Nominal Rigidity and the Microeconomic Origin of Aggregate Fluctuations*

Ernesto Pasten[†], Raphael Schoenle[‡], Michael Weber[§]

January 10, 2016

Abstract

We study the aggregate propagation of idiosyncratic, sectoral shocks in a multi-sector new-Keynesian model with intermediate inputs featuring sectoral heterogeneity in price stickiness, sectoral GDP, and input-output linkages. Price rigidity distorts the "granular" effect of large sectors on aggregate volatility as well as the "network" effect of central sectors in the production network. This distortion affects both the magnitude of GDP volatility due to sectoral shocks and the identity of sectors that drive aggregate fluctuations. Thus, the friction not only creates conventional aggregate inertia; it can also distort the sign of fluctuations. Sector total sales are also no longer a sufficient statistic for their importance for GDP volatility, as in Hulten (1978). Calibrating our model to 348 sectors using U.S. data suggests that (1) sectoral heterogeneity of price rigidity alone generates sizable GDP volatility from sectoral shocks, (2) price rigidity amplifies both the "granular" and the "network" effects, and (3) price rigidity substantially changes the identity and relative contributions of the most important sectors – especially those operating through the production network.

JEL classification: E31, E32, O04

Keywords: sticky prices, idiosyncratic shocks, business cycles

*We thank comments and suggestions by Saki Bigio, Eduardo Engel, Pete Klenow, John Cochrane, and participants in seminars at the Central Bank of Chile, UCLA, Stanford and UChile-Econ as well as the 2016 SED Meeting (Toulouse). Pasten thanks the support of the Université de Toulouse Capitole during his stays in Toulouse. Weber gratefully acknowledges financial support from the University of Chicago, the Neubauer Family Foundation, and the Fama-Miller Center. The views expressed herein are those of the authors and do not necessarily represent the position of the Central Bank of Chile.

[†]Central Bank of Chile and Toulouse School of Economics. Email: epasten@bcentral.cl

[‡]Brandeis University. Email: schoenle@brandeis.edu.

[§]Booth School of Business, University of Chicago. Email: michael.weber@booth.edu.

I Introduction

A recent literature advances the possibility that idiosyncratic shocks, shocks at the firm or sector level, may be the origin of aggregate fluctuations. This view stands in contrast to the "diversification argument" (Lucas, 1977) which conjectures that idiosyncratic shocks at a highly disaggregated level wash out at the aggregate by application of the Central Limit Theorem. A similar view has been taken by Dupor (1999) and virtually anyone who conceptualizes aggregate fluctuations as the result of aggregate shocks. In contrast, Gabaix (2011) argues when firm size distribution follows a fat tailed distribution, as in U.S. data, the Central Limit Theorem does not readily apply. Intuitively, when there are some "granular" firms – i.e., disproportionately large firms – shocks to them matter for the aggregate. In similar vein, Acemoglu, Carvalho, Ozdaglar and Tahbaz- Salehi (2012) focus on sectoral shocks. They show that the Central Limit Theorem also does not apply when firms are linked by input-output relationships and measures of sectoral centrality in the production network follow a fat-tailed distribution, as suggested by U.S. Input-Output Tables. Thus, either through granularity or network centrality, microeconomic shocks to a handful of firms/sectors may drive aggregate fluctuations, instead of aggregate shocks which might be difficult to identify (Cochrane (1994)). A fast-growing literature has followed.[1]

In this paper, we study the importance of nominal rigidities for the importance of granular shocks to aggregate fluctuations. To fix ideas, let's abstract from input-output linkages and think about a positive productivity shock to one sector. Prices in this sector should decrease, but assume they remain constant. Demand in this sector remains unchanged, so production remains unchanged. Therefore, regardless of the size of the sector, the contribution of its shocks to aggregate fluctuations is zero except for some general equilibrium effects.[2] A similar logic applies to production networks. A price cut in one sector due to a positive productivity shock should spread downstream by decreasing production costs, triggering price cuts in other sectors. But, if prices do not change in the

[1]Some few examples are Acemoglu, Akcigit and Kerr (2016), Acemoglu, Ozdaglar and Tahbaz-Salehi (2016), Atalay (2015), Baqaee (2016), Bigio and La'O (2016), Caliendo, Parro, Rossi-Hansberg and Sarte (2016), Carvalho and Gabaix (2013), Carvalho and Grassi (2016), Di Giovanni, Levchenko and Mejean (2014, 2016), and Foester, Sarte and Watson (2011).

[2]First, lower demand for inputs in the shocked sector decreases wages. Second, higher profits of firms in the shocked sector increase household incomes. However, these effects turn out to be minor when taking a first-order approximation.

shocked sector, the marginal costs of downstream firms remain unchanged and there is no propagation regardless of the centrality of the shocked sector in the production network, except for the general equilibrium effects pointed out above.

In the data, prices are neither fully rigid nor fully flexible. As a matter of fact, substantial heterogeneity of price rigidity across sectors in the U.S. economy has been documented (Bils and Klenow, 2004; Krytsov and Klenow, 2008). How does this heterogeneity in nominal price rigidity interact with the "granularity effect" of Gabaix (2011) and the "network effect" of Acemoglu et al. (2012) on the ability of microeconomic shocks to generate sizable aggregate fluctuations? If such a capacity remains strong, how does price rigidity distort the identity of the sectors that are the origin of aggregate fluctuations?

We answer these questions in a multi-sector new-Keynesian model in which firms produce output using labor and intermediate inputs. Our model follows those of Basu (1995) and Carvalho and Lee (2011) but makes no simplification on the steady-state distribution of sectoral value-added production (sectoral GDP) or input-output linkages as well as the sectoral distribution of the price-setting friction (modeled à la Calvo.) Given the questions we want to pursue, we allow only for sectoral productivity shocks. We calibrate our model by using Input-Output Tables provided by the Bureau of Economic Analysis (BEA) at the most disaggregated level available and the data underlying the Producer Price Index (PPI) computed by the Bureau of Labor Statistics (BLS). After merging these two datasets, we end up with 348 sectors.

Before conducting quantitative analysis, we analytically study in a simplified version of our model the distortionary role of price rigidity on the "granular" and "network" origins of aggregate fluctuations. Up to a log-linear approximation around the steady state, GDP is a linear combination of sectoral shocks that nests the solutions of the models of Gabaix (2011) and Acemoglu et al. (2012) as special cases.

When abstracting from intermediate inputs and price rigidity, our model recovers the granularity effect of Gabaix (2011): the ability of microeconomic shocks to generate aggregate fluctuations depends on how fat-tailed the sector size distribution. Price rigidity has two effects in this context. First, the average price rigidity in the economy dampens the level of aggregate volatility generated by shocks, either sectoral or aggregate. Second, the sectoral distribution of price rigidity distorts the relative importance of sectors for aggregate fluctuations. In particular, a sector is important when it is large, as in Gabaix (2011), and/or when it's prices are much more flexible than most prices in the economy. To illustrate the implications of the sectoral distribution of price rigidity, assume the sector size distribution is fat-tailed and negatively correlated with price rigidity. Larger sectors are more likely to be more flexible, so the effect of their shocks on aggregate volatility is even larger than in a frictionless economy. The distribution of the multiplier of sectoral shocks is more fat-tailed than implied by the sector size distribution alone. The opposite holds if sector size is positively correlated with price rigidity. In this case it is in fact possible that the diversification argument of Lucas (1977) holds in an economy with sticky prices under the otherwise same conditions that the granularity argument of Gabaix (2011) holds in a frictionless economy. From a different angle, price rigidity distorts the identity of the most important sectors provided that there is no monotone negative relationship between price rigidity and size. Since shocks are idiosyncratic, the friction may thus distort the sign of the business cycle and not only generate inertia, as standard with aggregate shocks.

We reach similar results for the distortionary role of price rigidity on the network effect of Acemoglu et al. (2012). With flexible prices, microeconomic shocks are more important for GDP volatility as the distribution of sector centrality is more fat-tailed. Sectors that matter the most are large suppliers of intermediate inputs (first-order interconnection) and/or large suppliers of large suppliers of intermediate inputs (second-order interconnection). When we introduce price stickiness, the most important sectors are the most flexible sectors that are also large suppliers of the most flexible sectors. Thus, the multipliers of sectoral shocks may be more or less fat-tailed than the distribution of sector centrality dictates. Heterogeneity in price rigidity also invalidates the result shown by Hulten (1978), which also holds in Gabaix (2011) and Acemoglu et al. (2012), that sector (or firm) total sales are a sufficient statistic of their importance for GDP volatility.

In our quantitative analysis, we start by establishing/confirming the following facts for the US: (i) sectoral GDP is distributed fat-tailed Pareto; (ii) our measures of centrality in the U.S. Input-Output Tables are distributed fat-tailed Pareto; (iii) sectoral gross output (summing value added and intermediate goods) is distributed fat-tailed Pareto;[3]

[3]This distribution is also remarkably similar to the distribution of firm total sales reported by Gabaix (2011). Gross output is conceptually the closest in our data to total sales.

(iv) the sectoral frequency of price changes in the U.S. is distributed Pareto, but not fat-tailed; (iv) the correlation between frequency of price changes and sectoral GDP in its upper tail is only 6.73% and its correlation with measures of network centrality ranges from 22.63% to 33.33%.

In light of these results, we should find little role for price rigidity. This is not the case. In our baseline calibration with 348 sectors,[4] the multiplier of sectoral productivity shocks on GDP volatility relative to that of aggregate productivity shocks[5] increases from 11.3% when prices are flexible to 22.8% when prices are sticky (matching the sectoral frequency of price changes in the data). It also increases the effect of network centrality from 8% to 11.5%. The distribution of stickiness alone yields a multiplier of sectoral shocks of 10.8%. This change suggests a "frictional" origin of aggregate fluctuations conceptually different from granularity or network centrality. Overall, the multiplier on GDP volatility of sectoral shocks relative aggregate shocks increases from 17.5% to 24%. For comparison, this relative multiplier is 5.43% if all sectors are perfectly homogeneous.

Price rigidity also has strong effects on distorting the identity and contribution of sectors driving aggregate fluctuations. For instance, the identity of the two most important sectors when we only consider network effects shifts from "Real Estate" and "Wholesale Trading" with flexible prices to "Petroleum Refineries" and "Oil and Gas Extraction" with sticky prices. However, after adding in sectoral GDP heterogeneity, the two most important sectors with flexible and sticky prices are the same again, "Real Estate" and "Wholesale Trading." At the same time, the relative contribution of the former is now cut in half while that of the latter is doubled.

Literature review. At an abstract level, this paper shows that not only the size or centrality of nodes in the network matter for the macro effect of micro shocks, but also the frictions that affect the capacity of nodes to propagate shocks. This point clearly goes beyond sticky prices. We choose sticky prices because prices are at the core of the transmission mechanism of sectoral shocks in production networks, and stickiness is a measurable friction at a highly disaggregated level. Our point also goes beyond production networks in a closed economy; it is suitable for any context characterized as a network with

[4]This calibration assumes linear disutility of labor and monetary policy targeting on constant nominal aggregate demand. Results are not much different when we allow for curvature in disutility of labor and a standard Taylor rule for monetary policy.

[5]We report these multipliers in relative terms because the effect of an aggregate shock on GDP volatility is not invariant to the distributions of price rigidity, sectoral GDP and input-output linkages.

heterogeneous effects of frictions across nodes, e.g. in international networks, financial networks or social networks. Our work is thus related to an extensive literature that we do not attempt to summarize here.

The microeconomic origin of aggregate fluctuations was pioneered by Long and Plosser (1983) and pushed among others by Horvath (1998, 2000). Dupor (1999) argued against that microeconomic shocks matter only due to poor disaggregation. The discussion has been settled down by Gabaix (2011) appealing to firms' size distribution and Acemoglu et al. (2012) to the sectoral network structure of the economy. Both show that, under assumptions consistent with the data, microeconomic shocks do matter, not because our levels of disaggregation are too low. All cites in footnote 1 apply here.

The distortionary role of frictions (and price rigidity, in particular) is at the core of the business-cycle literature that conceptualizes aggregate productivity shocks as the main driver of aggregate fluctuations, including the new-Keynesian literature. However, to the best of our knowledge, there is no parallel study of the distortionary role of frictions when aggregate fluctuations have microeconomic origins. That said, there are few papers that include frictions in their analyses. Baqaee (2016) shows that entry and exit of firms coupled with love-for-variety preferences may amplify the aggregate effect of microeconomic shocks. Carvalho and Grassi (2016) study the effect of large firms in a quantitative business cycles model with entry and exit of firms. Bigio and La'O (2016) study the aggregate effect of the tightening of financial frictions in a production network. In spite of a different focus, we share our finding with Baqaee (2016) and Bigio and La'O (2016) that Hulten's result referred to above does not apply when there are frictions.

Our model shares building blocks with previous work studying pricing frictions in production networks. Basu (1995) shows that the misallocation introduced by the friction implies that nominal aggregate demand shocks look like aggregate productivity shocks. Carvalho and Lee (2011) show that a new-Keynesian model naturally generates firms' prices responding slowly to aggregate shocks and fast to idiosyncratic shocks, as documented by Boivin, Giannoni and Mihov (2011). Besides answering different questions, our model differs from theirs by allowing for a more flexible production structure. This allows us to pursue more fine-tuned quantitative exercises.

Nakamura and Steinson (2010), Midrigan (2012), Alvarez, Le Bihan and Lippi (2016) and Karadi and Reiff (2016) endogenize price rigidity to study monetary non-neutrality in multi-sector menu cost models. Computational burden and calibration issues of such an approach make analysis unfeasible for our highly disaggregated model. Exactly for this reason, we study in a companion paper the effect of disaggregation on monetary non-neutrality predicted by the new-Keynesian model (Pasten, Schoenle and Weber, 2016).

Outline. Section 2 presents the model. Section 3 uses a simplification of it to provide intuition for our main findings. Section 4 describes the data and stylized facts, explains the calibration and presents quantitative results. Section 5 concludes. Appendices contain proofs, tables and detailed derivations.

II Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms with sticky prices demanding labor and intermediate inputs, and a monetary authority following a Taylor rule. Sectors are heterogeneous in three dimensions: their final goods production (interpreted as value added or simply GDP), input-output linkages, and the frequency of price changes.

A. Households

The problem that a representative household must solve is given by

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1+\varphi} dj \right)$$

subject to

$$\begin{aligned} \sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t &= P_t^c C_t \\ \sum_{k=1}^{K} L_{kt} &\leq 1 \end{aligned}$$

where C_t and P_t^c respectively are aggregate consumption and aggregate prices to be specified below, L_{kt} and W_{kt} are labor employed and wages paid in sector $k = 1, ..., K$, Π_{kt} is transfers from firms in sector k, I_{t-1} is the gross interest rate paid by bonds held at the beginning of period t, B_{t-1}. Total labor supply equals a constant normalized to one.

Households' demand of final goods is given by

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{1}$$

where C_{kt} is household demand for goods produced in sector k which is given by

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

where C_{jkt} is households' demand of good j that is categorized in sector k. There is a continuum of goods indexed by $j \in [0,1]$ with total measure one. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into subsets $\{\Im_k\}_{k=1}^{K}$ respectively with measure $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$. Although the sectoral subindex is redundant, it clarifies exposition.

We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ. The first key ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in (1). These weights show up in households' sectoral demand:

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t. \tag{3}$$

In steady state (solved in the Appendix) all prices are identical, so $\omega_{ck} \equiv \frac{C_k}{C}$ where C_k and C are steady state levels. In our economy, C_{kt} and C_t represent the sectoral and total production of final goods, i.e., we interpret them as sectoral and total value-added (in short, GDP). Thus, we refer to Ω_c as the vector of steady-state sectoral GDP shares satisfying $\Omega_c' \iota = 1$ where ι denotes a column-vector of ones. Outside steady state, sectoral GDP shares depend on the gap between sectoral prices and the aggregate price P_t^c:

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4}$$

Given the interpretation of C_t as GDP, we interpret P_t^c as the GDP deflator.

Households' demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k} \left(\frac{P_{jkt}}{P_{kt}} \right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K, \tag{5}$$

such that, in steady state, sectoral GDP is equally shared among goods belonging to that sector. Outside the steady state, the demand of goods within a sector is distorted by the gap between their price and the sectoral price defined as

$$P_{kt} = \left[\frac{1}{n_k} \int_{\Im_k} P_{jkt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{6}$$

From the first order conditions for households we also get labor supply and the Euler equation:

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{7}$$

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c} \right] = 1 \tag{8}$$

Implicitly in our setup there is the sectoral segmentation of the labor market, so labor supply in (7) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^{K}$. The parameters $\{g_k\}_{k=1}^{K}$ are conveniently chosen to ensure a symmetric steady state across all firms. In turn, the Euler equation is standard.

B. Firms

Goods are produced by monopolistically competitive firms. We refer as "firm jk" to the firm that produces good j in sector k. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^{\delta}, \tag{9}$$

where a_{kt} is a i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor hired and Z_{jkt} is an aggregator of intermediate inputs demanded by firm jk defined as

$$Z_{jkt} \equiv \left[\sum_{k'=1}^{K} \omega_{kk'}^{\frac{1}{\eta}} Z_{jk} (k')^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}; \tag{10}$$

$Z_{jk}(k')$ denotes the amount of goods from sector k' demanded by firm jk at period t. The second key ingredient of our model is heterogeneity in aggregator weights $\{\omega_{kk'}\}_{k,k'}$ These weights are denoted in matrix form by Ω, satisfying $\Omega\iota = \iota$. The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}(k') = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k}\right)^{-\eta} Z_{jkt}. \tag{11}$$

such that $\omega_{kk'}$ can be interpreted as the steady-state intensity of use of goods categorized in sector k' for production in sector k, i.e., the steady-state input-output linkages of sector k as customer with sector k'. Outside the steady state, input-output linkages are distorted by the gap between the price of goods in sector k' and the aggregate price relevant for a firm in sector k:

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \text{for } k = 1, ..., K \tag{12}$$

which uses its sector-specific steady-state input-output linkages as customer to aggregate sectoral prices.

In turn, the demand of firm jk for goods in sector k' is given by the aggregator

$$Z_{jk}(k') \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}(j', k')^{1-\frac{1}{\theta}} \, dj'\right]^{\frac{\theta}{\theta-1}} \tag{13}$$

such that firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}(j', k') = \frac{1}{n_{k'}} \left(\frac{P_{j'k't}}{P_{k't}}\right)^{-\theta} Z_{jk}(k'). \tag{14}$$

Within sectors, in steady state all firms share equally the demand of sectors as intermediate inputs. Outside the steady state firms' shares are distorted by the gap between their own price and their respective sectoral price, which are given by (6).

Note that our economy has $K + 1$ different versions of aggregate prices, depending on whether the demanding decision maker is a household or a firm in either of the K sectors of the economy. In contrast, there is only one version of sectoral prices.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. Specifically, we model price rigidity *á la* Calvo with parameter $\{\alpha_k\}_{k=1}^{K}$ such that the pricing problem of the firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^{\tau} \left[P_{jkt} Y_{jkt+s} - MC_{kt+s} Y_{jkt+s} \right]$$

where marginal costs are given by $MC_{kt} = \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta} \left(P_t^k \right)^{\delta}$ in reduced form after imposing the optimal mix of labor and intermediate inputs:

$$\delta W_{kt} L_{jkt} = (1-\delta) P_t^k Z_{jkt}. \tag{15}$$

We assume in our setup that the elasticity of substitution across and within sectors, although allowed to be different among them, is the same for households and all firms. This assumption shuts down the incentives of firms to discriminate among different customers so firms choose a single price given by

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau} \alpha_k^s Y_{jkt+\tau} \left[P_{kt}^* - \frac{\theta}{\theta - 1} MC_{kt+\tau} \right] = 0 \tag{16}$$

where $Y_{jkt+\tau}$ is the total production of firm jk at period $t+\tau$.

Since idiosyncratic shocks $\{a_{kt}\}_{k=1}^{K}$ are defined at the sectoral level, the optimal adjusting price, P_{kt}^*, is the same for all firms in a given sector. Thus, aggregating among all prices within sectors yields

$$P_{kt} = \left[(1-\alpha_k) P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{17}$$

C. Monetary policy, equilibrium conditions and definitions

Monetary policy controls I_t which is set according to the Taylor rule:

$$I_t = \frac{1}{\beta} \left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_y} \tag{18}$$

Note that monetary policy reacts to inflation using the GDP deflator P_t^c and deviations from steady state of total value-added (GDP) C_t. There is no idiosyncratic monetary shock.

We also impose that bonds are in zero net supply, $B_t = 0$, labor market clears, and the goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k't}(j,k)\, dj', \tag{19}$$

implying a wedge between gross output Y_t and GDP C_t.

III The core idea

To provide intuition for our main results, this section solves a closed-form, simplified version of our model. Given our focus on the effect of sectoral idiosyncratic shocks on aggregate fluctuations, we restrict attention to the log-linear deviation from the steady state of GDP. The steady state solution and the full log-linear system that solves the equilibrium up to a first order approximation are relegated to Appendix A.

Simplifying assumptions. All variables in lower cases denote log-linear deviations from steady state.

(i) Households have log utility of consumption, $\sigma = 1$ and linear disutility of labor, $\varphi = 0$, thus

$$w_{kt} = p_t^c + c_t;$$

the labor market is integrated and nominal wages are proportional to nominal GDP.

(ii) Instead of following the Taylor rule in (18), monetary policy targets on constant nominal GDP, so

$$p_t^c + c_t = 0.$$

(iii) We replace price rigidity *á la* Calvo by a simple specification: All prices are flexible although there is probability λ_k that a firm in sector k must set its price before observing shocks; otherwise it does so after. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P_{jkt}^*\right] & \text{with probability } \lambda_k, \\ P_{jkt}^* & \text{with probability } 1-\lambda_k, \end{cases}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on all information up to $t-1$.

Solution. As Appendix B shows, c_t under assumptions (i), (ii) and (iii) is given by

$$c_t = \chi' a_t \tag{20}$$

where $\chi \equiv (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta \Omega' (\mathbb{I} - \Lambda)\right]^{-1} \Omega_c$, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]$ as diagonal, and $a_t \equiv [a_1, ..., a_K]'$ is the vector of realizations of sectoral productivity shocks.

Equation (20) shows that the log-deviation of GDP from its steady state up to a first-order approximation is given by a linear combination of sectoral shocks. Thus, given that by assumption $\mathbb{V}[a_{kt}] = v^2$ for all k,

$$v_c = v \sqrt{\sum_{k=1}^{K} \chi_k^2} = \|\chi\|_2 \, v \tag{21}$$

with $\|\chi\|_2$ denoting the Euclidean norm of χ. Thus, χ represents the vector of *multipliers of the volatility of productivity shocks specific to each sector for GDP volatility* (in short, *sectoral multipliers*). In the following, we study the effect of heterogeneous price rigidity on the scale of v_c in an economy with a given number K of sectors, as well as the effect on the rate of decay of v_c to zero as the economy becomes increasingly more disaggregated, $K \to \infty$. We use the following definition:

Definition 1 *A given random variable X follows a* ***power-law distribution with shape parameter*** *β when $\Pr(X > x) = (x/x_0)^{-\beta}$ for $x \geq x_0$ and $\beta > 0$.*

A. The granular effect and price rigidity

We now study the interaction of price rigidity with the "granularity effect" in Gabaix (2011) which refers to the role of firms' size distribution on the importance of idiosyncratic shocks as origin of aggregate volatility. Gabaix measures firms' size by their total sales which includes sales as final goods and as intermediate inputs. Because of data availability, we study sectors instead of firms. Further, partially because we want to make the distinction between final good sales and intermediate inputs sales and partially because of expositional purposes, we start our analysis by setting $\delta = 0$. This way we ignore input-output linkages such that sector size only depends on sectoral value added (sectoral

GDP). Setting $\delta = 0$ also allows us to get functional forms that parallel the analysis in Gabaix (2011); the next subsection relaxes this assumption, mimicking the networks analysis of Acemoglu et al. (2012).

When $\delta = 0$,

$$\chi = (\mathbb{I} - \Lambda')\,\Omega_c$$

or, simply, $\chi_k = (1 - \lambda_k)\,\omega_{ck}$ for all k. Recall that $\omega_{ck} = C_k / \sum_{k=1}^{K} C_k$.

This expression clearly shows that sectoral multipliers are fully determined by steady state sectoral GDP shares only when prices are flexible. In general, sectoral multipliers also depend on the sectoral distribution of the nominal pricing friction. This observation contradicts the Hulten (1978) result in Gabaix (2011) that sales are a sufficient statistic for the importance of sectors on aggregate volatility.

The following lemma introduces our first result by assuming homogeneity in price rigidity.

Lemma 1 *When $\delta = 0$ and $\lambda_k = \lambda$ for all k,*

$$v_c = \frac{(1-\lambda)\,v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(C_k\right) + \overline{C}_k^2}$$

where $\overline{C}_k$ and $\mathbb{V}(\cdot)$ are the sample mean and sample variance of $\{C_k\}_{k=1}^{K}$.[6]

This lemma follows from (21) when $\delta = 0$. Exactly as in Gabaix (2011), the volatility of GDP in an economy with K sectors depends on the cross-sectional dispersion of sector size. The only role here for price rigidity is to scale up volatility whether productivity shocks are sectoral or aggregate. This can be seen from (20): If $\delta = 0$, and if all sectoral shocks are perfectly correlated, then $v_c = (1-\lambda)\,v$.

The next proposition presents our results for the rate of decay of v_c as the economy is increasingly more disaggregated, $K \to \infty$, under the assumption of homogeneous price rigidity.

Proposition 1 (Granular effect) *If $\delta = 0$, $\lambda_k = \lambda$ for all k, and $\{C_k\}_{k=1}^{K}$ follows a*

[6] We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^{K}$ as $\mathbb{V}(X_k) \equiv \frac{1}{K}\sum_{k=1}^{K}\left(X_k - \overline{X}\right)^2$. In turn, the definition of the sample mean is standard.

power-law distribution with shape parameter $\beta_c \geq 1$, *then*

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1 \end{cases}$$

where u_0 *is a random variable independent of* K *and* v.

Proof. See Appendix C. ■

Proposition 1 revisits the central idea of the "granularity effect:" When the size distribution of sectors is fat-tailed, given by $\beta_c < 2$, v_c converges to zero at a slower rate than the standard rate $K^{1/2}$ dictated by the Central Limit Theorem. The rate of decay of v_c is slower as $\beta_c \to 1$. In intuitive terms, when the size distribution of sectors is fat-tailed, few sectors remain disproportionately large at any level of disaggregation. Gabaix (2011) documents that the upper tail of the distribution of firms' size can be empirically characterized by a power-law distribution with shape parameter close to one. We find the same result with sectoral data (presented in Section 4). Thus, contrary to the claim in Dupor (1999), sectoral shocks can generate sizable aggregate fluctuations even if sectors are defined at a quite disaggregated level. Homogeneous price rigidity plays no role for this result, except for the scale effect pointed out in Lemma 1.

We now move to the more interesting case when price rigidity is heterogeneous across sectors.

Lemma 2 *When* $\delta = 0$ *and price rigidity is heterogeneous across sectors,*

$$v_c = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(\left(1-\lambda_k\right) C_k\right) + \left[\left(1-\overline{\lambda}\right)\overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2}$$

where $\overline{\lambda}$ *is the sample mean of* $\{\lambda_k\}_{k=1}^K$ *and* $\mathbb{COV}(\cdot)$ *is the sample covariance of* $\{\lambda_k\}_{k=1}^K$ *and* $\{C_k\}_{k=1}^K$.[7]

Lemma 2 states that, for a given level of disaggregation, the volatility of GDP depends on the sectoral dispersion of the convoluted variable $\{(1-\lambda_k) C_k\}_{k=1}^K$ as well as the covariance between sectoral price rigidity and sectoral GDP. This result is valid regardless whether price rigidity and sectoral GDP are independent or not – in the former

[7] We define $\mathbb{COV}(X_k, Q_k)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}(X_k, Q_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

case presumably their sample covariance is smaller than in the latter yet not exactly zero. For the rate of decay of v_c as $K \to \infty$, independence is, however, an important distinction to make.

Proposition 2 *If $\delta = 0$, $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$ are independently distributed, the distribution of $\{\lambda_k\}_{k=1}^K$ satisfies*

$$\Pr[1-\lambda_k > y] = \frac{y^{-\beta_\lambda}-1}{y_0^{-\beta_\lambda}-1} \text{ for } y \in [y_0, 1], \beta_\lambda > 0$$

and $\{C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_c \geq 1$, then

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1 \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Appendix C. ■

This Proposition shows that price rigidity plays no role for the rate of decay of v_c as $K \to \infty$ when $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$ are independently distributed. The reason is the lower bound in the support of the distribution of λ_k. If λ_k had no lower bound, $(1-\lambda_k) C_k$ would be Pareto with the lesser shape parameter from among those of C_k and $1-\lambda_k$. But, in the case if the proposition, $(1-\lambda_k) C_k$ is Pareto with the same shape parameter as the distribution of C_k. Despite of this result, price rigidity is still highly relevant. A direct implication of Lemma 2 is that price rigidity distorts the identity and the contribution of the most important sectors for the volatility of GDP. This identity effect is actually a crucial result when the goal is to uncover the microeconomic origin of aggregate fluctuations.

Proposition 2 assumes a specific functional form for the distribution of $\{\lambda_k\}_{k=1}^K$. At this point the analysis can be done only via examples. However, we find in a subsequent section that this distribution characterizes well the empirical distribution of the frequency of price changes across sectors; it is basically a Pareto with a theoretically bounded support (that is not binding in our sample of sectors.) We now move to our next result, which is the most important in the section.

Proposition 3 *Let $\delta = 0$, the distributions of $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$ are not independent* *such that the following relationships holds*

$$\lambda_k = \max\left\{0, 1 - \phi C_k^{\mu}\right\} \text{ for some } \mu \in (-1,1)\,,\ \phi \in \left(0, x_0^{-\mu}\right) \tag{22}$$

and $\{C_k\}_{k=1}^{K}$ *follows a power-law distribution with shape parameter* $\beta_c \geq 1$.

If $\mu < 0$,

$$v_c \sim \begin{cases} \frac{u_1}{K^{\min\{1-(1+\mu)/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{\min\{-\mu, 1/2\}} \log K} v & \text{for } \beta_c = 1 \end{cases} \tag{23}$$

If $\mu > 0$,

$$v_c \sim \begin{cases} \frac{u_2}{K^{\min\left\{1-\frac{1+\mathbb{1}\{K \leq K^*\}\mu}{\beta_c}, \frac{1}{2}\right\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{-\mathbb{1}\{K \leq K^*\}\mu} \log K} v & \text{for } \beta_c = 1 \end{cases} \tag{24}$$

for $K^* \equiv x_0^{-\beta_c} \phi^{-\beta_c/\mu}$.

Proof. See Appendix C. ■

This proposition illustrates the implications of the interaction between sector size and price rigidity on the rate of decay of GDP volatility v_c as sectors are defined at a increasingly disaggregated level. First, consider the case when $\mu < 0$, that is, when larger sectors have more rigid prices. When $\beta_c \in (\max\{1, 2(1+\mu)\}, 2)$, v_c decays at the standard rate $K^{1/2}$. More generally, when $\beta_c \in [1, 2)$ and $\mu < 0$, a positive relationship between sectoral size and price stickiness slows down the rate of decay of v_c.

Second, consider the case when larger sectors have more flexible prices ($\mu > 0$.) In this case, the functional form (22) assumed in Proposition 3 has a kink such that sectors with value added larger than $\phi^{-1/\mu}$ have perfectly flexible prices. This kink also generates a kink in the rate of decay of v_c. If $\beta_c \in [1, 2)$, v_c decays at a faster rate than when sector size and price stickiness are independently distributed for $K \leq K^*$. If the number of sectors is sufficiently large, price rigidity is irrelevant for the rate of decay of v_c. Intuitively, the relationship assumed in (22) implies that sector size and price rigidity are independent for any sector with GDP higher than $\phi^{-1/\mu}$. For $K > K^*$, there is a high enough probability that the upper tail of the distribution of the multipliers $\chi_k = (1 - \lambda_k) C_k$ is dominated by sectors with value added larger than $\phi^{-1/\mu}$. A key question then is what an empirically sensible number of sectors is. In other words, how high K^* is. In the context of Proposition 3, this question can be answered by noticing that (22) implies that, for $K > K^*$, there is high density of sectors with fully flexible prices. In our data for 348 sectors, which is the finest level of disaggregation our data allows for, there is not a single sector with fully flexible prices. This observation suggests that, if larger sectors tend to have more flexible prices, the friction slows down the decay of aggregate volatility v_c for any level of disaggregation of the U.S. economy with at most 348 sectors. If there were no kink on the relationship between sectoral GDP and price stickiness for large C_k, as for $\mu < 0$ in Proposition 3, then the friction would slow down the rate of decay of v_c for any K.

For expositional convenience, we have assumed a deterministic relationship between sectoral GDP and price stickiness. However, if this relationship is stochastic, trivially we would find that price rigidity distorts the identity of the most important sectors for GDP volatility – even if price rigidity is irrelevant for the rate of decay of GDP volatility v_c.

The next corollary wraps up results in this section.

Corollary 1 *In an economy where sectors have heterogeneous sectoral GDP but no input-output linkages, sectoral heterogeneity of price rigidity distorts the magnitude of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

B. The network effect and price rigidity

We now make similar points as above in the context of the "network" effect of input-output linkages on amplifying the impact of idiosyncratic shocks on aggregate volatility (Acemoglu et al, 2012). We assume $\delta \in (0,1)$ and shut down the heterogeneity of sectoral GDP, so $\Omega_c = \frac{1}{K}\iota$ where ι is a column-vector of ones. In this case, the vector of multipliers solves

$$\chi = \frac{1}{K}\left(\mathbb{I} - \Lambda\right)\left[\mathbb{I} - \delta\Omega'\left(\mathbb{I} - \Lambda\right)\right]^{-1}\iota. \tag{25}$$

This expression nests the solution for the "influence vector" in Acemoglu et al (2012) when prices are fully flexible, i.e. $\lambda_k = 0$ for all $k = 1, ..., K$.[8] In general, however, there is a non-trivial interaction between price rigidity and input-output linkages across sectors. To study this interaction, we follow Acemoglu et al (2012) by using an approximation of the vector of multipliers that truncates the effect of input-output linkages up to a

[8]The only difference is that here $\chi'\iota = 1/\left(1-\delta\right)$ since Acemoglu et al. (2012) normalize the scale of shocks such that the sum of the influence vector equal to one.

second-order of interconnection:

$$\chi \simeq \frac{1}{K}\left(\mathbb{I}-\Lambda\right)\left[\mathbb{I}+\delta\Omega'\left(\mathbb{I}-\Lambda\right)+\delta^{2}\left[\Omega'\left(\mathbb{I}-\Lambda\right)\right]^{2}\right]\iota.$$

Then, as a first step, we start by assuming homogeneous price rigidity across sectors.

Lemma 3 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$ and $\lambda_k = \lambda$ for all k,*

$$v_c \geq \frac{(1-\lambda)\,v}{K^{1/2}}\sqrt{\kappa+\delta'^{2}\mathbb{V}\left(d_k\right)+2\delta'^{3}\mathbb{COV}\left(d_k,q_k\right)+\delta'^{4}\mathbb{V}\left(q_k\right)}$$

where $\kappa \equiv 1+2\delta'+3\delta'^{2}+2\delta'^{3}+\delta'^{4}$, $\delta' \equiv \delta(1-\lambda)$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample cross-sectoral variance and covariance statistics, and $\{d_k\}_{k=1}^{K}$ and $\{q_k\}_{k=1}^{K}$ respectively are the ***outdegrees*** *and* ***second-order outdegrees*** *defined for all $k = 1,...,K$ as*

$$\begin{aligned} d_k &\equiv \sum_{k'=1}^{K}\omega_{k'k}, \\ q_k &\equiv \sum_{k'=1}^{K} d_{k'}\omega_{k'k}. \end{aligned}$$

This lemma follows from (21), $d = \Omega'\iota$ and $q = \Omega'^{2}\iota$. The inequality comes from the fact that the exact solution of χ is strictly larger than the approximation we use. The terms *outdegrees* and the *second-order outdegrees* are statistics coined by Acemoglu et al. (2012) to measure the centrality of sectors in the production network. In particular, d_k is large when goods from sector k are intensively used for production in the economy; in short, when sector k is a large supplier of intermediate inputs. In turn, q_k is large when goods from sector k are intensively used for production in sectors that are large suppliers of intermediate inputs.

Similarly to Lemma 1, for a given level of disaggregation, price rigidity that is homogeneous across sectors only scales up aggregate volatility. Thus, as in Acemoglu et al. (2012), aggregate volatility is higher if the production network is more asymmetric given by higher dispersion of outdegrees and second-order outdegrees across sectors.

The next proposition shows results for the rate of decay of v_c as $K \to \infty$ under the assumption of homogeneous price rigidity.

Proposition 4 (Network effect) *If $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$,* *the distribution of outdegrees* $\{d_k\}$ *and second-order outdegrees* $\{q_k\}$ *follow power-law distributions with respective power parameter* $\beta_d, \beta_q > 1$ *then*

$$v_c \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d, \beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1,2) \end{cases}$$

where u_3 *is a random variable independent of* K *and* v*.*

Proof. See Appendix C. ■

This proposition summarizes the essence of the "network effect" in Acemoglu et al. (2012): The rate of decay of aggregate volatility is bounded below by the fattest-tailed distribution from among the distribution of outdegrees, and second-order outdegrees. Thus, if some sectors are disproportionately central in the production network, sectoral idiosyncratic shocks have sizable effects on aggregate volatility even if sectors are defined at a quite disaggregated level. Acemoglu et al. (2012) document in the U.S. data that $\beta_d \approx 1.4$ and $\beta_q \approx 1.2$. We find slightly higher numbers.[9] Thus, this result complements the reply of Gabaix (2011) to Dupor (1999) who argues that idiosyncratic shocks matter for aggregate volatility only due to poor disaggregation.

As in Section 3.1, homogeneous price rigidity across sectors only has a scale effect on GDP volatility. However, it has one implication worth noticing. Since $\beta_q < \beta_d$ in U.S. data, the distribution of second-order outdegrees contributes the most to the slow decay of v_c when K is large. Lemma 3 implies that this contribution is quantitatively less important as price rigidity increases because $\delta'/\delta = 1 - \lambda$.

We turn now to present results for the case of heterogeneous price rigidity.

Lemma 4 *If* $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, *price rigidity is heterogeneous across sectors, then*

$$v_c \geq \frac{v}{K^{1/2}} \left[\begin{array}{c} \left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[2\delta^2\left(1+\widetilde{\delta}+\widetilde{\delta}^2\right)\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ +\mathbb{COV}\left((1-\lambda_k)^2, \left(1+\delta\widetilde{d}_k + \delta^2\widetilde{q}_k\right)^2\right) \end{array} \right]^{\frac{1}{2}}$$

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta} + 2\widetilde{\delta} + \widetilde{\delta}$, $\widetilde{\delta} \equiv \delta\left(1-\overline{\lambda}\right)$, $\overline{\lambda}$ *is the sample mean of* $\{\lambda_k\}_{k=1}^{K}$, $\mathbb{V}(\cdot)$ *and*

[9]This is why we abstract from the case when $\min\{\beta_d, \beta_q\} = 1$ in Proposition 4.

$\mathbb{COV}(\cdot)$ *are the sample across-sectors variance and covariance statistics, and* $\left\{\widetilde{d}_k\right\}_{k=1}^{K}$ *and* $\{\widetilde{q}_k\}_{k=1}^{K}$ *respectively are the* ***modified outdegrees*** *and* ***modified second-order outdegrees*** *defined for all* $k = 1, ..., K$ *as*

$$
\begin{aligned}
\widetilde{d}_k &\equiv \sum_{k'=1}^{K} (1 - \lambda_{k'})\, \omega_{k'k}, \\
\widetilde{q}_k &\equiv \sum_{k'=1}^{K} (1 - \lambda_{k'})\, \widetilde{d}_{k'} \omega_{k'k}.
\end{aligned}
$$

This result follows from (21), $\widetilde{d} = \Omega' (\mathbb{I} - \Lambda)\, \iota$ and $\widetilde{q} = [\Omega' (\mathbb{I} - \Lambda)]^2\, \iota$ which we respectively label the vector of *modified outdegrees* and the *modified second-order outdegrees*. These statistics measure centrality of sectors in the production network after adjusting nodes by their degree of price rigidity. In particular, $\widetilde{d}_k$ is high either when sector k is a large supplier of intermediate inputs and/or when it is a large supplier of the most flexible sectors. Similarly, $\widetilde{q}_k$ is large when sector k is a large supplier of the most flexible sectors which are in turn large suppliers of the most flexible sectors.

The lower bound obtained for v_c in Lemma 4 collapses to the one in Lemma 3 if price rigidity is homogeneous across sectors. The first line in the right-hand side of this expression has similar components of the lower bound in Lemma 3 with two differences. The first difference is that by Jensen's inequality

$$
\frac{1}{K} \sum_{k=1}^{K} (1 - \lambda_k)^2 \geq \left(1 - \overline{\lambda}\right)^2;
$$

therefore the effect of price rigidity muting the scale of aggregate volatility is weaker if price rigidity is heterogeneous across sectors relative to an economy with $\lambda_k = \overline{\lambda}$ for all k. The second difference is that now the key statistics are computed using $\widetilde{d}$ and $\widetilde{q}$ instead of d and q. To see the implications of this, we use that

$$
\begin{aligned}
\widetilde{d}_k &= \left(1 - \overline{\lambda}\right) d_k - K\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right), \\
\widetilde{q}_k &= \left(1 - \overline{\lambda}\right)^2 q_k - K\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k \omega_{k'k}\right) - \left(1 - \overline{\lambda}\right) \sum_{k'=1}^{K} \omega_{k'k} \mathbb{COV}\left(\lambda_s, \widetilde{d}_s \omega_{sk'}\right)
\end{aligned}
$$

such that the dispersion of $\widetilde{d}$ relative to the dispersion of $\left(1 - \overline{\lambda}\right) d$ is higher when

$\mathbb{COV}(\lambda_{k'}, \omega_{k'k})$ is more dispersed across sectors and when it is negatively correlated with d. In words, the dispersion of $\widetilde{d}$ is high when the intensity of use of intermediate inputs of the most flexible sectors is highly unequal across supplying sectors, and when sectors that are large suppliers of intermediate inputs are also large suppliers of the most flexible sectors. Similarly, the dispersion of $\widetilde{q}$ relative to $\left(1-\overline{\lambda}\right)^2 q$ is higher when there is higher dispersion of $\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k\omega_{k'k}\right)$, the last term on the right-hand side of the expression for $\widetilde{q}_k$, and there is negative covariance between them and q.

The second and the third line on the right-hand side of the lower bound for v_c in Lemma 4 captures new effects. In particular, volatility of GDP is higher when $\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) < 0$. That is, if sectors with high $\widetilde{d}_k$ are the most flexible sectors (second line), and if Jensen's inequality is stronger (third line).

Analyzing the rate of decay of v_c as $K \to \infty$ is now less straight-forward than in the case of $\delta = 0$ studied in Section 3.1. The reason is that the interaction of price rigidity and input-output linkages on GDP volatility is much intricate than its interaction with sectoral GDP shares.

Proposition 5 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, price rigidity is heterogeneous across sectors, the distribution of modified outdegrees $\left\{\widetilde{d}_k\right\}$ and modified second-order outdegrees $\{\widetilde{q}_k\}$ follow power-law distributions with respective power parameter $\widetilde{\beta}_d, \widetilde{\beta}_q > 1$, then*

$$v_c \geq \begin{cases} \frac{u_4}{K^{1/2}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\ \frac{u_4}{K^{1-1/\min\{\widetilde{\beta}_d,\widetilde{\beta}_q\}}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2) \end{cases}$$

where u_4 is a random variable independent of K and v.

Proof. Identical to the proof of Proposition 4: see Appendix C. ■

This proposition has similar message as Proposition 3 but in the context of production networks. If sectors with the most sticky (flexible) prices are also the most central in the price rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} > (<) \min\{\beta_d, \beta_q\}$, GDP volatility decays at a faster (slower) rate than when price rigidity is homogeneous across sectors or independent of network centrality. Also as in Section 3.1, regardless of the effect of price rigidity on the rate of decay of v_c as $K \to \infty$, price rigidity distorts the identity of the most important sectors driving GDP volatility through the network effect.

To close the section, we wrap results up in the following corollary.

Corollary 2 *In an economy characterized as a production network, sectoral heterogeneity of price rigidity may affect the scale of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

The details of the analysis are different from in Section 3.1, but the main message is the same. The inefficiency introduced by price rigidity shows up by dampening aggregate fluctuations, exactly as when shocks are aggregate, but changes the sectoral origin of aggregate fluctuations. Thus, when shocks are idiosyncratic, the inefficiency of price rigidity may even change the sign of cycles relative to a frictionless economy.

C. Relaxing simplifying assumptions

We now discuss the implications of relaxing some of the simplifying assumptions we make to get our results in Sections 3.1 and 3.2. For this we impose no restrictions on $\delta \in (0,1)$, steady-state sectoral GDP shares, Ω_c, or input-output linkages, Ω.

Non-linear disutility of labor. Appendix B contains details of the derivations. When $\varphi > 0$, wages are jointly determined by demand and supply such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}$$

is the log-linear version of (7). Thus, under the assumption that monetary policy imposes $c_t + p_t^c = 0$, it is not longer true that sectoral productivity shocks have no effect on sectoral wages. In the following we describe these effects one by one.

First, the log-linear version of the production function yields deviations of labor demand in a given sector k as:

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right),$$

so shocks in sector k have direct effects on labor demand in sector k and indirectly on labor demand of all other sectors via the effect of the sector k on the aggregate price index relevant for firms in each sector $\left\{ p_t^k \right\}_{k=1}^K$ (which depends on input-output linkages).

Second, aggregating demand for goods by households and firms implies that deviations in sectoral gross output depend on deviations in total gross output y_t and prices according to

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi)\, p_t^c - \psi \widetilde{p}_t \right] \right)$$

so shocks in sector k affect the share of sector k in total gross output to the extent that p_{kt} responds to these shocks. In turn, shocks to sector k affect gross output of all other sectors to the extent that p_{kt} affects the GDP deflator p_t^c and aggregate prices for intermediate goods $\widetilde{p}_t$:

$$\widetilde{p}_t = \sum_{k'=1}^{K} \zeta_{k'} p_{k't}$$

which uses as weights the steady-state shares of sectors in the aggregate production of intermediate inputs:

$$\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}$$

where $\{n_k\}_{k=1}^{\infty}$ in turn is the share of sectors in aggregate gross output (which coincides by construction with the measure of firms in each sector):

$$n_k = (1-\psi)\, \omega_{ck} + \psi \zeta_k \text{ for all } k = 1, ..., K,$$

with $\psi \equiv \frac{Z}{Y}$ being the fraction of total gross output used as intermediate input in steady state.

Third, deviations in aggregate gross output y_t can be decomposed into deviations of aggregate value-added c_t (GDP) and aggregate production of intermediate inputs, z_t,

$$y_t = (1-\psi)\, c_t + \psi z_t$$

such that z_t solves

$$z_t = (1+\Gamma_c)\, c_t + \Gamma_p \left(p_t^c - \widetilde{p}_t \right) - \Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't}$$

with $\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$. Thus, another channel in which sectoral productivity shocks affect labor demand is through their effects on the aggregate demand for intermediate inputs.

Overall, the solution for c_t is still given by (20) but the vector of multipliers χ is now defined as

$$\chi \equiv (\mathbb{I} - \Lambda) \left[\gamma_1 \mathbb{I} + \gamma_2 \aleph \iota'\right] \left[\mathbb{I} - \varphi \left[\gamma_3 \iota \Omega_c' + \gamma_4 \iota \vartheta' - \gamma_5 \iota'\right] (\mathbb{I} - \Lambda) - \gamma_6 \Omega' (\mathbb{I} - \Lambda)\right]^{-1} \Omega_c \quad (26)$$

with $\gamma_1 \equiv \frac{1+\varphi}{1+\delta\varphi}$, $\gamma_2 \equiv \frac{\psi(1-\delta)\Gamma_a}{1+\delta\varphi}$, $\gamma_3 \equiv \frac{(1-\delta)[(1-\psi)\eta-1]}{1+\delta\varphi}$, $\gamma_4 \equiv \frac{\psi(1-\delta)(\eta-\Gamma_p)}{1+\delta\varphi}$, $\gamma_5 \equiv \frac{\gamma_2}{\Gamma_a}$, $\gamma_6 \equiv \delta\gamma_1$, $\aleph \equiv (n_1, ..., n_K)'$, and $\vartheta = (\zeta_1, ..., \zeta_K)'$.

Relative to the solution of χ in (25), multipliers take a richer functional form, capturing all the new channels introduced through labor demand. However, as we show in the next section, it turns out that allowing for a finite labor supply elasticity ($\varphi > 0$) has little effect on our quantitative results.

Pricing frictions and monetary policy rule. Restoring the pricing friction *à la Calvo* adds serial correlation to the response of prices even when shocks are i.i.d.:

$$p_{kt} = (1 + \beta + \kappa_k)^{-1} \left[\kappa_k mc_{kt} + \beta \mathbb{E}\left[p_{kt+1}\right] + p_{kt-1}\right] \text{ for } k = 1, ..., K$$

where $\kappa_k \equiv (1 - \alpha_k)(1 - \beta\alpha_k)/\alpha_k$ denotes price flexibility.

In turn, a Taylor rule of the form

$$i_t = \phi_\pi^c \left(p_t^c - p_{t-1}^c\right) + \phi_c c_t$$

offsets some of the serial correlation introduced by price rigidity.

Serial correlation in price responses implies that log-linear deviations of GDP are now given by

$$c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}$$

so multipliers χ_k for $k = 1, ..., K$ must be redefined as

$$\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2} \quad (27)$$

such that $v_c = \|\chi\|_2 \, v$ still holds.

In our simplified model, multipliers capture both effect of shocks in sector k on c_t and on aggregate volatility v_c. This conclusion now no longer holds. We choose this definition for χ for a better comparison between our simplified model and our quantitative results that come next.

IV Quantitative results

This section shows results for a calibration of our model to a 348-sector U.S. economy.

A. Data and calibrations

This section describes the data we use to construct the input-output linkages, and the micro-pricing data we use to construct measures of price stickiness at the sectoral level.

Input-output linkages. The Bureau of Economic Analysis (BEA) produces Input-Output Tables detailing the dollar flows between all producers and purchasers in the US. Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the Input-Output Tables using Census data that are collected every five years. The BEA has published Input-Output tables every five years beginning in 1982 and ending with the most recent tables in 2012. The Input-Output tables are based on NAICS industry codes. Prior to 1997, the Input-Output Tables were based on SIC codes.

The Input-Output tables consist of two basic national-accounting tables: a "make" table and a "use" table. The make table shows the production of commodities by industry. Rows present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries adds up the output of a commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of "value added": compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column adds up to industry output.

We utilize the Input-Output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use both levels of aggregation to create industry-by-industry trade flows.

The BEA provides concordance tables between NAICS codes and Input-Output industry codes. We follow the BEA's Input-Output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as Input-Output codes. In some cases, an identical set of NAICS codes defines different Input-Output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix which details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity C that each industry k produces. We define the market share ($SHARE$) of industry k's production of commodity C as

$$SHARE = MAKE \odot (\mathbb{I} - MAKE)^{-1}_{k,k'}$$

We multiply the share and use tables (USE) to calculate the dollar amount that industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE$$

We then use the revenue share matrix to calculate the percentage of industry k' inputs purchased from industry k and label the resulting matrix $SUPPSHARE$:

$$SUPPSHARE = REVSHARE \odot \left((\mathbb{I} - MAKE)^{-1}_{k,k'}\right)' \tag{28}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model. The BEA also provides a direct-requirement. This table is a commodity-by-industry matrix, and the mapping to our theoretical model is therefore less straightforward. A commodity-by-commodity direct-requirements table would be an alternative to our approach of modeling input-output relations, but is not readily available. We report calibration results using direct requirements in the appendix for comparison with the literature (see, e.g., Acemoglu et al, 2012).

Price rigidity. We use the confidential microdata underlying the producer price data (PPI) from the BLS to calculate the frequency of price adjustment at the industry level.[10] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the US, the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about $25,000$ establishments for approximately $100,000$ individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price adjustment at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry level.

The overall mean monthly frequency of price adjustment is 22.15%, which implies an average duration, $-1/\log(1 - FPA)$, of 3.99 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 4.01% for the semiconductor manufacturing sector (duration of 24.43 months) to 93.75% for dairy production (duration

[10] The data have been used before in Nakamura and Steinsson (2008), Goldberg (2011), Gilchrist et al. (2015) among others.

of 0.36 months).

Calibration. The steady state input-output linkages captured by Ω in our model are calibrated to match the U.S. input-share matrix in 2002 computed as described above. The Calvo parameters are set to match the frequency of price changes between 2005 and 2011 also computed as described above. After we merge the input-output and the frequency of price adjustment data, we end up with 348 sectors. The input-output tables has 407 sectors in total. We miss some sectors mainly because of two reasons. The first is that there are some sectors that produce almost only final goods, so there are not enough observations of such goods in the PPI data to compute frequency. Second, there are some sectors whose goods are not traded in a formal market, so there is no price to record. Examples of missing sectors are 'video tape and disc rentals' (532230), 'bowling centers' (713950), 'military armored vehicle, tank, and tank component manufacturing' (336992), and 'religious organizations' (813100).

We show results for several calibrations of our model. $MODEL1$ has linear disutility of labor, $\varphi = 0$, and monetary policy targeting constant nominal GDP. This is the closest parametrization of our new-Keynesian model to the simple model studied in Section 3 with the modeling of the pricing friction as the only difference.[11] $MODEL2$ is a somewhat intermediate case where $\varphi = 0$, and monetary policy follows a Taylor rule specified in section 2 with parameters $\phi_c = .33/12 = .0275$ and $\phi_\pi = 1.34$. In $MODEL3$ monetary policy follows this same Taylor rule and the inverse-Frisch elasticity is set to $\varphi = 2$.

These calibrations are at a monthly frequency, so the discounting rate is $\beta = .9975$ (implying an annual risk-free interest rate of about 3%). The labor share $1 - \delta$ is assumed .5 and the elasticity of substitution across sectors is $\theta = 2$ and within sectors is $\eta = 6$.

B. Quantitative results

Power laws. We estimate the shape parameter of the power law distribution for various variables of interest following the procedure suggested by Gabaix and Ibragimov (2011). In particular, we compute the OLS estimator of the empirical log-counter

[11] We also calibrate our simplified model studied in Section 3 as $MODEL1$ interpreting the sectoral probability of setting prices after observing shocks by the sectoral frequency of price changes. Although the price friction assumed in the simplified model of Section 3 has no empirical counterpart in the frequency of price changes, our interest of doing so is to check the implications of the pricing friction *a la* Calvo. Results are remarkably similar to those of $MODEL1$.

cumulative distribution on the log sequence of the variable of interest using the data in the upper 20% tail.

Using all our sectoral data for 348 sectors, the shape parameter of sectoral GDP is .8859 (st dev .1497). For the whole sample, with 407 sectors, the shape parameter of sectoral GDP is 1.021 (st dev .1604). The exclusion of some sectors turns out to enhance our estimation of the granular effect due to the distribution of sectoral GDP, at least with frictionless prices. In any case, in our data the shape parameter of sectoral GDP is always significantly different from 0 and 2, and not significantly different from 1 at a 5% significance level. As a robustness check, we report some results below for a sample of 345 sectors which excludes the three sectors with the largest GDP share: "Retail trade" ($4A0000$), "Real Estate" (531000) and "Wholesale trade" (420000). The shape parameter of this adjusted sample is 1.006 (standard deviation .1713).

In turn, the shape parameter of the outdegrees in our 348 sector sample is 1.5676 (st dev .2797) while Acemoglu et al. (2012) report 1.4559 (st dev .2461) for 416 sectors. In addition to including a different number, they treat the input-output linkages in a different way. Our shape parameters for outdegrees are not statistically different from theirs at 5% significance level. Our estimates are also significantly different from 0 and 2. The same result holds for the second-order outdegrees, with our estimate of the shape parameter being 1.2834 (st dev .2017) while Acemoglu et al. (2012) report 1.3019 (st dev .2201).

Combining data for both sectoral GDP and input linkages, the shape parameter of sectoral gross output (value added plus intermediate inputs) is estimated to be 1.0603 (st dev .1792). This estimate is quite similar to the shape parameter of firm sales in the U.S. economy of 1.054 reported by Gabaix (2011). Gross output in our sectoral data conceptually is the closest counterpart to sectoral total sales in Gabaix (2011)..

Turning to price rigidity, the shape parameter of the sectoral distribution of frequency of price changes is 2.5773 (st dev .4050), so this distribution is not fat-tailed.

Correlations. The correlation between the frequency of price changes with sectoral GDP $\{C_k\}_{k=1}^{K}$, outdegrees d and second-order outdegrees q respectively is 5.1%, 18.8%, 22.2% for the whole sample and 6.7%, 22.6%,and 33.3% in their 20% upper tail. The correlation between the degree of price flexibility of sectors and their GDP is low, but we will see that there is still an interesting interaction between them. In addition, although the correlation between sectoral price flexibility and our measures of sectoral centrality in the production network is much higher than with sectoral GDP, our simple model studied in Section 3 suggests that correlation is a statistic that does not fully capture the intrincate interaction between the pricing friction and the input-output structure of the economy.

Multipliers. The picture that emerges so far is that the sectoral distribution of frequency of price adjustment is not really fat-tailed and there is a stronger interaction between price rigidity and centrality in the production network than between price rigidity and sector size. We show next this still leads to economically important effects. Table 1 reports results under alternative calibrations for the multiplier of volatility of sectoral productivity shocks for GDP volatility, $\|\chi\|$, with χ defined as in (27), both in levels and relative to the multiplier of volatility of aggregate productivity shocks on GDP volatility (called the "aggregate multiplier"). The former gives information about the effect of the pricing friction on the level of GDP volatility due to sectoral productivity shocks. The latter controls by the effect of the friction on aggregate volatility generated by an aggregate shock. It provides information about the rate of decay of the multiplier of sectoral productivity shocks as the economy becomes increasingly more disaggregated.

We start by reporting results for a specification of our model labeled as $MODEL1$. This specification has the same specification as the simplified model in section 3 except for the modeling of the pricing friction (i.e., it features Calvo price stickiness, a monetary policy targeting constant nominal GDP, and linear disutility of labor). We first focus on the assumption that prices are frictionless (first column of Table 1), so we can isolate the quantitative strength of the granular effect due to the empirical distribution of sectoral GDP, the network effect from the empirical input-output structure of the U.S. economy, and their interaction. The multiplier is .0538 when all sectors are assumed to have equal sectoral GDP and uniform input-output linkages among them. As suggested by our analytical model in Section 3, this multiplier equals $K^{-1/2}$ for $K = 348$ and it is 5.38% of the aggregate multiplier, which is just 1. When we set $\delta = 0$ (no intermediate inputs) and calibrate steady-state sectoral shares of GDP in the model, captured by Ω_c, to the empirical GDP shares in 2002, the multiplier is now .1994 (19.94% of the aggregate multiplier), so GDP volatility is multiplied by a factor of 4 relative to the case when sectoral GDP shares are calibrated to be uniform. This result shows the strong granular effect due to the distribution of sectoral GDP. This strong effect is somewhat mitigated when we allow for intermediate inputs ($\delta = .5$) but require that input-output linkages in steady state, captured by Ω, are assumed homogeneous across sectors. In this case, the multiplier is .11 (11% of the aggregate multiplier). Next we shut down the granular effect (GDP shares are equal for all sectors) and turn to the network effect. We calibrate Ω to match the U.S. input-output tables in 2002. The multiplier is only .0795 (7.95% of the aggregate multiplier) showing that the network effect has less quantitative strength than the granular effect of GDP shares. In turn, when we calibrate both Ω_c and Ω to respectively match sectoral GDP shares and the input-output data, the multiplier is .1745 (17.45% of the aggregate multiplier). Therefore, the granular and the network effects add up to generate large GDP volatility from idiosyncratic shocks for $K = 348$ sectors which suggests that the rate of decay of GDP volatility is slow as the economy becomes increasingly more disaggregated.

The second column in Table 1 reports results when price rigidity is homogeneous across sectors: the Calvo parameter in all sectors is calibrated to match the average frequency of price changes in the PPI data. As predicted by our simple model of section 3, the level of GDP volatility generated by sectoral shocks is smaller than when prices are flexible. However, in relative terms to the multiplier due to an aggregate productivity shock, results are not much different from those presented above with two exceptions: (1) introducing homogeneity in input-output linkages now has less power for offsetting the granular effect than when prices are flexible: the multiplier goes from 11% to 16.4% relative to the multiplier of an aggregate productivity shock; and (2) input-output linkages calibrated to the U.S. economy have now an even weaker impact on generating GDP volatility from sectoral productivity shocks relative to aggregate productivity shocks (going from 7.95% to 6.07%). Our analysis in Section 3 suggests that the reason for these results is that the pricing friction more strongly mitigates the network effect of second-order outdegrees than of outdegrees. Since the distribution of second-order outdegrees is more fat-tailed than outdegrees, even a homogeneous price friction reduces the quantitative strength of the network effect.

We now turn to show results for $MODEL1$ once sectoral Calvo parameters are calibrated to match the sectoral frequency of price changes in the U.S. data. These are the main results of this section, which quantitatively confirm our analysis using analytical model in Section 3. First, sectoral heterogeneity of the price rigidity increases the magnitude of the multiplier of sectoral productivity shocks in all cases relative to the case when the pricing friction is homogeneous (compare the third and second columns in Table 1 for $MODEL1$). Second, heterogeneity of price rigidity by itself increases the relative multiplier of sectoral shocks on GDP volatility: The multiplier goes from 5.36% to 10.77% when sectoral GDP shares and input-output linkages are homogeneous. Note that the pricing friction increases the multiplier by more than the network effect. Thus, heterogeneous price rigidity creates a "frictional" source of aggregate volatility independent of the "granular" or "network" sources already described in the literature. Third, the interaction between the granular effect of GDP shares and the frictional effect is quite strong in spite of the fact that the correlation between sectoral GDP and frequency of price changes is quite mild in the data. If $\delta = 0$ (no intermediate inputs), the multiplier is 24.87% of the aggregate multiplier instead of 19.94% when prices rigidity is equal for all sectors. If $\delta = .5$ and input-output linkages are homogeneous, the relative multiplier goes to 22.77% relative – while in a frictionless economy it is only 11% and for homogeneous price rigidity it is 16.4%. Fourth, there is also a strong interaction between the network effect and the frictional effect since the relative multiplier is 11.52% – while with frictionless prices it is 7.95% and with homogeneous price rigidity is 6.07%. Overall, when our model is calibrated to replicate the sectoral GDP shares and input-output tables in the U.S. economy, the relative multiplier of sectoral productivity shocks on aggregate volatility is 23.95% which is a sizable increase relative to the 17.45% in a frictionless economy.

Table 1 reports very similar results for another two specifications of our model: $MODEL2$ assumes a standard Taylor rule instead of a monetary policy targeting constant nominal GDP while $MODEL3$ has a Taylor rule and additionally drops the assumption of linear disutility of labor. The level of the multipliers are different but not so much relative to the multiplier of an aggregate productivity shock. The main observation is that the effect of heterogeneous price rigidity on relative multipliers is less strong in $MODEL2$ than $MODEL3$ where multipliers are in turn less strong than $MODEL1$. We then conclude that a Taylor rule for monetary policy somewhat mitigates, and the

Frisch elasticity $\varphi = 2$ somewhat amplifies the effect of price rigidity on the relative multiplier of volatility of sectoral productivity shocks on GDP volatility.

Table 2 also reports multipliers in levels and relative to the aggregate multiplier for the same cases but taking into account only the impact effect of sectoral shocks on GDP. Numbers are slightly different from those reported in Table 1. This finding suggests the small role of persistence introduced by the Calvo assumption relative to the simple specification used in the simplified model of Section 3.

Table 3 presents the same results than Table 1 after excluding the three sectors with the largest GDP: "Retail trade" ($4A0000$), "Real Estate" (531000) and "Wholesale trade" (420000). This way the shape parameter of the sectoral GDP distribution used for our calibrations is very close to its shape parameter using the whole sample of 407 sectors, as reported above in this section. The quantitative strength of the granular effect of the GDP distribution of course is somewhat diminished, but it is still strong by contributing half the amplification of the multiplier of sectoral shocks on GDP volatility when prices are assumed flexible; the other half is accounted for by the network effect. However, all our key results obtained in Table 1 regarding the introduction of the price rigidity remain unchanged.

Sectoral sources of aggregate fluctuations. From a different angle, Table 4 reports the identity and the relative contribution of the 5 most important sectors on the multiplier in the different calibrations of $MODEL1$.[12] Results are very similar either using $MODEL2$ or $MODEL3$. In a nutshell, the sectoral heterogeneity of price rigidity distorts the identity and the relative contribution of the most important sectors for aggregate fluctuations generated from sectoral productivity shocks.

To get column (1) Calvo parameters are calibrated to match sectoral frequency of price changes in U.S. data while sectoral GDP shares and input-output linkages are assumed homogeneous. The most important sectors are those with the most flexible prices, which are mostly farming products: "Dairy cattle and milk production" (112120), "All other crop farming" ($1119B0$), "Cattle ranching and farming" ($1121A0$), "Primary smelting and refining of copper" (331411), "Poultry and egg production" (112300) with a contribution ranging from 6.05% to 4.59% of the multiplier of sectoral productivity

[12] The relative contribution of all sectors sum one.

shocks on GDP volatility in our model. If all sectors would have perfectly symmetrical, the contribution of a sector would be .29% (i.e., 348^{-1}).

Comparing Columns (2) and (3) in Table 4 gives information about the distortion of price rigidity on the granular effect of the empirical distribution of sectoral GDP. The calibration to get column (2) assumes flexible prices and steady-state sectoral GDP shares that match GDP shares in the data while for column (3) the calibration additionally matches the sectoral frequency of price changes. The three most important sectors in both calibrations remain the same but not quite so their relative contribution: With flexible prices, the relative contribution of "Retail trade" ($4A0000$), "Real estate" (531000), "Wholesale trade" (420000) respectively is 22.58%, 21.3% and 18.42%; once the price rigidity is introduced, their relative contribution respectively is 11.86%, 17.31% and 36.13%. In words, the contribution of "Wholesale trade" (420000) doubles while the contribution of "Retail trade" ($4A0000$) is cut by half. The fourth most important sector, "Monetary authorities and depository credit intermediation" ($52A000$), has a relative contribution that increases from 4.92% when prices are assumed frictionless to 10.93% once the pricing friction is introduced. In turn, the fifth most important sectors changes its identity: when prices are flexible, it is "Offices of physicians, dentists, and other health practitioners" ($621A00$) with 3.6% while with the pricing friction it is "Telecommunications" (517000) contributing with a 8.47% of the multiplier of sectoral productivity shocks on GDP volatility.

Next we compare columns (4) and (5) in Table 4 to show the distortion of price rigidity on the network effect of the input-output structure of the US economy. Column (4) assumes flexible prices and steady-state input-output linkages calibrated to match U.S. input-output tables while in column (5) the model additionally matches sectoral frequency of price changes. Now the identity of the five most important sectors is completely different in both calibrations. Without frictions, the most important sector by far is "Wholesale trade" (420000), with a contribution of 25.22%, followed by far by "Real estate" (531000), "Monetary authorities and depository credit intermediation" ($52A000$), "Electric power generation, transmission, and distribution" (221100), and "Petroleum refineries" (324110) with respective relative contributions of 9.44%, 3.59%, 3.42% and 2.87%. Once the pricing friction is added in, the contribution of the five most important sectors ranges from 6.65% to 5.5% which in descending order are "Petroleum refineries" (324110), "Electric power generation, transmission, and distribution" (221100), "Cattle ranching and farming" (1121A0), "All other crop farming" (1119B0) and "Primary smelting and refining of copper" (331411). In short, energy sectors become the most important followed by farming sectors.

Finally, we compare columns (6) and (7) in Table 4 reporting the five most important sectors with and without the pricing friction when steady state the sectoral GDP shares are calibrated to match sectoral GDP shares in U.S. data and steady-state input-output linkages match the input-output tables. The relative contribution of the most important sector when prices are flexible, "Real estate" (531000), decreases from 33.92% to 19.32% when prices are rigid while the contribution of the second-most important sector when prices are flexible, "Wholesale trade" (420000), increases from 16.71% to 32.76% when prices are rigid. The identity of the following three most important sectors remains unchanged; only their order and relative contribution change when prices are rigid. They are "Monetary authorities and depository credit intermediation" (52A000), "Retail trade" (4A0000) and "Telecommunications" (517000) respectively with a 12.09%, 9.56% and 9.04% when prices are rigid.

V Concluding remarks

This paper studies the distortionary effect of nominal price rigidity when microeconomic shocks lead to aggregate fluctuations. We do so theoretically and quantitatively in the context of a multi-sector new-Keynesian model where production needs intermediate inputs. The model is calibrated as a production network with 348 sectors that replicates in steady state the distribution of sectoral GDP and input-output linkages in the data as well as the sectoral frequency of price changes outside the steady state.

Using a simplified model, we show analytically that the aggregate propagation of a sectoral productivity shock depends on the sector' size (measured by its GDP), its centrality in the production network, and the distribution of price rigidity among sectors. In particular, a shock to a sector has stronger aggregate effects when it hits a large/central sector with highly flexible prices that sells to large/central sectors with highly flexible prices. We find conditions due to which the interaction between the frictional, granular and network sources may amplify or mitigate the scale of aggregate fluctuations from microeconomic shocks. We also show that the pricing friction changes the identity and relative contribution of the most important sectors driving aggregate fluctuations. Thus, price rigidity not only generates aggregate inertia, as standard when shocks are aggregate, but may also distorts the sign of aggregate fluctuations given the idiosyncratic nature of microeconomic shocks.

Quantitatively, the friction alone creates sizable effects of microeconomic shocks on GDP volatility. Thus, there is a "frictional" origin for aggregate fluctuations that is conceptually different from the granular or network mechanisms already described in the literature. The friction also has a strong impact on increasing the multiplier of idiosyncratic shocks on aggregate volatility due to the granular and network mechanisms. It also plays an important role determining which sectors drive aggregate fluctuations, especially those generated by the production network.

VI References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations," Econometrica, 80:5, 1977-2016.

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2016. "Networks and the Macroeconomy: An Empirical Exploration," NBER Macroeconomics Annual 2016, 276 - 335.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2016. "Microeconomic Origins of Macroeconomic Tail Risks," American Economic Review, forthcoming.

Bigio, Saki and Jeniffer La'O. 2016. "Financial Frictions in Production Networks," NBER Working Paper Series No. 22212.

Bils, Mark and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices," Journal of Political Economy 112:5, 947-985.

Boivin, Jean, Mark Giannoni and Ilian Mihov. 2009. "Sticky Prices and Monetary Policy: Evidence from Disaggregated U.S. Data," American Economic Review, 99:1, 350-384.

Atalay, Enghin. 2015. "How Important Are Sectoral Shocks?," mimeo.

Baqaee, David. 2016. "Cascading Failures in Production Networks," mimeo.

Basu, Susanto. 1995. "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," American Economic Review, 85:3, 512-531.

Caliendo, Lorenzo, Fernando Parro, Esteban Rossi-Hansberg and Pierre-Daniel Sarte. 2016. "The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy," mimeo.

Carvalho, Carlos and Jae Won Lee. 2011. "Sectoral Price Facts in a Sticky-Price Model," mimeo.

Carvalho, Vasco and Xavier Gabaix. 2013. "The Great Diversification and Its Undoing," American Economic Review 103:5, 1697-1727.

Carvalho, Vasco. 2015. "From Micro to Macro via Production Networks," Journal of Economic Perspectives 28:4, 23-48.

Carvalho, Vasco and Basile Grassi. 2016. "Large Firms Dynamics and the Business Cycle," mimeo.

Di Giovanni, Julian, Andrei Levchenko and Isabelle Mejean. 2014. "Firms, Destinations, and Aggregate Fluctuations," Econometrica, 82:4, 1303-1340.

Di Giovanni, Julian, Andrei Levchenko and Isabelle Mejean. 2016. "The Micro Origins of International Business Cycle Comovement," mimeo.

Dupor, William. 1999. "Aggregation and Irrelevance in Multi-Sector Models," Journal of Monetary Economics, 43:2, 391-409.

Durret, Richard. 2013. "Probability: Theory and Examples," Edition 4.1, Edition 4 published by Cambridge University Press in 2010. `https://services.math.duke.edu/~rtd/PTE/PTE4_1.pdf`

Foerster, Andrew, Pierre-Daniel Sarte and Mark W. Watson. 2011. "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production," Journal of Political Economy 119:1, 1-38.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations," Econometrica, 79:3, 733-772.

Hall, Robert E. 2009. "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," Journal of Political Economy, 117:2, 281–323.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks," Review of Economic Dynamics, 1:4, 781-808.

Horvath, Michael. 2000. "Sectoral shocks and Aggregate Fluctuations," Journal of Monetary Economics, 45:1, 69-106.

Klenow, Peter and Oleksiy Kryvtsov. 2008. "State-Dependent or Time-Dependent

Pricing: Does It Matter for Recent U.S. Inflation?," Quarterly Journal of Economics 123, 863-904.

Long, John and Charles I. Plosser. 1983. "Real Business Cycles," Journal of Political Economy 91, 39-69.

Lucas, Robert E. Jr. 1977. "Understanding Business Cycles," Carnegie-Rochester Conference Series on Public Policy 5, 7-29.

Nakamura, Emi and Jon Steinsson. 2010. "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," Quarterly Journal of Economics, 125:3, 961-1013.

Pasten, Ernesto, Raphael Schoenle and Michael Weber. 2016. "The Propagation of Monetary Policy Shocks in a heterogeneous Production Economy," mimeo.

Appendix A: Steady state solution and log-linear system

A.1 Steady state solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms where

$$W_k = W,\ Y_{jk} = Y,\ L_{jk} = L,\ Z_{jk} = Z \text{ and } P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (1), (2), (10) and (13),

$$\begin{aligned}
C_k &= \omega_{ck} C, \\
C_{jk} &= \frac{1}{n_k} C_k, \\
Z_{jk}(k') &= \omega_{kk'} Z, \\
Z_{jk}(j', k') &= \frac{1}{n_{k'}} Z_{jk}(k').
\end{aligned}$$

such that the vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$\begin{aligned}
C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\
Z_{jk} &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}(j', k')\, dj' = Z
\end{aligned}$$

From Walras law in (19) and symmetry across firms, it holds

$$Y = C + Z. \tag{29}$$

Walras law and results above yield, for all j, k,

$$
\begin{aligned}
Y_{jk} &= C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}(j', k')\, dj' \\
Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z
\end{aligned}
$$

so $\aleph$ satisfies

$$
\begin{aligned}
n_k &= \psi \omega_{ck} + (1-\psi) \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \\
\aleph &= (1-\psi) \left[I - \psi \Omega' \right]^{-1} \Omega_c
\end{aligned}
$$

for $\psi \equiv \frac{Z}{Y}$. Note that by construction $\aleph' \iota = 1$, which must hold given that the total measure of firms is one.

Steady-state labor supply from (7) is

$$
\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.
$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exist if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$
\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{30}
$$

In turn, households' budget constraint, firms' profits, production function, efficiency of production (from equation (15)) and optimal prices in steady state respectively are

$$
CP = WL + \Pi \tag{31}
$$

$$
\Pi = PY - WL - PZ \tag{32}
$$

$$
Y = L^{1-\delta} Z^{\delta} \tag{33}
$$

$$
\delta WL = (1-\delta) PZ \tag{34}
$$

$$
P = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{35}
$$

for $\xi \equiv \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}$. Equation (35) solves

$$\frac{W}{P} = \left(\frac{\theta-1}{\theta\xi}\right)^{\frac{1}{1-\delta}}. \tag{36}$$

This latter result together with equations (32), (33) and (34) solve

$$\frac{\Pi}{P} = \frac{1}{\theta}Y.$$

Plugging this result in (31) and using (29) yields

$$\begin{aligned} C &= \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]Y, \\ Z &= \delta\left(\frac{\theta-1}{\theta}\right)Y \end{aligned} \tag{37}$$

such that $\psi \equiv \delta\left(\frac{\theta-1}{\theta}\right)$. This result and (34) solves

$$L = \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\delta}{1-\delta}} Y$$

where Y solves from this latter result, (30), (37) and (36):

$$Y = \left(\frac{\theta-1}{\theta\xi}\right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{\delta\varphi}{(1-\delta)(\sigma+\varphi)}} \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

A.2 Log-linear system

Aggregation:

Aggregate and sectoral consumption (interpreted as value-added) given by (1), (2) are

$$\begin{aligned} c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\ c_{kt} &= \frac{1}{n_k}\int_{\Im_k} c_{jkt}\,dj \end{aligned} \tag{38}$$

Aggregate and sectoral production of intermediate inputs are given by

$$
\begin{aligned}
z_t &= \sum_{k=1}^{K} n_k z_{kt}, && (39)\\
z_{kt} &= \frac{1}{n_k}\int_{\Im_k} z_{jkt}dj
\end{aligned}
$$

where (10) and (13) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r}\int_{\Im_r} z_{jk}(j', r)\, dj'$.

Sectoral and aggregate prices are given from (4), (6), and (12),

$$
\begin{aligned}
p_{kt} &= \int_{\Im_k} p_{jk}dj \text{ for } k = 1, ..., K\\
p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt},\\
p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}.
\end{aligned}
$$

Aggregation of labor is

$$
\begin{aligned}
l_t &= \sum_{k=1}^{K} l_{kt}, && (40)\\
l_{kt} &= \int_{\Im_k} l_{jkt}dj.
\end{aligned}
$$

Demands:

Households' demand for sectors and goods in (3) and (5) for all $k = 1, ..., K$ become

$$
\begin{aligned}
c_{kt} - c_t &= \eta\left(p_t^c - p_{kt}\right), && (41)\\
c_{jkt} - c_{kt} &= \theta\left(p_{kt} - p_{jkt}\right)
\end{aligned}
$$

In turn, firm jk's demand for sectors and goods in (11) and (14) for all $k, r = 1, ..., K$,

$$
\begin{aligned}
z_{jkt}(k') - z_{jkt} &= \eta\left(p_t^k - p_{k't}\right), && (42)\\
z_{jkt}(j', k') - z_{jkt}(k') &= \theta\left(p_{k't} - p_{j'k't}\right).
\end{aligned}
$$

Firms' gross output satisfies Walras' law

$$y_{jkt} = (1-\psi)\, c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't}\,(j,k)\, dj' \tag{43}$$

Total gross output solves from the aggregation of (19),

$$y_t = (1-\psi)\, c_t + \psi z_t. \tag{44}$$

IS and labor supply:

Households' Euler equation in (8) becomes

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left\{i_t - \left(\mathbb{E}_t\left[p^c_{t+1}\right] - p_t\right)\right\}$$

Labor supply condition in (7) is

$$w_{kt} - p^c_t = \varphi l_{kt} + \sigma c_t \tag{45}$$

Firms:

Production function:

$$y_{jkt} = a_{kt} + (1-\delta)\, l_{jkt} + \delta z_{jkt} \tag{46}$$

Efficiency condition:

$$w_{kt} - p^k_t = z_{jkt} - l_{jkt} \tag{47}$$

Marginal costs:

$$mc_{kt} = (1-\delta)\, w_{kt} + \delta p^k_t - a_{kt} \tag{48}$$

Optimal reset price:

$$p^*_{kt} = (1-\alpha_k\beta)\, mc_{kt} + \alpha_k\beta\mathbb{E}_t\left[p^*_{kt+1}\right]$$

Sectoral prices:

$$p_{kt} = (1-\alpha_k)\, p^*_{kt} + \alpha_k p_{kt-1}$$

Taylor rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

Appendix B: Solution of equations (25) and (26) in Section 3

B.1. Solution equation (25)

Setting $\sigma = 1$ and $\varphi = 0$ in (45) yields

$$w_{kt} = c_t + p_t^c = 0$$

where the last equality is due to the assumed monetary policy rule, so (48) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}$$

Here sectoral prices for all $k = 1, ..., K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k)\, mc_{kt} \\ &= \delta\,(1 - \lambda_k)\, p_t^k - (1 - \lambda_k)\, a_{kt} \end{aligned}$$

which solve in matrix form

$$p_t = -\left[\mathbb{I} - \delta\,(\mathbb{I} - \Lambda)\,\Omega\right]^{-1} (\mathbb{I} - \Lambda)\, a_t$$

where $p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ in its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies that $c_t = -p_t^c$, so

$$c_t = (\mathbb{I} - \Lambda')\left[\mathbb{I} - \delta\,(\mathbb{I} - \Lambda')\,\Omega'\right]^{-1} \Omega_c' a_t.$$

B.2 Solution equation (26)

Setting $\sigma = 1$ and $\varphi > 0$ in (45) yields

$$w_{kt} = \varphi l_{kt}^s + c_t + p_t^c = \varphi l_{kt}^d$$

where the last equality is due to the assumed monetary policy rule.

Labor demand is obtained from the production function in (46), the efficiency condition for production in (47) and aggregation of labor in (40) is

$$l_{kt}^{d} = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right)$$

where y_{kt} solves from (38), (39), (41), (42), and (43)

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi) \, p_t^c + \psi \sum_{k=1}^{K} n_k p_t^k \right] \right)$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^{K} n_k p_t^k = \sum_{k=1}^{K} \zeta_k p_{kt}$$

with $\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}$.

To solve for y_t, we use (39), (40), (44) and $y_t = \sum_{k=1}^{K} \int_{\Im_k} y_{jkt} dj$ to get

$$y_t = c_t + \psi \left[\Gamma_c c_t - \Gamma_p \left(\widetilde{p}_t - p_t^c \right) - \Gamma_a \sum_{k=1}^{K} n_k a_{kt} \right]$$

where $\Gamma_c \equiv \frac{(1-\delta)(1+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}$.

Putting together all these equations, sectoral wages solve

$$w_{kt} = \frac{\varphi}{1+\delta\varphi} \left[\begin{array}{c} (1+\psi\Gamma_c) \, c_t - a_{kt} - \psi \Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't} \\ \left[(1-\psi) \, \eta + \psi \Gamma_p \right] p_t^c + \psi \left(\eta - \Gamma_p \right) \widetilde{p}_t + \delta p_t^k - \eta p_{kt} \end{array} \right]$$

With this expression, equation (26) solves following the same steps than the solution of equation (25).

Appendix C: Proofs

Most proofs below are modifications of the argument in Gabaix (2011), Proposition 2, which heavily relies on the Lévy's Theorem (as in Theorem 3.7.2 in Durrett, 2013, p. 138)

Theorem 5 (Lévy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

(i) $\lim_{x\to\infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0,1)$

(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L(tx)/L(x) = 1$.

Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \leq 1/K\} \text{ and } b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1| \leq a_K}\right] \tag{49}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$ *where* u *has a nondegenerated distribution.*

Proof of Proposition 1. When $\delta = 0$ and $\lambda_k = \lambda$ for all k,

$$\|\chi\|_2 = \frac{1-\lambda}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}\sum_{k=1}^{K} C_k^2}$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2}\|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}[C_k^2]}}{\mathbb{E}[C_k]}.$$

In contrast, when $\beta_c \in (1,2)$ only the first moment exists. In such cases, by the Lévy's theorem,

$$K^{-2/\beta_c}\sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2$$

where u_0^2 is a random variable following a Lévy's distribution with exponent $\beta_c/2$ since $\Pr[C_k^2 > x] = x_0^{\beta} x^{-\beta_c/2}$. Thus,

$$K^{1-1/\beta_c}\|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}[C_k]}.$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by the Lévy's theorem,

$$\left(\overline{C}_k - K\log K\right) \xrightarrow{d} g$$

where g is a random variable following a Lévy's distribution.

Since the second moment is equivalent to the result above,

$$(\log K)\,\|\chi\|_2 \xrightarrow{d} u'$$

Proof of Proposition 2. Let λ_k and C_k be two independent random variables distributed as specified in the Proposition, the counter-cumulative distribution of $z_k = (1-\lambda_k)\,C_k$ is given by

$$f_Z(z) = \int_z^{z/y_0} f_{C_k}(z/y)\, f_{1-\lambda_k}(y)\, dy$$

which solves as Pareto distribution with shape parameter β_c.The proof of the Proposition then follows exactly as the proof of Proposition 1 for

$$\|\chi\|_2 = \frac{1}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}z_k^2} \tag{50}$$

Proof of Proposition 3. Given the relationship between λ_k and C_k specified in the Proposition, $Z_k = (1-\lambda_k)\,C_k = \phi C_k^{1+\mu}$. When $\mu < 0$, Z_k is distributed Pareto with shape parameter $\beta_c/(1+\mu)$. Proceeding similarly to the proof of Proposition 1, when $\beta_c \geq \max\{1, 2(1+\mu)\}$, both $\mathbb{E}[Z_k^2]$ and $\mathbb{E}[C_k]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, 2(1+\mu))$, $\mathbb{E}[C_k]$ exist but $\mathbb{E}[Z_k^2]$.does not. Applying Lévy's theorem,

$$K^{-2(1+\mu)/\beta_c}\sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2$$

Thus, $v_c \sim \frac{u_1}{K^{1-(1+\mu)/\beta}}v$.

When $\beta_c = 1$, this last result also holds. But now $\mathbb{E}[C_k]$ does not exist. As in Proposition 1, $\left(\frac{1}{K}\sum_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$. Thus, if $\mu \in [-1/2, 0]$, $v_c \sim \frac{u_2}{K^{-\mu}\log K}v$ while if $\mu \in (-1, -1/2)$, $v_c \sim \frac{u_2}{K^{1/2}\log K}v$. The proposition for $\mu < 0$ if then obtained by rearranging terms.

When $\mu > 0$, Z_k is distributed piece-wise Pareto such that

$$\Pr\left[Z_k \geq z\right] = \begin{cases} x_0^{\beta_c} z^{-\beta_c} & \text{for } z > \phi^{-2/\mu} \\ z_0^{-\beta_c/(1+\mu)} z^{-\beta_c/(1+\mu)} & \text{for } z \in \left[z_0^2, \phi^{-2/\mu}\right] \end{cases}$$

We now follow the same steps in the proof of Proposition 1.When $\beta_c > 2(1+\mu)$, $\mathbb{E}\left[Z_k^2\right]$ and $\mathbb{E}\left[C_k\right]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, 2(1+\mu))$, $\mathbb{E}\left[C_k\right]$ exist but $\mathbb{E}\left[Z_k^2\right]$.does not . Applying Lévy's theorem,

$$\frac{1}{a_K}\sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2$$

where

$$a_K = \begin{cases} x_0^2 K^{2/\beta_c} & \text{for } K > K^* \\ z_0^2 K^{2(1+\mu)/\beta_c} & \text{for } K \leq K^* \end{cases}$$

for $K^* \equiv x_0^{-\beta_c}\phi^{-\beta_c/\mu}$. Thus, $v_c \sim \frac{u_1}{K^{1-\frac{1+1\{K\leq K^*\}\mu}{\beta_c}}} v$ for some random variable u_1.

When $\beta_c = 1$, $\left(\frac{1}{K}\sum\limits_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$, so now $v_c \sim \frac{u_2}{K^{-1\{K\leq K^*\}\mu}\log K} v$ for some random variable u_2, completing the proof.

Proof of Proposition 4. When $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know from (**??**) that

$$\|\chi\|_2 \geq \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}\left[1 + (1-\lambda)\,\delta d_k + (1-\lambda)^2\,\delta^2 q_k\right]^2}$$

If $\{d_k\}$ and $\{q_k\}$ are independent random variables, we can ignore cross-terms to get the rate of decay of the expression on the right-hand size of this inequality. Following the same argument than in Proposition 2,

$$K^{-2/\beta_d}\sum_{k=1}^{K} d_k^2 \longrightarrow u_d^2,$$
$$K^{-2/\beta_q}\sum_{k=1}^{K} q_k^2 \longrightarrow u_q^2,$$

where u_d^2 and u_q^2 respectively are random variables following Lévy's distribution with exponent $\beta_d/2$ and $\beta_q/2$. Thus,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v$$

where u_3^2 is a random variable following a Lévy's distribution with exponent $\min\{\beta_d, \beta_q\}/2$.

Appendix D: Tables

	flex prices		hom. rigidity		het rigidity	
$MODEL1:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0028	(5.36%)	.0052	(10.77%)
het Ω_c, $\delta = 0$	.1994	(19.94%)	.0167	(19.94%)	.0249	(24.87%)
het Ω_c, hom Ω	.1100	(11.00%)	.0085	(16.40%)	.0126	(22.77%)
hom Ω_c, het Ω	.0795	(7.95%)	.0032	(6.07%)	.0063	(11.52%)
het Ω_c, het Ω	.1745	(17.45%)	.0098	(18.86%)	.0143	(23.95%)
$MODEL2:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0050	(5.36%)	.0060	(7.54%)
het Ω_c, $\delta = 0$	.1994	(19.94%)	.0343	(19.94%)	.0432	(25.96%)
het Ω_c, hom Ω	.1100	(11.00%)	.0138	(14.81%)	.0177	(18.66%)
hom Ω_c, het Ω	.0795	(7.95%)	.0059	(6.38%)	.0077	(8.62%)
het Ω_c, het Ω	.1745	(17.45%)	.0171	(18.40%)	.0219	(21.45%)
$MODEL3:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0048	(5.36%)	.0076	(8.26%)
het Ω_c, $\delta = 0$	.1994	(19.94%)	.0401	(19.94%)	.0517	(25.78%)
het Ω_c, hom Ω	.1135	(11.00%)	.0137	(15.40%)	.0196	(18.72%)
hom Ω_c, het Ω	.0775	(7.95%)	.0056	(6.23%)	.0093	(9.09%)
het Ω_c, het Ω	.1755	(17.45%)	.0166	(18.55%)	.0234	(21.10%)

Table 1 – Multiplier of sectoral productivity shocks on GDP volatility, $\|\chi\|$ In parenthesis, $\|\chi\|$ relative of the multiplier of aggregate productivity shocks on GDP volatility using $MODEL1$, $MODEL2$ and $MODEL3$.

	flex prices	hom. rigidity	het rigidity
$MODEL1:$			
hom Ω_c, hom Ω	5.36%	5.36%	11.71%
het Ω_c, $\delta = 0$	19.94%	19.94%	23.53%
het Ω_c, hom Ω	11.00%	18.58%	22.40%
hom Ω_c, het Ω	7.95%	5.43%	12.37%
het Ω_c, het Ω	17.45%	19.51%	22.94%
$MODEL2:$			
hom Ω_c, hom Ω	5.36%	5.36%	7.87%
het Ω_c, $\delta = 0$	19.94%	19.94%	25.58%
het Ω_c, hom Ω	11.00%	16.01%	18.58%
hom Ω_c, het Ω	7.95%	5.91%	8.77%
het Ω_c, het Ω	17.45%	18.71%	21.00%
$MODEL3:$			
hom Ω_c, hom Ω	5.36%	5.36%	7.72%
het Ω_c, $\delta = 0$	19.94%	19.94%	25.50%
het Ω_c, hom Ω	11.35%	15.28%	18.44%
hom Ω_c, het Ω	7.75%	6.12%	8.64%
het Ω_c, het Ω	17.55%	18.51%	20.99%

Table 2 – Impact multiplier of sectoral productivity shocks on GDP volatility relative to impact multiplier of aggregate productivity shocks in $MODEL1$, $MODEL2$ and $MODEL3$.

	flex prices		hom. rigidity		het rigidity	
$MODEL1:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0024	(5.36%)	.0052	(10.82%)
het Ω_c, $\delta = 0$	.1496	(14.96%)	.0108	(14.96%)	.0204	(21.61%)
het Ω_c, hom Ω	.0880	(8.80%)	.0056	(12.52%)	.0103	(20.05%)
hom Ω_c, het Ω	.0702	(7.02%)	.0026	(5.76%)	.0064	(11.69%)
het Ω_c, het Ω	.1384	(13.84%)	.0064	(14.42%)	.0121	(21.47%)
$MODEL2:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0043	(5.36%)	.0060	(7.57%)
het Ω_c, $\delta = 0$	.1496	(14.96%)	.0224	(14.96%)	.0313	(21.68%)
het Ω_c, hom Ω	.0880	(8.80%)	.0091	(11.42%)	.0134	(15.40%)
hom Ω_c, het Ω	.0702	(7.02%)	.0048	(5.95%)	.0076	(8.49%)
het Ω_c, het Ω	.1386	(13.84%)	.0114	(14.19%)	.0169	(17.94%)
$MODEL3:$						
hom Ω_c, hom Ω	.0536	(5.36%)	.0039	(5.36%)	.0076	(8.26%)
het Ω_c, $\delta = 0$	.1496	(14.96%)	.0251	(14.96%)	.0390	(22.14%)
het Ω_c, hom Ω	.0904	(9.04%)	.0086	(11.86%)	.0156	(15.78%)
hom Ω_c, het Ω	.0686	(6.86%)	.0043	(5.86%)	.0094	(9.08%)
het Ω_c, het Ω	.1386	(13.86%)	.0104	(14.27%)	.0190	(17.89%)

Table 3 – Multiplier of sectoral productivity shocks on GDP volatility, $\|\chi\|$ In parenthesis, $\|\chi\|$ relative of the multiplier of aggregate productivity shocks on GDP volatility using $MODEL1$, $MODEL2$ and $MODEL3$ for a sample of sectors that excludes the three sectors with the largest GDP: Retail trade ($4A0000$), Real Estate (531000) and Wholesale trade (420000). All reported numbers are adjusted by a factor $\left(\frac{345}{348}\right)^{1/2}$.

(1)		(2)		(3)	
6.05	(112120)	22.58	(4A0000)	36.13	(420000)
5.13	(1119B0)	21.30	(531000)	17.31	(531000)
5.11	(1121A0)	18.42	(420000)	11.86	(4A0000)
5.04	(331411)	4.92	(52A000)	10.93	(52A000)
4.59	(112300)	3.60	(621A00)	8.47	(517000)

(4)		(5)		(6)		(7)	
25.22	(420000)	6.65	(324110)	33.92	(531000)	32.76	(420000)
9.44	(531000)	6.54	(211000)	16.71	(420000)	19.32	(531000)
3.59	(52A000)	5.92	(1121A0)	10.27	(4A0000)	12.09	(52A000)
3.42	(221100)	5.62	(1119B0)	8.13	(52A000)	9.56	(4A0000)
2.87	(324110)	5.50	(331411)	5.85	(517000)	9.04	(517000)

Table 4 – Contribution of the top-5 most important sectors to the multiplier of sectoral shocks on GDP volatility in $MODEL1$ in a calibration that matches (1) only the frequency of price changes, (2) sectoral GDP shares, (3) frequency of price changes and sectoral GDP shares, (4) only input-output matrix, (5) frequency of price changes and input-output matrix, (6) sectoral GDP shares and input-output matrix, (7) frequency of price changes, sectoral GDP shares and input-output matrix.

Identity of sectors in parenthesis: 112120 (Dairy cattle and milk production), 1119B0 (All other crop farming), 1121A0 (Cattle ranching and farming), 331411 (Primary smelting and refining of copper), 112300 (Poultry and egg production), 4A0000 (Retail trade), 531000 (Real estate), 420000 (Wholesale trade), 52A000 (Monetary authorities and depository credit intermediation), 621A00 (Offices of physicians, dentists, and other health practitioners), 517000 (Telecommunications), 221100 (Electric power generation, transmission, and distribution), 324110 (Petroleum refineries), 211000 (Oil and gas extraction), 331411 (Primary smelting and refining of copper).