Ambiguity, Monetary Policy, and Trend Inflation

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This paper: Knightian uncertainty about monetary policy

- **Q:** What happens if market participants are extremely uncertain about monetary policy?
  - This paper: effects of **Knightian uncertainty**, also called **ambiguity**
  - That is, market participants **cannot attribute probabilities** to monetary policy stance

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    - May consider **multiple priors** about monetary policy stance
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  - That is, market participants **cannot attribute probabilities** to monetary policy stance
    - May consider **multiple priors** about monetary policy stance
    - **Ambiguity aversion**: they fear that true prior is the worst one
- **Q:** What are the implications for **trend inflation** if market participants are averse to ambiguity about monetary policy?
- **Q:** What is **optimal monetary policy** if market participants are averse to ambiguity about monetary policy?

Motivation: Persistent changes in inflation

1. Large regime changes in monetary policy
   - Depression → Keynesianism → Stagflation → Disinflation → Moderation → Financial Crisis
2. Low frequency component in inflation variation
   - Would be challenging to forecast effects of policy changes even if *macroeconomic theory* were unchanging
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   - Would be challenging to forecast effects of policy changes even if *macroeconomic theory* were unchanging
   - But theoretical consensus changes too!
3. Importance of transparency in monetary policy communication
4. Concerns about de-anchoring of inflation expectations
5. Empirical indicators of time-varying market uncertainty
6. Persistent failures to hit inflation target

Model: Monetary policy

- **Monetary policy rule:**

$$R_t \;=\; R_t^n e^{\epsilon_t} \left(\Pi_t\right)^{\phi}$$

  - R_t^n is natural rate of interest
  - ϕ is reaction of nominal interest rate to inflation
  - ϵ_t is monetary policy shock:

$$\epsilon_t \;=\; \rho^{\epsilon}\epsilon_{t-1} + u_t^{\epsilon} + \mu_t^*$$

- Stochastic component: $u_t^{\epsilon} \sim N(0, \sigma_u)$
- **Deterministic component:** μ_t^*
  - Sample moments of μ_t^* converge as $T \rightarrow \infty$ to $N(0, \sigma^{\mu})$
- **What does that mean?**

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- Stochastic component: $u_t^\epsilon \sim N(0, \sigma_u)$
  - Example: quantum mechanics
- **Deterministic component:** μ_t^*
  - Sample moments of μ_t^* converge as $T \to \infty$ to $N(0, \sigma^\mu)$
  - Example: normrnd.m in MATLAB
- **Why?**

Model: Monetary policy – allowing for ambiguity

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$$\epsilon_t \;=\; \rho^{\epsilon}\epsilon_{t-1} + u_t^{\epsilon} + \mu_t^{*}$$

- Agents have prior about first component: $u_t^{\epsilon} \sim N(0, \sigma_u)$
- **Agents have ambiguous views about** μ_t^{*}
  - They believe $E\mu_t^{*} \in [\underline{\mu}, \overline{\mu}]$
  - But they don't assign probabilities to these alternatives

Model: Ambiguity aversion

- **Recursively-defined utility function** implies **ambiguity aversion** (Gilboa-Schmeidler 1989)
  - Let $\mathcal{P}_t(s^t)$ be *set of possible priors* over next state s_{t+1}
  - Let $\vec{C} \equiv \{c_t(s^t), n_t(s^t)\}_{t=0}^T$ be a *contingent plan*
  - Define the utility function recursively as follows:

$$U_t(\vec{C}; s^t) \equiv \min_{p \in \mathcal{P}_t(s^t)} \log\left(c_t(s^t)\right) - \frac{n_t(s^t)^{1+\psi}}{1+\psi} + \beta E^p U_{t+1}(\vec{C}; s^{t+1})$$

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- By maximizing $U_0(\vec{C}; s^0)$, agents are choosing a **robust plan** $\vec{C}$ that performs well even under the **worst scenario** that they contemplate

Rest of model: Plain vanilla NK-DSGE

- Monetary policy rule
  - **Ambiguous component**
- Utility function
  - **Ambiguity aversion / robustness incentive**
- Household decisions: Consume / save / labor supply
- Firms: Monopolistic competitors / set sticky prices / labor demand

Solving the model

1. Guess *which state* $\mu_t \in \left[\underline{\mu}, \overline{\mu}\right]$ is the *worst one*
2. Can then **evaluate expectations** under **pessimistic beliefs** μ_t
   - For example:

$$\frac{1}{C_t} = E^{\mu_t}\left(\frac{\beta R_t}{C_{t+1}\Pi_{t+1}}\right) = E\left(\frac{\beta R_t^n \exp(\rho^\epsilon \epsilon_{t-1} + \mu_t)\Pi_t^\phi}{C_{t+1}\Pi_{t+1}}\right)$$

   - E^{μ_t} represents *pessimistic beliefs*
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3. Find *steady state* under *pessimistic beliefs*

4. *Linearize dynamics*, under pessimistic beliefs, around pessimistic steady state

Results: Steady state inflation

- Solving Euler equation:

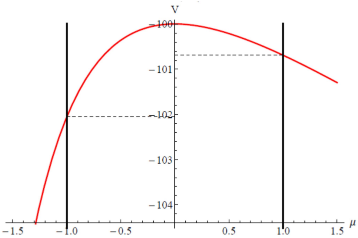
$$\frac{1}{C_t} = E^{\mu_t}\left(\frac{\beta R_t}{C_{t+1}\Pi_{t+1}}\right) = E\left(\frac{\beta R_t^n \exp(\rho^\epsilon \epsilon_{t-1} + \mu_t)\Pi_t^\phi}{C_{t+1}\Pi_{t+1}}\right)$$

- Steady state under pessimistic beliefs:

$$1 = \beta R^n \exp(\mu)\Pi^{\phi-1} \quad \Pi = \exp^{\frac{-\mu}{\phi-1}}$$

Results: persistent deviations from target

- max V is at $\mu = 0$
- $V'' < 0$, $V''' > 0$
- Therefore **worst** μ is at a **corner.**



- **Symmetric case**: $\mu_t^* \in [-\overline{\mu}, \overline{\mu}]$.
  - Worst case is **excessive expansion**: $\mu = -\overline{\mu}$.
  - Steady state **inflation above target**: $\Pi > 1$.
- **Asymmetric case**: $|\underline{\mu}| << \overline{\mu}$.
  - Worst case is **excessive contraction**: $\mu = \overline{\mu}$.
  - Steady state **deflation**: $\Pi < 1$.

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- **Reduced ambiguity** $\rightarrow$ corners closer to zero $\rightarrow$ Π **closer to target**
- **More hawkish rule** $\rightarrow$ ϕ larger $\rightarrow$ Π **closer to target**

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- **Reduced ambiguity** $\rightarrow$ corners closer to zero $\rightarrow$ Π **closer to target**
  - Transparency is good!
- **More hawkish rule** $\rightarrow$ ϕ larger $\rightarrow$ Π **closer to target**
  - Conservatism is good!

Comments (1)

- **Well-motivated question**. Ambiguity about monetary policy is relevant.

- Closely following Ilut-Schneider (2014): Ambiguity about TFP.
  - Highly tractable because *worst case* is always *low TFP*.
  - Dynamic, quantitative linearized model of risk shocks.

- Here, **worst case varies**: may be excessive expansion or contraction.
  - Potentially *more interesting* but much *less tractable* than Ilut-Schneider.
  - Therefore studied **steady state trend only**.

- Would be extremely interesting to study entry/exit from ZLB, as "pessimism" switches from excessive inflation to excessive deflation (and back). But seems beyond the scope of the methods used here.

Comments (2)

- **Clear, qualitatively robust** findings about long-run.
  - **Persistent deviations** of inflation from target.
  - **Greater transparency** reduces deviations from target.
  - More **hawkish rule** reduces deviations from target.

- Unfortunately, quantitative results are entirely driven by **arbitrary corners**.
  - Predicted inflation trend: $\Pi = e^{-\mu/(\phi-1)}$.
    - Depends on "worst possible" shock inflation μ.
  - Predicted inflation highly sensitive to rule coefficient ϕ
  - Optimal monetary policy: set $\phi = \bar{\phi}$.
    - "Highest possible" inflation reaction coefficient $\bar{\phi}$.

- Corners also play a big role in the dynamics.
  - Sudden switches from focus only on excessive expansion to excessive contraction... plausible?
  - These jumps are also likely to make computing dynamics harder...

An alternative: Ambiguity aversion with less extreme uncertainty

- **Recursively-defined utility function** with **entropy constraint** (Hansen-Sargent book, 2008)

$$U_t(\vec{C}; s^t) \equiv \min_{p \in \mathcal{P}_t(s^t)} \log\left(c_t(s^t)\right) - \frac{n_t(s^t)^{1+\psi}}{1+\psi} + \beta E^p U_{t+1}(\vec{C}; s^{t+1})$$

$$s.t.\ \mathcal{D}(p || \mathcal{Q}_t) \leq \theta^{-1}$$

- Implies p cannot be too far from the "benchmark" distribution $\mathcal{Q}_t$.

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- Resulting distribution of $s_{t+1} \equiv (\underline{\mu}_t, \overline{\mu}_t)$ is regarded as a logit:

$$p_t(s) \;=\; \frac{\mathcal{Q}_t(s)\exp(-U(s)/\theta)}{\int \mathcal{Q}_t(s')\exp(-U_{t+1}(s')/\theta)ds'}$$

- Value function $E^p U$ is Hansen-Sargent "risk-sensitive expectation"

$$E^p U_{t+1}(\vec{C}; s) \;=\; \theta \log\left(\int \mathcal{Q}_t(s')\exp\left(\frac{U_{t+1}(s')}{\theta}\right)\right) ds'$$

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- Value function $E^p U$ is Hansen-Sargent "risk-sensitive expectation"...
- **Smoother, more tractable model** for calculating dynamics, less sensitive to assumed corners?

THANKS FOR YOUR ATTENTION!

Minor comments

- μ is confusing notation for the *contractionary* money shock, since μ often represents money growth
- Having notation (μ^e?) for the pessimistic expectation would be clearer
- Prop. 4.1 is hard to read because definition of δ^* is not immediately obvious
- You can simplify equation (3) by moving E^p after the β
- I'm confused about why (8) is valid. Doesn't Π_t depend on u_t^e? If so, how did you you cancel out u_t^e by taking expectations?