Integrated Assessment in a Multi-region World with Multiple Energy Sources and Endogenous Technical Change

John Hassler, Per Krusell, Conny Olovsson and Michael Reiter
IIES, IIES, Riksbank and IHS

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Abstract

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We construct an integrated assessment model with multiple energy sources—including fossil fuels and "green energy"—and multiple world regions. The energy sources are imperfect substitutes and their production involve structures that are endogenous. In particular, firms can decide to lower the marginal cost of producing one form of energy at the expense of the marginal costs of other energy sources: there is directed technical change. In the lowering of these marginal costs, there are also spillovers, which are international. We analyze how (potentially region-specific) taxes affect output and the climate with and without the endogeneity of technology. We emphasize the second-best nature of taxation when optimal world-wide technology subsidies are not implemented.

1 Introduction

Climate change is often portrayed as a key challenge for the survival of mankind. In principle, the analysis of climate change and how to handle it with economic policy is straightforward and based on two pieces of century-old knowledge.

The first piece of knowledge is due to Arrhenius (1896), who described and quantified the greenhouse effect. Carbon dioxide (CO_2) is released into the atmosphere when fossil fuel is burnt. In contrast to gases where molecules have two atoms, like oxygen and hydrogen, CO_2 lets through high-frequency electromagnetic radiation, like sunlight, but it reflects low-frequency infrared heat radiation. Arrhenius examined this effect in a laboratory setting and its key quantitative findings still hold up.

The other piece of knowledge is from Pigou (1920). Emitted CO_2 mixes quickly into the atmosphere, affecting the energy budget, and therefore the climate, independently of the identity and geographic location of the emitter. Thus, if climate change has economic consequences, this creates a pure externality that implies that unregulated markets do not function perfectly. Pigou's general solution to externality problems, which is to apply a tax that is equal to the marginal societal effect that markets do not internalize, can straightforwardly be applied here: one needs to adopt a global tax on emissions equal to the marginal economic effects on the economy. This principle applies whether these effects are positive or negative; in this area, the estimates suggest that the effects are negative on average: climate change causes damages.

Neither of these pieces of knowledge can reasonably be questioned (except by members of the Flat Earth Society).[1] However, the issue of climate change is far from settled. One reason for this is arguably that the two quoted pieces of evidence are not sufficient for calculating a quantitative value for the tax with sufficient precision. To devise an implementable policy, i.e., specify the right tax on fossil fuels, we thus need to know (i) how much climate change emissions induce, (ii) how long carbon stays in the atmosphere, and (iii) how large the economic consequences are. Although there is much research on these issues, significant uncertainty remains. Another reason is likely political: the economic impacts of climate change differ significantly by geographic location. For example, there are reasons to believe that countries/regions which are currently in climate zones with a very low average temperature stand to gain, thus potentially making their populations averse to carbon taxes, at least unless they are compensated by those who would gain from the tax.

[1] Joining is free of charge at `http://theflatearthsociety.org/home/index.php/about-the-society/joining-the-`

In terms of the quantitative uncertainties, it is well established that the direct effect of a doubling the CO_2 concentration is a perturbation of the energy budget (incoming energy flows from the sun minus outgoing flows from earth to space) by around 4 W/m^2. In itself, this is not a large amount and would not cause dramatic warming. However, we have good reasons to believe that this direct effect leads to a multitude of feedback effects of uncertain strengths, and even signs.[2] A reasonable interval of uncertainty includes both feedback mechanisms that are so weak as to make the effect of CO_2 emissions on climate small and the very opposite: very large feedback effects causing dramatic warming.

There is also substantial uncertainty about how emissions affect the CO_2 concentration over time. There is a constant large flow of carbon between a large number of reservoirs, including the atmosphere, plants in the biosphere, the top layer of earth, the surface of the oceans, and the deep oceans. The flows between these reservoirs and their respective storage capacities are affected directly by emissions and, more importantly, indirectly by climate change.

However, despite the substantial uncertainty regarding the natural-science mechanisms behind emission-induced climate change, the uncertainties in the social-science domain are far greater. This paper aims at developing a model that could, potentially at least, increase our understanding of some of the relevant economic mechanisms. Our paper build on previous work, in particular Golosov et al. (2014) and Hassler and Krusell (2014), but more fundamentally it builds on the work by Nordhaus (see, e.g., Nordhaus 1977, 1994, and Nordhaus and Boyer, 2000, as well as the overview in Nordhaus, 2011) who constructed simple but reasonably accurate representations of the natural-science elements—the circulation between carbon reservoirs and climate change induced by the greenhouse effect—and integrated them into a growth model building on Ramsey, Solow, and Dasgupta & Heal (1974). A key feature of his analysis was the aim to make quantitative statements both on the natural-science and social-science sides of the analysis. This explains the choice of basing the economic analysis on the neoclassical growth model, which is the established framework for analyzing long-run macroeconomics, here in its global version. Nordhaus thus contributed the first *Integrated Assessment Models* with his sequence of papers.

A key question in our previous work was about the quantitative determination of the optimal carbon tax. The present paper has a different focus. One set of questions addressed

[2] Airborne particles in combination with cloud formation is a potentially strong feedback mechanism with uncertain sign. Recent overestimates of global warming on the basis of some climate models are sometimes argued to be accounted for by this kind of feedback mechanism having a negative sign, i.e., dampening the direct warming effect.

relates to the consequences of introducing sub-optimal taxes. Specifically, how important is it that all fossil fuels are taxed? Would it be enough to tax only some, like coal, and disregarding oil? A motivation for this question is that oil taxes have obvious and strong consequences for the international distribution of income while this is arguably not the case for taxes on coal. Furthermore, while the greenhouse effect is independent of the source of the emitted carbon the market for conventional oil, non-conventional fossil reserves and coal are very different. At one end of the spectrum, conventional oil is very cheap to extract but exists in quite limited supply. The opposite is true for coal (and non-conventional oil and natural gas); here, the price is close the extraction cost but exists in huge quantities all over the world. Our answer to this question is that whether a global carbon tax applies to oil or not is immaterial for the effect of the tax on climate change.

A related question is how effective taxes are if they are only applied in parts of the world. For example, suppose only Europe introduces a carbon tax (or, equivalently, adopts a well-functioning quantity regulation, with emission trading with an implied market price corresponding to the tax). We show that such a partial tax has effects as long as it is levied on coal, but that the effects are weak.

The second set of questions relate to technical change in energy production. As already mentioned, many sources of fossil fuel are costly to extract. These costs change over time. For example, coal production per unit of labor input increased by an average of 3.2% per year between 1949 and 2011.[3] More recently, technological advancement in hydraulic fracturing ("fracking") has made large reserves profitable to extract.[4] Also, the cost of producing renewable energy has recently fallen at a high pace. These trends have been portrayed as a great threat to the climate (cheaper fossil fuels) and as the salvation (cheaper green energy): a technology race, thus, with high stakes! Our aim is thus to analyze the consequences of different trends in the production of different sources of energy. We do this by including the different energy sources and their corresponding cost-reduction technologies into the neoclassical growth framework in a way that is quantitatively motivated, i.e., such that one can calibrate the relevant parameters.

Under the assumption of exogenous trends for the extraction technologies, we find that that if the technology for extracting fossil fuel evolves slowly, thus leading to increasing relative prices, this has an effect similar to the introduction of the optimal tax. Cheaper green energy, on the other hand, is not in itself sufficient for mitigating global warming in this case.

[3] Source: US Energy Information Administration, (20??).

[4] For a discussion of fracking and its effects on the oil market, see Bornstein, Krusell, and Rebelo (2017).

The assumption of exogenous technology trends is clearly unsatisfactory, however. Therefore, we endogenize technical change in energy production using a framework of directed technical change.[5] In this endeavor, we follow Hassler, Krusell, and Olovsson (2016) and simply add the choice of what extraction technologies to improve. Here, we consider international spillovers: advances in one region in, say, coal production are fully available (with a lag) in all other regions. We then use the constructed model to analyze how energy taxes (on fossil fuel-based energy or, if levied on green energy, subsidies) interact with cost-reducing technical change.

Our first, and quite surprising, finding is that coal taxes that are proportional to the coal price are completely impotent. A tax on coal increases the incentive to reduce coal production costs and the combined effect is to make after-tax costs independent of the tax. However, this result is overturned when we consider per-unit taxes (tax per ton of coal). Then, it turns out that with our calibration of the inter-fuel elasticity of substitution, an increase in the coal tax has a large negative effect on the incentives to reduce costs. We find that also a quite modest coal tax would make it unprofitable to spend resources on reducing the cost of coal production. Endogenous technical change will then reinforce the effectiveness of carbon taxes.

Our little contribution stands on the shoulders of giants. We have already mentioned Nordhaus's long line of work and Dasgupta and Heal (1976). Hémous (2016) analyzes unilateral environmental policies in a world with several regions. Another recent contribution with a global growth model with several regions analyzing climate policy is Hildebrand and Hildebrand (2017). Our extension to endogenous technical change draws fundamentally on Romer (1986) but a more proximate precursor is the seminal paper on the importance of directed technical innovation for climate change is Acemoglu et al. (2012) and also Acemoglu et al. (2014).

The paper is organized as follows: in section 2 the basic model with exogenous growth is presented. Section 3 describes how we calibrate the model and how we use it to answer the questions discussed above. In section 4 we introduce endogenous technical change in energy production and analyze how the incentives to reduce costs of energy production interact with energy taxes. Section 5 discusses our results and concludes.

[5] For a recent discussion of the importance of endogenous cost reductions in fossil fuel production, see IMF (2017).

2 Model

2.1 Economy

The world consists of r regions of which $r-1$ have no endowments of conventional oil. These regions are called *oil consumers.* Region 1 is endowed with a finite amount of oil that it extracts and sells to the rest of the world. It is called the oil producer. Oil is the only commodity that is traded on the world market and there no international capital market. The oil consuming countries are similar in the sense that the only difference between them is parametric (size and productivity).

In all regions $i \in \{1, 2, ..., r\}$, there is a representative consumer with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_{i,t}). \tag{1}$$

The oil consuming regions use an aggregate production functions for the final good $Y_{i,t}$ given by

$$Y_{i,t} = A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}$$

where $A_{i,t}$ is factor productivity, $L_{i,t}$ is labor used in final-good production, $K_{i,t}$ is the capital stock and $E_{i,t}$ energy services.

Energy services are provided by firms that act competitively with a CRS production function in the n different energy inputs

$$E_{i,t} = \mathcal{E}(e_{1,i,t}, \ldots, e_{n,i,t}) = \left(\sum_{k=1}^{n} \lambda_k \left(e_{k,t}\right)^{\rho} \right)^{\frac{1}{\rho}}. \tag{2}$$

Here, $e_{1,i,t}$ is country $i's$ import of oil in period t. The other energy sources $e_{2,i,t}, ..e_{n,i,t}$ are energy sources produced domestically within the region using a production technology that is linear in the final good. To produce $e_{k,i,t}$ units of energy source $k \in \{2, ..., n\}$ in region i and period t, $p_{k,i,t}$ units of the final good is required. We will later endogenize productivity in energy production but for now let it be exogenous.

Final goods not used for energy production are consumed or invested. Therefore, the resource constraint for the final good is

$$C_{i,t} + K_{i,t+1} = A_{i,t} L_{i,t} K_{i,t}^{\alpha} E_{i,t}^{\nu} - p_{1,t} e_{1,i,t} - \sum_{k=2}^{n} p_{k,i,t} e_{k,i,t} + (1-\delta) K_{i,t}.$$

We assume the final good is identical in all countries allowing us to express the world market price of oil $p_{1,t}$ in terms of the global final good.

Finally, region 1 produces oil without any resource cost but faces the constraints where R_t is the remaining stock of oil in ground

$$\begin{aligned} R_{t+1} &= R_t - \sum_{i=2}^{n} e_{1,i,t}, \\ R_t &\geq 0 \forall t, \\ C_{1,t} &= p_{1,t} \left(R_t - R_{t+1} \right). \end{aligned} \tag{3}$$

2.2 Carbon circulation

The use of energy leads to emission of CO_2. Specifically, emissions from region i in period t are

$$M_{i,t} = \sum_{k=1}^{n} g_k e_{k,i,t}$$

where g_k measure how dirty energy source k is.[6] We measure fossil energy sources in terms of their carbon content, implying that for them $g_{k,i,t} = 1$. Conversely, purely green energy sources have $g_{k,i,t} = 0$ and we could also have intermediate cases.

We follow Golosov et al. (2014), and model the law-of-motion for the atmospheric excess stock of carbon S_t as

$$S_t = \sum_{s=0}^{t} (1 - d_{t-v}) \sum_{i} M_{i,t}$$

where

$$1 - d_s = \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^s$$

measures carbon depreciation from the atmosphere. Specifically, the share of emissions that remains forever in the atmosphere is φ_L, the share that leaves the atmosphere within a period is $1 - \varphi_0$ and the remainder $(1 - \varphi_L) \varphi_0$ depreciates geometrically at rate φ.

2.3 Climate

The climate is affected by the concentration of CO_2 via the greenhouse effect. Golosov et al. (2014) show that the effect of the CO_2 concentration on productivity is fairly well captured

[6] It is straightforward to allow time and region dependence of g_k.

by a log-linear specification. Therefore, we assume that

$$A_{i,t} = e^{\left(z_{i,t} - \gamma_{i,t} S_{t-1}\right)}. \tag{4}$$

$\gamma_{i,t}$ is region specific parameter capturing the region-specific and possibly time-varying sensitivity to atmospheric CO_2 concentration. Note that this specification implies that the marginal damage per unit of excess carbon in the atmosphere is a constant share of net-of-damage output given by $\gamma_{i,t}$. The value $\gamma_{i,t}$ is affected positively by i) the sensitivity of the global mean temperature to changes in the CO_2 concentration, ii) the sensitivity of the regional climate to global mean temperature, and iii) the sensitivity of the regional economy to climate change.

We assume that the climate damage appears with a lag of one period. Given that climate change is a slow-moving process and that the evolution of the atmospheric CO_2-concentration is very sluggish this is immaterial for the dynamics of climate damages. However, as we will see below, the computation of the equilibrium allocation is simplified by the introduction of this lag.

Behind the specification of climate damages in (4) is a logarithmic effect of CO_2 on the energy budget (Arrhenius, 1896). This, in turn, gives a logarithmic relation between the global mean temperature T_t and the stock of carbon in the atmosphere

$$T_t = \frac{\lambda}{\ln 2} \ln\left(\frac{S_t + S_0}{S_0}\right).$$

Here, λ is the climate sensitivity and S_0 the pre-industrial atmospheric carbon stock. We disregard dynamics in the relation between S_t and T_t although that would be straightforward to include.

2.4 Markets and equilibrium

We assume all agents are price takers. Markets within regions are complete and perfect. Trade between regions is only allowed in oil.

Consider first the oil producing region. We assume that there are many oil producers operating under perfect competition. The representative oil producer chooses how much oil to keep in the ground for next period, R_{t+1} taking the world market price of oil as given. Using the last line of (3) to substitute for $C_{1,t}$ in (1) and taking the first order condition with respect to subject to R_{t+1} yields

$$\frac{1}{R_t - R_{t+1}} = E_t \frac{\beta}{R_{t+1} - R_{t+2}}.$$

This is solved by $R_{t+1} = \beta R_t$ implying $C_{1,t} = p_{1,t} (1 - \beta) R_t$. Note that even if $p_{1,t}$ is stochastic, it has no effect on oil supply. The reason is that the income and substitution effects exactly cancel with logarithmic preferences.

The assumption of competitive and frictionless markets imply that we can write the behavior of energy service providers as the solution to the cost-minimization problem

$$\min_{e_{k,i,t}} \sum_{k=1}^{n} p_{k,i,t} e_{k,i,t} - \Lambda_{i,t} \left(\left(\sum_{k=1}^{n} \lambda_k (e_{k,t})^{\rho} \right)^{\frac{1}{\rho}} - E_{i,t} \right) \tag{5}$$

where we note that by construction the Lagrange multiplier $\Lambda_{i,t} = P_{i,t}$, the price index of energy services.

The FOC for $e_{k,i,t}$ yields for $k \in \{2, n\}$

$$e_{k,i,t} = E_{i,t} \left(\frac{P_{t,i} \lambda_k}{p_{k,i,t}} \right)^{\frac{1}{1-\rho}} \tag{6}$$

and similarly oil consumption

$$e_{1,i,t} = E_{i,t} \left(\frac{P_{t,i} \lambda_1}{p_{1,i,t}} \right)^{\frac{1}{1-\rho}} \tag{7}$$

Using this in the expenditure function, we have

$$P_{t,i} = \left(p_{1,t}^{\frac{\rho}{\rho-1}} \lambda_1^{\frac{1}{1-\rho}} + \sum_{k=2}^{n} p_{k,i,t}^{\frac{\rho}{\rho-1}} \lambda_k^{\frac{1}{1-\rho}} \right)^{\frac{\rho-1}{\rho}} \tag{8}$$

Producers of the final good maximize profits taking $P_{t,i}$ as given, implying that

$$P_{t,i} = \nu \frac{A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}}{E_{i,t}},$$

which we can solve for energy service use

$$E_{i,t}=\left(\nu\frac{A_{i,t}L_{i,t}^{1-\alpha-\nu}K_{i,t}^{\alpha}}{P_{t,i}}\right)^{\frac{1}{1-\nu}}.$$

Output net of energy expenses is $(1-\nu)\,Y_{i,t}\equiv\hat{Y}_{i,t}$.Note, however, that the shares of spending on the different energy sources is not necessarily constant unless $\rho=0$, implying that the overall production function is Cobb-Douglas in all inputs.

Households supply labor inelastically and maximize (1) subject to the budget constraint

$$C_{i,t}+K_{i,t+1}=w_{i,t}L_{i,t}+r_{i,t}K_{i,t}+(1-\delta)\,K_{i,t}$$

where $w_{i,t}=(1-\alpha-\nu)\,\frac{Y_{i,t}}{L_{i,t}}$ and $r_{i,t}=\frac{\alpha Y_{i,t}}{K_{i,t}}$ implying that $w_{i,t}L_{i,t}+r_{i,t}K_{i,t}=\hat{Y}_{i,t}$

Now, assume $\delta=1$, and define the savings rate out of net output as $s_{i,t}=\frac{\hat{Y}_{i,t}-C_{i,t}}{\hat{Y}_{i,t}}$then we can write the Euler equation for the households in oil consuming countries as

$$\begin{aligned}\frac{C_{i,t+1}}{C_{i,t}} &= \beta\left(\frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}}\right)\\ \frac{(1-s_{i,t+1})\,(1-\nu)Y_{i,t+1}}{(1-s_{i,t})\,(1-\nu)Y_{i,t}} &= \beta\frac{\alpha Y_{i,t+1}}{s_{i,t}(1-\nu)Y_{i,t}}\end{aligned}$$

implying that the savings rate is constant across time and regions at $s=\frac{\alpha\beta}{1-\nu}$. Finally, assume labor supply in each country is exogenous and normalized to unity.[7]

Proposition 1 *In each period the allocation is determined by the state variables* $K_{i,t},R_t$ *and* S_{t-1} *such that i) the capital savings rate is constant at* $\frac{\alpha\beta}{1-\nu}$*, ii) oil supply is* $(1-\beta)R_t$*, iii) energy prices are* $P_{i,t}=\left(p_{1,t}^{\frac{\rho}{\rho-1}}\lambda_1^{\frac{1}{1-\rho}}+\sum_{k=2}^{n}p_{k,i,t}^{\frac{\rho}{\rho-1}}\lambda_k^{\frac{1}{1-\rho}}\right)^{\frac{\rho-1}{\rho}}$*, iv) energy service demand is* $E_{i,t}=\left(\nu\frac{e^{\left(z_{i,t}-\gamma_{i,t}S_{t-1}\right)}L_{i,t}^{1-\alpha-\nu}K_{i,t}^{\alpha}}{P_{t,i}}\right)^{\frac{1}{1-\nu}}$*, v) domestic fuel demand is* $e_{k,i,t}=E_{i,t}\left(\frac{P_{t,i}\lambda_k}{p_{k,i,t}}\right)^{\frac{1}{1-\rho}}$*, vi) and regional oil demand is* $e_{1,i,t}=E_{i,t}\left(\frac{P_{t,i}\lambda_1}{p_{1,i,t}}\right)^{\frac{1}{1-\rho}}$*. The price of oil is determined from equilibrium in the world oil market* $\sum_{i=2}^{r}e_{1,i,t}=(1-\beta)\,R_t$*. The laws-of-motion for the state variables are* $K_{i,t}=\frac{\alpha\beta}{1-\nu}\hat{Y}_{i,t}$*,* $R_{t+1}=\beta R_t$ *and* $S_t=\sum_{v=0}^{t}(1-d_{t-v})\sum_i M_{i,t}$

Two things should be noted. First, the allocation is determined sequentially without any

[7]It would very simple to include endogenous labor supply.

forward-looking terms. Second, given a world market price of oil, all equilibrium conditions have closed form solutions. Thus, in each period, finding the equilibrium is only a matter of finding the equilibrium oil price where supply is predetermined at $(1-\beta) R_t$.

2.5 Taxation

A key purpose of the analysis is to analyze the consequences of taxing fossil fuel and in particular how effective partial taxation is. To this end, we allow each oil consuming region to tax the use of fossil energy inputs. Specifically, we allow regions to set a carbon tax rate $\tau_{i,t}$ implying that the cost for the energy service provider of using energy type k, becomes $(1+\tau_{i,t}g_k) p_{k,i,t}$ in the case of *ad valorem* taxes $\tau_{i,t}g_k + p_{k,i,t}$ with per unit taxes.

The direct consequence of adding taxes is that the price of energy and the fuel mix changes. These changes are straightforward to calculate. The prices $p_{1,t}$ and $p_{k,i,t}$ are replaced by after tax prices in (8), (7) and (6). The aggregate use of energy services remain

$$E_{i,t} = \left(\nu \frac{A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha}}{P_{t,i}}\right)^{\frac{1}{1-\nu}}$$

but using the tax-inclusive energy price.

A complication that arises with taxes is that it matters what is done with the revenues. If they are redistributed lump-sum to the households, the savings rate will no longer be exactly $\alpha\beta/(1-\nu)$. However, our numerical analysis suggests that the consequence of this for the savings rate is negligible. The reason is that since the income share of energy is small (ν is in the order of a few percent), energy taxes cannot generate much revenue. We are thus neglecting the consequence of taxes on savings rates. Since households are not allowed to re-optimize there savings, this underestimates the value of introducing taxes, but we argue that the consequence of this is minimal. The alternative of simply assuming that government revenues are waisted will not be used since it would seriously underestimate the welfare value of carbon taxes. Given the approximation, all other features of proposition 1 remains intact.

3 Calibration and simulation

Let us now calibrate the model in order to understand its quantitative implications. We set $r = 5$ in order to consider a world consisting of four oil-consuming regions with aim of representing Europe, U.S., China and Africa. The number of fuel inputs into energy production n is set to three. As mentioned above, the first fuel represents oil. In addition we let fuel type two represent coal and type three green (renewable) energy. Measuring oil and coal in carbon units imply that $g_{k,i,t} = 1$ for $k = 1, 2$ and 0 for $k = 3$.

We use a discount factor of 0.985^{10} with the understanding that a period is a decade. In the final good production function, we set $\alpha = 0.3$ and the fuel income share ν to 0.04. The production of energy services is calibrated in the following way. For the elasticity of substitution between the three sources of energy, we use a metastudy (Stern, 2012) of 47 studies of interfuel substitution. The unweighted mean of the oil-coal, oil-electricity and coal-electricity elasticities is 0.95. This elasticity imply $\rho = -0.058$ which we use as the main case. In the next section, we will return to the calibration of this parameter.

To calibrate the $\lambda' s$ we need prices and quantities of the three types of fuel. Using the demand equations (6) and (7) we derive

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{e_{1,t}}{e_{2,t}}\right)^{1-\rho} \frac{p_{1,t}}{p_{2,t}}.$$

We use world market prices from Golosov at al (2014), where the coal price is \$74/ton and the (pre-financial crises) oil price \$70/barrel, corresponding to \$70*7.33 per ton. This implies that the relative price between oil and coal in units of carbon is 5.87. Using the same source for the ratio of global oil to coal use in carbon units we find that $\frac{\lambda_1}{\lambda_2} = 5.348$. For green we use data for the sum of nuclear, hydro, wind, waste and other renewables from Golosov at al (2014) and stick with their (somewhat arbitrary assumption) of a unitary relative price between oil and renewables. This gives $\frac{\lambda_1}{\lambda_3} = 1.527$. Together with the normalization $1 = \lambda_1 + \lambda_2 + \lambda_3$ this yields $\lambda_1 = 0.543, \lambda_2 = 0.102$, and $\lambda_3 = 0.356$. Perhaps not fully consistent with the calibration of common $\lambda' s$ from global prices and use, we allow for different extraction costs of coal in the different regions. Based on IES World Energy Outlook 2010, we set the relative extraction cost of coal in China and the US to 1 and that in Africa to 0.8. Based on evidence in Radetzki, 1995, "Elimination of West-European coal subsidies", Energy Policy, 23:6, we set the relative cost in Europe to 2.

BP (2010) reports that global proved reserves of oil are 181.7 Gigatons. However, these figures only aggregate reserves that are economically profitable to extract at current economic and technical conditions. Thus, they are not aimed at measuring the total resource base taking into account in particular technical progress, and they do not take into account the chance that new profitable oil reserves will be discovered. Rogner (1997) instead estimates global reserves taking into account technical progress, ending up at an estimate of over

5000 Gigatons of oil equivalents.[8] Of this, around 16% is oil, i.e., 800 Gtoe. We take as a benchmark that the existing stock of oil is 400 Gtoe, i.e., somewhere well within the range of these two estimates. We set the carbon content per unit of weight to unity in both oil and carbon.

We allow different climate damage parameters. Based on Hassler and Krusell (2012), we set the damage coefficients to $\gamma_{Ch} = 0.147\bar{\gamma}, \gamma_{Eu} = 1.89\bar{\gamma}, \gamma_{US} = 0.3\bar{\gamma}, \gamma_{Af} = 2.61\bar{\gamma}$, and use Golosov et al. (2014) calculation for the global parameter $\bar{\gamma} = 2.38 * 10^{-5}$. We also follow Golosov et al. (2014) for the carbon cycle parameters setting $\varphi_L = 0.2$, $\varphi_0 = 0.393$ and $\varphi = 0.0228$. We assume initial GDP is 70 trillion US$ and set initial productivity so the initial GDP is approximately the same in the Europe, the U.S. and China, while it is a third as high in Africa. Initial capital is chosen so that the economies approximately are on their balanced growth paths. Productivity in final good production, $e^{z_{i,t}}$, is assumed to grow at 1.5% per year in all regions and in the base case, we assume the cost of producing coal and green fuel is constant in terms of the final good. This rate of productivity increase imply a GDP growth rate of about 2% per year.

3.1 Results

3.1.1 Taxes

In this section we present simulation results for a set of different policies. We start by comparing a *laissez faire* scenario to four different tax policies. In all cases we use an initial tax of 56.9 US$ per ton of carbon, which following Golosov et al. (2014) internalizes the externality given the chosen parameters. In all cases, the tax increases by 2% per year, which is approximately equal to the growth rate of GDP. The four different policy scenarios differ in terms of the coverage of the tax. In the first, the tax is applied universally to all fossil fuel. In the second, only Europe applies the fossil fuel tax. The third scenario assumes the tax is applied globally, but only on coal. Finally, the fourth scenario considers a unilateral European coal tax only.

Let us first consider the effectiveness of the different policies in affecting climate change. Figure 1 shows the increase in global mean temperature over the coming 200 years for the five different scenarios. A couple of important results are immediate from the figure. First, global fossil fuel taxes are effective in mitigating climate change. Towards the end of this

[8] By expressing quantities in oil equivalents, the difference in energy content between natural gas, oil, and various grades of coal is accounted for.

century, the difference between laissez faire and global taxation in terms of temperature is 1.4 degrees Celsius and the difference increases substantially thereafter. Second, taxes in Europe only is not effective. The difference in terms of temperature between the two scenarios with taxes only in Europe and *laissez faire* negligible. Third, what matters is taxes on coal. Both the cases with global taxes and only European taxes are identical in terms of climate change regardless of whether oil is taxed or not. The explanation is straightforward. The supply of oil is inelastic and does not respond to taxes, while the opposite is true for coal.

Figure 1. Increase in global mean temperature

Let us now compare the consumption levels in the different scenarios. We express consumption relative to *laissez faire.* Figure 2 represents Europe. As we see, a global carbon tax increases welfare for consumers in Europe. Consumption is always higher and substantially so far into the future. The case of a global coal tax also improves welfare substantially but not really as much as if the tax also includes oil. Since climate change is the same in these two scenarios, the reason for the difference is something else, namely that an oil tax redistributes oil rents from the oil producing to the oil consuming region. This is the reason for why a fossil fuel tax in Europe only increases consumption in Europe while having a negligible impact on the climate.

Figure 2. Consumption relative to no taxes – Europe.

The distributional consequences of the oil tax is clear when we consider the consumption path of the oil producing region, shown in Figure 3. As we see, a tax that includes oil reduces consumption for most of the time considered. Interestingly, a tax on coal only has a small positive impact on the consumption of the oil producers towards the end of the simulation period.

Figure 3. Consumption relative to no taxes – Oil Producers.

The global tax improves welfare in the U.S. but only marginally, see Figure 4. However, the comparison between the global fossil tax and the global coal tax reveals that most of the benefits comes from taxing the oil rents and quite little from dealing with climate change. In China consumption is slightly higher in the current century but slightly lower thereafter as seen in Figure 5.

Figure 4. Consumption relative to no taxes – U.S.

Figure 5. Consumption relative to no taxes – China.

Finally, Figure 6 reveals that the stakes are highest in Africa. The difference between the scenarios are quite large during the 22nd century.

Figure 6. Consumption relative to no taxes – Africa.

3.1.2 Technical change

The previous section indicated that a global coal tax is an efficient way of mitigating climate change. Specifically, while both coal and green increases by a factor of 13 over the coming 200 years without taxes a coal tax that increases by 2% per year effectively stops coal use from growing at all. It peaks towards the end of this century, about 30% higher than today, and then falls at about the same rate as it increased previously.

If we use equation (6) to compute the relative use of coal and green we get

$$\frac{e_{2,i,t}}{e_{3,i,t}} = \left(\frac{\lambda_2}{\lambda_3} \frac{p_{3,i,t}}{(1+\tau_{i,t})\, p_{2,i,t}} \right)^{\frac{1}{1-\rho}} .$$

This equation shows that what drives the relative use of coal and green fuel is the relative price including taxes. Furthermore, this suggests that if technology develops in a way that increases the relative price of coal, this would have similar effects as the tax. We therefore consider two other scenarios. In the first one, green energy becomes cheaper to produce over time by 2% per year ($p_{3,i,t}$ falls by 2% per year in all regions). In the the second scenario we add to this that coal becomes more expensive over time, i.e., $p_{2,i,t}$ increases over time. If we interpret the growth rates of production costs as coming from technological change, it may seem odd that costs increase over time. However, recall that we assume that TFP in the final good sector grows over time, implying that the production of final goods grow by 2% per year. We measure $p_{k,i,t}$ in terms of the final good. Therefore, if technology in the extraction of coal grows slower than that in the final good sector, the cost of coal, $p_{3,i,t}$, would grow over time. Our second scenario is therefore to be interpreted as one in which coal extraction technology is stagnant leading to an increasing relative price of coal, while technological advancements in the green energy sector grows even faster than that in the final good sector.

The results of the simulations are presented in Figure 7. The intuition is, at least partially, supported. The second scenario, in which the relative price of coal increases while the relative price of green falls leads to a path of the global mean temperature that is virtually indistinguishable from the one with global taxes. However, only reducing the price of green does not work. In that scenario coal use is in fact even larger than in the base line *laissez faire* case.

Figure 7. Climate change with different rates of technical change.

We can also compare consumption in the second scenario to that in the case of a global fossil fuel tax. This is done in Figure 8, where we see that the scenario with the alternative technological change produces consumption paths quite little relative to the case of a global coal tax. For oil producers, the scenario with technical change provides higher consumption for most of the simulation period. Also adding a tax on oil shifts consumption from oil producers to oil consumers.

Figure 8. Consumption with tech change relative to that with coal tax.

The finding that directed green technical change is a powerful means of overcoming the problems associated with climate change suggests that we need to analyze the determinants of technical change. Doing this, still in a stylized a transparent way is the purpose of the next section.

4 Endogenous technical change

We now turn to an analysis of endogenous technical change. The purpose of this section is to provide a simple framework for endogenizing the cost of producing the different sources of energy. We maintain that oil is imported and the other sources of energy are domestically produced at costs $p_{k,i,t}$. We allow a tax $\tau_{k,i,t}$ on each fuel. We will consider taxes per unit of fuel and proportional **ad-valorem** taxes.

As in the previous section, there is an energy producing representative firm selling energy services on a competitive market. The only difference relative to the previous section is that we now give the energy producer a possibility to improve its technology. Specifically, it can improve its technology for producing the domestic energy inputs. It can reduce the costs of energy inputs, except oil, subject to a constraint $RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right) \geq 0$.

The problem of the representative energy service provider is now

$$\min_{\{e_{k,i,t}\}_1^n,\{p_{k,i,t}\}_2^n} \sum_{k=1}^{n} (1+\tau_{k,i,t})\, p_{k,i,t} e_{k,i,t} \\ -P_{i,t}\left(\mathcal{E}(e_{1,i,t},\ldots,e_{n,i,t}) - E_{i,t}\right) - \Lambda_{i,t} RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right), \tag{9}$$

where we note that the firm can affect $p_{k,i,t}$ for $k \neq 1$ but takes the oil price, $p_{1,i,t}$ as given. Comparing (9) to (5) reveals that we now have a a set of new choice variables $\{p_{k,i,t}\}_2^n$ and a new constraint $RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)$ with a Lagrange multiplier $\Lambda_{i,t}$. All features of proposition 1 still remain.

The first-order conditions for $e_{k,i,t}$ imply

$$e^*_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{(1+\tau_{k,i,t})\, p_{k,i,t}}\right)^{\frac{1}{1-\rho}} \tag{10}$$

as in the previous section and now, additionally, for $p_{k,i,t}$, $k \neq 1$,

$$(1+\tau_{k,i,t})\, e^*_{k,i,t} = \Lambda_{i,t}\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}. \tag{11}$$

In the case taxes are per unit of the energy source, the objective function (9) is instead

$$\sum_{k=1}^{n} (\tau_{k,i,t} + p_{k,i,t})\, e_{k,i,t}$$

with first-order conditions

$$e^*_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\tau_{k,i,t} + p_{k,i,t}}\right)^{\frac{1}{1-\rho}}$$

and

$$e^*_{k,i,t} = \Lambda_{i,t}\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}. \tag{12}$$

Comparing (11) and (12) we find an important difference. The LHS of both equations represent the value of reducing costs of producing a particular fuel. In the case of *ad-valorem* taxes, this value increases in the tax for given $e^*_{k,i,t}$. This is note the case with taxes per unit. We will return to this below.

4.1 Specializing some assumptions

We now make more specific assumptions on the technology available to the producers of energy services. First, as in the section on exogenous technology, we consider the case of three energy sources, oil, coal and green. Second, we assume that production and extraction cost of coal and green can be improved upon subject to the constraint on the weighted average of the relative improvements. Formally, the constraint is assumed to be

$$RD_{i,t}\left(p_{2,i,t},p_{3,i,t}\right) = \min\left(\varepsilon_{2,i}\ln\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}},0\right)+\min\left(\varepsilon_{3,i}\ln\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}},0\right)+a\geq 0, \quad (13)$$

$$\sum_{k}\varepsilon_{k,i} = 1. \quad (14)$$

It is, of course, not obvious how the R&D production technology should be modelled. However, we think of (13) as an interesting starting point in the spirit of Romer (1986). There a the number of researchers active in R&D determined the rate of technological change. We think of our specification as an extension to directed technical change where the number of R&D workers is fixed and the productivity in improving technology is different for different energy sources. In (13), $\bar{p}_{k,i,t-1}$ are costs if no cost-reductions occur. We assume they are determined by aggregate innovation decisions in the previous period, either in the region or globally. Cost-reductions thus spill over with a one period lag, implying that firms are myopic in their choices of technology. Given that we express costs in terms of final goods, where some productivity increases occur. It is natural to think that $\bar{p}_{k,i,t-1}$ is higher than average costs in the previous period in line with the discussion in the previous section. However, it appears natural to assume that $p_{k,i,t}$ cannot be increased by making $p_{j,i,t}$ larger than the cost no cost-reductions occur. This motivates the min operator in (13).

Now, the specification in (13) implies that for $k=2,3$

$$\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}=\frac{\varepsilon_k}{p_{k,i,t}},$$

which in turn implies

$$\left(1+\tau_{k,i,t}\right)p_{k,i,t}^{*}e_{k,i,t}^{*}=\varepsilon_k\Lambda_{i,t}. \quad (15)$$

Proposition 2 *When first-order conditions for the technology choice are satisfied and taxes are* ad-valorem*, spending on green energy is a fixed fraction of all spending on domestically* *produces energy sources, i.e.,*

$$\frac{(1+\tau_{3,i,t})\, p^*_{3,i,t} e^*_{3,i,t}}{P_{t,i} E_{i,t} - (1+\tau_{1,i,t})\, p_{1,i,t} e^*_{1,i,t}} = \varepsilon_3.$$

Proof: Follows directly from noting that $(1+\tau_{1,i,t})\, p_{1,i,t} e^*_{1,i,t} + \sum_{k=2}^{3} (1+\tau_{k,i,t})\, p^*_{k,i,t} e^*_{k,i,t} = P_{i,t} E_{i,t}$ and using (15).

Furthermore, using the expression (10) in (15) yields for $k = 2, 3$

$$(1+\tau_{k,i,t})\, p^*_{k,i,t} = \left(\frac{\varepsilon_k \Lambda_{i,t}}{E_{i,t}}\right)^{\frac{\rho-1}{\rho}} (P_{t,i} \lambda_k)^{\frac{1}{\rho}}$$

and

$$\frac{(1+\tau_{2,i,t})\, p^*_{2,i,t}}{(1+\tau_{3,i,t})\, p^*_{3,i,t}} = \left(\frac{\varepsilon_2}{\varepsilon_3}\right)^{\frac{\rho-1}{\rho}} \left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho}}.$$

Since the RHS are technological constants, this implies that taxes are unable to change the after tax relative price of domestic fuels provided the FOC's are satisfied. Furthermore, the growth rates of the two production costs are identical.

Since both the price and spendings are independent of taxes, also volumes are independent of taxes, i.e.,

$$\frac{e^*_{2,i,t}}{e^*_{3,i,t}} = \left(\frac{\lambda_3}{\lambda_2} \frac{\varepsilon_2}{\varepsilon_3}\right)^{\frac{1}{\rho}}.$$

We can get an intuition for this result by considering the first-order condition for the choice $p_k, k \neq 1$ given by (11). The LHS of the equation represents the marginal value of cost reductions. As we see, for a given $e^*_{k,i,t}$, this value increases in the tax rate $\tau_{k,i,t}$. The fact that taxes are proportional to the cost of production implies that costs reductions are worth more the higher is the tax, *ceteris paribus.* However, an increased tax also reduces $e^*_{k,i,t}$ and this reduces the value of cost reductions. With the log-linear specification of R&D costs, it turns that the two effects balances each other in such a was as to make R&D nullify the effect of taxes on after tax prices.

This intuition also suggests that if taxes are per unit of the energy sources, rather than proportional to the price (cost), higher taxes should lead to higher costs. This since the positive effect of taxes on the value of cost reductions just described disappears. Therefore, let us now turn to the case of taxes that are levied per unit of the source of energy. In this case, the first-order condition is

$$p_{k,i,t}e^{*}_{k,i,t} = \Lambda_{i,t}\varepsilon_k$$

where

$$e^{*}_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\tau_{k,i,t}+p_{k,i,t}}\right)^{\frac{1}{1-\rho}}.$$

Thus,

$$\frac{p_{2,i,t}\left(\tau_{2,i,t}+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(\tau_{3,i,t}+p_{3,i,t}\right)^{\frac{-1}{1-\rho}}} = \frac{\varepsilon_2}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}}. \tag{16}$$

Now,consider the effect of changes in taxes on coal. Totally differentiating the previous expression and noting that the R&D constraint implies $\frac{dp_{3,i,t}}{dp_{2,i,t}} = -\frac{1-\varepsilon_3}{\varepsilon_3}\frac{p_{3,i,t}}{p_{2,i,t}}$ and evaluating at both taxes being zero yields

$$\left.\frac{dp_{2,i,t}}{d\tau_{2,i,t}}\right|_{\tau_{3,i,t}=\tau_{2,i,t}=0} = -\frac{\varepsilon_3}{\rho}.$$

Thus, when ρ is negative but close to zero, an increase in coal taxes leads to a large increase in the relative price of coal.

4.1.1 Calibration

In addition to the calibration done for the case of exogenous technology, we will use the assumption that first-order conditions are satisfied before the introduction of carbon taxes. Obviously, this assumption is not satisfied in reality – there are both taxes on and substantial subsidies to energy production. However, this calibration serves the purpose of illustrating the quantitative implications of our model.

Consider first Europe, expressed per ton of carbon, the price of coal in Europe is calibrated to 165 US$ and the price of green in carbon oil equivalents to 600 US$. Using the already calibrated values of for λ_2 and λ_3 we can use these prices to calibrate the R&D parameter ε_3 from

$$\left[\frac{(p_{2,i,t})^{1-\frac{1}{1-\rho}}}{(p_{3,i,t})^{1-\frac{1}{1-\rho}}} = \frac{1-\varepsilon_{EU,3}}{\varepsilon_{EU,3}}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}}\right]_{p_{2,i,t}=165,p_{3,i,t}=600,\lambda_2=0.102,\lambda_3=0.356,\rho=-0.058}$$
$$\rightarrow \quad \varepsilon_{EU,3} = 0.778.$$

Now, consider the implications of introducing a carbon tax on coal. Using the R&D constraint, and initially disregarding the constraint on cost increases ($p_{2,i,t} \leq \bar{p}_{2,i,t-1}$), the relation between the two domestic fuel prices must imply that $(p_{2,i,t})^{(1-\varepsilon_{EU,3})}(p_{3,i,t})^{\varepsilon_{EU,3}}$ is constant. Using the calibrated $\varepsilon_{EU,3}$ and the initial prices, this constant is 450.5, yielding $p_{3,i,t}$ as a function of $p_{2,i,t}$

$$p_{3,i,t}(p_{2,i,t}) = 450.5^{\frac{1}{\varepsilon_{EU,3}}}(p_{2,i,t})^{-\frac{1-\varepsilon_{EU,3}}{\varepsilon_{EU,3}}}.$$

Using this relation in (16) yields

$$\frac{p_{2,i,t}(\tau_{2,i,t}+p_{2,i,t})^{\frac{-1}{1-\rho}}}{p_{3,i,t}(p_{2,i,t})^{\frac{2-\rho}{1-\rho}}} = \frac{\varepsilon_2}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}},$$

which implicitly defines $p_{2,i,t}$ as a function of the coal tax. As we see, the coal price is an increasing function of the tax and the green energy price price is falling. The fact that we calibrated a value of $\varepsilon_3 > 1/2$ implies that it requires more R&D resources to reduce the cost of producing green energy than reducing the cost of producing coal. However, both curves are quite steep. Already for a coal tax of around 35 US$ per ton carbon, the coal price becomes as high as the price of green energy. For the coal tax used in the previous section, 56.9 US$ per ton carbon, the coal price is substantially higher than the price of green energy (574 vs. 420 Us $ per ton) rather than only a fourth of the green price without taxes.

Figure 9. Coal and green energy prices in interior R&D optimum as function of coal tax.

Let us now return to the constraint $p_{2,i,t} \leq \bar{p}_{2,i,t-1}$. As discussed above, it seems reasonable to assume that technology cannot regress. With no technical advances in the coal industry, we may assume that costs in terms of the final good increase at about the rate of GDP-growth. Over a decade, the would be a bit over 20% implying that coal prices cannot increase by more than that in a period. The conclusion is then that also a very modest tax on coal would lead to a stagnant coal technology and instead all R&D focusing on green energy.

Let us therefore finally consider the implications of a global coal tax of the same size as in the previous section but noting that this will lead all R&D to focus on reducing the cost of green energy. We use the R&D-parameter calibrated for Europe and endogenize the growth rates of coal and green energy prices from an initial situation of both of them being constant in terms of the output good (the same rate of technical advances as in the final good sector). Then, redirecting R&D towards green, implies (by assumption) that coal prices growth at 2% per year. The price of green energy falls by $\frac{1-\varepsilon_{EU}}{\varepsilon_{EU}} * 2\% = 0.57\%$ per year. In Figure 9, we show the path of the global mean temperature for the case of taxes and endogenous technical change as well as the *laissez faire* scenario (which remains identical to the case of exogenous technical change by assumption). As we see, the temperature rises substantially less with endogenous technical change due to the fact that coal use starts falling immediately. In the case exogenous technical change and a global coal tax, coal used increased over the coming century and peaked around 2090.

Figure 10. Increase in global mean temperature global coal tax with and without endogenous technical change and laissez faire.

Figure 10 shows the path of consumption relative to *laissez faire.* In comparison to the case of exogenous technical change, Europe and Africa gains even more by the introduction of the global coal tax, while the consumption of all other regions remains largely unaffected

Figure 11. Consumption under global coal tax and endogenous technical change relative to laissez faire.

Results so far have been derived under the assumption that first-order conditions define the optimum. However, if the elasticity of substitution $(1-\rho)^{-1}$ is large enough, the energy producing firm may specialize in some fuels. Examining this is the purpose of the next sub-section.

4.2 Corner solutions

Cost-minimization amounts to minimizing the price index $P_{i,t}$ as defined by (8) under the relevant constraints, i.e., solving

$$\min_{p_{2,i,t}\leq \bar{p}_{2,i,t-1}, p_{3,i,t}\leq \bar{p}_{3,i,t-1}} P_{i,t}$$
$$s.t. \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{\varepsilon_2}\left(\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}}\right)^{\varepsilon_3} = e^{-a}.$$

Using the constraint to replace $p_{3,i,t}$ and maximizing over $p_{2,i,t}$, the second derivative evaluated at the FOC is

$$\frac{\rho}{\rho-1}\frac{\varepsilon_2+\varepsilon_3}{\varepsilon_3}\frac{\varepsilon_2}{\varepsilon_3}\frac{1}{(p_{2,i,t})^2}\left((1+\tau_{3,i,t})\,\bar{p}_{3,i,t-1}\left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{-\frac{\varepsilon_2}{\varepsilon_3}} e^{-\frac{a}{\varepsilon_3}}\right)^{\frac{\rho}{\rho-1}} \lambda_3^{\frac{1}{1-\rho}}$$

which is negative iff$\rho > 0$ implying that the FOC then identifies a maximal cost. Then, minimum energy cost arises at one of the corners[9]

$$\left(\bar{p}_{2,i,t-1}e^{\frac{-a}{\varepsilon_2}}, \bar{p}_{3,i,t-1}\right) \text{ and } \left(\bar{p}_{2,i,t-1}, \bar{p}_{3,i,t-1}e^{\frac{-a}{\varepsilon_3}}\right).$$

It is immediate that iff

$$X_{2/3} \equiv \frac{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} e^{\frac{a}{\varepsilon_2}\frac{\rho}{1-\rho}} + 1}{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} + e^{\frac{a}{\varepsilon_3}\frac{\rho}{1-\rho}}} < 1 \tag{17}$$

where $\hat{p}_{2,3} \equiv \frac{(1+\tau_{2,i,t})\bar{p}_{2,i,t-1}}{(1+\tau_{3,i,t})\bar{p}_{3,i,t-1}}\left(\frac{\lambda_2}{\lambda_3}\right)^{-\frac{1}{\rho}}$, costs are lower if the first corner (reducing the cost of producing fuel of type 2) is used. The cut-off where energy firms are indifferent between the corners is

$$\bar{p}_{2,3} \equiv \left(\frac{e^{\frac{\rho}{1-\rho}\frac{a}{\varepsilon_2}} - 1}{e^{\frac{\rho}{1-\rho}\frac{a}{\varepsilon_3}} - 1}\right)^{\frac{\rho-1}{\rho}}.$$

Since $X_{2/3}$ is increasing in $\hat{p}_{2,3}$ when $\rho > 0$, costs are lower (higher) in the corner where costs of fuel 2 are reduced if $\hat{p}_{2,3} < (>) \, \bar{p}_{2,3}$.

A key difference between the result here and in the previous subsection is that the direction of R&D can be permanently shifted by a temporary tax. We can see this by noting that the base-line costs enter in the optimality condition (17). A temporary tax may be required to make R&D switch to renewables. Over time, this leads to $\frac{\bar{p}_{2,i,t-1}}{\bar{p}_{3,i,t-1}}$ increasing and eventually, not tax is required to keep R&D directed towards reducing the cost of renewables. This is the mechanism in Acemoglu et al. (2012).

5 Discussion and concluding remarks

The models we provide in this paper are highly stylized but build on well established macroeconomic principles. Therefore, we do think that the quantitative implications of the models contain valuable information. In particular, development along the lines of these models can complement more complicated existing integrated assessment models. Building on a standard macroeconomic growth model makes it straightforward to extend the model in various dimensions. An example of this is our extension to consider endogenous directed technical change. Here, however, we sail on less well-chartered waters. Our calibration, as well as the specification of the R&D technology for energy production, is rudimentary and preliminary.

[9] We have not yet proved global concavity, but that seems straightforward.

By using microeconomic evidence on the R&D process as in Popp (2002) and Aghion et al. (2016), we believe that our analysis could be improved. Thus, rather than providing trustworthy quantitative answers for the questions we ask, we think of our extension as providing a framework for further analysis.

What are particularly important extensions or robustness checks?

First, we assume away an international bond market. Allowing bond trade between oil-consuming countries will not matter if they are all on a common balanced growth path. In reality, however, it is hard to argue that the key regions in the world are all on a balanced growth path. China grows much faster than Europe and the U.S. since some time and, although there is slowdown in China's growth rate, it is slow and still occurring at a high growth rate. On the other hand, we do not see large current account deficits in China; rather, we observe the opposite. Allowing bond trade between oil consumers and oil producers would change the analysis. In particular, the oil supply would become more fundamentally endogenous, though still in line with Hotelling's general principles. In this regard, it is not clear that bond trade would make the model more realistic.

We have used the clear distinction between oil (existing in fixed supply and have zero extraction costs) and coal (existing in perfectly elastic supply). In reality, there is a spectrum of intermediate fossil fuels, including e.g., deep sea oil, tar sands, and gas and oil available with hydraulic fracturing. It seems likely that technological developments making the extraction of these sources of fuel cheaper is where our focus should be, rather than on coal. Therefore, including more sources of fossil fuel and working toward a serious calibration of the technologies for extracting these kinds of fuel is of first-order importance for future work. Here, work like the one presented in McGlade and Ekins (2015) can be very valuable.

The way we introduced R&D could, obviously, also benefit from further development. In particular, our prediction of the consequences of taxes for how R&D is directed relies on functional form assumptions (the log-linear R&D constraint) and the value of the substitution between different sources of energy. Regarding the former, it is clear that the validity of our specification can and should be examined and possibly changed. Regarding the elasticity of substitution, Acemoglu et al. (2012) demonstrated the strong difference in long-run implications between the elasticity being above or below unity. We also note these differences but find that they are mitigated. In particular, even if the elasticity is below unity, a tax will be effective in moving the direction of R&D for a long time forward. In contrast to Acemoglu et al. (2012), our model imply that a permanent tax is required in order to permanently deal with climate change, but also a quite modest tax can work to redirect R&D.

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Figure 1.

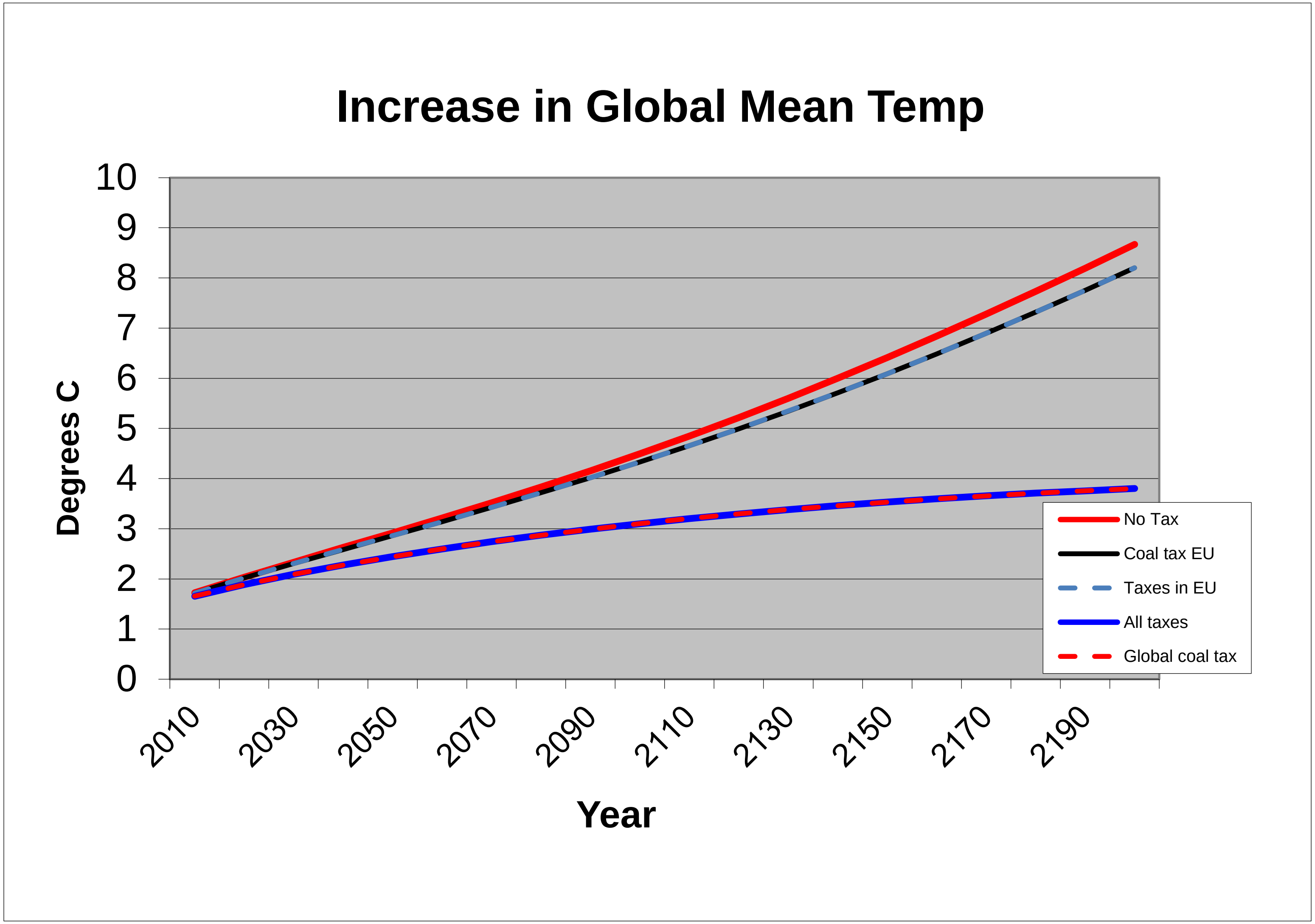
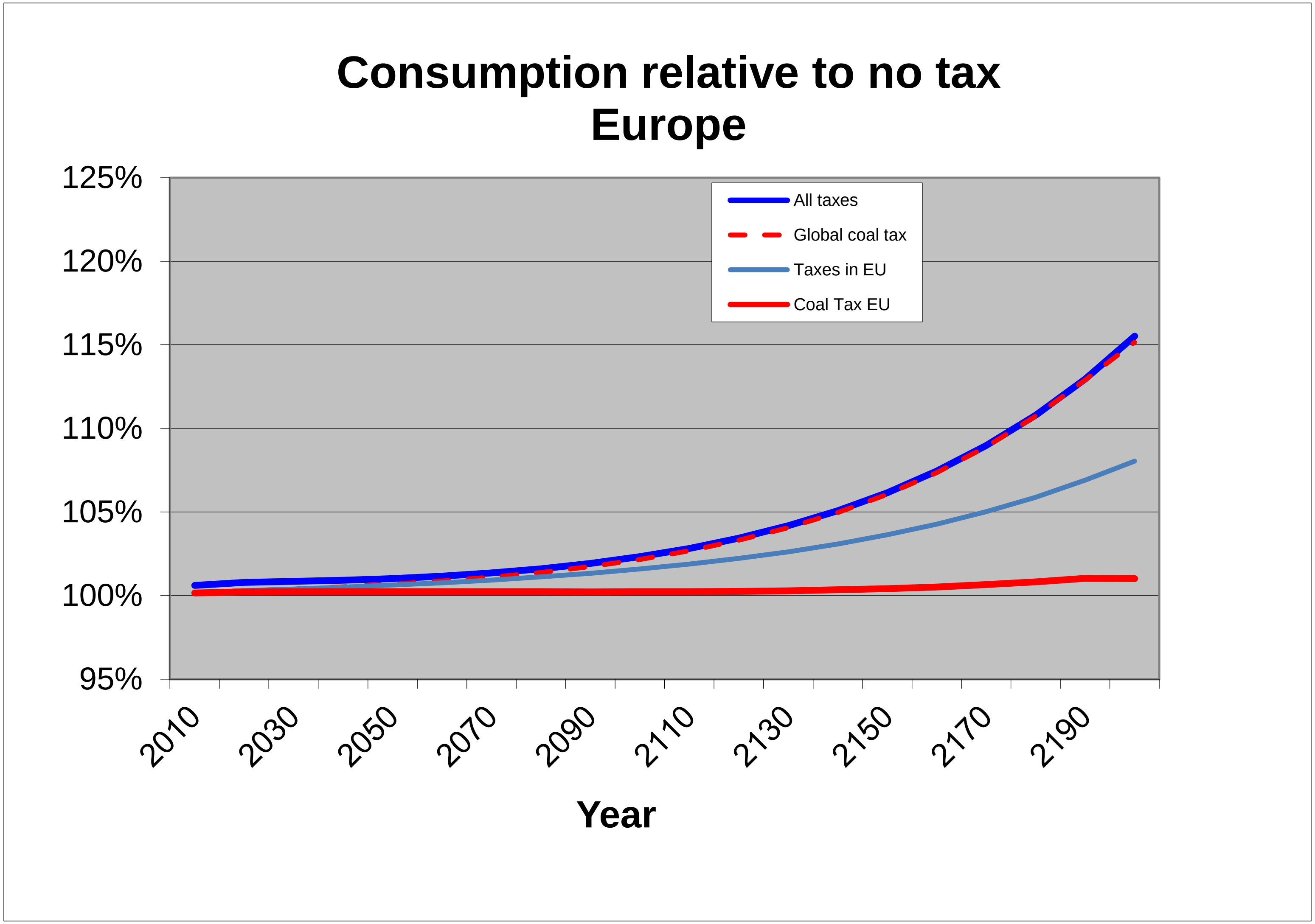
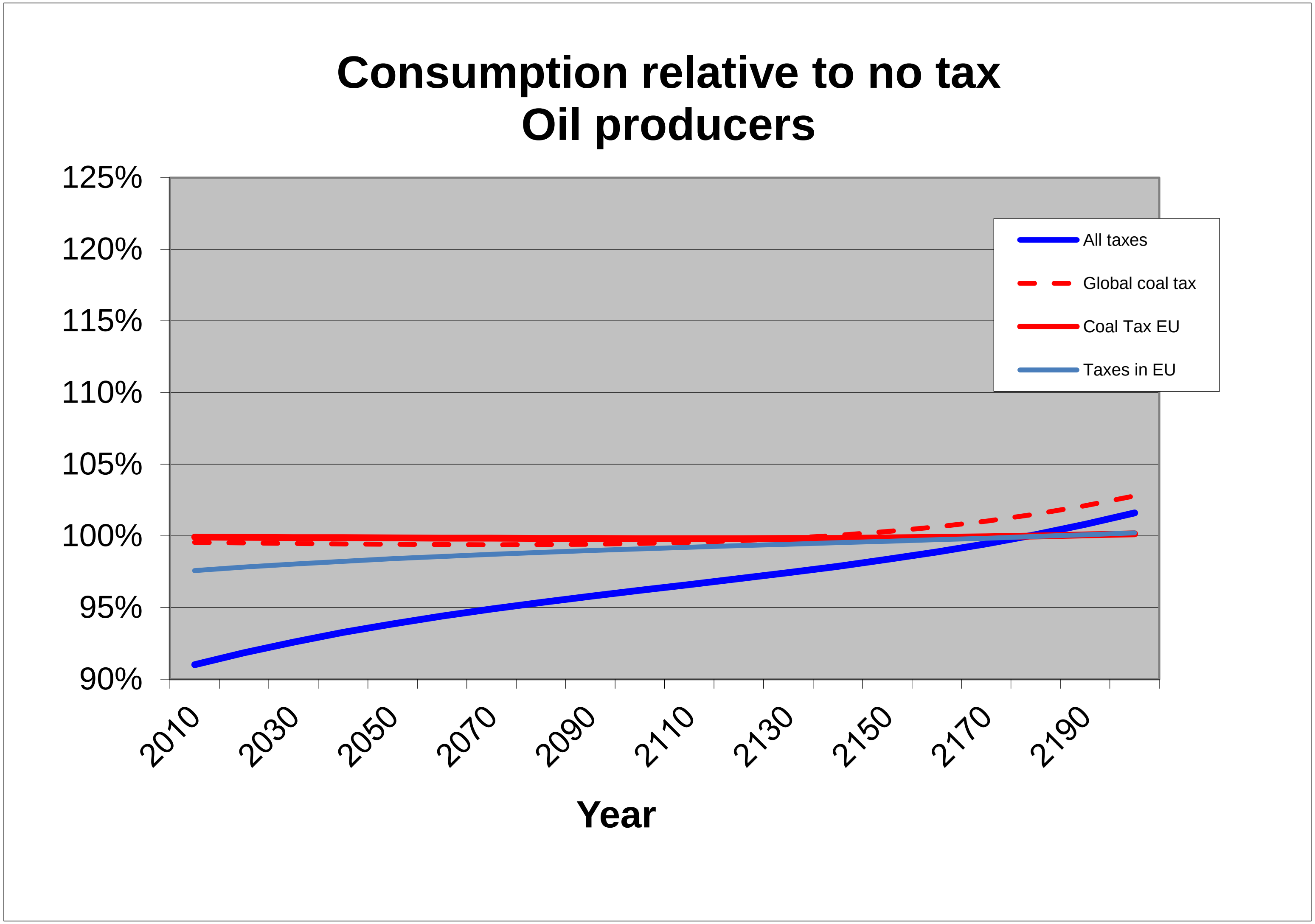

Increase in Global Mean Temp
Degrees C
0
1
2
3
4
5
6
7
8
9
10
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
No Tax
Coal tax EU
Taxes in EU
All taxes
Global coal tax

Figure 2.



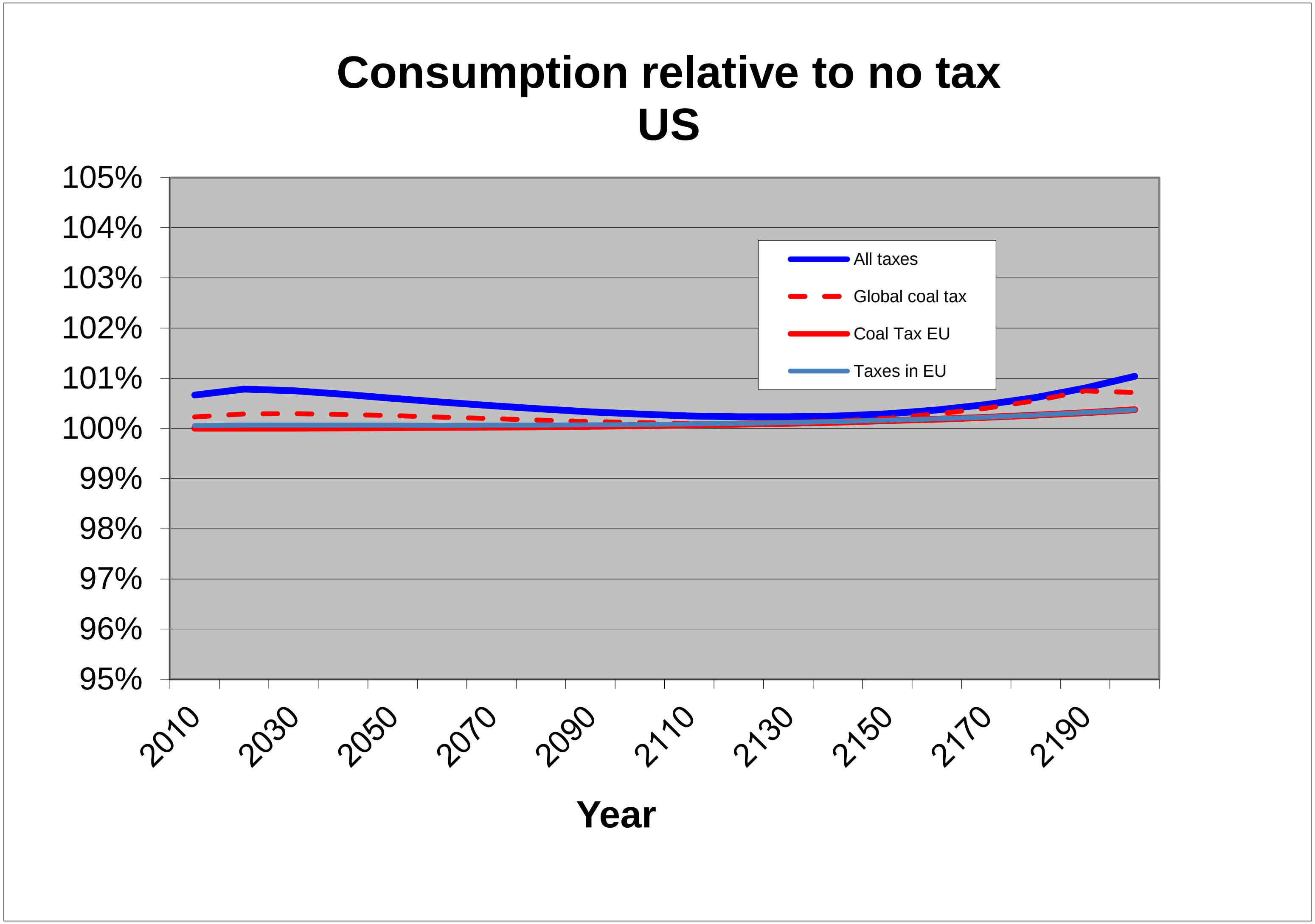
Consumption relative to no tax
Europe
125%
120%
115%
110%
105%
100%
95%
All taxes
Global coal tax
Taxes in EU
Coal Tax EU
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year

Figure 3.



Consumption relative to no tax
Oil producers
125%
120%
115%
110%
105%
100%
95%
90%
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
All taxes
Global coal tax
Coal Tax EU
Taxes in EU

Figure 4.



Consumption relative to no tax
US
105%
104%
103%
102%
101%
100%
99%
98%
97%
96%
95%
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
All taxes
Global coal tax
Coal Tax EU
Taxes in EU

Figure 5.

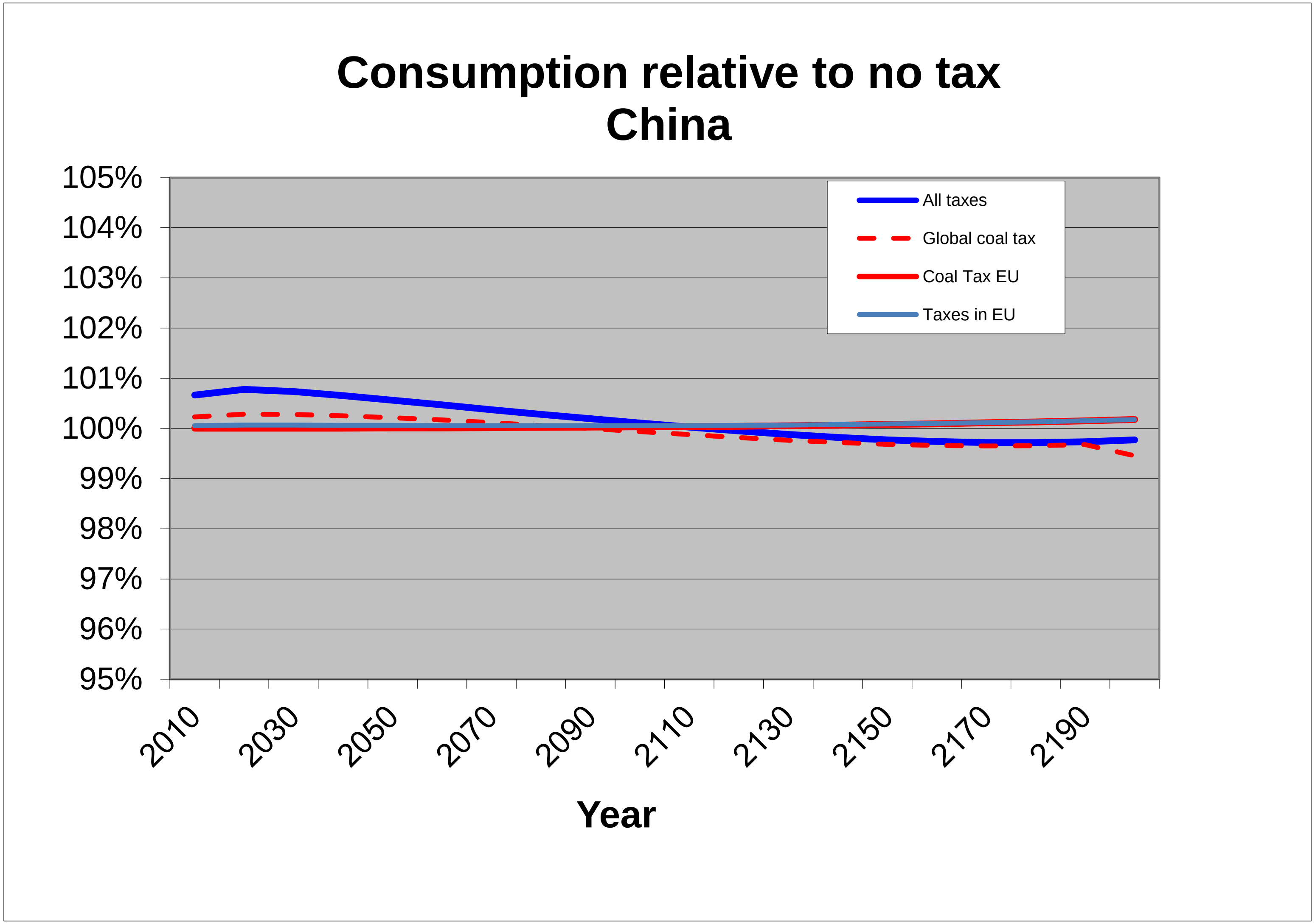

Consumption relative to no tax
China
105%
104%
103%
102%
101%
100%
99%
98%
97%
96%
95%
All taxes
Global coal tax
Coal Tax EU
Taxes in EU
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year

Figure 6.

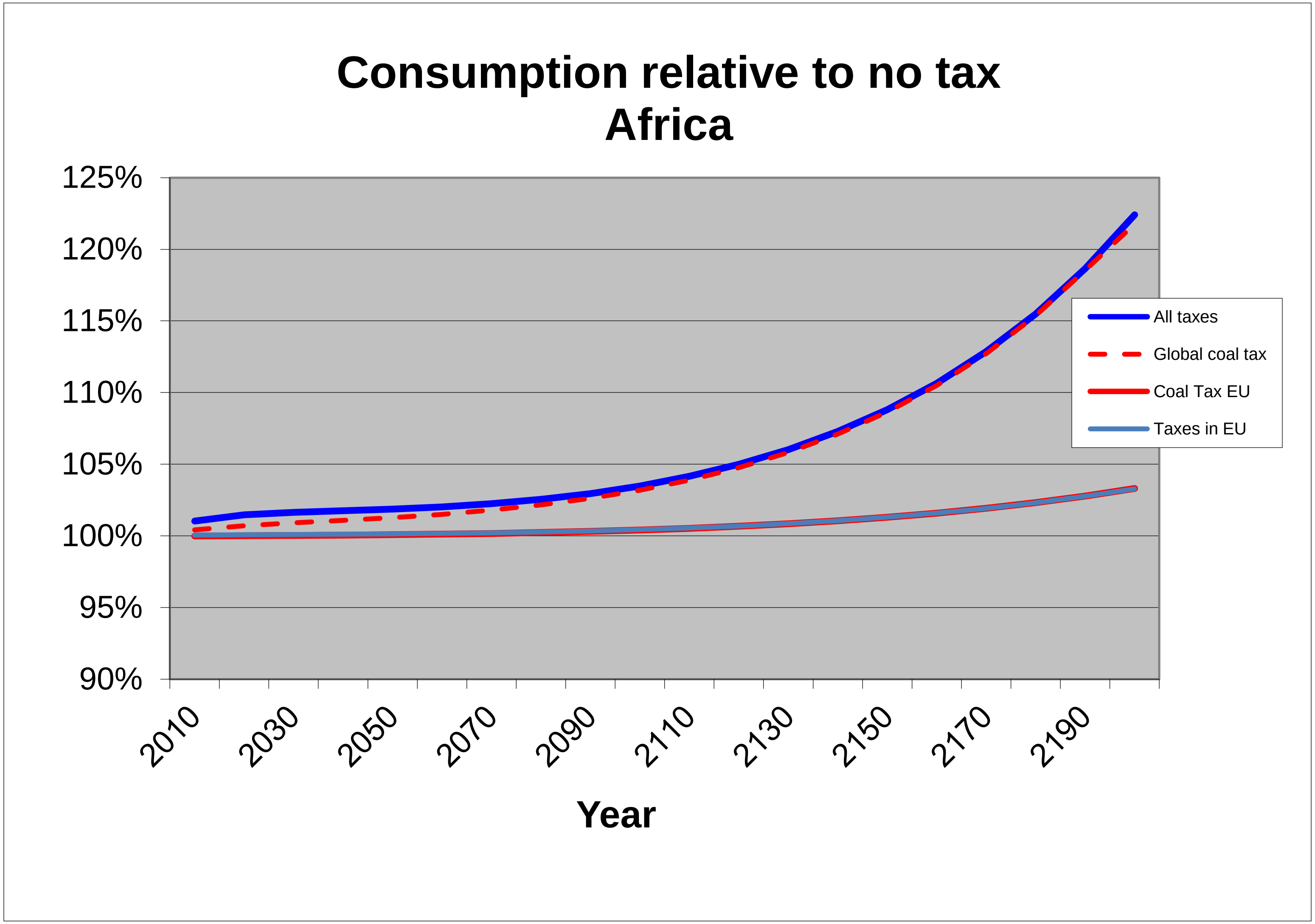

Consumption relative to no tax
Africa
125%
120%
115%
110%
105%
100%
95%
90%
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
All taxes
Global coal tax
Coal Tax EU
Taxes in EU

Figure 7.

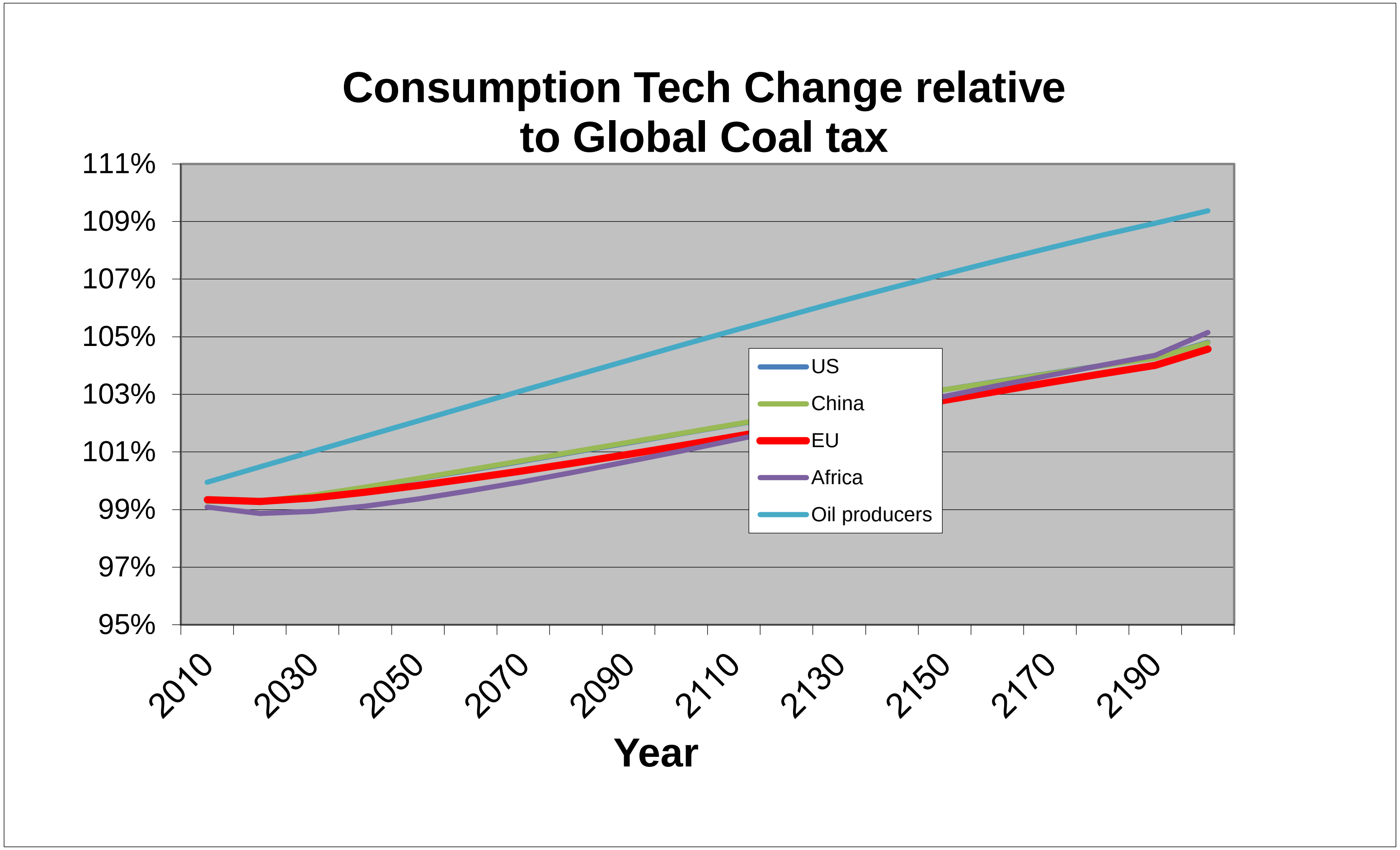

Consumption Tech Change relative to Global Coal tax
111%
109%
107%
105%
103%
101%
99%
97%
95%
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
US
China
EU
Africa
Oil producers

Figure 8.

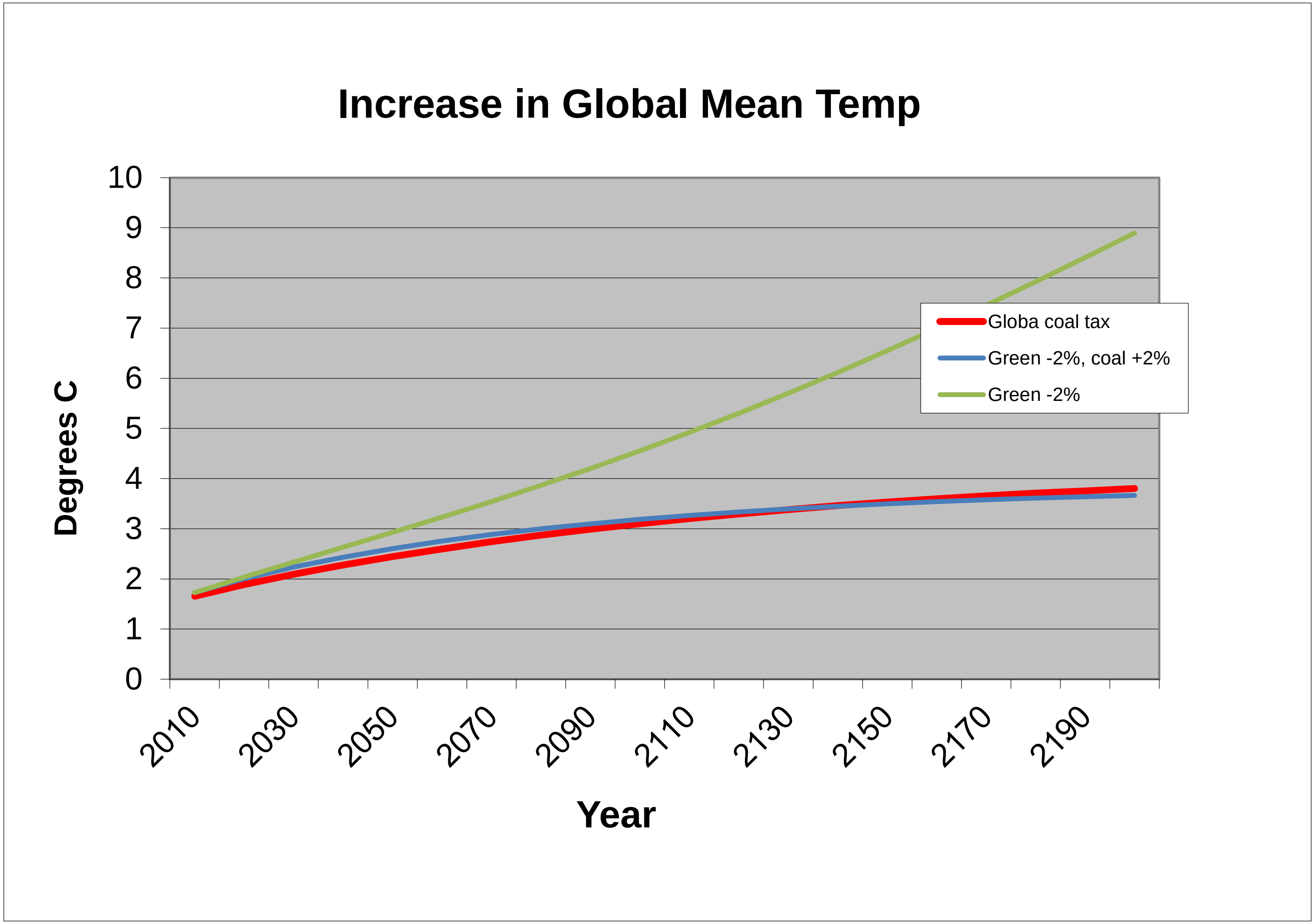
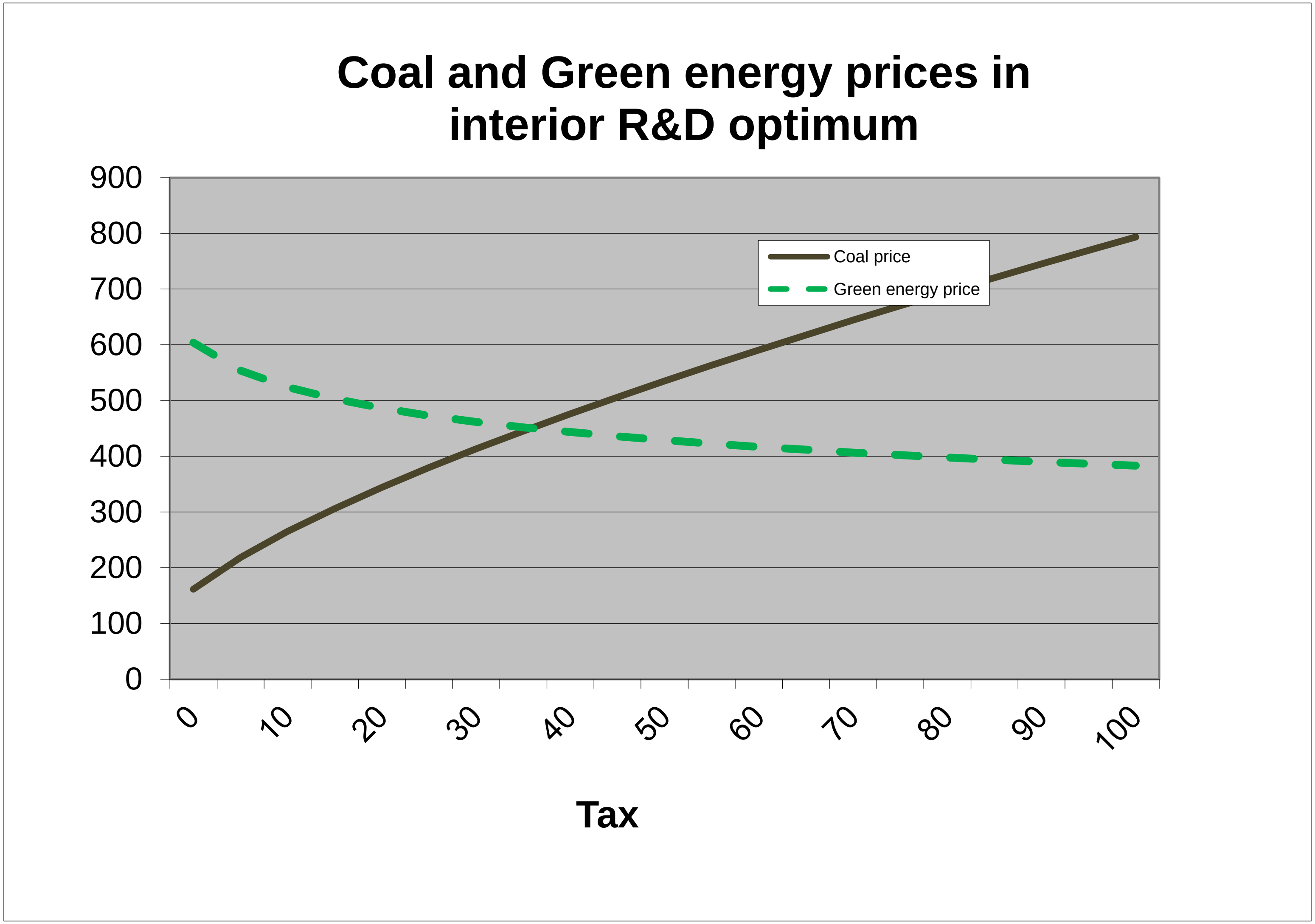
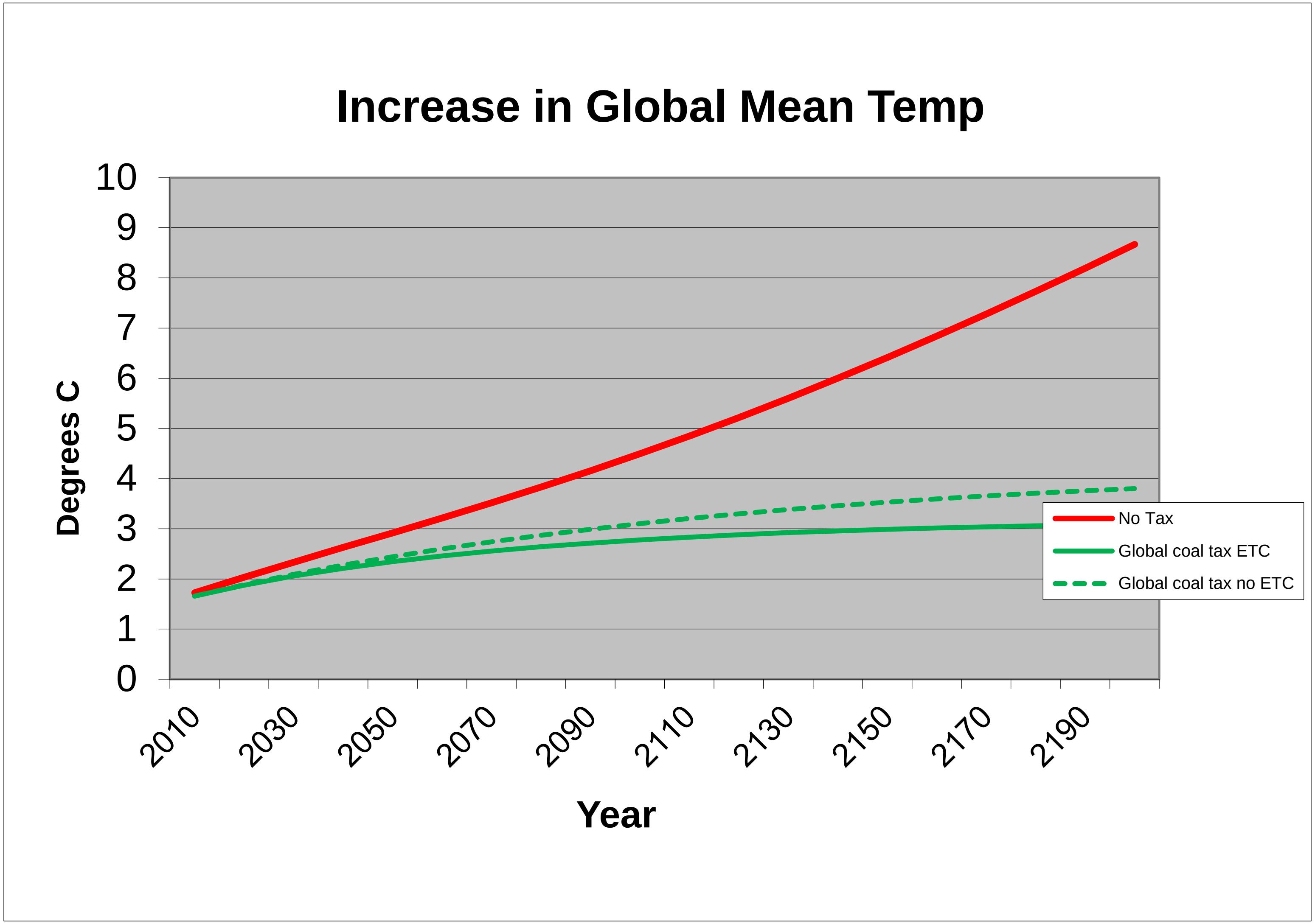

Increase in Global Mean Temp
Degrees C
10
9
8
7
6
5
4
3
2
1
0
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
Globa coal tax
Green -2%, coal +2%
Green -2%

Figure 9.



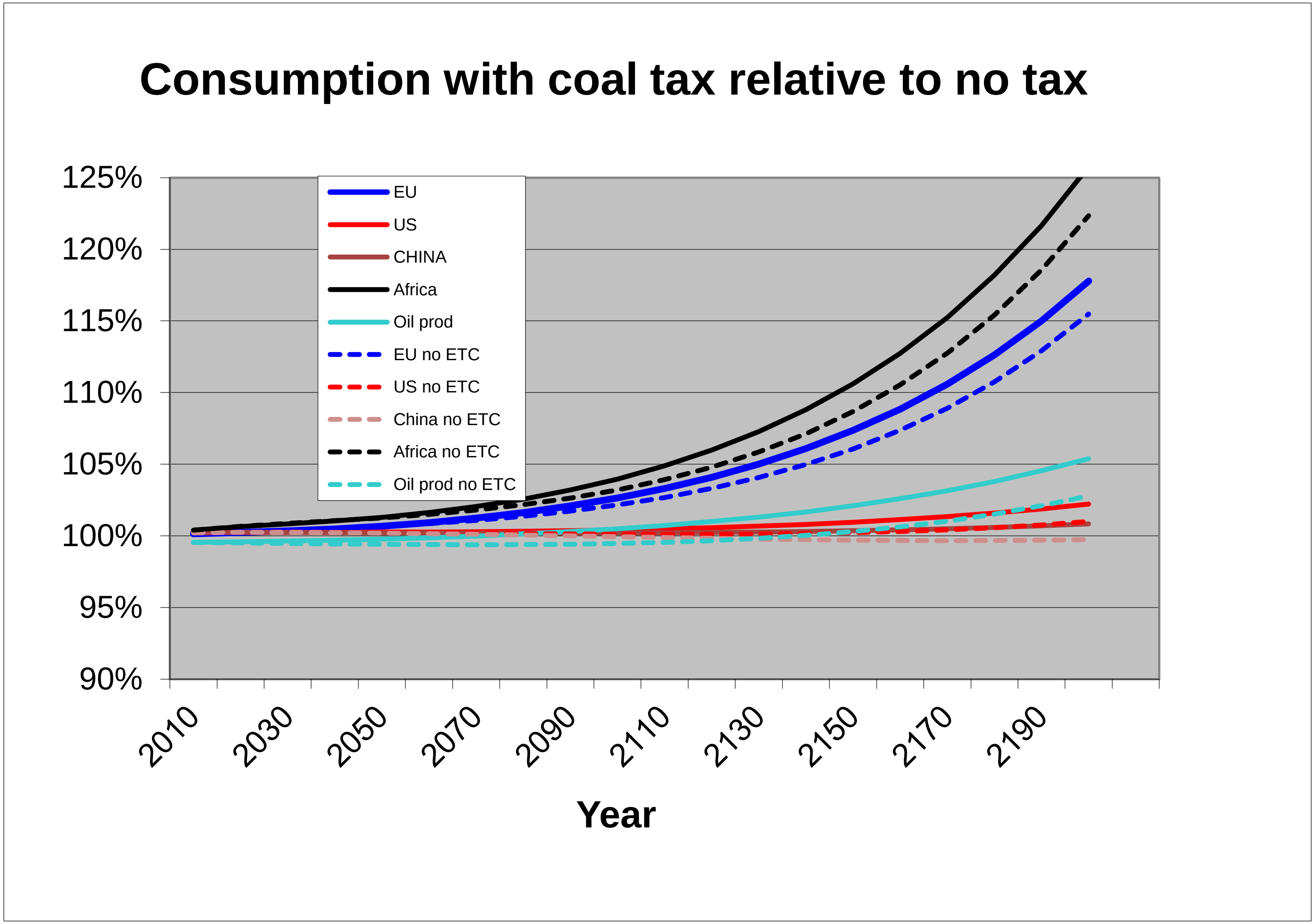
Coal and Green energy prices in interior R&D optimum
900
800
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80
90
100
Tax
Coal price
Green energy price

Figure 10.



Increase in Global Mean Temp
Degrees C
10
9
8
7
6
5
4
3
2
1
0
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year
No Tax
Global coal tax ETC
Global coal tax no ETC

Figure 11


Consumption with coal tax relative to no tax
125%
120%
115%
110%
105%
100%
95%
90%
EU
US
CHINA
Africa
Oil prod
EU no ETC
US no ETC
China no ETC
Africa no ETC
Oil prod no ETC
2010
2030
2050
2070
2090
2110
2130
2150
2170
2190
Year