

Monetary Policy, Bounded Rationality and Incomplete Markets

Emmanuel Farhi, Harvard

Iván Werning, MIT

Motivation

- How is monetary policy affected by
 - Bounded Rationality?
 - Incomplete Markets?
 - Combination?

Paper: complementarities!

Motivation

- Helps fix “bugs” of standard NK model
 - indeterminacy given interest rate paths (Taylor principle)
 - Neo-Fisherian controversies
 - effectiveness of monetary policy
 - dependence on horizon (“forward guidance puzzle”)
 - effects of fiscal policy at ZLB (“fiscal multipliers puzzle”)
 - explosive nature of long-lasting liquidity traps
 - ...

Bounded Rationality

- Expectations management major (main) channel of policy transmission in NK model under RE
- Realistic?
 - incomplete information or inattention to policy announcement?
 - less than full understanding of its future effects?

Bounded Rationality

- “Inductive”

- learning: extrapolate from past data rationally or irrationally (Sargent; Evans; Honkapohja; Shleifer)
- incomplete info and inattention: ignore, underweight, cost to process info (Sims; Mankiw-Reis; Maćkowiak-Wiederholt; Gabaix; Angeletos-Lian)

- “Eductive”

- robustness (Hansen-Sargent)
- **level-k thinking**: think through reaction of others (Stahl-Wilson; Nagel; Crawford-Costa-Gomes-Iriberri; Evans-Ramey; Woodford; García-Schmidt-Woodford)

- **Level-k thinking**

- credible and clear announcement policy change
- with little past experience
- agents think through consequences, with bounded rationality

Incomplete Markets

- Standard NK model: representative agent or complete markets
- Incomplete markets alternative (Bewley-Huggett-Aiyagari)
 - lack of insurance to idiosyncratic shocks
 - borrowing constraints
- Key for effects and channels of monetary policy
 - high Marginal Propensity to Consume (MPC)
 - low intertemporal substitution
- Large and active area in macro (Guerrieri-Lorenzoni, Farhi-Werning, Chamley, Beaudry-Galizia-Portier, Ravn-Sterk, Sheedy, McKay-Nakamura-Steinsson, Auclert, Werning, Kaplan-Moll-Violante etc.)

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark →	?
Bounded Rationality	↓ ?	???

Outline

- General concept of level-k
- Representative agent with level-k
- Incomplete markets without level-k
- Incomplete markets with level-k
- Start: rigid prices or effects of real interest rates
- End: sticky prices and inflation

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Comparative static $\{R_{t+s}\} \longrightarrow \{\hat{R}_{t+s}\}$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Comparative static $\{R_{t+s}\} \longrightarrow \{\hat{R}_{t+s}\}$

$$\hat{C}_t - C_t = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\})$$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Comparative static $\{R_{t+s}\} \longrightarrow \{\hat{R}_{t+s}\}$

$$\begin{aligned}\hat{C}_t - C_t &= C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \\ &= C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \\ &\quad + C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\})\end{aligned}$$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Comparative static $\{R_{t+s}\} \longrightarrow \{\hat{R}_{t+s}\}$

$$\begin{aligned} \hat{C}_t - C_t &= C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \\ &= C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \\ &\quad \text{PE} \\ &\quad + C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \end{aligned}$$

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\})$$

$$C_t = Y_t$$

R.E. Equilibria. Solution for $\{C_t, Y_t\}$ with
 $Y_{t+s}^e = Y_{t+s}$

Comparative static $\{R_{t+s}\} \longrightarrow \{\hat{R}_{t+s}\}$

$$\begin{aligned} \hat{C}_t - C_t &= C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \\ &= \underbrace{C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\})}_{\text{PE}} \\ &\quad + \underbrace{C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\})}_{\text{GE}} \end{aligned}$$

Level-k Thinking

Level-1 thinking: $\hat{C}_t^1 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\})$

$$\hat{C}_t^1 = \hat{Y}_t^1$$



status quo REE

Level-k Thinking

Level-1 thinking: $\hat{C}_t^1 = C^* (\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\})$

$$\hat{C}_t^1 = \hat{Y}_t^1$$




status quo REE

(almost PE effect! continuous time...)

Level-k Thinking

Level-1 thinking: $\hat{C}_t^1 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\})$


$$\hat{C}_t^1 = \hat{Y}_t^1$$

 status quo REE

(almost PE effect! continuous time...)

Level-2 thinking: $\hat{C}_t^2 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^2, \{\hat{Y}_{t+1+s}^1\})$


$$\hat{C}_t^2 = \hat{Y}_t^2$$

 level-1 thinking

Level-k Thinking

Level-1 thinking: $\hat{C}_t^1 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\})$


$$\hat{C}_t^1 = \hat{Y}_t^1$$

 status quo REE

(almost PE effect! continuous time...)

Level-2 thinking: $\hat{C}_t^2 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^2, \{\hat{Y}_{t+1+s}^1\})$

$$\hat{C}_t^2 = \hat{Y}_t^2$$

 level-1 thinking

Level-k thinking: $\{\hat{Y}_t^{k+1}\} = \Gamma(\{\hat{Y}_t^k\})$

Note: REE is a fixed point!

Level- k Thinking

- Coincides with PE for $k = 1$
- Mitigates GE, less and less as k increases
- Converges to RE as $k \rightarrow \infty$
- Determinate for any k , without Taylor rule
- Can generalize to aggregate consumption functions depending on state variable Ψ for incomplete markets (wealth distribution)

Effects of Monetary Policy

- Elasticities of output to interest rates
 - at different horizons
 - PE, GE, level-k

$$\epsilon_{t,\tau} = \lim_{\Delta R_\tau \rightarrow 0} -\frac{R_\tau}{Y_t} \frac{\Delta Y_t}{\Delta R_\tau}$$

$$\epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^k = \lim_{\Delta R_\tau \rightarrow 0} -\frac{R_\tau}{Y_t} \frac{\Delta Y_t^k}{\Delta R_\tau}$$

$$\epsilon_{t,\tau}^k = \epsilon_{t,\tau}^{k,PE} + \epsilon_{t,\tau}^{k,GE}$$

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark →	?
Bounded Rationality	?	???

Representative Agent

- Representative agent (= complete markets)
- Continuous time
 - not crucial, but...
 - ...partial equilibrium = level-1 thinking

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

$$\frac{\Delta \log C_t}{\Delta \log \alpha}$$

start at
steady state

t

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

$$\frac{\Delta \log C_t}{\Delta \log \alpha}$$

$$\epsilon_{t,\tau} = \sigma^{-1}$$

change interest
rate at τ

$$\hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases}$$

τ

t

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

$$\frac{\Delta \log C_t}{\Delta \log \alpha}$$

$$\epsilon_{t,\tau} = \sigma^{-1}$$

$$\epsilon_{t,\tau}^{PE} = \sigma^{-1} e^{-r(t-\tau)}$$

change interest
rate at τ

$$\hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases}$$

τ

t

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

$$\frac{\Delta \log C_t}{\Delta \log \alpha}$$

$$\epsilon_{t,\tau} = \sigma^{-1}$$

$$\epsilon_{t,\tau}^{PE} = \sigma^{-1} e^{-r(t-\tau)}$$

change interest
rate at τ

$$\hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases}$$

Bottom line: weak mitigation and horizon effects from level-k thinking.

τ

t

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark →	?
Bounded Rationality	?	???

Incomplete Markets

- See e.g. Werning (2015)
- Benchmark neutrality result: “as if” rep. agent
- Subtle dependence on cyclicality of
 - income risk
 - liquidity

Keynesian Cross

- Liquidity constrained cannot substitute, so...
- Q: How can incomplete markets not affect aggregate response?
- A: General Equilibrium vs. Partial Equilibrium
 - some do substitute and increase their spending...
 - ...increases income all around...
 - ...raises spending of liquidity constrained more...
 - ... increases income.... etc.

$$\downarrow PE + \uparrow GE = \text{constant}$$

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark	small effects
Bounded Rationality	small effects	???

Perpetual
Youth
+
Aiyagari
Simulations

Perpetual Youth Model

- Tractable model to easily visit all 4 squares!
- Continuum measure 1 of agents
- OLG with Poisson death and arrival $\pi \geq 0$

- Preferences

$$\int_0^\infty e^{-(\rho+\pi)s} \log(c_{t+s}^i) ds$$

- Income

- labor income: $(1 - \delta)Y_t$
- Lucas tree dividend: δY_t

- Budget with annuities

$$\frac{da_t^i}{dt} = (r_t + \pi)a_t^i + Y_t - c_t^i$$

Perpetual Youth Model

- Alternative interpretation
 - agents do not die
 - life separated by stochastic “periods”
 - heavy discount across periods:
 - wish to borrow against future periods
 - but cannot do so!
- OLG ~ borrowing constraints
 - short or interrupted time horizons
 - no precautionary savings
 - linear consumption function and aggregation

Perpetual Youth Model

$$V_t = \int_t^\infty e^{-\int_t^s r_u du} \delta Y_s^e ds$$

$$H_t = \int_t^\infty e^{-\int_t^s (r_u + \pi) du} (1 - \delta) Y_s^e ds$$

individual
consumption function $\longrightarrow c_t^i = (\rho + \pi)(a_t^i + H_t)$

$$\int_0^1 a_t^i di = V_t \quad \text{equilibrium} \quad \int_0^1 c_t^i di = Y_t$$

aggregate

$$C_t = (\rho + \pi)(V_t + H_t)$$

$$C_t = Y_t$$

consumption function \longleftarrow

Steady State

- Steady state

$$Y_t = Y$$

$$1 = (1 - \delta) \frac{\rho + \pi}{r + \pi} + \delta \frac{\rho + \pi}{r}$$

- Comparative static (“MIT shock”)
 - new path for interest rate
 - compute
 - rational expectations equilibrium
 - k-level thinking

Mitigation and Horizon

$$1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^{PE} = (1 - \delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau-t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau-t)}$$

Mitigation and Horizon

$$1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^{PE} = (1 - \delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau-t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau-t)}$$

Mitigation and Horizon

$$1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^{PE} = (1 - \delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau-t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau-t)}$$

$$\frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0$$

$$\frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} < 0$$

$$\frac{\partial^2 \epsilon_{t,\tau}}{\partial \pi \partial \tau} = 0$$

$$\frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} < 0$$

Result. Complementarity between incomplete markets and bounded rationality.

Speed of Convergence

- Recall, level-1 = PE, level- ∞ = RE
- Level-k

$$\epsilon_{t,\tau}^k = (1 - \delta)e^{-(\rho + \pi)(\tau - t)} \left[\sum_{\ell=1}^k \frac{(\rho + \pi)^{\ell-1} (\tau - t)^{\ell-1}}{(\ell - 1)!} \right] \\ + \delta e^{-\rho(\tau - t)} \left[\sum_{\ell=1}^k \frac{\rho^{\ell-1} (\tau - t)^{\ell-1}}{(\ell - 1)!} \right].$$

Speed of Convergence

- Recall, level-1 = PE, level- ∞ = RE
- Level-k

$$\epsilon_{t,\tau}^k = (1 - \delta)e^{-(\rho + \pi)(\tau - t)} \left[\sum_{\ell=1}^k \frac{(\rho + \pi)^{\ell-1} (\tau - t)^{\ell-1}}{(\ell - 1)!} \right] \\ + \delta e^{-\rho(\tau - t)} \left[\sum_{\ell=1}^k \frac{\rho^{\ell-1} (\tau - t)^{\ell-1}}{(\ell - 1)!} \right].$$

Complementarity: Asymptotic convergence to RE slower for higher π .

Bewley-Aiagari-Hugget

- Assumptions:

- idiosyncratic income uncertainty
- no insurance
- borrowing constraints

- Results:

- occasionally binding borrowing constraints
- precautionary savings
- concave consumption functions (varying MPC)

- Monetary policy and bounded rationality?

- general theoretical characterization
- numerical simulations

Bewley-Aiagari-Hugget

- Assumptions:

- idiosyncratic income uncertainty
- no insurance
- borrowing constraints

- Results:

- occasionally binding borrowing constraints
- precautionary savings
- concave consumption functions (varying MPC)

- Monetary policy and bounded rationality?

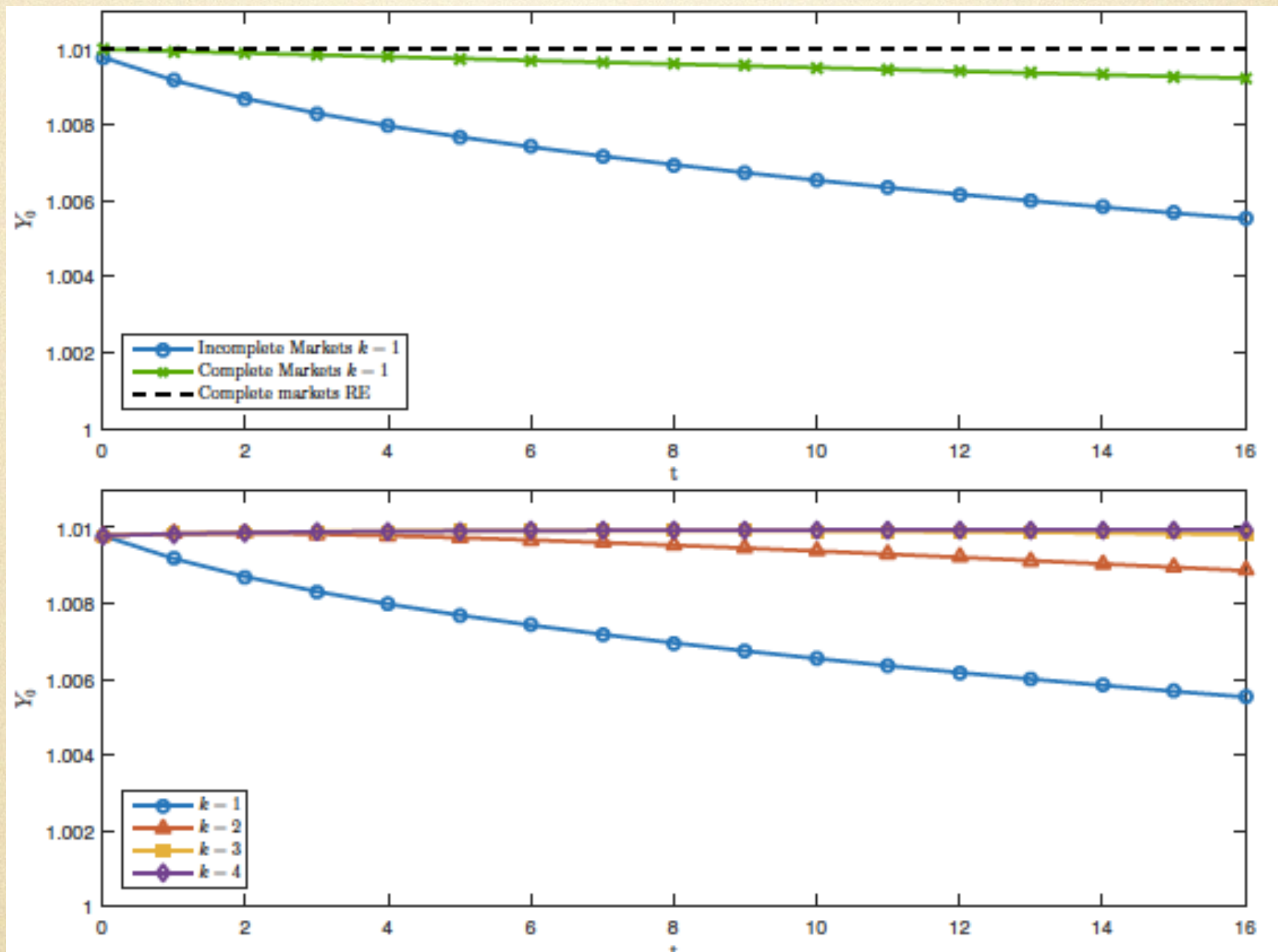
- general theoretical characterization

Result. Complementarity between incomplete markets and bounded rationality.

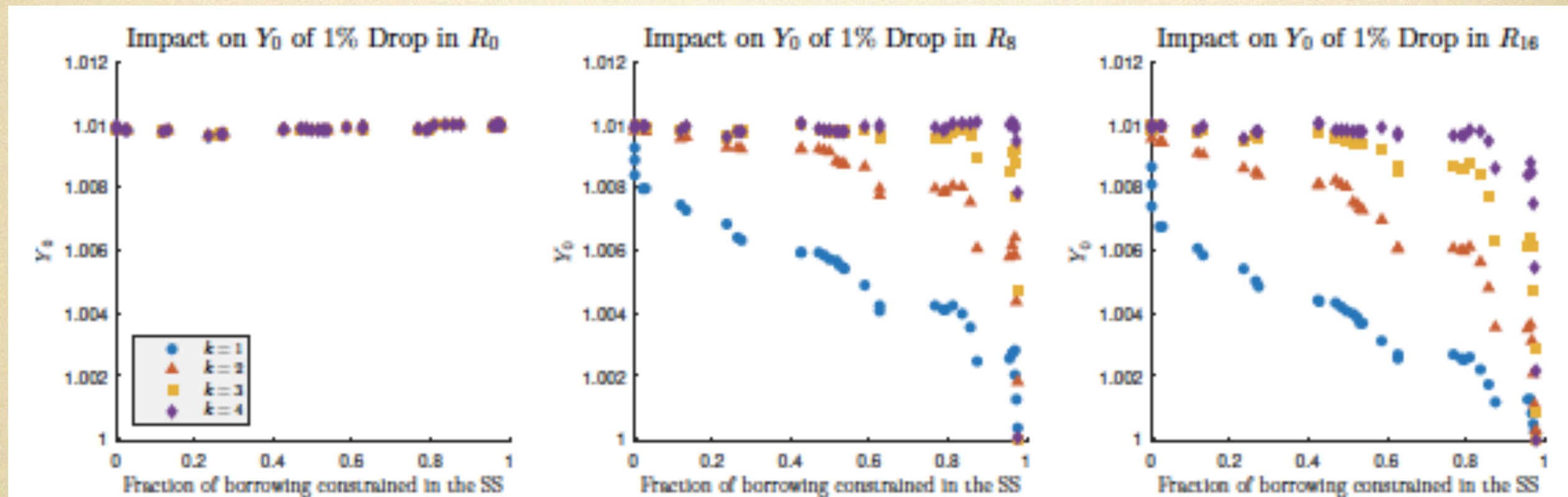
Bewley-Aiyagari-Huggett Model

- Bewley-Aiyagari-Huggett economy
- Discrete periods (quarters)
- Calibration
 - income process $\log y_t = \rho \log y_{t-1} + \epsilon_t$
 $\rho = 0.966 \quad \sigma_\epsilon = 0.017$
 - steady state interest rates at 2%
 - choose δ to match outside liquidity to output 1.44
(fraction of borrowing constrained agents 15%), as
in McKay et al. (2016)

Simulations



Simulations



Sticky Prices

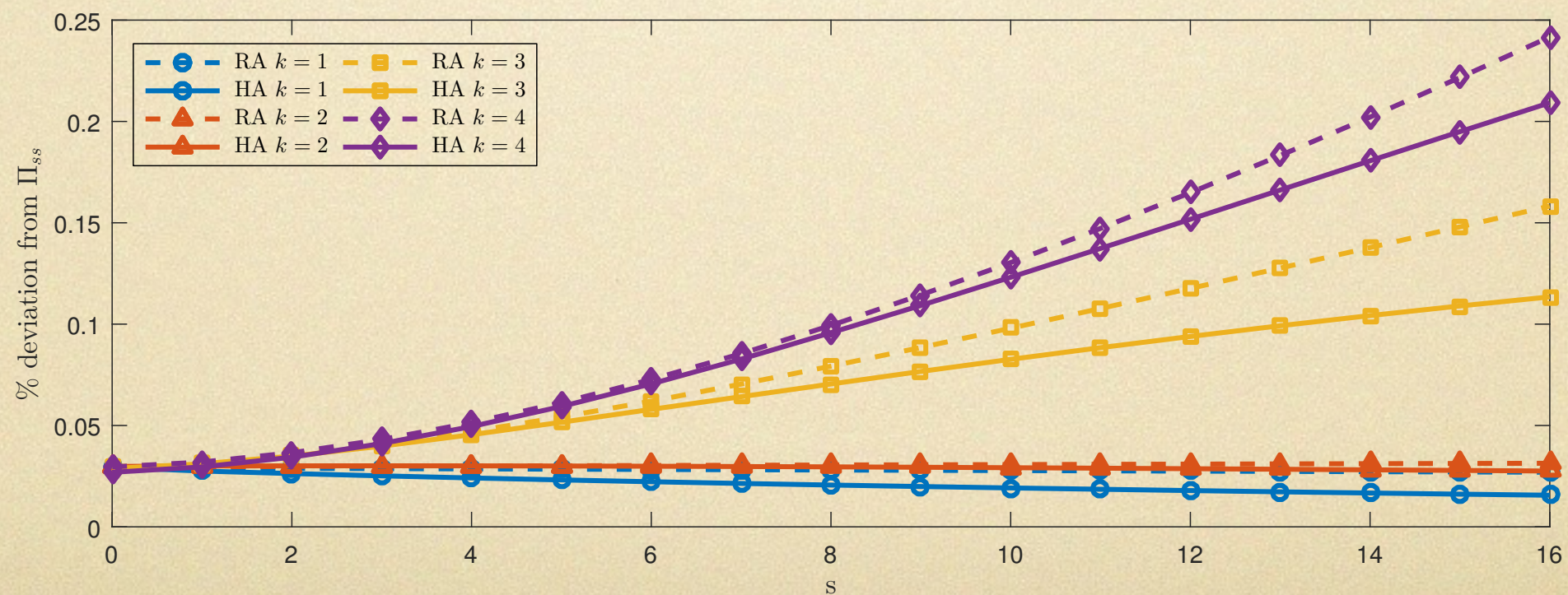
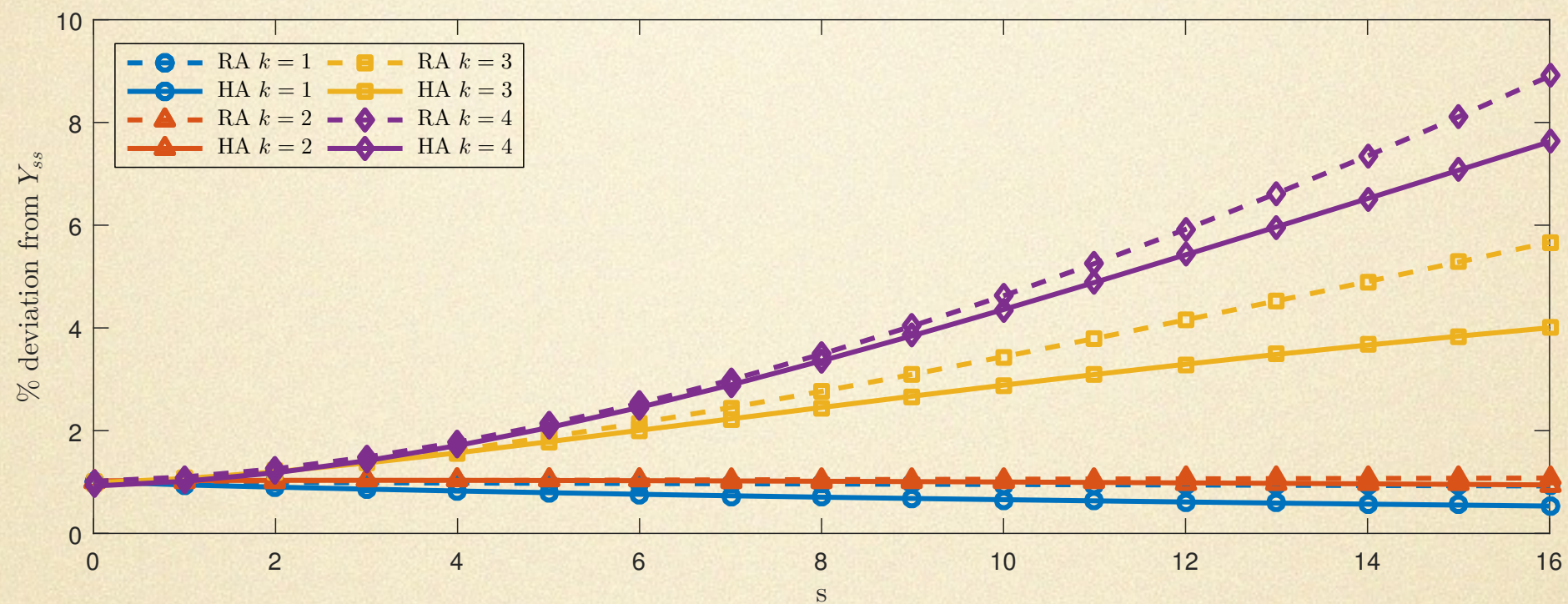
- So far: rigid prices or equivalently real interest rates
- Now: sticky prices
- Differences:
 - additional GE effect: output-inflation feedback loop
 - baseline representative agent features anti-horizon
 - can get big difference from level-k alone

Sticky Prices

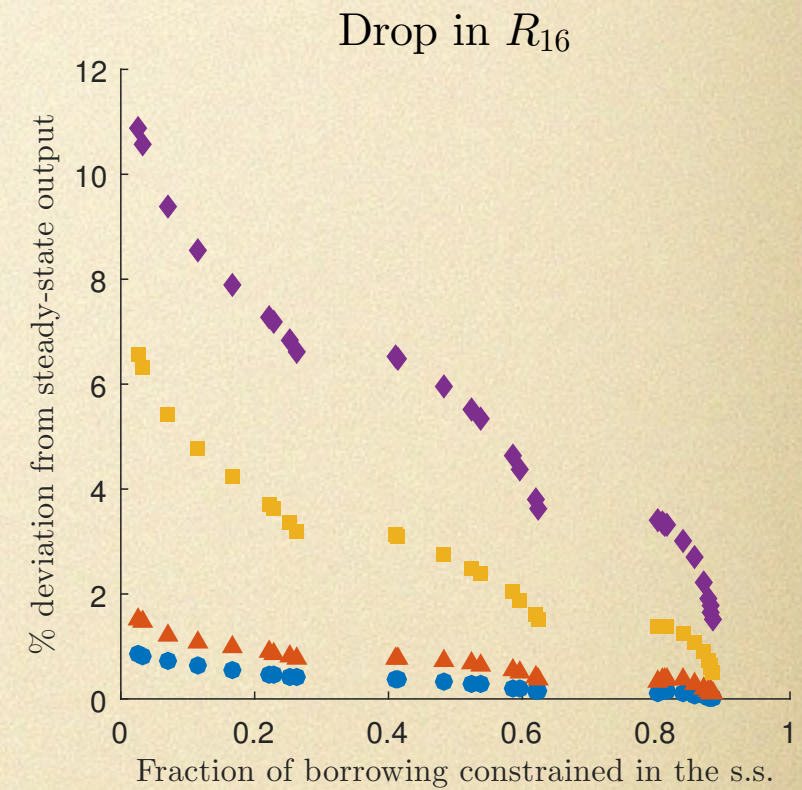
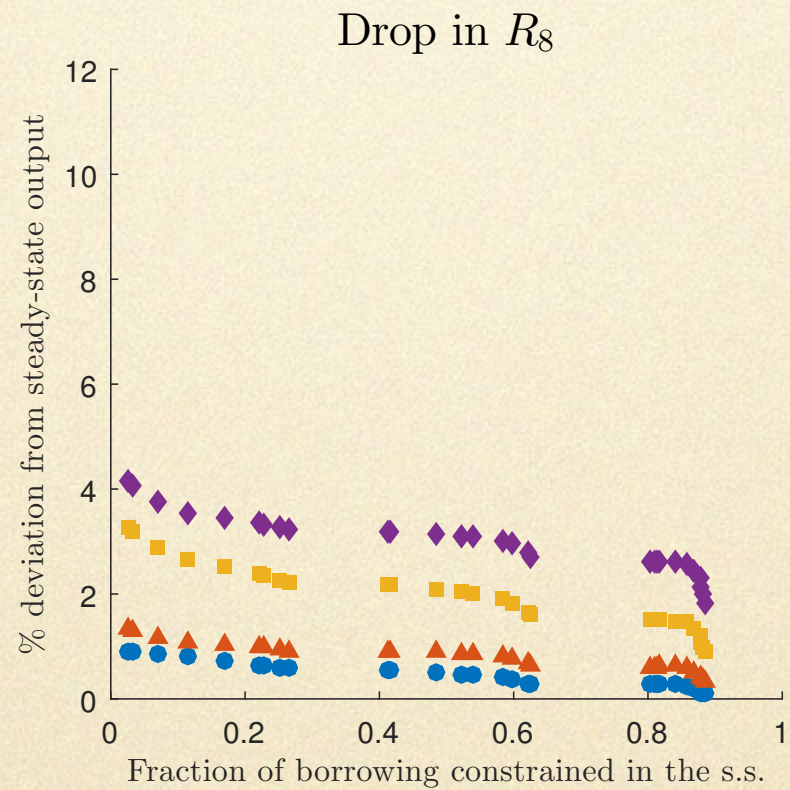
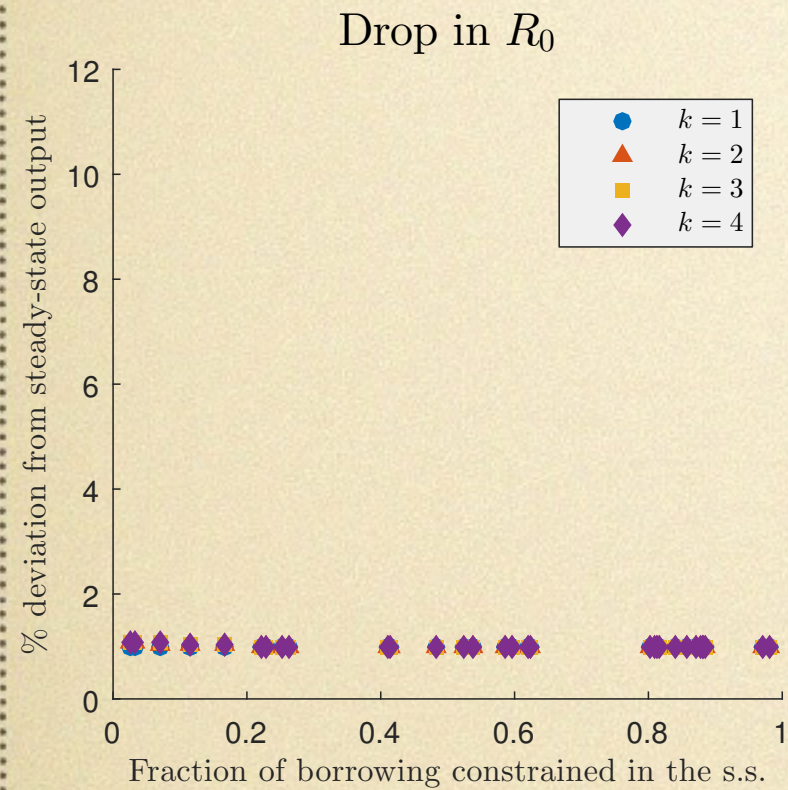
- So far: rigid prices or equivalently real interest rates
- Now: sticky prices
- Differences:
 - additional GE effect: output-inflation feedback loop
 - baseline representative agent features anti-horizon
 - can get big difference from level-k alone

Result. Complementarity between incomplete markets and bounded rationality.

Simulations



Simulations



Conclusion

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark	small effects
Bounded Rationality	small effects	large effects

