

International spillovers of quantitative easing*

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PRELIMINARY

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Abstract

This paper investigates the effects of quantitative easing implemented by the major central banks on a typical small open economy that follows independent monetary policy. To this end, we develop a two-country model with asset market segmentation, where agents can trade in long-term bonds issued by the two governments. We next use the calibrated version of the model to simulate the effects of quantitative easing in a (much) larger economy. We find that the model is able to replicate the key empirical facts on emerging markets' response to large scale asset purchases conducted by the major central banks. In particular, it generates an inflow of foreign capital to the small economy's sovereign bond market that matches the data very well. The model also successfully accounts for the observed very strong comovement of the term premia in the two regions during the period of quantitative easing, a much higher one than in the preceding years. According to our simulations, quantitative easing abroad boosts domestic demand in the small economy, but depresses its aggregate output, at least in the short run. This is in contrast to the effects of conventional monetary easing in the large economy, which has positive spillovers to output in other economies.

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1 Introduction

Following the financial turmoil in the second half of 2008 and after decreasing their short-term interest rates close to zero, the major central banks, notably the Federal Reserve, the European Central Bank and the Bank of England, implemented several rounds of non-standard policy measures. These measures included purchases of long-term assets, the scale of which was unprecedented in the modern economic history. Figure 1 offers one way of documenting this process by showing that asset purchases in the United States, the United Kingdom and the euro area substantially lowered the share of long-term government bonds held by private agents in these economies in total supply of public sector liabilities: by mid-2016 this share had decreased by nearly 10 percentage points as compared with 2009.

The primary goal of these operations, popularly referred to as quantitative easing, was to reduce long-term interest rates and overcome the ongoing slump in economic activity. A number of papers have discussed the effects of quantitative easing across these dimensions, especially in the context of the US economy, see e.g. Gagnon et al. (2011), Joyce et al. (2011), D’Amico et al. (2012), Baumeister and Benati (2013), Kiley (2014). While the scale of estimated effects differs among various studies, they are usually unanimous in concluding that the unconventional measures benefited those economies that implemented them. At the same time, it has been both acknowledged by the policymakers¹ and confirmed by empirical studies that their impact on other countries, including small open economies (SOEs), is more complicated. Neely (2015), Fratzscher et al. (2013), Ahmed and Zlate (2014), Lim and Mohapatra (2016) and Tillmann (2016) all stress that quantitative easing by the Federal Reserve resulted in procyclical capital inflows into emerging markets and appreciation of their exchange rates. Falagiarda et al. (2015) finds similar effects of the ECB non-standard monetary policy measures on Central and Eastern European countries that are tightly integrated with the euro area. Figure 2 illustrates the magnitude of capital inflows to sovereign bond markets in SOEs. It shows that the share of non-resident investors holdings in the outstanding bonds issued by emerging market governments (in their currencies) has increased by about 15 percentage points since 2009. Naturally, flows of this type and scale have affected the prices of long-term bonds issued by the recipient countries. As Figure 3 strikingly reveals, the comovement between the term premium on 10-year US treasuries and 10-year bonds issued by the governments of SOEs has significantly increased since 2009.

Against this backdrop, a natural question to ask is whether the quantitative easing programs pursued by the major central banks have been on net beneficial to other economies, and especially those following independent monetary policy. While it is clear that the induced compression of long-term yields has been stimulating for the economic activity in countries not themselves engaged in quantitative easing, this effect could have been well offset by the

¹See e.g. speeches by Ben Bernanke or Mario Draghi invoked in Bhattarai et al. (2015) and Falagiarda et al. (2015).

exchange rate appreciation resulting from increased capital inflows. Bhattarai et al. (2015) offer some support for this hypothesis. Using a VAR analysis, they fail to detect a significantly positive impact of the Fed non-standard measures on GDP in emerging markets, and some of their specifications actually suggest negative effects.

In this paper we contribute to this debate by proposing a model that helps us understand and quantitatively analyze international spillovers of long-term asset purchase programs pursued in the major economies. We build on the segmented asset markets framework considered by Andres et al. (2004), and more recently further developed by Chen et al. (2012). Our main modeling extension is to formulate this environment in an open economy, two-country setup. In the model we propose, agents can trade long-term bonds issued by the two governments so that changes in their supply trigger portfolio adjustments that have real effects on both economies. Importantly, in our model financial linkages are defined in terms of gross rather than net international positions in assets, as recently advocated by Passari and Rey (2015).

We calibrate the two-country model to Poland, a typical small open economy pursuing independent monetary policy and deeply integrated with the rest of the world, and a conglomerate of three big economies whose central banks engaged in long-term asset purchases during the last decade, namely the United States, the United Kingdom and the euro area. We next use this model to simulate the quantitative easing in the large economies. We find that the model is able to replicate the salient features of the data discussed above. In particular, it generates an inflow of foreign capital to the small economy's sovereign bond market that matches the data very well. In line with what we show in Figure 3, the model also implies very strong comovement of the term premia in the two regions during the period of quantitative easing, but not necessarily during normal times. According to the model-based simulations, quantitative easing abroad boosts domestic demand in the small economy, but usually depresses the level of economic activity as measured with GDP, at least in the short run. This is in contrast to the effects of conventional monetary easing abroad, which positively affects the small economy's output. Our model is hence consistent with the empirical findings on conventional and unconventional monetary policy spillovers reported by Mackowiak (2007) and Bhattarai et al. (2015).

Our paper is related to the recent work by Alpanda and Kabaca (2015), who also develop a two-country model with portfolio balance effects to investigate international spillovers of quantitative easing. However, they assume that only US bonds are traded internationally, and hence their model cannot account for the observed inflow of capital into the markets for bonds issued by the rest of the world. According to their results, output in the rest of the world reacts positively to quantitative easing in the US, even though its impact on the domestic term premium is relatively weak. They also find that the scale of domestic output response to foreign quantitative easing is actually stronger than under conventional monetary policy easing abroad.

The rest of this paper is structured as follows. Section two presents the model and section three discusses its calibration. The main results and robustness checks are covered in section four. Section five concludes.

2 Model

We develop a two-country DSGE model where agents can trade long- and short-term government bonds issued by the two governments. The world population is normalized to unity and the relative size of the domestic (small) economy is $\omega \in (0; 0.5)$. Each country is populated by two types of households, as well as final and intermediate goods producers that supply domestic and foreign markets. The government in each country controls the short-term interest rate, exogenous spending and the supply of long- and short-term bonds, both issued in local currency.

Similarly to Andres et al. (2004) and others who followed their modeling approach (e.g. Chen et al., 2012; Kiley, 2014; Alpanda and Kabaca, 2015), we introduce market segmentation between short- and long-term bonds in a parsimonious way. It allows us to analyze the impact of quantitative easing on macrovariables without modeling imperfect asset substitutability in detail.

As the model structure is largely symmetric, in what follows we focus on the problems faced by agents populating the domestic (small) economy, and discuss those related to foreign agents only when they are distinct. We also adopt a standard convention of indicating variables related to the foreign economy with an asterisk. A full list of equations defining the equilibrium in our model can be found in the Appendix.

2.1 Households

There are two types of households in our model, which we call restricted and unrestricted, and which we index with $j = \{r, u\}$. The mass of restricted households is $\omega_r \in (0; 1)$ and that of unrestricted agents is $1 - \omega_r$ so that the measure of households in the economy is normalized to unity.

Household types differ in two ways. First, they are distinguished by their access to bond markets. Restricted households trade only in long-term bonds, reflecting the observation that in the real world some agents hold mostly long-term assets (e.g. pension funds). Unrestricted households, in turn, conduct transactions in all types of bonds. Second, while trading in long-term bonds, unrestricted households have to pay transaction costs, whereas restricted households do not bear such expenses. As argued by Chen et al. (2012), since the latter agents specialize in trading only in long-term bonds and their investment horizon is likely to be longer, their transaction costs are believed to be minor.

The introduction of segmented markets and transaction costs limits the arbitrage between short and long-term bonds. As a result, fluctuations in transaction costs have effects on real activity since, by impacting yields on long-term bonds, they affect intertemporal consumption allocation of restricted households who trade only in these assets. However, they do not directly influence the stochastic discount factor of unrestricted households since it is determined by the short-term interest rates.

2.1.1 Households in the small economy

Restricted agents in the small economy have access only to domestic long-term bonds, while unrestricted households can also trade domestic short-term bonds and foreign long-term bonds. Both types of households rent their labor services to firms at the nominal wage rate W_t , receive dividends from monopolistically competitive firms D_t^j and pay lump sum taxes T_t^j .

A representative household of type j maximizes her lifetime utility that depends on consumption c_t^j and labor effort n_t^j

$$U_t^j = E_t \sum_{s=0}^{\infty} \beta_j^s \exp\{\varepsilon_{t+s}^d\} \left[\frac{(c_{t+s}^j)^{1-\sigma}}{1-\sigma} - \frac{(n_{t+s}^j)^{1+\varphi}}{1+\varphi} \right] \quad (1)$$

where ε_t^d is the preference shock, $\beta_j \in (0; 1)$ is the subjective discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and φ is the inverse of the Frisch elasticity of labor supply.

Following Woodford (2001), we model long-term bonds as perpetuities that pay an exponentially decaying coupon κ^s every period $s + 1$ ($s \geq 0$) after the issuance, where $\kappa \in (0; 1]$. Then the current price of a bond issued s periods ago is related to the price of currently issued bonds with $P_{L-s,t} = \kappa^s P_{L,t}$. This allows us to write the budget constraint of restricted households as (see Chen et al., 2012)

$$P_t c_t^r + P_{L,t} B_{H,L,t}^r + T_t^r = P_{L,t} R_{L,t} B_{H,L,t-1}^r + W_t n_t^r + D_t^r \quad (2)$$

where P_t is the aggregate price level, $B_{H,L,t}^r$ denotes bonds issued by the domestic government and held by domestic restricted households, and $R_{L,t} \equiv P_{L,t}^{-1} + \kappa$ is the gross yield to maturity on these bonds.

Unrestricted households additionally have access to domestic short-term bonds and long-term bonds issued by the foreign government. Whenever they trade in long-term bonds, unrestricted households are required to pay transaction costs. Their budget constraint can be written as

$$P_t c_t^u + B_{H,t}^u + (1 + \zeta_{H,t}) P_{L,t} B_{H,L,t}^u + (1 + \zeta_{F,t}) S_t P_{L,t}^* B_{F,L,t}^u + T_t^u = R_{t-1} B_{t-1}^u + P_{L,t} R_{L,t} B_{H,L,t-1}^u + S_t P_{L,t} (1 + \Gamma_{t-1}) R_{L,t}^* B_{F,L,t-1}^u + W_t n_t^u + D_t^u + \Xi_t^u \quad (3)$$

where R_t is the short-term interest rate controlled by the domestic monetary authority, S_t is the nominal exchange rate expressed as the home currency price of one unit of foreign currency, B_t^u and $B_{H,L,t}^u$ stand for short and long-term domestic bond holdings, while $B_{F,L,t}^u$ denotes holdings of bonds issued by the foreign government, the price and yield to maturity of which are $P_{L,t}^*$ and $R_{L,t}^*$, respectively.

Unrestricted households are subject to the following three types of transaction costs, all of which are external to an individual household (i.e. depend on aggregate positions) and rebated back in a lump sum fashion as Ξ_t^u . Two of these costs are related to transactions in long-term bonds and are given by

$$\frac{1 + \zeta_{h,t}}{1 + \zeta_h} = \left(\frac{P_{L,t} b_{h,L,t}^u}{P_L b_{h,L}^u} \right)^{\xi_h} \quad (4)$$

for $h = \{H, F\}$, where $\xi_h > 0$, $b_{H,L,t}^u \equiv B_{H,L,t}^u / P_t$, $b_{F,L,t}^u \equiv B_{F,L,t}^u / P_t^*$, and variables without time subscripts indicate their steady state values. The third transaction cost is a standard country premium as in Schmitt-Grohe and Uribe (2003), which we introduce only to make the model stationary

$$1 + \Gamma_t = \exp \{ -\xi (a_t - a) + \varepsilon_t^\Gamma \} \quad (5)$$

where ξ is a small positive number, ε_t^Γ is the country risk premium shock, and a_t is the ratio of domestic economy's net foreign assets to output defined as

$$a_t = \frac{(1 - \omega_r)(B_{H,t}^u + S_t B_{F,t}^u + P_{L,t} B_{H,L,t}^u + S_t P_{L,t}^* B_{F,L,t}^u) + \omega_r P_{L,t} B_{H,L,t}^r - B_{H,t}^g - P_{L,t} B_{H,L,t}^g}{P_t y_t} \quad (6)$$

where $B_{H,t}^g$ and $B_{H,L,t}^g$ stand for the supply of domestic short and long-term bonds controlled by the local government, while y_t is the economy-wide output that we define later.

2.1.2 Households in the large economy

Households in the large (foreign) economy are modeled analogously, except that restricted agents trade in both domestic and foreign long-term bonds. We allow them to hold small economy's assets relying on two pieces of evidence. First, OECD data on foreign investment by pension funds indicates that a substantial portion of their assets is allocated in foreign debt securities.² Secondly, we draw on anecdotal evidence suggesting that yield-seeking pension

²See Figure 9 in OECD (2015).

funds are increasingly interested in acquiring emerging market fixed-income securities (see e.g. Fixsen, 2016).³

The budget constraint of restricted households in the large economy can be written as

$$P_t^* c_t^{r*} + P_{L,t}^* B_{F,L,t}^{r*} + (1 + \Gamma_t^{r*}) \frac{P_{L,t}}{S_t} B_{H,L,t}^{r*} + T_t^{r*} = P_{L,t}^* R_{L,t}^* B_{F,L,t-1}^{r*} + \frac{P_{L,t}}{S_t} R_{L,t} B_{H,L,t-1}^{r*} + W_t^* n_t^{r*} + D_t^{r*} + \Xi_t^{r*} \quad (7)$$

where Γ_t^{r*} is an external adjustment cost given by

$$1 + \Gamma_t^{r*} = \exp \left\{ \xi_r^* \left(\frac{P_{L,t} B_{H,L,t}^{r*}}{S_t P_{L,t}^* B_{F,L,t}^{r*}} - \kappa^{r*} \right) \right\} \quad (8)$$

and rebated back as Ξ_t^{r*} , where $\kappa^{r*} > 0$ is the steady state proportion of restricted households' holdings of bonds issued by the small and large economies. This adjustment cost is introduced only to make the steady state portfolio problem of restricted households in the large economy determinate, and we parametrize it such that it does not affect the model dynamics, i.e. by setting ξ_r^* to a very low positive number.

2.2 Firms

To introduce price stickiness and imperfect substitution between domestic and imported goods, we consider three stages of production. At the final stage, perfectly competitive final goods producers combine homogeneous home-made goods $y_{H,t}$ and imported goods $y_{F,t}$ according to the following technology

$$\tilde{y}_t = \left(\eta^{\frac{1}{\nu}} y_{H,t}^{\frac{\nu-1}{\nu}} + (1 - \eta)^{\frac{1}{\nu}} y_{F,t}^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \quad (9)$$

where $\eta \in (0; 1)$ is the home-bias parameter and $\nu > 0$ is the elasticity of substitution between domestic and imported goods.

At the previous production stage homogeneous goods are produced by perfectly competitive aggregators according to

$$y_{h,t} = \left(\int_0^1 y_{h,t}(i)^{\frac{1}{\mu}} di \right)^{\mu} \quad (10)$$

for $h = \{H, F\}$, where $\mu > 1$ controls the degree of substitution between intermediate inputs $y_{h,t}(i)$.

³It is important to note that the assumed asymmetry in the model structure does not have any significant impact on our results, i.e. they are very similar if we allow also the home restricted agents to hold both domestic and foreign long-term bonds.

Intermediate inputs are produced by monopolistically competitive firms indexed by i that operate a linear production function in local labor

$$y_{H,t}(i) + y_{H,t}^*(i) = \exp\{\varepsilon_t^z\}n_t(i) - \phi \quad (11)$$

where ε_t^z is the productivity shock and ϕ is the fixed cost of production. These firms set their prices in the buyer's currency, separately for the domestic market and exports, in a staggered fashion that is similar to the Calvo scheme. More specifically, every period a firm operating in the domestic economy faces a fixed probability θ_H of price reoptimization for the domestic market and probability θ_H^* of price reset for exports. Firms that cannot reoptimize index their prices to steady state CPI inflation.

We assume that firms using local labor are owned by local restricted and unrestricted households in a proportion equal to their shares in population. The problem of reoptimizing firms is hence to maximize

$$E_t \sum_{s=0}^{\infty} (\theta_H)^s \Lambda_{t+s} \left(P_{H,t}(i) \pi^s - \frac{W_{t+s}}{\exp\{\varepsilon_{t+s}^z\}} \right) y_{H,t+s}(i) \quad (12)$$

$$E_t \sum_{s=0}^{\infty} (\theta_H^*)^s \Lambda_{t+s} \left(S_{t+s} P_{H,t}^*(i) (\pi^*)^s - \frac{W_{t+s}}{\exp\{\varepsilon_{t+s}^z\}} \right) y_{H,t+s}^*(i) \quad (13)$$

where $\Lambda_{t+s} \equiv P_{t+s}^{-1}[\omega_r \beta_r^s (c_t^r)^{-\sigma} + (1 - \omega_r) \beta_u^s (c_t^u)^{-\sigma}]$ is the stochastic discount factor for nominal payoffs that is consistent with the assumed firm ownership structure, $P_{H,t}(i)$ is the price set by intermediate producer i for the domestic market, $P_{H,t}^*(i)$ is the price set for the foreign market, while $\pi_t \equiv P_t/P_{t-1}$ and $\pi_t^* \equiv P_t^*/P_{t-1}^*$ are the domestic and foreign inflation rates for final goods. This maximization problem is subject to the demand schedules consistent with aggregators' optimization sketched above.

2.3 Government

The monetary authority follows a Taylor-like feedback rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_r} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{y_t}{y} \right)^{\gamma_y} \right]^{1-\gamma_r} \exp\{\varepsilon_t^r\} \quad (14)$$

where ε_t^r is the monetary policy shock, $\gamma_r \in (0, 1)$ controls the degree of interest rate smoothing, while γ_π and γ_y control, respectively, the degree of interest rate response to deviations of inflation from the target and to the output gap.

The fiscal authority sets exogenous spending on final goods $g_t \equiv g \exp\{\varepsilon_t^g\}$, where ε_t^g is the government spending shock, and finances it with lump sum taxes levied on domestic households $T_t \equiv \omega_r T_t^r + (1 - \omega_r) T_t^u$ and with net debt issuance. We assume that both types

of households pay the same amount of taxes in per capita terms so that $T_t^r = T_t^u = T_t$. The government budget constraint is

$$B_{H,t}^g + P_{L,t}B_{H,L,t}^g + T_t = R_{t-1}B_{H,t-1}^g + P_{L,t}R_{L,t}B_{H,L,t-1}^g + P_t g_t \quad (15)$$

and the market value of total (short and long-term) government debt is

$$B_t^g = B_{H,t}^g + P_{L,t}B_{H,L,t}^g \quad (16)$$

The government of the small country holds the real market value of debt $b_t^g \equiv B_t^g/P_t$ and its composition $\frac{B_{H,t}^g}{P_{L,t}B_{H,L,t}^g}$ constant. Total real debt of the large country b_t^{g*} is also fixed, but its composition may change according to the following rule

$$\frac{P_{L,t}^* b_{F,L,t}^{g*}}{P_L^* b_{F,L}^{g*}} = \left(\frac{P_{L,t-1}^* b_{F,L,t-1}^{g*}}{P_L^* b_{F,L}^{g*}} \right)^{\gamma_L^*} \exp\{\varepsilon_t^{L*}\} \quad (17)$$

where $\gamma_L^* > 0$ is a smoothing parameter and ε_t^{L*} is the quantitative easing shock.

2.4 Goods market clearing

We impose a set of market clearing conditions. Equilibrium on the goods market requires

$$\tilde{y}_t = \omega_r c_t^r + (1 - \omega_r) c_t^u + g_t \quad (18)$$

and

$$y_t \equiv y_{H,t} \Delta_{H,t} + \frac{1 - \omega}{\omega} y_{H,t}^* \Delta_{H,t}^* = \exp\{\varepsilon_t^z\} n_t - \phi \quad (19)$$

where $n_t \equiv \omega_r n_t^r + (1 - \omega_r) n_t^u$ is aggregate labor input, y_t defines aggregate output while

$$\Delta_{H,t} = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{\frac{\mu}{1-\mu}} di \quad (20)$$

$$\Delta_{H,t}^* = \int_0^1 \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{\frac{\mu^*}{1-\mu^*}} di \quad (21)$$

are the measures of price dispersion resulting from staggered pricing by intermediate goods producing firms.

Given our assumptions on market segmentation, the market clearing conditions for domestic bonds are

$$(1 - \omega_r) B_{H,t}^u = B_{H,t}^g \quad (22)$$

$$\omega_r B_{H,L,t}^r + (1 - \omega_r) B_{H,L,t}^u + \frac{1 - \omega}{\omega} \omega_r^* B_{H,L,t}^{r*} + \frac{1 - \omega}{\omega} (1 - \omega_r^*) B_{H,L,t}^{u*} = B_{H,L,t}^g \quad (23)$$

Using these market clearing conditions together with the budget constraints of the households and government, as well as the zero-profit condition of the final goods producers and aggregators, we obtain the following law of motion for the small economy's net foreign assets position $A_t \equiv a_t P_t y_t$

$$A_t = R_{L,t}^* \frac{S_t}{S_{t-1}} \frac{P_{L,t}^*}{P_{L,t-1}^*} A_{t-1}^+ - R_{L,t} \frac{P_{L,t}}{P_{L,t-1}} A_{t-1}^- + NX_t \quad (24)$$

where

$$A_t^+ = (1 - \omega_r) S_t P_{L,t}^* B_{F,L,t}^u \quad (25)$$

$$A_t^- = \frac{1 - \omega}{\omega} P_{L,t} \left((1 - \omega_r^*) B_{H,L,t}^{u*} + \omega_r^* B_{H,L,t}^{r*} \right) \quad (26)$$

$$NX_t = \frac{1 - \omega}{\omega} S_t P_{H,t}^* y_{H,t}^* - P_{F,t} y_{F,t} \quad (27)$$

are, respectively, small economy's gross foreign assets, gross foreign liabilities and net exports.

2.5 Term premium

As in Chen et al. (2012), we define the term premium on long-term bonds as

$$TP_t = R_{L,t} - R_{L,t}^{EH} \quad (28)$$

where $R_{L,t}^{EH}$ is the counterfactual yield to maturity on a long-term bond in the absence of transaction costs, which we price using unrestricted households' stochastic discount factor, and κ^{EH} is such that this counterfactual bond has the same steady state duration D_L as the actual bond, i.e. the following must hold

$$D_L = \frac{R_L}{R_L - \kappa} = \frac{R_L^{EH}}{R_L^{EH} - \kappa^{EH}} \quad (29)$$

As shown by Chen et al. (2012), the term premium can be approximated up to first order as the discounted sum of expected values of transaction costs $\xi_{H,t}$ associated with trade in domestic long-term bonds

$$TP_t \approx D_L^{-1} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s E_t \xi_{H,t+s} \quad (30)$$

Hence, fluctuations in the term premium essentially reflect the current and planned portfolio rebalancing decisions made by unrestricted agents.

2.6 Exogenous shocks

The key driving force in our model are exogenous shifts in the composition of public debt in the large economy ε_t^{L*} . The model also features a set of standard shocks used in open economy DSGE models. These are the country pairs of shocks to productivity (ε_t^z and ε_t^{z*}), time preferences (ε_t^d and ε_t^{d*}) and monetary policy (ε_t^r and ε_t^{r*}), as well as country risk premium (ε_t^Γ). All shocks are modeled as independent first-order autoregressions, except for monetary and quantitative easing shocks that we assume to be i.i.d.

3 Calibration

We calibrate our two-country model to Poland and a block of three big economies that engaged in quantitative easing during the last decade, namely the United States, the United Kingdom and the euro area (BIG3 henceforth). We set the parameters to match some key steady state proportions observed in the data or take them from the previous literature. Table 1 shows the calibrated parameter values while Table 2 presents the targeted steady state ratios. The time period is one quarter.

If we measure the country size with GDP in current US dollars, the relative size of the small economy ω is 0.014. The home bias parameter η is calibrated at 0.75 to capture the average share of imports in the Polish GDP, corrected for the import content of exports estimated by the OECD. The elasticity of substitution between domestically produced goods and imports is set to 3, which can be seen as a compromise between the micro and macro estimates found in the literature.

In our model the key transmission channel of international policy spillovers relies on gross bond holdings and their adjustment. Hence, the crucial part of our calibration concerns the steady state composition of the bond portfolios held by agents in the small and large economy. Our targets for these proportions are based on the averages observed over the period 2004-2015, which are calculated by combining several data sources. The shares of sovereign bonds in quarterly GDP in Poland and in BIG3 are calibrated to 1.25 and 2.65, respectively. These values are derived using the nominal value of government debt securities reported by the Eurostat (for Poland) and by the World Bank in its Quarterly Public Sector Debt Database (for BIG3).⁴ These databases also allow us to distinguish between long- and short-term bonds, where we follow Chen et al. (2012) and treat sovereign debt securities that mature in one year or less as short-term bonds. For BIG3, the latter category also includes money holdings since these are very close substitutes of short-term safe debt securities when short-term interest rates are close to zero. This gives the share of long-term bonds in total sovereign bonds of 0.71 in Poland and 0.63 for BIG3. The steady state share of resident

⁴In the World Bank database the time series on debt securities for Poland are only available from 2010, therefore we chose to complement this source with the Eurostat.

holdings in total long-term bonds issued by the small economy is set to 0.76, which is in line with the data published by the Polish Ministry of Finance. The ratio of foreign bonds to total bonds held by small economy's agents is calibrated at 0.05, reflecting average portfolio investment in foreign bonds by Polish residents according to the International Investment Position statistics published by the Polish central bank. Finally, we assume that the share of small economy's bonds in the long-term bonds portfolio held by foreign households is the same for their two types, which pins down the value of κ^{r*} at 0.0018.

Another important group of parameters determine the degree of market segmentation and sensitivity of transaction costs, and hence the term premia, to adjustments in agents' portfolios. While calibrating these parameters, we rely heavily on Chen et al. (2012), but later verify that our simulation outcomes are consistent with empirical evidence on the effects of quantitative easing on domestic asset prices. In particular, the shares of restricted households ω_r and ω_r^* , transaction costs on long-term bonds ξ_H and ξ_F , as well as the degree of smoothing of the portfolio composition γ_L and γ_L^* are all set in accordance with the econometric estimates presented in this source paper, assuming symmetric values for both countries.

The following parameters determine the steady state levels of the interest rates and bond prices, and hence the term premia and bond duration. We set the inflation targets π and π^* to 1.005 (2% annualized) so that they are consistent with those adopted by the three major central banks (the Fed, the Bank of England and the European Central Bank) and the average inflation rate in Poland since 2004. These, together with the discount factors for restricted and unrestricted households β_r and β_u , pin down the steady state long- and short-term interest rates. We target them to match the US averages of 5.2% and 4%, respectively, and symmetrically between the two regions. Since long-term bonds are modeled as perpetuities, we need to specify their coupons κ and κ^* . We do it to match the duration of long-term bonds, which is equal to 3 years in Poland according to MoF (2015), and 7.5 years in the US according to Chen et al. (2012).

The remaining parameters are either relatively well established in the literature or do not have important effects on our key results. The steady state government spending in both countries is set to 20% of GDP, roughly in line with the long-run averages observed in the data. The elasticity of intertemporal substitution σ , the Frisch elasticity of labor supply φ , the Calvo probabilities θ_H and θ_H^* , price markups μ , interest rate rule coefficients γ_r , γ_π and γ_y , as well as their large economy counterparts, are all set to standard values considered in the DSGE literature. As explained before, we set the portfolio adjustment cost ξ_r^* and the country risk premium parameter ξ to small positive numbers that ensure determinacy and stationarity, and at the same time do not affect significantly the model dynamics.

4 Results

We use the model calibrated as described above to run several simulations. We first generate a quantitative easing scenario in the large economy that resembles the large scale asset purchase programs implemented in the major central banks, and discuss its impact on the small economy. We next compare these international spillovers to those associated with conventional policy easing. We argue that our model-based outcomes are consistent with the estimates documented in the empirical literature. We also show that our model can account for the increased international synchronization of the term premia observed since 2009. Finally, we discuss the robustness of our results to alternative model parametrization and assumptions.

4.1 Unconventional monetary stimulus in the large economy

We design our quantitative easing scenario to mimic the evolution of the share of long-term bonds in the total supply of bonds issued by the BIG3 as in Figure 1 from 2009 through 2016, i.e. the central bank of the large economy buys domestic long-term sovereign bonds in exchange for short-term securities, reducing the share of the former in the private sector portfolio by nearly 10 percentage points. We assume that after 2016 this policy is withdrawn at the same pace as it was implemented so that by 2024 the composition of outstanding bonds is the same as before the crisis. The scenario is implemented by designing an appropriate sequence of (fully anticipated) shocks to ε_t^{L*} in equation (17). The outcomes for selected variables in both economies are presented in Figure 4.

Long- and short-term bonds are imperfect substitutes, thus quantitative easing is not neutral for other macrovariables. In particular, it drives down the term premium in the large economy by 60 bps on impact, stays around this level for about 6 years, and then gradually rises, coming back close to its steady state value several years before the asset purchase program is withdrawn. The magnitude of this response is consistent with that obtained by Chen et al. (2012) in a closed economy setup if one takes into account the differences in the size and length of the impulse, and well within the bounds implied by the empirical literature they summarize.⁵ Our model also predicts a similar response of inflation, but much weaker expansion in output, suggesting lower efficiency of quantitative easing in stimulating the real activity.⁶

⁵Chen et al. (2012) obtain a reduction of the term premium by 30 bps following the LSAP II in the United States. However, the ultimate scale of this program (600 bn USD, i.e. 4% of the US GDP) is about three times smaller than in our case (12% of the combined GDP of the US, the UK and the euro area) and its implementation is less spread over time.

⁶This result contrasts with most of the previous studies and, as we discuss later, is driven mainly by an endogenous increase in the short-term interest rates in the large economy. As we show in Section 4.4, once we assume that the central bank in the large economy keeps them fixed (as under the zero lower bound), the response of output in the large economy is much larger and in line with the existing estimates.

We now turn to the main focus of our paper, which is the response of the small economy. A lower term premium in the large economy induces its investors to search for yield abroad. As a consequence, the share of non-residents in the small economy’s long-term bond market increases by 20 pp after around 8 years. Both the scale and the timing of this process match the data very well, as can be seen by comparing our simulation outcomes to Figure 2. The inflow of foreign capital into local bond market is accompanied by a drop in the domestic term premium by 30 bps on impact and 60 bps at its trough around 5 years after the program started. In this way the program is expansionary as lower long-term interest rates stimulate aggregate demand. Furthermore, improved demand in the large economy supports small economy’s exports. At the same time, however, the massive inflow of foreign capital leads to persistent appreciation of the small economy’s real exchange rate, which deteriorates price competitiveness and leads to a fall in the trade balance.

Overall, the capital inflow induced by quantitative easing abroad affects the small economy via two channels that have opposite effects on its output. According to our simulations, the unfavorable one prevails. This is in contrast to Alpanda and Kabaca (2015), whose model implies strong output growth outside the US after the Fed starts quantitative easing. The source of this difference might be related to their assumption that only US bonds are traded internationally so they cannot capture the inflow of non-residents into the sovereign bond market of the small economy. It is important to note that our results seem to fit better the empirical evidence from VARs presented in Bhattarai et al. (2015), who find that long-term asset purchases by the Fed had insignificant, and in some specifications negative, effect on industrial output in emerging economies.

4.2 Conventional monetary stimulus in the large economy

Our next step is to compare the effects of quantitative easing to a conventional monetary policy easing. Figure 5 depicts the dynamic responses to an expansionary monetary policy shock in the large economy. The results are consistent with what is well documented in the literature, also in the context of emerging markets’ reaction, see e.g. Mackowiak (2007). Monetary accommodation in the large economy boosts aggregate demand in this country, which leads to an increase in its output and imports. As regards international spillovers, the mechanism at play is different from that described in the previous section. The reaction of long-term bond prices and the term premia in both regions is small, and the home currency value of foreign bond holdings by small country’s agents decreases following the exchange rate appreciation so that their balance sheets are negatively affected. As a consequence,

Notwithstanding the scale of increase in GDP growth in the large economy, the reaction of output in the small economy that we discuss below is not significantly affected. We chose to allow for endogenous fluctuations of the short-term interest rates in our baseline scenario to be consistent with the observed tightening of conventional policy by the European Central Bank in 2011.

conventional monetary easing in the large economy actually depresses aggregate demand abroad. At the same time, however, strong expansion in foreign demand stimulates the small economy's exports. As in the case of the quantitative easing scenario, appreciation of the exchange rate deteriorates the price competitiveness of the small economy. This time, however, the net effect on its trade balance and output is clearly positive which, together with a rise in inflation, leads to a slight tightening of monetary policy in the small economy.

In the empirical literature on the effects of asset purchase programs, they are sometimes presented as equivalence of short-term interest rate cuts. For example, Gambacorta et al. (2014) use a back of the envelope calculation to express the impact of doubling of the central bank balance sheets on output as equivalent to a decrease in the policy rate by 300 bps. Our simulations suggest that an analogous way of thinking about international monetary policy spillovers is not valid as the channels through which conventional and unconventional measures operate are different, and their effect on output abroad may be even of different sign.

4.3 Term premia comovement

As we have stressed while discussing Figure 3, one of the striking features of the quantitative easing period is a dramatic increase in the cross-country synchronization of the term premia. Moreover, as shown by Jablecki et al. (2016), the term premium in Poland has been following almost one-to-one its counterpart in Germany since the outbreak of the crisis. In this section we show that our model is consistent with this observation.

We have already seen from our model simulations that, following the quantitative easing scenario, the term premia in the large and small economy react very similarly and exhibit a similar degree of persistence. We now investigate this comovement more formally, using the model-implied correlations between the term premia in the large and small economy, conditional on the type of shocks included in our model. The calculations are based on the first-order approximation to the model equilibrium conditions. In line with evidence from estimated DSGE models considered in the literature, autoregressive shocks are allowed to exhibit considerable degree of inertia (we set their autoregressive coefficients to 0.9). The smoothing coefficient in the debt composition rule for the large economy (17) is calibrated at 0.99 to reflect high persistence of asset purchase programs. Shocks of same type are assumed to have the same volatility in both countries.

Table 3 reports the outcomes. As expected, shocks to the composition of the large economy's bond supply imply tight comovement between the term premia. None of the other disturbances can generate similarly high correlation, and productivity shock imply even negative comovement. Moreover, when we allow the volatility of shocks in the small economy to be higher than in the large economy,⁷ these correlations become even lower.

⁷See e.g. Kolasa (2009) for evidence based on a two-country DSGE model estimated for Poland and the

Hence, we can conclude that our model is able to account for the observed increase in the cross-country term premia comovement during the period of quantitative easing compared to the pre-crisis times.

4.4 Robustness checks

In this section we show that our assessment of international spillovers of unconventional monetary is robust to a number of modeling choices and assumptions underlying our simulations.

4.4.1 The role of QE implementation

First, we relax the assumption on the fully-anticipated path of asset purchases. This modification aims to bring our simulations more towards how unconventional monetary policy in the BIG3 was conducted in reality. In fact, subsequent rounds of asset purchases were announced every one-two years. Even though they might have been anticipated to some extent by the agents, as pointed out e.g. by Krishnamurthy and Vissing-Jorgensen (2011) or Joyce et al. (2011), they were definitely not perfectly foreseen in 2009. In this paper we do not attempt to model in detail how expectations of central bank unconventional actions are formed. Instead, we compare how our results differ between two extreme approaches. In the first one, all rounds of quantitative easing are assumed to be perfectly anticipated by the agents from the moment the large economy's monetary authority starts purchasing long-term bonds. This is our baseline scenario that we presented in section 4.1. As an alternative, we assume that the agents are fully taken by surprise each time the central bank announces a new round of asset purchases. More specifically, we assume that quantitative easing is initially announced for a period of two years, after which it is expected to be gradually withdrawn at the same pace as it was introduced. Then, every year the program is extended by one more year, with the last extension announced two years before the program starts being withdrawn.

Figure 6 presents the simulated responses under these alternative assumptions. The scenario is designed such that the actual path of asset purchases is the same as in our baseline, so the differences reflect only the effect of imperfect anticipation. Since the consecutive extensions of quantitative easing take agents by surprise, we observe kinks in the simulated paths of all variables except those reflecting actual aggregate bond holdings. The responses of both financial and macroeconomic variables are much more gradual than in the baseline and their peaks or troughs are delayed since for most of the time agents expect a smaller scale of total asset purchases. However, the maximum responses are of similar size to those

euro area.

observed in the baseline scenario, suggesting that our main conclusions are little sensitive to the degree to which quantitative easing was anticipated by the agents.

This conclusion is backed by our second robustness check, in which we assume that the central bank in the large economy announces the whole path of quantitative easing one year in advance. The dynamic responses are plotted in Figure 7. Comparing this exercise with the baseline simulation reveals very similar patterns for most of variables. In particular, output in the small economy reacts negatively and the term premia in both countries tightly comove.

Third, we consider the binding zero lower bound (ZLB) for the short term interest rates in the large economy, which was a potentially important feature of the monetary policy conducted by the three major economies that we consider, at least for some period included in our analysis. We combine it with the 'surprises' scenario described in our first robustness check so that when the asset purchase program or its extension is announced, the central bank of the large economy commits to keep the short-term interest rates constant for one year.⁸ As a result, the ZLB period effectively lasts until one year before the withdrawal of quantitative easing. Figure 8 plots the responses of the key variables to this alternative scenario. As expected, accounting for the ZLB strengthens the positive impact of asset purchases on the large economy since it eliminates endogenous tightening of its conventional monetary policy present in the baseline. In particular, the response of output is now of similar magnitude to that found in the previous studies, which is conducive to higher GDP in the small economy. However, the appreciation of its exchange rate is now stronger as well. These two forces offset each other to a large extent, and the paths of output and absorption in the small economy are very similar to those obtained in our baseline scenario.

4.4.2 The role of modelling assumptions

In our benchmark model agents' financial decisions do not affect the economies' productive capacity. We now modify the setup by including physical capital as the second production factor and allowing for its endogenous accumulation.⁹ The outcomes are presented in Figure 9. We find that allowing for capital accumulation reduces the impact of quantitative easing on the term premia (and thus on bond prices) in both economies as investment in non-financial assets offers to the agents an additional way of transferring their wealth intertemporally. Since this type of investment is assumed to affect the productive capacity, its expansion affects positively output in both countries. As a result, GDP of the small economy eventually rises above the level observed before quantitative easing abroad started, but only after about

⁸As discussed by Chen et al. (2012), this expected duration of the ZLB is consistent with the evidence from Blue Chip surveys conducted in the US.

⁹We implement this extension as in Chen et al. (2012). Full details of the extended model are available from the authors upon request.

four years.¹⁰

Finally, we check to what extent our results depend on some assumptions regarding the values of calibrated parameters. In a model like ours, two crucial parameters that govern the response of the term premium and its transmission to other macroeconomic variables are ω_r , ξ_H and their large economy counterparts. The first one controls the share of restricted households and is calibrated at a rather small level of 0.1 under our baseline. It might be argued that in emerging economies the degree of bond market segmentation is larger than in developed countries, so as an alternative we consider a higher value of ω_r equal to 0.2. The second parameter determines the slope of the bond transaction cost function. We check how our results change if we decrease its value in both economies by a half so that $\xi_H = \xi_F = 0.0075$. The outcomes of these two robustness checks are plotted in Figure 10. With asymmetric bond market segmentation, quantitative easing in the large economy generates a stronger outflow of its capital to foreign bond markets. The consumption boom in the small economy is bigger, but the exchange rate appreciates more sharply and the fall in output is even deeper than under our baseline parametrization. At the same time, the comovement of the term premia in both economies is still very high. As regards the role of transaction costs, decreasing them does not change the direction of responses of any macrovariable of interest, but their magnitude becomes lower.

One of our paper's key messages are the negative spillovers of quantitative easing on other economies' output. One might be concerned that this finding crucially depends on the parametrization of the trade block in our model. In particular, one of the key parameter could be the price elasticity of demand for imports ν . Some papers suggest that a lower value of this parameter helps achieve a better fit to the international business cycle, see Bodenstein (2010) for a review. Therefore, we check how our results change if we assume a Cobb-Douglas form of the final goods basket by picking $\nu = \nu^* \approx 1$. We also examine the sensitivity of our main findings on the degree of exchange rate pass-through by considering higher values of Calvo stickiness whenever firms in both countries serve foreign markets, i.e. $\theta_H^* = \theta_F = 0.9$. As can be seen from Figure 11, none of our qualitative results hinge on the parametrization of the trade block. As regards the magnitude of the responses, non-negligible differences relative to our baseline concern only the case of low elasticity of substitution between home goods and imports as it implies a lower inflow of capital and weaker response of the term premium in the small economy, but a stronger appreciation of its exchange rate.

Overall, we can conclude that our main findings, i.e. a negative effect of quantitative easing abroad on output in the small economy (at least in the short-run) and tightening of the cross-country comovement of the term premia, are robust to the alternative assumptions

¹⁰One can conjecture that the mitigating effect of the presence of non-financial investment on the drop in small economy's output following quantitative easing abroad would be weaker if we allowed some of it to be not directly related to the productive capacity, e.g. residential investment or purchase of other durables.

considered in this section.

5 Conclusions

We have developed a two-country DSGE model with segmented asset markets and used it to quantitatively analyze international spillovers of large scale asset purchase programs pursued in some large economies in the aftermath of the Great Recession. We showed that this framework can replicate key empirical facts observed in small economies pursuing independent monetary policy during the period of quantitative easing abroad. First, it accurately mimics the inflow of foreign capital to small economies' sovereign bond markets. Second, it accounts for the very strong cross-country comovement of the term premia during the period of quantitative easing, but not necessarily during normal times.

Our main finding is that, notwithstanding the positive effects of quantitative easing abroad on domestic demand in the small economy, the impact on its GDP is likely to be negative, at least in the short run. This is in contrast to the effects of conventional monetary easing abroad, which positively affects the small economy's output. Our results thus offer support for concerns expressed by some policy makers about possibly negative international spillovers of non-standard measures adopted by the major central banks to emerging markets.

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Tables and figures

Table 1: Calibrated parameters

Parameter	Value
Size of the small economy; ω	0.014
Share of restricted households; ω_r, ω_r^*	0.1
Inv. elasticity of intertemporal substitution; σ, σ^*	2
Inv. Frisch elasticity of labor supply; φ, φ^*	2
Discount factor, unrestricted households; β^u, β^{u*}	0.992
Discount factor, restricted households; β^r, β^{r*}	0.995
Coupon; κ, κ^*	0.929, 0.979
Transaction cost on long-term bonds (unrestricted households); ξ_H, ξ_F	0.015
Portfolio adjustment cost (large economy's restricted households); ξ_r^*	10^{-5}
Country risk premium; ξ	0.01
Calvo probability; $\theta_H, \theta_H^*, \theta_F, \theta_F^*$	0.75
Price markup; μ, μ^*	1.15
Elasticity of substitution btw. home and imported goods; ν, ν^*	3
Home-bias; η	0.75
Steady-state inflation; π, π^*	1.005
Interest rate smoothing; γ_r, γ_r^*	0.9
Interest rate response to inflation; γ_π, γ_π^*	2
Interest rate response to output gap; γ_y, γ_y^*	0.125

Table 2: Targeted steady state ratios

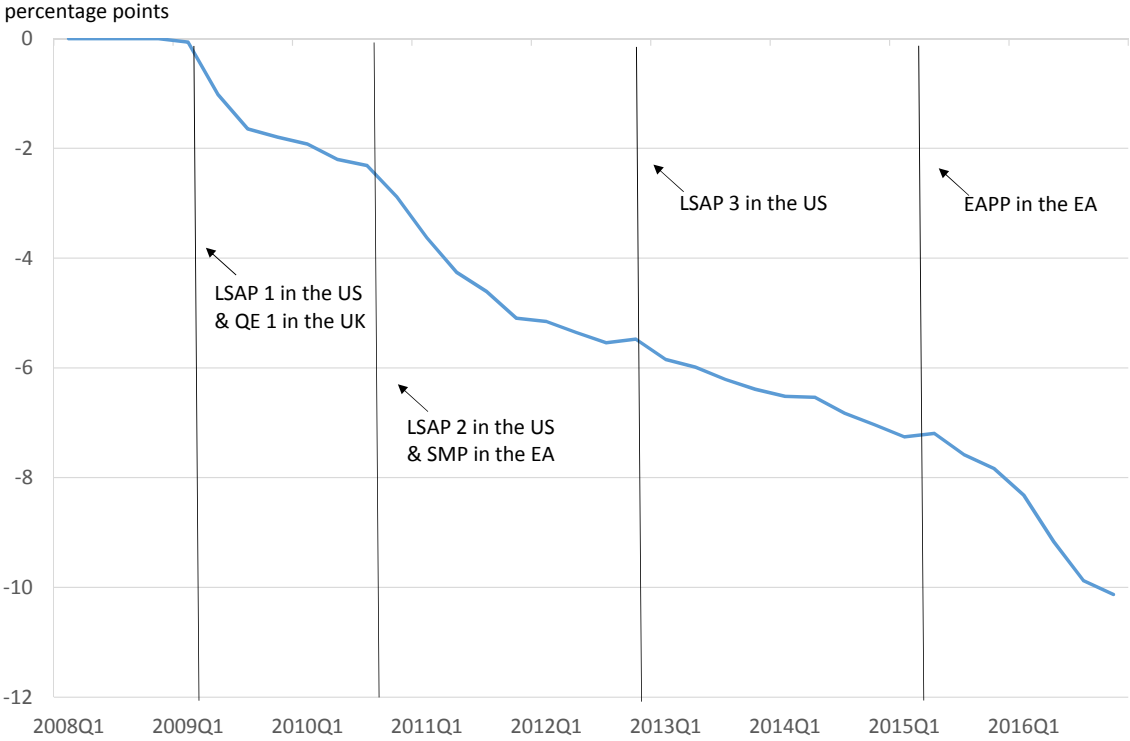
Steady state ratio	Value
Share of government spending in GDP; $\frac{g}{y}, \frac{g^*}{y^*}$	0.2
Share of government bonds in GDP; $\frac{b^g + P_L b_{H,L}^g}{y}, \frac{b^{g*} + P_L^* b_{H,L}^{g*}}{y^*}$	1.25, 2.65
Share of long-term bonds in total bonds; $\frac{P_L b_{H,L}^g}{b^g + P_L b_{H,L}^g}, \frac{P_L^* b_{H,L}^{g*}}{b^{g*} + P_L^* b_{H,L}^{g*}}$	0.71, 0.65
Share of residents in small economy's long-term bonds; $\frac{P_L(\omega_r b_{H,L}^r + (1-\omega_r)b_{H,L}^u)}{b_{H,L}^g}$	0.76
Share of foreign bonds in small country's portfolio; $\frac{(1-\omega_r)s P_L^* b_{F,L}^u}{\omega_r P_L b_{H,L}^r + (1-\omega_r)(s P_L^* b_{F,L}^u + P_L b_{H,L}^u)}$	0.05

Table 3: Conditional cross-country correlation of the term premia

Shock	correlation
QE in large economy ε_t^{L*}	0.94
Productivity; $\varepsilon_t^z, \varepsilon_t^{z*}$	-0.20
Time preference; $\varepsilon_t^d, \varepsilon_t^{d*}$	0.22
Government spending; $\varepsilon_t^g, \varepsilon_t^{g*}$	0.24
Monetary policy; $\varepsilon_t^r, \varepsilon_t^{r*}$	0.60
Country risk premium; ε_t^Γ	0.73

Note: The correlations are calculated using the first-order approximation to the model equilibrium conditions. The inertia of productivity, time preference, government spending and risk premium shocks are all set to 0.9. QE and monetary shocks are assumed to be i.i.d. The smoothing coefficient γ_L^* in equation (17) is calibrated at 0.99.

Figure 1: QE impact on the share of long-term government bonds (excluding central bank holdings) in total public sector liabilities in the US, UK and EA

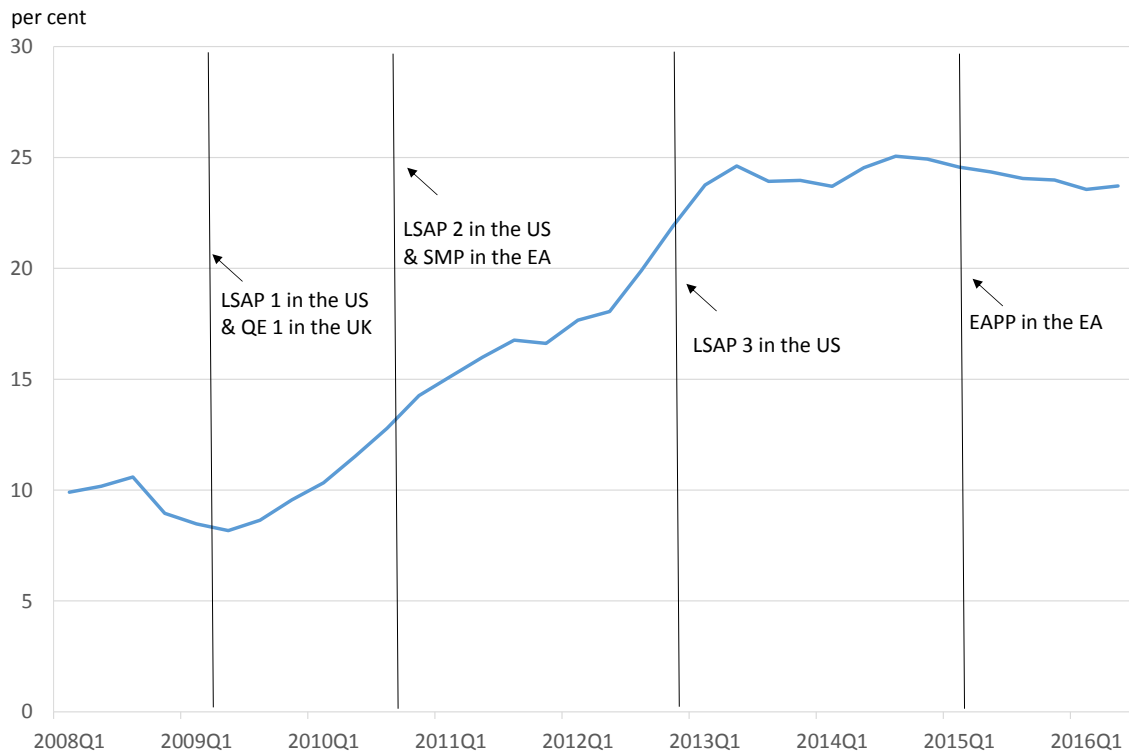


Note: The presented QE impact is calculated as the difference between (i) the share of long-term bonds in total public sector liabilities (that we call bonds in what follows) issued in the BIG3 countries (United States, United Kingdom and euro area) excluding central bank holdings, and (ii) the share of long-term bonds including central bank holdings in total bonds issued by the BIG3 governments. Central bank asset purchases reduce the outstanding amount of long-term bonds, but do not impact the total government debt - when a central bank buys long-term bonds, it creates new central bank reserves, replacing de facto long-term public liabilities with short-term ones. Short-term debt comprises short-term government bonds, as well as central bank short-term interest-bearing liabilities and cash in circulation. Both long- and short-term bonds are calculated as a sum of outstanding bonds in the BIG3.

Source: World Bank, central bank web pages, authors' calculations.

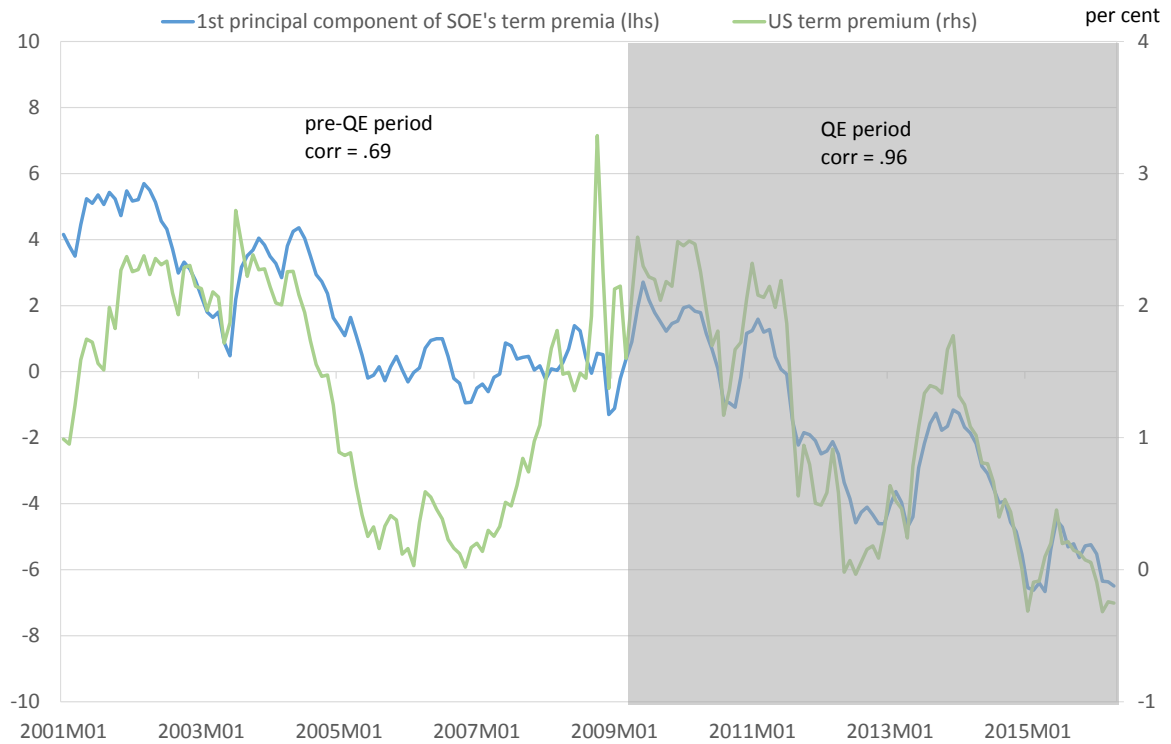
Memo: LSAP - Large-Scale Asset Purchases; SMP - Securities Markets Programme; EAPP - Expanded Asset Purchase Programme.

Figure 2: Share of foreign investors in sovereign bond markets of emerging economies



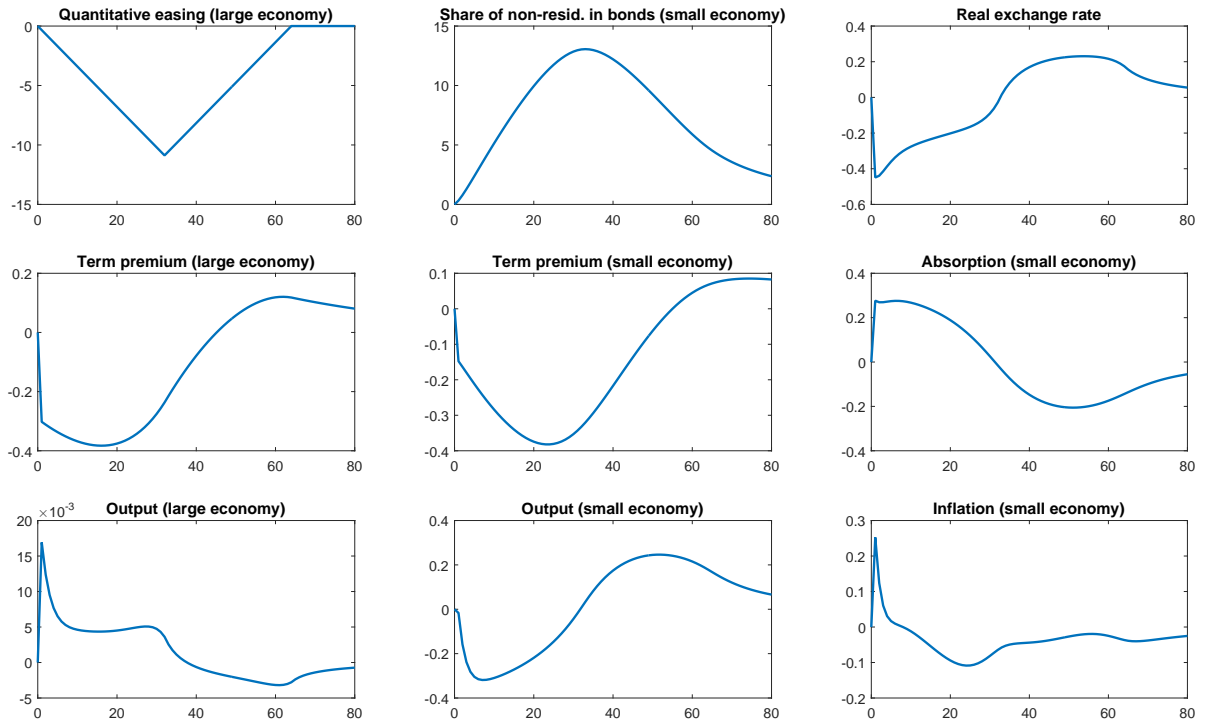
Note: This plot presents the share of bonds issued by the emerging economies' governments held by non-residents. The emerging market economies included are: Brazil, Colombia, Czech R., Hungary, Indonesia, Israel, Malaysia, Mexico, Peru, Poland, Russia, South Africa, South Korea, Thailand, Turkey. Source: Credit Suisse. For exact definitions, calculation and data sources, see Credit Suisse monthly note "Emerging Markets: Non-residents' holdings in local currency government bonds".

Figure 3: Term premium on 10-year bonds in the US and small open economies



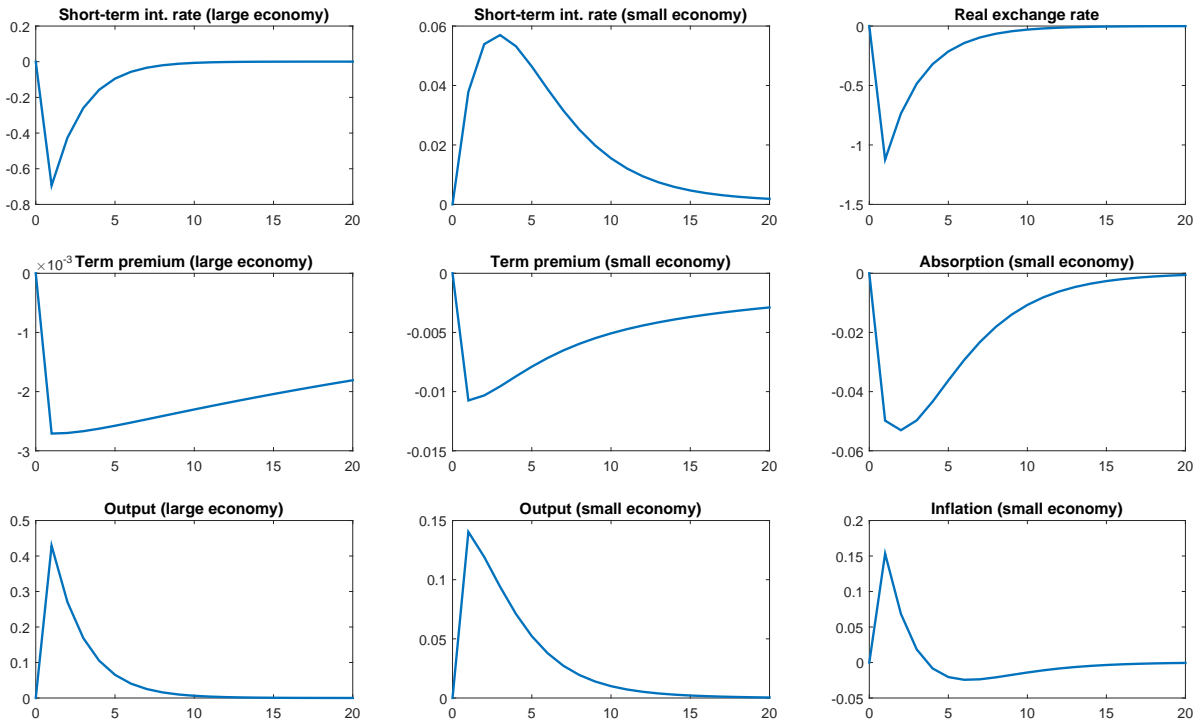
Note: This plot presents the 10-year term premium in the United States and the first principal component of 10-year term premia in small open economies (SOEs). SOEs comprise: Australia, Canada, Chile, Czech Republic, Hong Kong, Hungary, Indonesia, Israel, Japan, South Korea, Malaysia, New Zealand, Norway, Philippines, Poland, Singapore, South Africa, Sweden, Turkey. The term premia in SOEs were calculated based on the Adrian et al. (2013) model. Source: Bloomberg, New York Fed, authors' calculations.

Figure 4: Effects of quantitative easing in the large economy



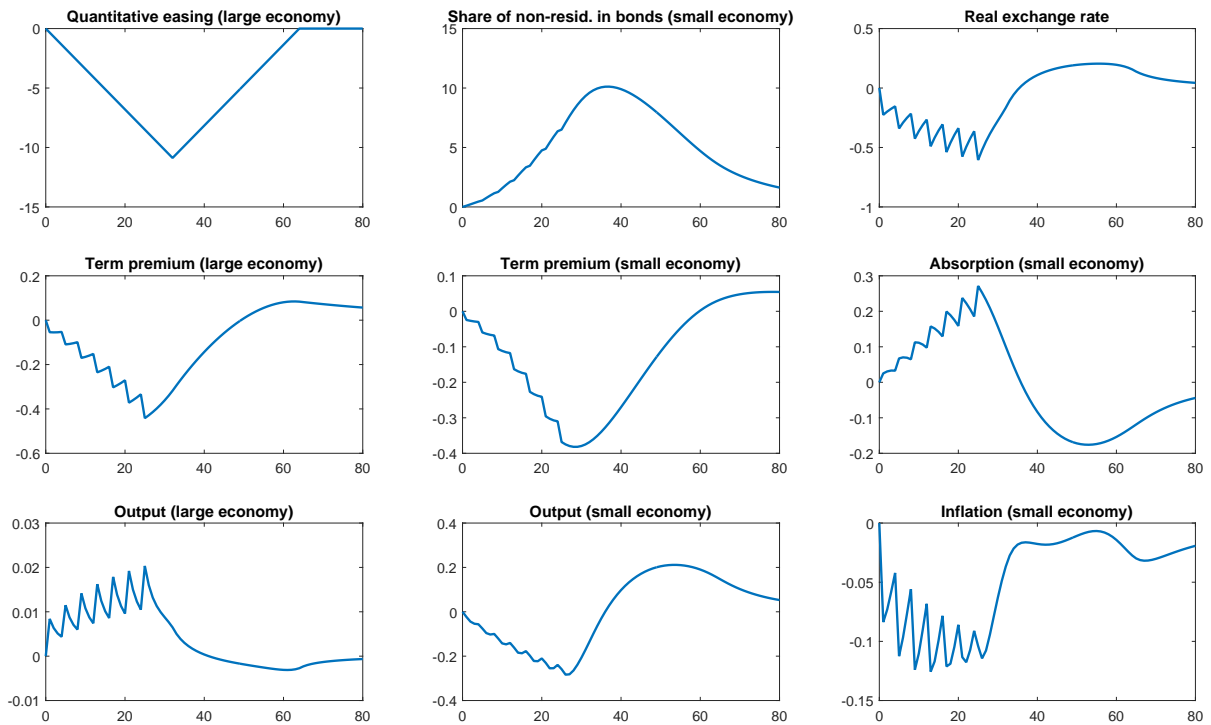
Note: This figure presents the effects of quantitative easing in the large economy, calibrated to mimic the evolution of the share of foreign long-term bonds in total foreign government debt as in Figure 1, and assuming that it will be withdrawn at the same pace as it was implemented. All responses are in percent deviations from the steady state. The responses of the term premium and inflation are annualized.

Figure 5: Effects of conventional monetary policy easing in the large economy



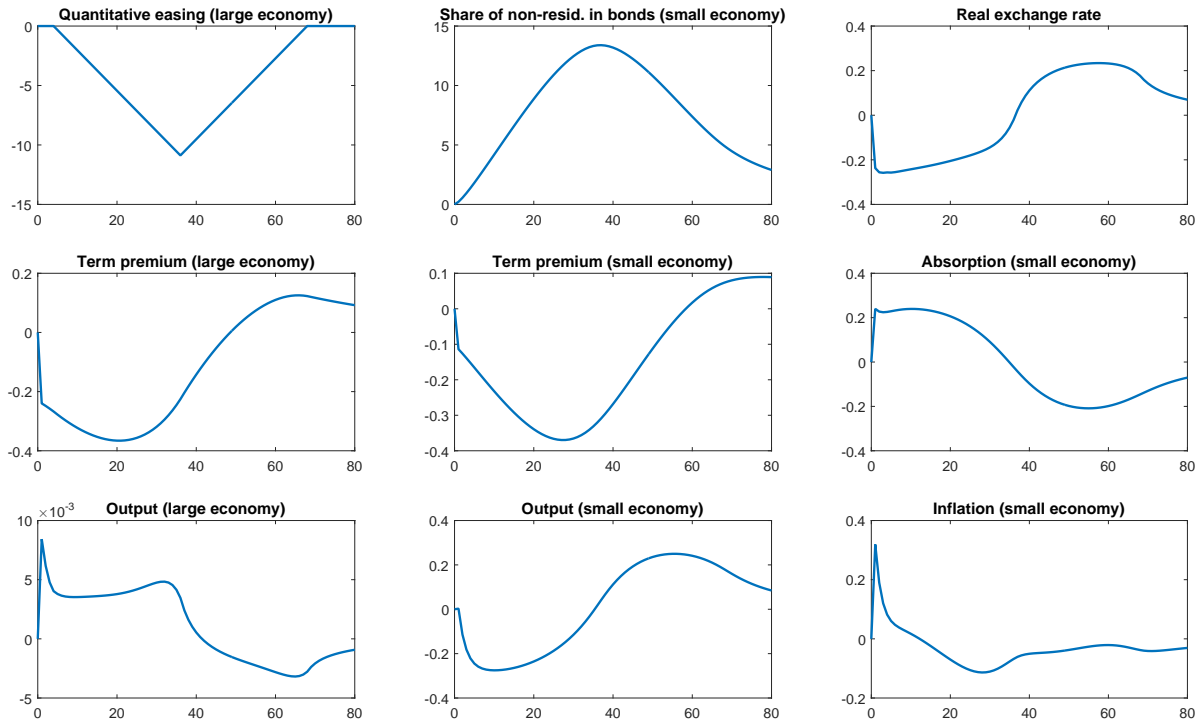
Note: This figure presents the effects of an expansionary monetary policy shock in the large economy of 1 percentage point (annualized). All responses are in percent deviations from the steady state. The responses of the term premium, interest rates and inflation are annualized.

Figure 6: Effects of quantitative easing in the large economy with asset purchases extended every year



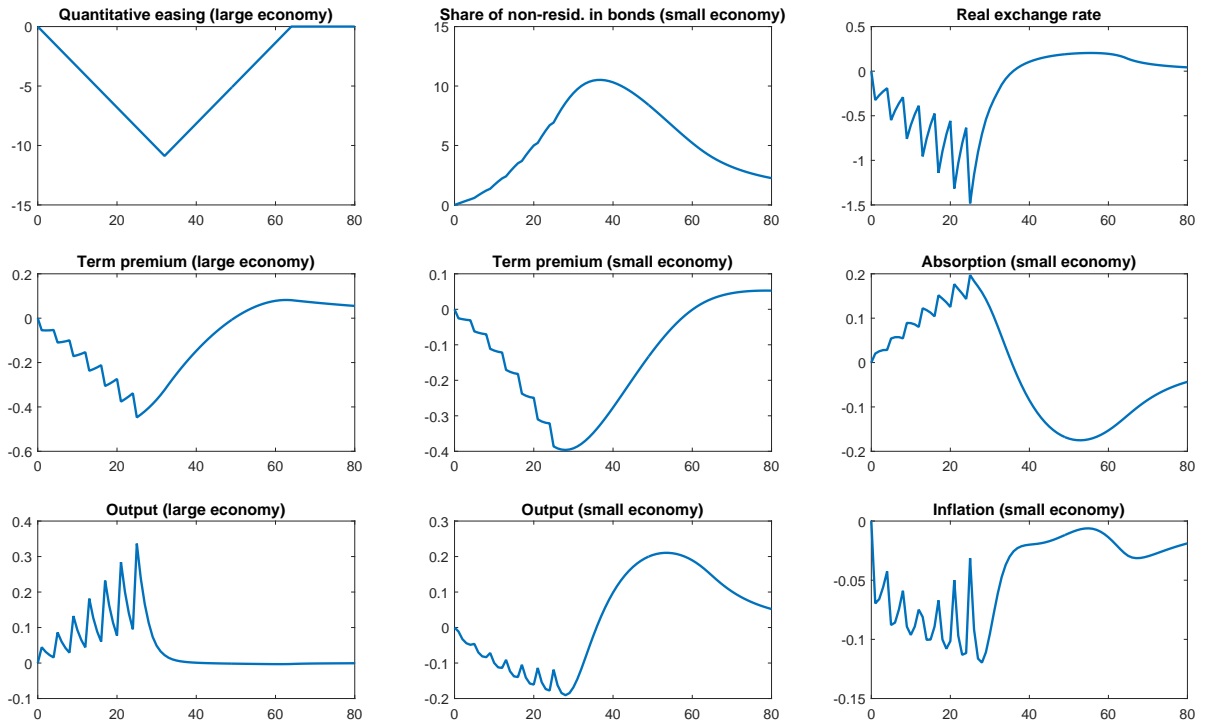
Note: This figure presents the effects of quantitative easing in the large economy under the assumption that the central bank announces a subsequent round of asset purchases every year. All responses are in percent deviations from the steady state. The responses of the term premium and inflation are annualized.

Figure 7: Effects of quantitative easing in the large economy announced one year in advance



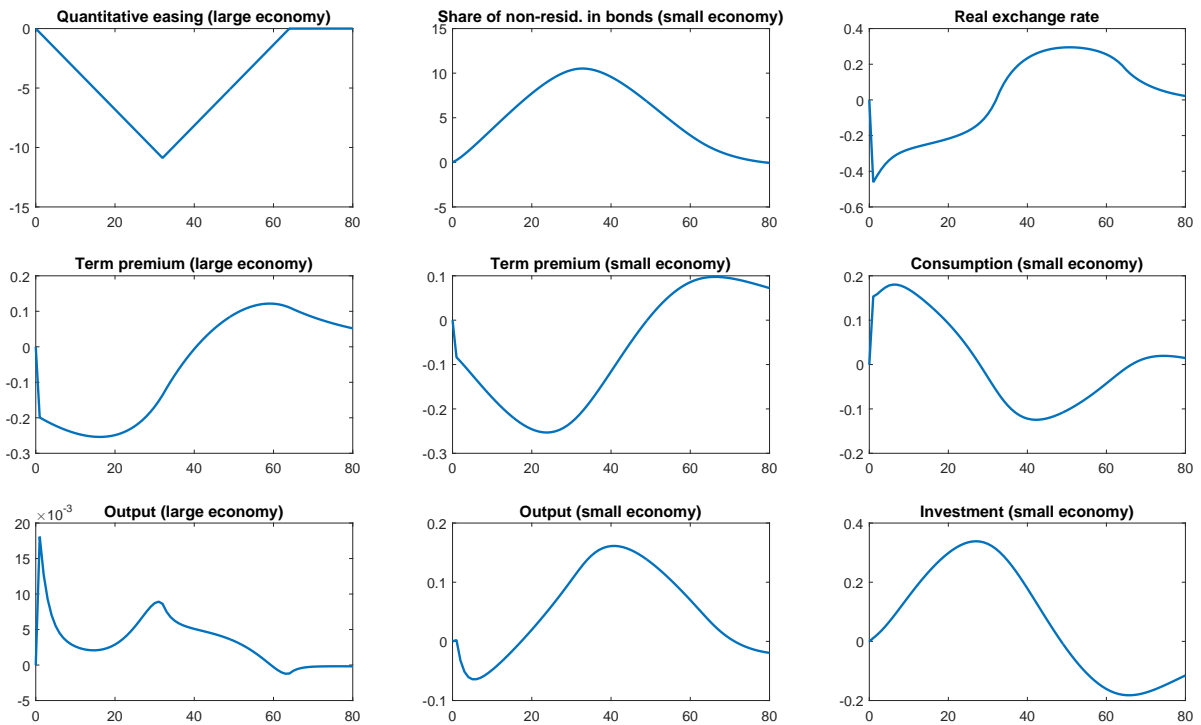
Note: This figure presents the effects of quantitative easing in the large economy under the assumption that the central bank announces the path of asset purchases one year before they start. All responses are in percent deviations from the steady state. The responses of the term premium and inflation are annualized.

Figure 8: Effects of quantitative easing in the large economy with asset purchases announcements every year at the zero lower bound



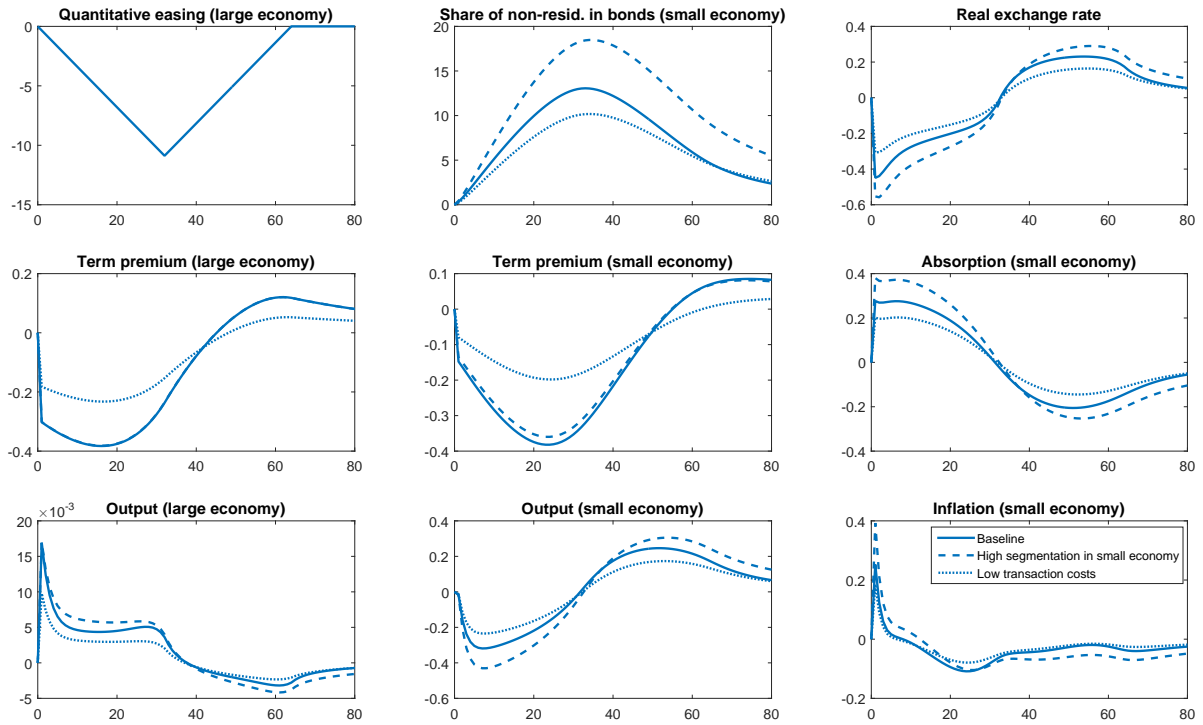
Note: This figure presents the effects of quantitative easing in the large economy under the assumption that the central bank announces a subsequent round of asset purchases every year and the short-term interest rates are expected to stay at the zero lower bound for one year after every announcement. All responses are in percent deviations from the steady state. The responses of the term premium and inflation are annualized.

Figure 9: Effects of quantitative easing in the large economy in the model with endogenous formation of productive capital



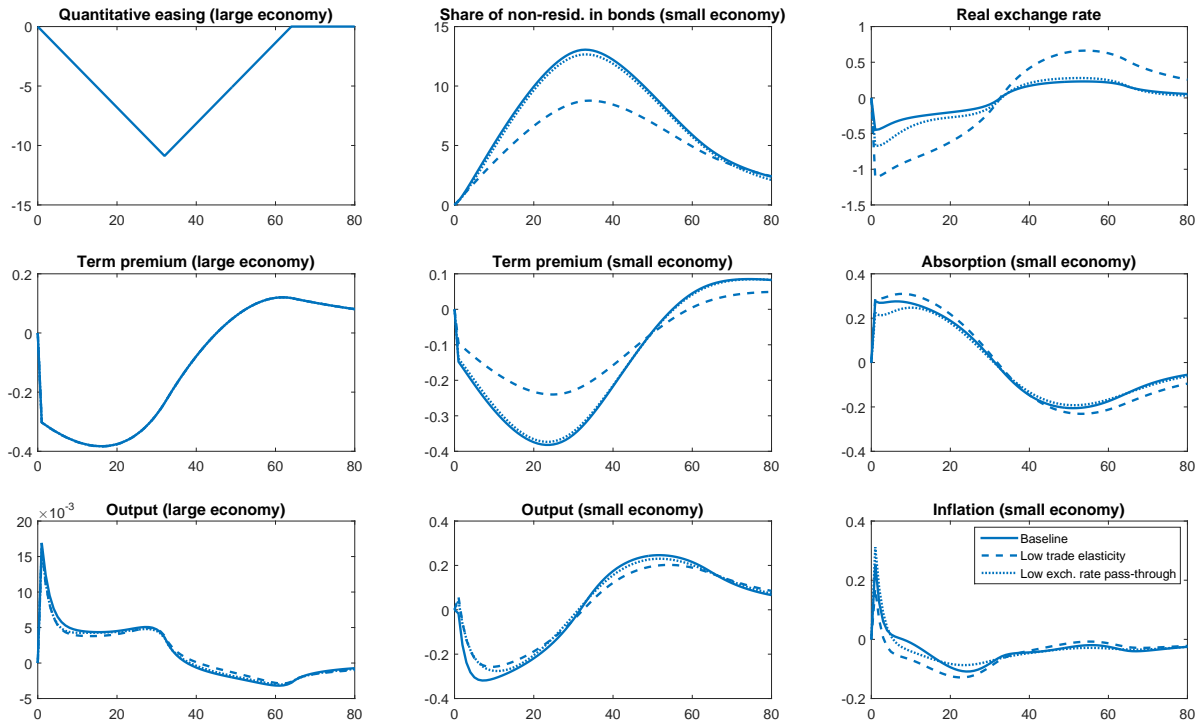
Note: This figure presents the effects of quantitative easing in the large economy according to the model with a two-factor production function and endogenous capital formation. All responses are in percent deviations from the steady state. The term premium responses are annualized.

Figure 10: Effects of quantitative easing in the large economy - role of market segmentation and transaction costs



Note: This figure presents the effects of quantitative easing in the large economy for the baseline and two alternative parametrizations: high bond market segmentation in the small economy ($\omega_r = 0.2$) and low transaction costs ($\xi_H = \xi_F = 0.0075$). All responses are in percent deviations from the steady state. The term premium responses are annualized.

Figure 11: Effects of quantitative easing in the large economy - role of trade elasticity and exchange rate pass-through



Note: This figure presents the effects of quantitative easing in the large economy for the baseline and two alternative parametrizations: low trade elasticity ($\nu = \nu^* \approx 1$) and low exchange rate pass-through ($\theta_H^* = \theta_F = 0.9$). All responses are in percent deviations from the steady state. The term premium responses are annualized.

Appendix

A List of model equations

The following equations describe the equilibrium in our two-country model. Small letters for variables defined in the main text indicate their real counterparts.

A.1 Households

Marginal utility

$$\lambda_t^r = \exp\{\varepsilon_t^d\}(c_t^r)^{-\sigma} \quad (\text{A.1})$$

$$\lambda_t^{r*} = \exp\{\varepsilon_t^{d*}\}(c_t^{r*})^{-\sigma^*} \quad (\text{A.2})$$

$$\lambda_t^u = \exp\{\varepsilon_t^d\}(c_t^u)^{-\sigma} \quad (\text{A.3})$$

$$\lambda_t^{u*} = \exp\{\varepsilon_t^{d*}\}(c_t^{u*})^{-\sigma^*} \quad (\text{A.4})$$

$$\lambda_t = \omega_r \lambda_t^r + (1 - \omega_r) \lambda_t^u \quad (\text{A.5})$$

$$\lambda_t^* = \omega_r^* \lambda_t^{r*} + (1 - \omega_r^*) \lambda_t^{u*} \quad (\text{A.6})$$

Bond prices

$$P_{L,t} = \frac{1}{R_{L,t} - \kappa} \quad (\text{A.7})$$

$$P_{L,t}^* = \frac{1}{R_{L,t}^* - \kappa^*} \quad (\text{A.8})$$

Restricted households' budget constraint

$$c_t^r + P_{L,t} b_{H,L,t}^r + t_t^r = P_{L,t} \frac{R_{L,t}}{\pi_t} b_{H,L,t-1}^r + w_t n_t^r + d_t^r \quad (\text{A.9})$$

$$\begin{aligned} & c_t^{r*} + P_{L,t}^* b_{F,L,t}^{r*} + s_t^{-1} P_{L,t} b_{H,L,t}^{r*} + t_t^{r*} = \\ & P_{L,t}^* \frac{R_{L,t}^*}{\pi_t^*} b_{F,L,t-1}^{r*} + s_t^{-1} P_{L,t}^* \frac{R_{L,t}^*}{\pi_t^*} b_{H,L,t-1}^{r*} + w_t^* n_t^{r*} + d_t^{r*} \end{aligned} \quad (\text{A.10})$$

Unrestricted households' budget constraint

$$\begin{aligned} & c_t^u + b_{H,t}^u + P_{L,t} b_{H,L,t}^u + s_t P_{L,t}^* b_{F,L,t}^u + t_t^u = \\ & \frac{R_{t-1}}{\pi_t} b_{H,t-1}^u + P_{L,t} \frac{R_{L,t}}{\pi_t} b_{H,L,t-1}^u + s_t P_{L,t}^* \frac{R_{L,t}^*}{\pi_t^*} b_{F,L,t-1}^u + w_t n_t^u + d_t^u \end{aligned} \quad (\text{A.11})$$

$$c_t^{u^*} + b_{F,t}^{u^*} + s_t^{-1} P_{L,t}^* b_{H,L,t}^{u^*} + P_{L,t}^* b_{F,L,t}^{u^*} + t_t^{u^*} = \frac{R_{t-1}^*}{\pi_t^*} b_{F,t-1}^{u^*} + s_t^{-1} P_{L,t} \frac{R_{L,t}}{\pi_t} b_{H,L,t-1}^{u^*} + P_{L,t}^* \frac{R_{L,t}^*}{\pi_t^*} b_{F,L,t-1}^{u^*} + w_t^* n_t^{u^*} + d_t^{u^*} \quad (\text{A.12})$$

Consumption-leisure choice

$$(n_t^r)^\varphi = \lambda_t^r w_t \quad (\text{A.13})$$

$$(n_t^{r^*})^{\varphi^*} = \lambda_t^{r^*} w_t^* \quad (\text{A.14})$$

$$(n_t^u)^\varphi = \lambda_t^u w_t \quad (\text{A.15})$$

$$(n_t^{u^*})^{\varphi^*} = \lambda_t^{u^*} w_t^* \quad (\text{A.16})$$

Restricted households' optimal bond holdings

$$\lambda_t^r P_{L,t} = \beta^r E_t \left\{ \lambda_{t+1}^r P_{L,t+1} \frac{R_{L,t+1}}{\pi_{t+1}} \right\} \quad (\text{A.17})$$

$$\lambda_t^{r^*} P_{L,t}^* = \beta^{r^*} E_t \left\{ \lambda_{t+1}^{r^*} P_{L,t+1}^* \frac{R_{L,t+1}^*}{\pi_{t+1}^*} \right\} \quad (\text{A.18})$$

$$\lambda_t^{r^*} (1 + \Gamma_t^{r^*}) s_t^{-1} P_{L,t} = \beta^{r^*} E_t \left\{ \lambda_{t+1}^{r^*} s_{t+1}^{-1} P_{L,t+1} \frac{R_{L,t+1}}{\pi_{t+1}} \right\} \quad (\text{A.19})$$

Unrestricted households' optimal bond holdings

$$\lambda_t^u = \beta E_t \left\{ \lambda_{t+1}^u R_t \pi_{t+1}^{-1} \right\} \quad (\text{A.20})$$

$$\lambda_t^u s_t = \beta E_t \left\{ \lambda_{t+1}^u s_{t+1} (1 + \Gamma_t) R_t^* \pi_{t+1}^{*-1} \right\} \quad (\text{A.21})$$

$$\lambda_t^u (1 + \zeta_{H,t}) P_{L,t} = \beta E_t \left\{ \lambda_{t+1}^u P_{L,t+1} \frac{R_{L,t+1}}{\pi_{t+1}} \right\} \quad (\text{A.22})$$

$$\lambda_t^u s_t (1 + \zeta_{F,t}) P_{L,t}^* = \beta E_t \left\{ \lambda_{t+1}^u s_{t+1} (1 + \Gamma_t) P_{L,t+1}^* \frac{R_{L,t+1}^*}{\pi_{t+1}^*} \right\} \quad (\text{A.23})$$

$$\lambda_t^{u^*} = \beta E_t \left\{ \lambda_{t+1}^{u^*} R_t^* \pi_{t+1}^{*-1} \right\} \quad (\text{A.24})$$

$$\lambda_t^{u^*} s_t^{-1} (1 + \zeta_{H,t}^*) P_{L,t} = \beta E_t \left\{ \lambda_{t+1}^{u^*} s_{t+1}^{-1} P_{L,t+1} \frac{R_{L,t+1}}{\pi_{t+1}} \right\} \quad (\text{A.25})$$

$$\lambda_t^{u^*} (1 + \zeta_{F,t}^*) P_{L,t}^* = \beta E_t \left\{ \lambda_{t+1}^{u^*} P_{L,t+1}^* \frac{R_{L,t+1}^*}{\pi_{t+1}^*} \right\} \quad (\text{A.26})$$

Transaction costs

$$\frac{1 + \zeta_{H,t}}{1 + \zeta_H} = \left(\frac{P_{L,t} b_{H,L,t}^u}{P_L b_{H,L}^u} \right)^{\xi_H} \quad (\text{A.27})$$

$$\frac{1 + \zeta_{H,t}^*}{1 + \zeta_H^*} = \left(\frac{P_{L,t} b_{H,L,t}^{u^*} s}{P_L b_{H,L}^{u^*} s_t} \right)^{\xi_H} \quad (\text{A.28})$$

$$\frac{1 + \zeta_{F,t}}{1 + \zeta_F} = \left(\frac{P_{L,t}^* b_{F,L,t}^u s_t}{P_L^* b_{F,L}^u s} \right)^{\xi_F} \quad (\text{A.29})$$

$$\frac{1 + \zeta_{F,t}^*}{1 + \zeta_F^*} = \left(\frac{P_{L,t}^* b_{F,L,t}^{u*}}{P_L^* b_{F,L}^{u*}} \right)^{\xi_F} \quad (\text{A.30})$$

$$1 + \Gamma_t = \exp \{ -\xi(a_t - a) + \varepsilon_t^\Gamma \} \quad (\text{A.31})$$

$$1 + \Gamma_t^{r*} = \exp \left\{ \zeta_r^* \left(\frac{P_{L,t} B_{H,L,t}^{r*}}{S_t P_{L,t}^* B_{F,L,t}^{r*}} - \kappa^{r*} \right) \right\} \quad (\text{A.32})$$

A.2 Firms

Final goods basket

$$\tilde{y}_t = \left[\eta^{\frac{1}{\nu}} (y_{H,t})^{\frac{\nu-1}{\nu}} + (1-\eta)^{\frac{1}{\nu}} (y_{F,t})^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \quad (\text{A.33})$$

$$\tilde{y}_t^* = \left[\eta^{*\frac{1}{\nu^*}} (y_{H,t}^*)^{\frac{\nu^*-1}{\nu^*}} + (1-\eta^*)^{\frac{1}{\nu^*}} (y_{F,t}^*)^{\frac{\nu^*-1}{\nu^*}} \right]^{\frac{\nu^*}{\nu^*-1}} \quad (\text{A.34})$$

Optimal composition of final goods basket

$$y_{H,t} = \eta (p_{H,t})^{-\nu} \tilde{y}_t \quad (\text{A.35})$$

$$y_{F,t} = (1-\eta) (p_{F,t})^{-\nu} \tilde{y}_t \quad (\text{A.36})$$

$$y_{H,t}^* = \eta^* (p_{H,t}^*)^{-\nu^*} \tilde{y}_t^* \quad (\text{A.37})$$

$$y_{F,t}^* = (1-\eta^*) (p_{F,t}^*)^{-\nu^*} \tilde{y}_t^* \quad (\text{A.38})$$

Real price indices

$$p_{H,t}^{\frac{1}{1-\mu}} = \theta_H \left(p_{H,t-1} \frac{\pi}{\pi_t} \right)^{\frac{1}{1-\mu}} + (1-\theta_H) (\tilde{p}_{H,t})^{\frac{1}{1-\mu}} \quad (\text{A.39})$$

$$p_{F,t}^{\frac{1}{1-\mu^*}} = \theta_F \left(p_{F,t-1} \frac{\pi}{\pi_t} \right)^{\frac{1}{1-\mu^*}} + (1-\theta_F) (\tilde{p}_{F,t})^{\frac{1}{1-\mu^*}} \quad (\text{A.40})$$

$$p_{H,t}^{*\frac{1}{1-\mu}} = \theta_H^* \left(p_{H,t-1}^* \frac{\pi^*}{\pi_t^*} \right)^{\frac{1}{1-\mu}} + (1-\theta_H^*) (\tilde{p}_{H,t}^*)^{\frac{1}{1-\mu}} \quad (\text{A.41})$$

$$p_{F,t}^{*\frac{1}{1-\mu^*}} = \theta_F^* \left(p_{F,t-1}^* \frac{\pi^*}{\pi_t^*} \right)^{\frac{1}{1-\mu^*}} + (1-\theta_F^*) (\tilde{p}_{F,t}^*)^{\frac{1}{1-\mu^*}} \quad (\text{A.42})$$

Optimal reset prices

$$\tilde{p}_{H,t} = \mu \frac{\Omega_{H,t}}{\Upsilon_{H,t}} \quad (\text{A.43})$$

$$\Omega_{H,t} = \lambda_t \frac{w_t}{\exp\{\varepsilon_t^z\}} p_{H,t}^{\frac{\mu}{\mu-1}} y_{H,t} + \beta \theta_H E_t \left(\frac{\pi}{\pi_{t+1}} \right)^{\frac{\mu}{1-\mu}} \Omega_{H,t+1} \quad (\text{A.44})$$

$$\Upsilon_{H,t} = \lambda_t p_{H,t}^{\frac{\mu}{\mu-1}} y_{H,t} + \beta \theta_H E_t \left(\frac{\pi}{\pi_{t+1}} \right)^{\frac{1}{1-\mu}} \Upsilon_{H,t+1} \quad (\text{A.45})$$

$$\tilde{p}_{F,t} = \mu^* \frac{\Omega_{F,t}}{\Upsilon_{F,t}} \quad (\text{A.46})$$

$$\Omega_{F,t} = \lambda_t^* \frac{w_t^*}{\exp\{\varepsilon_t^{z*}\}} p_{F,t}^{\frac{\mu^*}{\mu^*-1}} y_{F,t} + \beta \theta_F E_t \left(\frac{\pi}{\pi_{t+1}} \right)^{\frac{1}{1-\mu^*}} \Omega_{F,t+1} \quad (\text{A.47})$$

$$\Upsilon_{F,t} = \lambda_t^* s_t^{-1} p_{F,t}^{\frac{\mu^*}{\mu^*-1}} y_{F,t} + \beta \theta_F E_t \left(\frac{\pi}{\pi_{t+1}} \right)^{\frac{1}{1-\mu^*}} \Upsilon_{F,t+1} \quad (\text{A.48})$$

$$\tilde{p}_{H,t}^* = \mu \frac{\Omega_{H,t}^*}{\Upsilon_{H,t}^*} \quad (\text{A.49})$$

$$\Omega_{H,t}^* = \lambda_t \frac{w_t}{\exp\{\varepsilon_t^z\}} p_{H,t}^{\frac{\mu}{\mu-1}} y_{H,t}^* + \beta^* \theta_H^* E_t \left(\frac{\pi^*}{\pi_{t+1}^*} \right)^{\frac{1}{1-\mu}} \Omega_{H,t+1}^* \quad (\text{A.50})$$

$$\Upsilon_{H,t}^* = \lambda_t s_t p_{H,t}^{\frac{\mu}{\mu-1}} y_{H,t}^* + \beta^* \theta_H^* E_t \left(\frac{\pi^*}{\pi_{t+1}^*} \right)^{\frac{1}{1-\mu}} \Upsilon_{H,t+1}^* \quad (\text{A.51})$$

$$\tilde{p}_{F,t}^* = \mu^* \frac{\Omega_{F,t}^*}{\Upsilon_{F,t}^*} \quad (\text{A.52})$$

$$\Omega_{F,t}^* = \lambda_t^* \frac{w_t^*}{\exp\{\varepsilon_t^{z*}\}} p_{F,t}^{\frac{\mu^*}{\mu^*-1}} y_{F,t}^* + \beta^* \theta_F^* E_t \left(\frac{\pi^*}{\pi_{t+1}^*} \right)^{\frac{1}{1-\mu^*}} \Omega_{F,t+1}^* \quad (\text{A.53})$$

$$\Upsilon_{F,t}^* = \lambda_t^* p_{F,t}^{\frac{\mu^*}{\mu^*-1}} y_{F,t}^* + \beta^* \theta_F^* E_t \left(\frac{\pi^*}{\pi_{t+1}^*} \right)^{\frac{1}{1-\mu^*}} \Upsilon_{F,t+1}^* \quad (\text{A.54})$$

Dividends

$$d_t = p_{H,t} y_{H,t} + \frac{1-\omega}{\omega} s_t p_{H,t}^* y_{H,t}^* - w_t n_t \quad (\text{A.55})$$

$$d_t^* = \frac{\omega}{1-\omega} p_{F,t} y_{F,t} \frac{1}{s_t} + p_{F,t}^* y_{F,t}^* - w_t^* n_t^* \quad (\text{A.56})$$

$$d_t^r = \omega_r d_t \quad (\text{A.57})$$

$$d_t^u = (1-\omega_r) d_t \quad (\text{A.58})$$

$$d_t^* = \omega_r^* d_t^* \quad (\text{A.59})$$

$$d_t^{u*} = (1-\omega_r^*) d_t^* \quad (\text{A.60})$$

A.3 Government

Monetary policy rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y}\right]^{1-\gamma_r} \exp\{\varepsilon_t^r\} \quad (\text{A.61})$$

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\gamma_r^*} \left[\left(\frac{\pi_t^*}{\pi^*}\right)^{\gamma_\pi^*} \left(\frac{y_t^*}{y^*}\right)^{\gamma_y^*}\right]^{1-\gamma_r^*} \exp\{\varepsilon_t^{r^*}\} \quad (\text{A.62})$$

Government budget constraint

$$b_{H,t}^g + P_{L,t} b_{H,L,t}^g + t_t = \frac{R_{t-1}}{\pi_t} b_{H,t-1}^g + P_{L,t} \frac{R_{L,t}}{\pi_t} b_{H,L,t-1}^g + g \exp\{\varepsilon_t^g\} \quad (\text{A.63})$$

$$b_{F,t}^{g*} + P_{L,t}^* b_{F,L,t}^{g*} + t_t^* = \frac{R_{t-1}^*}{\pi_t^*} b_{F,t-1}^{g*} + P_{L,t}^* \frac{R_{L,t}^*}{\pi_t^*} b_{F,L,t-1}^{g*} + g^* \exp\{\varepsilon_t^{g*}\} \quad (\text{A.64})$$

Total government debt

$$b_{H,t}^g + P_{L,t} b_{H,L,t}^g = b_H^g + P_L b_{H,L}^g \quad (\text{A.65})$$

$$b_{F,t}^{g*} + P_{L,t}^* b_{F,L,t}^{g*} = b_F^{g*} + P_L^* b_{F,L}^{g*} \quad (\text{A.66})$$

Composition of government debt

$$P_{L,t} b_{H,L,t}^g = P_L b_{H,L}^g \quad (\text{A.67})$$

$$\frac{P_{L,t}^* b_{F,L,t}^{g*}}{P_L^* b_{F,L}^{g*}} = \left(\frac{P_{L,t-1}^* b_{F,L,t-1}^{g*}}{P_L^* b_{F,L}^{g*}}\right)^{\gamma_L^*} \exp\{\varepsilon_t^{L^*}\} \quad (\text{A.68})$$

A.4 Aggregation and market clearing

Aggregate labor

$$n_t = \omega_r n_t^r + (1 - \omega_r) n_t^u \quad (\text{A.69})$$

$$n_t^* = \omega_r^* n_t^{r^*} + (1 - \omega_r^*) n_t^{u^*} \quad (\text{A.70})$$

Goods market clearing

$$\tilde{y}_t = \omega_r c_t^r + (1 - \omega_r) c_t^u + g_t \quad (\text{A.71})$$

$$\tilde{y}_t^* = \omega_r^* c_t^{r^*} + (1 - \omega_r^*) c_t^{u^*} + g_t^* \quad (\text{A.72})$$

Aggregate production function

$$y_t = \exp\{\varepsilon_t^z\} n_t - \phi_t \quad (\text{A.73})$$

$$y_t^* = \exp\{\varepsilon_t^{z^*}\} n_t^* - \phi_t^* \quad (\text{A.74})$$

Aggregate output

$$y_t = y_{H,t} \Delta_{H,t} + \frac{1 - \omega}{\omega} y_{H,t}^* \Delta_{H,t}^* \quad (\text{A.75})$$

$$y_t^* = \frac{\omega}{1-\omega} y_{F,t} \Delta_{F,t} + y_{F,t}^* \Delta_{F,t}^* \quad (\text{A.76})$$

Aggregate taxes

$$t_t = \omega_r t_t^r + (1 - \omega_r) t_t^u \quad (\text{A.77})$$

$$t_t^* = \omega_r^* t_t^{r*} + (1 - \omega_r^*) t_t^{u*} \quad (\text{A.78})$$

Price dispersion

$$\Delta_{H,t} = \left(\frac{p_{H,t}}{p_{H,t-1}} \right)^{\frac{\mu}{\mu-1}} \theta_H \Delta_{H,t-1} \left(\frac{\pi}{\pi_t} \right)^{\frac{\mu}{1-\mu}} + (1 - \theta_H) \left(\frac{\tilde{p}_{H,t}}{p_{H,t}} \right)^{\frac{\mu}{1-\mu}} \quad (\text{A.79})$$

$$\Delta_{H,t}^* = \left(\frac{p_{H,t}^*}{p_{H,t-1}^*} \right)^{\frac{\mu}{\mu-1}} \theta_H^* \Delta_{H,t-1}^* \left(\frac{\pi^*}{\pi_t^*} \right)^{\frac{\mu}{1-\mu}} + (1 - \theta_H^*) \left(\frac{\tilde{p}_{H,t}^*}{p_{H,t}^*} \right)^{\frac{\mu}{1-\mu}} \quad (\text{A.80})$$

$$\Delta_{F,t} = \left(\frac{p_{F,t}}{p_{F,t-1}} \right)^{\frac{\mu^*}{\mu^*-1}} \theta_F \Delta_{F,t-1} \left(\frac{\pi}{\pi_t} \right)^{\frac{\mu^*}{1-\mu^*}} + (1 - \theta_F) \left(\frac{\tilde{p}_{F,t}}{p_{F,t}} \right)^{\frac{\mu^*}{1-\mu^*}} \quad (\text{A.81})$$

$$\Delta_{F,t}^* = \left(\frac{p_{F,t}^*}{p_{F,t-1}^*} \right)^{\frac{\mu^*}{\mu^*-1}} \theta_F^* \Delta_{F,t-1}^* \left(\frac{\pi^*}{\pi_t^*} \right)^{\frac{\mu^*}{1-\mu^*}} + (1 - \theta_F^*) \left(\frac{\tilde{p}_{F,t}^*}{p_{F,t}^*} \right)^{\frac{\mu^*}{1-\mu^*}} \quad (\text{A.82})$$

Short-term bond market clearing

$$(1 - \omega_r) b_{H,t}^u = b_{H,t}^g \quad (\text{A.83})$$

$$(1 - \omega_r^*) b_{F,t}^{u*} = b_{F,t}^{g*} \quad (\text{A.84})$$

Long-term bond market clearing

$$\omega(1 - \omega_r) b_{H,L,t}^u + (1 - \omega)(1 - \omega_r^*) b_{H,L,t}^{u*} + \omega \omega_r b_{H,L,t}^r + (1 - \omega) \omega_r^* b_{H,L,t}^{r*} = \omega b_{H,L,t}^g \quad (\text{A.85})$$

$$\omega(1 - \omega_r) b_{F,L,t}^u + (1 - \omega)(1 - \omega_r^*) b_{F,L,t}^{u*} + (1 - \omega) \omega_r^* b_{F,L,t}^{r*} = (1 - \omega) b_{F,L,t}^{g*} \quad (\text{A.86})$$

Net foreign assets to output ratio

$$a_t = \frac{(1 - \omega_r)(b_{H,t}^u + s_t b_{F,t}^u + P_{L,t} b_{H,L,t}^u + s_t P_{L,t}^* b_{F,L,t}^u) + \omega_r P_{L,t} b_{H,L,t}^r - b_{H,t}^g - P_{L,t} b_{H,L,t}^g}{y_t} \quad (\text{A.87})$$