Optimal Monetary Policy with Heterogeneous Agents

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Outline

Motivation

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- Little progress on the **normative** front: the entire wealth distribution is a state in the policy-maker's problem

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  - ... that relies on **infinite-dimensional calculus**

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  - in continuous time

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  - **Anticipated** inflation **raises the nominal costs of new debt** issuances
  - **Nonzero** inflation also entails a **direct welfare loss**

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- Discretion implies **first-order welfare losses** relative to commitment for creditors *and* debtors

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Output, prices and net assets

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$$Q_t A_{kt}^{new} = P_t \left(y_{kt} - c_{kt}\right) + \delta A_{kt}$$

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- Borrowing constraint

$$a_{kt} \geq \phi, \quad \phi \leq 0.$$

Households

- Separable preferences
- Individual value function

$$v(t,a,y) = \max_{\{c_s\}_{s\in[t,\infty)}} \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \left[u(c_{ks}) - x(\pi_s)\right] ds$$

International investors (bond pricing)

- Risk-neutral investors can invest elsewhere at riskless real rate $\bar{r}$
- Unit price of the **nominal non-contingent bond**

$$Q(t) = \int_t^\infty \delta e^{-(\bar{r}+\delta)(s-t)-\int_t^s \pi_u du} ds$$

Dynamics of the income-wealth density

- *Kolmogorov Forward* (KF) equation

$$\frac{\partial f_i(t,a)}{\partial t} = -\frac{\partial}{\partial a}\left[s_i\left(t,a\right) f_i(t,a)\right] - \lambda_i f_i(t,a) + \lambda_j f_j(t,a),$$

$i, j = 1, 2,\ j \neq i$, where

$$s_i\left(t,a\right) \equiv \left(r_t - \pi_t\right) a + \frac{y_i - c_i\left(t,a\right)}{Q_t}$$

is the *drift function*

Assumption

- The value of parameters is such that the country is a **net debtor** against the rest of the World

$$\int_{\phi}^{\infty} a\left(f_{1t}\left(a\right)+f_{2t}\left(a\right)\right)da \leq 0$$

- For tractability: it avoids introducing a foreign bond

Outline

Central bank

- The central bank chooses the **inflation rate** $\pi(t)$
- Central bank's utilitarian welfare criterion

$$U_0^{CB} \equiv \int_{\phi}^{\infty} \sum_{i=1}^{2} v_i(0,a) f_i(0,a) da$$

Commitment

Statement of the problem

- The **Ramsey problem** is

$$J^R\left[f\left(0,\cdot\right)\right]=\max_{\left\{\pi_s,Q_s,f\left(s,\cdot\right),v\left(s,\cdot\right),c\left(s,\cdot\right)\right\}_{s\in[0,\infty)}}U_0^{CB}$$

  subject to the **law of motion** of the distribution, the **bond pricing** equation, the individual **HJB** equation and the **first-order condition**

- J^R and π are not ordinary functions, but **functionals** as they map a distribution $f\left(t,\cdot\right)$ into $\mathbb{R}$

- The problem is **time-inconsistent**

Methodology: commitment

- In the case of commitment we construct a functional **Lagragian**
- This is a problem of **constrained optimization** in an infinite-dimensional Hilbert space $\longrightarrow$ **Gateaux derivative**

Gateaux derivative

Definition (Gateaux derivative)

Let $J\left[f\right]$ be a functional and let h be arbitrary in $L^2\left(\Omega\right)$, where $\Omega \subset \mathbb{R}^n$. If the limit

$$\delta J\left[f;h\right] = \lim_{\alpha \to 0} \frac{J\left[f + \alpha h\right] - J\left[f\right]}{\alpha} = \frac{d}{d\alpha} J\left[f + \alpha h\right]\Big|_{\alpha=0} \qquad (1)$$

exists, it is called the *Gateaux derivative* of J at f with increment h.

- *Intuition*: a perturbation $h(\cdot)$, as in Calculus of Variations

Optimal inflation under commitment

Optimal inflation at time t satisfies

$$x'\left(\pi\left(t\right)\right)=\underbrace{\sum_{i=1}^{2}\int_{\phi}^{\infty}\frac{\partial v_i}{\partial a}\left(-a\right)f_i\left(t,a\right)da}_{\textbf{redistributive motive}}+\underbrace{\mu(t)Q(t)}_{\textbf{(commitment)}},$$

where $\mu\left(t\right)$ is a costate with law of motion

$$\frac{d\mu\left(t\right)}{dt}=\left(\rho-\bar{r}-\pi(t)-\delta\right)\mu\left(t\right)+\sum_{i=1}^{2}\int_{\phi}^{\infty}\frac{\partial v_{it}}{\partial a}\frac{\delta a+y_i-c_i\left(t,a\right)}{Q\left(t\right)^2}f_i\left(t,a\right)da$$

and initial condition

$$\mu\left(0\right)=0\ \textbf{(no precommitments)}.$$

Proposition (Optimal long-run inflation under commitment)

In the limit as $\rho \longrightarrow \bar{r}$, the **steady-state inflation is zero***:*

$$\lim_{\rho \longrightarrow \bar{r}} \pi\left(\infty\right) = 0$$

Discretion

- A **Markov Perfect Stackelberg Equilibrium** is defined as the limit as $\Delta t \to 0$ of a sequence of problems in which the central bank chooses policy with commitment in each interval $(t, t + \Delta t]$ but not across intervals
- The problem is **time-consistent**

Discretion

Statement of the problem

- The value functional of the central bank at time t is given by

$$J^M\left[f_t\left(\cdot\right)\right] = \lim_{\Delta t \to 0} J^M_{\Delta t}\left[f_t\left(\cdot\right)\right],$$

where

$$\begin{aligned} J^M_{\Delta t}\left[f_t\left(\cdot\right)\right] &= \max_{\{\pi_s, Q_s, v(s,\cdot), c(s,\cdot), f(s,\cdot)\}_{s\in(t,t+\Delta t]}} \\ & \int_t^{t+\Delta t} e^{-\rho(s-t)} \left[\int_\phi^\infty \sum_{i=1}^{2} u\left(c_{is}\left(a\right), \pi_s\right) f_i(s,a) da\right] ds \\ & + e^{-\rho\Delta t} J^M_{\Delta t}\left[f_{t+\Delta t}\left(\cdot\right)\right], \end{aligned}$$

subject to the **law of motion** of the distribution, the **bond pricing** equation, the individual **HJB** equation and the **first-order condition**

Methodology: discretion

We proceed in two steps:

1. First we solve the commitment problem over $(t, t + \Delta t]$ using infinite-dimensional calculus
2. Then we take the limit as $\Delta t \to 0$

Optimal inflation under discretion

- Optimal inflation at time t satisfies

$$x'(\pi(t)) = \sum_{i=1}^{2} \int_{\phi}^{\infty} \frac{\partial v_i}{\partial a} (-a) f_i(t,a)\, da > 0.$$

- As long as $x'(\pi) > 0$ only for $\pi > 0$, we have

$$\pi(t) > 0 \textbf{ for all } t \quad \text{(inflationary bias)}$$

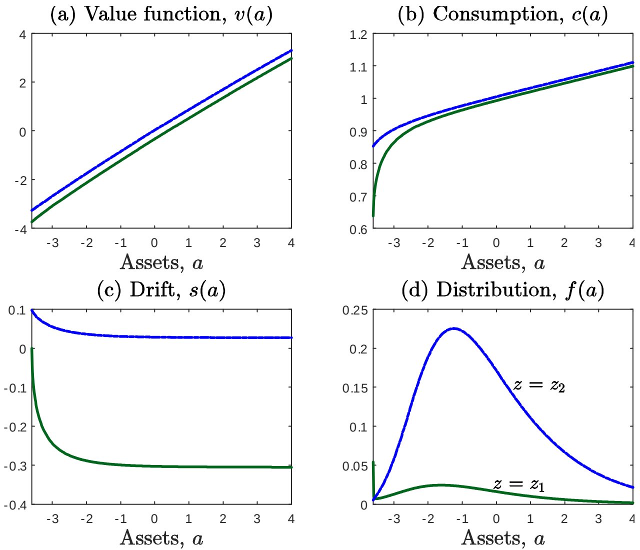
Outline

Calibration

- Calibrate to a prototypical European small open economy, time unit = 1 year
- $u(c) = \log(c)$, $x(\pi) = \frac{\psi}{2}\pi^2$ (Rotemberg pricing)

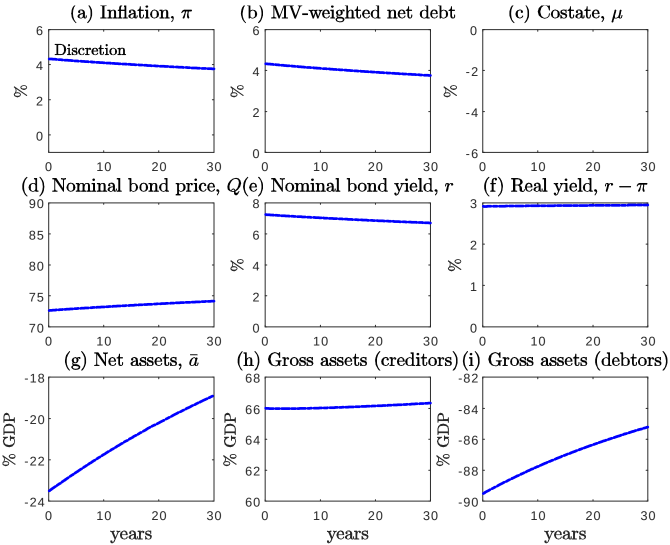
	Value	Description	Source/Target
$\bar{r}$	0.03	world real interest rate	standard
ψ	5.5	scale inflation disutility	slope NKPC in Calvo model
δ	0.19	bond amortization rate	Macaulay duration = 4.5 yrs
λ_1	0.72	transition rate U to E	monthly job finding rate 0.1%
λ_2	0.08	transition rate E to U	unemployment rate 10%
y_1	0.73	income in U state	Hall & Milgrom (2008)
y_2	1.03	income in E state	$E(y) = 1$
ρ	0.0302	subjective discount rate	{ NIIP/GDP (-25%)
ϕ	-3.6	borrowing limit	HH debt/GDP (90%)

Steady-state (zero inflation)

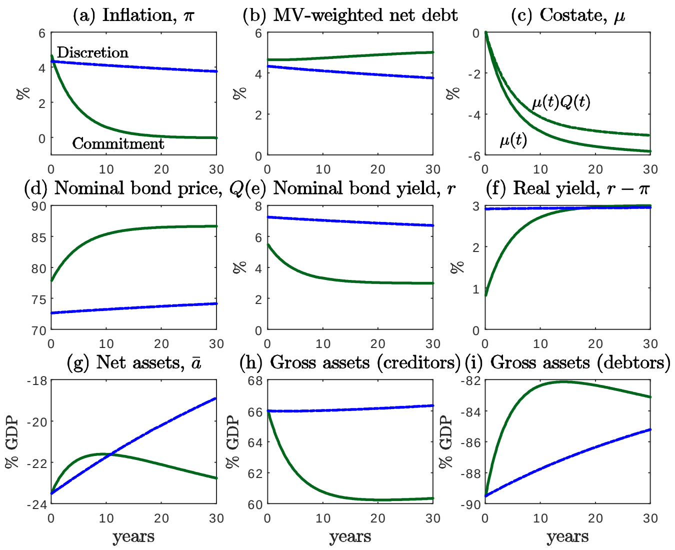


- We will use the wealth distribution in the zero-π steady state as initial condition

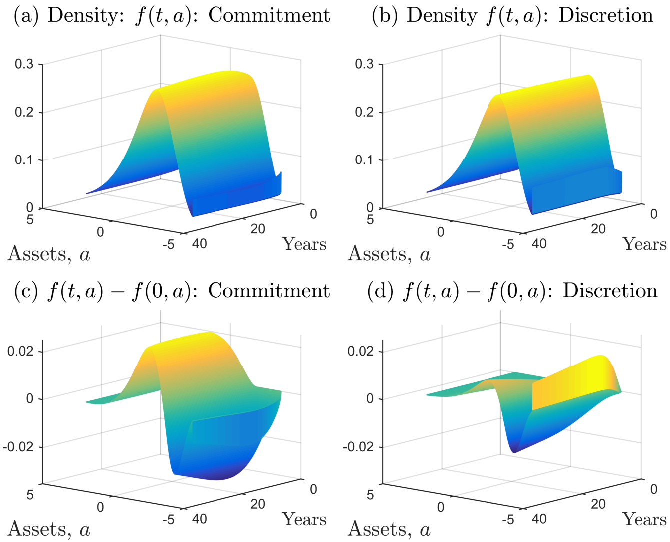
Optimal transitional dynamics: discretion



Optimal transitional dynamics: commitment



Density dynamics



Welfare analysis

Welfare losses relative to the optimal commitment

	Economy-wide	Creditors	Debtors
Discretion	0.31	0.23	0.08
Zero inflation	0.05	-0.17	0.22

Note: welfare losses are expressed as a % of permanent consumption

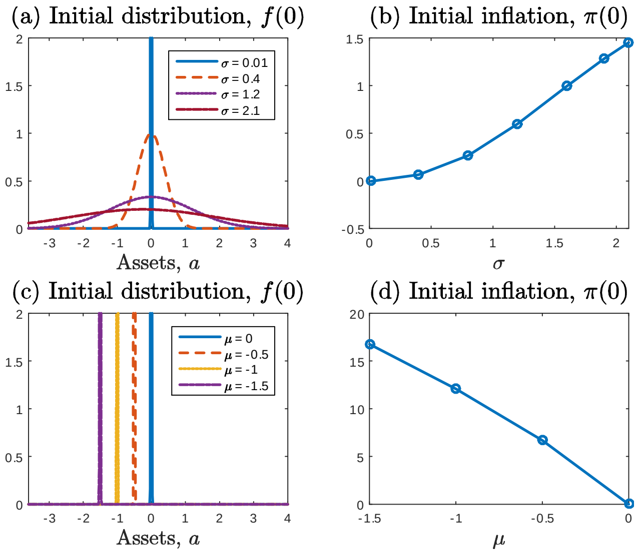
Robustness: initial distribution and initial inflation

- Initial net wealth distribution is a key determinant of optimal policy
- We have assumed so far that $f(0, a)$ is the one corresponding to the zero inflation steady state
- **How does $\pi(0)$ change with initial distribution?**
- To make the analysis operational, assume Normal distributions truncated at borrowing limit:

$$f(0, a) = \begin{cases} \phi(a; \mu, \sigma) / [1 - \Phi(\phi; \mu, \sigma)], & a \geq \phi \\ 0, & a < \phi \end{cases}$$

  ϕ, Φ: Normal pdf and cdf
- Sensitivity wrt σ and μ
  - When varying σ, adjust μ such that $\sum_{i=1}^{2} \int_{\phi}^{\infty} a f_i(0, a)\, da = 0 \rightarrow$ isolate **domestic redistribution** channel

Robustness: initial distribution and initial inflation



Aggregate shocks

- Consider a one-time, unanticipated, temporary increase in the World real interest rate, after which

$$d\bar{r}_t = \eta_r \left(\bar{r} - \bar{r}_t\right) dt,$$

- Up to a first order approximation $\Leftrightarrow$ model with stochastic process $d\bar{r}_t = \eta_r \left(\bar{r} - \bar{r}_t\right) dt + \sigma dZ_t$
  - The IRFs coincide with the solution by first-order perturbation around the det. steady state, as in Ahn, Kaplan, Moll and Winberry (2017)
- We focus on the *commitment* case
  - Initial condition: steady state of the Ramsey problem $\rightarrow$ optimal responses from a *timeless perspective*

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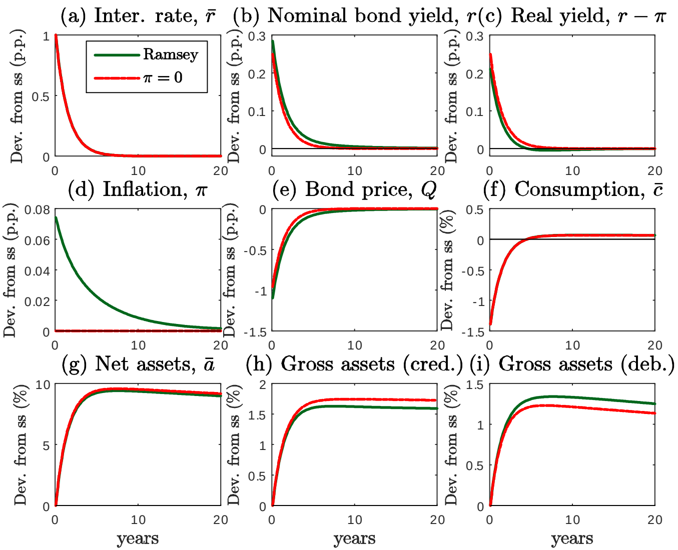

Outline

Conclusions

- Analyze **optimal monetary policy** in an economy with nontrivial household heterogeneity (uninsurable idiosyncratic risk)
  - **Methodological contribution**: Novel methodology based on **infinite-dimensional calculus**
- Under discretion there is an **inflationary bias** for **redistributive motives**
  - Intuition: the central bank gives more weight to debtors as they have larger marginal consumption utilities
- Under commitment there is **inflation front-loading**
  - Avoid inflation expectations from being priced into bond issuances
  - Long-run inflation rate is zero under certain conditions

Outline

Competitive equilibrium

Given $\{\pi(t)\}_{s\in[0,\infty)}$ and the initial density $f(0,a,y)$, a competitive equilibrium is $v(t,a,y)$, $c(t,a,y)$, $Q(t)$ and $f(t,a,y)$ such that:

1. Given π, the price of bonds set by investors is Q
2. Given Q and π, v is the solution of the households' problem and c is the optimal consumption policy
3. Given Q, π, and c, f is the solution of the KF equation

Why continuous time?

- **Numerical advantages**
  - The discretized HJB

$$\left[\left(\frac{1}{\Delta t}+\rho\right)\mathbf{I}-\mathbf{A}_n\right]\mathbf{V}^n=\mathbf{u}^n\Delta t+\mathbf{V}^{n+1},$$

    where $\left[\left(\frac{1}{\Delta t}+\rho\right)\mathbf{I}-\mathbf{A}_n\right]$ is a sparse (tridiagonal matrix) $\rightarrow$ **efficient solution**

  - Analytical mapping from the value function to the optimal policy function $\rightarrow$ **no numerical maximization**

$$c_i\left(t,a\right)=\left[\frac{1}{Q\left(t\right)}\frac{\partial v_i(t,a)}{\partial a}\right]^{-1/\gamma}$$

  - Trivial solution of the **KF equation**

$$\left(\mathbf{I}-\Delta t\mathbf{A}_n^{\mathbf{T}}\right)\mathbf{f}^{n+1}=\mathbf{f}^n.$$

Steady state results

	units	Ramsey	MPE
Inflation, π	%	-0.05	1.68
Nominal yield, r	%	2.95	4.68
Net assets, $\bar{a}$	% GDP	-24.1	-0.6
Gross assets (creditors)	% GDP	65.6	80.0
Gross debt (debtors), $\bar{b}$	% GDP	89.8	80.6
Current acc. deficit, $\bar{c} - \bar{y}$	% GDP	-0.63	-0.01

Robustness: interest rate gap