Term Structure and Real-Time Learning*

Pablo Aguilar[a,b,c] and Jesús Vázquez[c]

October, 2017 (First version: February, 2016)

ABSTRACT: This paper introduces the term structure of interest rates into a medium-scale DSGE model. This extension results in a multi-period forecasting model that is estimated under both adaptive learning and rational expectations. Term structure information enables us to characterize agents' expectations in real time, which addresses an imperfect information issue mostly neglected in the adaptive learning literature. Relative to the rational expectations version, our estimated DSGE model under adaptive learning largely improves the model fit to the data, which include not just macroeconomic data but also the yield curve and the consumption growth and inflation forecasts reported in the Survey of Professional Forecasters. Moreover, the estimation results show that most endogenous sources of aggregate persistence are dramatically undercut when adaptive learning based on multi-period forecasting is incorporated through the term structure of interest rates.

JEL classification: C53, D84, E30, E44

Keywords: Real-time adaptive learning, term spread, multi-period forecasting, short- versus long-sighted agents, SPF forecasts, medium-scale DSGE model

a Bank of Spain

b IRES, Université catholique de Louvain

c Universidad del País Vasco (UPV/EHU). Corresponding author: jesus.vazquez@ehu.es

*We are very grateful to Raf Wouters and Sergey Slobodyan for their close guidance on this work. We also thank Steve Cassou, Dean Croushore, David de la Croix, Stefano Eusepi, George Evans, Chris Gibbs, Cars Hommes, Rigas Oikonomou, Luca Pensieroso, Bruce Preston and seminar participants at the 2016 Learning Conference "Expectations in Dynamic Macroeconomic Models" (University of Amsterdam), the 91st Annual Conference of the Western Economic (Portland, Oregon), the 23th International Conference on Computing in Economics and Finance (Fordham University, New York), the 70th European Summer Meetings of the Econometric Society (Lisbon), the 2016 THUIT Workshop (Universidad Complutense de Madrid) and the IRES Macroeconomic Workshop (Université catholique de Louvain) for comments and suggestions. This research was mainly supported by the Bank of Spain. Additional support funding was provided by the Spanish Ministry of Economy and Competition and the Basque Government (Spain) under grant numbers ECO2013-43773P and IT-793-13, respectively.

1 Introduction

Since the pioneering publications by Marcet and Sargent (1989) and Evans and Honkapohja (2001) a growing literature (among others, Preston, 2005; Milani, 2007, 2008, 2011; Eusepi and Preston, 2011; Slobodyan and Wouters, 2012a,b) has considered adaptive learning (AL) as an alternative to the rational expectations (RE) assumption in the characterization of macroeconomic dynamics. This literature has shown that AL results in a sizable amplification mechanism in the transmission of shocks by introducing persistent learning dynamics which takes over other sources of aggregate persistence.[1] However, most estimated AL models typically consider forecasting models based on variables whose observable counterparts are *final revised* data. This neglects an imperfect information issue in the estimation of learning dynamics.[2] Namely, learning dynamics are in reality driven by data truly available to agents when forming their expectations in real time. It is important to recognize that macroeconomic dynamics are affected by the actual information available during the learning process. In particular, macroeconomic dynamics can be affected by term structure information as well as by other sources of information observed in real time—such as real time information on macroeconomic data, studied in a growing real-time literature (Croushore, 2011)—through the learning process.

This paper deals with this imperfect information issue by assuming that agents form *all* their expectations using term structure information observed in real time. The use of term

[1] In particular, Eusepi and Preston (2011) show that AL based on forecasts over an infinite horizon—i.e. considering the maintained beliefs hypothesis as in Marcet and Sargent (1989) and Preston (2005)—results in a huge amplification mechanism of technology shocks in a standard real business cycle model. Milani (2007) finds similar results in a small-scale New Keynesian model.

[2] An exception is Milani (2011). He focuses on real-time data on output and inflation and the forecasts from the Survey of Professional Forecasters (SPF) recorded in real time when estimating a small-scale DSGE model. However, he ignores revised data on macroeconomic variables, which more accurately describe the actual economy when estimating and assessing model fit. More recently, Slobodyan and Wouters (2017) estimate a medium-scale DSGE model considering real-time inflation to model agents' inflation forecasts as well as SPF inflation data as observable to discipline inflation expectations. However, their remaining small forecasting models, as in Slobodyan and Wouters (2012a,b), are based on either revised data or variables that do not have an observable counterpart such as Tobin's-q or the rental rate of capital. This feature certainly introduces asymmetry into their learning processes, which we overcome in our AL model by using only real time information in the expectation formation of all forward-looking variables.

structure information to characterize agents' learning process can be rationalized in two ways. From a theoretical perspective, consumption-based asset pricing models show a close connection between term spreads and expectations of both consumption and inflation. From an empirical perspective, the use of term structure information in agents' forecasting models is further rationalized by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997).[3]

We build on the AL model of Slobodyan and Wouters (2012a), which aims to investigate the role of term structure information in the learning process. They introduced AL into the medium-scale DSGE model suggested by Smets and Wouters (2007) (hereinafter called the SW model). We extend the Slobodyan and Wouters (2012a) framework by incorporating the term structure of interest rates. The extended model results in a multi-period forecasting model. Our model deviates from the two main approaches in the recent literature to AL. One approach (called the Euler equation approach) focuses on *short-sighted* agents, for whom optimal current decisions are based on just one-period-ahead expectations showing up in the standard Euler equations (e.g. Milani, 2007; Slobodyan and Wouters, 2012a,b), while the other approach—based on the maintained beliefs hypothesis (e.g. Preston, 2005; Eusepi and Preston, 2011; Sinha, 2015; and Sinha; 2016)—focuses on *long-sighted* agents, as under RE, taking into account infinite-horizon forecasts driven by their intertemporal decision problem.[4] By including the term structure of interest rates, our approach certainly goes beyond the one-period-ahead expectations but, as in the Euler equation approach and in contrast to the maintained beliefs hypothesis, we consider finite horizon AL expectations models.

Despite the highly restrictive forecasting models considered, our estimated AL-DSGE model extended with term structure information fits the data much better than the RE version of the model. In particular, it provides a good fit of macroeconomic dynamics, the yield

[3]The idea of using only term structure information to predict business cycle conditions goes back at least to McCallum (1994), who emphasizes that term spreads can be used as simple predictors regarding future macroeconomic conditions for defining monetary policy.

[4]As shown by Eusepi and Preston (2011), this distinction can be crucial because the latter results in a much stronger source of persistent dynamics.

curve, and the consumption growth and inflation forecasts reported in the SPF. Moreover, our estimated model shows that the importance of most endogenous sources of aggregate persistence (such as habit formation, Calvo probabilities, the elasticity of the cost of adjusting capital, and the elasticity of capital utilization adjustment cost) declines substantially when the hypothesis of multi-period forecasting is incorporated through the term structure of interest rates. Furthermore, we find that consumer decision making based on a 1-year time forecast horizon (i.e. medium-sighted agents) is sufficient to induce a great deal of aggregate persistence through the learning mechanism. These results show the importance of the multi-period forecasting hypothesis in an estimated medium-scale DSGE model together with the characterization of learning based on real-time information, assessing results found by Eusepi and Preston (2011) in a prototype real business cycle model and Milani (2007, 2011) and Slobodyan and Wouters (2012a) in New Keynesian frameworks.

The rest of the paper is organized as follows. Section 2 introduces a DSGE model with real-time AL under the multi-period forecasting hypothesis, focusing on the short end of the yield curve. Section 3 shows the estimation results and discusses their implications. Section 4 extends the analysis in two important directions: First, we consider the information available in the longer maturity term structure. Second, we include real-time inflation data in addition to term structure information in the small forecasting models. Section 5 concludes.

2 A real-time adaptive learning DSGE model

This paper investigates the interaction of the multi-period forecasting hypothesis with the term structure of interest rates—an important source of information observed in real time—in the characterization of the agents' learning process in an estimated DSGE model. Our model builds on the SW model and its AL extension studied by Slobodyan and Wouters (2012a). This standard medium-scale estimated DSGE model contains both nominal and real frictions affecting the choices of households and firms. In order to deal with the limited information issue affecting AL in real time, we extend the medium-scale DSGE model in three directions:

- First, we extend the model to account for the term structure of interest rates, which is perfectly observable in real time. More precisely, we consider the standard consumption-based asset pricing equation associated with each maturity. This approach is different from the approach followed in a parallel paper—Aguilar and Vázquez (2017)—which imposes the term structure expectation hypothesis obtained by iterating the asset pricing equations forward. Our approach enables us to abandon the term structure expectation hypothesis while still considering the possibility that multiple-period-ahead expectations under AL may matter to agents' decisions. In this way, the current consumption in our AL framework (i) depends on a finite set of expectations instead of depending on an infinite-horizon expectations set as occurs under the maintained beliefs hypothesis—Sargent and Marcet (1989), Preston (2005)—and (ii) is consistent with the term structure of interest rates.

- Second, and more importantly given the important informational issue addressed in this paper, the small forecasting models of all forward-looking variables in the medium-scale DSGE model under AL only include lagged term structure information, which is truly observed in real time. We thus overcome an informational issue often neglected in the AL literature, namely the use of revised data—which are hardly observable in real time—to identify the features of learning dynamics.

- Finally, beyond taking into account term structure information, both AL and RE expectations are further disciplined by requiring that the deviations of the estimated model expectations from the corresponding forecasts reported in the SPF follow AR(1) processes. We relax the assumption of Ormeño and Molnár (2015) that these deviations are identically, independent distributed—i.i.d.—measurement errors. The rationale for this alternative assumption is that in our extended model term structure information is used in addition to SPF forecasts to discipline the expectations. As is well known in the related literature, market and SPF expectations may differ. Indeed, there is evidence that term structure information—e.g. Rudebusch and Williams (2009)—is not consistently used by professional forecasters. Hence, by allowing for the possibility of stationary but persistent deviations between AL expectations and those reported in the SPF we let term structure speak more freely when disciplining agents' expectations.

Next, we present these extensions of the model. The remaining log-linearized equations of the model are presented in Appendix 1.

2.1 The DSGE model

Our model is based on the standard SW model. Households maximize their utility functions, which depend on their levels of consumption relative to an external habit component and leisure. Labor supplied by households is differentiated by a union with monopoly power setting sticky nominal wages *à la* Calvo (1983). Households rent capital to firms and decide how much capital to accumulate depending on the capital adjustment costs they face. Moreover, households also determine the degree of capital utilization as a positive function of the rental rate of capital, which depends on the capital utilization adjustment costs. Intermediate firms decide how much differentiated labor to hire to produce differentiated goods and set their prices *à la* Calvo. In addition, both wages and prices are partially indexed to lagged inflation when they are not re-optimized, introducing additional sources of nominal rigidity. As a result, current prices depend on current and expected marginal cost and past inflation whereas current wages are determined by past and expected future inflation and wages. We deviate from the monetary policy rule in the SW model by assuming that the monetary authorities follow a Taylor-type rule reacting to expected inflation and lagged values of output gap, output gap growth, and a term spread as defined below.

2.2 The term structure extension

This section introduces the term structure of interest rates in the SW model. This can be viewed as an alternative approach of introducing the multi-period forecasting hypothesis to assuming the maintained beliefs hypothesis, which considers the intertemporal budget constraint associated with the infinite-horizon problem faced by long-sighted optimal agents as in Preston (2005) and Eusepi and Preston (2011). A key aspect of our alternative approach is that the characterization of forward-looking behavior (i.e. multi-period expectations) is disciplined by observable outcomes determined in fixed-income security markets (i.e. term structure information).

Following De Graeve, Emiris and Wouters (2009) and Vázquez, María-Dolores and Londoño (2013), we extend the DSGE model by explicitly considering the interest rates associated with alternative bond maturities indexed by j (i.e. $j = 1, 2, ..., n$). From the first-order conditions describing the optimal decisions of the representative consumer, one can obtain the standard consumption-based asset pricing equation associated with each maturity:

$$E_t \left[\beta^j \frac{U_C(C_{t+j}, N_{t+j}) exp(\xi_t^{\{j\}})(1 + R_t^{\{j\}})^j}{U_C(C_t, N_t) \prod_{k=1}^{j}(1 + \pi_{t+k})} \right] = 1, \ for \ j = \ 1, \ 2, \ ..., \ n,$$

where E_t stands for the RE or the AL operator depending on the estimated model analyzed below, β is the discount factor, U_C denotes the marginal utility of consumption, and C_t, N_t, $R_t^{\{j\}}$, π_t and $\xi_t^{\{j\}}$ denote consumption, labor, the nominal yield of a j-period maturity bond, the inflation rate, and the term premium shock associated with a j-period maturity bond, respectively. The inclusion of a term premium shock for each maturity is in line with the view of many authors of interpreting the gap between the pure-expectations-hypothesis-implied yield, $R_t^{\{j\}}$, and the observed yield as a measure of fluctuations in the risk premium (e.g. De Graeve, Emiris and Wouters, 2009). Moreover, since we focus on government bonds in our empirical analysis, $\xi_t^{\{j\}}$ can be understood as a *convenience yield term* (see, among others, Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Del Negro et al., 2017) defined as a risk premium associated with the safety and liquidity features of government bonds relative to assets with the same payoff, but without such outstanding properties.

Assuming that the utility function is logarithmic in consumption,[5] after some algebra, the (linearized) consumption-based asset pricing equations can be written as

$$\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right)c_t - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right)c_{t-1} =$$

$$E_t\left[\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right)c_{t+j} - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right)c_{t+j-1}\right] - \left[jr_t^{\{j\}} - E_t\sum_{k=1}^{j}\pi_{t+k} + \xi_t^{\{j\}}\right],\ for j = 1, 2, ..., n, \quad (1)$$

where lower case variables denote log-deviations from balanced-growth values or, alternatively, deviations from steady-state values. In particular, for $j = 1$ the last expression results in the standard IS-curve equation in the SW model assuming a logarithmic utility function in consumption—i.e. Equation (2) in Smets and Wouters (2007, p.588) when the risk aversion parameter, σ_c, is one. Subtracting the expression in (1) for $j = 1$ from (1), we obtain the following expression linking the term spread defined by the j- and the 1-period yields with consumption growth and inflation expectations:

$$r_t^{\{j\}} - r_t^{\{1\}} = \left(\frac{j-1}{j}\right)r_t^{\{1\}} + \frac{1}{j}E_t\left[c\left(c_{t+j} - c_{t+1}\right) + (1-c)\left(c_{t+j-1} - c_t\right)\right]$$

$$+\frac{1}{j}E_t\sum_{k=2}^{j}\pi_{t+k} - \frac{1}{j}\left(\xi_t^{\{j\}} - \xi_t^{\{1\}}\right),$$

where $c = \frac{1}{1-\frac{h}{\bar{\gamma}}}$. This optimality condition clearly shows that term spreads must be linked to consumption and inflation expectations in equilibrium, which rationalizes our modeling

[5]This assumption greatly simplifies the estimation procedure below by avoiding the characterization of labor supply expectations over multiple time horizons. Moreover, while inflation and consumption expectations can be disciplined with the SPF, reported at least since 1981, labor forecasts in the SPF started to be released much more recently, in the fourth quarter of 2003, which prevents us from using them. Another problem of considering SPF labor forecasts is that they are based on total payroll employment (extensive margin of labor) whereas the counterpart in the model is defined as total hours worked.

approach of using term structure information to characterize the formation of agents' expectations in real time.

Under RE, the optimality conditions in (1) for $j > 1$ are somewhat redundant because long-term bonds are redundant assets in equilibrium. However, agents face much greater model uncertainty under AL and consequently long term bonds may help agents to hedge against future sources of uncertainty. Moreover, as is clear from the set of consumption-based asset pricing equations (1), optimal consumption household decisions under AL explicitly involve the multi-period forecasting hypothesis since current consumption depends, among other things, on the expected paths of both future consumption and inflation. In contrast to the maintained belief hypothesis considered in Preston (2005) and Eusepi and Preston (2011), the multi-period forecasting hypothesis does not necessarily impose the condition that today's consumption decision must depend on the entire (infinite path) of future consumption and inflation, but only on a finite number of periods, say n.[6] We further assume that the risk premium shock $\xi_t^{\{1\}}$ follows an AR(1) process: $\xi_t^{\{1\}} = \rho^{\{1\}}\xi_{t-1}^{\{1\}} + \eta_t^{\{1\}}$, whereas the term premium shocks $\xi_t^{\{j\}}$, for $j > 1$, follow AR(1) processes augmented with an additional term that allows for an interaction with the risk premium shock: $\xi_t^{\{j\}} = \rho^{\{j\}}\xi_{t-1}^{\{j\}} + \rho_\xi^{\{j\}}\eta_t^{\{1\}} + \eta_t^{\{j\}}$. That is, $\rho_\xi^{\{j\}}$ captures the interaction of the risk premium innovation, $\eta_t^{\{1\}}$, with the term premium shock, $\xi_t^{\{j\}}$, associated with the j-period maturity bond.

2.3 Real-time adaptive learning

Most papers in the AL literature rely on *revised* aggregate data when estimating the small forecasting models that agents (are assumed to) follow to update their expectations, the so-called "perceived law of motion" (PLM). This assumption is rather problematic because revised aggregate data are not available to economic agents when they are forming their expectations. As shown in a substantial body of real-time literature,[7] macroeconomic data

[6] In the econometric analysis below, we consider alternative values of n to assess the empirical importance of considering medium- versus long-sighted agents.

[7] See Croushore (2011) and references therein for an analysis of aggregate data revisions and their consequences.

revisions are sizable, which means that estimated AL-DSGE models based on revised data may somewhat distort the characterization of both learning and macroeconomic dynamics. An exception is Aguilar and Vázquez (2017), which introduces the *current* term spread between the 1-year constant maturity rate and the Fed funds rate as the only explanatory variable in the definition of the PLM. In this paper we take an additional step forward by considering only *lagged* term structure information *truly* available to agents at the time they are forming their expectations.[8]

Appendix 2 outlines how AL expectation formation works and how AL interacts with the rest of the economy. Here, we described the small forecasting models agents use to forecast the forward-looking variables of the DSGE model.

A PLM with only term structure information

We consider a specific PLM based on term structure information, which is truly observed when agents form their expectations in real time. As emphasized above, a PLM based on term structure information is rationalized, from a theoretical perspective, by the interaction between term spreads and the expectations of both consumption and inflation implied by the set of optimality conditions (1). From an empirical perspective, the use of term structure information in the PLM is further motivated by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997).

More precisely, we focus our attention on the role of 2-quarter and 4-quarter term spreads to characterize the PLM of all forward-looking variables of the model as follows:[9]

[8] The importance of including real-time inflation data in addition to term structure information in the small forecasting models is analyzed below.

[9] At first sight, one might think that it would be useful to consider the whole term structure of interest rates to characterize AL. However, considering term spreads associated with long-horizon bonds means estimating the large number of parameters associated with the whole set of expectations of consumption and inflation from the 1-period horizon up to a long horizon. Without additional restrictions on the learning process, this task cannot be accomplished under the Euler equation AL approach because the number of parameters defining the PLM dramatically increases with the number of expectation functions of consumption and inflation defined over alternative forecast horizons in (1), which results in a curse of dimensionality problem. Moreover, there is evidence (Mishkin, 1990) showing that at maturities longer than just two quarters the

$$
\begin{cases}
E_t y_{t+1} = \theta_{y,t-1} + \beta^{\{2\}}_{y,t-1} sp^{\{2\}}_{t-1}, \quad for\ y = i,\ r^k,\ q,\ w \\
E_t y_{t+j} = \theta^{\{j\}}_{y,t-1} + \beta^{\{j,2\}}_{y,t-1} sp^{\{2\}}_{t-1}, \quad for\ y =\ c,\ \pi\ and\ j =\ 0,\ 1,\ 2,\ 3 \\
E_t y_{t+j} = \theta^{\{j\}}_{y,t-1} + \beta^{\{j,4\}}_{y,t-1} sp^{\{4\}}_{t-1}, \quad for\ y =\ c,\ \pi\ and\ j = 4
\end{cases}
\tag{2}
$$

where i, r^k, q, w, c and π stand for (in deviation from their respective steady-state values or detrended by the balanced growth rate) investment, rental rate of capital, Tobin's q, real wage, consumption, and inflation respectively; and $sp^{\{2\}}_{t-1} = r^{\{2\}}_{t-1} - r^{\{1\}}_{t-1}$ and $sp^{\{4\}}_{t-1} = r^{\{4\}}_{t-1} - r^{\{1\}}_{t-1}$ denote the term spreads associated with the 2-quarter and 1-year term spreads, respectively, measured with respect to the 1-quarter interest rate.[10]

Notice that lagged term spreads are already known at the beginning of period t when agents form their expectations in this period. According to the baseline PLM set (2), the PLM of all forward looking variables are characterized by the 2-quarter term spread except those characterizing the expectations of consumption and inflation four-quarter-ahead, which are based on the 1-year term spread.[11] Moreover, the presence of intercepts $\theta^{\{j\}}_{y,t-1}$ relaxes the RE assumption of agents having perfect knowledge about a common deterministic growth rate and a constant inflation target made in the SW model. Thus, considering a time-varying intercept coefficient allows expectations to trace growth rate shifts in the data as well as changes in the inflation target.

term structure of interest rates helps to anticipate future inflationary pressures. For these reasons, we start our analysis by focusing on medium-sighted agents in our baseline model by assuming that $n = 4$ in both (1) and (2). Below, we extend our analysis by considering a much longer forecast horizon: $n = 40$.

[10] In the estimation exercise below, we consider that the federal funds interest rate is equivalent to the 1-quarter interest rate. This assumption is standard in the related literature since these two interest rates are highly correlated.

[11] Empirical results are robust to alternative specifications which consider either only the 2-quarter term spread or only the 1-year term spread for all forecasting horizons instead of the term-spread mix considered in (2). However, these alternative specifications produce a worse-fitting model. Moreover, in preliminary estimation attempts, we also considered other PLM specifications that included the two term spreads together, but the model estimated resulted in a much worse fit, probably due to multicollinearity problems driven by the high correlation between the two term spreads. Furthermore, Section 4 investigates the robustness of results by estimating an alternative formulation of the PLM that includes not just term structure information but also real-time inflation data.

By considering small forecasting models as in Slobodyan and Wouters (2012a,b), we deviate from the minimum state variable (MSV) approach to AL followed by Eusepi and Preston (2011) and others (Orphanides and Williams, 2005a; Milani, 2007, 2008, 2011; Sinha 2015, 2016) where agents' expectations are based on a function of the state variables of the model. In contrast, small forecasting models assume that agents form their expectations based on the information provided by observable endogenous variables, such as those showing up in the Euler equations of a DSGE model. Small forecasting models based only on observable variables can arguably be viewed as a more appealing approach to AL than the MSV approach for characterizing learning dynamics on several grounds. Small forecasting models are robust to alternative models characterized by different MSV sets. It is important to recognize this feature because one of the main motivations for moving from the RE assumption and assuming some sort of AL is that in reality agents do not know the true model (i.e. the true data generating process). Consequently, they can not know the actual MSV set.[12] In short, a small forecasting model approach recognizes that each type of agent might, in reality, be endowed with much less information regarding the structure of the actual economy than the MSV approach presumes.[13]

As in the Euler equation learning approach, the PLM for each horizon is separately estimated according to (2), and thus they do not have to be consistent with each other.[14] These features of the PLM (2) are in line with the conclusions in Stark (2013) about how

[12]Indeed, state-of-the-art DSGE models have no common MSV set. Moreover, the MSV approach to AL requires that each type of agent (household, firms, and the government) perfectly observe the realizations of all state variables. For instance, this implies that when consumers are forming their expectations about the future paths of consumption and inflation that determine their decisions they have to observe the capital stock of the firms. Certainly, these assumptions are not in line with the limited information scenario faced by different agents in actual economies.

[13]Other papers (Adam, 2005; Orphanides and Williams, 2005b; Branch and Evans, 2006; Hommes and Zhu, 2014, Ormeño and Molnár, 2015) have also provided support for the use of small forecasting models on several grounds such as their relative forecast performance, their ability to facilitate coordination, and their ability to approximate well the Survey of Professional Forecasters. In particular, Ormeño and Molnár (2015) use a small forecasting model to characterize inflation expectations, but they rely on the MSV approach to characterize the rest of the forward-looking variables of the SW model.

[14]Hence, the Euler equation learning approach stands in contrast to the maintained beliefs hypothesis considered in Preston (2005) and Eusepi and Preston (2011), which imposes not only an infinite forecast horizon but also a consistent PLM over all forecast horizons used under the MSV approach.

SPF panelists behave: *"SPF panelists are quite flexible in their approach to forecasting... They use a combination of models in forming their expectations, rather than just one model. And, they vary their methods with the forecast horizon... the panelist update their projections frequently, suggesting that their projections incorporate the most recent information available on the economy around the survey's deadline.*" The flexibility of the PLM (2) across forecast horizons allows us to overcome the log-linear approximation typically imposed in DSGE modeling to some extent. Thus, this flexibility improves model fit by capturing non-linear features as well as transitory non-stationary patterns in the data (e.g. the downtrend of inflation in the last three decades).[15]

By restricting our attention to the four-period (quarter) forecasting horizon we certainly move beyond the 1-period forecast horizon considered in many related AL models, but we still do not impose infinite-horizon forecasting learning as under the maintained beliefs hypothesis, which may induce very strong persistent dynamics by linking the current consumption to the entire path of expectations about future consumption and inflation. In this way, we can assess how much aggregate persistence can be generated with a multi-period forecasting model based on *medium-sighted* consumers focusing on a medium-term (four-quarter) forecasting horizon. Below, we also investigate the case of *finite-long-sighted* consumers by using a 10-year forecasting horizon in their decision making as well as four additional medium- and long-term yields (up to the 10-year Treasury yield) as observables in order to discipline long-term expectations.

PLM disciplined by the Survey of Professional Forecasters (SPF)

AL is often criticized because it introduces additional degrees of freedom resulting in an arbitrary improvement in model fit. However, there is not much room for this type of criticism

[15] Notwithstanding the flexibility of having agents forecasting the real yield associated with each maturity by using a different forecasting model, one might ask what ensures that there is a stationary solution to this extended model. In practice, this is not much of an issue since unbounded solutions are ruled out in the implementation of the Kalman filter learning algorithm used in this paper. Technically, the reason is that the so called *projection facility* used in the learning algorithm prevents the changing beliefs from moving the system dynamics into an unstable region.

in our characterization of learning dynamics since we are considering a rather restrictive information set: term structure information observed in real time. Nevertheless, as a way of further disciplining expectations, we assume that the deviations of agents' expectations on both inflation, $E_t\pi_{t+j}$, and consumption growth, $E_t\left(c_{t+j}-c_{t+j-1}\right)$, from the (observed) forecasts reported in the SPF follow AR(1) processes: $\epsilon_{\pi,t}^{\{j\}}=\rho_{\pi}^{\{j\}}\epsilon_{\pi,t-1}^{\{j\}}+\eta_{\pi,t}^{\{j\}}$ and $\epsilon_{\Delta c,t}^{\{j\}}=\rho_{\Delta c}^{\{j\}}\epsilon_{\Delta c,t-1}^{\{j\}}+\eta_{\Delta c,t}^{\{j\}}$, respectively, for $j=1,2,3,4$. We consider the expectations of consumption and inflation from the 1-quarter to the 4-quarter forecast horizon, because in our baseline empirical analysis below we restrict our attention to the short-term horizon of the yield curve (i.e. the 1-year maturity bond yield) in order to overcome the curse of dimensionality problem mentioned above.

Unlike Ormeño and Molnár (2015), we do not impose the more restrictive assumption that agents' expectations must match those of the SPF up to a white noise error. In short, we allow for persistent deviations between AL expectations and those reported in the SPF. The reason is that our extended model uses term structure information to characterize model expectations, which disciplines them in addition to SPF forecasts. As pointed out by Stekler and Ye (2017), there is evidence that term structure information is not consistently used by professional forecasters—this is called "the yield spread puzzle" in the related literature (Rudebusch and Williams, 2009; Lahiri et al., 2013). Thus, by allowing for the possibility of stationary but persistent deviations between AL expectations and those reported in the SPF we let market expectations through term structure data speak more freely when disciplining agents' expectations.

2.4 Real-time monetary policy rule

In line with the limited information assumption considered in this paper, the monetary policy rule is assumed to be determined by inflation expectations and lagged values of output gap, output gap growth, and the 1-year term spread, which are assumed to be available to the policymaker at the time of implementing monetary policy. Formally,

$$r_t^{\{1\}} = \rho_r r_{t-1}^{\{1\}} + (1-\rho_r)\left[r_\pi E_t \pi_{t+1} + r_y \hat{y}_{t-1}\right] + r_{\Delta y}\Delta\hat{y}_{t-1} + r_{sp} sp_{t-1}^{\{4\}} + \varepsilon_t^r,$$

where $\hat{y}_{t-1,t} = y_{t-1,t} - \Phi\varepsilon_t^a$. That is, following Slobodyan and Wouters (2012a) the output gap, $\hat{y}_{t-1,t}$, is defined as the deviation of output from its underlying neutral productivity process.[16] ε_t^r is assumed to follow an AR(1) process: $\varepsilon_t^r = \rho_R \varepsilon_{t-1}^r + \eta_t^r$. In contrast to Slobodyan and Wouters (2012a), we assume a forward looking policy rule and include a potential reaction of the policy rate to the term spread. The first assumption introduces a kind of symmetry about the information sets of the central banker and the private sector (households and firms) and the way in which they forecast inflation. The second assumption allows the term spread to be a direct determinant in the policy rule. That is, the term spread may play a role beyond the indirect role played by being a determinant of inflation expectations as described in (2).

3 Estimation results

This section starts by describing the data and the estimation approach, then discusses the model fit, estimation results, the transmission of shocks, and the PLM of the AL.

3.1 Data and the estimation approach

Our AL model extended with term structure and the associated RE model are estimated using US data for the great moderation period running from 1984:1 until 2007:4.[17] The set of observable variables is identical to the set considered by Slobodyan and Wouters (2012a) (i.e. the quarterly series of the inflation rate, the Fed funds rate, the log of hours worked, and

[16] In this way, we avoid characterizing a large number of additional forward-looking variables associated with the frictionless economy, which describes the level of potential output needed to obtain the standard definition of the output gap.

[17] Our estimated model considers consumption growth forecasts from the SPF in addition to the inflation forecasts used by Ormeño and Molnár (2015). While inflation forecasts are reported back to the late 1960's, the consumption growth forecast time series starts at 1981:3. We decided to start our sample at 1984:1. In this way, we ignore the inflationary episode right after the Volcker monetary experiment and focus on the great moderation period when the policy rule was well characterized by a Taylor-type rule.

the quarterly log differences of real consumption, real investment, real wages, and real GDP) with the addition of the 1-year Treasury constant maturity yield and the SPF forecasts about inflation and the growth rate of consumption from 1- to 4-quarter horizons. GDP, consumption, investment, and hours worked are measured in per-working age population terms.[18]

The measurement equation is

$$X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ One\ year\ TB\ yield_t \\ dlCONS_t^{SPF\{j\}} \\ dlP_t^{SPF\{j\}} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \bar{l} \\ \overline{r} \\ \overline{r}^{\{4\}} \\ \overline{\gamma} \\ \overline{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ E_t\left(c_{t+j} - c_{t+j-1}\right) + \epsilon_{c,t}^{\{j\}} \\ E_t \pi_{t+j} + \epsilon_{\pi,t}^{\{j\}} \end{bmatrix}, \tag{3}$$

where l and dl denote the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment, and real wages, which are the variables featuring a long-run trend. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ and $\overline{r}^{\{4\}}$ are the steady-state levels of hours worked, inflation, the federal funds rate, and the 1-year (four-quarter) constant maturity Treasury yield, respectively. The superscripts SPF and $\{j\}$ in the last two rows of the measurement equation denote actual forecasts from the SPF and the corresponding forecast horizon for $j = 1,\ 2,\ 3,\ 4$; respectively.

The estimation approach follows a standard two-step Bayesian estimation procedure.

[18] Section 4 extends the analysis to consider four additional yields up to the 10-year Treasury constant maturity yield and the initial release of (real-time) inflation. Both SPF forecasts and real-time inflation data were downloaded from the website of the Federal Reserve Bank of Philadelphia.

First, the log posterior function is maximized by combining prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012a). Moreover, we consider rather loose priors for the parameters characterizing both the 1-year yield dynamics and the stationary processes characterizing the deviations of inflation and consumption growth model expectations from the corresponding forecasts reported in the SPF. The second step implements the Metropolis-Hastings algorithm, which runs a huge sequence of draws of all the possible realizations for each parameter in order to obtain its posterior distribution.[19]

3.2 Posterior estimates

Our estimated real-time AL—henceforth, RT-AL—model differs from the Slobodyan and Wouters (2012a)—henceforth, SlW—model in three important features, as highlighted at the beginning of Section 2, so it is important to study the effect of each of these departures from their estimated DSGE model on both model fit and parameter estimates.

Table 1 shows the estimation results for the alternative samples and specifications considered:

- Our sample period corresponds to the great moderation period (1984:1-2007:4) whereas Slobodyan and Wouters (2012a) focus on the 1966:1-2008:4 period. Fortunately, they also report their model's fit for 1984:1-2008:4, which is really close to the great moderation period. We can therefore compare our model's fit with theirs. Thus, the first column of Table 1 shows the estimation results of the SlW model for the sample period 1966:1-2007:4, whereas the second column reports the estimation results of the SlW model for the great moderation period.[20]

[19] The DSGE models are estimated using Dynare codes kindly provided by Sergey Slobodyan and Raf Wouters with a few modifications to accommodate the presence of term structure information in both the structural model and the small forecasting models, as described in equation (2).

[20] While the end of the great moderation period may be an issue debated in the related literature, it is a fact that our estimation results for the SlW model based on the sample period omitting the 2008 quarterly observations result in estimates almost identical to those reported in Slobodyan and Wouters (2012a), as discussed below.

- Our model uses the 1-year Treasury bill as an observable. Consequently, we also estimate the SlW model including the 1-year Treasury bill as observable—i.e. using 8 time series as observables. The corresponding estimates are reported in column 3. Column 4 contains the estimation results for our RT-AL model using these 8 time series as observables.

- Column 5 contains the estimation results for our RT-AL model when the four forecasts of both inflation and consumption growth from the SPF are included as described in the measurement equation (3)—i.e. using 16 time series as observables—and assuming that the discrepancies between model expectations and SPF forecasts are i.i.d. measurement errors.

- Finally, columns 6 and 7 show the estimation results for the RT-AL model and the RE model, respectively; when the 16 time series are used as observables and the discrepancies between model's expectations and SPF forecasts are allowed to follow AR(1) processes.

For each model estimated, Table 1 firstly reports the number of observable time series, the sample period, and the model fit based on marginal likelihood. The remaining rows show the posterior mean and the corresponding 90 percent interval of the posterior distribution—in parentheses—for four groups of selected parameters. The first and second groups include the parameters featuring real and nominal rigidities, respectively. The third group includes the parameters describing the ARMA coefficients that describe price and wage markup shocks. Finally, the fourth group includes the policy rule parameters.

A comparison of column 1 in Table 1 with the figures reported in Slobodyan and Wouters (2012a, Table 1, p. 74) shows both a similar fit and almost identical parameter estimates. This suggests that including or ignoring the quarterly observations of 2008 has no impact on the estimation results. By restricting the sample period to the great moderation episode, we observe (column 2) that a few sources of real rigidity (habit formation and the cost of adjusting capital accumulation) and nominal rigidity (price Calvo probability, indexation parameters and wage markup shock parameters) become more important. Moreover, the value of the marginal log-likelihood (-424.86) is close to the value (-411.00) reported in Slobodyan and Wouters (2012a, Table 8, p. 93) for the 1984-2008 sample period.

A comparison of columns 2 and 3 shows that including the 1-year Treasury bill as an observable in the SlW model decreases the importance of most sources of endogenous rigidity; the only exception is the wage indexation parameter. The rationale for this generalized decrease is that by considering multi-period expectations as in (1), current consumption is linked to longer expectation paths of both inflation and consumption, which induces greater endogenous persistence and hence takes over other sources of persistence. Moreover, there is a large increase in persistence driven by the overall increase in ARMA coefficients featuring price and wage markup shocks.

A comparison of the marginal likelihood values in columns 3 and 4 shows that the switch from the SlW learning scheme to our real-time AL (RT-AL) model results in a large improvement in model fit [-474.92-(-614.55)=139.63]. Regarding the posterior estimates of parameters, the RT-AL model results in (i) a reduction in the parameter estimates featuring real rigidities even further; and (ii) a minor increase in the importance of the exogenous persistence driven by price and wage markup shocks.

When expectations are strongly restricted by requiring that the discrepancies between model consumption growth and inflation expectations and the corresponding SPF forecasts be i.i.d errors (column 5), most parameter estimates become closer to those estimated for the SlW model. This finding suggests that model expectations in the SlW model are fairly close to those in the SPF as shown in Slobodyan and Wouters (2012a). By imposing—the less restrictive assumption—that the deviations between model's consumption growth and inflation expectations and the corresponding SPF forecasts follow AR(1) processes (column 6), our AL model allows the term structure information to discipline model expectations more freely, which results in sizeable changes in the relative importance of alternative sources of persistence, as discussed in detail below.

Next, we discuss the main differences between our estimated baseline AL (column 6) and the estimated RE version of the model (column 7). The posterior log data densities of the AL and RE models are 216.70 and 186.20, respectively. The difference between their marginal likelihood values is 30.50 points, which results in a very high posterior odd of 1.76E+13.[21]

Regarding the posterior estimates of parameters, most sources of endogenous persistence can be seen in general to lose a great deal of importance under AL based on term structure information. Thus, the estimates of the habit formation parameter, h, and the elasticity of the cost of adjusting capital, φ, are much smaller under AL (0.31 and 1.02, respectively) than under RE (0.92 and 8.88, respectively). Similarly, the price and wage probabilities and the elasticity of capital utilization adjusting cost, ψ, are much smaller under AL than under RE, whereas the opposite occurs for the price and wage indexation parameters (ι_p and ι_w, respectively). Regarding exogenous sources of price and wage markup persistence, we find that price markup shock persistence (both autoregressive and moving average coefficients) is lower under AL than under RE, whereas for the wage markup shock the autoregressive coefficient is higher under AL than under RE, but the opposite is true for the moving average coefficient.

[21]Is this difference large compared to other estimated AL DSGE models in the related literature? Our estimated difference in favor of the AL learning specification is roughly half the difference found in Ormeño and Molnár (2015), but is higher than the one obtained in Slobodyan and Wouters (2012a) for their sample periods. Several reasons may explain these differences. First, our AL model is much more restrictive: learning is based only on the information content of the *lagged* (1-year) term spread which is observed in *real time*, whereas Slobodyan and Wouters (2012a) and Ormeño and Molnár (2015) base the PLM in their models on *revised* and *current* aggregate data, which results in a much richer but also rather unrealistic information set available to agents when forming their expectations. Second, we introduce the multi-period forecasting hypothesis, which is not taken into account in the two previous papers. Third, as mentioned above, our learning process is less restrictive in one dimension than the one assumed in Ormeño and Molnár (2015) because we require that the deviations of model expectations from their SPF counterparts be stationary instead of a (more restrictive) white noise process as they assume. However, our model is more restricted in another dimension since we use term structure information to discipline model expectations, which are not considered in their paper. Fourth, we also consider the consumption growth expectations from the SPF as observables to discipline model expectations of consumption. However, Ormeño and Molnár (2015) only use inflation expectations from the SPF. Slobodyan and Wouters (2012a) do not use data from the SPF as observables in their estimation procedure. Finally, we consider the great moderation period (1984-2007) whereas the other two papers use samples periods starting in the second half of the 1960's.

Table 1. Selected parameter estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	SlW	SlW	SlW-TS	RT-AL	RT-AL	RT-AL	RE
					i.i.d. error	AR(1) error	AR(1) error
Number of observables	7	7	8	8	16	16	16
Sample period	1966-2007			1984-2007			
log data density	-960.22	-424.86	-614.55	-474.92	-403.31	216.70	186.20
Parameters associated with real rigidities							
habit formation	0.69	0.83	0.44	0.35	0.50	0.31	0.92
(h)	(0.63,0.75)	(0.78,0.87)	(0.41,0.47)	(0.30,0.40)	(0.46,0.55)	(0.21,0.44)	(0.91,0.93)
cost of adjusting capital	3.35	6.53	3.63	2.34	3.26	1.02	8.88
(φ)	(1.88,3.87)	(4.81,8.23)	(3.58,3.69)	(2.19,2.49)	(3.20,3.30)	(0.69,1.37)	(8.46,9.50)
capital utilization adjusting cost	0.51	0.53	0.29	0.21	0.28	0.22	0.37
(ψ)	(0.31,0.71)	(0.30,0.76)	(0.25,0.33)	(0.14,0.28)	(0.19,0.37)	(0.14,0.29)	(0.31,0.43)
Parameters associated with nominal rigidities							
price Calvo probability	0.65	0.78	0.57	0.62	0.69	0.58	0.94
(ξ_p)	(0.59,0.69)	(0.74,0.82)	(0.54,0.61)	(0.54,0.69)	(0.61,0.81)	(0.51,0.66)	(0.93,0.95)
wage Calvo probability	0.82	0.73	0.35	0.60	0.58	0.60	0.75
(ξ_w)	(0.77,0.86)	(0.64,0.80)	(0.32,0.37)	(0.52,0.69)	(0.50,0.65)	(0.53,0.67)	(0.70,0.81)
price indexation	0.27	0.40	0.35	0.46	0.35	0.85	0.11
(ι_p)	(0.12,0.39)	(0.21,0.60)	(0.32,0.38)	(0.28,0.62)	(0.23,0.46)	(0.73,0.95)	(0.09,0.13)
wage indexation	0.18	0.26	0.77	0.19	0.33	0.56	0.21
(ι_w)	(0.07,0.26)	(0.11,0.41)	(0.67,0.86)	(0.07,0.35)	(0.26,0.38)	(0.39,0.77)	(0.15,0.27)

Table 1. (*Continued*)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	SlW	SlW	SlW-TS	RT-AL	RT-AL	RT-AL	RE
					i.i.d. error	AR(1) error	AR(1) error
Number of observables	7	8	8	8	16	16	16
Sample period	1966-2007				1984-2007		
Parameters associated with price and wage markups							
markup price AR coef.	0.33	0.16	0.77	0.83	0.54	0.67	0.997
(ρ_p)	(0.07,0.58)	(0.03,0.28)	(0.71,0.82)	(0.73,0.93)	(0.49,0.60)	(0.41,0.91)	(0.994,0.999)
markup wage AR coef.	0.54	0.73	0.96	0.96	0.98	0.94	0.83
(ρ_w)	(0.32,0.81)	(0.55,0.91)	(0.95,0.97)	(0.93,0.99)	(0.96,0.998)	(0.91,0.97)	(0.79,0.89)
markup price MA coef.	0.47	0.58	0.69	0.64	0.52	0.56	0.996
(μ_p)	(0.29,0.68)	(0.45,0.73)	(0.65,0.73)	(0.53,0.75)	(0.47,0.57)	(0.36,0.79)	(0.993,0.998)
markup wage MA coef.	0.42	0.56	0.43	0.27	0.47	0.28	0.54
(μ_w)	(0.12,0.71)	(0.33,0.80)	(0.41,0.46)	(0.20,0.34)	(0.36,0.54)	(0.13,0.41)	(0.46,0.65)
Policy rule parameters							
inertia	0.89	0.88	0.94	0.87	0.85	0.90	0.81
(ρ_r)	(0.85,0.91)	(0.84,0.93)	(0.93,0.95)	(0.80,0.93)	(0.73,0.94)	(0.87,0.94)	(0.77,0.85)
inflation	1.74	1.48	1.83	1.91	1.58	1.35	2.00
(r_π)	(1.39,2.05)	(1.10,1.86)	(1.78,1.89)	(1.79,2.03)	(1.40,1.72)	(1.17,1.53)	(1.86,2.16)
output gap	0.14	0.11	0.10	0.14	-0.01	0.15	0.07
(r_y)	(0.07,0.19)	(0.04,0.19)	(0.05,0.15)	(0.06,0.20)	(0.30,0.40)	(0.09,0.23)	(0.05,0.08)
output gap growth	0.14	0.06	0.03	0.05	0.13	0.04	0.06
($r_{\Delta y}$)	(0.11,0.17)	(0.02,0.09)	(0.01,0.05)	(0.03,0.08)	(0.10,0.16)	(0.01,0.06)	(0.04,0.07)
term spread	-	-	0.10	0.11	0.19	0.06	0.16
(r_{sp})	-	-	(0.09,0.11)	(0.04,0.16)	(0.13,0.25)	(-0.01,0.14)	(0.12,0.21)

Notes: parameter notation and standard deviation in parentheses

The overall conclusion is that AL under the multi-period forecasting hypothesis takes over other sources, which have strong support under the RE hypothesis, in explaining aggregate persistence. These results assess the findings in Milani (2007) using a small-scale DSGE model and Slobodyan and Wouters (2012a) using a medium-scale DSGE model, since these two papers only consider one-period-ahead expectations and revised data in shaping current decisions of agents and neglect both the contribution of forecasts over longer horizons and the importance of relying only on imperfect information when agents form their expectations in real time, as emphasized in this paper. In spite of the important differences in modeling learning, our estimation results are also in line with those found in Eusepi and Preston (2011) in the sense that AL with multi-period forecasting induces a much stronger mechanism for explaining aggregate persistence.

3.3 Model fit

Along with the overall model fit based on the posterior log data density, we also analyze the performance of our estimated baseline AL model in reproducing selected second-moment statistics obtained from actual data as shown in Table 2. We focus on three types of moment: standard deviations, contemporaneous correlations with inflation, and first-order autocorrelations. Regarding the actual size of fluctuations, we observe that the AL model is able to match reasonably well the standard deviation of inflation and the growth rates of real variables: output, consumption, investment, and wages. A similar conclusion can be drawn by looking at the correlation of inflation with these real variables. Thus, the AL model is able to reproduce negative and low correlations of inflation with real variables very well. However, the AL model has more trouble in quantitatively replicating actual persistence. Our baseline AL model generates too much persistence. This is particularly true for inflation and to a lesser extent for the real macroeconomic variables studied.

Table 2. Actual and simulated second moments

Actual data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.51	1.68	0.62	0.54	0.24
Correlation with π	-0.30	-0.28	-0.29	-0.29	1
Autocorrelation	0.19	0.51	0.22	0.21	0.69

Simulated data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.53	1.61	0.63	0.70	0.26
Correlation with π	-0.29	-0.26	-0.10	-0.30	1.0
Autocorrelation	0.26	0.70	0.57	0.48	0.97

3.4 Impulse responses

Impulse responses to a term premium shock

Figure 1 shows the impulse responses to a term-spread innovation. The stability of learning coefficients associated with the PLM characterized only by the term spread shown below means that the impulse response functions barely change over time.[22] A positive term-spread shock increases the future interest rate relative to the contemporaneous interest rate, which brings forward consumption and investment decisions and results in greater economic activity (output, consumption, and investment), higher inflation, and a higher (short-term) nominal interest rate. The impulse responses of all variables are hump-shaped, capturing the gradual process of learning. This hump-shaped feature is more pronounced in the nominal variables (inflation and nominal interest rate) than in the real variables (output and consumption). As emphasized in Aguilar and Vázquez (2017), the introduction of AL extended with term structure information allows for a feedback from the term structure to the macroeconomy

[22]This explains why we only report the average impulse responses instead of showing the time-varying impulse response functions. The stability of learning coefficients may also be due to the great moderation period studied in this paper.

through the learning dynamics that is missing under RE.

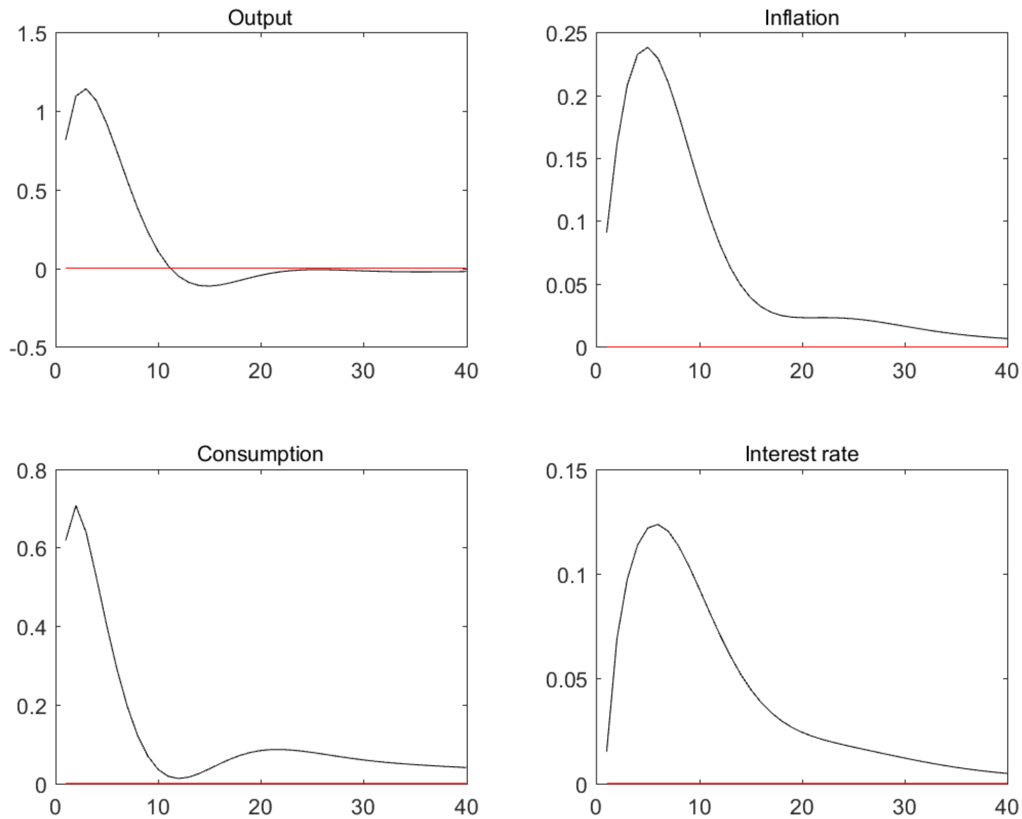


Figure 1. Impulse responses to a term-spread innovation

Impulse responses of output and inflation to alternative shocks

Figure 2 shows the responses of output and inflation to a technology shock, a risk premium shock, a wage markup shock, and a monetary policy shock. The gradual learning of AL dynamics results in hump-shaped (or alternatively U-shaped) impulse responses, as occurred for the responses to a term-spread innovation studied above. A positive technology impulse decreases inflation and increases potential output more than output initially, which results in a lower output measured in deviations from the balanced growth path, as shown in the top-left graph. A positive risk premium innovation decreases aggregate demand, which results in falls in output and inflation. A positive wage markup shock initially stimulates aggregate demand by raising wages, which results in increases in both output and inflation. However, the increase in output does not last long as higher labor costs lead to lower production. Finally, a positive interest rate shock results in both lower output and inflation as expected.

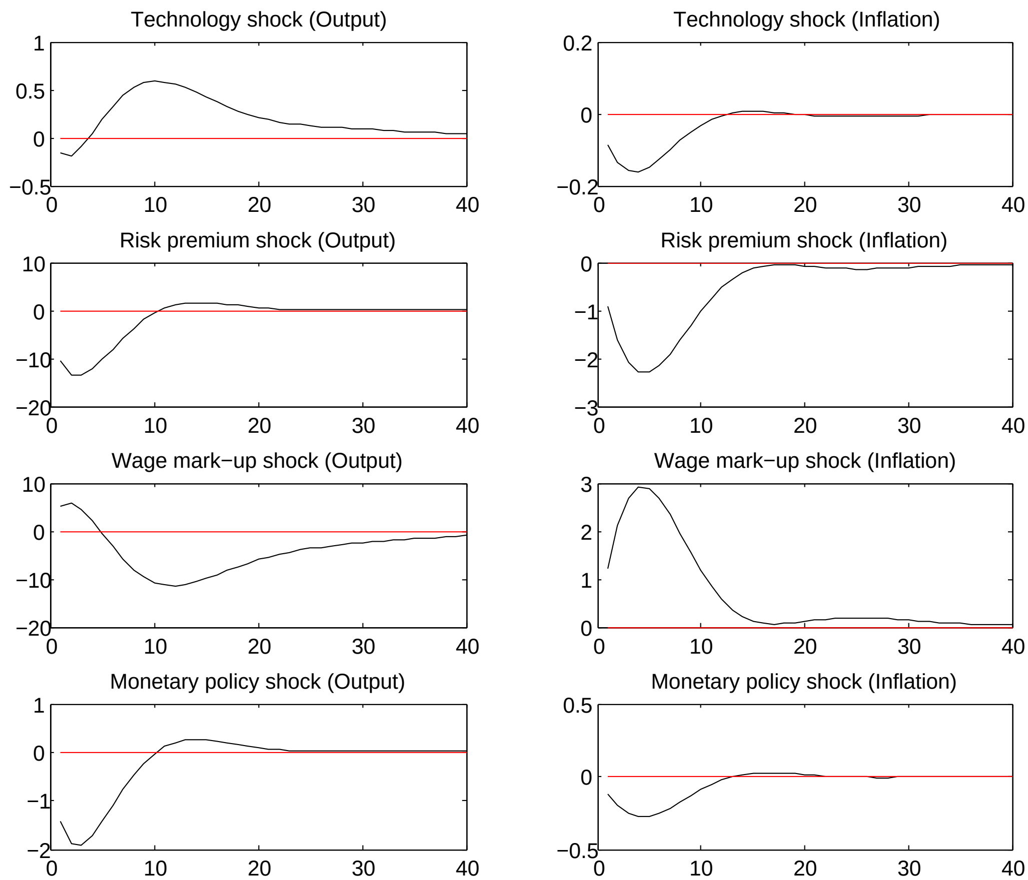


Figure 2. Impulse responses of output and inflation

3.5 Variance decomposition

Table 3 shows the (long-run) decomposition for inflation and the growth rates of output and consumption. The estimated AL model shows that risk premium and wage markup shocks between them explain more than 97% of the total variability of inflation and the growth rates of output and consumption. The risk premium shock explains roughly two thirds of the total variability of the real variables and one third of that of inflation, while the relative importance of wage markup shocks is the opposite. These results are in sharp contrast to the findings of Slobodyan and Wouters (2012a) that the wage markup shock becomes significantly less important under AL. Moreover, although term spread shocks affect short-run dynamics of both real and nominal variables, as shown in the impulse-response functions displayed in Figure 1, their contribution to their long-run variability is negligible (less than 0.5%). Furthermore, the measurement errors associated with inflation and consumption growth SPF forecasts (i.e. $\eta_{\pi,t}^{\{j\}}$ and $\eta_{\Delta c,t}^{\{j\}}$, respectively) play no role in explaining the unconditional variance of any aggregate variable (so their associated values are not reported in Table 3).

Table 3. Variance decomposition (long-run)

	dy	dc	π
Productivity	0.06	0.10	0.16
Risk premium	65.92	75.59	38.22
Exogenous spending	0.02	0.05	0.01
Investment specific technology	0.19	0.29	0.01
Monetary policy	1.30	1.44	0.52
Price markup	0.09	0.09	0.34
Wage markup	31.99	21.99	60.29
Term spread	0.42	0.45	0.46

3.6 Analysis of the PLM

Figure 3 shows the trend over time of the PLM coefficients for four-quarter-ahead expectations of inflation and consumption. We focus on the four-quarter expectations horizon because the 1-year term spread is the only observable term spread considered in the PLM. The time-varying intercept of the PLM of inflation (consumption) shows how agents' perception about steady-state inflation (balanced-growth consumption) changes over time. Thus, the intercept of inflation expectations captures the fall of inflation expectations over the sample period, whereas the (roughly constant) intercept of consumption captures the fairly constant balanced-growth consumption expectations. The term-spread coefficients associated with these two PLM are negative, indicating that a higher 4-quarter bond yield today (relative to the 1-quarter interest rate) anticipates tighter financial conditions in the future, resulting in lower 4-quarter-ahead expectations of both inflation and consumption. Notice that the term-spread coefficient in the PLM of consumption is roughly eight times larger than the one associated with the PLM of inflation, which results in much larger swings in consumption growth expectations than in inflation expectations due to term spread changes.

This last feature is consistent with the corresponding four-quarter-ahead forecasts reported by the SPF, as shown in Figure 4. Moreover, the graph at the top of this figure shows that the AL expectations generated by the model (red line) reproduce the SPF (blue line) inflation downtrend. Nevertheless, there is a positive gap between the inflation expectations from the SPF and the inflation expectations from the AL model. The mean of this gap is partially explained by the difference between the average of the 4-quarter-ahead inflation forecasts in the SPF (0.69) and the average of actual inflation (0.63) over the sample period, indicating that the AL inflation expectations generated by the model are closer to matching actual inflation than SPF inflation (see the first panel of Table 3 below). As emphasized above, another important reason explaining the deviation of the inflation expectations in the model from SPF inflation forecasts is that the model's inflation expectations are driven by term structure information whereas SPF panelists do not seem to use this market information at all (Rudebusch and Williams, 2009).

In contrast to inflation expectations, the graph at the bottom of Figure 4 shows that the AL model (red line) does a great job in reproducing both the timing and the large swings of consumption growth expectations reported in the SPF (blue line). This indicates that term structure information is closely in line with that used by SPF panelists when forecasting consumption growth.

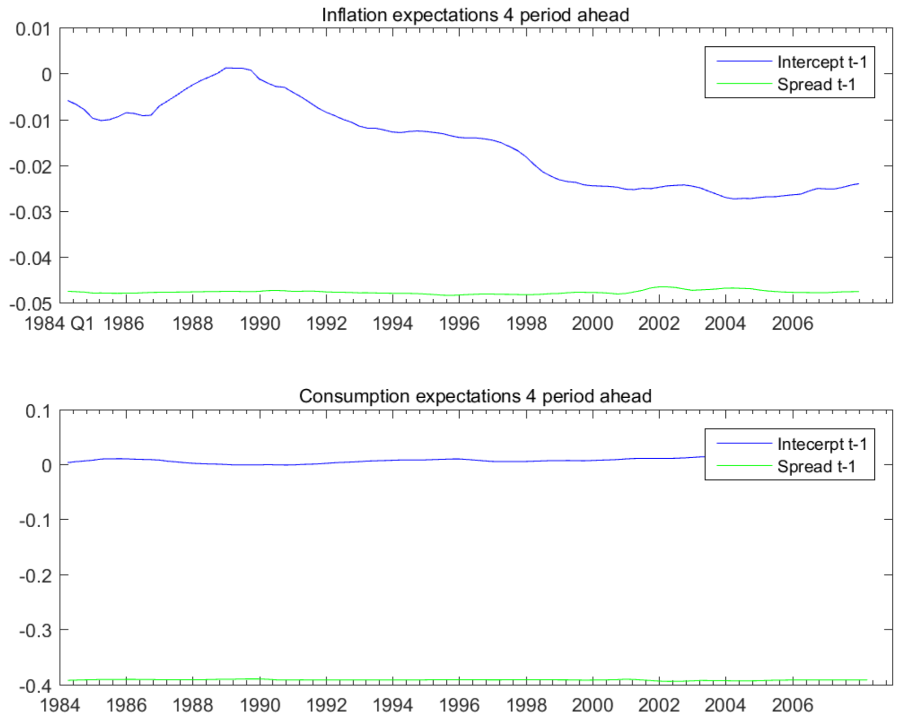


Figure 3. PLM of inflation and consumption expectations

The estimated learning coefficients of the PLM suggest that only the inflation intercept changes somewhat over time while the coefficients on the term spreads stay constant. This finding is consistent with the estimated posterior confidence interval of the learning coefficient, $\rho \in [0.69, 0.77]$. As shown in Slobodyan and Wouters (2012a), in their framework a low ρ is typically associated with no time variation of beliefs. This situation typically shows up when the model is initially close to the stability boundary and the estimation algorithm forces ρ to a number significantly less than one in order to prevent changing beliefs from moving the system dynamics into the unstable region.[23] We investigated these issues in our framework by requiring ρ to be lower than 0.5. In this case, the estimated posterior confidence interval of the learning coefficient is $[0.00, 0.35]$. As expected, the estimated PLM with ρ close to zero shows no variation in the learning coefficients, somewhat similar to those found when

[23]We thank Raf Wouters and Sergey Slobodyan for pointing out these insights.

ρ is a free parameter in the estimation procedure. However, the model fit, measured by the posterior log data density, deteriorates substantially (198.16 versus 216.70). These results indicate that even a small variation in the learning coefficients implied by a ρ far lower than one but larger than 0.5 results in a large improvement in the model's fit.

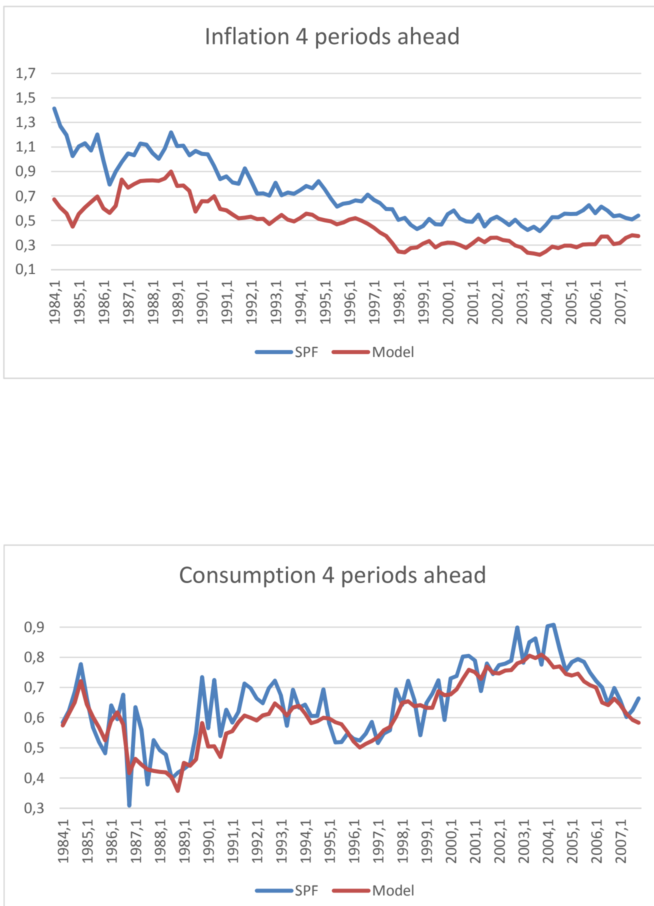


Figure 4. Model expectations versus SPF forecasts of inflation and consumption

Table 4 shows descriptive statistics (mean, standard deviation and first-order autocorrelation) of inflation and consumption growth four-quarter-ahead expectations from the SPF and the model together with inflation and consumption growth statistics from actual data and the model estimated. The first panel shows that the model's expectations of inflation and consumption growth are lower on average than those reported in the SPF and actual data, but are closer to the latter. Regarding standard deviations, it is interesting to observe that SPF forecasts and model expectations of consumption growth show a similar volatility, but a much smoother behavior than that associated with both actual and simulated data. This last feature is not shared by inflation expectations. In this case, inflation expectations from the model are slightly smoother than both actual data and the SPF inflation forecasts. Regarding persistence, both inflation and consumption expectations from the SPF and the model are much more persistent than for actual data. Moreover, SPF forecasts and model expectations share a similar degree of persistence. These last two features might be seen as explaining—by forcing it to reproduce the persistence of the forecasts from the SPF—why the model estimated is generating higher persistence than the one associated with actual data, as shown above in Table 2. We show that this is not the case by re-estimating the model without considering the SPF forecasts as observables in the estimation procedure.

Table 4. Descriptive statistics of inflation and consumption growth

Mean	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.63	0.73	0.57	0.65
Model	0.70	0.53	0.53	0.52
Standard deviation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.24	0.24	0.51	0.12
Model	0.26	0.16	0.53	0.10
Autocorrelation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecast	0.69	0.96	0.19	0.70
Model	0.96	0.96	0.26	0.78

Table 5. Selected parameter estimates without SPF forecasts as observables

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.35	(0.30,0.40)	0.70	(0.63,0.77)
φ: cost of adjusting capital	2.34	(2.19,2.49)	6.13	(4.38,7.85)
ψ: capital utilization adjusting cost	0.21	(0.14,0.28)	0.78	(0.65,0.91)
ξ_p: price Calvo probability	0.62	(0.54,0.69)	0.69	(0.63,0.76)
ξ_w: wage Calvo probability	0.60	(0.52,0.69)	0.52	(0.43,0.62)
ι_p: price indexation	0.46	(0.28,0.62)	0.13	(0.05,0.22)
ι_w: wage indexation	0.19	(0.07,0.35)	0.45	(0.21,0.69)
ρ_p: persistence of price markup shock	0.83	(0.73,0.93)	0.94	(0.89,0.99)
ρ_w: persistence of wage markup shock	0.96	(0.93,0.99)	0.97	(0.96,0.99)
μ_p: MA coef. price markup shock	0.64	(0.53,0.75)	0.79	(0.72,0.86)
μ_w: MA coef. wage markup shock	0.27	(0.20,0.34)	0.65	(0.51,0.82)
log data density	-474.92		-375.77	

Table 5 shows the estimation results *without* considering the forecasts from the SPF as observables for both the AL (already reported in Column 5 of Table 1) and the RE versions of the model.[24] Two main conclusions emerge from this analysis. First, as in the empirical results obtained in the baseline AL and RE cases (Columns 6 and 7 of Table 1), ignoring the SPF forecasts also results in most sources of endogenous persistence losing a great deal of importance under AL with multi-period forecasting. Second, the model's fit under AL is much worse than under RE when SPF forecasts are ignored to discipline expectations.[25] This latter result, together with a much larger improvement in the AL model than in the RE model—the log data density increases when SPF forecasts are used as observables in the

[24] As shown in Table 1, policy parameter estimates are fairly robust across specifications. Hence, we do not report their estimates in Table 5 and the remaining tables of the paper.

[25] See Del Negro and Eusepi (2011) for a discussion of the econometric framework based on log marginal likelihood differences for assessing how a model estimated to fit a given set of time series performs on fitting additional time series.

AL model and the RE model are 691.62(=216.70-(-474.92)) and 561.97(=186.20-(-375.77)), respectively—suggests that AL using only term structure information observed in real time contributes substantially to characterizing SPF forecasts along with the persistence deviations of these forecasts from the market expectations involved in the term structure of interest rates.

Our empirical results also suggests that using SPF forecasts as observables do not induce too much persistence. Thus, inflation persistence drops slightly from 0.97 to 0.95 when SPF forecasts are removed from the set of observables whereas a slight increase in persistence is obtained for the real variables (e.g. the growth rates of output, consumption, investment and real wages).[26] Again, these findings provide additional support for the view that learning based on multi-period forecasting is a major source of aggregate persistence.

4 Two extensions

This section extends the model in two important directions. First, we consider a few more yields beyond the 1-year maturity yield studied so far. Second, we study a combination of alternative small forecasting models where not only term structure information but also real-time inflation data are taken into account.

4.1 Long maturity term structure

This subsection extends the AL and RE models to incorporate information from longer maturity term structure. More precisely, we consider 3-year, 5-year, 7-year, and 10-year constant maturity Treasury bond yields as additional observables in the measurement equation considered in the baseline model in equation (3). Moreover, four additional consumption-Euler

[26] The corresponding table with second-moment statistics when the SPF forecasts are not included in the set of observables can be obtained from the authors upon request.

equations are considered in the model estimated, each associated with one of these additional yields. For instance, as implied by equation (1), the equation for 10-year yield is given by

$$\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right) c_t - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right) c_{t-1} = E_t \left[\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right) c_{t+40} - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right) c_{t+39}\right] - \left[40 r_t^{\{40\}} - E_t \sum_{k=1}^{40} \pi_{t+k} + \xi_t^{\{40\}}\right].$$

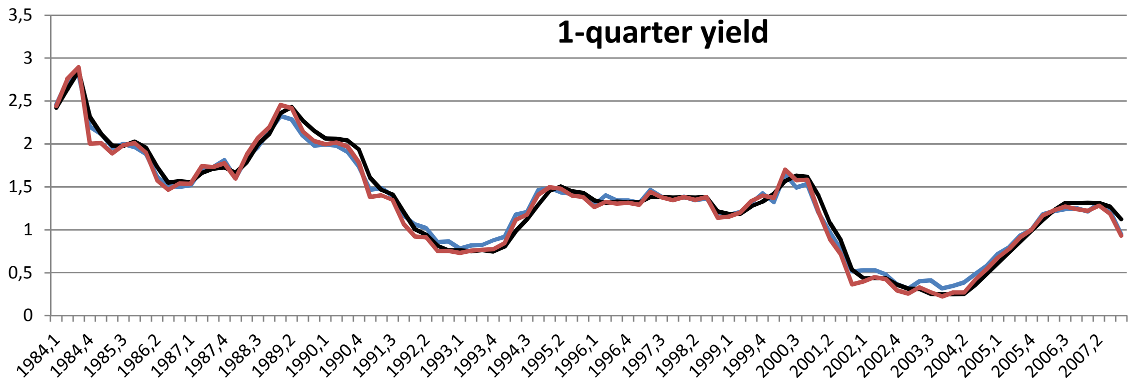
Considering a long term maturity yield such as the 10-year yield means characterizing expectations of consumption and inflation up to a 40-quarter horizon. As mentioned above, a curse of dimensionality problem will arise if attempts are made to estimate the whole sequence of expectations without imposing a structure linking short-horizon with long-horizon expectations. We assume the following simple recursive structure for consumption and inflation expectations on forecast horizons beyond the four-quarter horizon

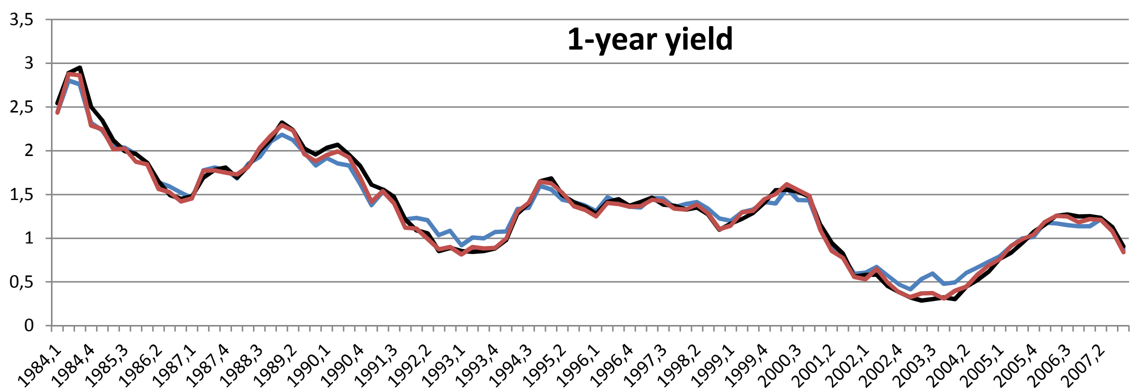
$$\begin{cases} E_t c_{t+j} = \mu_c E_t c_{t+j-1}, \\ E_t \pi_{t+j} = \mu_\pi E_t \pi_{t+j-1}, \end{cases} \tag{4}$$

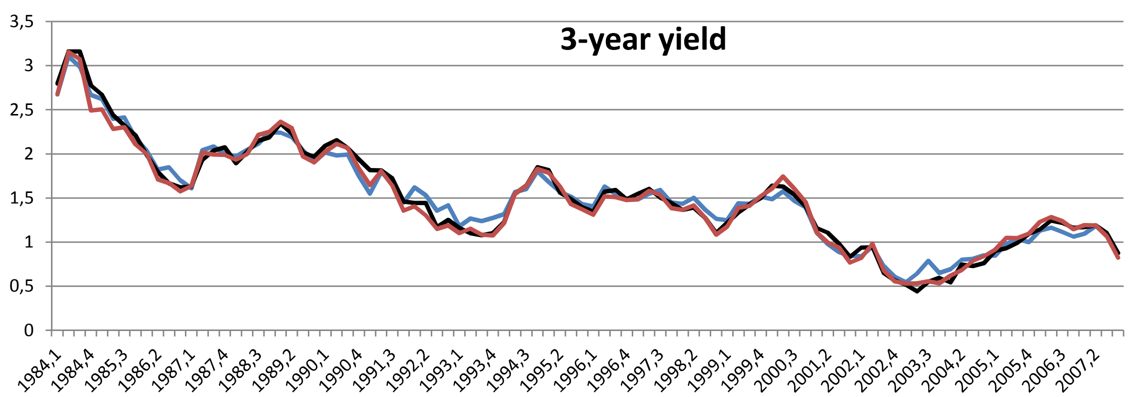
for $j > 4$. This structure builds on the forecasting rules described in equation (2) above which, among others, characterize $E_t c_{t+4}$ and $E_t \pi_{t+4}$, The parameters μ_c and μ_π are estimated jointly with the rest of model parameters. Since consumption is expected to be more persistent than inflation, the prior distribution assumed for these two parameters is a Beta-distribution with mean 0.9 and 0.8, respectively, and standard deviation 0.15.

Table 6. Selected parameter estimates with the 1-year (***up to the 10-year***) yield as observable(***s***)

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.31	(0.21,0.44)	0.92	(0.91,0.93)
	0.37	***(0.33,0.41)***	***0.85***	***(0.82,0.87)***
φ: cost of adjusting capital	1.02	(0.69,1.37)	8.88	(8.46,9.50)
	1.19	***(1.01,1.44)***	***7.60***	***(6.14,9.20)***
ψ: capital utilization adjusting cost	0.22	(0.14,0.29)	0.37	(0.31,0.43)
	0.01	***(0.00,0.01)***	***0.81***	***(0.68,0.91)***
ξ_p: price Calvo probability	0.58	(0.51,0.66)	0.94	(0.93,0.95)
	0.56	***(0.53,0.59)***	***0.92***	***(0.90,0.94)***
ξ_w: wage Calvo probability	0.60	(0.53,0.67)	0.75	(0.70,0.81)
	0.57	***(0.52,0.63)***	***0.88***	***(0.82,0.92)***
ι_p: price indexation	0.85	(0.73,0.95)	0.11	(0.09,0.13)
	0.26	***(0.17,0.31)***	***0.07***	***(0.03,0.12)***
ι_w: wage indexation	0.56	(0.39,0.77)	0.21	(0.15,0.27)
	0.43	***(0.34,0.52)***	***0.28***	***(0.09,0.48)***
ρ_p: persistence of price markup shock	0.67	(0.41,0.91)	0.997	(0.994,0.999)
	0.95	***(0.92,0.99)***	***0.03***	***(0.00,0.06)***
ρ_w: persistence of wage markup shock	0.94	(0.91,0.97)	0.83	(0.79,0.89)
	0.99	***(0.98,0.99)***	***0.63***	***(0.34,0.99)***
μ_p: MA coef. price markup shock	0.56	(0.36,0.79)	0.996	(0.993,0.998)
	0.73	***(0.64,0.80)***	***0.94***	***(0.89,0.99)***
μ_w: MA coef. wage markup shock	0.28	(0.13,0.41)	0.54	(0.46,0.65)
	0.22	***(0.18,0.26)***	***0.53***	***(0.18,0.99)***
μ_π: persistence of inflation rule expect.	***0.982***	***(0.980,0.985)***		
μ_c: persistence of consumption rule expect.	***0.978***	***(0.937,1.000)***		
log data density	216.70		186.20	
	727.80		***235.29***	
log data density difference	511.10		49.09	







Data: black line RE: blue line AL: red line

Figure 5. Term structure fitting

Figure 5. (*Continued*)

In order to facilitate comparison, Table 6 shows the estimation results for selected parameters obtained with the baseline model (already reported in Table 1) together with the estimation results (***with italic/bold font***) incorporating the information on the term structure up to the 10-year yield. The last three rows in Table 6 show that AL characterizes the term structure much better than the RE model. The log data data density improvement obtained by using term structure information up to the 10-year yield as an observable is more than ten times greater in the AL model (511.10) than in the RE model (49.09). This clearly shows the importance of using term structure information for modeling learning expectations in fitting the yield curve.

Figure 5 shows the term structure fitting by plotting actual yields (black lines) together with the estimated yields from the RE (blue lines) and the AL (red lines) models. In general, the AL model gives a closer fit to the actual yields than the RE model. This is clear in the proximity of local yield peaks and especially around troughs. The latter finding is more evident for the 1-year, 3-year, 5-year and 7-year yields.[27]

Table 6 also provides other interesting findings. First, as occurs with the baseline estimates, the parameter estimates characterizing the endogenous sources of persistence fall dramatically when AL and the information on longer maturity term structure are considered. This fall is particularly great for the estimate of the elasticity of capital utilization adjusting cost, ψ, implying a quasi-perfectly elastic capital utilization demand which in turn results in a roughly constant rental rate in equilibrium (see equation (11) in Appendix 1). Second, the information on longer maturity term structure also results in a fall of price and wage indexation parameters under AL. Third, the persistence of both price and wage markup shocks, ρ_p and ρ_w, increases when long-term yields are considered as observables under AL. However, the opposite occurs under RE. Finally, the posterior means of the two parameters measuring the persistence of the recursive structure of consumption and inflation expectations are similar and close to one, although the confidence interval of μ_c is larger than that

[27]A more in-depth analysis of how our AL model fits the yield curve is being carried out in an ongoing paper.

associated with μ_π.

4.2 Real-time inflation

Following Slobodyan and Wouters (2008, 2012a), we investigate robustness by allowing agents to combine alternative forecasting models at the same time, tracking their forecasting performance, and using a variant of the Bayesian model averaging method to generate an aggregate forecast from the alternative forecasting models that is used to characterize their decisions.[28] In this robustness exercise, we combine three forecasting models for each forward-looking variable of the model. In line with equation (2), the first two formulations rely on the *lagged* 2-quarter and 1-year term spreads, respectively, while the third formulation relies on *lagged* real-time inflation. More precisely, the three models are as follows:

$$\begin{aligned} m_1: \quad & E_t y_{t+j} = \theta^{\{j\}}_{1,y,t-1} + \beta^{\{j\}}_{1,y,t-1} sp^{\{2\}}_{t-1}, \\ m_2: \quad & E_t y_{t+j} = \theta^{\{j\}}_{2,y,t-1} + \beta^{\{j\}}_{2,y,t-1} sp^{\{4\}}_{t-1}, \\ m_3: \quad & E_t y_{t+j} = \theta^{\{j\}}_{3,y,t-1} + \beta^{\{j\}}_{3,y,t-1} \pi^r_{t-1,t}, \end{aligned} \tag{5}$$

where $\pi^r_{t-1,t}$ is the first announcement of inflation corresponding to time $t-1$, which is released at time t. The combination of different small forecasting models helps to circumvent the multicollinearity problem that arises when a single forecasting model which includes highly correlated regressors is used on the one hand. On the other hand it adds flexibility, which is in line with how SPF panelists forecast (Stark, 2013), as discussed above.

[28]More precisely, for each forecasting model m_i, the agents track the value of

$$B_{i,t} = t \cdot log\left(det\left(\frac{1}{t}\sum_{i=1}^{t} u_i u_i^T\right)\right) + \kappa_i \cdot log(t),$$

where κ_i is the number of degrees of freedom in the forecasting model m_i, and u_i is the i-th model forecasting error. As pointed out in Slobodyan and Wouters (2008), this expression is a generalization of the sum of squared errors adjusted for degrees of freedom using the Bayesian information criterion penalty. Thus, given values of $B_{i,t}$, the weight of a model i at time t is proportional to $exp\left(-\frac{1}{2}B_{i,t}\right)$.

Table 7. Estimates using yields up to the 10-year yield and real-time inflation as observables

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.37	(0.33,0.41)	0.85	(0.82,0.87)
	0.31	***(0.26,0.36)***	***0.84***	***(0.81,0.86)***
φ: cost of adjusting capital	1.19	(1.01,1.44)	7.60	(6.14,9.20)
	1.28	***(1.13,1.46)***	***8.19***	***(6.48,9.92)***
ψ: capital utilization adjusting cost	0.01	(0.00,0.01)	0.81	(0.68,0.91)
	0.22	***(0.15,0.28)***	***0.84***	***(0.74,0.93)***
ξ_p: price Calvo probability	0.56	(0.53,0.59)	0.92	(0.90,0.94)
	0.51	***(0.47,0.55)***	***0.90***	***(0.88,0.92)***
ξ_w: wage Calvo probability	0.57	(0.52,0.63)	0.88	(0.82,0.92)
	0.42	***(0.36,0.48)***	***0.82***	***(0.77,0.87)***
ι_p: price indexation	0.26	(0.17,0.31)	0.07	(0.03,0.12)
	0.22	***(0.14,0.29)***	***0.04***	***(0.01,0.07)***
ι_w: wage indexation	0.43	(0.34,0.52)	0.28	(0.09,0.48)
	0.39	***(0.32,0.47)***	***0.33***	***(0.13,0.51)***
ρ_p: persistence of price markup shock	0.95	(0.92,0.99)	0.03	(0.00,0.06)
	0.94	***(0.89,0.98)***	***0.89***	***(0.87,0.92)***
ρ_w: persistence of wage markup shock	0.99	(0.98,0.99)	0.63	(0.34,0.99)
	0.93	***(0.90,0.95)***	***0.59***	***(0.44,0.75)***
μ_p: MA coef. price markup shock	0.73	(0.64,0.80)	0.94	(0.89,0.99)
	0.65	***(0.57,0.73)***	***0.86***	***(0.83,0.90)***
μ_w: MA coef. wage markup shock	0.22	(0.18,0.26)	0.53	(0.18,0.99)
	0.24	***(0.18,0.30)***	***0.35***	***(0.13,0.57)***
log data density	727.80		235.29	
	698.01		***423.01***	
log data density difference	-29.79		187.72	

Following Casares and Vázquez (2016), we consider the following identity relating revised inflation, π_t, to both the initial announcement of inflation (i.e. real-time inflation), $\pi^r_{t,t+1}$, and the final revision of inflation, $rev^{\pi}_{t,t+S}$:

$$\pi_t = \pi^r_{t,t+1} + rev^{\pi}_{t,t+S}.$$

Here, S denotes the number of periods (quarters) of delay for the final release.[29] Many papers (e.g. Aruoba, 2008) have shown that US data revisions of many aggregate time series (e.g. inflation) are not rational forecast errors. More precisely, revisions are correlated to their initial (real-time) announcements and show persistence. Thus, we assume that[30]

$$rev^{\pi}_{t,t+S} = b^r_{\pi}\pi^r_{t,t+1} + \epsilon^{\pi}_{t,t+S},$$

$$\epsilon^{\pi}_{t,t+S} = \rho^r_{\pi}\epsilon^{\pi}_{t-1,t+S-1} + \eta^{\pi r}_{t,t+S}.$$

Table 7 shows the estimation results for a group of selected parameters for two specifications using the information for the term structure up to the 10-year yield: First, the baseline model (already reported in Table 6); and second, a model specification based on the PLM resulting from the mix of the three alternative forecasting models (6) and using real-time inflation data as observable (***written with italic/bold font***). The last three rows in Table 7 again show that the AL model fits the data much better than the RE model. However, the marginal log likelihood improvement obtained by using real-time inflation data is positive in the RE model (187.72) whereas in the AL model it is negative (-29.79), indicating that the AL model has more trouble fitting real-time inflation than the RE model. Moreover, Table 7 confirms one of the main findings of the paper, namely that parameter estimates featuring endogenous persistence are much lower when our real-time AL framework with term structure

[29] See, for instance, Croushore (2011) for details about the timing of macroeconomic data releases.

[30] As pointed out in Casares and Vázquez (2016), this revision process does not seek to provide a structural characterization of how actual statistical agencies really behave, but to provide a simple framework to assess whether departures from the hypothesis of a well-behaved revision process (i.e. a white noise process) might affect parameter estimates.

is considered instead of the RE formulation or other AL formulations that ignore real-time information issues.

5 Conclusions

This paper considers an estimated DSGE model with adaptive learning (AL) based on information actually available at the time when expectations are forming. We extend the AL model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates, which results in multi-period forecasting. Our extension retains the feature of AL based on small forecasting models, but enables the term spread of interest rates to fully characterize the expectations of all forward-looking variables of the model in real time. We view the use of real time information alone as a crucial step forward in the characterization of learning in estimated DSGE models. Moreover, the introduction of term structure information into small forecasting models results in a stable perceived law of motion for forward-looking variables. This finding is important because AL schemes are often criticized for being arbitrary (see, for instance, Adam and Marcet (2011) and references therein) and potentially amplifying the size of fluctuations in an ad hoc manner.

We use a rather comprehensive data set. The largest data set considered contains a much larger number of observable time series (twenty-one) than the seven observables used in the estimation of standard medium-scale DSGE models by including (i) yield curve time series; (ii) macroeconomic forecasts; and (iii) real-time data. Our estimation results show that the extended AL model with term structure provides a much better fit to the data than the rational expectations version. This finding is made very clear when fitting the whole yield curve. Moreover, the models estimated show that the importance of most endogenous sources of aggregate persistence (such as habit formation, Calvo probabilities, the elasticity of the cost of adjusting capital, and the elasticity of capital utilization adjustment costs) is dramatically reduced when learning based on multi-period forecasting and real-time data is incorporated into optimal decision-making through the term structure of interest rates, whereas the estimated persistence of markup shocks remains high. This last feature is somewhat in contrast with the findings in Slobodyan and Wouters (2012a).

Our empirical results also show that even medium-sighted consumers (agents who base their decisions on 1-year-ahead forecast horizons) are able to induce a great deal of aggregate persistence through the learning mechanism similar to that generated by long-sighted agents (agents who base their decisions on 10-year-ahead forecast horizons), but greater than that of short-sighted consumers (agents who base their decisions on 1-quarter-ahead forecast horizons) as considered in Milani (2007) and Slobodyan and Wouters (2012a).

References

- Adam, Klaus. 2005. "Learning to forecast and cyclical behavior of output and inflation." *Macroeconomic Dynamics* 9, 1-27.

- Adam, Klaus, and Albert Marcet. 2011. "Internal rationality, imperfect market knowledge and asset prices." *Journal of Economic Theory* 146, 1224-1252.

- Aguilar, Pablo A., and Jesús Vázquez. 2017. "An estimated DSGE model with learning based on term structure information." Universidad del País Vasco (UPV/EHU) *mimeo.*

- Branch, William A., and George W. Evans. 2006. "A simple recursive forecasting model." *Economics Letters* 91, 158-66.

- Croushore, Dean. 2011. "Frontiers of real-time data analysis". *Journal of Economic Literature* 49, 72-100.

- De Graeve, Ferre, Marina Emiris and Rafael Wouters. 2009. "A structural decomposition of the US yield curve." *Journal of Monetary Economics* 56, 545-559.

- Del Negro, Marco, and Stefano Eusepi. 2011. "Fitting observed inflation expectations." *Journal of Economic Dynamics and Control* 35, 2105-2131.

- Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti. 2017. "Safety, liquidity, and the natural rate of interest." Federal Reserve Bank of New York, *mimeo.*
- Estrella, Arturo, and Gikas A. Hardouvelis. 1991. "The Term Structure as a Predictor of Real Economic Activity." *Journal of Finance* 46, 555-576.
- Estrella, Arturo, and Frederic S. Mishkin. 1997. "The predictive power of the term structure of interest rates in Europe and the United States: Implications for the European Central Bank." *European Economic Review* 41,1375–1401.
- Evans, George W., and Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.
- Eusepi, Stefano, and Bruce Preston. 2011. "Expectations, learning, and business cycle fluctuations". *American Economic Review* 101, 2844-2872.
- Greenwood, Robin, Samuel G. Hanson, and Jeremy C Stein. 2015. "A comparative-advantage approach to government debt maturity." *The Journal of Finance* 70, 1683-1722.
- Hommes, Cars, and Mei Zhu. 2014. "Behavioral learning equilibria". *Journal of Economic Theory* 150, 778-814.
- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The aggregate demand for Treasury debt." *Journal of Political Economy* 120, 233-267.
- Lahiri, Kajal, George Monkroussos, and Yongchen Zhao. 2013. "The yield spread puzzle and the information content of SPF forecasts." *Economics Letters* 118, 219-221.
- Marcet, Albert, and Thomas J. Sargent. 1989. "Convergence of least-squares learning in environments with hidden state variables and private information." *Journal of Political Economy* 97, 1306–1322.
- McCallum, Bennett T. 1994. "Monetary policy and the term structure of interest rates". NBER Working Paper 4938. Reprinted in the *Federal Reserve Bank of Richmond Economic Quarterly*, Volume 91/4 Fall 2005.

- Milani, Fabio. 2007. "Expectations, learning and macroeconomic persistence." *Journal of Monetary Economics* 54, 2065-2082.

- Milani, Fabio. 2008. "Learning, monetary policy rules, and macroeconomic stability." *Journal of Economic Dynamics and Control* 32, 3148-3165.

- Milani, Fabio. 2011. "Expectation shocks and learning as drivers of the business cycle." *Economic Journal* 121, 379-401.

- Mishkin, Frederic S. 1990. "The information in the longer maturity term structure about future inflation." *Quarterly Journal of Economics* 105, 815-828.

- Ormeño, Arturo, and Krisztina Molnár. 2015. "Using survey data of inflation expectations in the estimation of learning and rational expectations models." *Journal of Money, Credit, and Banking* 47, 673-699.

- Preston, Bruce. 2005. "Learning about monetary policy rules when long-horizon expectations matter." *International Journal of Central Banking* 1, 81-126.

- Orphanides, Athanasios, and John C. Williams. 2005a. "Inflation scares and forecast-based monetary policy." *Review of Economic Dynamics* 8, 498-527.

- Orphanides, Athanasios, and John C. Williams. 2005b. "The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations." *Journal of Economic Dynamics and Control* 29, 1927-1950.

- Rudebusch, Glenn D, and John C. Williams. 2009. "Forecasting recessions. The puzzle of the enduring power of the yield curve." *Journal of Business Economics and Statistics* 27, 492-503.

- Sinha, Arunima. 2015. "Government debt, learning and the term structure." *Journal of Economic Dynamics and Control* 53, 268-289.

- Sinha, Arunima. 2016. "Learning and the yield curve." *Journal of Money, Credit, and Banking* 48, 513-547.

- Smets, Frank R., and Rafael Wouters. 2007. “Shocks and frictions in US business cycles: A Bayesian DSGE approach.” *American Economic Review* 97, 586-606.
- Slobodyan, Sergey, and Rafael Wouters. 2008. “Estimating a medium-scale DSGE model with expectations based on small forecasting models.” *Mimeo.*
- Slobodyan, Sergey, and Rafael Wouters. 2012a. “Learning in a medium-scale DSGE model with expectations based on small forecasting models.” *American Economic Journal: Macroeconomics* 4, 65-101.
- Slobodyan, Sergey, and Rafael Wouters. 2012b. “Learning in an estimated medium-scale DSGE model.” *Journal of Economic Dynamics and Control* 36, 22-46.
- Slobodyan, Sergey, and Rafael Wouters. 2017. “Adaptive learning and survey expectations of inflation.” National Bank of Belgium *mimeo*.
- Stark, Tom. 2013. “SPF panelists’ forecasting methods: A note on the aggregate results of a November 2009 special survey.” Federal Reserve Bank of Philadelphia, Research Department Report.
- Stekler, Herman O., and Tianyu Ye. 2017. “Evaluating a leading indicator: an application—the term spread”. *Empirical Economics* 53, 183-194.
- Vázquez, Jesús, Ramón María-Dolores, Juan M. Londoño. 2013. “On the informational role of term structure in the US monetary policy rule.” *Journal of Economic Dynamics and Control* 37, 1852-1871.

Appendix 1

Set of the remaining log-linearized dynamic equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{6}$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{7}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \tag{8}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1-\delta) = \frac{(1-\delta)}{(r^k + 1 - \delta)}$.

- Log-linearized aggregate production function:

$$y_t = \Phi\left(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a\right), \tag{9}$$

where $\Phi = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[31]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{10}$$

[31] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

- Capital utilization:

$$z_t = z_1 r_t^k, \tag{11}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1) i_t + k_2 \varepsilon_t^i, \tag{12}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2 \varphi$.

- Marginal cost:

$$mc_t = (1-\alpha) w_t + \alpha r_t^k - \varepsilon_t^a. \tag{13}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 mc_t + \pi_4 \varepsilon_t^p, \tag{14}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{A}{1+\overline{\beta}\iota_p}\left[\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p}\right]$, and $\pi_4 = \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p}$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

- Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{15}$$

- Wage markup equation:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c_{t-1}\right)\right). \tag{16}$$

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w. \tag{17}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{(1-\overline{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007)

Appendix 2

This appendix provides a brief explanation of how AL expectation formation works.[32] A DSGE model can be represented in matrix form as follows:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+j} + B_0 \epsilon_t = 0,$$

where y_t is the vector of endogenous variables at time t, $E_t y_{t+j}$ contains multi-period-ahead expectations, and w_t is the exogenous driving force following a VAR(1):

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t,$$

where ϵ_t is the vector of innovations.

Agents are assumed to have a rather limited view of the economy under AL. More precisely, their PLM process is generally defined as follows:

$$y_{t+j} = X_{t-1}\beta_{t-1}^{\{j\}} + u_{t+j},\ for\ j =\ 1,\ 2,\ ...,\ n,$$

where y is the vector containing the forward-looking variables of the model, X is the matrix of regressors, $\beta^{\{j\}}$ is the vector of updating parameters, which includes an intercept, and u is a vector of errors. These errors are linear combinations of the true model innovations. So, the variance-covariance matrices, $\Sigma = E[u_{t+j}u_{t+j}^T]$, are non-diagonal.

Agents are further assumed to behave as econometricians under AL. In particular, it is assumed that they use a linear projection scheme in which the parameters are updated to form their expectations for each forward-looking variable:

[32] For a detailed explanation see Slobodyan and Wouters (2012a,b).

$$E_t y_{t+j} = X_{t-1} \beta^{\{j\}}_{t-1}.$$

In line with Jordà (2005), we assume that agents make multi-period-ahead forecasts using local projections conditional on the information set available at the end of period $t-1$. Among the numerous advantages of using local projections for characterizing multi-period-ahead forecasts pointed out by Jordà (2005), we highlight two of them. First, they are easy to implement, which is a sensible approach when deviating from the RE hypothesis. Second, local projections are robust to model misspecifications. As discussed above, this is also a sensible feature to characterize agents' forecasts in a context where they face uncertainty about the true (highly non-linear) model economy.

The updating parameter vector, β, which results from stacking all the vectors $\beta^{\{j\}}$, is further assumed to follow an autoregressive process where agents' beliefs are updated through a Kalman filter. This updating expectation process can be represented as in Slobodyan and Wouters (2012a) by the following equation:

$$\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t,$$

where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V.

Once the expectations of the forward-looking variables,$E_t y_{t+j}$, are computed they are plugged into the matrix representation of the DSGE model to obtain a backward-looking representation of the model as follows

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t,$$

where the time-varying matrices μ_t, T_t and R_t are nonlinear functions of structural parameters (entering in matrices A_0, A_1, A_2 and B_0) together with learning coefficients discussed below.

Updating expectations

The Kalman-filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by

$$\beta_{t|t} = \beta_{t|t-1} + R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} \Big(y_t - X_{t-1} \beta_{t|t-1} \Big),$$

where $(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta})$. $\beta_{t|t-1}$ is the estimate of β using the information up to time $t-1$ (but further considering the autoregressive process followed by β), $R_{t|t-1}$ is the mean squared error associated with $\beta_{t|t-1}$. Therefore, the updated learning vector $\beta_{t|t}$ is equal to the previous one, $\beta_{t|t-1}$, plus a correction term that depends on the forecast error, $\Big(y_t - X_{t-1} \beta_{t|t-1} \Big)$. Moreover, the mean squared error, $R_{t|t}$, associated with this updated estimate is given by

$$R_{t|t} = R_{t|t-1} - R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} X_{t-1}^T R_{t|t-1}^{-1},$$

with $R_{t+1|t} = F R_{t|t} F^T + V$.

Table A.1.A: Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ: cost of adjusting capital	Normal	4.00	1.50	1.02	0.69	1.37	8.88	8.46	9.50
h: habit formation	Beta	0.70	0.10	0.31	0.21	0.44	0.92	0.91	0.93
σ_l : Frisch elasticity	Normal	2.00	0.75	1.47	0.63	2.21	1.23	1.11	1.35
ξ_p: price Calvo probability	Beta	0.50	0.10	0.58	0.51	0.66	0.94	0.93	0.95
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.60	0.53	0.67	0.75	0.70	0.81
ι_w: wage indexation	Beta	0.50	0.15	0.56	0.39	0.77	0.21	0.15	0.27
ι_p: price indexation	Beta	0.50	0.15	0.85	0.73	0.95	0.11	0.09	0.13
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.22	0.14	0.29	0.37	0.31	0.43
Φ : steady state price mark-up	Normal	1.25	0.12	1.52	1.36	1.68	1.63	1.49	1.73
r_π: policy rule inflation	Normal	1.50	0.25	1.35	1.17	1.53	2.00	1.86	2.16
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.90	0.87	0.94	0.81	0.77	0.85
r_y: policy rule output gap	Normal	0.12	0.05	0.15	0.09	0.23	0.07	0.05	0.08
$r_{\Delta y}$: policy rule output gap growth	Normal	0.12	0.05	0.04	0.01	0.06	0.06	0.04	0.07
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.06	-0.01	0.14	0.16	0.12	0.21
π: steady-state inflation	Gamma	0.62	0.10	0.77	0.70	0.84	0.69	0.60	0.78
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	0.22	0.09	0.38	0.17	0.12	0.22
l: steady-state labor	Normal	0.00	2.00	1.01	0.07	1.78	0.17	-0.14	0.53
γ: one plus st-state rate of output growth	Normal	0.40	0.10	0.52	0.50	0.53	0.47	0.45	0.49
$\bar{r}^{\{4\}}$: steady-state 1-year yield	Normal	1.00	0.50	1.18	0.90	1.44	1.38	1.29	1.47
α: capital share	Normal	0.30	0.05	0.13	0.09	0.17	0.23	0.19	0.26
ρ: learning parameter	Beta	0.50	0.28	0.73	0.69	0.77	-	-	-

Table A.1.B: Priors and estimated posteriors of the structural shock process parameters

	Priors			Posterior					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innovation	Invgamma	0.10	2.00	0.41	0.36	0.47	0.39	0.34	0.42
σ_b: Std. dev. risk premium innovation	Invgamma	0.10	2.00	0.99	0.71	1.34	5.16	4.81	5.48
σ_g: Std. dev. exogenous spending innovation	Invgamma	0.10	2.00	0.40	0.35	0.43	0.42	0.37	0.47
σ_i: Std. dev. investment innovation	Invgamma	0.10	2.00	1.38	1.23	1.50	0.32	0.27	0.37
σ_R: Std. dev. monetary policy innovation	Invgamma	0.10	2.00	0.10	0.09	0.12	0.11	0.09	0.12
σ_p: Std. dev. price mark-up innovation	Invgamma	0.10	2.00	0.20	0.18	0.22	0.21	0.18	0.23
σ_w: Std. dev. wage mark-up innovation	Invgamma	0.10	2.00	0.65	0.56	0.73	0.20	0.16	0.24
$\sigma_{\eta^{\{2\}}}$: Std. dev. 2-quarter yield innovation	Invgamma	0.10	2.00	2.86	1.32	4.21	0.06	0.03	0.08
$\sigma_{\eta^{\{3\}}}$: Std. dev. 3-quarter yield innovation	Invgamma	0.10	2.00	0.10	0.03	0.17	0.14	0.04	0.22
$\sigma_{\eta^{\{4\}}}$: Std. dev. 1-year yield innovation	Invgamma	0.10	2.00	0.46	0.40	0.54	0.33	0.29	0.36
ρ_a: Autoregressive coef. productivity shock	Beta	0.50	0.20	0.93	0.89	0.96	0.92	0.88	0.96
ρ_b: Autoregressive coef. risk-premium shock	Beta	0.50	0.20	0.93	0.91	0.95	0.19	0.16	0.22
ρ_g: Autoregressive coef. exog. spending shock	Beta	0.50	0.20	0.97	0.95	0.99	0.96	0.94	0.98
ρ_i: Autoregressive coef. investment shock	Beta	0.50	0.20	0.90	0.84	0.95	0.73	0.67	0.81
ρ_R: Autoregressive coef. monetary policy shock	Beta	0.50	0.20	0.56	0.46	0.65	0.37	0.29	0.46
ρ_p: Autoregressive coef. price markup shock	Beta	0.50	0.20	0.67	0.41	0.91	0.997	0.994	0.999
ρ_w: Autoregressive coef. wage markup shock	Beta	0.50	0.20	0.94	0.91	0.97	0.83	0.79	0.89
$\rho^{\{2\}}$: Autoregressive coef. 2-quarter yield shock	Beta	0.50	0.20	0.992	0.990	0.995	0.44	0.32	0.55
$\rho^{\{3\}}$: Autoregressive coef. 3-quarter yield shock	Beta	0.50	0.20	0.57	0.29	0.84	0.33	0.23	0.43
$\rho^{\{4\}}$: Autoregressive coef. 1-year yield shock	Beta	0.50	0.20	0.92	0.89	0.96	0.20	0.18	0.23
μ_p: MA coef. price markup shock	Beta	0.50	0.20	0.56	0.36	0.79	0.996	0.993	0.998
μ_w: MA coef. wage markup shock	Beta	0.50	0.20	0.28	0.13	0.41	0.54	0.46	0.65
ρ_{ga}: Interact. betw. product. and spending shocks	Beta	0.50	0.25	0.48	0.36	0.59	0.50	0.42	0.58

Table A.1.B: (Continued)

	Priors			Posterior					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\rho^{\{2\}}{}_{\xi}$: Interact. betw. 1- and 2-quarter yield shocks	Beta	0.50	0.25	0.22	0.06	0.40	0.49	0.37	0.60
$\rho^{\{3\}}{}_{\xi}$: Interact. betw. 1- and 3-quarter yield shocks	Beta	0.50	0.25	0.49	0.18	0.85	0.52	0.39	0.62
$\rho^{\{4\}}{}_{\xi}$: Interact. betw. 1- and 4-quarter yield shocks	Beta	0.50	0.25	0.81	0.70	0.90	1.24	1.21	1.29

Table A.1.C: Estimated parameters describing the deviations of model and SPF expectations

	Priors			Posterior					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\sigma_{\pi}^{\{1\}}$: Std. dev. 1-q-a inflation expect. innov.	Invgamma	0.10	2.00	0.08	0.07	0.09	0.08	0.07	0.09
$\sigma_{\pi}^{\{2\}}$: Std. dev. 2-q-a inflation expect. innov.	Invgamma	0.10	2.00	0.06	0.06	0.07	0.07	0.06	0.08
$\sigma_{\pi}^{\{3\}}$: Std. dev. 3-q-a inflation expect. innov.	Invgamma	0.10	2.00	0.07	0.06	0.08	0.07	0.06	0.07
$\sigma_{\pi}^{\{4\}}$: Std. dev. 4-q-a inflation expect. innov.	Invgamma	0.10	2.00	0.06	0.05	0.07	0.07	0.06	0.07
$\sigma_{\Delta c}^{\{1\}}$: Std. dev. 1-q-a cons. growth expect. innov.	Invgamma	0.10	2.00	0.50	0.44	0.55	0.17	0.15	0.19
$\sigma_{\Delta c}^{\{2\}}$: Std. dev. 2-q-a cons. growth expect. innov.	Invgamma	0.10	2.00	0.10	0.09	0.11	0.12	0.11	0.13
$\sigma_{\Delta c}^{\{3\}}$: Std. dev. 3-q-a cons. growth expect. innov.	Invgamma	0.10	2.00	0.08	0.07	0.10	0.11	0.10	0.13
$\sigma_{\Delta c}^{\{4\}}$: Std. dev. 4-q-a cons. growth expect. innov.	Invgamma	0.10	2.00	0.06	0.06	0.07	0.09	0.08	0.10
$\rho_{\pi}^{\{1\}}$: persist. 1-q-a inflation expect. shock	Beta	0.50	0.20	0.90	0.85	0.94	0.63	0.54	0.73
$\rho_{\pi}^{\{2\}}$: persist. 2-q-a inflation expect. shock	Beta	0.50	0.20	0.89	0.85	0.93	0.74	0.69	0.80
$\rho_{\pi}^{\{3\}}$: persist. 3-q-a inflation expect. shock	Beta	0.50	0.20	0.92	0.88	0.96	0.76	0.70	0.82
$\rho_{\pi}^{\{4\}}$: : persist. 4-q-a inflation expect. shock	Beta	0.50	0.20	0.89	0.86	0.92	0.82	0.74	0.90
$\rho_{\Delta c}^{\{1\}}$: persist. 1-q-a cons. growth expect. shock	Beta	0.50	0.20	0.93	0.89	0.97	0.67	0.60	0.74
$\rho_{\Delta c}^{\{2\}}$: persist. 2-q-a cons. growth expect. shock	Beta	0.50	0.20	0.18	0.05	0.30	0.59	0.51	0.68
$\rho_{\Delta c}^{\{3\}}$: persist. 3-q-a cons. growth expect. shock	Beta	0.50	0.20	0.17	0.03	0.29	0.61	0.54	0.68
$\rho_{\Delta c}^{\{4\}}$: persist. 4-q-a cons. growth expect. shock	Beta	0.50	0.20	0.23	0.07	0.39	0.66	0.57	0.75