Monetary Policy, Bounded Rationality and Incomplete Markets

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Motivation

- How is monetary policy affected by
  - Bounded Rationality?
  - Incomplete Markets?
  - Combination?

**Paper**: complementarities!
Motivation

- Helps fix “bugs” of standard NK model

  - indeterminacy given interest rate paths (Taylor principle)
  - Neo-Fisherian controversies
  - effectiveness of monetary policy
  - dependence on horizon (“forward guidance puzzle”)
  - effects of fiscal policy at ZLB (“fiscal multipliers puzzle”)
  - explosive nature of long-lasting liquidity traps
  - ...

Bounded Rationality

- Expectations management major (main) channel of policy transmission in NK model under RE

- Realistic?
  - incomplete information or inattention to policy announcement?
  - less than full understanding of its future effects?
Bounded Rationality

“Inductive”
- learning: extrapolate from past data rationally or irrationally (Sargent; Evans-Honkapohja; Shleifer)
- incomplete info and inattention: ignore, underweight, cost to process info (Sims; Mankiw-Reis; Maćkowiak-Wiederholt; Gabaix; Angeletos-Lian)

“Eductive”
- robustness (Hansen-Sargent)
- level-k thinking: think through reaction of others (Stahl-Wilson; Nagel; Crawford-Costa-Gomes-Iriberri; Evans-Ramey; Woodford; García-Schmidt-Woodford)

Level-k thinking
- credible and clear announcement policy change
- with little past experience
- agents think through consequences, with bounded rationality
Incomplete Markets

- Standard NK model: representative agent or complete markets

- Incomplete markets alternative (Bewley-Huggett-Aiyagari)
  - lack of insurance to idiosyncratic shocks
  - borrowing constraints

- Key for effects and channels of monetary policy
  - high Marginal Propensity to Consume (MPC)
  - low intertemporal substitution

- Large and active area in macro (Guerrieri-Lorenzoni, Farhi-Werning, Chamley, Beaudry-Galizia-Portier, Ravn-Sterk, Sheedy, McKay-Nakamura-Steinsson, Auclert, Werning, Kaplan-Moll-Violante etc.)
<table>
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Outline

- General concept of level-k
- Representative agent with level-k
- Incomplete markets without level-k
- Incomplete markets with level-k
- Start: rigid prices or effects of real interest rates
- End: sticky prices and inflation
Rational Expectations

\[ C_t = C^* (\{ R_{t+s} \}, \gamma_t, \{ \gamma_{t+1+s}^e \}) \]
\[ C_t = \gamma_t \]
Rational Expectations

\[ C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\}) \]
\[ C_t = Y_t \]

R.E. Equilibria. Solution for \( \{C_t, Y_t\} \) with
\[ Y_{t+s}^e = Y_{t+s} \]
Rational Expectations

\[
C_t = C^*(\{R_{t+s}\}, \gamma_t, \{\gamma^e_{t+1+s}\})
\]

\[
C_t = \gamma_t
\]

R.E. Equilibria. Solution for \( \{C_t, \gamma_t\} \) with \( \gamma^e_{t+s} = \gamma_{t+s} \)

Comparative static \( \{R_{t+s}\} \rightarrow \{\hat{R}_{t+s}\} \)
Rational Expectations

\[ C_t = C^* (\{R_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}^e\}) \]

\[ C_t = \gamma_t \]

**R.E. Equilibria. Solution for \{C_t, \gamma_t\} with**

\[ \gamma_{t+s}^e = \gamma_{t+s} \]

**Comparative static \{R_{t+s}\} \rightarrow \{\hat{R}_{t+s}\}**

\[ \hat{C}_t - C_t = C^* (\{\hat{R}_{t+s}\}, \hat{\gamma}_t, \{\hat{\gamma}_{t+1+s}\}) - C^* (\{R_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}\}) \]
Rational Expectations

\[ C_t = C^*\left(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}\right) \]
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R.E. Equilibria. Solution for \( \{C_t, Y_t\} \) with \[ Y^e_{t+s} = Y_{t+s} \]

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\[ = C^*\left(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}\right) - C^*\left(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}\right) + C^*\left(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}\right) - C^*\left(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}\right) \]
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R.E. Equilibria. Solution for \( \{C_t, \gamma_t\} \) with \( \gamma_{t+s}^e = \gamma_{t+s} \)

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\[ = C^*(\{\hat{R}_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}\}) - C^*(\{R_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}\}) \]

\[ + C^*(\{\hat{R}_{t+s}\}, \hat{\gamma}_t, \{\hat{\gamma}_{t+1+s}\}) - C^*(\{\hat{R}_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}\}) \]

PE
Rational Expectations

\[ C_t = C^* (\{R_{t+s}\}, Y_t, \{Y^e_{t+1+s}\}) \]

\[ C_t = Y_t \]

R.E. Equilibria. Solution for \( \{C_t, Y_t\} \) with \( Y^e_{t+s} = Y_{t+s} \)

Comparative static \( \{R_{t+s}\} \rightarrow \{\hat{R}_{t+s}\} \)

\[ \hat{C}_t - C_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^* (\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \]

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PE

GE
Level-1 thinking:

\[ \hat{C}_t^1 = C^* (\{ \hat{R}_{t+s} \}, \hat{Y}_t^1, \{ Y_{t+1+s} \} ) \]

\[ \hat{C}_t^1 = \hat{Y}_t^1 \]

status quo REE
Level-1 thinking:

\[ \hat{C}_t^1 = C^* (\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\}) \]

\[ \hat{C}_t^1 = \hat{Y}_t^1 \]

(status quo REE)

(almost PE effect! continuous time...)

Level-k Thinking

[Image of a slide with a title and mathematical equations]
Level-k Thinking

Level-1 thinking:
\[
\hat{C}^1_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}^1_t, \{\hat{Y}_{t+1+s}\})
\]
\[
\hat{C}^1_t = \hat{Y}^1_t
\]

(almost PE effect! continuous time...)

Level-2 thinking:
\[
\hat{C}^2_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}^2_t, \{\hat{Y}^1_{t+1+s}\})
\]
\[
\hat{C}^2_t = \hat{Y}^2_t
\]

status quo REE

level-1 thinking
Level-k Thinking

Level-1 thinking:
\[
\hat{C}_t^1 = C^*\left(\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{\hat{Y}_{t+1+s}\}\right)
\]
\[
\hat{C}_t^1 = \hat{Y}_t^1
\]
(status quo REE

(almost PE effect! continuous time...)

Level-2 thinking:
\[
\hat{C}_t^2 = C^*\left(\{\hat{R}_{t+s}\}, \hat{Y}_t^2, \{\hat{Y}_{t+1+s}^1\}\right)
\]
\[
\hat{C}_t^2 = \hat{Y}_t^2
\]
(level-1 thinking

Level-k thinking:
\[
\{\hat{Y}_t^{k+1}\} = \Gamma(\{\hat{Y}_t^k\})
\]

Note: REE is a fixed point!
Level-k Thinking

- Coincides with PE for \( k = 1 \)
- Mitigates GE, less and less as \( k \) increases
- Converges to RE as \( k \to \infty \)
- Determinate for any \( k \), without Taylor rule
- Can generalize to aggregate consumption functions depending on state variable \( \Psi \) for incomplete markets (wealth distribution)
Effects of Monetary Policy

- Elasticities of output to interest rates
  - at different horizons
  - PE, GE, level-k

\[ \epsilon_{t,\tau} = \lim_{\Delta R_{\tau} \to 0} - \frac{R_{\tau}}{Y_t} \frac{\Delta Y_t}{\Delta R_{\tau}} \]

\[ \epsilon_{k,\tau} = \lim_{\Delta R_{\tau} \to 0} - \frac{R_{\tau}}{Y_t} \frac{\Delta Y_t^k}{\Delta R_{\tau}} \]

\[ \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE} \]

\[ \epsilon_{k,\tau} = \epsilon_{k,\tau}^{PE} + \epsilon_{k,\tau}^{GE} \]
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Representative Agent

- Representative agent (= complete markets)

- Continuous time
  - not crucial, but...
  - ...partial equilibrium = level-1 thinking
\[ \max_{\{c_t\}} \frac{1}{1 - \sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt \]

\[ p_t = e^{-\int_0^t r_s ds} \]
\[
\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t}c_t^{1-\sigma} \, dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t \, dt = \int_0^\infty p_t y_t \, dt
\]

\[p_t = e^{-\int_0^t r_s \, ds}\]

\[\frac{\Delta \log C_t}{\Delta \log \alpha}\]

\text{start at steady state}
\[
\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} \, dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t \, dt = \int_0^\infty p_t y_t \, dt
\]

\[
p_t = e^{-\int_0^t r_s \, ds}
\]

\[
\Delta \log C_t
\]

\[
\Delta \log \alpha
\]

\[
\epsilon_{t,\tau} = \sigma^{-1}
\]

\[
\hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases}
\]

change interest rate at \( \tau \)
\[ \max \left\{ c_t \right\} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt \]

\[ p_t = e^{-\int_0^t r_s ds} \]

\[ \epsilon_{t,\tau} = \sigma^{-1} \]

\[ \epsilon_{t,\tau}^{PE} = \sigma^{-1} e^{-r(t-\tau)} \]

change interest rate at \( \tau \)

\[ \hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases} \]
Bottom line: weak mitigation and horizon effects from level-k thinking.
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Incomplete Markets

- See e.g. Werning (2015)

- Benchmark neutrality result: “as if” rep. agent

- Subtle dependence on cyclicality of
  - income risk
  - liquidity
Keynesian Cross

- Liquidity constrained cannot substitute, so...

Q: How can incomplete markets not affect aggregate response?

A: General Equilibrium vs. Partial Equilibrium

- some do substitute and increase their spending...
- ...increases income all around...
- ...raises spending of liquidity constrained more...
- ... increases income.... etc.

\[ \downarrow \text{PE} + \uparrow \text{GE} = \text{constant} \]
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| **Bounded Rationality**    | small effects    | ???

Perpetual Youth + Aiyagari Simulations
Perpetual Youth Model

- Tractable model to easily visit all 4 squares!
- Continuum measure 1 of agents
- OLG with Poisson death and arrival $\pi \geq 0$
- Preferences
  \[ \int_0^{\infty} e^{-(\rho+\pi)s} \log(c_{t+s}^i) ds \]
- Income
  - labor income: $(1 - \delta)Y_t$
  - Lucas tree dividend: $\delta Y_t$
- Budget with annuities
  \[ \frac{da_t^i}{dt} = (r_t + \pi)a_t^i + Y_t - c_t^i \]
Perpetual Youth Model

- Alternative interpretation
  - agents do not die
  - life separated by stochastic “periods”
  - heavy discount across periods:
    - wish to borrow against future periods
    - but cannot do so!

- OLG ~ borrowing constraints
  - short or interrupted time horizons
  - no precautionary savings
  - linear consumption function and aggregation
Perpetual Youth Model

\[ V_t = \int_t^\infty e^{-\int_t^s r_u du} \delta Y_s^e ds \]

\[ H_t = \int_t^\infty e^{-\int_t^s (r_u + \pi) du} (1 - \delta) Y_s^e ds \]

individual consumption function

\[ c_t^i = (\rho + \pi)(a_t^i + H_t) \]

\[ \int_0^1 a_t^i di = V_t \quad \text{equilibrium} \quad \int_0^1 c_t^i di = Y_t \]

aggregate consumption function

\[ C_t = (\rho + \pi)(V_t + H_t) \]

\[ C_t = Y_t \]
Steady State

- Steady state
  \[ Y_t = Y \]
  \[ 1 = (1 - \delta) \frac{\rho + \pi}{r + \pi} + \delta \frac{\rho + \pi}{r} \]

- Comparative static ("MIT shock")
  - new path for interest rate
  - compute
    - rational expectations equilibrium
    - k-level thinking
Mitigation and Horizon

\[ 1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE} \]

\[ \epsilon_{t,\tau}^{PE} = (1 - \delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau-t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau-t)} \]
Mitigation and Horizon

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\[ \frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0 \]
\[ \frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} < 0 \]

\[ \frac{\partial^2 \epsilon_{t,\tau}}{\partial \pi \partial \tau} = 0 \]
\[ \frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} < 0 \]

Result. Complementarity between incomplete markets and bounded rationality.
Speed of Convergence

- Recall, level-1 = PE, level-∞ = RE
- Level-k

\[ e_{t,\tau}^k = (1 - \delta)e^{-(\rho + \pi)(\tau - t)} \left[ \sum_{\ell=1}^{k} \frac{(\rho + \pi)^{\ell-1}(\tau - t)^{\ell-1}}{\ell!} \right] \]

\[ + \delta e^{-\rho(\tau - t)} \left[ \sum_{\ell=1}^{k} \frac{\rho^{\ell-1}(\tau - t)^{\ell-1}}{\ell!} \right] \]
Speed of Convergence

- Recall, level-1 = PE, level-∞ = RE
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\[
\epsilon^k_{t,\tau} = (1 - \delta)e^{-(\rho+\pi)(\tau-t)} \left[ \sum_{\ell=1}^{k} \frac{(\rho + \pi)^{\ell-1}(\tau - t)^{\ell-1}}{(\ell - 1)!} \right] \\
+ \delta e^{-\rho(\tau-t)} \left[ \sum_{\ell=1}^{k} \frac{\rho^{\ell-1}(\tau - t)^{\ell-1}}{(\ell - 1)!} \right].
\]

Complementarity: Asymptotic convergence to RE slower for higher π.
Bewley-Aiagari-Hugget

Assumptions:
- idiosyncratic income uncertainty
- no insurance
- borrowing constraints

Results:
- occasionally binding borrowing constraints
- precautionary savings
- concave consumption functions (varying MPC)

Monetary policy and bounded rationality?
- general theoretical characterization
- numerical simulations
Assumptions:
- idiosyncratic income uncertainty
- no insurance
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Results:
- occasionally binding borrowing constraints
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Monetary policy and bounded rationality?
- general theoretical characterization

Result. Complementarity between incomplete markets and bounded rationality.
Bewley-Aiyagari-Huggett Model

- Bewley-Aiyagari-Huggett economy
- Discrete periods (quarters)

Calibration

- income process \( \log y_t = \rho \log y_{t-1} + \epsilon_t \)
  \( \rho = 0.966 \quad \sigma_\epsilon = 0.017 \)

- steady state interest rates at 2%

- choose \( \delta \) to match outside liquidity to output 1.44 (fraction of borrowing constrained agents 15%), as in McKay et al. (2016)
Simulations
Simulations
Sticky Prices

- So far: rigid prices or equivalently real interest rates
- Now: sticky prices
- Differences:
  - additional GE effect: output-inflation feedback loop
  - baseline representative agent features anti-horizon
  - can get big difference from level-k alone
So far: rigid prices or equivalently real interest rates

Now: sticky prices

Differences:
- additional GE effect: output-inflation feedback loop
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- can get big difference from level-k alone

Result. Complementarity between incomplete markets and bounded rationality.
Simulations

Figure 3: Proportional output response $e_{k,0}, t$ and inflation response $e_{P,k,0}, t$ at date 0 to a 1% interest rate cut at different horizons $t$ for the baseline incomplete-markets economy (dashed lines) and the complete-markets or representative-agent economy (solid lines). Different colors represent equilibrium output under level-$k$ thinking with different values of $k$.

Figure 4: Proportional output response $e_{k,0}, t$ at date 0 to a 1% interest rate cut at a horizon of $t = 0$, $t = 8$ quarters, and $t = 16$ quarters. Different colors represent equilibrium output under level-$k$ thinking with different values of $k$. Different dots of the same color correspond to economies with different fractions of borrowing-constrained agents in steady state. This variation is achieved by varying the discount factor $b$ and amount of liquidity $d$ and keeping the steady-state annual interest rate constant at 2%.
Simulations

Figure 3: Proportional output response \(e_{k0, t}\) and inflation response \(e_{P0, k0, t}\) at date 0 to a 1% interest rate cut at different horizons \(t\) for the baseline incomplete-markets economy (dashed lines) and the complete-markets or representative-agent economy (solid lines). Different colors represent equilibrium output under level-\(k\) thinking with different values of \(k\). Different dots of the same color correspond to economies with different fractions of borrowing-constrained agents in steady state. This variation is achieved by varying the discount factor \(b\) and amount of liquidity \(d\) and keeping the steady-state annual interest rate constant at 2%.

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## Conclusion

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