Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations

Jordi Galí

CREI, UPF and Barcelona GSE

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Motivation

- Asset price bubbles: ubiquitous in the policy debate...
  - key source of macro instability
  - monetary policy: cause and cure

...but absent in workhorse monetary models
  - no room for bubbles in the New Keynesian model
  - no discussion of possible role of monetary policy
Motivation

- Asset price bubbles: ubiquitous in the policy debate...
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- Present paper: modification of the basic NK model to allow for bubbles

- Key ingredients:
  (i) overlapping generations of finitely-lived agents
  (ii) transitions to inactivity ("retirement")
Related Literature

- Monetary models with bubbles: Samuelson (1958),..., Asriyan et al. (2016) ⇒ flexible prices
- Monetary policy and bubbles in sticky price models:
  - Bernanke and Gertler (1999, 2001): ad-hoc bubble
  - Galí (2014): 2-period OLG, constant output
  - Present paper:
    - many-period lifetimes
    - variable employment and output
    - nests standard NK model as a limiting case
A New Keynesian Model with Overlapping Generations

- Individual survival rate: $\gamma$ (Blanchard (1985), Yaari (1965))
- Size of cohort "at birth": $1 - \gamma$
- Total population size: 1

- Two types of individuals:
  - "Active": manage own firm, work for others.
  - "Retired": consume financial wealth

- Probability of remaining active: $\upsilon$ (Gertler (1999))
- Labor force (and measure of firms): $\alpha \equiv \frac{1 - \gamma}{1 - \upsilon \gamma} \in (0, 1]$
Consumers

- Complete markets (including annuity contracts)
- Consumer’s problem:

\[
\max E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t \log C_{t|s} \\
\frac{1}{P_t} \int_0^{\alpha} P_t(i) C_{t|s}(i) di + E_t \{ \Lambda_{t,t+1} Z_{t+1|s} \} = A_{t|s} \left[ + W_t N_{t|s} \right] \\
A_{t|s} = Z_{t|s} / \gamma
\]
Firms

- Technology:
  \[ Y_t(i) = \Gamma_t N_t(i) \]
  where \( \Gamma = 1 + g \geq 1 \).

- Price-setting à la Calvo
  - incumbent firms: a fraction \( \theta_I \) keep prices unchanged
  - newly created firms: a fraction \( \theta_N \) set price equal to \( P_{t-1} \)
  - average price rigidity index: \( \theta = \nu \gamma \theta_I + (1 - \nu \gamma) \theta_N \)
Labor Markets and Inflation

- Wage equation:
  \[ \mathcal{W}_t = \left( \frac{N_t}{\alpha} \right)^\varphi \]
  where \( \mathcal{W}_t \equiv W_t / \Gamma_t \) and \( N_t \equiv \int_0^\alpha N_t(i) \, di \).

- *Natural* level of output: setting \( 1 / \mathcal{W}_t = \mathcal{M} \)
  \[ Y_t^n = \Gamma_t \mathcal{Y} \]
  with \( \mathcal{Y} \equiv \alpha \mathcal{M}^{-\frac{1}{\varphi}} \). *Remark*: invariant to bubble size.

- New Keynesian Phillips curve
  \[ \pi_t = \Phi E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t \]
  where \( \Phi \equiv \beta \gamma \theta_1 / \theta, \kappa \equiv \lambda \varphi \), and \( \hat{y}_t \equiv \log \left( Y_t / Y_t^n \right) \).
Asset Markets

- Aggregate stock market

\[ Q_t^F = \sum_{k=0}^{\infty} (\nu \gamma)^k E_t \{ \Lambda_{t,t+k} D_{t+k} \} \]

*Remark*: same discount rate as labor income.

- Bubbly asset

\[ Q_t^B = E_t \{ \Lambda_{t,t+1} Q_{t+1}^B \} \]

with \( Q_{t|s}^B \geq 0 \) for all \( t \) and \( s \leq t \).

- Aggregate bubble:

\[ Q_t^B = B_t + U_t \]

where \( B_t \equiv \sum_{s=-\infty}^{t-1} Q_{t|s}^B \geq 0 \) and \( U_t \equiv Q_{t|t}^B \geq 0 \)

- Equilibrium condition:

\[ Q_t^B = E_t \{ \Lambda_{t,t+1} B_{t+1} \} \]
Characterization of Equilibria

- Balanced Growth Paths
- Equilibrium Dynamics around a Balanced Growth Path

Remark: key role for consumption function (individual and aggregate) in the determination of equilibria
Balanced Growth Paths

- Consumption function (consumer of age \( j \); normalized by productivity)
  
  (i) active individuals (letting \( \Lambda \equiv \frac{1}{1+r} \)):

  \[
  C_j = (1 - \beta \gamma) \left[ A_j^a + \frac{1}{1 - \Lambda \Gamma \nu \gamma} \left( \frac{WN}{\alpha} \right) \right]
  \]

  (ii) retired individuals

  \[
  C_j = (1 - \beta \gamma) A_j^r
  \]

- Aggregate consumption function

  \[
  \mathcal{C} = (1 - \beta \gamma) \left[ Q^F + Q^B + \frac{WN}{1 - \Lambda \Gamma \nu \gamma} \right]
  \]

  \[
  = (1 - \beta \gamma) \left[ Q^B + \frac{\mathcal{Y}}{1 - \Lambda \Gamma \nu \gamma} \right]
  \]

  using \( Q^F = \mathcal{D} / (1 - \Lambda \Gamma \nu \gamma) \) and \( \mathcal{Y} = WN + D \).
Balanced Growth Paths

- In equilibrium ($C = Y$):

$$1 = (1 - \beta \gamma) \left[ q^B + \frac{1}{1 - \Lambda \Gamma \nu \gamma} \right]$$

where $q^B \equiv Q^B / Y$.

- Bubbleless BGP ($q^B = 0$)

$$\Lambda \Gamma \nu = \beta$$

or, equivalently,

$$r = (1 + \rho)(1 + g) \nu - 1 \equiv r$$

*Remark #1*: $r$ increasing in $\nu$

*Remark #2*: $\nu < \beta \iff r < g$
Balanced Growth Paths

- Bubbly BGP:
  \[ q^B = \frac{\gamma(\beta - \Lambda \Gamma v)}{(1 - \beta \gamma)(1 - \Lambda \Gamma v \gamma)} > 0 \]

  \[ u = \left(1 - \frac{1}{\Lambda \Gamma}\right) q^B \geq 0 \]

where

\[ \Lambda \Gamma \geq 1 \iff r \leq g \]

\[ \Lambda \Gamma v < \beta \iff r > r \]

- Existence condition:
  \[ v < \beta \]
Figure 1. Balanced Growth Paths

- Bubbly BGP without bubble creation
- Bubbly BGP with bubble creation
- Bubbleless BGP
Some Numbers

- Life expectancy (at 16): $63 \times 4 = 252$ quarters $\Rightarrow \gamma \approx 0.996$
- Average employment rate: $0.6 \Rightarrow \nu = 0.9973$
- Condition for existence of bubbles: $\beta > 0.9973$
- Average real interest rate (1960-2015): $r = 1.4\% \div 4 = 0.35\%$
- Average growth rate (1960-2015): $g = 1.6\% \div 4 = 0.4\%$
Equilibrium Dynamics (I)

- Goods market clearing:
  \[
  \hat{y}_t = \hat{c}_t
  \]

- Aggregate consumption function:
  \[
  \hat{c}_t = (1 - \beta \gamma)(\hat{q}_t^B + \hat{x}_t)
  \]

  where

  \[
  \hat{x}_t = \Lambda \Gamma \nu \gamma E_t\{\hat{x}_{t+1}\} + \hat{y}_t - \frac{\Lambda \Gamma \nu \gamma}{1 - \Lambda \Gamma \nu \gamma}(i_t - E_t\{\pi_{t+1}\})
  \]

- Aggregate bubble dynamics:
  \[
  \hat{q}_t^B = \Lambda \Gamma E_t\{\hat{q}_{t+1}^B\} - q^B(i_t - E_t\{\pi_{t+1}\})
  \]
Equilibrium Dynamics (II)

- New Keynesian Phillips curve
  \[ \pi_t = \Phi E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t \]

- Monetary Policy
  \[ \hat{i}_t = \phi_{\pi} \pi_t + \phi_q \hat{q}_t^B \]

- Assumption: no fundamental shocks, focus on bubble-driven fluctuations
- Stationary solutions?
Figure 4.
Determinacy and Indeterminacy Regions around Bubbly BGPs

$r = 0.00335$

$r = 0.0035$

$r = 0.0037$

$r = 0.004$
Assumed bubble process:

\[
q_t^B = \begin{cases} 
\frac{\beta}{\delta} q_{t-1}^B + u_t & \text{with probability } \delta \\
u_t & \text{with probability } 1 - \delta
\end{cases}
\]

where \(\{u_t\} > 0\) is white noise with mean \(\bar{u} \gtrsim 0\).

**Remark:** it satisfies the relevant bubble condition

\[
q_t^B = (\beta / \nu) E_t \{q_{t+1}^B\}
\]

Equilibrium dynamics:

\[
\hat{y}_t = E_t \{\hat{y}_{t+1}\} - (\hat{i}_t - E_t \{\pi_{t+1}\}) + \Theta q_t^B
\]

\[
\pi_t = \Phi E_t \{\pi_{t+1}\} + \kappa \hat{y}_t
\]

\[
\hat{i}_t = \phi_{\pi} \pi_t + \phi_q \hat{q}_t^B
\]

where \(\Theta \equiv (1 - \beta \gamma)(1 - \nu \gamma) / \beta \gamma > 0\).
Remark: alternative representation of the dynamic IS equation

\[ \hat{y}_t = E_t\{\hat{y}_{t+1}\} - (\hat{i}_t - E_t\{\pi_{t+1}\} - \hat{r}_t^n) \]

where

\[ \hat{r}_t^n = \Theta q_t^B \]

is the natural rate of interest.
Determinacy condition (for any given bubble):

\[ \phi_{\pi} > \max \left[ 1, \frac{1}{\kappa} (\Phi - 1) \right] \]

Remark: independent of \( \phi_q \)

Equilibrium output and inflation (assuming determinacy)

\[ \hat{y}_t = (1 - v\gamma \theta_\ell / \theta) \Psi (\Theta - \phi_q) q_t^B \]

\[ \pi_t = \kappa \Psi (\Theta - \phi_q) q_t^B \]

where \( \Psi \equiv \frac{1}{(1 - v\gamma \theta_\ell / \theta)(1 - v / \beta) + \kappa (\phi_{\pi} - v / \beta)} > 0. \)

Simulated bubble driven fluctuations (\( \phi_{\pi} = 1.5, \phi_q = 0 \)) (*)
Figure 2. Simulated Bubble-Driven Fluctuations around the Bubbleless BGP
An optimal "leaning against the bubble" monetary policy:

\[ \phi_q = \Theta > 0 \]

\[ \Rightarrow \hat{y}_t = \pi_t = 0 \]

Remark: in the present environment, bubble fluctuations are not affected by "leaning against the bubble" policies

Remark: same outcome can be attained by directly targeting inflation \((\phi_q = 0, \phi_\pi \to +\infty)\), with no need to observe the bubble or knowing \(\Theta\) accurately

Monetary policy and macro volatility (*)
Figure 3. Bubble-driven Fluctuations: Monetary Policy and Macro Volatility in a Neighborhood of the Bubbleless BGP.
Equilibrium dynamics

\[
\hat{y}_t = \frac{\Lambda \Gamma}{\beta} E_t \{ \hat{y}_{t+1} \} - \frac{Y}{\beta} (\hat{i}_t - E_t \{ \pi_{t+1} \}) + \Theta \hat{q}_t^B
\]

\[
\hat{q}_t^B = \Lambda \Gamma E_t \{ \hat{q}_{t+1}^B \} - q^B (\hat{i}_t - E_t \{ \pi_{t+1} \})
\]

\[
\pi_t = \Phi E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t
\]

\[
\hat{i}_t = \phi_\pi \pi_t + \phi_q q_t^B
\]

where \( \Theta \equiv \frac{(1-\beta \gamma)(1-\nu \gamma)}{\beta \gamma} > 0 \) and \( Y \equiv 1 + \frac{(1-\beta \gamma)(\Lambda \Gamma - 1)}{1-\Lambda \Gamma \nu \gamma} \geq 1 \)

Remark: \( \{ \hat{q}_t^B \} \) no longer independent of monetary policy.
An optimal "leaning against the bubble" policy

\[ \phi_q = \frac{\Theta \beta}{\Psi \upsilon} > 0 \]

\[ \phi_\pi > 1 - \frac{(1 - \Phi)(1 - \Lambda \Gamma \upsilon / \beta)}{\kappa \Psi \upsilon / \beta} \approx 1 \]

then:

\[ \Rightarrow \hat{y}_t = \pi_t = 0 \]

but no elimination bubble fluctuations:

\[ \Rightarrow \hat{q}_t^B = \left( \frac{1}{\chi} \right) \hat{q}_{t-1}^B + \xi_t \]

where \( \xi_t \equiv \hat{b}_t - E_{t-1}\{\hat{b}_t\} + \hat{u}_t \) and \( \chi \equiv \left( \frac{\Lambda \Gamma \upsilon}{\beta} \right) \frac{\Lambda \Gamma (1 - \beta \gamma) + \gamma (\beta - \Lambda \Gamma \upsilon)}{1 - \Lambda \Gamma \upsilon \gamma} > 1 \)

if \( r < \bar{r} \).

Remark: same outcome with strict inflation targeting policy.

Simulated bubble-driven fluctuations
Figure 5. Simulated Bubble-driven Fluctuations Around a Bubbly BGP
Figure 6.b Bubble-driven Fluctuations: Monetary Policy and Bubble Volatility in a Neighborhood of a Bubbly BGP
Conclusions, Caveats and Possible Extensions

- Bubbly equilibria may exist in the NK model once we depart from the infinitely-lived representative consumer assumption. Room for bubble-driven fluctuations.
- More likely in an environment of low natural interest rates.
- No obvious advantages of "leaning against the bubble" policies (relative to inflation targeting), plus some risks (e.g. may amplify bubble fluctuations)

Caveats/potential extensions

(i) *Rational* bubbles. But non-rational bubbles can be readily accommodated.
(ii) ZLB has been ignored. Potential interesting interaction with bubbles (e.g. by raising underlying natural rate, bubbles may lower the risk of hitting the ZLB).
(iii) No role for credit supply factors; may be needed to boost "bubble multiplier".
Equilibrium dynamics

\[ \hat{y}_t = E_t{\{\hat{y}_{t+1}\}} - (\hat{i}_t - E_t{\{\pi_{t+1}\}}) \]

\[ \pi_t = \Phi E_t{\{\pi_{t+1}\}} + \kappa \hat{y}_t \]

\[ \hat{i}_t = \phi_{\pi} \pi_t \]

Monetary policy and equilibrium determinacy:

\[ \phi_{\pi} > \max \left[ 1, \frac{1}{\kappa} (\Phi - 1) \right] \]

Thus, if \( \theta/\theta_1 < \frac{\beta \gamma}{1+\kappa} \Rightarrow \phi_{\pi} > (\beta \gamma \theta_1 / \theta - 1)/\kappa > 1 \)

⇒ "reinforced Taylor principle"

Remark: forward guidance puzzle still present in equilibrium despite finitely-lived agents.
Figure 6.a  Bubble-driven Fluctuations: Monetary Policy and Output Volatility in a Neighborhood of a Bubbly BGP