Macroeconomic Fluctuations with HANK&SAM: An Analytical Approach

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BdE, September 2017

Introduction

Offer a new **framework** for understanding macroeconomic fluctuations and policy: **HANK+SAM**

Ingredients

- Heterogeneous agents and incomplete markets, (HA)
- Sticky prices (NK)
- Search and matching, (SAM)

Aim: Characterize equilibrium outcomes analytically in order to understand mechanism(s)

Key point: **HANK&SAM provides new insights for a host of issues**

HANK & SAM: Key Insights

1. HA + NK + SAM $\Rightarrow$ endogenous countercyclical earnings risk

unemployment

(SAM) $\nearrow$ $\searrow$ (HA)

labor demand — precautionary saving

(NK) $\nwarrow$ $\swarrow$ (HA)

goods demand

- **Endogenous countercyclical earnings risk**: Demand contracts in recessions $\Rightarrow$ **AMPLIFICATION**
- **Endogenous countercyclical** earnings risk empirically relevant
- Exogenous earnings risk matters little
- Incomplete markets matters little w/o sticky prices

HANK & SAM: Key Insights

2. **Show Block Recursivity**: Can separate analysis into
   - **Macroblock**: 2x3-equations macro blocks which determine aggregate outcome
   - **Distribution block**: Subsystem which determines wealth distribution

Appealing because it allows for characterization of aggregate outcomes (and estimation)

- Heterogeneity still matters for aggregate outcomes so do not get "approximate aggregation" (Krusell and Smith, 1998)
- Also straightforward to allow for capital accumulation

HANK & SAM: Key Insights

3. NK + **HA** + **SAM**: Nine major results

1. Emergence of **Unemployment Trap**
2. **Breakdown of Taylor Principle**
3. **Amplification** of Productivity Shocks
4. **Inflationary** Impact of Productivity Shocks
5. **Tightness - Real Interest Rate** Nexus
6. Sources of **Liquidity Trap**
7. **Missing Deflation** at the ZLB
8. Eliminates **Supply Shock Paradox**
9. **Endogenous Risk Premia**

- All derive from endogenous risk due to HA+NK+SAM

Literature

NK+SAM: Walsh (2005), Gertler, Sala and Trigari (2008), Blanchard and Gali (2010), Christiano, Eichenbaum, Trabandt (2016)
HANK: Auclert (2015), Bayer, Luetticke, Pham-Dao, and Tjaden (2015), Beaudry, Galizia, and Portier (2015), Berger, Dew-Becker, Schmidt and Takahasi (2016), Bhandari, Evans, Golosov and Sargent (2017), Bilbiie and Ragot (2016), den Haan, Rendahl and Riegler (2016), Farhi and Werning (2017), Gornemann, Kuester and Nakajima (2012), Guerrieri and Lorenzoni (2016), Hagedorn, Manovskii and Mitman (2016), Heathcote and Perri (2017), Kaplan, Moll, Violante (2015), Kim (2016), Luetticke (2016), McKay and Reis (2016a), Nakamura, Steinsson, and McKay (2015), Nuno and Thomas (2016), Werning (2015), Wong (2016)
HANK+SAM: Challe and Ragot (2016), Challe, Matheron, Ragot and Rubio-Ramirez (2014), Cho (2016), Kekre (2015), McKay and Reis (2016b), Ravn and Sterk (2012, 2017)

HANK & SAM: Building Blocks

Households

- Search for jobs
- Face uninsurable unemployment risk
- Save in bonds and equity

Firms

- Monopolistically competitive
- Face Rotemberg (1982) quadratic price adjustment costs
- Hire labor in frictional matching market

Monetary Authority

- Set short term nominal interest rate subject to ZLB

Households - Preferences

Preferences

$$\mathcal{V}_{is} = \max \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{\mathbf{c}_{i,s}^{1-\sigma} - 1}{1-\sigma} - \zeta \mathbf{n}_{i,s} \right),$$

Consumption

$$\mathbf{c}_{i,s} = \left(\int \left(c_{i,s}^{j} \right)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}$$

Employment Status and Earnings

$$\mathbf{n}_{i,s} = \begin{cases} 0 \text{ if not employed at date } s, \textbf{ home production } \vartheta \\ 1 \text{ if employed at date } s, \text{ earns wage } w_{i,s} \end{cases}$$

Technology - Production and Hiring

Technology

$$
\begin{aligned}
y_{j,s} &= \exp\left(A_s\right) k_{j,s}^{\mu} n_{j,s}^{1-\mu} \\
A_s &= \rho A_{s-1} + \sigma_A \varepsilon_s^A
\end{aligned}
$$

Employment Dynamics

$$
n_{j,s} = (1-\omega) n_{j,s-1} + q_s v_{j,s}
$$

Capital Accumulation

$$
k_{j,s+1} = (1-\delta)\, k_{j,s} + i_{j,s}
$$

- firms own capital

Matching technology

Timing: (i) job losses, (ii) hiring, (iii) production

Matching function:

$$\mathbf{M}(e_s, v_s) = \psi e_s^{\alpha} v_s^{1-\alpha},$$

- $v_{j,s} \geq 0$, flow cost $\kappa > 0$ per unit

Matching rates: Let $\theta_s = v_s / e_s$ denote labor market tightness

$$\begin{aligned} \text{job finding rate} \quad &: \quad \eta_s = \mathbf{M}_s / e_s = \psi \theta_s^{1-\alpha} \\ \text{vacancy filling rate} \quad &: \quad q_s = \mathbf{M}_s / v_s = \psi^{1/(1-\alpha)} \eta_s^{-\alpha/(1-\alpha)} \end{aligned}$$

Prices, Wages, Interest Rates

Price Setting: Rotemberg quadratic price adjustment costs

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{t,t+s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - i_{j,s} - \frac{\phi}{2} \left(\frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right]$$

Wages

$$w_{i,s} = \mathbf{w}\left(\eta_s\right)$$

Monetary Policy: Interest Rate Rule:

$$R_s = \max \left\{ \overline{R} \left(\frac{\Pi_s}{\overline{\overline{\Pi}}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\overline{\theta}} \right)^{\delta_\theta}, 1 \right\}$$

Assets and Borrowing Constraints

Assets: Two assets in the economy

- Bonds: $b_{i,s}$ - in zero net supply
- Equity: $x_{i,s}$ - positive net supply

Borrowing Constraints

$$\begin{aligned}\mathbf{b}_{i,s} &\geq -\varkappa \mathbf{w}_{i,s}\mathbf{n}_{i,s} \\ \mathbf{x}_{i,s} &\geq 0\end{aligned}$$

Budget Constraint

Households face budget constraint

$$\mathbf{c}_{i,s} + \mathbf{b}_{i,s} + \mathbf{x}_{i,s} \geq \mathbf{w}_{i,s}\mathbf{n}_{i,s} + (1 - \mathbf{n}_{i,s})\,\vartheta + \frac{R_{s-1}}{\Pi_s}\mathbf{b}_{i,s-1} + \frac{(1-\tau_i)\,R_{x,s}}{\Pi_s}\mathbf{x}_{i,s-1}$$

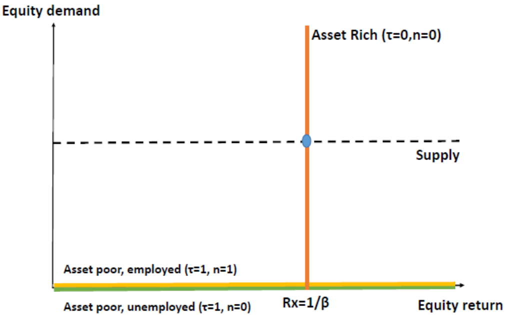
1. **No access to unemployment insurance**: households face idiosyncratic unemployment risk
2. **Heterogeneous equity returns**: τ_i varies across households

- Benhabib, Bisin and zhu (2011), Gabaix, Lasry, Lions and Moll (2016), Fagereng, Guiso, Malacrino and Pistaferri (2016)

Limited Participation: Special case when $\tau_i = 0, 1$ (Christiano, Eichenbaum and Evans, 1997)

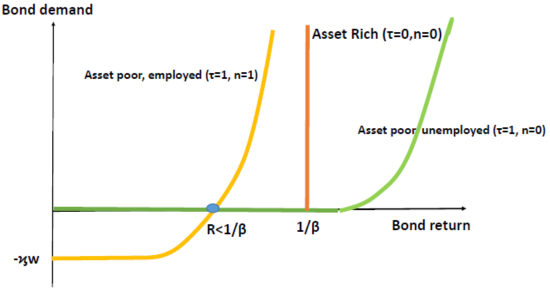
Limited-Participation: Steady-State

Equity Market



- Households with $\tau_i = 1$: Exit equity market, become asset poor
- Households with $\tau_i = 0$: Hold equity, become asset rich, stop working

Bond Market



- Real interest rate uniquely determined by asset poor employed agents $\Rightarrow$ asset rich leave bond market
- Asset poor unemployed can't borrow $\Rightarrow$ in equilibrium asset poor consume labor income
- From now on: $\varkappa = 0 \Rightarrow$ no bond supply in equilibrium

Equilibrium

Demand-Supply Interaction:

$$c_{e,s}^{-\sigma} = \mathbb{E}_s \frac{\mathbf{R}_s}{\Pi_{s+1}} \left[c_{e,s+1}^{-\sigma} + \omega \left(1 - \eta_{s+1}\right) \left(c_{u,s+1}^{-\sigma} - c_{e,s+1}^{-\sigma} \right) \right]$$

$$\mathbf{R}_s = \overline{R} \left(\frac{\Pi_s}{\overline{\Pi}} \right)^{\delta_\pi}$$

$$\gamma \mathbf{mc}_s = \phi \left(\Pi_s - 1\right) \Pi_s - \mathbb{E}_s \Lambda_{s,s+1} \frac{y_{s+1}}{y_s} \phi \left(\Pi_{s+1} - 1\right) \Pi_{s+1} + \gamma - 1$$

$$\mathbf{mc}_s = \frac{w_s}{\exp\left(\mathbf{A}_s\right)} + \frac{\kappa}{q_s \exp\left(\mathbf{A}_s\right)} - \left(1 - \omega\right) \mathbb{E}_s \Lambda_{s,s+1} \frac{\kappa}{q_{s+1} \exp\left(\mathbf{A}_{s+1}\right)}$$

- **HANK & SAM**: Endogenous countercyclical earnings risk
- **Feedback mechanism**: job uncertainty ⇒ precautionary savings (fall in goods demand at current real rate) ⇒ decline in real rate and in inflation ⇒ less hiring ⇒ job uncertainty

Steady-States

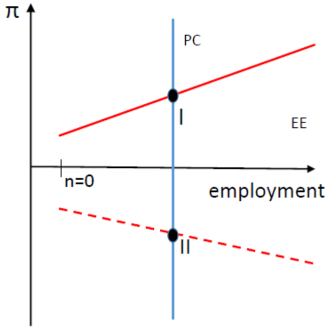
- Assume $\delta_\pi > 1, \ \delta_\theta = 0$
- Focus for now on limited participation, no capital

3 cases:

- Endogenous risk + flexible prices
- Exogenous risk + sticky prices (complete markets case)
- Endogenous risk + sticky prices

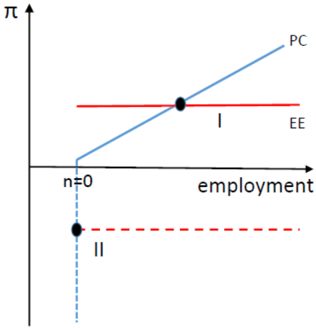
New Steady-State Emerges

Endogenous Risk + Flexible Prices



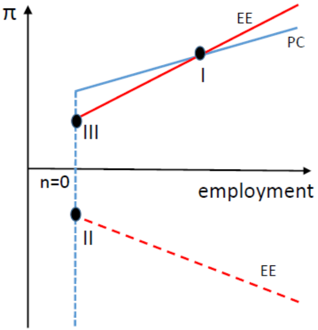
- I: Intended equilibrium
- II: Liquidity trap

Exogenous Risk + Sticky Prices



- I: Intended equilibrium
- II: Liquidity trap

Endogenous Risk + Sticky Prices



- I: Intended equilibrium
- II: Liquidity trap
- III: Unemployment trap
- **Unemployment Trap** arises If IM wedge is sufficiently large

Steady-states

Result 1: Emergence of Unemployment Trap : **Inflationary low activity steady-state** - unrelated to ZLB - non-existence under

- Complete markets
- Exogenous earnings risk
- Flexible prices

Unemployment trap / **Secular stagnation outcome** Steady-state with low output, high unemployment, low inflation

- Hansen (1939): Low productivity growth + population ageing
- Eggertsson & Mehrotra & Robbins, Summers etc: Deleveraging + low productivity and ZLB produce steady state equilibria with negative 'natural real interest rates'

Fluctuations

Properties Close to Intended Steady-State

To simplify formulae, assume

- $\delta_\pi = 1/\beta > 1$
- $\mu = 0$ (no capital)
- $\Lambda_{s,s+1} = \beta$
- Log-linearize macro block(s)

Properties of Equilibria: Local Dynamics

Log linearize around the intended steady-state

- **Wage from Nash bargaining**

$$\widehat{w}_s = \chi \widehat{\eta}_s$$

- **Employed agents' Euler equation** :

$$-\sigma \widehat{c}_{e,s} + \sigma \beta \overline{R} \mathbb{E}_s \widehat{c}_{e,s+1} = \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \underbrace{\beta \overline{R} \Theta^I \mathbb{E}_s \widehat{\eta}_{s+1}}_{\text{incomplete-markets wedge}}$$

$$\Theta^I \equiv \omega \eta \left((\vartheta / w)^{-\sigma} - 1 \right)$$

- **NK Phillips Curve** :

$$\underbrace{\frac{\phi}{\gamma} \widehat{\Pi}_s - \beta \frac{\phi}{\gamma} \mathbb{E}_s \widehat{\Pi}_{s+1}}_{\text{sticky-price wedge}} = mc \widehat{mc}_s$$

One-equation model

Result 2: **Breakdown of "Taylor Principle"** - **Local determinacy iff**

$$\underbrace{\frac{\phi\beta}{\gamma}}_{\text{price rigidity}} \left(\underbrace{\beta\overline{R}\Theta^{l}}_{\text{end. risk}} - \underbrace{\frac{\delta_\theta}{1-\alpha}}_{\text{mon.policy}} - \sigma\left(1-\beta\overline{R}\right)\chi \right) < \underbrace{\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right)}_{\text{search and matching}} + w\chi$$

$$\chi = \frac{d \ln w(\eta)}{d\eta} \geq 0$$

1. **Flexible prices or complete markets**: **local determinacy**
2. **Sticky prices AND incomplete markets**: Policy needs to be more aggressive than Taylor principle to stabilize feedback mechanism
3. Real wage flexibility χ stabilizing

Productivity shocks

Result 3: Amplification of technology shocks

$$\widehat{\eta}_s = \Gamma_\eta A_s$$
$$\Gamma_\eta = \Psi\left(\phi, \Theta^l, .\right) > 0$$

Complementarity between IC and NK:

- $\frac{\partial \Psi}{\partial \Theta^l} \geq 0$ and $\frac{\partial^2 \Psi}{\partial \Theta^l \partial \phi} > 0$
- Higher productivity stimulates demand because of decline in precautionary savings due to endogenous earnings risk
- $\Gamma_\eta = \frac{w - \frac{\kappa}{q}\beta(1-\omega)(1-\rho)}{\frac{\beta\phi}{\gamma}\left(\frac{\delta_\theta}{1-\alpha} - \rho\beta\overline{R}\Theta^l\right) + \frac{\kappa}{q}\frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha}}$

Productivity shocks

Result 4: Inflationary impact of technology shocks

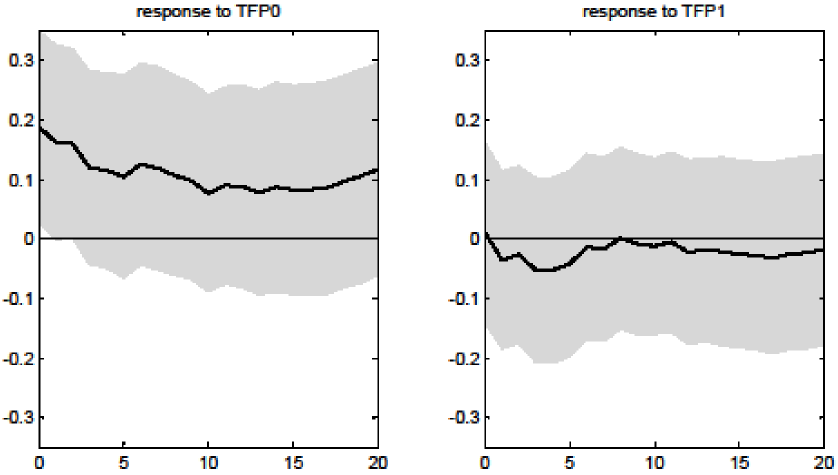
$$\begin{aligned}\widehat{\Pi}_s &= \Gamma_{\Pi} A_s \\ \Gamma_{\Pi} &= \frac{\beta^2 \overline{R} \Theta^I \rho - \frac{\beta \delta_\theta}{1-\alpha} - \sigma w \left(1 - \rho \beta^2 \overline{R}\right)}{1 - \beta\rho} \Gamma_\eta\end{aligned}$$

- $\Gamma_{\Pi} < 0$ under **complete markets** ($\Theta = 0$).
- $\Gamma_{\Pi} \gtreqless 0$ under **incomplete markets** ($\Theta > 0$)

Higher productivity triggers rise in inflation when:

$$\beta \overline{R} \Theta^I \rho > \frac{\delta_\theta}{1-\alpha} + \sigma w \left(1 - \rho \beta^2 \overline{R}\right)$$

LP Estimates of Inflation Response to Prod. Shock



Real Interest Rates and Tightness

Result 5: Tightness - real rate nexus

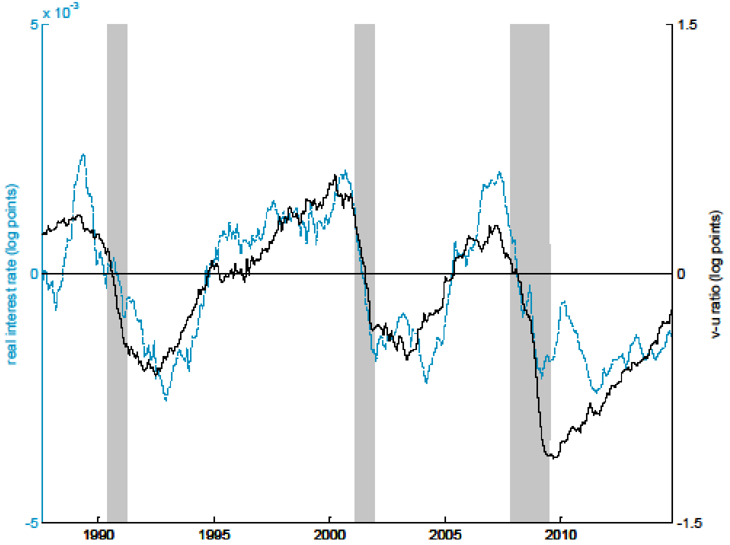
$$\widehat{R}_s^r = (1-\alpha)\left(-\sigma\chi + \beta\overline{R}\left(\sigma\chi + \Theta^I\right)\right)\rho\widehat{\theta}_s$$

$$\left(\frac{\partial \widehat{R}_s^r}{\partial \widehat{\theta}_s}\right)^{CM} = -(1-\alpha)\rho\left(1-\beta\overline{R}\right)\sigma\chi < 0$$

$$\left(\frac{\partial \widehat{R}_s^r}{\partial \widehat{\theta}_s}\right)^{IM} \gtrless 0$$

- **CM** (Intertemporal savings motive): Recessions associated with low tightness and high real rates
- **IM** and $\Theta \gg 0$ (precautionary and intertemporal savings motive): Recessions associated with low tightness and low real rates (employed workers have incentive to save)

Real Interest - Tightness Nexus



Liquidity Trap Sources

Result 6: **Productivity shocks and liquidity traps**

- **Complete markets**: Negative productivity shocks stimulates inflation and desire for borrowing
- **Incomplete markets**: Negative productivity shocks stimulate precautionary savings due to endogenous earnings risk

HANK&SAM: ZLB may be induced by negative productivity shocks if nominal interest rate declines in response to contractionary shock

- This can happen when

$$\Theta^{I} \geq \frac{\sigma\chi}{\beta\overline{R}\rho}\left(1-\beta\overline{R}\rho\right)$$

Missing Deflation

Result 7: Inflation at the ZLB

Complete markets: ZLB steady-state

$$\Pi_{LT}^{CM} = \beta < 1$$

- Temporary ZLB: Even more deflationary

HANK and SAM: ZLB steady-state

$$\Pi_{LT}^{IM} = \beta\Theta\left(\eta\right) \gtrless 1$$

- Job finding rate declines which triggers low demand - endogenous earnings risk channel again

Supply Shock Paradox at ZLB

Result 8: Supply shock paradox:

Complete Markets: Eggertsson and Krugman (2012), Wieland (2015):

- Positive productivity shock increase real interest rate $\Rightarrow$ increase in consumption growth $\Rightarrow$ decline in current consumption.

HANK and SAM: Supply shock stimulates demand by reducing precautionary savings due countercylical endogenous earnings risk (rise in job finding rate)

$$\frac{d\eta_s^{ZLB}}{d\mathbb{E}_s\Pi_{s+1}} = \frac{1}{\sigma\chi\left(1-\beta\overline{R}\mathbf{p}\right)-\beta\overline{R}\Theta^l\mathbf{p}}$$

- $\frac{d\widehat{\eta}_s}{d\mathbb{E}_s\widehat{\Pi}_{s+1}} < 0$ if $\Theta^l > \frac{\sigma\chi}{p}\left(\frac{1}{\beta\overline{R}} - p\right) \geq 0$

Asset pricing

Result 9: Endogenous Risk Premia

- Consider risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s + 1$.
- Price of risky asset:

$$z_s = \mathbb{E}_s \left\{ \Lambda_{e,s,s+1} \left(1 + A_{s+1} - \rho A_s \right) \right\}$$

- Price real bond that pays of 1 in period $s + 1$ is given by:

$$\widetilde{z}_s = \mathbb{E}_s \Lambda_{e,s,s+1}$$

- *Positive risk premium when markets are incomplete*

$$\beta\omega \left(\left(\vartheta / w \right)^{-\sigma} - 1 \right) \eta \Gamma_\eta \sigma_A^2$$

- Monetary policy affects risk premium via Γ_η: "stochastic discount factor channel"

Extensions

A. Amplification with Capital

- Does introduction of capital stabilize or further amplify
- **Stabilization**: Capital "cheap" in recession due to precautionary savings leading to downward pressure on real interest rate
- **Amplification**: Capital not very useful in recessions - firms face low goods demand

B. Block Recursivity and Wealth Distribution

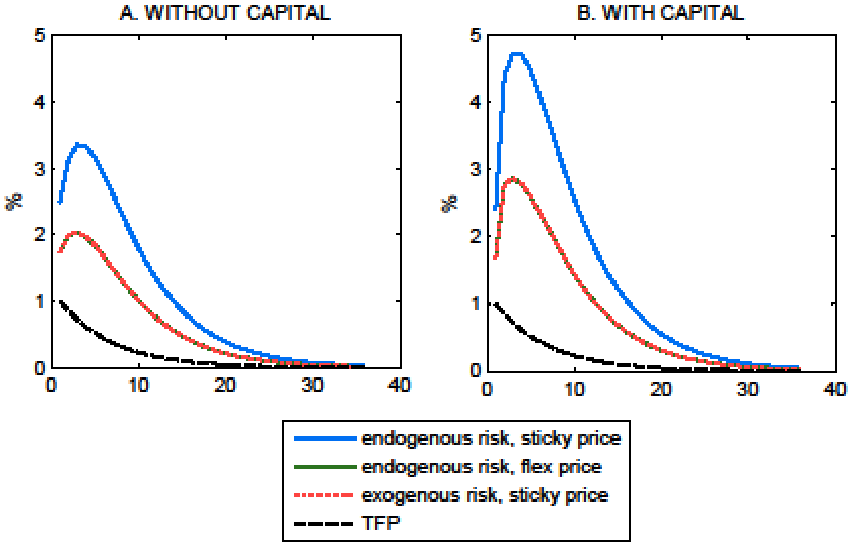
- Allow for distribution of τ_i
- Show that equilibrium is block recursive - macro block solves independently of wealth distribution
- Everything above still true but full wealth distribution in the background

Capital Accumulation

- Monthly model

Steady-state Targets			Parameter values		
u	0.05	unempl. rate	δ_π	1	int. rule
η	0.30	job find. rate	δ_θ	0	int. rule
$\frac{\kappa}{\psi 3w}$	0.05	hiring costs	σ	2	risk aversion
		(% of quart. wage)	χ	0	wage elast.
$1-\frac{\vartheta}{w}$	0.15	Income loss upon	δ	0.005	depr. rate
		job loss	ω	0.02	job term. rate
Π	1	gross infl. rate	ρ	0.85	TFP pers.
$R^{12}-1$	0.03	annual int. rate	α	0.5	match. func.
$\frac{k}{12y}$	2	Capital outp. ratio	μ	0.3	prod. elast.

Capital Accumulation



Steady-State:

General Case: τ_i follows some distribution with support on $[0,1]$

1. **Threshold** $\overline{\tau}$: Households with $\tau_i > \overline{\tau}$ exit equity market, hold zero wealth in equilibrium

$$\mathbf{c}_{i,s} = \mathbf{w}_{i,s}\mathbf{n}_{is} + \vartheta\left(1 - \mathbf{n}_{is}\right)$$

2. Let employed asset poor who never invest in equity be given by set $\mathcal{I} = \left(i : \tau_i > \overline{\tau}, n_{i,s} = 1\right)$: real interest rate determined from

$$1 = \left[\beta \frac{R}{\Pi} \mathbb{E}\left(\frac{c_{i,s}}{c_{i,s+1}}\right)^{\sigma}\right]_{i \in \mathcal{I}} \geq \left[\beta \frac{R}{\Pi} \mathbb{E}\left(\frac{c_{i,s}}{c_{i,s+1}}\right)^{\sigma}\right]_{i \notin \mathcal{I}}$$

3. Equity return must be consistent with Euler equation of those who never buy bonds:

$$1 = \beta \frac{R_x}{\Pi}$$

   - Shareholder agree to discount profits at rate β

Block Recursivity

Macro Block 1: The Essentials: Interest rate, inflation and job finding rate determined from:

$$1 = \beta \frac{R}{\Pi} \Theta(\eta) \tag{EE}$$

$$1 - \gamma + \gamma \mathbf{mc}(\eta) = \phi(1-\beta)(\Pi - 1)\Pi \tag{PC}$$

$$R = \max\left\{ \overline{R} \left(\frac{\Pi}{\overline{\Pi}} \right)^{\delta_\pi} \left(\frac{\eta}{\overline{\eta}} \right)^{\delta_\theta/(1-\alpha)}, 1 \right\} \tag{IR}$$

where

$$\Theta(\eta) = 1 + \omega(1-\eta)\left[\left(\frac{\mathbf{w}(\eta)}{\vartheta} \right)^{-\sigma} - 1 \right] \geq 1$$

$$\mathbf{mc}(\eta) = \frac{\mathbf{w}(\eta) + \left(\kappa \eta^{\alpha/(1-\alpha)} - \lambda_f \right)(1 - \beta(1-\omega))}{1/\beta - 1 + \delta}$$

Block Recursivity

Macro Block 2: Other Variables: employment, investment, output:

$$\mathbf{n} = \frac{\eta}{\eta + \omega}$$

$$1 = \beta \left(1 - \delta + \mu A k^{\mu - 1} \mathbf{n}^{1-\mu}\right)$$

$$y = A k^{\mu} \mathbf{n}^{1-\mu}$$

where

$$\Theta(\eta) = 1 + \omega (1 - \eta) \left[\left(\frac{\mathbf{w}(\eta)}{\vartheta} \right)^{-\sigma} - 1 \right] \geq 1$$

$$\mathbf{mc}(\eta) = \frac{\mathbf{w}(\eta) + \left(\kappa \eta^{\alpha/(1-\alpha)} - \lambda_f \right) (1 - \beta (1 - \omega))}{1/\beta - 1 + \delta}$$

Block Recursivity

Micro Block: Wealth Distribution: Potentially large amount of inequality:

$$\mathbf{c}_{i,s}^{-\sigma} = \beta \frac{(1-\tau_i) R_x}{\Pi} \mathbb{E} \mathbf{c}_{i,s+1}^{-\sigma} + \lambda_{i,s}^{x} \tag{EE'}$$

$$c_{i,s} + x_{i,s} = n_{i,s} w_{i,s} + (1 - n_{i,s}) \vartheta + \frac{(1-\tau_i) R_x}{\Pi} x_{i,s-1} \tag{1}$$

$$x_{i,s} \geq 0, \tau_i \leq \overline{\tau}, \lambda_{i,s}^{x} \geq 0, \lambda_{i,s}^{x} x_{i,s} = 0$$

Individuals face uninsurable unemployment risk

- accumulate assets when employed
- decumulate assets when unemployed

Individuals face heterogeneity in equity return

- those with higher returns become wealthy
- those with lower returns remain poor

Summary

HANK with SAM: New framework for macroeconomic fluctuations

- Combines frictional markets approach of NK literature with incomplete markets models and SAM
- Key mechanism: **Endogenous countercyclical unemployment risk** induces amplification
- Key is interaction between frictions
- Results can be generalized

HANK + SAM

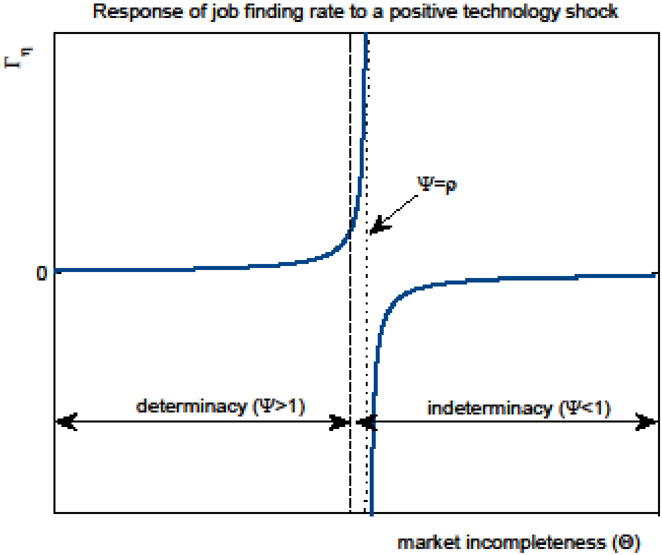
Dynamics in indeterminacy region

- Model in indeterminacy region:

$$\widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s - \Omega A_s + \Upsilon^A \varepsilon^A_{s+1} + \varepsilon^B_{s+1}$$

  where $\Omega \equiv \frac{w + \frac{\kappa}{q}(1-\rho)}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\sigma\beta^2 \overline{R}\chi + \frac{\phi}{\gamma}\beta^2 \overline{R}\Theta} > 0$ and ε^B_{s+1} is an i.i.d. belief shock (sunspot).
- Impact response to productivity shock (Υ^A) not determined.
- Follow Lubik and Schorfheide (2004) and assume responses are continuous at the border of the determinacy region, which in our case implies $\Upsilon^A = \Gamma_\eta$.
- Persistence of shocks maximized at border of indeterminacy region $(\Psi = 1)$.

Amplification and determinacy



Steady-states

Intended Steady-State : Inflation on target, nominal interest rate positive, moderate unemployment

$$\left(\frac{R}{\pi}\right)^{I} = \frac{1}{\beta}\frac{1}{\omega(1-\eta)\frac{c_e^{-\sigma}}{c_u^{-\sigma}}+1-\omega(1-\eta)} < \frac{1}{\beta}$$

- low real interest rate because of idiosyncratic job loss risk
- A fundamental difference between this model and standard NK model: Individuals face risk even if aggregates are constant
- Policy matters for long run real interest rate if it affects job uncertainty

Quantifying Market Incompleteness

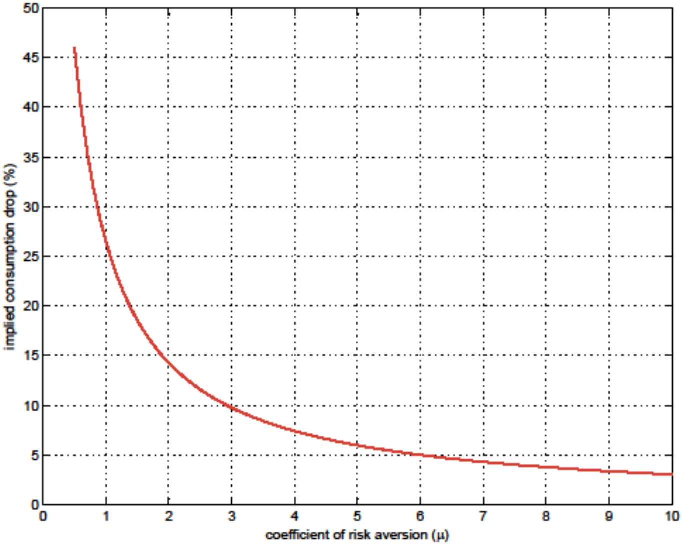
- We can compute the relative unconditional variances of real interest rates to tightness:

$$\lambda = \frac{\textbf{var}\left(\widehat{R}_s^r\right)}{\textbf{var}\left(\widehat{\theta}_s\right)} = (1-\alpha)^2\,\Theta^2\rho^2\beta^2\overline{R}$$

- Back of envelope:

$$\begin{aligned} \alpha &= 1/2 \\ \rho &= 0.99 \\ \beta\overline{R} &\simeq 1 \\ &\Longrightarrow \\ \Theta &\simeq 0.0061 \end{aligned}$$

Quantifying Market Incompleteness



One-equation model

Assume (i) $\delta_\pi = \frac{1}{\beta} > 1$ and (ii) risk neutral asset-rich households:

$$
\begin{aligned}
\mathbb{E}_s \widehat{\eta}_{s+1} &= \Upsilon_\eta \widehat{\eta}_s \\
\Upsilon_\eta &= \frac{\phi\gamma^{-1}\mu\chi\beta + \phi\gamma^{-1}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \phi\gamma^{-1}\mu\beta^2\overline{R}\chi + \phi\gamma^{-1}\beta^2\overline{R}\Theta}
\end{aligned}
$$

Dynamics depends on

- **Incomplete markets** wedge Θ
- **Goods market complementarities** through μ
- **Sticky price** wedges through ϕ
- **Wage flexibility** through χ
- **Labor market rigidities** through κ and α
- **Monetary policy** through δ_θ