An Equilibrium Theory of Determinate Nominal Exchange Rates, Current Accounts and Asset Flows

Marcus Hagedorn*

University of Oslo and CEPR

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Abstract

In standard open economy macro-models, where monetary policy in each country works through setting nominal interest rates, only the expected change but not the level of the nominal exchange rates is determinate. In contrast to this standard result (Kareken and Wallace (1981)), in this paper I show determinacy of the level in a large class of heterogenous agents incomplete markets models with aggregate risk. I then characterize the determinants of the nominal exchange rate: assets held by a country, the net foreign asset position, the nominal interest rate and productivity. I also show whether a change in one of the determinants leads to a depreciation or an appreciation. The incompleteness of markets implies that temporary shocks affect the long-run world distribution of assets and exchange rates with interesting feedback effects on the current exchange rate. The determinacy result also enables the researcher to answer many question in open economy macroeconomics within a coherent equilibrium model. I discuss some of these questions, such as how international asset flows affect exchange rates, how a country can divorce itself from these flows and how a country can manage its exchange rate. The model also implies that a country with an exchange rate peg and free asset mobility faces a tetralemma and not a trilemma as it not only loses monetary but also fiscal policy independence. This suggest a new way to think about fiscal coordination in a monetary union as a response to within union asset flows. I also provide some empirical evidence consistent with the theoretical predictions.

*University of Oslo, Department of Economics, Box 1095 Blindern, 0317 Oslo, Norway.
Email: marcus.hagedorn07@gmail.com

1 Introduction

How do fiscal and monetary policy, productivity shocks or a liquidity trap spill over to the rest of the world? What is the role of international asset flows in propagating regional policies and shocks to the rest of the world and how do they affect the nominal exchange rate? How does an increase in savings demand for US bonds affect asset flows, the exchange rate, the current account and the US economy? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate, for example engineer a depreciation? Finally, how independent is a country's policy in a monetary union with free capital mobility and is there a need for fiscal policy coordination among union members?

These are all classic questions in open economy macroeconomics but existing answers cannot be fully satisfactory since they have to impose strong assumptions on the behavior of nominal exchange rates. The exchange rate indeterminacy result by Kareken and Wallace (1981) (KW) explains why. Consider two countries where monetary policy sets nominal interest rates. The uncovered interest rate parity condition then determines the expected change in the exchange rate only but leaves the level of the exchange rate indeterminate. An equivalent type of price level indeterminacy also arises in closed economies (Sargent and Wallace (1975)), but as pointed out in KW, the open economy frameworks adds another subtle type of indeterminacy.[1] The KW indeterminacy arises if assets are fully mobile across borders and households' portfolio choices and net foreign asset positions are indeterminate. Households are then indifferent for example between a portfolio with a strong home bias and one which is perfectly internationally diversified. At the aggregate level, this portfolio indeterminacy turns into an indeterminacy of the demand for the assets supplied by each country. Both a high and low demand for a country's assets are equilibrium outcomes which are associated with different country price levels: the price level has to fall to absorb a high demand and has to increase if demand is low.

The researcher then has to select one out of many exchange rates. In the textbook Mundell Fleming model this typically amounts to a normalization of the future expected exchange rate, in modern dynamic models to fixing the long-run exchange rate. Such arbitrary assumptions on the long-run exchange rate anchor expectations to some arbitrary level with strong implications for agents' short-run and long-run behavior as well as for the full

[1] In a closed economy, the Fisher equation determines a country's inflation rate - the expected change in the price level - but leaves the price level in each country indeterminate.

path of the exchange rate in the short-run and in the medium-run. Nominal rigidities then imply that this nominal indeterminacy turns into a real indeterminacy. Different nominal exchange rates correspond to different real exchange rates and thus to different levels of exports and imports as well as different levels of output and employment at home and abroad. The implications for output and employment therefore depend on the researcher's choices on the nominal exchange rate and are also likely to affect the answers to the questions which motivate this research in the first place.

This paper shows that the nominal exchange rate is determinate and therefore these difficulties can be overcome if three assumptions,

1. Asset markets within each country are incomplete (Huggett (1993)),
2. Non-diversifiable aggregate country risk,
3. Assets are (partially) nominal,

are satisfied. The role of the latter assumption is clear. There is no role for nominal prices if assets are fully price-indexed. The first assumption - incomplete markets - implies a well-defined aggregate savings function in each country, which relates households' desired savings to the real interest rate, the income process, taxes and many other parameters.[2,3] Standard finance theory implies that the second assumption - non-diversifiable aggregate country risk and thus non-collinear country returns - entails well-defined portfolio choices in each country.[4] Combining assumptions 1 and 2 delivers the result that it is well-defined, first how much a country saves (incomplete markets within country) and second how the country allocates these savings to different countries (aggregate country risk). In particular, each country's demand for every other countries' assets is determined. Aggregating these country portfolios choices then yields the real world demand for the assets supplied by a country. The assets (nominal government bonds) are issued by governments and each country's price level is then

[2]Clarida (1990), Willen (2004) and Mendoza et al. (2009) were the first among many other contributions to integrate the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model into an open economy model and show that this model class helps to understand global capital flows and trade imbalances. Here I use this same type of model and show that this model class, in addition to its well documented appealing quantitative predictions, provides an additional benefit over complete markets models: nominal exchange rate determinacy.

[3]Kollmann (2012) and Coeurdacier et al. (2011) use a different class of incomplete markets models - limited participation in asset markets - to address the Kollmann-Backus-Smith Consumption-Real Exchange rate anomaly. Corsetti et al. (2008) address the same anomaly in a model with internationally incomplete but nationally complete markets.

[4]This assumption is generically satisfied but for knife-edge choices of technologies and preferences country returns can be collinear, see for example Kollmann (2006a).

determined as equating this nominal government asset supply to the well-defined real world demand for this asset. Prices and nominal exchange rates are thus determined as clearing the market for all government bonds for all countries or, by Walras' Law equivalently, the goods markets in all countries. The nominal exchange rates and prices thus ensure that in equilibrium the real value of government bonds for each country has to be equal to the real demand for it or equivalently the real demand for goods produced in each country has to be equal to its supply. It is important to point out that although prices determine the real value of bonds that this is not an open economy version of the Fiscal Theory of the Price Level (FTPL). Here the real value of bonds is such that the asset market clears whereas in the FTPL it is such that the government budget constraint holds. To clarify the difference to the FTPL, although not necessary for the results, I assume that the government budget constraint is fully specified in nominal terms, implying that the government budget constraint holds independent of the price level.[5]

In Section 2 I first explain that this nominal exchange rate determinacy result holds in a large class of incomplete markets models with aggregate risk. I then move to a simplified incomplete markets model, which not only, as follows from the general result, delivers determinacy but also allows for a closed-form solution of the nominal exchange rate, allows to characterize precisely the determinants of the exchange rate and how the exchange rate responds to shocks such as a tighter monetary policy at home or an increase in precautionary savings in the rest of the world. The simple model also has free asset mobility and all government bonds (the only assets) are all perfect substitutes. The key simplifying assumption is that households are members of families, which regularly pool their assets, rendering the asset distribution and therefore the whole model tractable.

The determinants of the steady-state nominal exchange rate can be readily read off the closed form solution: The nominal net foreign asset position, the nominal amount of home assets and productivity. The closed-form solution is also explicit on how changes in its determinants change the exchange rate across steady states, with a straightforward intuition.

A larger amount of nominal assets held by the home country, for example due to the issuance of more government bonds, leads to a depreciation (home prices increase relative to foreign prices): asset market clearing requires that the home price level increases such

[5]More generally it is sufficient that there is a fiscal rule such that the government budget constraint is always satisfied independently of the price level. Note that the FTPL makes the opposite assumption. The price level is determined as satisfying the government budget constraint and the asset market clears trivially (independent of the price level) since markets are assumed to be complete and the private sector is willing to hold any amount of government bonds supplied.

that the real value of home bonds decreases to match the real asset demand. A portfolio adjustment towards more foreign assets while keeping the total amount of assets unchanged leads to an appreciation: an increase in home net foreign assets is equivalent to a distribution of wealth towards the home country, inducing higher desired home savings. Asset market clearing requires that the home price level decreases such that the real value of home bonds increases to match the higher real asset demand. An increase in home productivity increases real desired savings and leads to an appreciation: asset market clearing requires the price level to fall to increase the real value of home assets to match the increase in desired savings. In terms of monetary policy, a permanent increase in the interest rate differential leads to a shift in the trend of the exchange rate, as in textbook models.

The steady-state nominal exchange rate and the world asset distribution are jointly determined. Conditionally on knowing these long-run values the current exchange rate is determined by iterating backwards on the uncovered interest rate parity condition as in any textbook. However, this somewhat mechanical determination of the short-run exchange rate overlooks some interesting interaction of the short-run and the long-run exchange rate arising from the interplay of incomplete markets and valuation effects. Suppose, that today the economy is hit by a shock. The incompleteness of markets implies that initial conditions and today's shock affect the long-run distribution of assets across countries, which in turn affects the long-run exchange rate. Applying the uncovered interest rate parity condition, this long-run change carries over to the short-run. But the interaction of the short-run and the long-run does not stop here. The change in today's nominal exchange rate induces valuation gains or losses on a country's international asset holdings.[6] These wealth gains or losses will again have effects on the long-run asset distribution and thus again on the long-run exchange rate. An equilibrium is reached if the long-run asset distribution and exchange rates and the short-run asset distribution and exchange rates are all mutually consistent. The literature has mainly focused on this valuation effect - how changes in exchange rates affect asset values - whereas this paper builds on these insights and adds the feedback from asset values to exchange rates, such that the distributions of assets and exchange rates are jointly determined in equilibrium.

A related history-dependence arises in standard small open economy models, where the

[6]For example, a country holding US dollar denominated bonds and appreciating vis-à-vis the US-dollar experiences a wealth loss. Several papers among them Lane and Milesi-Ferretti (2001, 2007), Tille (2003, 2008) Kollmann (2006b), Gourinchas and Rey (2007a,b), Devereux and Sutherland (2010), Pavlova and Rigobon (2012), Ghironi et al. (2015) have established the importance of such valuation effects. In particular the literature has documented that a large fraction of US foreign liabilities is denominated in US dollars whereas US foreign assets have a considerable non-dollar component.

interest rate is exogenous and incomplete markets add a random walk component to equilibrium dynamics implying an unbounded support for the endogenous variables (Schmitt-Grohé and Uribe (2003)). These problems do not arise here since the world is a closed economy, the asset distribution and the steady state interest rate are endogenous and the latter is smaller than the discount rate which ensures stationarity of the distribution of assets and consumption (Aiyagari (1994, 1995)).[7]

The interaction of short-run and long-run effects determines how monetary policy affects the exchange rate. A temporary tightening of home monetary policy leads, under a precise condition, to an accumulation of assets by the ROW and an asset de-accumulation by the home country. As those asset choices persist in the long-run they lead to an appreciation of the long-run exchange rate such that the current exchange rate appreciation is a combination of this long-run steady state response and uncovered interest parity. The condition which I characterize precisely is necessary since I consider the equilibrium response of the exchange rate and asset holdings. The equilibrium response is non-trivial since the change in exchange rates induces a revaluation of net foreign assets and thus wealth effects, which if strong enough can overturn the result that it is the ROW which accumulates more assets and instead the home country would save more. Under the same condition, I show that a temporary rise in the precautionary demand for savings in the ROW leads to an increase in long-run assets held by the foreign country, a decrease in long-run assets and in net foreign assets of the home country. These long-run shifts in world assets imply an appreciation of the steady state exchange rate and by uncovered interest rate parity again an appreciation of the current exchange rate.

Using data on US net foreign assets from Gourinchas and Rey (2007a,b) I then test some predictions of the model. A simple time-series regression is consistent with the theoretical relationship between the nominal exchange rate and its determinants. An increase in US debt holdings leads to a depreciation, whereas an increase in net foreign assets, an increase in the US money market interest rate as well as in US productivity all lead to an appreciation, where all effects are significant and ceteris paribus. The data also confirm model predictions on two determinants of future net foreign assets. Both an increase in US government debt and a tightening of US monetary policy lead to a deterioration of its net foreign asset position.

The empirical analysis detects the model predicted co-movement of key variables in the data although many frictions useful to explain short-run dynamics are missing in the theo-

[7]Cavallo and Ghironi (2002) and Ghironi (2008) adopt an overlapping generations instead of a representative agent model (within a country) to ensure stationarity. This departure from Ricardian equivalence assigns a role to the stock of real net foreign assets but does not overcome exchange rate indeterminacy.

retical analysis. The model is on purpose simple to focus on the determinacy result and to highlight the determinants of the exchange rate. Despite this simplicity, the model provides a good idea of what the answers to some of the motivating questions might be.

An outflow of assets leads to a depreciation whereas an inflow of assets, say due to an increase in precautionary savings demand for US bonds by emerging countries leads to an appreciation of the US exchange rate. The US can sterilize this latter effect on the exchange rate through acquiring foreign assets or just issuing government bonds. This suggests that a larger savings demand by the ROW for US bonds can be accommodated without any effects on US prices or exchange rates, provided that the ROW's demand does not persistently increase at a faster rate than US GDP. If it does, then stabilizing the exchange rate will require an exploding US debt/gdp ratio, which is infeasible due to the limited US fiscal capacity. The US would then have to accept falling prices and an appreciation of its currency, a flexible exchange post Bretton Woods version of Triffin's dilemma. Or the ROW diverts its savings to other currencies, the Euro or the Yuan.

The theory shows that various policies can be used to trigger a depreciation of a currency: Conduct an expansionary fiscal policy (increase debt issuance and lower taxes), loosen monetary policy (lower nominal interest rates) or buy, without sterilizing, foreign assets.

The model also suggest that the classic policy trilemma in international economics - at most two out of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting nominal interest rate independently (monetary policy independence); and (iii) a fixed exchange rate - turns into a tetralemma, as fixed exchange rates and free capital mobility not only imply the loss of monetary but also of fiscal policy independence. The argument is simple. Interest rate parity implies that monetary policy has to track foreign monetary policy to rule out anticipated changes in the exchange rate. Fiscal policy then has to ensure, for example through issuing more or less debt, that the exchange rate remains unchanged in response to unanticipated shocks. This restriction on fiscal policy is missing in the standard trilemma since there the level of the exchange rate is indeterminate and the focus is on the (anticipated) change only. Here, in contrast, monetary policy cannot stabilize the exchange rate on its own and fiscal policy has to step in when unanticipated shocks move the level of the exchange rate. The implication for monetary unions is that its members not only have to give up an independent monetary policy but de facto also an independent fiscal policy, at least if movements in the real exchange rate shall be avoided. This suggests a new perspective on the fiscal dimension of a monetary union: Fiscal policy coordination to jointly respond to asset and capital flows.

A full answer to all motivating question certainly requires to move to a quantitative analysis and add several features such as nominal rigidities and physical capital to the model. Nominal rigidities are important since this is why the indeterminacy of the nominal exchange rate matters for the real exchange rate. Different nominal exchange rates correspond to different real exchange rates and thus to different levels of exports and imports as well as different levels of output and employment at home and abroad. Adding capital, although irrelevant for determinacy, allows to obtain a full picture of a country's capital account which is in particular relevant for the US, the "Venture Capitalist of the World", which can be roughly described as issuing debt liabilities and investing in physical capital (equity and direct investment) abroad (Gourinchas and Rey (2007b,a)). Although these models are much richer than the simple one in this paper, it is important to notice that the same mechanism to determine the exchange rate is operating in the simple model and in more richer models. It is the mechanism proposed in this paper which enables the researcher to quantitatively and simultaneously account for the observed fall in US interest rates, the flow of capital and assets in and out of the US, the large current account US deficit and the evolution of exchange rates within a coherent equilibrium model. The mechanism also allows to consider different theories of "global imbalances" within a consistent framework. One theory put forth in Caballero et al. (2008) is that different regions of the world differ in their capacity to generate financial assets from real investments. Another explanation focuses on exchange rates and argues that emerging countries, mainly in Asia, have undervalued exchange rates, impose capital controls and accumulate reserve asset claims on the US (Dooley et al. (2003, 2014)). A joint assessment of these theories requires a model with a determinate equilibrium exchange rate; this is what this paper provides. This paper also enables to study spillovers of foreign fiscal and monetary policy as well as of foreign shocks and a foreign liquidity trap on the home macroeconomy. A key aspect when studying such policy or shock spillovers is the potential absorbing role of exchange rate adjustments, which requires a theory how the exchange rate is determined.

The rest of the paper is organized as follows. Section 2 shows that the nominal exchange rate is determinate in a large class of heterogeneous agents incomplete markets models with aggregate risk. Section 3 presents the simple model. Section 4 provides the determinacy result, derives the closed-form solution for the steady-state exchange rate, characterizes the determinants of the exchange rate and shows how the current and the long-run exchange rate respond to temporary shocks. Section 5 provides some empirical evidence confirming the predictions of the model. Section 6 discusses implications for the questions which motivate this paper and a large literature and concern many policy makers and finally provides concluding remarks. Most derivations, proofs and the data description are delegated to the appendix.

2 Exchange Rate Determinacy in Incomplete Markets Models with Aggregate Risk

In this Section I argue that three assumptions

1. Asset markets within each country are incomplete.
2. Presence of non-diversifiable aggregate country risk.
3. Assets are nominal.

imply nominal exchange rate determinacy. I first show that models where these assumptions are satisfied deliver determinacy before I turn to explaining the role of each of these assumptions. In this Section I only consider determinacy in a steady state. Once this step is accomplished, the uncovered interest rate parity condition implies that determinacy outside the steady state follows from determinacy of the steady state.

To show the determinacy of the steady-state nominal exchange rate, I consider a world with a measure one of small countries, each subject to aggregate shocks. Nothing depends on the absence of large countries but the small countries assumption implies a stationarity distribution of world assets, which simplifies the exposition substantially. There is a single good such that the law of one price implies a real exchange rate equal to one. The nominal exchange rate is the home price of foreign currency such that an increase is a depreciation. Each country is an endowment economy with uninsurable idiosyncratic labor income risk, based on Huggett (1993), where only one asset - a nominal government bond - can be traded subject to exogenously imposed borrowing limits. The aggregate world income is constant over time, but the aggregate income Y_c in each country c is not, i.e. there is aggregate uncertainty at the country but not at the world level. I consider a cashless economy (Woodford (2003)) where monetary policy in each country c sets nominal interest rates i_c. Fiscal policy sets nominal bonds B_c (denominated in their own currency), and nominal taxes T_c such that the steady-state government nominal budget constraints hold in all states of the world, $B_{c,+1} - (1+i_c)B_c = T_c$.[8] Bonds are fully mobile across borders and there are no transactions costs.

In each country, policy is exogenous and the growth rate of nominal debt and nominal taxes are constant in a steady state,

$$1+\gamma_c = \frac{B_{c,t+1} - B_{c,t}}{B_{c,t}} = \frac{T_{c,t+1} - T_{c,t}}{T_{c,t}},$$

so that bonds evolve as

$$B_{c,t} \quad = \quad B_c(1+\gamma_c)^t.$$

There is also a stationary distribution of the real value of national debt, implying a stationarity distribution μ_π of national inflation rates, [9]

$$1+\pi_{c,t+1} = \frac{P_{c,t+1} - P_{c,t}}{P_{c,t}} \sim \mu_\pi$$

Due to the aggregate country uncertainty inflation rates are not constant but fluctuate around their long-run value

$$1+\pi_{c,t+1} \approx 1+\gamma_c.$$

The incompleteness of markets implies a well-defined stationary distribution μ_S of aggregate country real savings,

$$S_c \sim \mu_S.$$

This property is well known to be generically the case in heterogenous agents incomplete market models with or without aggregate risk (Ljungqvist and Sargent (2004)). What aggregate country uncertainty in Y_c adds is well-defined portfolio choices by each country. Standard finance theory tells us that agents in each country hold a portfolio of home and

[8]Note that the government budget constraints are in nominal terms and therefore hold independent of the price level, implying that the exchange rate and prices are not determined trough some open economy modification of the Fiscal Theory of the Price Level.

[9]Note that this result says that the distribution of *changes* in prices and exchange rates is constant, which is consistent with a continuum of different price *levels* and associated real debt levels. For example, doubling all prices in all periods and countries yields the same inflation rates but cuts the real value of debt in half. In particular, this result does solve the indeterminacy issue yet.

foreign bonds to diversify the risk at home and abroad. The only relevant aspect of this large literature here is that it yields well-defined aggregate savings function for home and foreign bonds by each country.[10] The period t country c demand of country $\tilde{c}$ bonds (in real terms) is $S^{\tilde{c}}_{c,t}$. The world real demand for country $\tilde{c}$ bonds aggregates the demand of all countries c,

$$S^{\tilde{c}}_t(\ldots) = \int S^{\tilde{c}}_{c,t}(\ldots)dc. \tag{1}$$

The demand $S^{\tilde{c}}_t(\ldots)$ is not necessarily constant over time due to aggregate uncertainty but distributed according to a stationary distribution μ_c,

$$S^{\tilde{c}}_t \sim \mu_c. \tag{2}$$

The asset market clearing condition for country c bonds is then

$$\frac{B_{c,t}}{\mathbf{P_{c,t}}} = S^c_t, \tag{3}$$

such that the price level is uniquely determined as

$$P_{c,t} = \frac{B_{c,t}}{S^c_t}. \tag{4}$$

This shows that the price level $P_{c,t}$ in each country is determined at each point time and in every state of the world. But it is not constant and can change for two reasons. First, nominal country debt $B_{c,t}$ grows at rate γ_c and so do prices on average. Second, the demand for country c bonds, S^c_t, is not constant, but a draw from a stationary distribution, such that

$$\tilde{P}_{c,t} = \frac{P_{c,t}}{(1+\gamma_c)^t} = \frac{B_c}{S^c_t} \tag{5}$$

follows a unique stationary distribution μ_P, which is a simple transformation of the distribution μ_c of $S^{\tilde{c}}_t$.

[10]The determinacy argument does not rely on any specific properties of these savings functions and hence does not rely on any empirical prediction which might not be in line with the data (see e.g. Coeurdacier and Rey (2013) for a survey). The only requirement is that country returns are not collinear which would render households portfolio choices indeterminate. This assumption is generically satisfied but for knife-edge choices of technologies and preferences country returns might be collinear, see for example Kollmann (2006a).

Asset market clearing together with the government budget constraints characterize an equilibrium since Walras' Law ensures goods market clearing. Therefore the equilibrium steady-state price levels P_c and thus the exchange rates between country c and $\tilde{c}$,

$$\epsilon_{c,\tilde{c}} = P_c / P_{\tilde{c}}, \tag{6}$$

are characterized as the solution to the asset market clearing conditions (3) or equivalently (4) or (5).

To obtain a graphical representation of exchange rate determination I now move to a two-country version of this economy such that the world now consists of home and foreign. I also obtain an empirically more applicable characterization of prices and the exchange rate in terms of each countries observed asset holdings. As above each country is exposed to aggregate productivity shocks but now, since there are only two countries, uncertainty does not vanish at the world level, such that no stationary distribution for the world distribution of assets exists. Therefore, I consider the limit economy when this uncertainty vanishes. The limit aggregate steady-state savings in both the home and the foreign country are S_H and S_F. The savings of home households for home bonds converges to S_H^H and for foreign bonds to S_H^F . The savings of foreign households for foreign bonds converges to S_F^F and for home bonds to S_F^H.

As above, the asset market clearing conditions for country bonds - adapted to the two country and vanishing uncertainty environment - determines the price levels. The asset market clearing condition for home bonds is now[11]

$$\frac{B_H}{\mathbf{P_H}} \;=\; S_H^H + S_F^H, \tag{7}$$

where $S_H^H + S_F^H$ is the sum of the home and the foreign country demand for home real bonds. For foreign bonds the market clearing condition is

$$\frac{B_F}{\mathbf{P_F}} \;=\; S_F^F + S_H^F. \tag{8}$$

where $S_F^F + S_H^F$ is the sum of the home and the foreign country demand for foreign real bonds. As above, the equilibrium steady-state price levels P_H and P_F and thus the exchange rate $\epsilon = P_H/P_F$ are characterized as the solution to these two asset market clearing conditions

[11] Note that in steady-state $\frac{B_{H,t}}{P_{H,t}} = \frac{B_H(1+\pi_H)^t}{P_H(1+\pi_H)^t} = \frac{B_H}{P_H}$ and $\frac{B_{F,t}}{P_{F,t}} = \frac{B_F(1+\pi_F)^t}{P_F(1+\pi_F)^t} = \frac{B_F}{P_F}$.

(7) and (8).

Some simple algebra yields an equivalent but empirically more applicable characterization of prices and the exchange rate in terms of each countries observed asset holdings. Observe first that by definition nominal net foreign asset holdings by the home country, NFA_H, satisfy

$$\frac{NFA_H}{P_H} = S_H^F - S_F^H, \tag{9}$$

and by the foreign country, NFA_F, satisfy

$$\frac{NFA_F}{P_F} = S_F^H - S_H^F = -\frac{NFA_H}{P_H}. \tag{10}$$

Using this in (7) and (8) and rearranging yields:

$$\frac{B_H + NFA_H}{\mathbf{P_H}} = S_H^H + S_H^F = S_H, \tag{11}$$

$$\frac{B_F + NFA_F}{\mathbf{P_F}} = S_F^F + S_F^H = S_F, \tag{12}$$

which defines a mapping from assets to prices and exchange rates. The advantage of this characterization is that it is stated in terms of empirically observable assets B_H, NFA_H, B_F, NFA_F and depends only on a countries total savings S_H, S_F but not on the portfolio decisions $S_H^H, S_H^F, S_F^F, S_F^H$ separately. It is empirically more applicable since it is based on actually observed asset holdings without considering the underlying reasons for these portfolio choices.

The latter characterization also allows to use the Metzler diagram for a graphical illustration. Figure 1 shows how prices and the exchange rate are derived. The left and right panels of Figure 1 report the home and foreign savings curves S_H and S_F as a function of the world real interest rate $1 + r$. On the horizontal axis they also show the real value of home assets, $B_H/P_H + NFA_H/P_H$, and the real value of foreign assets, $B_F/P_F + NFA_F/P_F = B_F/P_F - NFA_H/P_H$, where I used that $NFA_F/P_F = -NFA_H/P_H$. The right panel tells us that the price level P_H can be determined as clearing the home market,

$$B_H/\mathbf{P_H} + NFA_H/\mathbf{P_H} = S_H\left(\frac{1 + i_H}{1 + \pi_H}, \ldots\right), \tag{13}$$

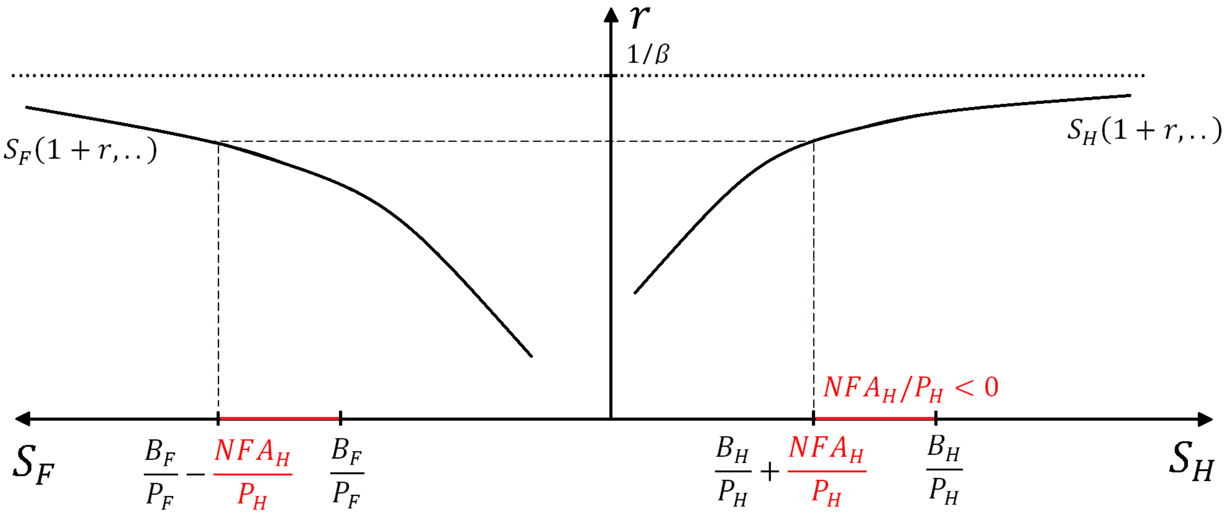


Figure 1: Exchange Rate Determination in Metzler Diagram

which then pins down the real value of net foreign assets, $NFA_F/P_F = -NFA_H/P_H$. Using this in the left panel pins down the price level P_F from asset market clearing in the foreign country,

$$B_F/\mathbf{P_F} - NFA_H/P_H = S_F(\frac{1+i_F}{1+\pi_F}, \ldots). \tag{14}$$

Therefore the exchange rate $\epsilon_t = P_{H,t}/P_{F,t}$ is determinate and solves

$$\epsilon_t = \frac{B_{H,t} + NFA_{H,t}}{S_H(\frac{1+i_H}{1+\pi_H}, \ldots)} \frac{S_F(\frac{1+i_F}{1+\pi_F}, \ldots)}{B_{F,t} - NFA_{H,t}/\epsilon_t}. \tag{15}$$

The determinacy results hinges on three properties:

1. Market incompleteness: asset markets within each country are incomplete.

   $\hookrightarrow$ Well-defined aggregate savings within each country.

2. Non-diversifiable Aggregate Risk

   $\hookrightarrow$ Well-defined international portfolios for each country.

3. Nominal assets

   $\hookrightarrow$ Assigns a role for nominal prices.

The necessity of the later property - assets are (partially) nominal - is clear. If assets were fully price-indexed, then there would be no role for prices since the whole economy would be specified in real terms only. It is however sufficient that assets are partially nominal, i.e. a fraction less than 100% could be indexed.

The role of the other two assumptions - market incompleteness and aggregate risk - is more subtle. To understand this, it is useful to consider a world without aggregate risk and where markets are complete. In such a world indeterminacies of the Sargent and Wallace (1975) (SW) and the Kareken and Wallace (1981) (KW) type arise. The steady state nominal interest rates i_H and i_F just determine the expected change of the nominal exchange rate, $E_t \frac{\epsilon_{t+1}}{\epsilon_t}$, but not the levels ϵ_t and ϵ_{t+1}. The uncovered interest rate parity condition,

$$1 + i_H = (1 + i_F) E_t \frac{\epsilon_{t+1}}{\epsilon_t}, \tag{16}$$

if satisfied for a pair $(\epsilon_t, \epsilon_{t+1})$, is also satisfied for any multiple $(\lambda\epsilon_t, \lambda\epsilon_{t+1})$ for all $\lambda > 0$. This is the analog for exchange rates of the price level indeterminacy pointed out by SW. Accordingly, the derivation illustrated in Figure 1 does not apply anymore. With complete markets the steady-state savings curve is degenerate and becomes a horizontal line at the steady-state real interest rate $1/\beta$ (for a discount factor β). As Figure 2 illustrates, asset market clearing in both countries is consistent with a continuum of prices, e.g. P_H^1, P_H^2, P_H^3 for the home country and P_F^1, P_F^2, P_F^3 for the foreign country, and hence with a continuum of exchange rates $\epsilon = P_H/P_F$.

What incomplete markets contribute are well defined steady-state aggregate savings function S_H and S_F as explained above. While adding incomplete markets overcomes the SW indeterminacy it still does not deliver determinacy as now the KW type indeterminacy kicks in. The world asset market clears when

$$S_H + S_F = \frac{B_H}{P_H} + \epsilon \frac{B_F}{P_H}, \tag{17}$$

which, for every exchange rate $\epsilon > 0$, has a different solution P_H. However, all of these different exchange rates and price levels are associated with different net foreign asset positions,

$$\frac{NFA_H}{P_H} = S_H - \frac{B_H}{P_H}. \tag{18}$$

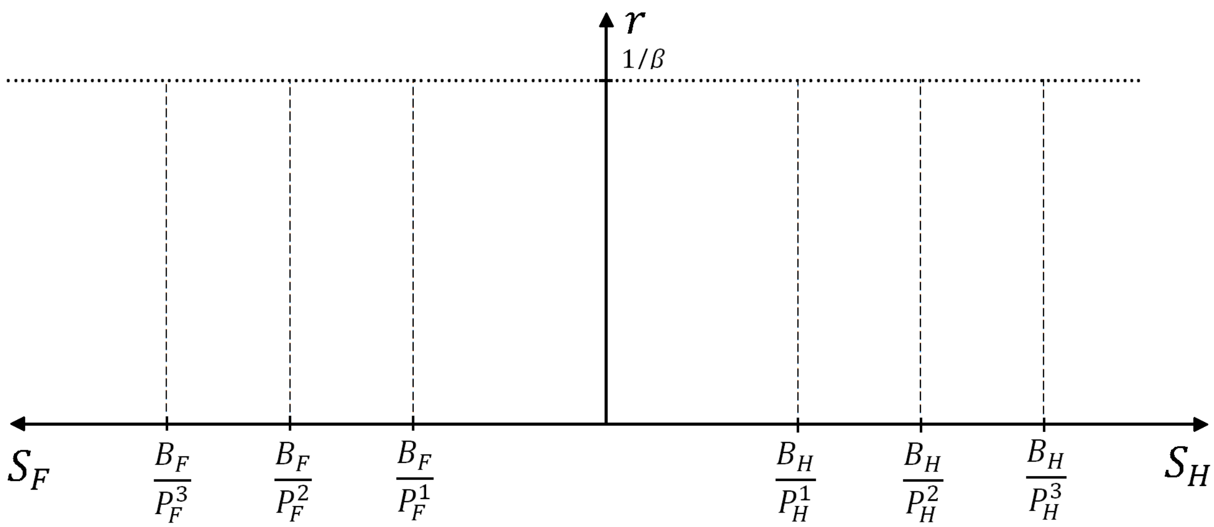


Figure 2: Complete Markets: Exchange Rate Indeterminacy of Sargent and Wallace (1975) type

Figure 5 illustrates the KW type indeterminacy. In panel a) of Figure 5 the price levels are P_H^- and P_F^- such that the world asset market clears. The exchange rate equals $\epsilon^- = P_H^-/P_F^-$ and the net foreign asset positions are

$$NFA_H^- = P_H^- S_H - B_H < 0 \tag{19}$$

$$NFA_F^- = P_F^- S_F - B_F > 0. \tag{20}$$

Panel b) and c) of Figure 5 show different combinations of home and foreign prices which also clear the world asset market but are associated with different net foreign asset positions. In panel b) prices are P_H^0 and P_F^0 and $NFA_H^0 = 0 = NFA_F^0$ whereas in panel c) prices are P_H^+ and P_F^+ and $NFA_H^+ > 0, NFA_F^+ < 0$. All these three panels show equilibrium outcomes but are associated with different exchange rates $\epsilon^- = P_H^-/P_F^- < \epsilon^0 = P_H^0/P_F^0 < \epsilon^+ = P_H^+/P_F^+$.

This is where assumption 2 (aggregate risk) becomes relevant. Aggregate country risk delivers well defined portfolio choices how to split a country's savings between home and foreign bonds. This adds NFAs to the list of equilibrium objects and eliminates it as a free parameter. In particular, total assets $A_H = B_H + NFA_H$ is an outcome of agents diversification of aggregate risk. Figure 1 then illustrates the mapping from $A_H = B_H + NFA_H$ to P_H and of $A_F = B_F + NFA_F$ into prices P_H and P_F and the exchange rate $\epsilon = P_H/P_F$.

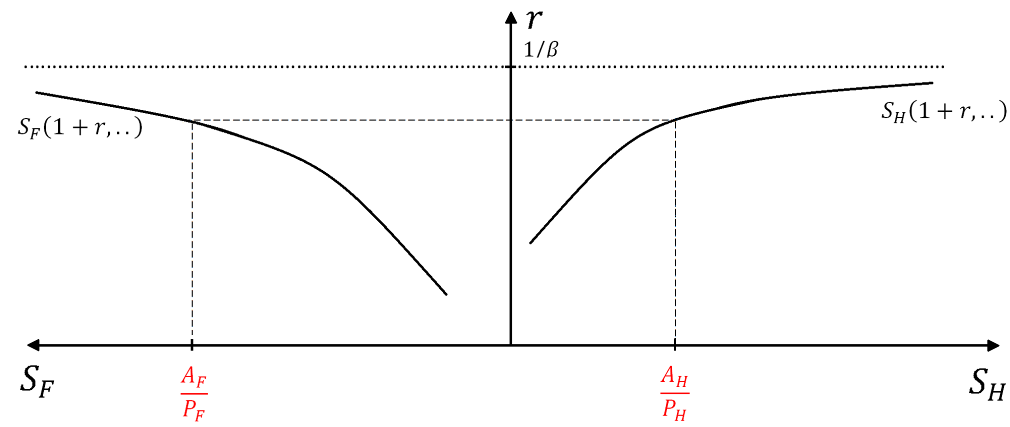
The graphical analysis is also informative on some of the determinants of the exchange rate. The exchange rate moves either because assets A_H or A_F change or because the savings curves S_H or S_F shift. Total assets $A_H = B_H + NFA_H$ in turn can change either because bond supply B_H changes or because net foreign assets NFA_H change.

Since, as argued above, diversification delivers a well-defined NFA, Figure 5 can also be used to give an idea how the exchange rate depends on a country's net foreign asset position. For a fixed supply of assets B_H and B_F, an increase in NFA_H shifts assets towards the home country so that asset market clearing in both countries requires a depreciation. Indeed, when $NFA_H^- < 0$ is negative (panel a), the exchange rate, $\epsilon^- = P_H^-/P_F^-$, is lower than when $NFA_H^0 = 0$ (panel b) which again is lower than the exchange rate when $NFA_H^+ > 0$ (panel c),

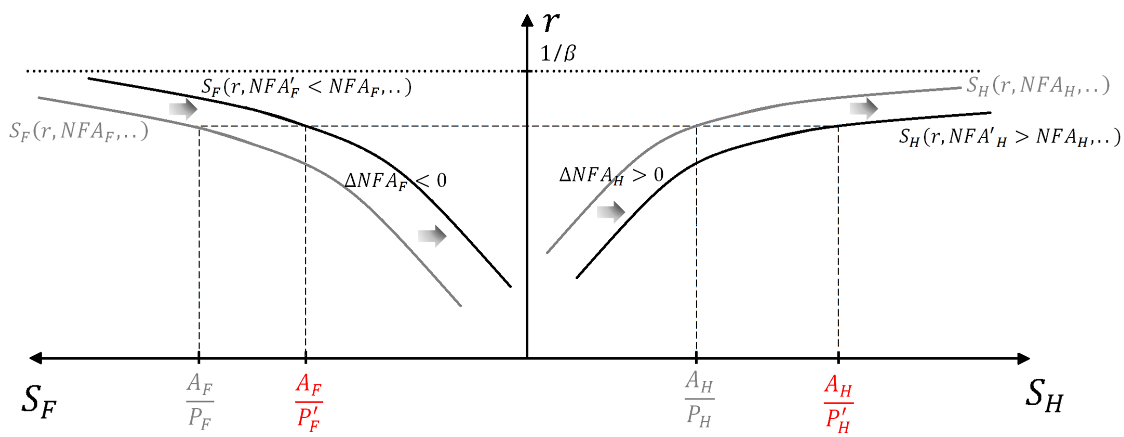
$$\epsilon^- = P_H^-/P_F^- < \epsilon^0 = P_H^0/P_F^0 < \epsilon^+ = P_H^+/P_F^+. \tag{21}$$

The Metzler diagram can also be used to understand how an increase in home supplied assets B_H affects the exchange rate. This leads to a depreciation or an appreciation depending on whether the home country or the foreign country absorbs those assets. Panel b) of Figure 4 shows the case when the increase in B_H is fully absorbed by the home country and Panel a) is the steady state before the policy change. A comparison of panel a) and b) shows that total home assets $A_H = B_H + NFA_H$ increase, the nominal NFA_H does not change since home absorbs the increase in B_H, and the home price level P_H increases to match the real savings of the home country, such that real NFA_H (the red line) falls. This drop in the real value of assets held by the foreign country leads to a fall in the foreign price level such that real desired savings are matched again. As a result the exchange rate $\epsilon = P_H/P_F$ depreciates (increases). Panel c) of Figure 4 shows the case when the increase in B_H is fully absorbed by the foreign country which leads to a fall in NFA_H and an increase in A_F. To clear the market for savings by foreigners, the price level P_F has to increase such that A_F/P_F matches real savings. Since the increase in B_H is fully absorbed abroad, $A_H = B_H + NFA_H$ is unchanged and so is the price level P_H.

A portfolio adjustment towards more foreign assets while keeping the total amount of assets A_H and A_F unchanged is equivalent to a distribution of wealth towards the home country, leading to higher savings at home and lower savings in the foreign country as illustrated in Figure 4. Panel a) of this figure shows the steady state before and panel b) after this portfolio shift from NFA_H' to NFA_H. The exchange rate appreciates in response



(a) Exchange Rate and Assets A_H, A_F.

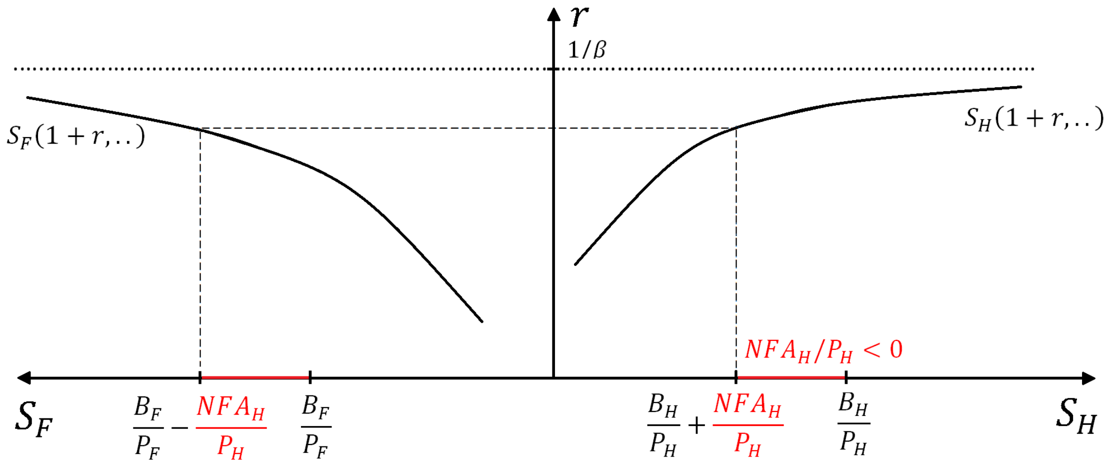


(b) Higher $NFA'_H > NFA_H$

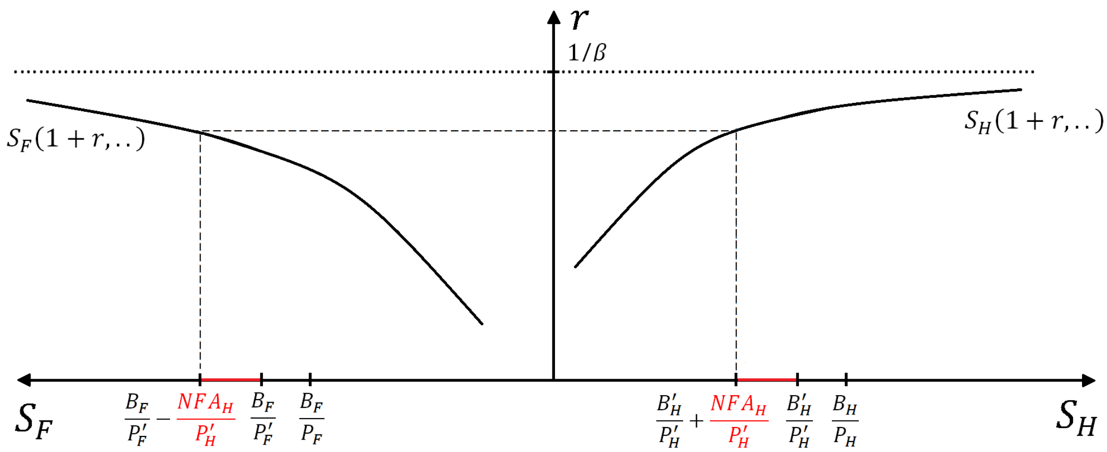
Figure 3: Portfolio Adjustments: NFA_H and Nominal Exchange Rates

to this increase in net foreign assets since the home price has to fall and the foreign price has to increase to clear all asset markets.

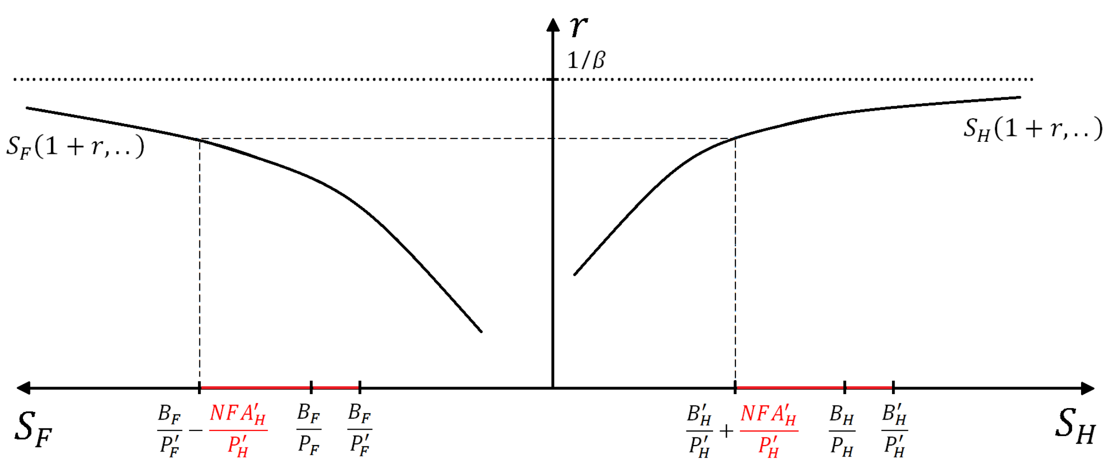
The determinacy result of the equilibrium exchange rate has two parts, one that is well understood and one that is new and the main contribution of this paper. The well understood part is the mapping from exchange rates to asset and portfolio choices. This is standard finance theory. The new part is the mapping from assets to the exchange rate which together with the portfolio choices determines the exchange rate. The remainder of the paper is devoted to this new part. I therefore develop a simpler incomplete markets open economy model in the next Section, which allows for a better understanding of the mapping of assets into prices and exchange rate not only in but also outside steady state and for a closed form solution for the exchange rate.



(a) Pre Expansion

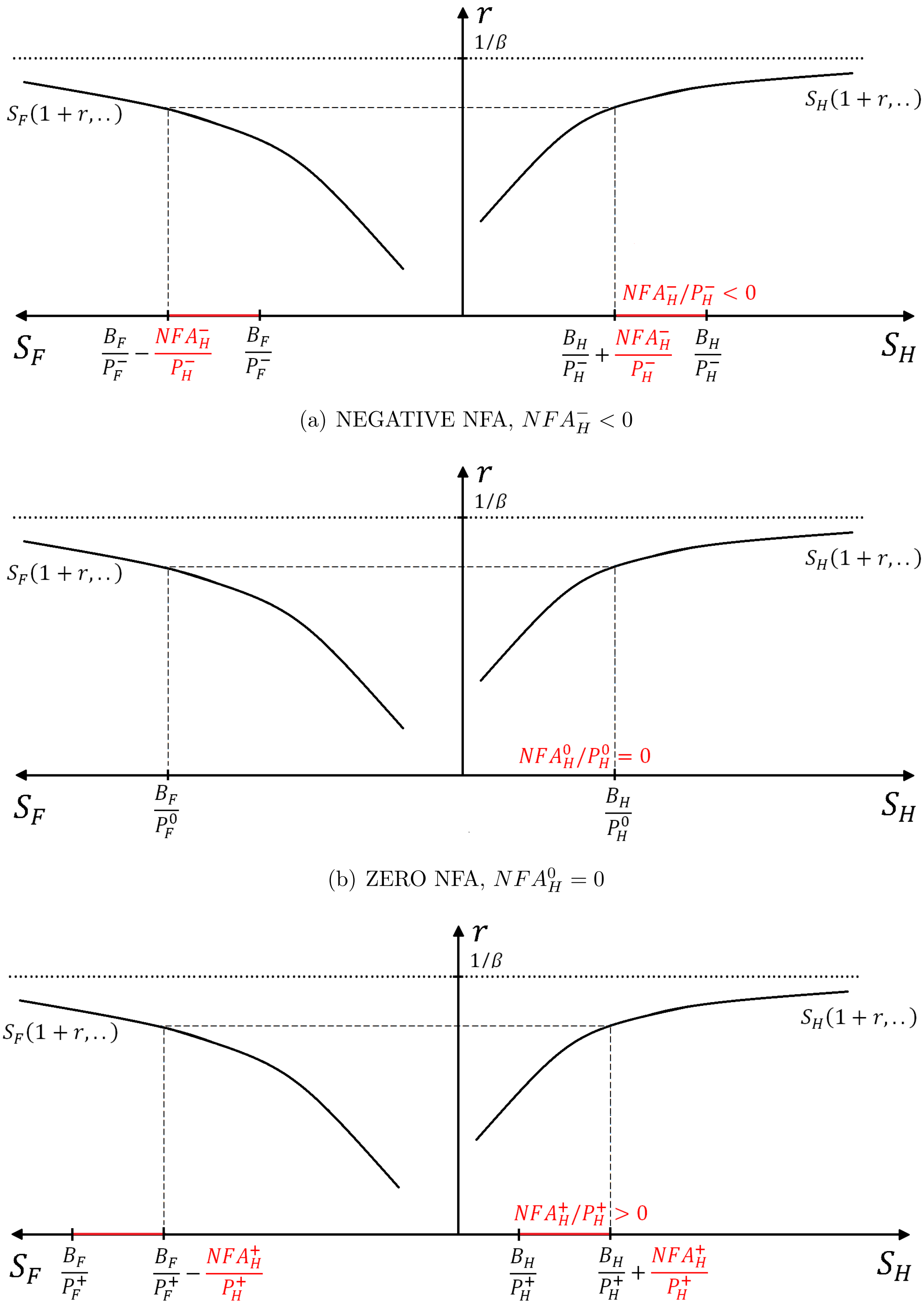


(b) Home Country Absorption of B_H



(c) Foreign Country Absorption of B_H

Figure 4: Home Asset Supply B_H and Nominal Exchange Rates



(a) NEGATIVE NFA, $NFA_H^- < 0$

(b) ZERO NFA, $NFA_H^0 = 0$

(c) POSITIVE NFA, $NFA_H^+ > 0$

Figure 5: Kareken and Wallace (1981) type Exchange Rate Indeterminacy

3 Model

In the main body of this paper I present the simplest version of a more general incomplete markets model, which can be found in the appendix. In this general model I relax various assumptions, which I make in the simple model to obtain easy to interpret and closed-form solutions: availability of credit to overcome spending needs, a more general distribution of shocks, general utility functions, government spending and an endogenous probability that the liquidity constraint binds. I will also prove the determinacy result in the general model, what is possible since it is still less general than the one discussed in the previous Section.

In the simple model to be presented next, households in this two-country one-good endowment economy are infinitely-lived and heterogenous in their spending needs. In terms of preferences and trading frictions each period resembles a Diamond and Dybvig (1983) economy. Markets are incomplete in both countries since the only available asset is a non-state-contingent bond. However, to keep the heterogeneity analytically tractable, households are members of large families which at the beginning of each period pool all family assets. This tractability allows me to derive results outside steady state which is crucial to understand the interaction of the short-run and long-run exchange rate and their responses to temporary shocks.

The main objective of the model is to first formalize and understand how the world distribution of assets maps into a determinate exchange rate and then to characterize how the exchange rate responds to policy changes and various shocks. As explained in the previous Section, an equilibrium model not only has to provide such a mapping but also a well-defined portfolio choice. To focus on the mapping and not on well-researched portfolio theory, I proceed as in the previous Section and first add aggregate uncertainty to the model which delivers well defined portfolio choices and then consider the limit economy when this uncertainty vanishes.

3.1 Households

Consider a two-country one-good endowment economy. I refer to the two countries as "Home" and "Foreign", denoted H and F. Time is discrete and extends from $t = 0, \ldots, \infty$. There is a continuum of measure one of households in each country. Each period $t \geq 0$ is divided into two distinct and successive sub-periods t_1 and t_2. The only source of uncertainty is an idiosyncratic i.i.d. emergency expenditure shock in the spirit of Diamond and Dybvig (1983), which realizes only in period t_2 with probability q. Households are exposed to liquidity risks at t_2, which leads to heterogeneity in consumption and asset holdings. To keep the model tractable, I make the assumption that each household is a family which consists of a continuum of individuals of measure one.

The timing of events is as follows: In subperiod t_1, before the realization of the the risk each household member consumes $C_{H,t}(C_{F,t})$. A member of the household with emergency expenditure needs consumes $c_{H,t}(c_{F,t})$ in the second subperiod and derives utility

$$\log(C_{\cdot,t}) + \theta_{\cdot} \log(c_{\cdot,t}),$$

where the "dot" stands for H or F respectively to avoid showing basically the same equation twice. The strength of these expenditures needs is allowed to be different in the home country, θ_H, and in the foreign country, θ_F. A member without these needs derives utility

$$\log(C_{\cdot,t}).$$

Because the household has a continuum of members the fraction of its members with spending needs at t_2 is equal to q. Each individual is endowed with $\tilde{Y}_H = z_H Y_H$ and $\tilde{Y}_F = z_F Y_F$ units of the consumption good respectively, where z_H and z_F are aggregate endowment shocks.

As in Diamond and Dybvig (1983), because the expenditures needs at t_2 are sudden, I assume that a liquid asset (bonds) is necessary to make these expenditures. The interpretation is that each member of the household has to acquire period t_2 consumption from the market and cannot obtain it from his or her own family because they are spatially separated.

In period t_1 each household chooses consumption in period t_1, $C_{\cdot,t}$, consumption at t_2, $c_{\cdot,t}$, and how many nominal foreign and home bonds to buy. Home agents buy $A^H_{H,t}$ home and $A^F_{H,t}$ foreign bonds and foreign agents buy $A^F_{F,t}$ foreign and $A^H_{F,t}$ home bonds. The Home price level at time t is $P_{H,t}$, the Foreign price level $P_{F,t}$ (same at t_1 and t_2 as goods are the same), and the nominal exchange rate is ϵ_t, the home price of foreign currency. The value of agents portfolios in the respective national currencies are denoted

$$A_{H,t} = A^H_{H,t} + \epsilon_t A^F_{H,t} \quad \text{and} \quad A_{F,t} = A^F_{F,t} + A^H_{F,t}/\epsilon_t, \tag{22}$$

so that the value of the nominal portfolios in terms of consumption goods are $A_{H,t}/P_{H,t}$ and $A_{F,t}/P_{F,t}$. The nominal return on one period home bonds acquired in period t that pay off in period in $t+1$ is $\rho_{H,t+1}$. The corresponding return on foreign bonds is $\rho_{F,t+1}$. The return on the home agents' portfolio is then

$$R_{H,t+1} := \frac{\rho_{H,t+1}A^H_{H,t} + \epsilon_{t+1}\rho_{F,t+1}A^F_{H,t}}{A^H_{H,t} + \epsilon_t A^F_{H,t}} \tag{23}$$

and on foreign agents' portfolio

$$R_{F,t+1} := \frac{\rho_{F,t+1}A^F_{F,t} + \rho_{H,t+1}A^H_{F,t}/\epsilon_{t+1}}{A^F_{F,t} + \epsilon_t A^H_{F,t}/\epsilon_t}. \tag{24}$$

Uncertainty in this economy is given by two productivity shocks

$$\mathcal{Z}_t := (z_{H,t}, z_{F,t}). \tag{25}$$

The no-uncertainty limit is obtained by letting these shocks converge to their mean of one.

Since all members of a household are identical each member has the same level of consumption at t_1 and enters period t_2 with the same amount of assets. During period t_2, each member has only access to his or her own assets to be spend on consumption c_t,

$$c_{H,t} \leq A_{H,t}/P_{H,t}, \tag{26}$$

$$c_{F,t} \leq A_{F,t}/P_{F,t}. \tag{27}$$

Excess bonds not needed for emergency expenditures, $\min(A_{\cdot,t} - P_{\cdot,t}c_{\cdot,t}, A_{\cdot,t})$, are returned to the family at the end of the period.

The household's budget constraint at t_1 is:

$$P_{\cdot,t}C_{\cdot,t} + A_{\cdot,t} = P_{\cdot,t}Y_{\cdot,t} - T_{\cdot,t} + R_{\cdot,t}A_{\cdot,t-1} - P_{\cdot,t}\overline{C}_{\cdot,t}, \tag{28}$$

where T_t are nominal tax obligations of the household to be paid at t_1, $\overline{C}^h_t$ is the sale of household consumption goods to members of other families who need consumption in period t_2. Since families are large, a fraction q of its members has spending needs at t_2, it follows that in a symmetric equilibrium expected spending on consumption in period t_2 is equal to sales in period t_2:

$$\overline{C}_{\cdot,t} = qc_{\cdot,t}, \tag{29}$$

so that the amount of assets owned by a household at the end of period t_2 equals

$$A_{\cdot,t_2} = q(A_{\cdot,t} - P_{\cdot,t}c_{\cdot,t}) + (1-q)A_{\cdot,t} + P_{\cdot,t}\overline{C}_{\cdot,t} = A_{\cdot,t}. \tag{30}$$

Thus, the household's flow budget constraints simplifies to

$$P_{\cdot,t}C_{\cdot,t} + qP_{\cdot,t}c_{\cdot,t} + A_{\cdot,t} = P_{\cdot,t}Y_{\cdot,t} - T_{\cdot,t} + R_{\cdot,t}A_{\cdot,t-1}. \tag{31}$$

The decision problem of a household with initial period bond holdings $A_{\cdot,t-1}$ is

$$V_t(A^H_{\cdot,t-1}, A^F_{\cdot,t-1}, \mathcal{Z}_t) = \max_{A^H_{\cdot,t},A^F_{\cdot,t},C_{\cdot,t},c_{\cdot,t}} \{\log(C_{\cdot,t}) + q\theta\log(c_{\cdot,t}) + \beta E_t V_{t+1}(A^H_{\cdot,t}, A^F_{\cdot,t}, \mathcal{Z}_{t+1})\} \tag{32}$$

subject to the flow budget constraint (31) and the liquidity constraints (26) and (27).
To simplify the analysis further, I assume that θ is large enough[12] so that the liquidity constraint is always binding,

$$c_{\cdot,t} = A_{\cdot,t}/P_{\cdot,t}. \tag{33}$$

The remaining decision how much home and foreign bonds to acquire is characterized through two first-order condition for Home households

$$A^H_{H,t}: \qquad \frac{1}{C_{H,t}} = q\theta_H \frac{1}{A_{H,t}/P_{H,t}} + (1-q)E_t\frac{\rho_{H,t+1}P_{H,t}}{P_{H,t+1}}\beta\frac{1}{C_{H,t+1}}, \tag{34}$$

$$A^F_{H,t}: \qquad \frac{1}{C_{H,t}} = q\theta_H \frac{1}{A_{H,t}/P_{H,t}} + (1-q)E_t\frac{\rho_{F,t+1}P_{F,t}}{P_{F,t+1}}\beta\frac{1}{C_{H,t+1}}, \tag{35}$$

and for Foreign households

$$A^F_{F,t}: \qquad \frac{1}{C_{F,t}} = q\theta_F \frac{1}{A_{F,t}/P_{F,t}} + (1-q)E_t\frac{\rho_{F,t+1}P_{F,t}}{P_{F,t+1}}\beta\frac{1}{C_{F,t+1}}, \tag{36}$$

$$A^H_{F,t}: \qquad \frac{1}{C_{F,t}} = q\theta_F \frac{1}{A_{F,t}/P_{F,t}} + (1-q)E_t\frac{\rho_{H,t+1}P_{H,t}}{P_{H,t+1}}\beta\frac{1}{C_{F,t+1}}. \tag{37}$$

[12] I assume and verify later that $\theta > A/PC$. In the general model (in the appendix) where the positive probability of a binding liquidity is endogenous such an assumption is not needed.

3.2 Fiscal and Monetary Policy

A standard way to represent monetary policy is as setting a sequence of nominal interest rates in a cashless economy,[13]

$$\mathcal{R}_\cdot := \{\rho_{\cdot,0} = 1 + i_{\cdot,0}, \rho_{\cdot,1} = 1 + i_{\cdot,1}, \rho_{\cdot,2} = 1 + i_{\cdot,2}, \ldots, \rho_{\cdot,t} = 1 + i_{\cdot,t}, \ldots\}. \tag{38}$$

One possibility that I explicitly allow for is that the interest is pegged in both countries. I will show that the nominal exchange rate is uniquely determined also in this case. Fiscal policy is represented by a sequence of nominal lump-sum taxes T_t,

$$\mathcal{T}_\cdot := \{T_{\cdot,0}, T_{\cdot,1}, \ldots, T_{\cdot,t}, \ldots\}. \tag{39}$$

The government's flow budget constraint has to satisfied at any point in time, which implicitly defines a sequence of nominal bonds

$$B_{\cdot,t+1} = \rho_{\cdot,t} B_{\cdot,t} - T_{\cdot,t}, \tag{40}$$

such that the intertemporal government budget constraint is satisfied:

$$B_{\cdot,0} = \sum_{t=0}^{\infty} T_{\cdot,0} \prod_{s=0}^{t} \frac{1}{\rho_{\cdot,s}} \tag{41}$$

and

$$\lim_{t \to \infty} B_{\cdot,t} \prod_{s=0}^{t-1} \frac{1}{\rho_{\cdot,s}} = 0. \tag{42}$$

Since fiscal and tax policies are expressed in nominal terms, this constraint holds for *all* sequences of prices,

$$\mathcal{P}_\cdot := \{P_{\cdot,0}, P_{\cdot,1}, \ldots, P_{\cdot,t}, \ldots\}. \tag{43}$$

[13]See Woodford (2003) for details. It would be straightforward to follow Woodford (2003) and add money to this model through assuming that agents derive utility from holding money. The results would be unchanged though.

In particular the price levels and the exchange rates are not determined such that the government budget constraint holds. Finally define the sequence of bonds

$$\mathcal{B}_{\cdot} := \{B_{\cdot,0}, B_{\cdot,1}, \ldots, B_{\cdot,t}, \ldots\}. \tag{44}$$

and the sequence of exchange rates

$$\mathcal{E} := \{\epsilon_0, \epsilon_1, \ldots, \epsilon_t, \ldots\}. \tag{45}$$

3.3 Resource constraints and asset markets

Total world savings, denominated in Home currency, equals $A_{H,t} + \epsilon_t A_{F,t}$, and total world supply of assets, again denominated in home currency, equals $B_{H,t} + \epsilon_t B_{F,t}$, so that the world asset market clearing equation equals.

$$A_{H,t} + \epsilon_t A_{F,t} = B_{H,t} + \epsilon_t B_{F,t} \tag{46}$$

The Home net nominal foreign asset position (in home currency) is equal to

$$NFA_{H,t} = A_{H,t} - B_{H,t}, \tag{47}$$

and the Foreign net nominal foreign asset position (in foreign currency) is equal to

$$NFA_{F,t} = A_{F,t} - B_{F,t}. \tag{48}$$

Asset market clearing is equivalent to

$$NFA_{H,t} = -\epsilon_t NFA_{F,t}. \tag{49}$$

The world resource constraint is

$$\tilde{Y}_{H,t} + \tilde{Y}_{F,t} = C_{H,t} + C_{F,t} + q(c_{H,t} + c_{F,t}) \tag{50}$$

which equates total world output $Y_H + Y_F$ to the sum of Home private consumption, $C_{H,t} + qc_{H,t}$, and Foreign private consumption, $C_{F,t} + qc_{F,t}$.

3.4 Competitive Equilibrium

In period $t = 0$, the initial conditions are that Home households hold $A^H_{H,0}$ of Home bonds and $A^F_{H,0}$ of Foreign bonds so that initial asset holding is equal to $A_{H,0} = A^H_{H,0} + \epsilon_0 A^F_{H,0}$ and the net foreign asset position equals $NFA_{H,0} = A_{H,0} - B_{H,0}$. Similarly, Foreign households hold $A^F_{F,0}$ of Foreign bonds and $A^H_{F,0}$ of Home bonds so that initial asset holding is equal to $A_{F,0} = A^F_{F,0} + A^H_{F,0}/\epsilon_0$ and the net foreign asset position equals $NFA_{F,0} = A_{F,0} - B_{F,0}$.

Since this is a one-good economy, the real exchange rate is equal to one, $\epsilon_t \frac{P_{F,t}}{P_{H,t}} = 1$.

Definition 1. *Given initial asset holdings $A^H_{H,0}, A^F_{H,0}$ and $A^F_{F,0}, A^H_{F,0}$ and sequences of nominal interest rates $\mathcal{R}_\cdot$, nominal taxes $\mathcal{T}_\cdot$, nominal bonds $\mathcal{B}_\cdot$ and shocks $\{\mathcal{Z}_{\cdot,t}\}_{t=0}^\infty$, a competitive equilibrium are sequences of consumption spending $\{C_{\cdot,t}\}_{t=0}^\infty$ at t_1 and $\{c_{\cdot,t}\}_{t=0}^\infty$ at t_2, bonds purchases $\{A^H_{\cdot,t}, A^F_{\cdot,t}\}_{t=0}^\infty$, prices $\mathcal{P}_\cdot$ and exchange rates $\mathcal{E}$, such that for all t:*

1. *Households take prices and policies as given and choose $\{C_{\cdot,t}, c_{\cdot,t}, A^H_{\cdot,t}, A^F_{\cdot,t}\}_{t=0}^\infty$ to maximize utility.*

2. *The government budget constraints (41) hold.*

3. *Market Clearing and Resource constraint:*

   (a) *Asset Markets*

$$\begin{aligned} World: &\quad A_{H,t} + \epsilon_t A_{F,t} = B_{H,t} + \epsilon_t B_{F,t} \\ Home: &\quad A^H_{H,t} + A^H_{F,t} = B_{H,t} \\ Foreign: &\quad A^F_{F,t} + A^F_{H,t} = B_{F,t} \end{aligned}$$

   (b) *Resource Constraint $\tilde{Y}_{H,t} + \tilde{Y}_{F,t} = C_{H,t} + C_{F,t} + q(c_{H,t} + c_{F,t})$.*

   (c) *Real exchange rate: $\epsilon_t \frac{P_{F,t}}{P_{H,t}} = 1$.*

From now on I will only consider the limit of this economy where all uncertainty has vanished, i.e. the variance of the shocks converges to zero.

4 Nominal Exchange Rate and Net Foreign Assets

This section shows that the nominal exchange rate is uniquely determined and how it depends on monetary and fiscal policy as well as on productivity and the demand for liquidity. I first derive the steady state nominal exchange rate before I move to the response to various shocks. A key finding is that the steady-state level of the exchange rate depends on the distribution of assets, which is the result of past asset accumulation decisions and past valuation gains. In particular, the exchange rate is non-stationary but depends on countries foreign and home asset holdings, the state variables. This history dependence also implies that in order to study the response to various shocks, one has to first compute the long-run response of the exchange rate. Once this is known, the uncovered interest rate parity condition allows to derive the short-run response as well. I therefore first consider steady states. Remember, that I only consider the limit economy when all uncertainty has vanished.

4.1 The Steady-State Exchange Rate

As in any incomplete market model with precautionary savings (e.g. Aiyagari (1995)), the steady-state real rate of return satisfies

$$\beta \frac{R.}{1+\pi.} = \beta \frac{1+i.}{1+\pi.} < 1, \tag{51}$$

since otherwise asset demand would grow without bound. Furthermore, in a steady state, the growth rate of nominal debt and nominal taxes as well as the real value of debt and taxes are constant, implying that the inflation rate is equal to

$$1+\pi. = \frac{T_{\cdot,t+1} - T_{\cdot,t}}{T_{\cdot,t}} = \frac{B_{\cdot,t+1} - B_{\cdot,t}}{B_{\cdot,t}}. \tag{52}$$

The growth rate and the real value of assets A and NFA are constant as well, implying that

$$A_{\cdot,t} = A.(1+\pi.)^t \tag{53}$$

$$NFA_{\cdot,t} = NFA.(1+\pi.)^t. \tag{54}$$

I also first consider steady states where the inflation rate is zero, $\pi = \pi_H = \pi_F = 0$, and the nominal interest rate is the same across countries, $R = R_H = R_F$. This simplifies notation as it avoids detrending variables and at the same time provides the full intuition. The more general case which allows for non-zero inflation rates and a steady-state interest-rate differential across countries is considered below. The only difference between the simpler and the more general one is that the exchange rate is constant in the first case whereas it can have a trend in the latter case.

I now turn to the derivation of the steady-state exchange rate ϵ_{ss}. The equilibrium exchange rate clears the world goods and asset market. Therefore, to characterize the world equilibrium, I need to derive the demand for goods by home, D_H, and foreign, D_F. Consider first the home country. It holds A_H nominal assets and has a net foreign nominal asset position NFA_H - the state variables - which are history-dependent as they are the result of past asset accumulation and portfolio decisions. In the limit case without uncertainty the uncovered interest parity condition $\rho_{H,t+1} = \rho_{F,t+1}\frac{\epsilon_{t+1}}{\epsilon_t}$ together with $R_{H,t+1} = \rho_{H,t+1} = \tilde{\rho}_{H,t+1}$ and $R_{F,t+1} = \rho_{F,t+1} = \tilde{\rho}_{F,t+1}$ imply that the two FOC per country collapse to one FOC per country. For the home country, the steady-state FOC for assets is

$$\frac{1}{P_H C_H} = q\frac{\theta_H}{A_H} + (1-q)\frac{\beta R_H}{1+\pi_H}\frac{1}{P_H C_H}, \tag{55}$$

which allows now to express nominal consumption as a function of nominal assets:

$$P_H C_H = A_H \underbrace{\frac{1-(1-q)\frac{\beta R_H}{1+\pi_H}}{q\theta_H}}_{=:\alpha_H}. \tag{56}$$

Total consumption in the second subperiod t_2, $qP_H c_H = qA_H$, so that aggregate demand in the home country

$$D_H = (q+\alpha_H)A_H. \tag{57}$$

The marginal propensity to consume out of wealth A_H is

$$mpc_H = q + \alpha_H \approx q + \frac{1}{\theta_H}. \tag{58}$$

Quite intuitively, the mpc_H is increasing in the fraction of households with sudden expenditure needs, q. A higher value of θ_H increases the utility of sudden consumption at t_2 and therefore increases precautionary savings and decreases consumption and the mpc_H. I assume that[14]

$$mpc_H > \frac{1}{\beta} - 1. \tag{59}$$

[14]Imposing $q + 1/\theta_H > 1/\beta - 1$ is sufficient.

I need to impose this parameter restriction here since the precautionary demand for savings can be so high (θ_H large) such that households consume only a fraction q of their liquid assets.

Similar derivations for the foreign country aggregate demand yield

$$D_F = (q + \alpha_F)A_F, \tag{60}$$

where $\alpha_F = \frac{1-(1-q)\frac{\beta R_F}{1+\pi_F}}{q\theta_F}$ and I assume that the marginal propensity to consume out of wealth A_F

$$mpc_F = q + \alpha_F \approx q + \frac{1}{\theta_F} > \frac{1}{\beta} - 1. \tag{61}$$

The supply of goods by the home country, Y_H, and the foreign country, Y_F are exogenously given. In a steady state nominal next exports NX equal the return on nominal foreign assets

$$NX_H = -(R-1)NFA_H, \tag{62}$$

$$NX_F = -(R-1)NFA_F, \tag{63}$$

so that in equilibrium

$$P_H Y_H = D_H + NX_H = D_H - (R-1)NFA_H, \tag{64}$$

$$P_F Y_F = D_F + NX_F = D_F - (R-1)NFA_F. \tag{65}$$

Using that $NFA_F = -NFA_H/\epsilon$ determines the steady state exchange rate

$$\epsilon\frac{Y_H}{Y_F} = \frac{D_H(A_H) - (R-1)NFA_H}{D_F(A_F) + (R-1)NFA_H/\epsilon}, \tag{66}$$

which can be conveniently represented in the top panel of Figure 6. To understand the location of the steady-state exchange rate note that an appreciation of the exchange rate (a fall in ϵ) means that foreign prices increase relative to home prices which contracts foreign demand relative to home demand, such that to the left of the equilibrium home demand is too high and ROW demand is too low. Symmetrically, a depreciation of the exchange rate (an increase in ϵ) means that home prices increase relative to foreign prices which contracts home demand relative to foreign demand, such that to the right of the equilibrium home demand is too low and ROW demand is too high. The steady-state exchange rate ϵ_{ss} is

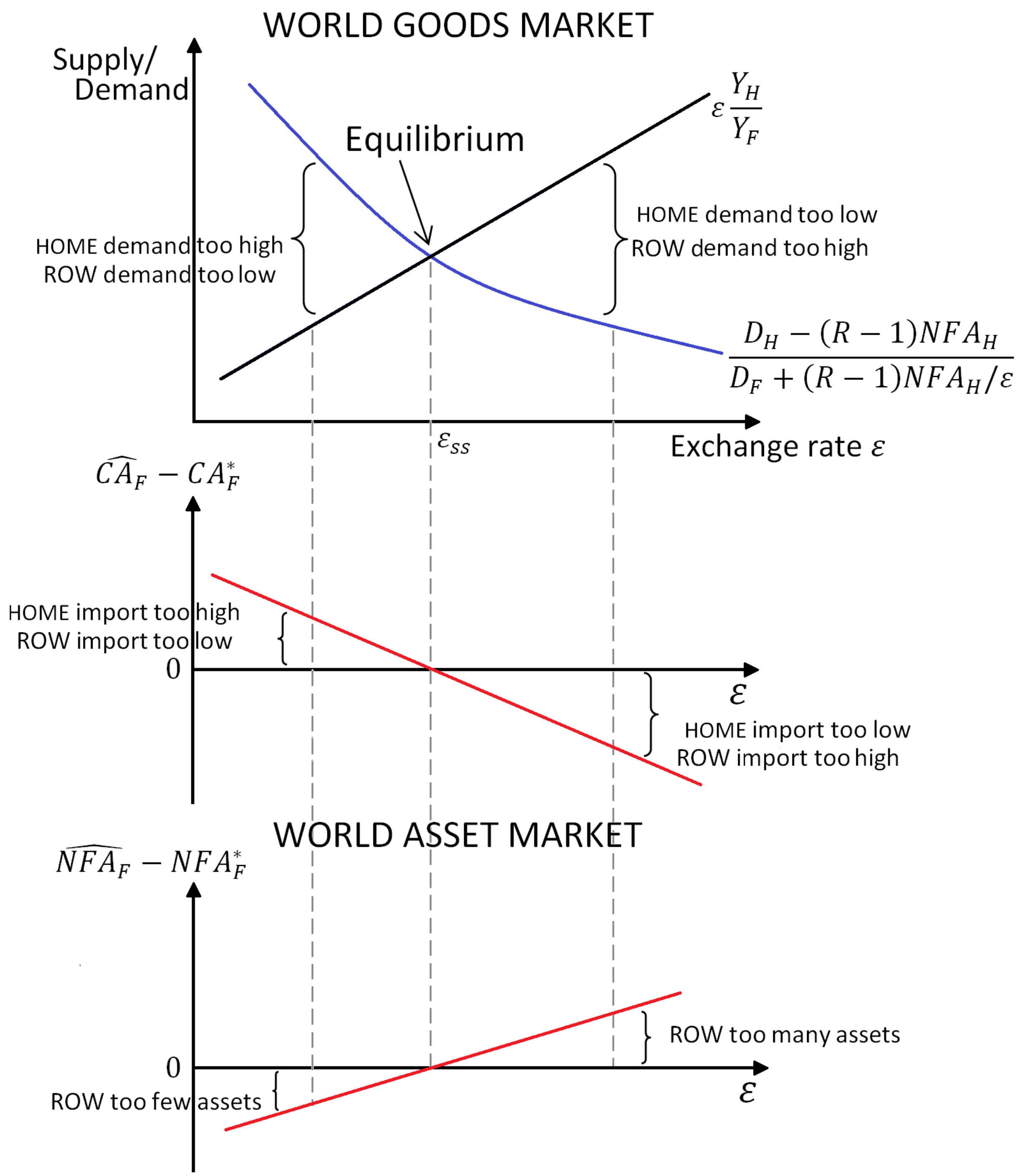


Figure 6: Exchange Rate Determination in World Goods and Asset Market

then such that world goods market clear and is located where the supply curve, $\epsilon\frac{Y_H}{Y_F}$, and the demand curve, $\frac{D_H(A_H)-(R-1)NFA_H}{D_F(A_F)+(R-1)NFA_H/\epsilon}$, intersect. Solving for the steady-state exchange rate ϵ_{ss}:

Result 1. *In a steady state where $R = R_H = R_F$ the nominal exchange rate equals*

$$\epsilon_{ss} = \frac{(q+\alpha_H)\frac{A_H}{Y_H} - (R-1)\frac{NFA_H}{Y_H}(1+\frac{Y_H}{Y_F})}{(q+\alpha_F)\frac{A_F}{Y_F}}. \tag{67}$$

The steady-state exchange rate in Result 1 was derived as clearing the world good market, which is equivalent to the condition $NX_F = -NX_H/\epsilon_{ss}$. An equivalent interpretation is thus that the exchange rate adjusts to ensure that exports and imports of the home and the foreign country are consistent with each other. To understand this adjustment better, define $\widehat{NX_F}(\epsilon)$ to be the net exports of the foreign country as a function of the exchange rate ϵ, which solves

$$\epsilon\frac{Y_H}{Y_F} = \frac{D_H(A_H) + NX_H}{D_F(A_F) + \widehat{NX_F}(\epsilon)}. \tag{68}$$

Equilibrium in world good markets requires that

$$\widehat{NX_F}(\epsilon) = -NX_H/\epsilon =: NX_F^*(\epsilon). \tag{69}$$

Excess NX (excess exports or insufficient imports),

$$\widehat{NX_F}(\epsilon) - NX_F^*(\epsilon) = (q+\alpha_F)A_F\frac{\epsilon_{ss}-\epsilon}{\epsilon}, \tag{70}$$

which is positive if the nominal exchange rate is below its equilibrium level and is negative if the nominal exchange rate is above its equilibrium level, as the middle panel of Figure 6 shows. The world goods market clears if and only if the nominal exchange rate $\epsilon = \epsilon_{ss}$. For values of $\epsilon > \epsilon_{ss}$, foreign consumption demand is too high relative to home consumption demand or equivalently foreign imports are too high relative to foreign exports (= home imports). Similarly, if $\epsilon < \epsilon_{ss}$, foreign consumption demand is too low relative to home consumption demand or equivalently foreign imports are too low relative to foreign exports (= home imports).

A third equivalent asset market interpretation uses that the steady state current account is zero, $NX_H = -(R-1)NFA_H$, such that ϵ_{ss} is clearing the market for net foreign assets.

To see this define $\widehat{NFA_F} = -\widehat{NX_F}/(R-1)$ to be the net demand for foreign assets by the foreign country as a function of the exchange rate ϵ.[15] Equilibrium in world asset markets requires that

$$\widehat{NFA_F}(\epsilon) = -NFA_H/\epsilon =: NFA_F^*(\epsilon), \tag{71}$$

so that excess asset demand,

$$\widehat{NFA_F}(\epsilon) - NFA_F^*(\epsilon) = \frac{(q+\alpha_F)}{R-1} A_F \frac{\epsilon - \epsilon_{ss}}{\epsilon}, \tag{72}$$

which inherits its interpretation from the corresponding NX equation and is plotted in the lower panel of Figure 6.

Using Result 1 it is easy to see how the steady-state exchange rate responds to changes in home assets, the net foreign asset position and productivity:

Result 2. *An increase in steady state*

- *Net foreign assets NFA_H leads to an appreciation,* $\frac{\partial \epsilon_{ss}}{\partial NFA_H} < 0$.
- *Home assets A_H leads to a depreciation,* $\frac{\partial \epsilon_{ss}}{\partial A_H} > 0$.
- *Home productivity Y_H leads to an appreciation,* $\frac{\partial \epsilon_{ss}}{\partial Y_H} < 0$.

The intuition is straightforward. An increase in net foreign assets, holding total assets fixed, is equivalent to a worsening of NX, exports relative to imports fall. This shifts demand towards foreign goods (relative to home goods) leading to an appreciation of the exchange rate. This appreciation contracts foreign demand relative to home demand such that world markets clear again. An increase in home assets stimulates home demand relative to foreign demand such that the exchange rate has to depreciate which contracts home demand such that world markets clear again. An increase in home productivity increases the supply of home goods so that the exchange rate has to appreciate to stimulate home demand relative to foreign demand.

So far, I assumed that the steady-state nominal interest rates in the home and foreign country are identical, equal to R, and that the intertemporal elasticity of substitution (IES) equals one. I now show how the previous results can be generalized if I relax these two

[15]For $R = 1$, $\widehat{NFA_F} = 0$.

assumptions. I now allow that, first, the steady-state nominal interest rates in the home and foreign country can be different, $R_H \neq R_F$. Second, the intertemporal elasticitiy of substitution (IES) does not have to equal one. Instead, the IES equals $1/\sigma$ an an individual with emergency expenditure needs now derives utility

$$\frac{(C_{\cdot,t})^{1-\sigma}}{1-\sigma} + \theta.\frac{(c_{\cdot,t})^{1-\sigma}}{1-\sigma} \tag{73}$$

from consuming $C_{\cdot,t}$ at t_1 and $c_{\cdot,t}$ at t_2, where $\sigma > 1$. A member without these needs derives utility

$$\frac{(C_{\cdot,t})^{1-\sigma}}{1-\sigma}. \tag{74}$$

I obtain the generalization of Result 1:

Result 3. [General Case] *In a steady state where* $R_H = 1 + i_H \neq R_F = 1 + i_F$ *and the IES equals* $1/\sigma > 0$ *the nominal exchange rate equals*

$$\epsilon_t = \left(\frac{1+\pi_H}{1+\pi_F}\right)^t \frac{(q + \alpha_H^{1/\sigma})\frac{A_H}{Y_H} - \frac{NFA_H}{Y_H}((R_H - 1) + (R_F - 1)\frac{Y_H}{Y_F})}{(q + \alpha_F^{1/\sigma})\frac{A_F}{Y_F}}. \tag{75}$$

which reduces to Result 1 if $R_H = R_F$, $\pi_H = \pi_F$ and $\sigma = 1$. Relaxing the two assumptions induces two changes in comparison to the simpler case in (67). First, there is an interest rate differential such that the exchange rate is now growing at rate $\frac{1+\pi_H}{1+\pi_F} = \frac{1+i_H}{1+i_F}$. This relationship between the change in the exchange rate and the inflation rate differential is well known from textbook models and holds here as well but with one important difference. Here, not only the change of the exchange rate is determined but also the level itself whereas textbook models speak only to the first property. As a result I can obtain the same responses of the exchange rate as in Result 2 with the only difference that now the change is relative to a non-zero trend whereas there was no trend before. Second, the marginal propensity to consume generalizes to $q + \alpha_{\cdot}^{1/\sigma}$. When σ gets large, $\alpha_{\cdot}^{1/\sigma} \to 1$ reflecting households increasing unwillingness to substitute between consumption C and $c()$. In the extreme case that $\sigma = \infty$, households do not substitute between consumption C and $c()$ but change both only proportionally, $\alpha_{\cdot}^{1/\sigma} = 1$.

4.2 Exchange Rates: Monetary Policy and Demand for Liquidity

I now consider another important determinant of exchange rates in the data, monetary policy and answer the question how a temporary unexpected change in nominal interest rates in one country affects the exchange rate. A key difference and complication in comparison with the previous section is that I now consider temporary changes rather than permanent changes, for monetary policy the relevant scenario. In the absence of shocks, I again assume a steady states with $R_H = R_F$, zero inflation and $\sigma = 1$. Outside steady state I will use that the uncovered interest parity condition holds in this model:

$$1 + i_{H,t+1} = (1 + i_{F,t+1}) E_t \frac{\epsilon_{t+1}}{\epsilon_t} = (1 + i_{F,t+1}) \frac{\epsilon_{t+1}}{\epsilon_t}, \tag{76}$$

where the last equality follows from the assumption that agents in this economy believe to live in a world without uncertainty, where all shocks are "MIT shocks". Starting from the new steady state exchange rate,

$$\epsilon_{ss} = \lim_{t \to \infty} \epsilon_t \tag{77}$$

and using uncovered interest parity to iterate backwards

$$\epsilon_t = \frac{1 + i_{F,t+1}}{1 + i_{H,t+1}} \epsilon_{t+1}, \tag{78}$$

yields for the period t exchange rate

$$\epsilon_t = \Big(\prod_{s=t+1}^{\infty} \frac{1 + i_{F,s}}{1 + i_{H,s}} \Big) \epsilon_{ss}. \tag{79}$$

This simple derivation has two implications, a theoretical one and one relevant for monetary policy. Theoretically it implies that the determinacy of the exchange rate at time t (outside steady state) follows from the determinacy of the steady state exchange rate ϵ_{ss}. For a response to a temporary monetary policy change it implies that one has to proceed in two steps. First one has to compute the response of the long-run exchange rate to the policy change. Then one can use the uncovered interest rate parity condition to find the period t response.

For the first step I use Result 1 of the previous section which provides a closed form solution for the steady state nominal exchange rate

$$\epsilon_{ss} = \frac{(q+\alpha_H)\frac{A_{H,ss}}{Y_H} - (R-1)\frac{NFA_{H,ss}}{Y_H}(1+\frac{Y_H}{Y_F})}{(q+\alpha_F)\frac{A_{F,ss}}{Y_F}}, \tag{80}$$

where $A_{H,ss}$, $A_{F,ss}$, $NFA_{H,ss}$ are steady state home assets, steady state foreign assets and the steady state net foreign asset position, respectively. Calculating the steady-state exchange rate and these assets is complicated (and interesting) since these differ from the pre-policy-change values, ϵ_{t-1}, $A_{H,t-1}$, $A_{F,t-1}$ and $NFA_{H,t-1}$. A change in home monetary policy while foreign monetary policy is unchanged affects the incentives of home and foreign households to accumulate assets, resulting eventually in new steady-state assets levels. As I have shown above, different steady-state assets holdings induce different market clearing exchange rates. I therefore have to characterize the asset accumulation path which depends on monetary policy but also on the path of exchange rates, which itself depends on the asset paths. In equilibrium the asset accumulation decisions and the induced change in the exchange rate have to be all consistent. The next result accomplishes this making for now the simplifying assumption that each country only holds its own assets before the policy change, $A^H_{H,t} = B_H$ and $A^F_{F,t} = B_F$. This assumption ensures the absence of valuation gains or losses and the corresponding wealth effects from the change of the nominal exchange rate t, which otherwise would affect the saving and consumption behavior. The next result states that in the absence of these wealth effects, a tightening of monetary policy always leads to an appreciation. Below I provide a sufficient condition for an appreciation in the presence of valuation effects.

Result 4. [Monetary Policy: No Valuation Gain] *An increase in $R_{H,t}$ leads to an appreciation (ϵ_t falls),*

$$\frac{\partial \epsilon_t}{\partial R_{H,t}} < 0, \tag{81}$$

and a fall in long-run home assets and in the net foreign asset position and an increase in foreign assets

$$\frac{\partial A_{H,ss}}{\partial R_{H,t}} < 0, \quad \frac{\partial A_{F,ss}}{\partial R_{H,t}} > 0 \quad \frac{\partial NFA_{H,ss}}{\partial R_{H,t}} < 0. \tag{82}$$

This result characterizes both the response of the steady-state exchange rate and of steady state asset holdings, which as explained above are tightly linked. The key to understand this result is therefore to understand why an increase in $R_{H,t}$ stimulates savings in the foreign country and depresses savings in the home country.

To this aim it is instructive to first consider the response of nominal demand D_H in the home country if it was a closed economy. In this case, an increase in R_H by Δi stimulates savings and contracts demand and leads to a percentage drop in prices by Δp. Equilibrium in the goods market in a closed economy requires that the percentage change Δp in nominal supply $P_H Y_H$ is equal to the percentage change in nominal demand $D_H(1 + i_H, ...)$:

$$\Delta p = \frac{\partial D_H}{\partial i} \Delta i. \tag{83}$$

In the appendix I show that in my incomplete markets model the demand elasticity, $-1 < \frac{\partial D_H}{\partial i} < 0$, so that $-\Delta p < \Delta i$, implying that the real interest rate increases, $\Delta r^{closed} = \Delta i + \Delta p > 0$, where for the sake of the argument I assume that the inflation rate change is equal to minus the price change, $-\Delta p$.[16]

In the open economy the real interest rate is equal in both countries. The arguments for the closed home economy therefore imply that the increase in R_H leads to an increase in the real interest rate in the home and the foreign country, implying an increase in foreign savings. If the real interest rate increased as much in the open economy (Δr^{open}) as in the closed economy (Δr^{closed}) then by construction savings would be unchanged in the home country but would increase in the foreign country, that is world asset market would not clear. Therefore the real interest has to increase by less than in its closed economy counterpart, $\Delta r^{open} < \Delta r^{closed}$, implying that home savings fall and foreign country savings increase but less then when $\Delta r^{open} = \Delta r^{closed}$. The increase in the interest rate Δr^{open} is then such that the increase in foreign savings is equal to the decrease in home savings and world goods and asset markets clear.

This asset accumulation by the foreign country and the corresponding de-accumulation by the home country implies an appreciation of the steady-state exchange rate.

The previous result depends on the interest rate elasticity of demand, $-1 < \frac{\Delta D_H}{\Delta i_H} < 0$, being not too high, which clearly depends on households' IES $1/\sigma$. A lower IES (a higher σ) decreases this elasticity whereas a higher IES (a lower σ) increases this elasticity. Result 4 shows that $-1 < \frac{\Delta D_H}{\Delta i_H} < 0$ for $\sigma = 1$, implying that it also holds for larger σ such that

[16]This line of argument would not be possible in a complete markets model since then the price level, the exchange rate and thus also nominal demand are indeterminate.

a tightening of monetary policy leads to an appreciation if $\sigma \geq 1$. For $\sigma < 1$, the demand elasticity can be larger, $\frac{\Delta D_H}{\Delta i_H} < -1$, the price drop can exceed the increase in the nominal interest rate, $-\Delta p > \Delta i$, and the real interest rate can fall. The next result provides a lower bound on σ which is sufficient (but not necessary) for $-1 < \frac{\Delta D_H}{\Delta i_H} < 0$, for an increase in the real interest rate and for an appreciation.

Result 5. [Monetary Policy: No Valuation Gain, IES $1/\sigma$] *For an utility function as in (73) and (74) with IES $1/\sigma$, an increase in $R_{H,t}$ leads to an appreciation (ϵ_t falls),*

$$\frac{\partial \epsilon_t}{\partial R_{H,t}} < 0, \tag{84}$$

and a fall in long-run home assets and in the net foreign asset position and an increase in foreign assets

$$\frac{\partial A_{H,ss}}{\partial R_{H,t}} < 0, \quad \frac{\partial A_{F,ss}}{\partial R_{H,t}} > 0, \quad \frac{\partial NFA_{H,ss}}{\partial R_{H,t}} < 0. \tag{85}$$

if $\frac{R}{1+\pi}\frac{(1-q)\beta}{\sigma} < 1$.

The closer the economy is to a complete markets model (probability that the credit constraint binds $q \to 0$), the closer σ has to be to one for an appreciation of the exchange rate. For intermediate values of q, values $\sigma < 1$ imply an appreciation if monetary policy is tightened. Note that a depreciation requires a fall in the real interest rate and a large fall in prices when nominal interest rates increase. Adding sticky prices to this model would clearly make such large price movements unlikely and would therefore rule out a depreciation. I will explore this in future research.

I now show how these results generalize when I allow for valuation effects. If the home country holds foreign assets, $A_H^F > 0$, or if the foreign country holds home assets , $A_F^H > 0$, then an increase of the nominal interest rates has valuation/wealth effects. There are two types of valuation/wealth effects. First, the value of foreign assets held by home households, $A_H^F \epsilon$, increases if the exchange rate depreciates (ϵ increases) and decreases if the exchange rate appreciates (ϵ decreases), inducing positive and negative wealth effects for home households, respectively. Second, the increase in the home nominal interest rate leads to an increase in interest rate payments $A_F^H(R_H - 1)$ from home to foreign households, inducing a negative wealth effect for home households and a positive wealth effect for foreign households.

Clearly these wealth effects affect demand and savings in both countries and can overturn the previous results if they are too strong. The next result provides sufficient conditions which bound the size of these wealth effects such that a tightening of monetary policy implies an appreciation.

Result 6. [Monetary Policy: With Valuation/Wealth Effects] *If*

$$\frac{A_H^F}{B_F} \leq \frac{Y_H(q+\alpha_F)}{Y_F(q+\alpha_H)+Y_H(q+\alpha_F)} \quad \text{and} \quad \frac{A_F^H}{B_H} \leq \frac{Y_F(1+\alpha_H)}{Y_F(1+\alpha_H)+Y_H(1+\alpha_F)}, \tag{86}$$

then an increase in $R_{H,t}$ leads to an appreciation (ϵ_t falls),

$$\frac{\partial \epsilon_t}{\partial R_{H,t}} < 0, \tag{87}$$

and a fall in long-run home assets and in the net foreign asset position and an increase in foreign assets

$$\frac{\partial A_{H,ss}}{\partial R_{H,t}} < 0, \quad \frac{\partial A_{F,ss}}{\partial R_{H,t}} > 0, \quad \frac{\partial NFA_{H,ss}}{\partial R_{H,t}} < 0. \tag{88}$$

The conditions in (86) restrict how many of a country's assets are held by the other country. This restriction bounds the size of the wealth effects $A_H^F\epsilon$ and $A_F^H(R_H-1)$ since these are naturally proportional to how many foreign assets a country holds. In the symmetric case where the home and the foreign country are identical the condition states that each country holds the majority of its own bonds.[17] In the asymmetric case the conditions depend on the relative magnitudes that these wealth shifts have on demand, which are parameterized by the marginal propensities to consume and the sizes of the two countries.

The model also has clear implications for the response of the exchange rate and asset holdings to a surge in the demand for liquidity in the home country, an increase in $\theta_{H,t}$.

Result 7. [Liquidity Demand Increase] *If (86) holds then an increase in $\theta_{H,t}$ leads to a depreciation (ϵ_t increases),*

$$\frac{\partial \epsilon_t}{\partial \theta_{H,t}} > 0, \tag{89}$$

[17]This assumption is well supported by the data which exhibit a strong home bias in bond holdings as documented in Section 7.1.1. in Coeurdacier and Rey (2013).

and an increase in long-run home assets and in the net foreign asset position and a fall in foreign assets

$$\frac{\partial A_{H,ss}}{\partial \theta_{H,t}} > 0, \quad \frac{\partial A_{F,ss}}{\partial \theta_{H,t}} < 0 \quad \frac{\partial NFA_{H,ss}}{\partial \theta_{H,t}} > 0. \tag{90}$$

The same sufficient conditions as above ensure a depreciation if valuations effects are present but here no assumption on σ is needed. In the special case when valuation effects are absent, $A_H^F = 0$ and $A_F^H = 0$, an increase in liquidity demand always leads to a depreciation.

The key to understand why the home currency depreciates is again to understand why home households accumulate assets and foreign households de-accumulate assets in response to an increase in liquidity demand (a higher $\theta_{H,t}$) since then the previous results imply a depreciation of the steady-state exchange rate. The intuition is straightforward. A higher $\theta_{H,t}$ increases precautionary savings and thus asset demand by home households, $\frac{\partial A_{H,ss}}{\partial \theta_{H,t}} > 0$. The real interest rate falls such that foreign households provide these assets, $\frac{\partial A_{F,ss}}{\partial \theta_{H,t}} < 0$, and world asset markets clear.[18]

5 Empirical Evidence

The previous theoretical analysis not only shows the determinacy of a unique exchange rate but is also explicit about its determinants: productivity, home and net foreign assets and monetary policy operating through setting nominal interest rates. Section 5.1 provides evidence which confirms these model predictions on the determinants of the exchange rate. The theoretical model also identifies the long-run net foreign asset position of a country as a key determinant of the current exchange rate and shows that net foreign assets depend on home debt issuance and current monetary policy. Section 5.2 provides evidence supporting these model predictions on the determinants of the long-run net foreign asset position.

[18]Using the notation from above, an increase in $\theta_{H,t}$ in a closed economy contracts demand such that prices drop in equilibrium

$$\Delta p = \underbrace{\frac{\partial D_H}{\partial \theta_H}}_{<0} \Delta \theta_H < 0,$$

implying that the real interest rate falls, $\Delta r^{closed} < 0$. In the open economy, $\Delta r^{closed} < \Delta r^{open} < 0$.

5.1 Evidence on the Determinants of the Exchange Rate

The theoretical analysis in Section 4 implies that the period t exchange rate ϵ_t depends on home assets $A_{H,t}$, net foreign assets $NFA_{H,t}$, the nominal interest rate $R_{H,t}$ and home productivity $Y_{H,t}$,

$$\epsilon_t \;=\; \mathcal{E}\big(\underset{(+)}{A_{H,t}}, \underset{(-)}{NFA_{H,t}}, \underset{(-)}{R_{H,t}}, \underset{(-)}{Y_{H,t}}\big). \tag{91}$$

The theoretical results also predict whether a change in $A_{H,t}, NFA_{H,t}, R_{H,t}, Y_{H,t}$ leads to an appreciation (ϵ falls) or a depreciation (ϵ increases). An increase in assets $A_{H,t}$, while all other determinants in particular $NFA_{H,t}$ are unchanged, leads to a depreciation. An increase in $NFA_{H,t}, R_{H,t}$ and $R_{H,t}$ all lead to an appreciation.

To test these predictions in the data I use the benchmark regression

$$\Delta\log(\epsilon_t) = \gamma_0 + \gamma^A\Delta\log(A_{H,t}) + \gamma^{NFA}\Delta\log(NFA_{H,t}) + \gamma^Y\Delta\log(Y_{H,t}) + \gamma^R\log(R_{H,t-1}) + \eta_t \tag{92}$$

where I difference non-stationarity data, denoted by Δ, so that for example $\Delta\log(\epsilon_t) = \log(\epsilon_t) - \log(\epsilon_{t-1})$.[19] Note that the interest rate is shifted by a period. The theory is confirmed in the data if $\gamma^A > 0$ and $\gamma^{NFA}, \gamma^Y, \gamma^R < 0$.

In the empirical analysis the US is the home country and the rest of the world (ROW) is the foreign country. The only asset in the model is debt, so that I use the quarterly US net foreign debt position from Gourinchas and Rey (2007b) (described in detail in Gourinchas and Rey (2007a)) who carefully construct US debt assets and liabilities for the time period 1952:Q1 - 2004:Q1 as my model consistent measure of NFA. The nominal effective trade weighted exchange rate is available for 1973:Q1-2004:Q1 from Gourinchas and Rey (2007b).[20] Assets $A_{H,t}$ is US government federal debt minus US foreign debt liabilities plus foreign debt assets, $Y_{H,t}$ is US output per worker and $R_{H,t}$ is the US Federal Funds Rate. The benchmark analysis uses all available data from 1973:Q1 to 2004:Q1. Appendix A.II provides a detailed data description.

Table 1 contains the main empirical result which confirms the theoretical predictions.

[19] To avoid problems arising from seasonalities in productivity, I consider the annual change $\Delta\log(Y_{H,t}) = \log(Y_{H,t}) - \Delta\log(Y_{H,t-4})$.

[20] For this time period $NFA_{US,t} < 0$ so that I obtain a well-defined growth rate of NFA, $\Delta NFA_{US,t} = -\big(\log(|NFA_{US,t}|) - \log(|NFA_{US,t-1}|)\big)$.

Table 1: Determinants of the US Exchange Rate

Assets A_{US}	Net foreign debt NFA_{US}	Federal Funds Rate R_{US}	Productivity Y_{US}
0.3497***	-0.1585***	-0.0034***	-0.3687***
(0.0006)	(0.0000)	(0.0000)	(0.0035)

Observations: 124, R^2: 0.1749
Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

An increase in US asset holdings leads to a depreciation of the US exchange rate whereas an increase in the US net foreign asset debt position, the US federal funds rate and US productivity lead to an appreciation. Although the theoretical model on purpose does not feature any of the frictions which are considered to be important for short-run dynamics and are apparently present in the data, it is remarkable that the relationship between the exchange rate and its theoretical determinants can nevertheless be detected in the data. While it is conceivable that other models with many rigidities and a rich set of structural shocks deliver the same empirical predictions, those alternative explanations cannot be ruled out using the available time series data only. However, for these alternative explanations to deliver well defined theoretical predictions for the nominal exchange rate, the mechanism to determine the nominal exchange rate in this paper's simple model has to be incorporated in these richer models as well, which suggests the same determinants of the exchange rate as in the simple model. I thus consider the empirical analysis while clearly far from a definitive answer a successful first pass of the data.

I now make a simple modification of the benchmark specification which can partly address the absence of realistic short-run dynamics in the model and at the same time deals with potential measurement error. Instead of considering the one period time difference I now consider k period time differences, so that the empirical specification is

$$\Delta_k \log(\epsilon_t) = \gamma_0 + \gamma_k^A \Delta_k \log(A_{H,t}) + \gamma^{NFA} \Delta_k \log(NFA^{H,t}) + \gamma_k^Y \Delta \log(Y_{H,t}) + \gamma_k^R \log(R_{H,t-k}) + \eta_{k,t}, \tag{93}$$

where $\Delta_k x_t = x_t - x_{t-k}$ is the k quarters difference of a variable x_t, so that the benchmark specification in (92) arises a special case for k = 1 quarter. The idea of using these longer

Table 2: Determinants of the US Exchange Rate: $\Delta_k \log(\epsilon)$.

VARIABLES	(1) k=4q	(2) k=6q	(3) k=8q	(4) k=10q	(5) k=12q
A_{US}	0.3252***	0.3635**	0.3700***	0.3192**	0.3093**
	(0.0035)	(0.0173)	(0.0075)	(0.0269)	(0.0357)
NFA_{US}	-0.1629**	-0.2210**	-0.2604***	-0.3254***	-0.4131***
	(0.0122)	(0.0172)	(0.0081)	(0.0019)	(0.0005)
R_{US}	-0.0122***	-0.0163***	-0.0207***	-0.0210***	-0.0228***
	(0.0000)	(0.0000)	(0.0000)	(0.0016)	(0.0064)
Y_{US}	-0.8464**	-0.9634*	-1.2129*	-1.2454	-1.0771
	(0.0106)	(0.0820)	(0.0986)	(0.1717)	(0.2479)
Observations	121	119	117	115	113
R^2	0.2393	0.2675	0.3281	0.3713	0.4482

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

time differences is that the short-run movements in the data and measurement partially average out and thus affect the results to a smaller degree. Table 2 reports the results for $k = 4, 6, 8, 10, 12$ quarters, which again confirm the theoretical predictions. Consistent wit the idea that time averaging should improve the fit of the model, the R^2 more than doubles when using the three year average instead of the one quarter difference. The estimated coefficients for all variables except for productivity stay highly significant even for long horizon such as 2 or 3 years. A conjecture for why productivity is not always significant might be that it is also driven by demand shocks in the data whereas it is driven by technology shocks in the model only. As said above answers to such type of question, which are not the focus of this paper, can only be expected from a richer structural model.

The theoretical model assumes that world financial and goods markets are perfectly integrated, at least for the US and its major trading partners. This suggest that the model mechanism is more likely to be operative in the second half of the data. Evidence on US capital flows, US current account and international interest rate differential surveyed in Bordo et al. (1998) suggests that financial integration started accelerating in 1983 and Gourinchas and Rey (2014) document that the United States became a capital importer in 1982. I therefore redo the benchmark analysis starting in 83:Q1 to maximize the size of the sample. Table 3 reports the result.

The results for the sample starting in 83:Q1 also confirm the theoretical predictions. All

Table 3: Determinants of the US Exchange Rate: Sample 83:Q1 - 04:Q1

Assets A_{US}	Net foreign debt NFA_{US}	Federal Funds Rate R_{US}	Productivity Y_{US}
0.4246***	-0.2064***	-0.0048***	-0.5475*
(0.0009)	(0.0005)	(0.0000)	(0.0867)

Observations: 85, R^2: 0.1858
Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

coefficients are of the expected sign and even larger in magnitude than for the full sample in Table 1 although not surprisingly less significant, since the sample size was cut by a third. It is conceivable, based on previous findings (Meese and Rogoff (1988) and Engel et al. (2007)), that taking into account the time-varying content of the fundamentals would improve the model fit.

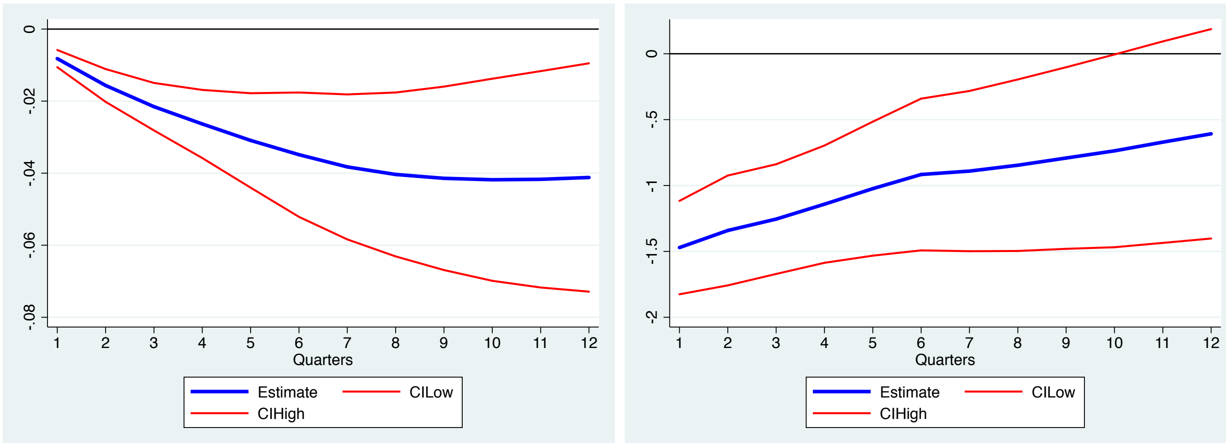
The conclusion in this Section is in line with the findings in Gourinchas and Rey (2007b) and Della Corte et al. (2010) on the importance of external imbalances for the dynamics of exchange rates. One difference is in the variables contributing to movements in the exchange rate. Gourinchas and Rey (2007b) and Della Corte et al. (2010) focus on exports, imports and all assets and liabilities - equity, FDI, bonds, ... - whereas here government bonds is the only asset determining the exchange rate. The focus here, in contrast, is on fiscal and monetary policy as well as productivity as the theory in this paper predicts those to be key determinants of the exchange rate.[21]

5.2 Evidence on the Determinants of Net Foreign Assets

A key mechanism underlying the theoretical results is that current policy changes in fiscal or monetary policy will affect the future net foreign asset position and thus the future and also the current exchange rate. In this Section I test empirically whether the determinants of future net foreign assets identified by the model - changes in nominal interest rate and debt issuance - can be detected in the data.

[21] The much richer empirical analysis in Gourinchas and Rey (2007b) also tests and confirms the asset pricing implication that today's external imbalances predict future nominal exchange rates, breaking "The curse of the random walk [...] for the dollar exchange rate". Although the focus here is not on forecasting, it is remarkable that past changes in net foreign assets (here bonds) are a statistically significant determinant of the current exchange rate with a quite high R^2, as reported in Table 5.

Figure 7: Determinants of the US net foreign asset position



(a) Monetary Policy, $R_{US} - R_{ROW}$ (b) Government Debt, B_{US}

More debt issued by the home country can be expected to be partly bought by the ROW, at last if financial markets are integrated enough, which I consider to be the case after 1983, as explained above. In addition to this mechanical determinant of NFAs, there is also a less mechanical effect on NFAs through monetary policy. An increase in the home monetary policy rate relative to the ROW leads to a deterioration of the net foreign asset position of the home country, so that current monetary policy forecasts future changes in NFAs.
To test these predictions I implement regressions

$$\log(NFA_{t+k}) - \log(NFA_t) = \delta_k + \delta_k^B(\log(B_{t+k}) - \log(B_t) + \delta_k^R(\log(R_{US,t-1}) - \log(R_{ROW,t-1})) + \mu_{k,t},\tag{94}$$

where k is the forecasting horizon. Whereas I assume a contemporaneous relationship between home bonds B and NFAs as those can be transacted quite quickly, the interest rate is lagged by one period to test whether it forecasts future movements in NFAs, which are not explained by changes in home assets. The theory predicts that both coefficients are negative, $\delta^B < 0$ and $\delta^R < 0$.
Figure 7 plots the estimated coefficients δ_k^B and δ_k^R and the 95% confidence intervals for $k = 1, 2, \ldots, 12$ quarters and Table 4 shows the estimates for $k = 1, 4, 8, 12$ quarters.
The estimates δ^B and δ^R are of the predicted negative sign across all horizons and almost all of them are significant although the confidence intervals widen - as expected - with a longer horizon k. It is again quite reassuring that despite the simplicity of the model its predicted determinants of the net foreign asset position can be detected in the data.

Table 4: NFA Assets: Debt and Monetary Policy

VARIABLES	k=1	k=4	k=8	k=12
Debt B	-1.4711***	-1.1416***	-0.8460***	-0.6076
	(0.0000)	(0.0000)	(0.0093)	(0.1263)
$R_{US} - R_{ROW}$	-0.0082***	-0.0263***	-0.0404***	-0.0412***
	(0.0000)	(0.0000)	(0.0004)	(0.0093)
Observations	84	81	77	73
R^2	0.3412	0.3866	0.4011	0.2878

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Exchange Rate and Past NFA

	$\log(\epsilon_{t+4}) - \log(\epsilon_t)$	
$NFA_t - NFA_{t-4}$	-0.2757**	-0.3372**
	(0.0377)	(0.0473)
R^2	0.1805	0.2227
$NFA_{t-4} - NFA_{t-8}$	-0.1143*	-0.2098***
	(0.0958)	(0.0040)
R^2	0.0466	0.0866
Observations	121	85
Time Period	1973:Q1-	1983:Q1-

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note - Reports results for (samples starting in 1973:Q1 and 1983:Q1):

$$\begin{aligned}
\log(\epsilon_{t+4}) - \log(\epsilon_t) &= \kappa_0 + \kappa_1(\log(NFA_t) - \log(NFA_{t-4})) + \eta_t, \\
\log(\epsilon_{t+4}) - \log(\epsilon_t) &= \kappa_0 + \kappa_1(\log(NFA_{t-4}) - \log(NFA_{t-8})) + \eta_t.
\end{aligned}$$

6 Implications and Concluding Remarks

Several lessons emerge from the previous analysis. The most important one being that the exchange rate like any other price is an endogenous object. Of course for this insight to be operative and applicable a theory is needed where this endogenous price is well defined in equilibrium. The exchange rate being an equilibrium object also means that it changes only if at least one of its potentially many determinants changes, implying that measuring exogenous shocks to the exchange rate in the data is likely to measure a combination of changes of its determinants. This paper provides a theory where the nominal exchange rate is determinate and and in addition characterizes its determinants.

In this Section I discuss the implications of these general lessons for several questions: How does a sudden asset outflow affect the exchange rate? How does an increase in savings demand in the rest of the world affect asset flows and the exchange rate? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate, for example engineer a depreciation? Finally, I argue that exchange rate determinacy transforms the open macroeconomics policy trilemma into a tetralemma: A country with a fixed exchange rate and free capital mobility loses both monetary and fiscal policy independence. In a monetary union, this tetralemma requires fiscal policy coordination.

Exchange rates and Asset outflows/inflows

How does a sudden asset outflow affect the exchange rate? The model can speak to this question although a final answer presumably requires to distinguish between bonds and capital what this paper does not. Although asset holdings are also endogenous, it is still instructive to assume that the rest of the world (ROW) "looses confidence" and pulls out assets, that is I consider first a thought experiment where NFAs move exogenously. What this means is that the ROW sells assets to the home country such that the assets held by the ROW decrease and assets held by the home country increase. This stimulates home demand relative to the ROW since $mpc_H > 1/\beta - 1$ and leads to a depreciation. Vice versa, an asset inflow to the home country, that is the ROW buys home assets, leads to an appreciation. An asset market based intuition can be grasped from the Metzler diagram, introduced in Section 2. Indeed, the same experiment of an exogenous change in NFA_H was considered in that Section using Figure 5 and I explained why an increase in NFA_H leads to a depreciation. Note that this results holds only if the total number of assets increases as well.

These implications are consistent with the basic Mundell Fleming model as well as more modern extensions of it. Both in this paper and in Mundell Fleming, it is important to remember the absence of capital when assessing the empirical validity of model predictions. There is however a key difference between my model and Mundell Fleming, a difference which motivates this paper. In this paper the exchange rate is determined as clearing the world asset and goods market whereas in the textbook Mundell Fleming model one has to fix expected future exchange rates to some arbitrary value.

An endogenous driver of international asset flows is changes in the demand for liquidity, an increase in θ in the model. As Result 7 shows, an increase in the demand for liquidity and therefore higher precautionary savings for example (and realistically) in developing countries leads to a depreciation of their currencies. Since this increase in savings was disproportionately absorbed by the United States relative to other industrial countries - maybe due to the depth of US financial markets or the US dollar being the leading reserve currency - the model predicts that these capital flows lead to an appreciation of the US dollar.

I now turn to discuss how policy can respond to changes in financial flows to or from abroad before I consider a more general exchange rate management.

Divorcing from global financial flows As explained above, an inflow of assets into the economy leads to an appreciation whereas an outflow leads to a depreciation. A policy maker who is concerned about appreciations and would like to avoid them, has to deal with the inflow of assets which caused the appreciation. The model framework in this paper suggest which policy measures are effective in neutralizing the asset inflow and thus the associated appreciation. The sterilized intervention is quite simple. In response to the ROW buying X home assets the home country has to buy the same amount of foreign assets, X/ϵ, where ϵ is the exchange rate before the capital inflow which the home country wants to maintain. As a result of this intervention, the net foreign asset position, the amount of assets held by home and by the ROW are unchanged and thus the exchange rate does not change either. The home and ROW portfolio are affected though. The home country now holds the same amount of bonds but more foreign bonds and the ROW holds the same amount of bonds but more home bonds.

This asset market intervention however does not address the underlying changes in fundamentals which have triggered this asset inflow in the first place. This policy just undoes changes in net foreign assets and is thus able to insulate the economy from global financial flows. In a richer model with various rigidities it is conceivable that these sterilizing policy measures while able to divorce an economies net foreign asset position from global asset markets, might have positive or negative effects on output.

A simpler strategy for the home country, if the objective is to only stabilize the exchange rate, is to issue more government debt to match the increase in demand for this asset. Whereas issuing the right amount of debt can fix the exchange rate, the net foreign asset position changes. This reasoning suggest that a larger savings demand by the ROW for US bonds can be accommodated without any effects on US prices or exchange rates. However, if the ROW's savings demand permanently increases at a faster rate than US output, the US debt/gdp ratio would eventually explode. Since the US fiscal capacity is bounded and the default probability on US bonds would become non-negligible at such high debt levels and render US bonds not safe anymore, this debt-issuing policy would not be feasible. The US would have to accept (permanently) falling prices and a (permanent) appreciation of its currency, a flexible exchange post Bretton Woods version of Triffin's dilemma. Or the ROW diverts its savings to other currencies - the Euros or the Yuan - provided those are considered safe.

Managing the Exchange rate The model is explicit about what policy can do, which instruments it can use and how to use them to induce changes in the exchange rate. These policy experiments are well defined since the exchange rate is an endogenous variable at all horizons (in the short-run, medium-run and long-run) without any exogenously imposed restrictions. If policy aims for a change in the exchange rate, it needs to change the amount of debt (the fiscal policy channel) or interest rates (monetary policy channel) or the amount of foreign assets (FX channel). A desired depreciation requires to either conduct an expansionary fiscal policy (increase debt issuance and lower taxes), to loosen monetary policy (lower nominal interest rates) or to buy foreign assets, which all stimulate home demand relative to foreign demand and lead to a depreciation. For an illustration of the first two channels using the Metzler diagram see again Section 2 and Figures 5 and 4.

Vice versa an appreciation requires to either conduct a contractionary fiscal policy (decrease debt issuance and increase taxes), to tighten monetary policy (increase nominal interest rates) or to sell foreign assets, which all depress home demand relative to foreign demand and lead to an appreciation. This exchange rate policy is available for any country independent of its size, including small open economies with a floating exchange rate, although it is less effective in smaller than in larger countries, reflected in the term $1 + \frac{Y_H}{Y_F}$ in Result 1. The size effect arises since a demand stimulus in a small country has a smaller impact on demand in the ROW than a stimulus in a larger country.

Tetralemma and Monetary Unions The classic policy trilemma in international economics is that at most two out of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting nominal interest rate independently (monetary policy independence); and (iii) a fixed exchange rate. The underlying logic is quite simple. Free asset flows imply that the uncovered interest rate parity holds such that a fixed exchange rate regime requires to set the domestic nominal interest rate equal to the ROW nominal interest rate. However, giving up an independent monetary policy is necessary but not sufficient to stabilize the level of the exchange rate. The reason is that the above logic is derived in a model where the level of exchange rate is indeterminate and therefore the uncovered interest rate parity condition can be used to rule out anticipated changes in the exchange rate only. But changes, for example in the savings demand at home (θ_H) or abroad (θ_F), lead to changes in the level of the exchange rate even if home monetary policy perfectly tracks foreign monetary policy. An exchange rate peg then requires that fiscal policy has to absorb these shocks and stabilize the exchange rate. Note that the uncovered interest rate parity still implies that monetary policy is not independent.

This suggest that a country faces a tetralemma. Unrestricted capital mobility and a fixed exchange rate imply that a country loses both monetary and fiscal policy independence, or more generally loses its ability to manage aggregate domestic demand.

The implications for a monetary union, where capital can freely move and the nominal exchange rate is fixed, are quite unpleasant. Not only do union-member countries have to give up monetary policy but they also lose an independent fiscal policy. Not implementing the fiscal policy necessary to stabilize the nominal exchange rate will in a monetary union lead to a change in the real exchange rate. For example a capital inflow, say into Spain, would require a contractionary fiscal policy in Spain or an expansionary fiscal policy in the rest of the Euro area. If instead Spanish fiscal policy is unchanged or even becomes expansionary, this inevitably leads to a real appreciation with the likely effects on exports, imports and output. This suggests a new perspective on the fiscal dimension of a monetary union: Fiscal policy coordination to respond to the capital flows which cause exchange rate movements or more precisely would have caused changes of the nominal exchange rate if it was flexible.

Concluding Remarks

In this paper I have shown that the nominal exchange rate is determinate in a large class of incomplete markets models with aggregate risk when government debt is nominal. While this theory is on purpose simple, it can nevertheless shed light on several questions as I have already discussed before. Questions which previous research has, due to the lack of a an equilibrium theory of exchange rates, struggled to answer in a fully satisfactory way.

But certainly the simple model by focusing on what determines the exchange rate lacks several features which are necessary to address many other questions in open economy macroeconomics. One important missing feature is nominal rigidities not only because it is important for short-run dynamics but because it is the reason why the indeterminacy of nominal exchange rate matters for the real exchange rate. Different nominal exchange rates correspond to different real exchange rates and thus to different levels of exports and imports as well as different levels of output and employment at home and abroad. Embedding price rigidities into this paper's framework also allows to consider spillovers of foreign fiscal and monetary policy as well as of foreign shocks and a foreign liquidity trap on the home macroeconomy. A key aspect when studying such policy or shock spillovers is the potential absorbing role of exchange rate adjustments, which requires a theory how the exchange rate is determined. This paper provides such a theory and enables to study these questions in a coherent framework.

Accounting for all aspects of the evolution of the US external positions requires to also add physical capital to the model. Capital, although irrelevant for determinacy, allows to obtain a full picture of a country's capital account in particular for the US, the "Venture Capitalist of the World", which can be roughly described as issuing debt liabilities and investing in physical capital (equity and direct investment) abroad (Gourinchas and Rey (2007b,a)). The researcher can use such a model to address questions how physical capital flows affect exchange rates and thus the current account. An extension with rigidities and capital is also necessary to quantitatively and simultaneously account for the observed fall in US interest rates, the flow of capital and assets in and out of the US, the large current account US deficit and the evolution of exchange rates within a coherent equilibrium model.

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APPENDIX

A.I Proofs and Derivations

Derivation of Equation (70)

To derive

$$\widehat{CA_F}(\epsilon) - CA_F^*(\epsilon) = (q + \alpha_F)A_F \frac{\epsilon_{ss} - \epsilon}{\epsilon}, \tag{A1}$$

note that $\widehat{CA_F}(\epsilon)$ is defined as solving

$$\epsilon \frac{Y_H}{Y_F} = \frac{D_H(A_H)CA_H}{D_F(A_F) + \widehat{CA_F}(\epsilon)}, \tag{A2}$$

so that

$$\widehat{CA_F}(\epsilon) = \frac{1}{\epsilon}\frac{Y_F}{Y_H}((q + \alpha_H)A_H + CA_H) - (q + \alpha_F)A_F \tag{A3}$$

Furthermore, steady-state $CA_F^*(\epsilon_{ss}) = -CA_H/\epsilon_{ss}$ solves

$$CA_F^*(\epsilon_{ss}) = \frac{1}{\epsilon_{ss}}\frac{Y_F}{Y_H}((q + \alpha_H)A_H + CA_H) - (q + \alpha_F)A_F \tag{A4}$$

so that

$$CA_F^*(\epsilon) = \frac{\epsilon_{ss}}{\epsilon}CA_F^*(\epsilon_{ss}) = \frac{1}{\epsilon}\frac{Y_F}{Y_H}((q + \alpha_H)A_H + CA_H) - \frac{\epsilon_{ss}}{\epsilon}(q + \alpha_F)A_F. \tag{A5}$$

Combining equations (A3) and (A5):

$$\widehat{CA_F}(\epsilon) - CA_F^*(\epsilon) = \frac{\epsilon_{ss} - \epsilon}{\epsilon}(q + \alpha_F)A_F. \tag{A6}$$

Derivation of Result 3

$$\frac{(C_{\cdot,t})^{1-\sigma}}{1-\sigma} + \theta.\frac{(c_{\cdot,t})^{1-\sigma}}{1-\sigma}$$

To show that in a steady state where $R_H \neq R_F$ and $1/\sigma \neq 1$, the nominal exchange rate equals

$$\epsilon_t = \frac{1+\pi_H}{1+\pi_F} \frac{(q+\alpha_H)\frac{A_H}{Y_H} - (R_H - 1)\frac{NFA^H}{Y_H}}{(q+\alpha_F)\frac{A_F}{Y_F}}, \tag{A7}$$

I proceed as in the main text where I consider $R_H = R_F$ and $\sigma = 1$.

The FOC for assets,

$$\frac{(C_{H,t})^{-\sigma}}{P_{H,t}} = q\theta_H \frac{(\frac{A_{H,t}}{P_{H,t}})^{-\sigma}}{P_{H,t}} + (1-q)R_H\beta \frac{(C_{H,t+1})^{-\sigma}}{P_{H,t+1}}, \tag{A8}$$

and equivalently in a steady state

$$(C_H)^{-\sigma} = q\theta_H(\frac{A_{H,t}}{P_{H,t}})^{-\sigma} + (1-q)\beta\frac{R_H}{1+\pi_H}\beta(C_H)^{-\sigma} \tag{A9}$$

allows to express nominal consumption as a function of nominal assets:

$$P_H C_H (1+\pi_H)^t = A_H\big(\frac{1-(1-q)\frac{R_H}{1+\pi_H}}{q\theta_H}\big)^{1/\sigma}(1+\pi_H)^t. \tag{A10}$$

for $P_{H,t} = P_H(1+\pi_H)^t$ and $A_{H,t} = A_H(1+\pi_H)^t$, so that total nominal home demand equals

$$D_{H,t} = (q+\alpha_H^{1/\sigma})A_H(1+\pi_H)^t. \tag{A11}$$

Similar derivations for the foreign country yield

$$D_{F,t} = (q+\alpha_F^{1/\sigma})A_F(1+\pi_F)^t. \tag{A12}$$

Good market clearing then requires, noting that $NFA_{H,t} = NFA_H(1+\pi_H)^t$ and $NFA_{F,t} = NFA_F(1+\pi_F)^t$ in steady state and using that $NFA_{F,t} = -NFA_{H,t}/\epsilon_t$,

$$\epsilon_t \frac{Y_H}{Y_F} = \frac{(D_{H,t} - (R_H-1)NFA_{H,t})}{D_{F,t} - (R_F-1)NFA_{F,t}} = \frac{(1+\pi_H)^t(D_H(A_H) - (R_H-1)NFA_H)}{(1+\pi_F)^t D_F(A_F) + (R_F-1)NFA_{H,t}/\epsilon_t}. \tag{A13}$$

Finally, solving for ϵ_t

$$\epsilon_t = \big(\frac{1+\pi_H}{1+\pi_F}\big)^t \frac{(q+\alpha_H^{1/\sigma})\frac{A_H}{Y_H} - \frac{NFA_H}{Y_H}((R_H-1)+(R_F-1)\frac{Y_H}{Y_F})}{(q+\alpha_F^{1/\sigma})\frac{A_F}{Y_F}}. \tag{A14}$$

Derivation of Result 4

I show that an increase in $R_{H,t}$ leads to a fall of ϵ_t in several steps. The economic mechanism underlying this result is that an increase in $R_{H,t}$ leads to an de-accumulation of assets by the home country and an accumulation of assets by the foreign country, which will lead to an appreciation os the steady state exchange rate. Interest parity implies that the currenf exchange rate appreciates as well. I therefore first derive how the steady state exchange rate depends on net foreign assets taking into account the induced changes in assets,

$$A_{H,ss} = B_{H,ss} + NFA_{H,ss}, \tag{A15}$$

$$A_{F,ss} = B_{F,ss} - NFA_H/\epsilon_{ss}(NFA_{H,ss}), \tag{A16}$$

and then show how the increase in $R_{H,t}$ changes asset accumulation in both countries.

Step 1: Exchange Rate and Assets

The exchange rate equals

$$\epsilon_{ss} = \frac{(q+\alpha_H)\frac{B_H+NFA_H}{Y_H} - (R-1)\frac{NFA_H}{Y_H}(1+\frac{Y_H}{Y_F})}{(q+\alpha_F)\frac{B_F-NFA_H/\epsilon_{ss}}{Y_F}}. \tag{A17}$$

Solving for ϵ_{ss},

$$\epsilon_{ss} = \frac{(q+\alpha_H)\frac{B_H+NFA_H}{Y_H} - (R-1)\frac{NFA_H}{Y_H}(1+\frac{Y_H}{Y_F})}{(q+\alpha_F)\frac{B_F}{Y_F}} + \frac{NFA_H}{B_F}. \tag{A18}$$

The NFA_H derivate of ϵ_{ss} is therefore

$$\frac{\partial \epsilon_{ss}}{\partial NFA_H} = \frac{(q+\alpha_H)Y_F}{(q+\alpha_F)Y_H B_F} - \frac{(R-1)(1+\frac{Y_F}{Y_H})}{(q+\alpha_F)B_F} + \frac{1}{B_F} \tag{A19}$$

$$= \frac{\frac{Y_F}{Y_H}(q+\alpha_H-(R-1)) + (q+\alpha_F-(R-1))}{(q+\alpha_F)B_F} \tag{A20}$$

The assumption that $mpc_H = q+\alpha_H > 1/\beta - 1$ implies that $q+\alpha_H-(R-1) > 0$ and $mpc_F = q+\alpha_F > 1/\beta - 1$ implies that $q+\alpha_F-(R-1) > 0$, so that

$$\frac{\partial \epsilon_{ss}}{\partial NFA_H} > 0. \tag{A21}$$

Step 2: Monetary Policy and Assets

I next show the effect of an increase in $R_{H,t}$ on the long-run net-foreign asset position which together with the previous result yields the effect of R_H on the steady state exchange rate.

I work backwards from the new steady state which features different asset holdings then the pre-policy change steady state. Since prices are flexible and policy changes only in period t, assets holdings, prices and exchange rates are equal to the new steady state values $NFA_{H,ss}$, $P_{H,ss}$ and ϵ_{ss} from period $t+1$ onwards. In particular

$$\epsilon_{t+1} = \epsilon_{ss}(NFA_{H,ss}), \tag{A22}$$

$$A_{H,t+1} = B_H + NFA_{H,ss}, \tag{A23}$$

$$A_{F,t+1} = B_F - NFA_{H,ss}/\epsilon_{ss}(NFA_{H,ss}). \tag{A24}$$

The objective is to understand how $NFA_{H,ss}$ depends on $R_{H,t}$. Therefore, consider the dynamic accumulation equation for net foreign assets

$$Rn_t^o = -P_{H,t}Y_{H,t} + E_{H,t} + qA_{H,t} + NFA_{H,ss}, \tag{A25}$$

where n_t^o are net foreign assets at the beginning of period t, $E_{H,t} = P_{H,t}C_{H,t}$ and $P_{H,t}c_{H,t} = A_t$. In a steady state $n^o = NFA_{H,ss}$, that is the net foreign asset position is constant. Due to the higher nominal interest rate period t households incentives to consume and save change and therefore $E_{H,t}$ and $P_{H,t}$ differ from their steady state counterparts $E_{H,ss}$ and $P_{H,ss}$. Note that I assume that each country only holds its own assets before the change in monetary policy implying that $n_t^o = 0$ and is independent of the exchange rate, i.e. there are no valuation gains. I allow for such valuation gains in Result 6. Households expenditures $E_{H,t}$ satisfy the FOC

$$\frac{1}{E_{H,t}} = q\frac{\theta_H}{B_{H,t} + NFA_{H,ss}} + (1-q)\beta R_{H,t}\frac{1}{E_{H,ss}}, \tag{A26}$$

which implicitly defines it as a function $E_{H,t}(R_{H,t}, NFA_{H,ss})$ since $E_{H,ss}$ can be written as a function of $NFA_{H,ss}$, $E_{H,ss}(NFA_{H,ss})$.

Finally I have to show how the price level depends on the interest rate and the net foreign asset position. The home price level $P_{H,t}$ satisfies

$$P_{H,t}(Y_{H,t} + Y_{F,t}) = E_{H,t} + \epsilon_t E_{F,t} + qA_{H,t} + q\epsilon_t A_{F,t} \tag{A27}$$

$$= E_{H,t} + \epsilon_t E_{F,t} + qB_{H,t} + q\epsilon_t B_{F,t}, \tag{A28}$$

where foreign expenditures $E_{F,t} = P_{F,t}C_{F,t}$ satisfies, using $\epsilon_t = R_{F,t}\epsilon_{t+1}/R_{H,t}$ the FOC

$$\frac{1}{E_{F,t}} = q\frac{\theta_H}{B_{F,t} - R_{H,t}NFA_{H,ss}/(\epsilon_{ss}(NFA_{H,ss})R_{F,t})} + (1-q)\beta R_{F,t}\frac{1}{E_{F,ss}(NFA_{H,ss})}, \quad \text{(A29)}$$

The new steady state net foreign asset position $NFA_{H,ss}$ therefore solves

$$\begin{aligned} 0 &= -P_{H,t}Y_{H,t} + E_{H,t}(R_{H,t}, NFA_{H,ss}) + q(B_{H,t} + NFA_{H,ss}) + NFA_{H,ss} \\ &= \frac{Y_{F,t}E_{H,t}(R_{H,t}, NFA_{H,ss}) - \epsilon_{ss}(NFA_{H,ss})\frac{R_{F,t}}{R_{H,t}}E_{F,t}(R_{H,t}, NFA_{H,ss})Y_{H,t}}{Y_{H,t} + Y_{F,t}} \\ &- \frac{qY_{H,t}(B_{H,t} + \epsilon_{ss}(NFA_{H,ss})\frac{R_{F,t}}{R_{H,t}}B_{F,t})}{Y_{H,t} + Y_{F,t}} + q(B_{H,t} + NFA_{H,ss}) + NFA_{H,ss} \\ &=: \Gamma(NFA_{H,ss}, R_{H,t}) \end{aligned} \quad \text{(A30)}$$

which implicitly defines $NFA_{H,ss}$ as a function of $R_{H,t}$. To obtain the sign of $\frac{\partial NFA_{H,ss}}{\partial R_{H,t}}$ I now compute the $NFA_{H,ss}$ and $R_{H,t}$ derivates of Γ, evaluated at the pre-policy change steady state values n^o and R,

$$\frac{\partial \Gamma}{\partial R_{H,t}}(n^o, R) = \frac{Y_{F,t}\frac{\partial E_{H,t}}{\partial R_{H,t}} + Y_{H,t}\epsilon_{ss}(n^o)(\frac{E_{F,t}}{R} - \frac{\partial E_{F,t}}{\partial R_{H,t}}) + \frac{\epsilon_{ss}(n^o)}{R}qY_{H,t}B_{F,t}}{Y_{H,t} + Y_{F,t}} \quad \text{(A31)}$$

Using the FOC (A26) and $E_{H,ss}(n^o) = \alpha_H B_{H,ss}$,

$$\frac{\partial E_{H,t}}{\partial R_{H,t}}(n^o, R) = -(1-q)\beta E_{H,ss}(n^o) = -(1-q)\beta\alpha_H B_{H,ss}, \quad \text{(A32)}$$

and using the FOC (A29),

$$\frac{\partial E_{F,t}}{\partial R_{H,t}}(n^o, R) = -\frac{E_{F,ss}(n^o)^2}{(B_{F,ss} - n^o/\epsilon_{ss}(n^o))^2}\frac{n^o q\theta_F}{R\epsilon_{ss}(n^o)} = -\alpha_F^2\frac{n^o q\theta_F}{R\epsilon_{ss}(n^o)} = 0. \quad \text{(A33)}$$

Plugging this into (A31) and using,

$$\epsilon_{ss}(n^o) = \frac{(q + \alpha_H)\frac{B_{H,ss}}{Y_H}}{(q + \alpha_F)\frac{B_{F,ss}}{Y_F}}, \quad \text{(A34)}$$

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial R_{H,t}}(n^o, R) &= \frac{-Y_{F,t}(1-q)\beta\alpha_H B_{H,ss} + \frac{\alpha_F}{R(q+\alpha_F)}(q+\alpha_H)B_{H,ss}Y_{F,t} + q\frac{q+\alpha_H}{R(q+\alpha_F)}B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&= \Big(\frac{\alpha_F(q+\alpha_H)/R + q(q+\alpha_H)/R - (1-q)\beta\alpha_H(q+\alpha_F)}{q+\alpha_F}\Big)\frac{B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&= \Big(\frac{(1/R-\beta)(q\alpha_H+\alpha_H\alpha_F) + q\beta\alpha_H(q+\alpha_F) + q\alpha_F/R + q^2/R}{R(q+\alpha_F)}\Big)\frac{B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&> 0. \qquad \text{(A35)}
\end{aligned}
$$

To calculate $\frac{\partial \Gamma}{\partial NFA_{H,ss}}(n^o, R)$ note that in a steady state with net foreign asset position $NFA_{H,ss}$ and interest rate R, the flow equation for net foreign assets is

$$
NFA_{H,ss}R = \Gamma(NFA_{H,ss}, R), \qquad \text{(A36)}
$$

so that

$$
\frac{\partial \Gamma}{\partial NFA_{H,ss}}(n^o, R) = R, \qquad \text{(A37)}
$$

implying that

$$
\frac{\partial NFA_{H,ss}}{\partial R_{H,ss}}(n^o, R) = -\frac{\frac{\partial \Gamma}{\partial R_{H,ss}}(n^o, R}{\frac{\partial \Gamma}{\partial NFA_{H,ss}}(n^o, R} < 0. \qquad \text{(A38)}
$$

I therefore have shown that

$$
\begin{aligned}
& \frac{\partial \epsilon_t}{\partial R_{H,t}}(n^o, R) \\
= \; & \frac{\partial\big(\epsilon(NFA_{H,ss})\frac{R_{F,t}}{R_{H,t}}\big)}{\partial R_{H,t}} = \frac{\partial \epsilon(NFA_{H,ss})}{\partial R_{H,t}} = \frac{\partial \epsilon_{ss}(NFA_{H,ss})}{\partial NFA_{H,ss}}\frac{\partial NFA_{H,ss}(R_{H,t})}{\partial R_{H,t}} \\
< \; & 0, \qquad \text{(A39)}
\end{aligned}
$$

that is the exchange rate appreciates and the net foreign asset position deteriorates in response to a tightening of monetary policy.

Derivation of Result 5

The proof follows the same steps as the proof of Result 4. A difference is that households expenditures $E_{H,t}$ now satisfy the FOC

$$
(\frac{E_{H,t}}{P_{H,t}})^{-\sigma}/P_{H,t} = q\theta_H(\frac{B_{H,t}+NFA_{H,ss}}{P_{H,t}})^{-\sigma} + (1-q)\beta R_{H,t}(\frac{E_{H,t+1}}{P_{H,t+1}})^{-\sigma}/P_{H,t+1}, \qquad \text{(A40)}
$$

which implies

$$\frac{\partial E_{H,t}}{\partial R_{H,t}}(n^o, R) = -(1-q)\beta E_{H,ss}(n^o) = -\frac{(1-q)\beta\alpha_H^{1/\sigma}}{\sigma}B_{H,ss}, \tag{A41}$$

$$\begin{aligned}
\frac{\partial \Gamma}{\partial R_{H,t}}(n^o, R) &= \frac{Y_{F,t}\frac{\partial E_{H,t}}{\partial R_{H,t}} + Y_{H,t}\epsilon_{ss}(n^o)\frac{E_{F,t}}{R} + \frac{\epsilon_{ss}(n^o)}{R}qY_{H,t}B_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&= \frac{-Y_{F,t}\frac{(1-q)\beta\alpha_H^{1/\sigma}}{\sigma}B_{H,ss} + \frac{\alpha_F^{1/\sigma}}{R(q+\alpha_F^{1/\sigma})}(q+\alpha_H^{1/\sigma})B_{H,ss}Y_{F,t} + q\frac{q+\alpha_H^{1/\sigma}}{R(q+\alpha_F^{1/\sigma})}B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&= \Big(\frac{\alpha_F^{1/\sigma}(q+\alpha_H^{1/\sigma})/R + q(q+\alpha_H^{1/\sigma})/R - \frac{(1-q)\beta\alpha_H^{1/\sigma}}{\sigma}(q+\alpha_F^{1/\sigma})}{q+\alpha_F^{1/\sigma}}\Big)\frac{B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&= \Big(\frac{\big(\frac{1}{R}-\frac{(1-q)\beta}{\sigma}\big)\big(q\alpha_H^{1/\sigma}+\alpha_H^{1/\sigma}\alpha_F^{1/\sigma}\big) + q\alpha_F^{1/\sigma}/R + q^2/R}{R(q+\alpha_F^{1/\sigma})}\Big)\frac{B_{H,ss}Y_{F,t}}{Y_{H,t}+Y_{F,t}} \\
&> 0.
\end{aligned} \tag{A42}$$

The rest of the proof is identical, so that $\frac{\partial \Gamma}{\partial NFA_{H,ss}}(n^o, R) = R$ and finally

$$\frac{\partial \epsilon_t}{\partial R_{H,t}}(n^o, R) < 0. \tag{A43}$$

Derivation of Result 6

The proof follows the same logic as the proof of Result 4.

The function $\Gamma(NFA_{H,ss}, R_{H,t})$ is the same as defined in (A30) so that

$$0 = \tilde{\Gamma}(NFA_{H,ss}, R_{H,t}) := \Gamma(NFA_{H,ss}, R_{H,t}) - Rn^0 = \Gamma(NFA_{H,ss}, R_{H,t}) - R(A_H^F\epsilon_t - A_F^H) \tag{A44}$$

implicitly defines $NFA_{H,ss}$ as a function of $R_{H,t}$. To derive the sign of $\frac{\partial NFA_{H,ss}}{\partial R_{H,t}}(n^o, R)$ I therefore compute, as above, $\frac{\partial \tilde{\Gamma}(NFA_{H,ss}, R_{H,t})}{\partial R_{H,t}}$ and $\frac{\partial \tilde{\Gamma}(NFA_{H,ss}, R_{H,t})}{\partial NFA_{H,ss}}$. The derivatives of expenditures with non-zero net foreign asset position

$$\frac{\partial E_{H,t}}{\partial R_{H,t}}(n^o, R) = -(1-q)\beta E_{H,ss}(n^o) = -(1-q)\beta\alpha_H(B_{H,ss}+n^o), \tag{A45}$$

and

$$\frac{\partial E_{F,t}}{\partial R_{H,t}}(n^o, R) = -\frac{E_{F,ss}(n^o)^2}{(B_{F,ss} - n^o/\epsilon_{ss}(n^o))^2}\frac{n^o q\theta_F}{R\epsilon_{ss}(n^o)} = -\alpha_F^2\frac{n^o q\theta_F}{R\epsilon_{ss}(n^o)}. \tag{A46}$$

I therefore obtain

$$\begin{aligned}
&\frac{\partial\tilde{\Gamma}(NFA_{H,ss}, R_{H,t})}{\partial R_{H,t}} \\
= \quad & \frac{-Y_{F,t}(1-q)\beta E_{H,ss}(n^o) + Y_{H,t}\epsilon_{ss}(n^o)\frac{E_{F,t}}{R} + \frac{\epsilon_{ss}(n^o)}{R}qY_{H,t}B_{F,t}}{Y_{H,t}+Y_{F,t}} + \frac{Y_H}{Y_H+Y_F}\frac{n^o\alpha_F^2 q\theta_F}{R} + A_H^F\epsilon_{ss}(n^o).
\end{aligned} \tag{A47}$$

Using that $\tilde{\Gamma}(n^0, R) = 0$, which is equivalent to

$$\frac{Y_{F,t}E_{H,ss} - Y_{H,t}\epsilon_{ss}(n^o)\frac{E_{F,t}}{R} + \frac{\epsilon_{ss}(n^o)}{R}qY_{H,t}B_{F,t}}{Y_{H,t}+Y_{F,t}} = Rn^o + q\frac{Y_{H,t}B_{H,ss}}{Y_{H,t}+Y_{F,t}} - qB_{H,ss} - (1+q)n^o,$$

yields (since $\beta < 1/R$)

$$\begin{aligned}
&\frac{\partial\tilde{\Gamma}(NFA_{H,ss}, R_{H,t})}{\partial R_{H,t}} \qquad \text{(A48)}\\
\geq \quad & \frac{qY_{F,t}\beta\alpha_H(B_{H,ss}+n^o)}{Y_{H,t}+Y_{F,t}} - \frac{1}{R}\big(Rn^o + q\frac{Y_{H,t}B_{H,ss}}{Y_{H,t}+Y_{F,t}} - qB_{H,ss} - (1+q)n^o\big) \\
+ \quad & \frac{Y_H}{Y_H+Y_F}\frac{n^o\alpha_F^2 q\theta_F}{R} + A_H^F\epsilon_{ss}(n^o) \\
\geq \quad & \frac{qY_{F,t}\beta\alpha_H(B_{H,ss} - A_F^H)}{Y_{H,t}+Y_{F,t}} - \frac{1}{R}\big(q\frac{Y_{H,t}B_{H,ss}}{Y_{H,t}+Y_{F,t}} - qB_{H,ss} + (1+q)A_F^H\big) \\
- \quad & \frac{Y_H}{Y_H+Y_F}A_F^H\frac{\alpha_F q}{R} + A_F^H \\
\geq \quad & \Big(qY_{F,t}\beta\alpha_H + qY_F/R\Big)\frac{B_{H,ss}}{Y_H+Y_F} - \Big(qY_{F,t}\beta\alpha_H + \frac{Y_H\alpha_F q}{R} + q(Y_H+Y_F)/R\Big)\frac{A_F^H}{Y_H+Y_F}
\end{aligned}$$

Thus

$$\Big(qY_{F,t}\beta\alpha_H + qY_F/R\Big)\frac{B_{H,ss}}{Y_H+Y_F} \geq \Big(qY_{F,t}\beta\alpha_H + \frac{Y_H\alpha_F q}{R} + q(Y_H+Y_F)/R\Big)\frac{A_F^H}{Y_H+Y_F} \tag{A49}$$

implies that $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},R_{H,t})}{\partial R_{H,t}} \geq 0$. Using that $1/R < \beta < 1$ shows that this follows from the sufficient condition

$$\frac{A_F^H}{B_{H,ss}} \leq \frac{Y_F(1+\alpha_H)}{Y_F(1+\alpha_H)+Y_H(1+\alpha_F)}. \tag{A50}$$

Next consider $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},R_{H,t})}{\partial NFA_{H,ss}}$.

$$\begin{aligned}
\frac{\partial\tilde{\Gamma}(NFA_{H,ss},R_{H,t})}{\partial NFA_{H,ss}}(n^o,R) &= \frac{\partial\Gamma(NFA_{H,ss},R_{H,t})}{\partial NFA_{H,ss}}(n^o,R) - RA_H^F\frac{\partial\epsilon_t}{\partial NFA_{H,ss}}(n^o,R) \\
&= R - RA_H^F\frac{\partial\epsilon_t}{\partial NFA_{H,ss}}(n^o,R),
\end{aligned}$$

which is positive if $A_H^F\frac{\partial\epsilon_t}{\partial NFA_{H,ss}}(n^o,R)<1$. This follows from the sufficient condition

$$\frac{A_H^F}{B_{F,ss}} \leq \frac{Y_H(q+\alpha_F)}{Y_F(q+\alpha_H)+Y_H(q+\alpha_F)}, \tag{A51}$$

which implies

$$\frac{Y_H B_{F,ss}(q+\alpha_F)}{A_H^F} \geq Y_F(q+\alpha_H)+Y_H(q+\alpha_F) > Y_F(q+\alpha_H-(R-1))+Y_H(q+\alpha_F-(R-1)),$$

so that I obtain

$$\begin{aligned}
&\frac{B_{F,ss}(q+\alpha_F)}{A_H^F} > \frac{Y_F}{Y_H}(q+\alpha_H-(R-1))+(q+\alpha_F-(R-1)) \\
\Leftrightarrow\ & A_H^F\frac{\partial\epsilon_t}{\partial NFA_{H,ss}}(n^o,R)<1.
\end{aligned}$$

To summarize, I have shown that $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},R_{H,t})}{\partial R_{H,t}}>0$ and $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},R_{H,t})}{\partial NFA_{H,ss}}>0$ which again implies that

$$\frac{\partial NFA_{H,ss}}{\partial R_{H,t}}<0, \quad \frac{\partial\epsilon_{ss}(NFA_{H,ss})}{\partial R_{H,t}}<0 \tag{A52}$$

and

$$\frac{\partial\epsilon_t}{\partial R_{H,t}}<0. \tag{A53}$$

Derivation of Result 7

The proof follows the line of argument as the proof of Result 6. Define therefore

$$\begin{aligned}
&\tilde{\Gamma}(NFA_{H,ss}, \theta_{H,t}) \\
:= \quad & \frac{Y_{F,t}E_{H,t}(\theta_{H,t}, NFA_{H,ss}) - \epsilon_{ss}(NFA_{H,ss})\frac{R_{F,t}}{R_{H,t}}E_{F,t}(R_{H,t}, NFA_{H,ss})Y_{H,t}}{Y_{H,t}+Y_{F,t}} \\
- \quad & \frac{qY_{H,t}(B_{H,t} + \epsilon_{ss}(NFA_{H,ss})\frac{R_{F,t}}{R_{H,t}}B_{F,t})}{Y_{H,t}+Y_{F,t}} + q(B_{H,t} + NFA_{H,ss}) + NFA_{H,ss} - R(A_H^F\epsilon_t - A_F^H),
\end{aligned} \tag{A54}$$

such that

$$\tilde{\Gamma}(NFA_{H,ss}, \theta_{H,t}) = 0$$

implicitly defines $NFA_{H,ss}$ as a function of $Y_{H,t}$. The derivative $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},\theta_{H,t})}{\partial NFA_{H,ss}} > 0$ as above. Noting that $\frac{\partial E_{H,t}}{\partial\theta_{H,t}}(n^o, \theta_H) < 0$, $\frac{\partial\tilde{\Gamma}(NFA_{H,ss},\theta_{H,t})}{\partial\theta_{H,t}} < 0$, so that

$$\frac{\partial NFA_{H,ss}}{\partial\theta_{H,t}} > 0, \tag{A55}$$

and thus

$$\frac{\partial\epsilon_t}{\partial\theta_{H,t}} > 0. \tag{A56}$$

A.II Data Sources

This appendix describes the data used in this paper. The exchange rate and US debt assets and liabilities are from Gourinchas and Rey (2007b) (described in detail in Gourinchas and Rey (2007a)).[22] Non-US Interest rates are from IMF International Financial Statistics,[23] US government debt, Real GDP, Employment and the Federal Funds Rate are from Federal Reserve Economic Data.[24]

Exchange Rate ϵ:
Nominal trade weighted effective exchange rate: Major currencies (Euro area, Canada,

[22] Downloadable at `http://socrates.berkeley.edu/~pog/academic/IFA_data.xls`.
[23] Downloadable at `https://fred.stlouisfed.org`
[24] Downloadable at `https://fred.stlouisfed.org`

Japan, United Kingdom, Switzerland, Australia, Sweden).

United States debt liabilities and assets:
End of period gross positions of debt assets (PDA) and liabilities (PDL), so that NFA = PDA - PDL.

US Federal Funds Rate R_{US}:
Effective Federal Funds Rate, Percent, Quarterly (FEDDUNDS).

ROW Interest Rate R_{ROW}:
Money Market Rate. Weighted average of all countries with full data availability: Australia, Canada, Germany, Italy, Japan, Sweden, Switzerland and United Kingdom.

Government debt:
Federal US Debt: Total Public Debt, Quarterly (GFDEBTN).

Real GDP:
Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly (GDPC1).

Employment:
CPS Civilian Employment Level, Quarterly, SA (CE16OV).

Productivity:
Real GDP divided by employment.

A.III The Generalized Model

In this section I first describe the generalized model and then show that a unique exchange rate exists. The general model differs from the model in the main text as it has:

- More General distribution for expenditure shocks.
- Credit available to pay for sudden expenditures needs.
- The fraction of households with binding liquidity constraints is endogenous (q in the main text).
- General utility functions.
- Government spending.

I now lay out these differences in detail.

More General distribution for expenditure shocks

The timing of events is as in the main text. Each member of the household has a need for spending in t_2 which is governed by the i.i.d. shock

$$\theta \in [\underline{\theta}, \infty] \sim \Phi, \tag{A57}$$

where $\underline{\theta} \geq 0$ and corresponding pdf ϕ.

General utility functions

A household who experiences a shock θ and consumes C_t^h in period t_1 and c_t^h at t_2 derives utility

$$u(C_t^h) + \theta v(c_t^h) \tag{A58}$$

in period t. The strictly concave utility function u and v satisfy the usual Inada Conditions.

Credit is available to pay for sudden expenditures needs.

During period t_2, each member has only access to his or her own bonds to be spend on consumption $c_t^h(\theta)$, but can obtain a credit up to some limit $\overline{A}$, such that the liquidity constraint

$$c_t(\theta) \leq A_t/P_t + \overline{A}. \tag{A59}$$

The fraction of households with binding liquidity constraints is endogenous

The optimal decision for $c_t(\theta)$ is described through threshold $\hat{\theta}_t$, which solves

$$\hat{\theta}_t v'(A_t/P_t + \overline{A}) = u'(C_t) \tag{A60}$$

such that

$$\begin{aligned} &i) \quad c_t(\theta) \text{ solves } \theta v'(c_t(\theta)) = u'(C_t) && \text{if } \theta \leq \hat{\theta}_t, \text{ i.e. } v'(A_t/P_t + \overline{A}) \leq u'(C_t), \\ &ii) \quad c_t(\theta) = A_t/P_t + \overline{A} && \text{if } \theta > \hat{\theta}_t, \text{ i.e. } v'(A_t/P_t + \overline{A}) > u'(C_t). \end{aligned}$$

The fraction of households with binding liquidity constraints is then endogenous and equal to $1 - \Phi(\hat{\theta}_t)$.

Government spending

I now allow for government spending given by the sequence of nominal government spending

$$\mathcal{G} = G_0, G_1, \ldots, G_t, \ldots, \tag{A61}$$

The government's flow budget constraint has to be satisfied at any point in time, which implicitly defines a sequence of nominal bonds

$$B_{t+1} = R_t B_t + G_t - T_t. \tag{A62}$$

To guarantee existence of an equilibrium I assume

$$\int_{\frac{u'(Y-\overline{A})}{v'(\overline{A})}}^{\infty} \theta v'(\overline{A}) d\Phi(\theta) > u'(Y - \overline{A}) \tag{A63}$$

for both countries.[25]

A.III.1 Unique Exchange Rate

I now show that there are uniquely determined steady-state price levels in the home country, $P_{H,ss}$, and in the foreign country, $P_{F,ss}$ and a unique steady state exchange rate, $\epsilon_{ss} = \frac{P_{H,ss}}{P_{F,ss}}$. The nominal steady-state interest rate R is the same in both countries. Fiscal policy variables in the steady-state are constant and denoted by G_H, G_F, B_F, B_H and T_H, T_F, respectively.

I first show properties of consumption $c()$ at t_2 and consumption C at t_1 before I proceed to the existence and uniqueness proof.

For a given level of consumption C_H at t_1 and a given threshold level $\hat{\theta}_H$, consumption in period t_2 equals

$$\begin{array}{lll} i) & c(\theta_H) = (v')^{-1}(\frac{u'(C_H)}{\theta}) & \text{if } \theta_H \leq \hat{\theta}_H, \\ ii) & c(\theta_H) = (B_H + NFA_H)/P_H + \overline{b} & \text{if } \theta_H > \hat{\theta}_H. \end{array}$$

The threshold level equals

$$\hat{\theta}_H(C_H, P_H) = \frac{u'(C_H)}{v'(\frac{B_H + NFA_H}{P_H} + \overline{b})}. \tag{A64}$$

I can first show, for each price level P_H, a consumption level C_H exists which solves the household (slightly rewritten) steady state FOC,

$$u'(C_H)(1 - \Phi(\hat{\theta}_H(C_H, P_H))\frac{R_H}{(1+\pi_H)}\beta) - \int_{\hat{\theta}_H}^{\infty} \theta v'((B_H + NFA_H)/P_{H,ss} + \overline{b}) d\Phi(\theta) = 0 \tag{A65}$$

such that I can write consumption as a function of the price level, $C_H(P_H)$.

[25] The interpretation is that the marginal value of acquiring a bond (LHS) exceeds its cost (RHS) when the household has zero bonds, that is buying the first bond is utility-enhancing.

This equation is positive for $C_H \to 0$ since $u'(C_H) \to \infty$ and all other terms are bounded and $(1 - \Phi(\hat{\theta}_H(C_H, P_H))\frac{R_H}{(1+\pi_H)}\beta) > (1 - \frac{R_H}{(1+\pi_H)}\beta) > 0$.

This equation is negative for $C_H \to \infty$ since $u'(C_H) \to 0$ and the last term is strictly positive since $\hat{\theta}_H(C_H, P_H) \to 0$.

The intermediate value theorem implies that there is at least one C_H which solves the steady-state FOC.

Uniqueness follows since the derivative of the FOC w.r.t. C_H,

$$u''(C_H)(1 - \Phi(\hat{\theta}_H(C_H, P_H))\frac{R_H}{(1+\pi_H)}\beta) + u'(C_H)\phi(\hat{\theta}_H(C_H, P_H))(1 - \frac{R_H}{(1+\pi_H)}\beta)\frac{\partial\hat{\theta}_H(C_H, P_H)}{\partial C_H}$$

is negative for all P_H and C_H since $\frac{\partial\hat{\theta}_H(C_H,P_H)}{\partial C_H} = \frac{u''(C_H)}{v'(\frac{B_H}{P_H}+\bar{b})} < 0$ and $(1 - \frac{R_H}{(1+\pi_H)}\beta) > 0$.

To show that $C'_H(P_H) < 0$ I calculate now the derivative of the FOC w.r.t. P_H,

$$u'(C_H)\phi(\hat{\theta}_H(C_H, P_H))(1 - \frac{R_H}{(1+\pi_H)}\beta)\frac{\partial\hat{\theta}_H(C_H, P_H)}{\partial P_H} + \frac{B_H}{P^2_{H,ss}}\int_{\hat{\theta}_H}^{\infty}\theta v''(B_H/P_{H,ss} + \bar{b})d\Phi(\theta) < 0$$

since $\frac{\partial\hat{\theta}_H(C_H,P_H)}{\partial P_H} < 0$ and $v'' < 0$.

This implies, that aggregate demand,

$$d_H(P_H) = C_H(P_H) + \frac{G_H}{P_H} + (1 - F(\hat{\theta}(P)))(\frac{B}{P} + \bar{b}) + \int_{\underline{\theta}}^{\hat{\theta}(P)}(v')^{-1}(\frac{u'(C(P))}{\theta})dF(\theta). \quad \text{(A66)}$$

is falling in P_H,

$$\frac{\partial d_H}{\partial P_H} = \frac{\partial C_H}{\partial P_H} - \frac{G_H}{P_H^2} - (1 - F(\hat{\theta_H}(P_H)))\frac{B_H}{P_H^2} < 0. \quad \text{(A67)}$$

Assuming that government spending is larger than net interest rate income from net foreign assets, $G_H > (R-1)NFA_H$, implies that

$$\begin{aligned}\frac{\partial d_H + (R-1)NFA_H/P_H}{\partial P_H} &= \frac{\partial d_H}{\partial P_H} - (R-1)\frac{NFA_H}{P_H^2}\\ &= \frac{\partial C_H}{\partial P_H} - \frac{G_H}{P_H^2} - (1 - F(\hat{\theta_H}(P_H)))\frac{B_H}{P_H^2} - (R-1)\frac{NFA_H}{P_H^2}\\ &< 0. \quad \text{(A68)}\end{aligned}$$

If $P_H \to \infty$, $B_H/P_H + \bar{b} \to \bar{b}$, $G_H/P_H \to 0$ and $NFA_H/P_H \to 0$ so that the assumption

$$u'(Y_H - \bar{b}) - \int_{\frac{u'(Y_H-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0 \tag{A69}$$

implies that for $C_H > Y_H - \bar{b}$

$$u'(C) - \int_{\frac{u'(C)}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) d\Phi(\theta_H) < u'(Y_H - \bar{b}) - \int_{\frac{u'(Y_H-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0 < u'(C_H)\Phi(\hat{\theta}_H(C_H, P_H)) \frac{R_H}{(1+\pi_H)}\beta.$$

This contradiction implies that

$$\lim_{P_H \to \infty} C_H(P_H) < Y_H - \bar{b}. \tag{A70}$$

Therefore

$$\lim_{P_H \to \infty} d(P_H) + (R-1)\frac{NFA_H}{P_H} < Y_H. \tag{A71}$$

Since

$$\lim_{P_H \to 0} d(P_H) + (R-1)\frac{NFA_H}{P_H} = \infty \tag{A72}$$

$$\tag{A73}$$

applying the intermediate value theorem implies the existence of at least one equilibrium price level P_H,

$$d_H(P_H) + (R-1)NFA_H/P_H = Y_H. \tag{A74}$$

Uniqueness follows from

$$\frac{\partial d_H + (R-1)NFA_H/P_H}{\partial P_H} < 0. \tag{A75}$$

I thus have shown that a unique steady-state price level $P_{H,ss}$ exists which solves

$$D_H(P_{H,ss}) + (R-1)NFA_H = P_{H,ss}Y_H, \tag{A76}$$

where $D_H = P_H d_H$ is nominal demand. Similar arguments show that a unique steady-state price level $P_{F,ss}$ exists which solves

$$D_F(P_{F,ss}) + (R-1)NFA_F = D_F(P_{F,ss}) - (R-1)\frac{NFA_H P_{F,ss}}{P_{H,ss}} = Y_F. \tag{A77}$$

I therefore have shown that a unique steady state exchange rate exists,

$$\epsilon_{ss} = \frac{P_{H,ss}}{P_{F,ss}}. \tag{A78}$$