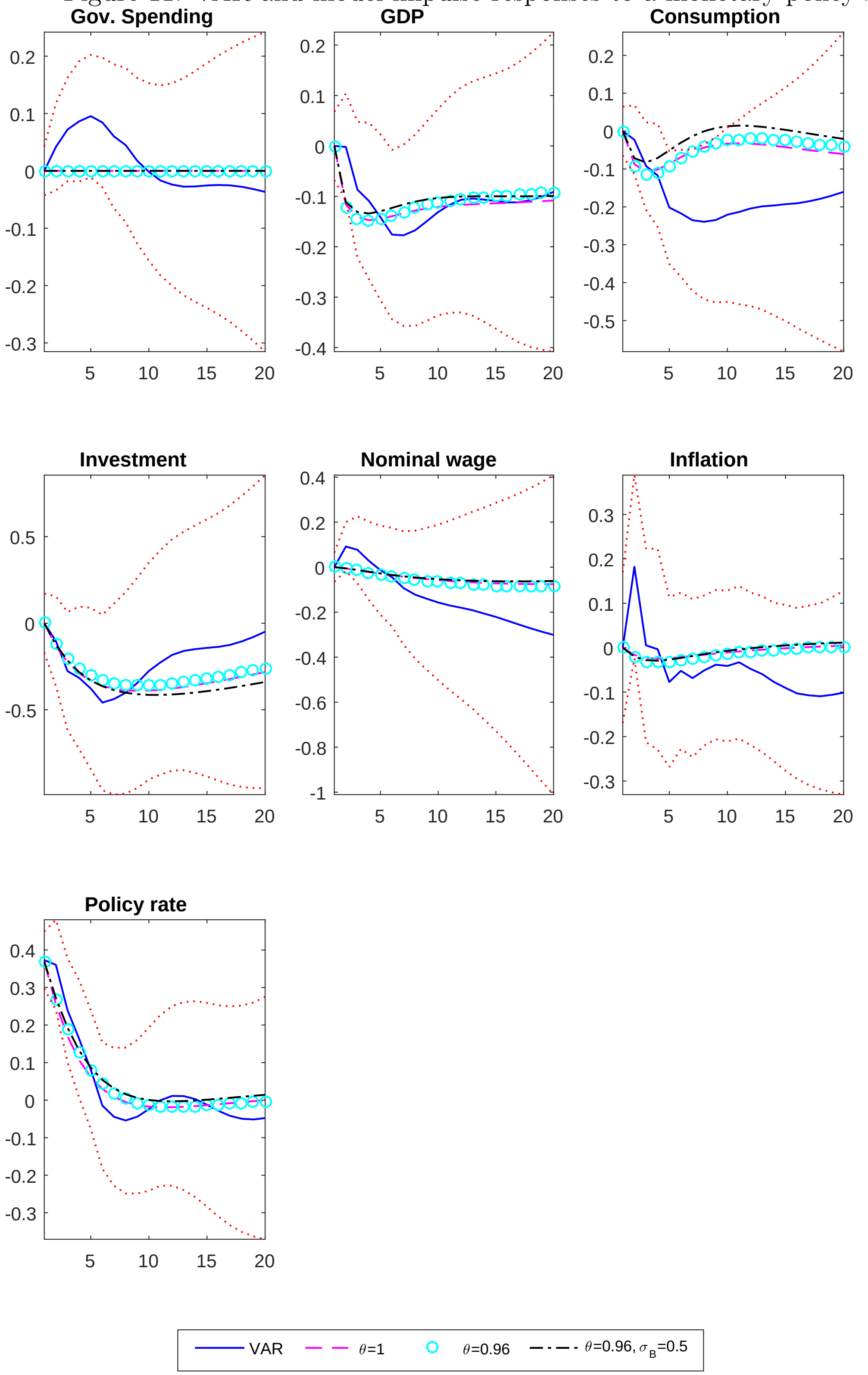
Table 7: Medium scale model: Estimated Parameters

Parameter name	Parameter	Estimation results for various versions of the model		
		$\theta = 1$	$\theta = 0.96, \sigma_B = 0$	$\theta = 0.96, \sigma_B = 1$
Investment adjustment cost	ϕ_I	7.9	9.9	9.8
Price markup coefficient	κ_p	0.007	0.013	0.015
Wage markup coefficient	κ_w	0.001	0.0014	0.0016
Persistence gov. spending shock	ρ_g	0.90	0.90	0.90
Interest rate smoothing	ρ_R	0.73	0.76	0.77
Sd. gov. spending shock	σ_g	0.29	0.29	0.29
Sd. monetary policy shock	σ_R	$\frac{0.37}{4}$	$\frac{0.37}{4}$	$\frac{0.37}{4}$

and price markup coefficients of a similarly small order of magnitude.[12] They are higher in the presence than in the absence of preferences over wealth ($\theta = 0.96$), although in absolute terms they are very close. The reason why price and wages are estimated to be more flexible for $\theta < 1$ is that reducing θ implicitly reduces β, implying that expectations of future variables (e.g. future marginal cost and the future wage markup) now have a smaller effect on price and wage setting. Matching the response of inflation and nominal wages requires a bigger role for current economic activity. By contrast, adding curvature ($\theta = 0.96, \sigma_B = 1$) has only a minor effect on the parameter estimates, as as due to their temporary nature, both shocks have only a moderate effect on the wealth of unconstrained households and thus on the marginal utility of wealth relative to consumption.

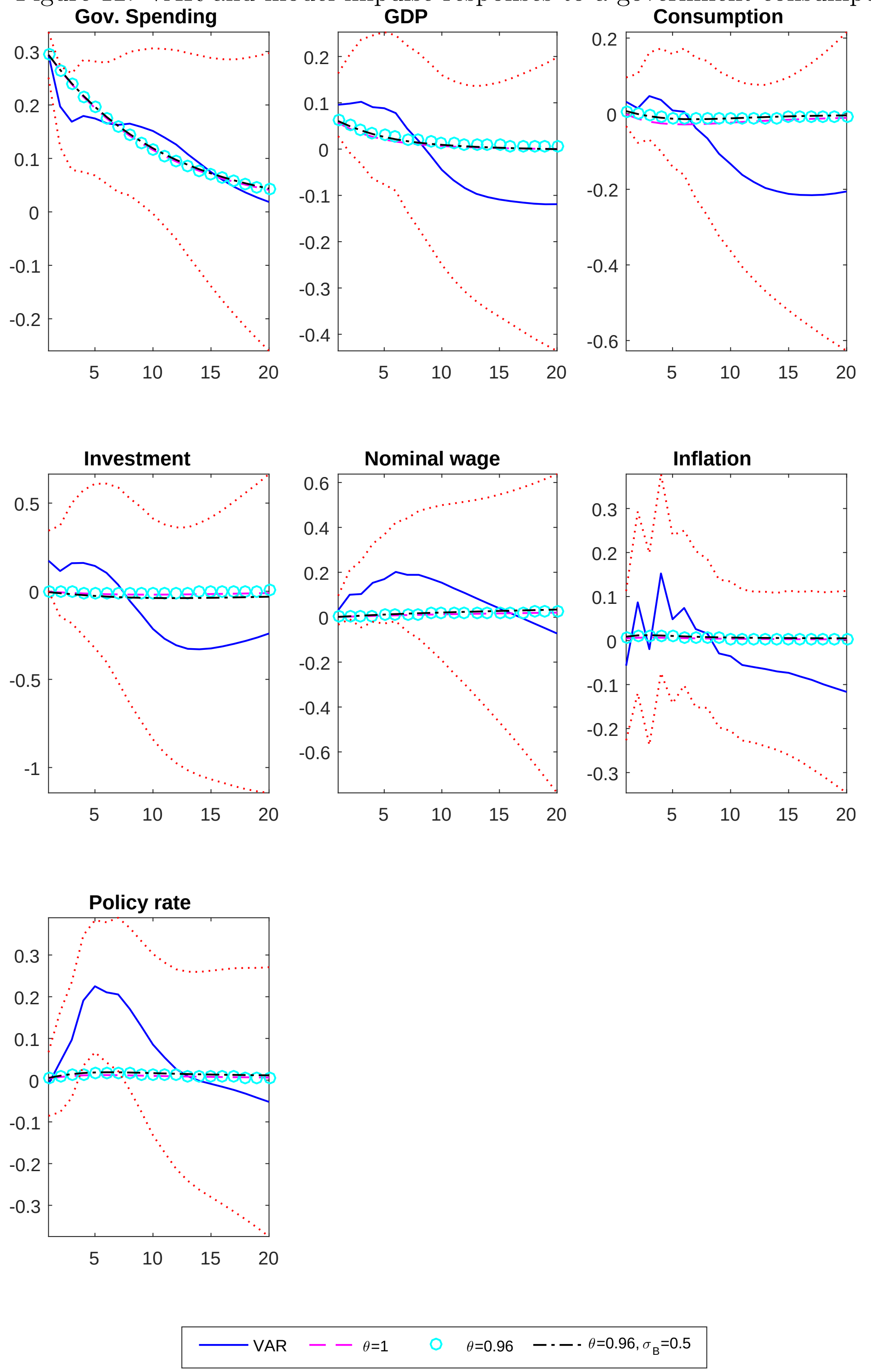
[12]For instance, their wage and price markup coefficient in their model with a financial accelerator equal 0.004 and 0.007, respectively.

Figure 11: VAR and model impulse responses to a monetary policy shock



Notes: The graph plots the impulse response function of the estimated VAR as well as each model variant to a one standard deviation policy rate shock. Dotted lines denote 95% confidence intervals.

Figure 12: VAR and model impulse responses to a government consumption shock



Notes: The graph plots the impulse response function of the estimated VAR as well as each model variant to a one standard deviation government consumption shock. Dotted lines denote 95% confidence intervals.

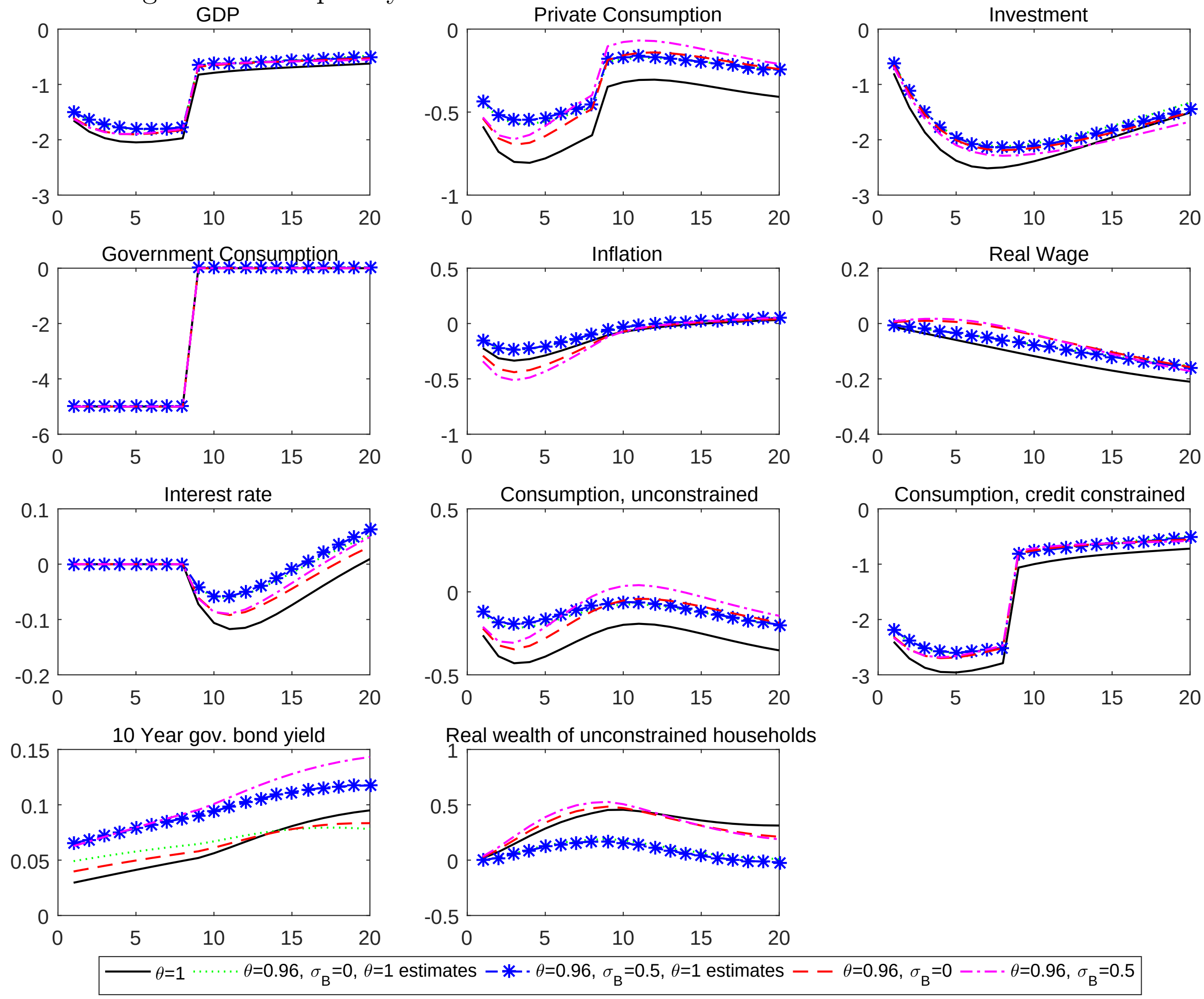
13.4 Temporary fiscal consolidation in the medium scale model

I first examine the effects of contraction of government consumption of 1% of GDP lasting eight quarters, 10 years and infinitely long, respectively. As a proxy for the ZLB, I assume that the central bank's response to the inflation and GDP effect of the consolidation given by equation (98) as well as the fiscal rule (equation (96)) are switched off for 2 years by setting $d_{R,t} = d_{\tau_C,t} = 1$ and during this period. I consider the three different model versions estimated in the previous section, as well as a model with linear preferences over wealth ($\theta = 0.96$) but all other parameters as estimated for $\theta = 1$. Doing so serves to highlight the impact of the higher nominal flexibility estimated for the model with preferences over wealth.

As can be obtained from Figure 13, the effect of temporary contraction are similar across the different values of θ. In all cases the decline in government consumption also reduces private consumption by lowering the disposable income of constrained households as well as inflation, which increases the real interest rate and thus lowers the consumption of unconstrained households. Furthermore, the increase in real interest rates and the decline in demand lower Tobin's Q and thus investment. As a result of the adverse effect on private expenditure, the multiplier exceeds one (see Table 8). These mechanisms are well documented by literature on fiscal policy changes when monetary policy is constrained (e.g. Coenen et al. (2012)) Eggertsson and Krugman (2012) Freedman et al. (2012), Carillo and Poilly (2013)). Lowering θ below one renders unconstrained households less sensitive to the effect of lower future real interest rates (see equation 6). It therefore somewhat attenuates their consumption decline, as can be obtained by comparing the model without preferences over wealth (black solid line in Figure 13) with the model with linear preferences over wealth but otherwise identical parameters (the green dotted line which is covered by the blue starred line), even after accounting for the larger decline of inflation and thus stronger real interest rate increase observed with preferences over wealth (red dashed line) as a consequence higher estimated nominal price and wage flexibility. However, for a two year fiscal contraction, the effect of lowering θ is too small to have a big effect on the multiplier.

Finally, the impact higher estimated nominal price and wage flexibility with preferences over wealth is largely restricted to the paths of inflation and the real wealth of unconstrained households.

Figure 13: Temporary fiscal contraction in the medium scale model



Note: The figure displays the effect of a contraction of government consumption of 1% of GDP lasting 8 quarters ($T_G = 8$) in the three model variants. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 8 for further details.

13.5 Permanent fiscal contraction in the medium scale model

I now turn to the effect of a permanent fiscal contraction under varying assumptions regarding preferences over wealth.Figure 14 displays the effect in the absence credit constraints in the household and firm sector. As in the simple model, without preferences over wealth and thus an effective infinite horizon ($\theta = 1$), unconstrained household consumption works as a powerful stabilizer against a permanent decline in government consumption and the associated decline in constrained household consumption. Unconstrained household consumption increases as a consequence of the monetary loosening after the economy's exit from the ZLB and the fact that in the new steady state, the share of government consumption in GDP is lower and the share

Table 8: Fiscal contraction in the medium scale model, cumulative multipliers

Baseline model			
	$T_G = 8$	$T_G = 40$	$T_G = \infty$
$\theta = 1$	2.0	0.6	1.0
$\theta = 0.96, \sigma_B = 0$, estimates for $\theta = 1$	1.8	1.6	1.5
$\theta = 0.96, \sigma_B = 0.5$, estimates for $\theta = 1$	1.8	1.6	1.6
$\theta = 0.96,\ \sigma_B = 0$	1.9	1.6	1.6
$\theta = 0.96,\ \sigma_B = 0.5$	1.9	1.6	1.7

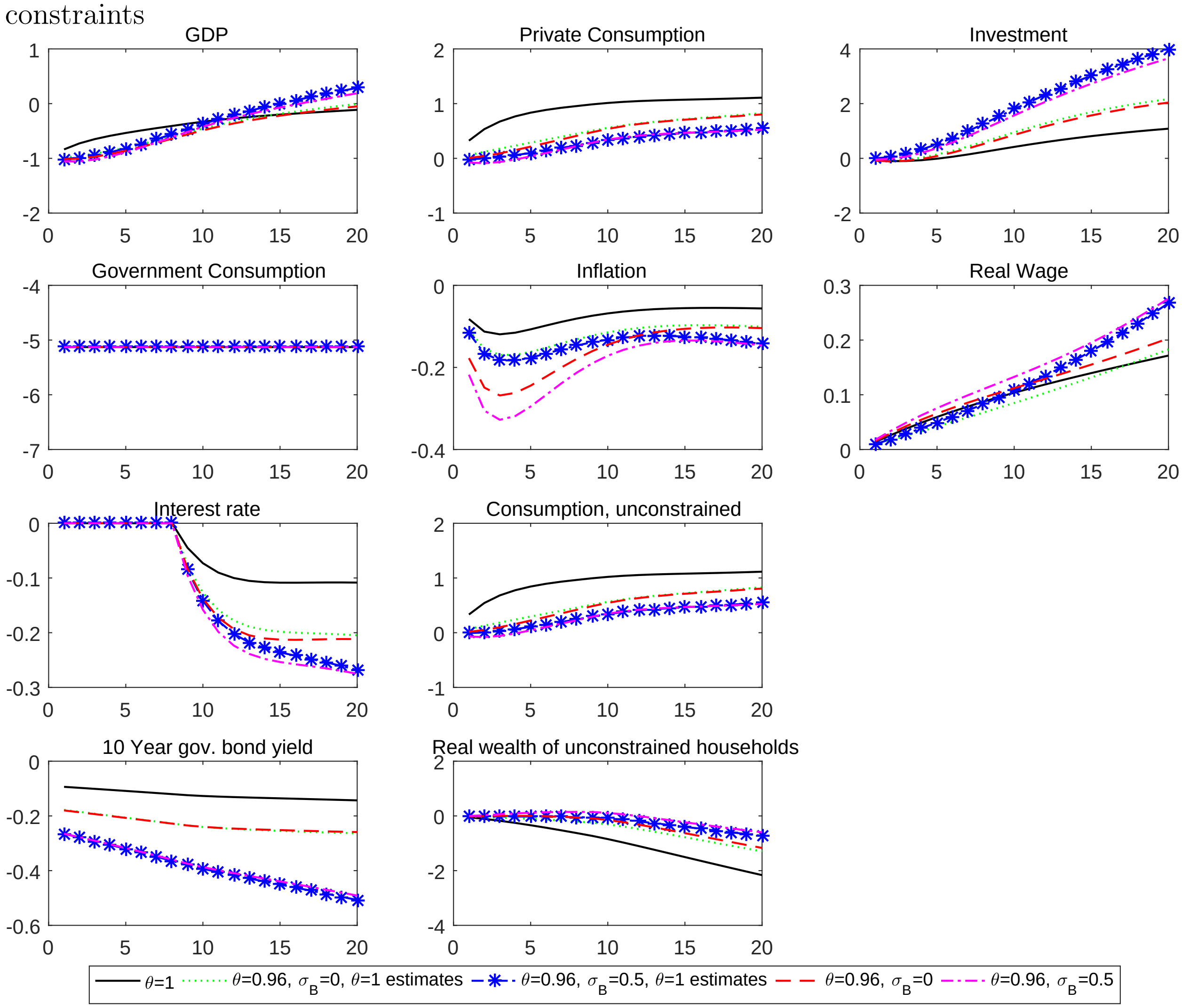
No credit constraints ($\omega = \mu = 0$)			
	$T_G = 8$	$T_G = 40$	$T_G = \infty$
$\theta = 1$	1.2	0.3	0.6
$\theta = 0.96, \sigma_B = 0$, estimates for$\theta = 1$	1.2	0.7	0.8
$\theta = 0.96, \sigma_B = 0.5$, estimates for$\theta = 1$	1.2	0.7	0.8
$\theta = 0.96, \sigma_B = 0$	1.2	0.8	0.9
$\theta = 0.96, \sigma_B = 0.5$	1.2	0.8	0.9

Note: The tables display the cumulative multiplier of increasing government spending for T_G quarters. In all scenarios, I assume that $d_{R,t} = 1$ for $t = 1 : D_L$ in equation (98), with $D_L = 8$, to proxy a zero lower bound duration of 8 quarters. The cumulative multiplier is calculated as $m_T = \frac{\sum_{t=1}^{D_L} \widehat{GDP}_t}{\sum_{t=1}^{D_L} \widehat{G}_t \frac{G}{GDP}}$. The calibrated parameters are as displayed in Table 6. σ_B is zero unless otherwise mentioned. The estimated parameters are displayed in Table 7 unless otherwise mentioned. In the row labeled "$\theta = 0.96$, $\sigma_B = x$, estimates for $\theta = 1$", I set $\theta = 0.96$ but use the parameters estimated for $\theta = 1$. In the model without credit constraints, I assume that there is no entrepreneurial consumption. Instead, entrepreneurs return their wealth to unconstrained households once they die.

of private consumption higher. With preferences over wealth ($\theta = 0.96$), household consumption increases much less than without ($\theta = 1$).

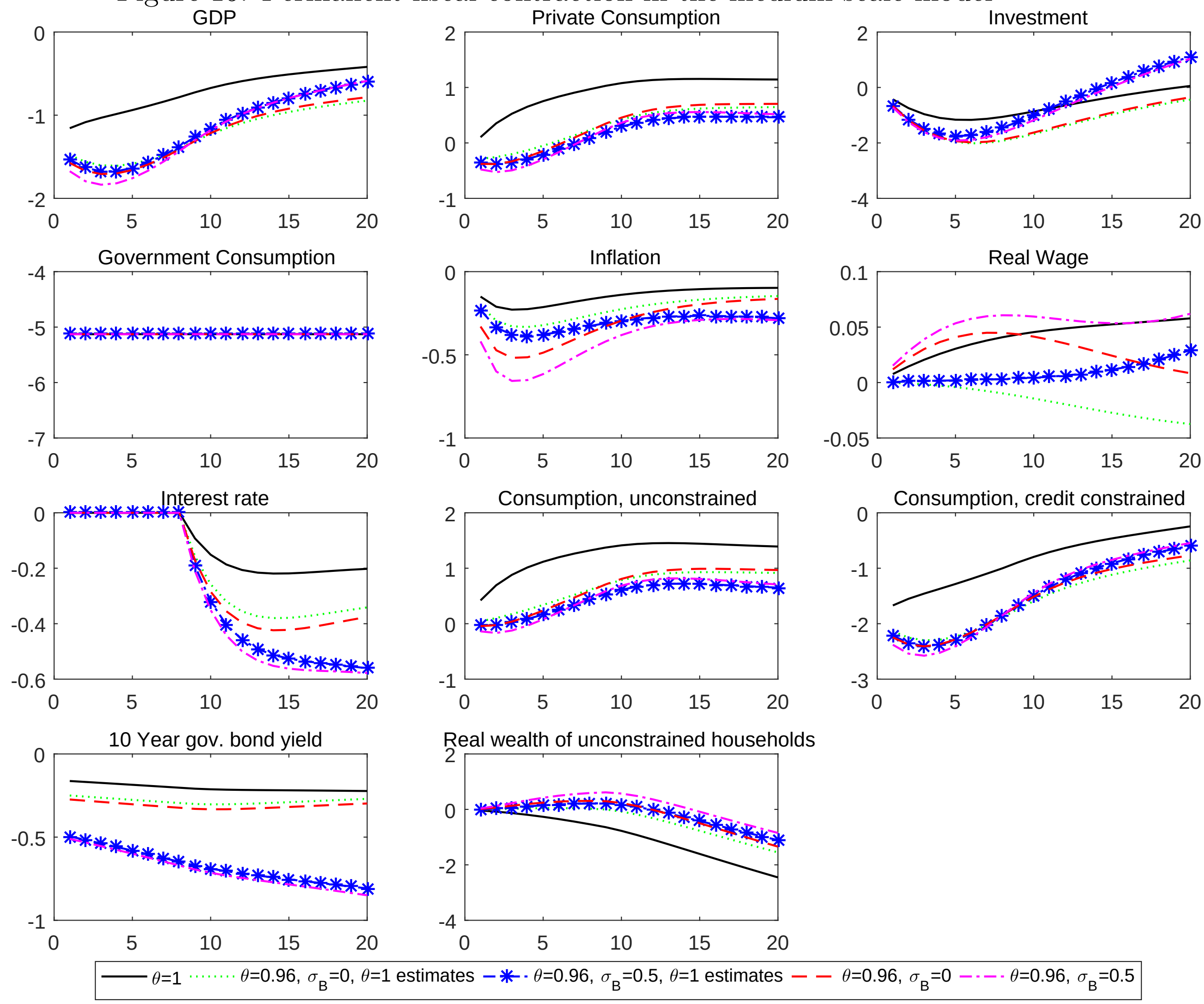
In the presence of credit constrained households and firms, the effect of the fiscal contraction increases (see Figure 15 and Table 8). The decline in employment directly lowers the consumption of credit constrained households. Lower inflation tends to increase the real debt burden of entrepreneurs and thus their cost of external finance, implying that investment now declines. However, the amplification provided by adding credit constraints to the model is substantially bigger with preferences over wealth than without. For $\theta = 1$, the increase in unconstrained household consumption is even larger than in the absence of constrained households and firms, implying that the path of private consumption is virtually unaffected by the introduction of credit constraints. With credit constrained households and firms, monetary policy is loosened more after the economy's exit from the ZLB, which causes a bigger crowding in of unconstrained household consumption if $\theta = 1$. With preferences over wealth ($\theta = 0.96$), the introduction of credit constraint households and firms does not cause such a stabilizing upward shift in the consumption trajectory of unconstrained households following a permanent government spending cut, as they are less sensitive to real interest rate movements, and thus the GDP effect of the decline in constrained household consumption and investment is diluted to a much smaller extent.

Figure 14: Permanent fiscal contraction in the medium scale model without credit constraints



Note: The figure displays the effect of a permanent contraction of government consumption of 1% of GDP ($T_G = \infty$) in the model without credit constraints. See the note below Table 8 for further details. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points.

Figure 15: Permanent fiscal contraction in the medium scale model



Note: The figure displays the effect of a permanent contraction of government consumption of 1% of GDP ($T_G = \infty$). in the baseline model. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 8 for further details.

13.6 Forward guidance

I now examine the effect of forward guidance on the path of the short term interest rate in the medium scale model. Analogously to the simple model, I assume that the Central Bank is initially constrained by the zero lower bound for 6 quarters, which I proxy by assuming $d_{R,t} = 1$ in equation (98) during this period. Furthermore, I assume that starting in quarter seven, the Central Bank voluntarily fixes $\hat{R}_f$ at an annualized value of 0.2% below its steady state and keeps it at this level for D_f quarters, i.e. $d_{R,t} = 1$ and $e_{R,t} = \frac{-0.2\%}{4}$ during quarters 7 to $6 + D_f$. This setup is broadly consistent with the evidence Del Negro et al. (2015) provide on financial market expectations regarding the timing of the exit of the Federal Funds rate from the ZLB prior to the forward guidance announcements of the US Federal Reserve in

Table 9: Peak GDP effect of forward guidance for varying duration of fixed interest rate period

Baseline model			
	$D_f = 6$	$D_f = 8$	$D_f = 10$
$\theta = 1$	1.3	1.9	2.2
$\theta = 0.96, \sigma_B = 0$, estimates for$\theta = 1$	0.8	1.0	1.2
$\theta = 0.96, \sigma_B = 0.5$, estimates for$\theta = 1$	0.7	0.8	0.9
$\theta = 0.96, \sigma_B = 0$	1.0	1.4	1.7
$\theta = 0.96,\ \sigma_B = 1$	0.8	1.0	1.1

Note: This table reports the peak GDP effect of forward guidance policies of the indicated duration T_f. In all cases, I assume that:

- for $t = 1:6$, $d_{R,t} = 1$ in equation (98) ,
- for $t = 7to7 + D_f$, $d_{R,t} = 1$ and $e_{R,t} = \frac{-0.2\%}{4}$.

the central bank then fixes the nominal interest rate at an annualized 0.2% below its steady state level, and keeps it there for the number of quarters indicated in the table. The calibrated parameters are as displayed in Table 6. σ_B is zero unless otherwise mentioned. The estimated parameters are displayed in Table 7 unless otherwise mentioned. In the row labeled "$\theta = 0.96$, estimates for$\theta = 1$", I set $\theta = 0.96$ but use the parameters estimated for $\theta = 1$.

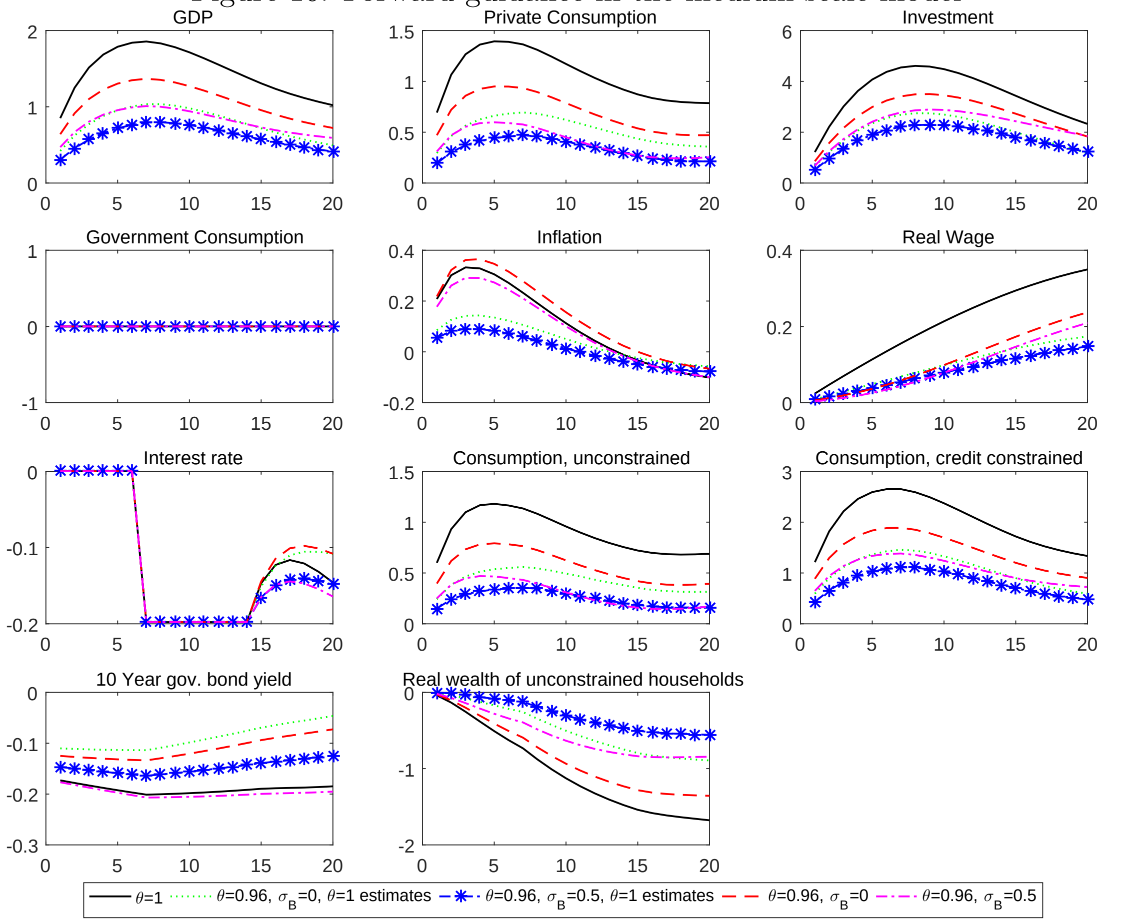
September 2011, January 2012 and September 2013, as well as their evidence on the effect of the announcement on private sector forecasts of three month and 10 year treasury bonds one to four quarters ahead, which gradually decline by 0.15 and 0.2 percentage points, respectively. The magnitude of D_f is less clear. I allow D_f to vary between 6 quarters, which closely follows Del Negro et al.'s (2015) simulations, and 10 quarters.

Figure 16 displays the effects of forward guidance for the $D_f = 8$ quarter case. The forward guidance policy stimulates private investment and consumption by lowering the expected real interest rate, increasing the value of the collateral of entrepreneurs, future expected demand and the disposable income of constrained households. While the policy is a powerful tool for all values of θ, just as in the simple model, the peak GDP response is lower with preferences over wealth ($\theta < 1$). Reducing θ to 0.96 but keeping all other parameters as in the model without preferences over wealth, the peak GDP effect is almost halved (0.8% with vs. 1.3% without preferences over wealth, Table 9/ green dotted line and black solid line in Figure 16). The expected decline in the future real interest rate has a smaller effect on unconstrained household consumption, and thus employment, constrained households consumption and investment. Once I use the parameters estimated for the $\theta = 0.96$, part of this attenuation is compensated by the higher price and wage flexibility estimated for the model with preferences over wealth, which implies a stronger decline in the real interest rate. Even after taking this compensating effect into account, the peak GDP effect still equals only two thirds of its value without preferences over wealth (see Table 9/ green dotted line vs. red dashed line in Figure 16) .

Once I allow for curvature in the preferences over wealth, the increase in GDP in response to the policy is attenuated even further (compare the blue starred line with the green dotted and the black solid line). The increase in unconstrained household consumption and inflation reduces their real financial wealth. With curvature ($\theta = 0.96, \sigma_B = 0.5$), the decline in their real wealth increases the marginal benefit of wealth accumulation relative to the marginal utility of consumption, which dampens their consumption increase (see equation 6). Increasing the period during which the central bank voluntarily reduces the interest rate D_f increases the peak GDP effect in all model variants, but considerably less so with preferences over wealth (see Table 9). With curvature in preferences over wealth, the marginal effect of lengthening the forward guidance episode actually declines. Allowing for the higher nominal flexibility estimated for the model with preferences over wealth implies only a marginally smaller attenuation effect.

Preferences over wealth parameterized based on micro evidence on personal discount rates and the saving behavior of high income households thus considerably alleviate the "Forward Guidance Puzzle" (Del Negro et al. (2015)), especially once curvature in the preferences over wealth is allowed for. For the $D_f = 8$ case, the simulated decline in the 10 year government bond yield by quarter four equals the empirical reduced form estimates of Del Negro et al. (2015) of 0.2 percentage points in both the model without preferences over wealth and curvature in preferences over wealth. However, with preferences over wealth and curvature, the cumulative increase in GDP in the fourth quarter after following the announcement of the policy is with 0.9% much closer to Del Negro et al.'s (2015) finding of 0.6% than it is in the model without preferences over wealth (1.7%).

Figure 16: Forward guidance in the medium scale model



Note: The figure displays the effects of forward guidance regarding the short term interest rate in the medium scale model ($D_f = 8$). Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 9 for further details.

14 Medium scale model equations

$$\hat{\lambda}_{ROT,t} = -\frac{\widehat{C}_{ROT,t} - h\widehat{C}_{ROT,t-1}}{\sigma_{ROT}(1-h)} - \frac{\hat{\tau}_{C,t}}{1-\tau_C}$$

$$(1+\tau_C) C_{ROT}\widehat{C}_{ROT,t} + C_{ROT}\widehat{\tau}_{C,t} = wl_{ROT}(1-\tau_{W_h})\left(\hat{w}_t + \hat{l}_{ROT,t} - \frac{\hat{\tau}_{W_h,t}}{1-\tau_{W_h}}\right) + TR_{ROT}\hat{TR}_{ROT,t}$$

$$\hat{\lambda}_{O,t} = -\frac{\widehat{C}_{O,t} - h\widehat{C}_{O,t-1}}{\sigma_O(1-h)} - \frac{\hat{\tau}_{C,t}}{1-\tau_C}$$

$$\hat{\lambda}_{O,t} = \theta E_t\left\{\hat{\lambda}_{O,t+1} + \widehat{R}_t - \widehat{\Pi}_{t+1}\right\} - (1-\theta)\sigma_B\hat{b}_{O,t}$$

$$\lambda\hat{\lambda}_t = (1-\omega)\lambda_O\hat{\lambda}_{O,t} + \omega\lambda_{ROT}\hat{\lambda}_{ROT,t}$$

$$\hat{l}_{ROT,t} = \hat{l}_{O,t}$$

$$\hat{l}_t = \hat{l}_{ROT,t}$$

$$\widehat{w}_t = \frac{1}{1+\beta}\left[\begin{array}{c} \beta E_t\widehat{w}_{t+1} + \widehat{w}_{t-1} + \beta E_t\widehat{\Pi}_{t+1} - (1+\beta\gamma_w)\,\widehat{\Pi}_t \\ +\gamma_w\widehat{\Pi}_{t-1} - \frac{(1-\beta\xi^w)(1-\xi^w)}{\xi^w(1+\varepsilon^w\varphi)}\left[\widehat{w}_t - \frac{\hat{\tau}_{W_h,t}}{1-\tau_{W_h}} + \hat{\lambda}_t - \varphi\widehat{l}_t\right] \end{array}\right] \text{Wage setting}$$

$$\widehat{\Pi}_t = \frac{1}{1+\beta\gamma_P}\left[\beta E_t\widehat{\Pi}_{t+1} + \gamma_P\widehat{\Pi}_{t-1} + \frac{\left(1-\beta\xi^P\right)\left(1-\xi^P\right)}{\xi^P}\widehat{mc}_t\right] \text{Price setting}$$

$$\widehat{w}_t + \frac{\hat{\tau}_{W_f,t}}{1-\tau_{W_f}} + \frac{\psi_l R\widehat{R}_t}{1+\psi_l\,(R-1)} = \widehat{mc}_t + \widehat{Y}_t - \widehat{l}_t$$

$$\frac{dr_t^k}{r^k} + \frac{\psi_K R\widehat{R}_t}{1+\psi_K\,(R-1)} = \widehat{mc}_t + \widehat{Y}_t - \widehat{K}_{t-1} - \widehat{U}_t$$

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t$$

$$\widehat{I}_t = \frac{1}{1+\beta}\left[\widehat{I}_{t-1} + \beta E_t\widehat{I}_{t+1} + \frac{\widehat{Q}_t}{\phi_I}\right]$$

$$\widehat{R}_t^K = \widehat{\Pi}_t + \frac{\Pi\left(dr_t^k\,(1-\tau_K) + \widehat{Q}_t\,(1-\delta)\right)}{R^k} - \widehat{Q}_{t-1}$$

$$dr_t^k = c^U r^k\widehat{U}_t$$

$$\widehat{\phi}_t^e = \widehat{Q}_t + \widehat{K}_t - \widehat{N}_t$$

$$E_t\widehat{R}_{t+1}^K - \widehat{R}_t = \chi^{\phi^e}\widehat{\phi}_t^e$$

$$\widehat{\varpi}_t + \widehat{R}_t^K = \widehat{R}_t^L + \frac{1}{\phi^e - 1}\widehat{\phi}_{t-1}^e$$

$$R\frac{\widehat{\phi}_{t-1}^e}{\phi^e} = e_t^{R^b} R^K Expr1 + \widehat{R}_t^K R^K Expr1 + R^K Expr2\varpi\widehat{\varpi}_t - R\frac{\phi^e - 1}{\phi^e}\widehat{R}_{t-1}$$

$$\widehat{N}_t = \gamma\frac{V}{N}\widehat{V}_t$$

$$\widehat{V}_t = \widehat{N}_{t-1} + \widehat{R}_t^K - \widehat{\Pi}_t + \widehat{\phi}_{t-1}^e - \frac{\Gamma'(\overline{\omega})\overline{\omega}}{1 - \Gamma(\overline{\omega})}\widehat{\varpi}_t + e_t^N$$

$$\widehat{L}_t^e = \widehat{N}_t + \frac{\phi^e}{\phi^e - 1}\widehat{\phi}_t^e$$

$$\widehat{C}_t^e = \widehat{V}_t$$

$$\widehat{R}_t R = (1-\rho_i)\left[\psi_\pi\widehat{\Pi}_t + \psi_y\left(\widehat{GDP}_t - \widehat{GDP}_{t-1}\right)\right] + \rho_i\widehat{R}_{t-1} + e_t^i$$

$$\widehat{Y}_t = \alpha\left(\widehat{U}_t + \widehat{K}_{t-1}\right) + (1-\alpha)\left(\widehat{a}_t + \widehat{l}_t\right)$$

$$\hat{b}_{G,t}b_G = b_G\frac{R}{\Pi}\left(\hat{b}_{G,t-1} + \hat{R}_{t-1} - \hat{\Pi}_t\right) + Gov\hat{G}ov_t + TRT\hat{R}_t - (\widehat{C}_t^P\tau_C + \tau_C C^P + (\tau_{W_h,t} + \tau_{W_f,t})wl + \tau_K r^k K\hat{K}_{t-1} +$$

$$\hat{b}_{O,t} = \frac{L^e}{L^e + b_G}\widehat{L}_t^e + \frac{b_G}{L^e + b_G}\hat{b}_{G,t}$$

$$\hat{\tau}_{C,t} = (1-\rho_\tau)\phi_\tau\hat{b}_{G,t} + \rho_\tau\hat{\tau}_{C,t-1}$$

$$\widehat{Y}_t = \frac{I}{Y}\widehat{I}_t + \frac{C^P}{Y}\widehat{C}_t^P + \frac{Gov}{Y}\widehat{G}ov_t + \frac{R^K}{\Pi}\frac{K}{Y}\mu G(\varpi)\left(\widehat{R}_t^K - \widehat{\Pi}_t + \widehat{Q}_{t-1} + \widehat{K}_{t-1} + \frac{G'(\varpi)}{G(\varpi)}\varpi\widehat{\varpi}_t\right) + r^k\frac{K}{Y}\widehat{U}_t$$

$$C\hat{C}_t = \omega C_{ROT}\hat{C}_{ROT,t} + (1-\omega)C_O\hat{C}_{O,t}$$

$$\widehat{C}_t^P = \frac{C}{C^P}\widehat{C}_t + \frac{C^e}{C^P}\widehat{C}_t^e$$

$$\widehat{GDP}_t = \frac{I}{GDP}\widehat{I}_t + \frac{C^P}{GDP}\widehat{C}_t^P + \frac{Gov}{GDP}\widehat{g}_t$$

$$Expr1 = [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]$$

$$Expr2 = [\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})]$$

$$R\frac{\widehat{Q}_{t-1} + \widehat{K}_{t-1} - \widehat{N}_{t-1}}{\phi^e} = e_t^{R^b} R^K Expr1 + \widehat{R}_t^K R^K Expr1 + R^K Expr2 \varpi \widehat{\varpi}_t - R\frac{\phi^e - 1}{\phi^e}\widehat{R}_{t-1}$$

$$\widehat{N}_t = \gamma\frac{V}{N}\widehat{V}_t$$

$$\widehat{V}_t = \hat{Q}_{t-1} + \widehat{K}_{t-1} + \widehat{R}_t^K - \widehat{\Pi}_t - \frac{\Gamma'(\overline{\omega})\overline{\omega}}{1 - \Gamma(\overline{\omega})}\widehat{\varpi}_t$$