The Macroeconomic Impact of Money Market Disruptions*

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Abstract

We build a general equilibrium model featuring unsecured and secured interbank markets, and collateralized central bank funding. The model is calibrated and used to analyse the macroeconomic impact of three key developments observed in the European money markets since 2008: i) the reduced ability of banks to access the unsecured market since the onset of the global financial crisis and the shift to secured market funding; ii) the impaired functioning of the secured market during the sovereign crisis; iii) the increased reliance of banks on central bank funding. We find that disruptions in interbank markets, as observed during the financial and sovereign debt crises, generate a sizeable impact on real activity.

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1 Introduction

In this paper, we construct a dynamic general equilibrium model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank providing funding against collateral. Interbank markets are essential to banks' liquidity management. They also play a key role in the transmission of monetary policy. These markets came under severe stress during the global financial crisis of 2007-2009 as well as during the euro area sovereign debt crisis of 2010-2012.[1] We use our model to assess the macroeconomic impact of the observed interbank market disruptions.

Our modelling approach is motivated by three stylized facts about euro area money markets. First, interbank money markets are an important funding source for banks in the euro area but their share in total interbank funding has been diminishing since 2008 (see Figure 1). In 2008, the ratio of interbank liabilities to total assets was about 30%. This ratio started to decline with the onset of the global financial crisis, dipping below 20% by 2013. This declining trend reflects disruptions in money markets, with some market segments "freezing"or drying-up altogether.

Second, there was a dramatic shift away from unsecured and towards secured money market funding since 2008 (see Figure 2). The secured money market segment was nearly double that of the unsecured segment in 2008 in terms of the transaction volumes. During the financial crisis, the share of secured funding grew, as some banks became unable to borrow in the unsecured markets due to perceptions of increased counterparty risk and shifted to secured borrowing instead. By 2013, the secured segment was ten times bigger compared to the unsecured segment.

Third, with private money markets malfunctioning, banks increasingly turned to the central bank for refinancing (see Figure 3). Reliance on central bank funding gradually rose in the euro area, particularly with the onset of the sovereign debt crisis. Banks located in euro area countries with vulnerable sovereigns started borrowing larger amounts from the ECB and pledging riskier collateral, taking advantage of the more favorable haircuts on risky assets imposed by the ECB relative to the secured market (see Table 1).

In our model, banks with a liquidity need can obtain funding in the secured or unsecured market, or at the central bank. Banks face an exogenous probability of being "connected,"

[1] The failure of the interbank market to redistribute liquidity was highlighted in a number of accounts of the recent crisis (see, for example, Allen and Carletti, 2008, and Brunnermeier, 2009).

defined as the ability to borrow in the unsecured market. Those banks that are unable to borrow in the unsecured market, the "unconnected" banks, can access the secured market or central bank funding. To access the secured market or central bank funding banks need to hold government bonds which can be pledged as collateral. At the beginning of each period, after knowing whether they are connected or unconnected, banks choose their liabilities (how much deposit and central bank funding to raise) and their assets (choosing between loans, bonds and cash). After making their asset-liability choices, banks face idiosyncratic deposit withdrawal shocks. Banks experiencing low withdrawals can lend funds in the secured or unsecured market. Banks experiencing high withdrawals can cover them with unsecured borrowing (connected banks), or the combination of collateralized borrowing and cash buffers (unconnected banks).

All collateralized borrowing is subject to a haircut, which can be different in the private secured market and at the central bank. If a bank loses access to the unsecured market but its government bond holdings are sufficiently valuable, it can replace unsecured funding with secured funding. Our model can therefore capture the shift from the unsecured to secured funding observed in the recent years. However, if private counterparties become reluctant to accept a particular government bond as collateral (due to, e.g., concerns about that sovereign's health), access to the secured market will become impaired as well. The possibility that banks face different haircuts on private and central bank funding allows us to capture the fact that haircuts set by the central bank can be more favorable in crisis times compared to the private market haircuts for low-quality collateral. In such circumstances, banks holding such collateral may replace their lost unsecured and secured funding with borrowing from the central bank. In our model, the choice between secured market funding and central bank funding is driven by the comparison of the respective interest rates and haircuts charged in the secured market and at the central bank.

We calibrate our model using euro area data to quantify the impact of money market disruptions on the economy. We analyze the macroeconomic impact of three alternative scenarios: 1) reduced access to the unsecured money market; 2) increased haircuts in the secured market, and 3) increased deposit withdrawals, which were reported in some euro area countries.

Our model suggests that the effects of these market disruptions on investment and output are sizeable. In the first scenario - when access to the unsecured money market is reduced - a higher proportion of banks becomes unconnected and needs to satisfy possible deposit withdrawals by holding bonds and/or money. These banks can therefore invest less in the productive asset, i.e., capital. It is this substitution from investing in productive capital when connected to investing in unproductive assets when unconnected that generates output effects of unsecured market disruptions in our model. In our benchmark calibration with moderate liquidity outflows in the afternoon, unconnected banks are not too much constrained and shifting from unsecured to secured funcing is only moderately costly. We find that an increase in the share of unconnected banks from 0.42 (corresponding to the average pre-2008 share of secured transactions in total volumes) to 0.24 (corresponding to the same average share during the crisis) generates a decline in output of around 0.4 percent. The adverse impact on real activity changes substantially if expected liquidity outflows in the afternoon increase with the disruptions experienced in the unsecured market. If those outflows are expected to double, the contraction in output is 4 percent.

In the second scenario - when haircuts in the secured market increase - the model economy moves between two regions. When private haircuts are low and collateral valuable, banks have access to the private funding markets (either secured or unsecured), and there is no recourse to central bank funding. When private haircuts increase beyond a certain threshold, banks relying on secured markets for funding become unable to cover their liquidity needs there as the value of their collateral drops. They access central bank funding instead. Specifically, as private haircuts increase, the value of bonds as collateral in the secured market decreases. Unconnected banks react by holding fewer bonds, raising fewer deposits, and reducing their investment in capital. Although connected banks maintain their investment in capital broadly unchanged, the overall effect is a decline in aggregate investment and output. We find that an increase in private haircuts from 3% to 25% generates an output contraction of 0.3 percent (0.8 percent if maximum liquidity outflows double). As private haircuts decline further, the economy moves to the second region where unconnected banks access central bank funding. As the central bank haircut is stable and more favorable compared to the private market haircut, banks receive more funding for the same amount of pledged bonds and they can stabilize their deposit intake and investment. Therefore, the availability of central bank funding puts a floor to the decline in deposits and capital. The contraction in real activity would be more severe in the absence of the central bank.

In the third scenario - when banks face increased fears of deposit withdrawals - unconnected banks react similarly by reducing their deposit, bond and cash holdings. As a result, the aggregate amount of deposits falls which in turn limits productive investment and reduces the capital stock. We find that doubling the expected maximum share of deposit withdrawals (from 0.1 to 0.2) generates an output loss of 3 percent.[2]

This paper is related to the literature on interbank markets and on the impact of sovereign risk on the macroeconomy. There is an extensive literature in banking on the role of interbank markets in banks' liquidity management, starting with Bhattacharya and Gale (1987). A number of recent papers focus on analyzing frictions that prevent interbank markets from distributing liquidity efficiently within the banking system. Frictions include asymmetric information about banks' assets (Flannery, 1996, Freixas and Jorge, 2007, Heider, Hoerova and Holthausen, 2015), imperfect cross-border information (Freixas and Holthausen, 2005), banks' free-riding on liquidity provision by the central bank (Repullo, 2005), and multiplicity of Pareto-ranked equilibria (Freixas, Martin and Skeie, 2011). Papers in this strand of literature tend to be partial equilibrium and static, with links to the real economy modeled in a reduced-form fashion.

Several recent papers build dynamic general equilibrium models which include interbank trade. For example, Afonso and Lagos (2015) develop an OTC model of the unsecured (Federal Funds) market and use it to study the intraday evolution of the distribution of reserve balances. Atkeson, Eisfeldt, and Weill (2015) analyse the trading decisions of banks in an OTC market and draw implications for policy. Bianchi and Bigio (2014) build a framework to study the implementation of monetary policy through the banking system. Bruche and Suarez (2009) show that unsecured interbank market freezes generate large output losses in the presence of deposit insurance, due to a distorted allocation of credit. Our paper contributes to this literature by considering both unsecured and secured interbank markets, and collateralized lending by the central bank.[3] In our setup, disruptions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding. We nevertheless find that disruptions in the unsecured market segment can generate sizable output costs. When also the secured market segment is severely malfunctioning, the possibility to tap into central bank can improve macroeconomic outcomes.

In our model, lending from the central bank is backed by government bonds, and increase in haircuts on sovereign bonds capture in a reduced-form the impact of sovereign default risk

[2] See, e.g., "Worrying about a Greek bank run," Reuters, April 15, 2010, http://blogs.reuters.com/felix-salmon/2010/04/15/worrying-about-a-greek-bank-run/.

[3] A related paper to ours is Schneider and Piazzesi (2017) which builds a framework where banks use reserves to settle interbank trades and to handle endusers' payment instructions. The authors show that key to the efficiency of a payment system is the provision and allocation of collateral.

on collateral value. Our paper is thus also related to the literature on the impact of sovereign default risk on financial intermediation and the macroeconomy. Recent contributions to this strand of literature study the impact of sovereign risk on the funding ability of banks and their lending decisions (Bocola, 2016) as well as the link between government default and financial fragility, including the question of why the banking system may become exposed to government bonds (e.g., Gennaioli, Martin, and Rossi, 2014). We do not model sovereign default risk explicitly, focusing instead on the implications of increased haircuts on government bonds for banks' ability to borrow in secured and unsecured interbank markets, as well as on how central bank policies can help alleviate bank funding problems arising due to increased private market haircuts.

The paper proceeds as follows. In section 2, we describe the model. In section 3, we define the equilibrium. In section 4, we characterize the system of equilibrium conditions. In section 5, we describe the steady state and present some analytical results. Section 6 illustrates the model predictions through a numerical analysis. Section 7 concludes.

2 The model

The economy is inhabited by a continuum of households, firms and banks. There is a government and a central bank. There is also a foreign sector that holds domestic bonds.

Time is discrete, $t = 0, 1, 2, \ldots$. We think of a period as composed of two sub-periods ("morning" and "afternoon"). Let us describe each in turn.

At the beginning of each period (in the morning), aggregate shocks occur. Households receive payments from financial assets and allocate their nominal wealth among money, long-term government bonds, and deposits at banks. Households also supply labor to firms, receiving wages in return. The government taxes the labor income of the households, makes payments on its debt and may change the stock of outstanding debt. Banks accept deposits from households and the central bank and make dividend payments to households. After accepting the deposits, banks learn their afternoon type in the morning. This latter can be either "connected," in which case banks can borrow in the unsecured interbank market, or "unconnected," in which case they cannot, and the only possibility is to borrow by pledging assets in the secured interbank market. Banks then lend to firms (more precisely, finance their capital) and they hold government bonds and reserves ("cash"). The central bank provides funding to banks that wish to borrow against collateral. As an additional policy tool, the central bank can choose “haircuts” on the collateral pledged to access those funds.

During the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks experience idiosyncratic deposit withdrawal shocks which average out to zero across all banks, Conceptually, these relate to random idiosyncratic consumption needs, additional economic activity and immediate payment for these services, which we shall refrain from modelling. Banks can accommodate those shocks by using their existing reserves, by selling government bonds or by borrowing in the unsecured market from other banks. They can only access the unsecured market, however, if they are connected. Banks are assumed to always position themselves so as to meet these liquidity withdrawals, i.e., bank failures are considered too costly and not an option. All banks meet as “one big banker family” at the end of the period. One can think of it as follows. First, the same bank-individual liquidity shock happens “in reverse”, so that banks enter the banker-family meeting in the same state they were in at the beginning of the afternoon. However, there would then still be bank heterogeneity left. Thus, banks all equate their positions at that point and restart the next period with the same portfolio. Alternatively, and equivalently, one can think that there are securities markets which open at the end of the period and allow banks to equate their portfolios. Banks during the period therefore are only concerned with the marginal value of an additional unit of net worth they can produce for the next period.

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that we assume that banks pay a fixed fraction of their net worth to households as a dividend in the morning of every period.

2.1 The households

There is a representative household, indexed by $i \in (0,1)$. At the beginning of time t, households hold an amount of cash, $\widetilde{M}_{t-1}^{H}$, brought from period $t-1$. They also receive repayment from banks of deposits opened in the previous period gross of the due interest, $R_{t-1}^{D}D_{t-1}$. Holding an amount H_t of nominal wealth at hand, each household chooses how to allocate it among existing nominal assets, namely money, M_t^H, and deposits, D_t.

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by household i into period $t+1$, $\widetilde{M}_t^H$, is thus

$$\widetilde{M}_t^H = M_t^H + (1-\tau_t) W_t l_t + E_t - P_t c_t, \tag{1}$$

where P_t is the price of the consumption good, l_t is hours worked, τ_t is the labor tax rate, W_t is the nominal wage level, and E_t is the profit payout ("earnings") by banks.

The nominal wealth available at the beginning of period $t+1$ for investment in nominal assets is given by

$$H_{t+1} = R_t^D D_t + \widetilde{M}_t^H. \tag{2}$$

The household then solves the problem

$$\max_{\left\{c_t>0, l_t>0, D_t\geq 0, M_t^H\geq 0\right\}} E_t \sum_{t=0}^{\infty} \beta^t \left[u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right)\right] \tag{3}$$

subject to (2) and

$$D_t + M_t^H \leq H_t.$$

2.2 Firms

A final-good firm j uses capital $k_{t-1,j}$ and labor $l_{t,j}$ to produce a homogeneous final output good $y_{t,j}$ according to the production function

$$y_{t,j} = \gamma_t k_{t-1,j}^{\theta} l_{t,j}^{1-\theta}$$

where γ_t is a country-specific productivity shock. It receives revenues $P_t y_{t,j}$ and pays wages $W_t l_{t,j}$. Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a real "rental rate" r_t per unit of capital or total nominal rental rate payments $P_t r_t k_{t-1,j}$ from firm j on their capital $k_{t-1,j}$.

Capital-producing firms buy old capital k_{t-1} from the banks and combine it with final goods I_t to produce new capital k_t, according to

$$k_t = (1-\delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

2.3 The government

The government has some outstanding debt with face value $\overline{B}_{t-1}$. It needs to purchase goods g_t and pays for it by taxing labor income as well as issuing discount bonds with a face value $\Delta\overline{B}_t$ to be added to the outstanding debt next period, obtaining nominal resources $Q_t\Delta\overline{B}_t$ for it in period t. We assume that some suitable no-Ponzi condition holds. The government discount bonds are repaid at a rate κ.

The outstanding debt at the beginning of period $t+1$ will be

$$\overline{B}_t = (1-\kappa)\,\overline{B}_{t-1} + \Delta\overline{B}_t \tag{4}$$

The government budget balance at time t is

$$P_t g_t + \kappa\overline{B}_{t-1} = \tau_t W_t l_t + Q_t\Delta\overline{B}_t + S_t \tag{5}$$

where S_t are seigniorage payments from the central bank and g_t is an exogenously given process for government expenditures.

We assume that the government conducts fiscal policy so as to stabilize the stock of debt at a targeted level $\overline{B}^*$. This way, we aim at disentangling the fiscal role of the government from that of issuer of bonds with collateral value. In our analysis, movements in collateral value will be unrelated to fiscal policy decisions.

The government stabilizes the debt by adopting the following rule for the income tax:

$$\tau_t - \tau^* = \alpha\left(B_t - \overline{B}^*\right), \tag{6}$$

where τ_t increases above its target level τ^*, if the debt level is above $\overline{B}^*$. We assume that α is such that the equilibrium is saddle-path stable and that the fiscal rule ensures a gradual convergence to the desired stock of debt, following aggregate disturbances.[4] The target value τ^* is the level of the income tax necessary to stabilizes the debt at $\overline{B}^*$.[5]

[4] In our quantitative section, we provide a comparison of steady state equilibria: in that analysis, the parameter α plays no role.

[5] Notice that τ^* can be obtained by combining equation (4) and (5) in steady state, together with the rule $B = \overline{B}^*$, to get

$$\tau^*(1-\theta)\,y = g + \kappa(1-Q)\frac{\overline{b}^*}{\pi} - Q\left(1-\frac{1}{\pi}\right)\overline{b}^* - s.$$

Here $\overline{b}^* = \frac{\overline{B}^*}{P}$, $s = \frac{S}{P}$ and π is the steady state inflation rate.

2.4 The central bank

The central bank chooses the total money supply $\overline{M}_t$ and interacts with banks in the "morning", providing them with funds. Banks come into the period with total liabilities (F="funds from the central bank") at face value $\overline{F}_{t-1}$. Banks make payments $\kappa^F \overline{F}_{t-1}$ on these liabilities and obtain new funds, at face value $\Delta\overline{F}_t$. Thus,

$$\overline{F}_t = \left(1 - \kappa^F\right) \overline{F}_{t-1} + \Delta\overline{F}_t \tag{7}$$

Banks obtain funds $Q_t^F \Delta\overline{F}_t$ for these new liabilities, at the common price or discount factor Q_t^F. This discount factor is a policy parameter set by the central bank. The central bank furthermore buys and sells government bonds outright. Let B_{t-1}^C be the stock of government bonds held by the central bank ("C") at the beginning of period t. The government makes payments on a fraction of these bonds, i.e., the central bank receives cash payments κB_{t-1}^C. The remaining government bonds in the hands of the central banks are $(1-\kappa) B_{t-1}^C$. The central bank then changes its stock to B_t^C, at current market prices Q_t, using cash. Thus,

$$B_t^C = (1-\kappa) B_{t-1}^C + \Delta B_t^c$$

The central bank balance sheet looks as follows at time t:

Assets	Liabilities
$Q_t^F \overline{F}_t$ (loans to banks)	M_t^H (currency held by HH)
$Q_t B_t^C$ (bond holdings)	M_t (bank reserves)
	S_t (seigniorage)

Let $\overline{M}_t = M_t^H + M_t$ be the total money stock before seigniorage is paid. Note that the seigniorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\begin{aligned}\overline{M}_t - \overline{M}_{t-1} \quad &= \quad S_{t-1} + Q_t^F \left(\overline{F}_t - \left(1-\kappa^F\right)\overline{F}_{t-1}\right) - \kappa^F \overline{F}_{t-1} \\ &\quad + Q_t \left(B_t^C - (1-\kappa) B_{t-1}^C\right) - \kappa B_{t-1}^C.\end{aligned} \tag{8}$$

Seigniorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t. \tag{9}$$

2.5 Banks

There is a continuum of banks ("Lenders"), indexed by $l \in (0,1)$. Consider a bank l.

2.5.1 Assets and liabilities

At the end of the morning, after earning income on its assets, paying interest on its liabilities and retrading, but just before paying dividends to share holders, the bank holds four type of assets. It additionally and briefly holds an asset in the afternoon, for a total of five. As an overview, the end-of-morning balance sheet of that bank is

Assets	Liabilities
$P_t k_{t,l}$(capital held)	$D_{t,l}$ (deposits by HH)
$Q_t B_{t,l}$ (bond holdings)	$Q_t^F F_{t,l}$ (secured loans)
$E_{t,l}$(cash dividends)	$N_{t,l}$ (net worth)
$M_{t,l}$(cash reserves)	

In detail:

1. Capital $k_{t,l}$ of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning. Capital evolves according to

$$k_{t,l} = (1-\delta)\, k_{t-1,l} + \Delta k_{t,l}$$

where $\Delta k_{t,l}$ is the gross investment of bank l in capital.

2. Bonds with a nominal face value $B_{t,l}$. A fraction κ of the government debt will be repaid. The bank changes its government bond position per market purchases or sales ("-") $\Delta B_{t,l}$ in the morning, so that

$$B_{t,l} = (1-\kappa)\, B_{t-1,l} + \Delta B_{t,l}$$

at the end of the morning. If the bank purchases (sells) bonds on the open market, it pays (receives) $Q_t \Delta B_{t,l}$. As a baseline, we allow $\Delta B_{t,l}$ to be negative, indicating a sale.

In the afternoon and after the first bank-individual liquidity shock, once again, the bank can change its government bond position per market purchases or sales ("-") $\widetilde{\Delta B}_{t,l}$, so that

$$\widetilde{B}_{t,l} = B_{t,l} + \widetilde{\Delta B}_{t,l}$$

When the second reverse liquidity shock hits, the trade is reversed as well, resulting in

$$B_{t,l} = \widetilde{B}_{t,l} - \widetilde{\Delta B}_{t,l}$$

One can instead think of this as a secured repo market, vis-a-vis other banks. To that end, it is useful to introduce haircut parameters $0 \leq \widetilde{\eta}_t \leq 1$, imposed by other lending banks. The bank then receives the cash amount $\widetilde{\eta}_t Q_t \widetilde{\Delta B}_{t,l}$ in the first of these two transactions, repaying the same amount in the second. Taken literally, there is no risk here that this haircut could reasonably insure against, but this is just due to keeping the model simple. The interest rate is zero.

3. Cash $E_{t,l}$ earmarked to be distributed to shareholders (E = "earmarked" or "earnings") at the end of the morning. Note that this does not mean that the households end up being forced to hold money, as everything happens "simultaneously" in the morning. If they want to hold those extra earnings as extra deposits, then D_t would simply already be higher before they receive the earnings from the banks, in "anticipation" of these earning payments.

4. Reserves (M="money") $M_{t,l} \geq 0$. They may add to cash (not earmarked for paying shareholders) in the morning,

$$M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$$

as well as in the afternoon,

$$\widetilde{M}_{t,l} = M_{t,l} + \widetilde{\Delta M}_{t,l} \geq 0$$

reversing the first-liquidity-shock transaction when the reverse liquidity shock hits,

$$M_{t,l} = \widetilde{M}_{t,l} - \widetilde{\Delta M}_{t,l}$$

5. Unsecured claims on other banks at face value, obtained during the first liquidity shock in the afternoon. They are repaid at zero interest rate during the second reverse-liquidity shock.

Bank l has four types of liabilities:

1. Deposits $D_{t,l}$. This is owed to household and subject to aggregate withdrawals and additions $\Delta D_{t,l}$ in the morning, so that

$$D_{t,l} = R^D_{t-1} D_{t-1,l} + \Delta D_{t,l}$$

where R^D_{t-1} is the return on one unit of deposits, agreed at time $t-1$. Additionally, there are idiosyncratic withdrawals and additions in the afternoon, to be described.

2. Secured loans (F="funding") from the central bank at face value $F_{t,l}$. Secured loans require collateral. A bank l with liabilities $F_{t,l}$ to the central banks needs to pledge an amount $0 \leq B^F_{t,l} \leq B_{t,l}$ of government bonds $B_{t,l}$ satisfying the collateral constraint

$$F_{t,l} \leq \eta_t \; Q_t B^F_{t,l} \tag{10}$$

where η_t is a haircut parameter and is set by the central bank. The collateral constraints are set in terms of the market value of securities, as is the case in ECB monetary policy operations. Secured loans from the central bank are obtained in the morning. The change in the secured loans $\Delta F_{t,l}$ provide the banks with change in liquidity ("cash") of $Q^F_t \Delta F_{t,l}$, in addition to the liquidity carried over from the previous period. Liquidity is needed in the afternoon. Therefore, the discount rate Q^F_t will not only relate to an intertemporal trade-off, as is common in most models, but importantly also to the intratemporal tradeoff of obtaining potentially costly liquidity in the morning in order to secure sufficient funding in the afternoon.

3. Outstanding unsecured liabilities to other banks issued at the time of the first liquidity shock in the afternoon. Only "connected" banks can issue them. They are repaid at zero interest rate at the time of the reverse liquidity shock.

4. Net worth $N_{t,l}$.

The sum of assets equals the sum of liabilities, at any point in time.

2.5.2 Liquidity needs in the afternoon

At the beginning of the afternoon, households hold total deposits D_t with banks. We seek to capture the daily churning of deposits at banks, due to cross-household and firm-household payment activities with inside money, as follows. At the start of the afternoon in period t, deposits get reshuffled across banks, so that bank l with pre-shuffle end-of-morning deposits $D_{t,l}$ experiences a withdrawal $\omega_{t,l} D_{t,l}$. Here, $\omega = \omega_{t,l} \in (-\infty, \omega^{\max}]$, with $0 \leq \omega^{\max} \leq 1$, is a random variable, which is iid across banks l and is distributed according to $F(\omega)$. The remaining post-shuffle beginning-of-afternoon deposits $\widetilde{D}_{t,l}$ are thus

$$\widetilde{D}_{t,l} = (1 - \omega_{t,l}) D_{t,l}$$

In order to meet withdrawals, banks need to have enough reserves at hand to cover them. We assume that banks will always find defaulting on the withdrawals worse than any precautionary measure they can take against it, and thus rule out withdrawal caps and bank runs by assumption. Reserves can be obtained in the morning by various trades, resulting in bank holdings $M_{t,l}$. In the afternoon, additional reserves can be obtained only by new unsecured loans from other banks, maturing at the end of the afternoon, or by selling government bonds. Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank funding in the morning in precaution to withdrawal demands in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen "fast", compared to central bank liquidity provision.

The withdrawal shock is exactly reversed with a second reverse liquidity shock, so that banks exit the period with the original level of deposits $D_{t,l}$ and can thus repay their unsecured loans or buy back the government securities originally sold. The same holds if the signs are reversed. Thus, the first liquidity shock creates only a very temporary liquidity need that banks must satisfy.

New unsecured loans can only be obtained by "connected" banks. We assume that banks face an exogenous iid probability ξ_t of being connected and being able to borrow on the unsecured loan market. We assume this probability to be iid across banks and time. The draw of the type of the bank (i.e., "connected" or "not connected", with probability ξ_t) happens early in the morning: thus, banks know in the morning, whether they are able to potentially borrow in the afternoon or whether they need to potentially sell government bonds instead. Every bank can lend unsecured, if they so choose.

If banks do not have access to the unsecured loan market, they will need to sell government bonds, in case of liquidity needs. They can only do so with the portion that has not yet been pledged to the central bank. With $\omega^{\max}$ as the maximal withdrawal shock, non-connected banks therefore have to hold government securities satisfying

$$\omega^{\max} D_{t,l} - M_{t,l} \leq \widetilde{\eta}_t Q_t \left(B_{t,l} - B_{t,l}^F \right) \tag{11}$$

where $0 \leq \widetilde{\eta}_t \leq 1$ is a haircut parameters imposed by other lending banks, if we interpret this sale of government bonds as a private repo or private secured borrowing, and where the constraint is in terms of the unpledged portion of the government bond holdings $B_{t,l} - B_{t,l}^F$.

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The only impact of these choices and restrictions is that banks need to plan ahead of time in the morning to make sure that they have enough funding in the afternoon, in the worst case scenario. If a bank is unconnected, that worse-case scenario is particularly bad, as it needs to have enough of cash reserves plus unpledged bonds to meet the maximally conceivable afternoon deposit withdrawal.

2.5.3 Objective function and leverage constraints

Banks are owned by households in their country. If net worth is nonnegative, they repay a portion ϕ of their net worth to households each period

$$E_{t,l} = \phi N_{t,l}$$

In terms of aggregate bank equity N_t and resulting dividend payments, the profit payments by banks are $E_t = \phi N_t$, if $N_t \geq 0$. If net worth is negative, banks declare bankruptcy. In that case, all assets are sold, and the proceeds are returned pro rata to the holders of bank liabilities. We shall consider only shocks and scenarios, so that net worth remains positive.

The net worth of bank l before payments to shareholders satisfies

$$\begin{aligned} N_{t,l} &= \max\{0, P_t\left(r_t+1-\delta\right)k_{t-1,l}+M_{t-1,l}+\left(\left(1-\kappa\right)Q_t+\kappa\right)B_{t-1,l}-R^D_{t-1}D_{t-1,l}-\kappa^F F_{t-1,l}\} \\ &= \max\{0, P_t k_{t,l}+\ Q_t B_{t,l}+M_{t,l}-D_{t,l}-Q^F_t F_{t,l}+E_{t,l}\} \end{aligned}$$

where the first equation is the net worth calculated on the balance of assets and their earnings and payments before the bank makes its portfolio decision, while the second equation exploits the equality of assets to liabilities after the portfolio decision.

From these two equations, one can calculate

$$\Delta M_{t,l} = M_{t,l} - M_{t-1,l}.$$

Given the draw of the type according to $\xi_t = P\left(\text{“connected”}\right)$, bank l can be either “connected” or “unconnected” (denoted with the subscripts “c" or “u", respectively).

Aggregate net worth at the beginning of the period is

$$\begin{aligned} N_t = \max\{0, \quad & P_t\left(r_t+1-\delta\right)\left(\xi_{t-1}k_{t-1,c}+\left(1-\xi_{t-1}\right)k_{t-1,u}\right) \\ & +\left(\xi_{t-1}M_{t-1,c}+\left(1-\xi_{t-1}\right)M_{t-1,u}\right) \\ & +\left(\left(1-\kappa^F\right)Q^F_t+\kappa^F\right)\left(\xi_{t-1}F_{t-1,c}+\left(1-\xi_{t-1}\right)F_{t-1,u}\right) \\ & +\left(\left(1-\kappa\right)Q_t+\kappa\right)\left(\xi_{t-1}B_{t-1,c}+\left(1-\xi_{t-1}\right)B_{t-1,u}\right) \\ & -R^D_{t-1}\left(\xi_{t-1}D_{t-1,c}+\left(1-\xi_{t-1}\right)D_{t-1,u}\right)\} \end{aligned}$$

which implies that $N_t = \xi_t N_{t,c} + \left(1-\xi_t\right)N_{t,u}$. In principle, the second expression could be negative and aggregate net worth could become zero, in which case the banking sector of an entire country becomes insolvent. For those cases, it would be important to specify what happens to the assets and liabilities, and the economy overall. To keep the analysis manageable, we shall entirely focus on shocks and equilibria for now, where this does not happen along the equilibrium paths.

We shall impose that sub-banks get the same net worth, regardless of type (“connected”, “unconnected”), effectively assuming that the net worth is assigned before the type is known[6], $N_{t,c} = N_{t,u} = N_t$,where $N_{t,c}$ is the net worth per connected bank, i.e., the total net worth

[6] If the net worth could be assigned after the type is known, obviously only connected banks would get any net worth, and the model would become rather uninteresting.

in all connected banks is $\xi_t N_{t,c}$, the total net worth in all unconnected banks is $(1-\xi_t) N_{t,u}$. Correspondingly, all assets and liabilities are likewise distributed equally, regardless of type (again, assuming that this redistribution is done before the new type is drawn for each sub-bank).

Summing this and imposing the two previous equations shows that total net worth is N_t, as it should be. Therefore, we shall drop the distinction between $N_{t,c}$, $N_{t,u}$ and N_t. The sub-bank budget constraint is

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t = D_{t,l} + Q_t^F F_{t,l} + N_t \tag{12}$$

As in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction of their assets in the morning, after their asset trades are completed and after dividends are paid to the household. The constraint is

$$\lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}\right) \leq V_{t,l}$$

where $0 \leq \lambda \leq 1$ is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral vis-a-vis the central bank[7].

2.6 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.[8] Because unconnected banks can

[7]Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. In that case, the collateral constraint would be

$$\lambda \left[P_t \left(k_{t,l} - k_{t,l}^F\right) + Q_t \left(B_{t,l} - B_{t,l}^F\right) + M_{t,l}\right] \leq V_{t,l}$$

or a version in between this and the in-text equation. Since collateral pledged to the ECB remains in the control of banks, we feel that the assumption used in the text is more appropriate.

[8]We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively rebalance their bond positions. For example, Koijen et al. (2016) document that during the Public Sector Purchase Programme implemented by the ECB since March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, therefore, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such direct link between the bond market and the unconnected banks' decisions would be unrealistic.

buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have a demand for domestic bonds that reacts to movements in the real return on these bonds,

$$B_t^W = P_t \left(\varkappa - \frac{1}{\varrho} \log Q_t \pi_t \right), \tag{13}$$

where $\varrho > 0$ and $\varkappa \geq 0$. Notice that this functional form allows foreign bond holdings to become negative, e.g., in case domestic bond demand exceeds government bond supply, while Q_t is always positive. If $\rho = 0$, bond demand becomes infinitely elastic. In that case, the real return $1/(Q_t \pi_t)$ is fixed and foreign holdings take whatever value is needed to clear the bond market. The flow budget constraint of the foreign sector is

$$Q_t B_t^W + P_t c_t^W = [\kappa + (1-\kappa) Q] B_{t-1}^W. \tag{14}$$

3 Equilibrium

An equilibrium is a vector of sequences such that:

1. Given $P_t, \tau_t, W_t, R_{t-1}^D, E_t,$ the representative household chooses $c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0$ to maximize their objective function

$$\max_{\left\{c_t>0, l_t>0, D_t \geq 0, M_t^H \geq 0\right\}} E_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) \right] \right]$$

   subject to

$$D_t + M_t^H \leq H_t$$

   where

$$H_{t+1} = R_t^D D_t + M_t^H + (1-\tau_t) W_t l_t + E_t - P_t c_t.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_{t,j} = \gamma_t k_{t-1,j}^{\theta} l_{t,j}^{1-\theta}.$$

3. Capital-producing firms choose how much old capital k_{t-1} to buy from banks and to combine with final goods I_t to produce new capital k_t, according to the technology

$$k_t = (1-\delta)\, k_{t-1} + I_t.$$

4. Bank families aggregate the assets and liabilities of the individual family members:

$$V_t = \xi_t V_{t,c} + (1-\xi_t)\, V_{t,u} \tag{15}$$

$$k_t = \xi_t k_{t,c} + (1-\xi_t)\, k_{t,u} \tag{16}$$

$$D_t = \xi_t D_{t,c} + (1-\xi_t)\, D_{t,u} \tag{17}$$

$$B_t = \xi_t B_{t,c} + (1-\xi_t)\, B_{t,u} \tag{18}$$

$$F_t = \xi_t F_{t,c} + (1-\xi_t)\, F_{t,u} \tag{19}$$

$$M_t = \xi_t M_{t,c} + (1-\xi_t)\, M_{t,u} \tag{20}$$

5. Given the stochastic paths for the endogenous variables c_t, l_t, r_t, P_t, Q_t, Q_t^F, η_t, and stochastic exogenous sequence for $\widetilde{\eta}_t$ and the draw of the type according to ξ_t, the representative date-t connected bank chooses $k_{t,c}$, $B_{t,c}$, $B_{t,c}^F$, $F_{t,c}$, $D_{t,c}$, $M_{t,c}$ and the representative date-t unconnected bank chooses $k_{t,u}$, $B_{t,u}$, $B_{t,u}^F$, $F_{t,u}$, $D_{t,u}$, $M_{t,u}$ to maximize the banks' objective function, i.e. to maximize

$$V_{t,l} = P_t E\left[\phi \sum_{s=0}^{\infty} \left(\beta\,(1-\phi)\right)^s \frac{u_c\left(c_{t+s}, l_{t+s}\right)}{u_c\left(c_t, l_t\right)} \frac{N_{t+s}}{P_{t+s}}\right] \tag{21}$$

where

$$\begin{aligned} N_t = \max\{0, \quad & P_t\,(r+1-\delta)\left(\xi_{t-1} k_{t-1,c} + \left(1-\xi_{t-1}\right) k_{t-1,u}\right) \\ & + \left(\xi_{t-1} M_{t-1,c} + \left(1-\xi_{t-1}\right) M_{t-1,u}\right) \\ & + \left(\left(1-\kappa^F\right) Q_t^F + \kappa^F\right)\left(\xi_{t-1} F_{t-1,c} + \left(1-\xi_{t-1}\right) F_{t-1,u}\right) \\ & + \left((1-\kappa)\, Q_t + \kappa\right)\left(\xi_{t-1} B_{t-1,c} + \left(1-\xi_{t-1}\right) B_{t-1,u}\right) \\ & - R_{t-1}^D \left(\xi_{t-1} D_{t-1,c} + \left(1-\xi_{t-1}\right) D_{t-1,u}\right)\} \end{aligned} \tag{22}$$

s.t. for $l = c, u$,

$$\begin{aligned} V_{t,l} &\geq \lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \\ 0 &\leq B_{t,l} - B^F_{t,l} \\ P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t &= D_{t,l} + Q^F_t F_{t,l} + N_t \\ F_{t,l} &\leq \eta_t Q_t B^F_{t,l} \end{aligned}$$

as well as

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \widetilde{\eta}_t Q_t \left(B_{t,u} - B^F_{t,u} \right)$$

for the unconnected banks.

6. The central banks chooses the total amount of money supply $\overline{M}_t$, the haircut parameter η_t, the discount factor on central bank funds Q^F_t , the bond purchases B^C_t as well as the seigniorage payment S_t. It satisfies the balance sheet constraint

$$S_t = Q^F_t \overline{F}_t + Q_t B^C_t - \overline{M}_t \tag{23}$$

and the budget constraint, rewritten as

$$\begin{aligned} \overline{M}_t &= Q^F_{t-1} \overline{F}_{t-1} + Q_{t-1} B^C_{t-1} + Q^F_t \left(\overline{F}_t - \left(1 - \kappa^F \right) \overline{F}_{t-1} \right) \\ &\quad -\kappa^F \overline{F}_{t-1} + Q_t \left(B^C_t - (1 - \kappa) B^C_{t-1} \right) - \kappa B^C_{t-1} \end{aligned} \tag{24}$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\overline{B}_t = (1 - \kappa) \overline{B}_{t-1} + \Delta \overline{B}_t \tag{25}$$

$$P_t \, g_t + \kappa \overline{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta \overline{B}_t + S_t \tag{26}$$

$$\tau_t - \tau^* = \alpha \left(B_t - \overline{B}^* \right), \tag{27}$$

where τ^* is the level of the income tax necessary to stabilizes the debt at $\overline{B}^*$.

8. The foreign sector chooses the amount of domestic bonds to hold

$$B_t^W = P_t \left(\varkappa - \frac{1}{\varrho} \log Q_t \pi_t \right), \tag{28}$$

and satisfies the budget constraint

$$Q_t B_t^W + P_t c_t^W = [\kappa + (1-\kappa) Q] B_{t-1}^W. \tag{29}$$

9. Markets clear:

$$c_t + \ g_t + I_t + c_t^W = y_t \tag{30}$$

$$\overline{B}_t = B_t + \ B_t^C + B_t^W \tag{31}$$

$$\overline{F}_t = F_t \tag{32}$$

$$\overline{M}_t = \ M_t + \ M_t^H \tag{33}$$

4 Analysis

We characterize the decision of households, firms and banks in turn.

4.1 Households

The household budget constraint at time t writes as

$$D_t + M_t^H \leq R_{t-1}^D D_{t-1} + M_{t-1}^H + (1-\tau_{t-1}) W_{t-1} l_{t-1} + E_{t-1} - P_{t-1} c_{t-1} \tag{34}$$

Note that the household's problem is subject to a non-negativity conditions,

$$M_t^H \geq 0, \tag{35}$$

Note also that $c_t > 0, l_t > 0$, and $D_t \geq 0$. We do not list these constraints separately for the following reasons. For $c_t > 0$ and $l_t > 0$, we can assure nonnegativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t > 0$, despite the possibility in principle that it could be zero or negative when allowing for more generality[9].

Let μ_t^{HH} denote a Lagrange multiplier on the period-t household budget constraint (34), and $\widehat{\mu}_{t,h}^M$ the multiplier on the constraint (35). The optimality conditions are given by:

$$\begin{aligned} -\frac{u_l(c_t,l_t)}{u_c(c_t,l_t)} &= (1-\tau_t)\frac{W_t}{P_t} \\ v_M\left(m_t^h\right) &= u_c(c_t,l_t)\left(R_t^D-1\right)-\mu_{t,h}^M \\ \frac{u_c(c_{t-1},l_{t-1})}{P_{t-1}} &= \beta R_t^D\left[\frac{u_c(c_t,l_t)}{P_t}\right] \end{aligned}$$

where $\mu_{t,h}^M = P_t\widehat{\mu}_{t,h}^M$.

4.2 Firms

First-order conditions arising from the problem of the firms are

$$\begin{aligned} y_t &= \gamma_t k_{t-1}^{\theta} l_t^{1-\theta}, \\ W_t l_t &= (1-\theta)\, P_t\; y_t, \\ r_{t,A}k_{t-1,A} &= \theta y_t, \\ k_t &= (1-\delta)\, k_{t-1} + I_t. \end{aligned}$$

4.3 Banks

The run-away constraint (assuming it always binds) is

$$V_{t,l} = \lambda\left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}\right) \tag{36}$$

The value of the mother bank is V_t, which is given by

$$V_t = \xi_t V_{t,c} + (1-\xi_t)\, V_{t,u} \tag{37}$$

[9] We have not yet fully analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

Proposition 1 (linearity) *The problem of bank l is linear in net worth and*

$$V_{t,l} = \psi_t N_{t,l} \tag{38}$$

for any bank l and some factor ψ_t. In particular, $V_{t,l} = 0$ if $N_{t,l} = 0$.

Proof: Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above.

We need to calculate $V_{t,l}$. The proposition above implies

$$V_t = \psi_t N_t \tag{39}$$

giving us a valuation of a marginal unit of net worth at the beginning of period t, for a representative bank.

Suppose, at the end of the period, the representative "mother bank" has various assets, k_t, B_t, and M_t, brought to it by the various sub-banks as they get together again at the end of the period. The end-of-period value $\tilde{V}_t$ of the "mother bank" then satisfies

$$\begin{aligned} \tilde{V}_t &= \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \frac{P_t}{P_{t+1}} \psi_{t+1} N_{t+1}\right] \\ &= \tilde{\psi}_{t,k}\; P_t k_t + \tilde{\psi}_{t,B} B_t \; + \tilde{\psi}_{t,M}\; M_t - \tilde{\psi}_{t,D} D_t - \tilde{\psi}_{t,F} F_t \end{aligned} \tag{40}$$

Per inspecting (22) as well (38), we obtain

$$\tilde{\psi}_{t,k} = \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \psi_{t+1}\left(r_{t+1} + 1 - \delta\right)\right] \tag{41}$$

$$\tilde{\psi}_{t,B} = \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \frac{P_t}{P_{t+1}} \psi_{t+1}\; \left(\left(1-\kappa\right) Q_{t+1} + \kappa\right)\right] \tag{42}$$

$$\tilde{\psi}_{t,D} = \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \frac{P_t}{P_{t+1}} \psi_{t+1}\; R_t^D\right] \tag{43}$$

$$\tilde{\psi}_{t,F} = \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \frac{P_t}{P_{t+1}} \psi_{t+1}\left(\left(1-\kappa^F\right) Q_{t+1}^F + \kappa^F\right)\right] \tag{44}$$

$$\tilde{\psi}_{t,M} = \beta\left(1-\phi\right) E_t\left[\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \frac{P_t}{P_{t+1}} \psi_{t+1}\right] \tag{45}$$

For the sub-banker of type l, write

$$V_{t,l} = \phi N_{t,A} + \tilde{V}_{t,l} \tag{46}$$

The sub-bankers contribute to $\tilde{V}_t$ per

$$\tilde{V}_{t,l} = \tilde{\psi}_{t,k} k_{t,l} + \tilde{\psi}_{t,B} B_{t,l} + \tilde{\psi}_{t,M} M_{t,l} - \tilde{\psi}_{t,D} D_{t,l} - \tilde{\psi}_{t,F} F_{t,l} \tag{47}$$

The run-away constraint for bank l can then be rewritten as

$$\phi N_{t,A} + \tilde{V}_{t,l} \geq \lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \tag{48}$$

Banks will pledge just enough collateral to the central bank to make the collateral constraint binding, nothing more (even if indifferent between that and pledging more: then, "binding" is an assumption). For both types of banks,

$$F_{t,l} = \eta_t Q_t B^F_{t,l} \tag{49}$$

with

$$0 \leq B_{t,l} - B^F_{t,l} \tag{50}$$

There are also nonnegativity constraints for investing in cash, bonds, loans and for financing from the central bank, for both types of banks:

$$0 \leq M_{t,l} \tag{51}$$

$$0 \leq B_{t,l} \tag{52}$$

$$0 \leq k_{t,l} \tag{53}$$

$$0 \leq F_{t,l}. \tag{54}$$

Note that we are interested in cases where banks choose to raise deposits and to extend loans. The former requirement ensures that banks have liquidity shocks in the afternoon and thus provides a meaningful role for interbank markets. The latter requirement generates an active link between financial intermediation and real activity in our economy.

We can have cases, however, when banks decide not to raise central bank finance, as in the case of connected banks that can always get afternoon zero-interest rate unsecured loans from other banks, if the need arises (this is assuming that $Q_t^F \leq 1$, otherwise there would be arbitrage possibilities for banks!). Similarly, banks can decide not to hold cash, if they have access to afternoon unsecured or secured finance, and if the expected return on capital is higher than the expected return on money.

To simplify the analysis, we assume (and verify in appendix A) that the economy is in an interior equilibrium for $D_{t,l}$, and $k_{t,l}$ in all the interesting cases we consider. In light of the considerations above, we explicitly allow for corner solutions for $F_{t,l}$, $B_{t,l}$ and $M_{t,l}$.

As for the afternoon, there is no need to keep track of trades, except to make sure that the afternoon funding constraints for the unconnected banks hold,

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \widetilde{\eta}_t Q_t \left(B_{t,u} - B_{t,u}^F \right). \tag{55}$$

Banks $l = u$ and $l = c$ who are given N_t maximize (47) subject to the sub-bank budget constraint (12) and the run-away constraint (48), the collateral constraints (49), (50), as well as (55) only for the unconnected banks. Let $\mu_{t,l}^{BC}$ denote the Lagrange multiplier on the budget constraint (12), $\mu_{t,l}^{RA}$ the Lagrange multiplier on the run-away constraint (48), $\mu_{t,l}^{CC}$ the Lagrange multiplier on the collateral constraint (49), $\mu_{t,u}$ the Lagrange multiplier on the afternoon funding constraint of the unconnected banks, $\mu_{t,l}^M \geq 0$, $\mu_{t,l}^F \geq 0$, $\mu_{t,l}^C \geq 0$ and $\mu_{t,l}^B \geq 0$ the Lagrange multipliers on the constraints $M_{t,l} \geq 0$, $F_{t,l} \geq 0$, the collateral constraint (50), and the non-negativity constraint for bonds $B_{t,l}$, respectively.

The first-order conditions characterizing banks' choices for capital, bonds, and money, are

$$\left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,k}}{P_t} = \mu_{t,l}^{BC} + \lambda \mu_{t,l}^{RA}$$

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^{C} \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^{C} - \mu_{t,u} \widetilde{\eta}_t \quad \text{for } l = u \end{aligned}$$

$$\begin{aligned} \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,u} \quad \text{for } l = u \end{aligned}$$

Those characterizing banks' choices for deposits, central bank funding, and bonds to be pledged at the central bank, are

$$\begin{aligned} \left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} \quad \text{for } l=c \\ \left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} - \omega^{\max}\mu_{t,u} \quad \text{for } l=u \end{aligned} \tag{56}$$

$$\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,F} = \mu_{t,l}^{BC} Q_t^F - \mu_{t,l}^{CC} + \mu_{t,c}^F \tag{57}$$

$$\begin{aligned} \mu_{t,l}^{CC}\eta_t &= \mu_{t,l}^{C} \quad \text{for } l=c \\ \mu_{t,l}^{CC}\eta_t &= \mu_{t,u}\widetilde{\eta}_t + \mu_{t,l}^{C} \quad \text{for } l=u \end{aligned}$$

The complementary slackness conditions are

$$\mu_{t,l}^F F_{t,l} = 0 \tag{58}$$

$$\mu_{t,l}^M M_{t,l} = 0 \tag{59}$$

$$\mu_{t,l}^C \left(B_{t,l} - B_{t,l}^F\right) = 0 \tag{60}$$

$$\mu_{t,l}^B B_{t,l} = 0. \tag{61}$$

Note these are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions.

5 Steady state analysis

We characterize a stochastic steady state where prices grow at the rate π and all shocks are zero except for the idiosyncratic liquidity shock ω faced by banks. The steady state is characterized by the set of conditions reported in Appendix A.

5.1 Analytical characterization of the bank problem

In this section, we provide some analytical results for the bank problem. We focus on the set of parameters such that:

1. Both bank types choose to extend loans and to raise deposits, $k_l > 0$ and $d_l > 0$. The requirement $k_l > 0$ ensures an active link between activity of all banks and the real activity. This requires capital to be sufficiently productive compared to the cost of deposits, $\tilde{\psi}_k > \tilde{\psi}_D$,

which after substituting for $\tilde{\psi}_k$ and $\tilde{\psi}_D$ yields:

$$\theta \frac{y}{(\xi k_c + (1-\xi) k_u)} + 1 - \delta > \frac{1}{\beta}. \tag{62}$$

The requirement that $d_l > 0$ means that both bank types will be subject to liquidity shocks in the afternoon and thus liquidity management will play an important role for both bank types. Different bank types may still choose to manage their liquidity differently (through interbank markets and/or by borrowing from the central bank and saving cash for the afternoon). For households to deposit with banks we need that $R^D > 1$ or, equivalently,

$$\frac{\pi}{\beta} > 1. \tag{63}$$

2. The central bank conducts monetary policy by conducting open market operations, i.e. by changing the amount of bonds b_C held on its balance sheet. It also sets the price of central bank funding, Q^F, and the haircut η.

3. Connected banks do not borrow from the central bank, $\mu_c^F > 0$ and $f_c = 0$. Note: In reality, when banks can easily borrow unsecured, they use central bank funding only to manage their expected liquidity needs, like reserve requirements. Those are set to zero in the model. In our model, banks will only access central bank funding when their access to interbank markets is impaired. Indeed, historically, banks have made precautionary use of central bank funding to satisfy (unexpected) liquidity needs only in crisis periods.

A sufficient condition for $\mu_c^F > 0$ and $f_c = 0$ is

$$\frac{\left(1-\kappa^F\right) Q^F + \kappa^F}{Q^F} > \frac{\pi}{\beta} = R^D. \tag{64}$$

Note that for $\kappa^F = 1$, this conditions is equivalent to

$$\frac{1}{Q^F} > \frac{\pi}{\beta}.$$

The condition is intuitive: if the interest rate on central bank funding is higher than the rate on deposits, central bank funding will not be used. It is both more expensive in terms of the interest rate and it requires collateral.

When conditions (62)-(64) hold, we can characterize decisions of connected banks as follows (the proof is in the Appendix).

Proposition 2 (connected banks) *Suppose conditions (62)-(64) hold. Then, a connected bank does not borrow from the central bank. A connected bank does not hold any cash. Moreover, if the afternoon constraint of unconnected banks binds,* $\mu_u > 0$*, then a connected bank does not hold any bonds, i.e.,* $b_c = 0$*.*

Connected banks have access to the unsecured market in which they can smooth out liquidity shocks without a need for collateral. Given condition (64), central bank funding is more expensive than the cost of deposits so connected banks will not use it for funding purposes. Similarly, connected banks will not hold any precautionary cash reserves since holding cash carries an opportunity cost. Whenever the afternoon constraint of unconnected banks binds, physical return on bonds is lower than the return on capital as bonds command a collateral premium. However, since connected banks do not need any collateral, they prefer to invest solely in capital.

Decisions of unconnected banks are as follows (the proof is in the Appendix).

Proposition 3 (unconnected banks) *Suppose conditions (62)-(64) hold. If the afternoon constraint is slack,* $\mu_u = 0$*, then an unconnected bank does not borrow from the central bank,* $\mu_u^F > 0$ *and* $f_u = 0$*. Also, if condition*

$$\widetilde{\eta} \geq \eta Q^F \omega^{\max} \tag{65}$$

holds, then an unconnected bank does not borrow from the central bank. If the afternoon constraint binds and condition

$$\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M} \tag{66}$$

holds, then an unconnected bank does not borrow in the secured market and instead it borrows only from the central bank and holds money, $m_u > 0$*.*

If the afternoon constraint is slack, unconnected banks are unconstrained in their afternoon borrowing in the secured market. Therefore, they do not borrow from the central bank. Similarly, they do not borrow from the central bank whenever (65) holds, as private haircut $\widetilde{\eta}$ on bonds is favorable compared to the cost of central bank funding ηQ^F weighted by the maximum afternoon withdrawals $\omega^{\max}$. Since $\eta Q^F \leq 1$ holds, an even simpler sufficient condition for (65) to hold is $\widetilde{\eta} \geq \omega^{\max}$. By contrast, whenever (66) holds, private haircut $\widetilde{\eta}$ is so unfavorable that unconnected banks do not use secured market and borrow from the central bank instead. Since unconnected banks borrow from the central bank, their afternoon constraint binds, $\mu_u > 0$. It follows that money holdings are positive, $m_u = \omega^{\max} d_u > 0$.

6 Numerical analysis

Throughout the numerical analysis, we assume the following functional form for utility: $u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) = \log c_t + \chi \log\left(\frac{M_t^H}{P_t}\right) - l_t$. Notice that we use the log specification also for real balances, so that the ratio $\frac{c_t}{m_t^H}$ depends on $\left(R_t^D - 1\right)$, consistent with a transaction technology specification.

We first describe our calibration strategy. We then use a numerical analysis to illustrate the properties of our model under specific disruptions of the interbank markets.

6.1 Calibration

In order to evaluate the macroeconomic impact of disruptions in funding markets, we calibrate the model to capture main features of the euro area economy in normal times.

In the model, each period is a quarter. We set the depreciation rate at $\delta = 0.02$,the capital income share θ at 0.33 and the discount factor at $\beta = 0.995$.[10]

The fraction of government bonds repaid each period, κ, is 0.11, corresponding to an average maturity of the outstanding stock of sovereign bonds of 9 years.[11] The parameters determining haircuts on bonds applied by the private market and by the central bank in normal times are set equal to each other, at $\widetilde{\eta} = \eta = 0.97$ (where $1 - \widetilde{\eta} = 1 - \eta = 0.03$ corresponds to the 3% haircut). The value for the central bank corresponds to the haircut imposed by the ECB on category 1 securities (fixed coupon bonds with high credit quality) in 2010.[12] The private market haircut value is taken from LCH.Clearnet , a large European-based multi-asset clearing house, and refers to an average haircut on French, German and Dutch bonds across all maturities in 2010.

[10] The inverse of the discount factor $1/\beta$ determines the real rate on household deposits. This rate has been very low in the euro area (in fact, it was negative for overnight deposits both before and after the onset of the financial crisis). To match this stylized fact, we choose a relatively high discount rate β.

[11] Andrade et al. (2016) compute an average remaining maturity of 9 years for all eligible sovereign bonds for purchases by the Eurosystem, in the context of the Public Sector Purchase Programme implemented since 2015. The data refer to the period from 9 March 2015 to 30 December 2015. Using data from the Securities Holding Statistics for the period 2013Q4-2014Q4, Koijen et al. (2016) compute that eligible sovereign bonds were a large fraction of the total outstanding stock of euro area public debt (around 70%).

[12] The ECB reports a haircut of 3% on this asset category also during the 2004-2009 period. Source: ECB dataset.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "connected" banks, ξ, and the maximum fraction of deposits that households can withdraw in the afternoon, $\omega^{\max}$. We compute the average pre-crisis value of ξ using data from Euro Area Money Market Survey (2013), which covers a panel of 98 euro area credit institutions.[13] We set ξ at 0.42, the average ratio of the annual cumulative quarterly turnover in the unsecured market segment over the sum of the annual cumulative turnover in the secured and unsecured segments, over the period 2004-2007 (where 2004 is the first year with an observation in the survey, and 2007 is the last year before the crisis). When we assess the impact of money market freezes, we also compute the same average for the crisis period, i.e. over the period 2008-2013 (where 2013 is the last available observation in the survey). The average value for that period is 0.24.

We compute $\omega^{\max}$ using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows banks to meet 30-days liquidity outflows under stress. As we are interested in maximum outflows, the "stressed" scenario as considered in the LCR appears to be an appropriate empirical counterpart for $\omega^{\max}$. The data is taken from the report of the European Banking Authority (December 2013), EBA henceforth. In 2013, the EBA conducted the first data exercise to assess the LCR impact on the EU banking sector. The report focuses on data as of the fourth quarter of 2012. The sample contains 357 EU banks from 21 EU countries, 50 Group 1 banks (large banks, CT1 capital of EUR 3 billion or above), and 307 Group 2 banks (CT1 capital below EUR 3 billion). Their total assets sum to EUR 33000 billion, the aggregate HQLA to EUR 3739 billion and their net monthly cash outflows to EUR 3251 billion. We calculate $\omega^{\max}$ as the ratio of the monthly net cash outflows over total assets so that $\omega^{\max} = 0.1$.[14]

We choose the parameter of the foreign demand for bonds, $\varkappa$, to ensure that, if foreign bond holdings take a value consistent with our target for the share of those holdings in total debt

[13] The survey provides information on annual cumulative quarterly turnover in the secured and unsecured market segments, as reported at the end of each year's second quarter. The unsecured segment comprises all unsecured transactions (total maturities and total turnover). The secured market segment is the repo market (and also includes total maturities and turnover).

[14] In our model, banks hold liquid assets in the amount of $\widetilde{\eta}Q\left(B_u - B_u^F\right) + M_u$ to cover afternoon withdrawals $\omega^{\max} D$. Since $F = 0$ in our calibrated steady state, and net worth is a small fraction of total liabilities, D can be approximated with total assets. Assuming that the afternoon constraint binds, we can calculate $\omega^{\max}$ as the share of the net monthly cash outflow over total assets.

(as discussed below), then Q and π also take the targeted value at that steady state.[15] The steady state calibration cannot inform us about ρ, so we pick a reasonably low value to ensure an elastic demand function that stabilizes the return on the bond, i.e. we set $\rho = 0.09$.[16] We check robustness to alternative values (not reported) and find little impact on the quantitative assessment of money market disruptions.

We are left with six parameters that we calibrate to jointly minimize the squared log-deviation of some model-based predictions on key variables from their empirical counterparts: the share of net worth distributed by banks as dividends, ϕ, the share of assets bankers can run away with, λ, the coefficient determining the utility from money holdings for households, χ, the expenditure on public goods, g, the amount of government bonds purchased by the central bank, B^C, and the targeted stock of debt in the economy, $\overline{B}^*$. The targeted variables are: i) average debt to GDP; ii) bank leverage ; iii) lending spread; iv) share of banks' bond holdings in total debt; v) share of foreign sector's bond holdings in total debt; and vi) average inflation. Table 2 summarizes all parameter values.

Table 3 reports the value taken by the six targets in the data (computed over the pre-crisis period, 1999-2006, unless otherwise indicated) and the model prediction under the chosen parameterization.[17]

6.2 Comparative statics

We assess the macroeconomic impact of disruptions in money markets by means of a comparative statics analysis. We analyze three alternative scenarios: 1) reduced access to the unsecured money market; 2) increased haircuts in the secured market, and 3) increased probability of deposit withdrawals.

[15] For Q, the target is 0.989, corresponding to an average yield on 10-year German government bonds of 4.5 percent, over the period 1999-2006. For π, the target is 2 percent annual, corresponding to the observed average HICP inflation in the euro area over the same period.

[16] A high eleasticity of the foreign demand for bonds is supported by the evidence provided in Koijen et al. (2016). The authors document the change in foreign holdings of euro area sovereign bonds during the Public Sector Purchase Programme implemented by the ECB in March 2015. They find that the foreign sector decreased its holdings by 5.4% during the period 2015Q2-2015Q4, relative to the average holdings during the period 2013Q4-2014Q4. Over the same period, the average yield on euro area sovereign bonds declined by 63 bps, indicating a high elasticity of the foreign demand to the nominal bond return.

[17] Both the value of leverage and the average lending spread for the euro area are taken from Andrade et al. (2016). The share of banks' and of foreign sector's bond holdings in total debt is computed using data for 2015 reported in Koijen et al. (2016).

6.2.1 Disruptions on the unsecured interbank market

The first exercise we conduct aims at analyzing the macroeconomic effects of the drying up of the euro area unsecured money markets during the initial phase of the financial crisis.

Figure 4 shows the results of a comparative statics exercise in which the share of connected banks (i.e., those with access to the unsecured market), ξ, decreases from 0.5 to 0.1. The figure reports with a solid red line the share of connected banks under our benchmark calibration ($\xi = 0.42$), and with a dashed black line the share observed on average during the crisis ($\xi = 0.24$).

It is useful to recall the definition of 'collateral premium' as the difference between the value to the bank of capital and of bonds, $\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}$. This difference is positive whenever the afternoon constraint of the unconnected banks is binding and bonds are held for their collateral value.

At $\xi = 0.5$, the collateral premium on bonds is positive but small. The afternoon constraint for unconnected banks binds. At the same time, the amount of deposits raised by connected and unconnected banks is of a comparable magnitude. Capital instead is lower for unconnected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. Due to the return on bonds being higher than the return on money, unconnected banks choose not to hold cash for the afternoon. As shown analytically, connected banks hold neither bonds, nor money, i.e. $b_c = m_c = 0$. Those banks combine deposits and net worth to finance the maximum possible amount of productive capital, k_c.

As the share of connected banks decreases (moving leftward in the figure), a larger number of banks now needs to satisfy possible afternoon liquidity withdrawals by holding bonds and/or money. Recall that the stock of bonds accepted as collateral in the secured market is in given supply, $\bar{b}$, and held by three types of agents: the central bank (in fixed amounts), the foreign sector, and the unconnected banks. When the share of unconnected banks rises, so does the aggregate demand for bonds by the domestic banking sector. The bond price (discount factor) increases, as does the collateral premium. The afternoon constraint becomes progressively more binding for unconnected banks. As the nominal (and real) return on the bond falls, foreign investors are induced to sell some of their holdings of domestic bonds. When ξ falls below 0.42, the afternoon constraint is so binding that unconnected banks start holding money in order to relax it. This decreases the demand for bonds by unconnected banks and the price of bonds declines gradually. At the same time, as unconnected banks start holding money, households decrease their money holdings, which is facilitated by an increase in inflation. This leads to a decrease in the real return on bonds and a gradual decrease of bond holdings by the foreign sector. The decrease is not enough, however, to free up a sufficient amount of bonds for unconnected banks to be able to raise the same amount of deposits. Therefore, as ξ falls, the overall effect is that unconnected banks contract deposits and reduce their investment in capital. Because the share of banks with such behavior increases, the aggregate deposits, capital and output fall.

Notice that unconnected banks choose not to fund themselves at the central bank. The private secured market provides a better alternative. In our benchmark calibration, the haircut applied by the private market and the central bank are identical. At the same time, the private market is active within the period and therefore funding in that market imposes a gross interest of one. The central bank, instead, provides funding whose repayment becomes due the following period and charges a higher interest rate.

In the benchmark calibration with $\omega^{\max} = 0.1$, the contraction in real activity induced purely by disruptions in the unsecured market is moderate. A fall in the share of connected banks from $\xi = 0.42$ (pre-crisis average share of unsecured transactions in total) to $\xi = 0.24$ (post-crisis average) generates a decline in real activity of around 0.4 percent. The reason is that planning for moderate liquidity outflows in the afternoon does not constrain the unconnected banks too much: The amount of resources diverted from investment in capital to the investment in unproductive assets (bonds) is limited.

The impact of the disruptions in the unsecured market changes substantially if expected liquidity outflows in the afternoon increase (i.e., if unsecured market disruptions go hand in hand with possibly higher customer withdrawals). We conduct the same comparative statics exercise in which we raise $\omega^{\max}$ from 0.1 to 0.2 (not reported). The results are qualitatively similar. However, the contraction of real activity when ξ is reduced from 0.42 to 0.24 is now 4 percent. This higher contraction is driven by a much larger distortion in the allocation of savings, whereby funds are diverted away from productive capital into unproductive assets.

6.2.2 Disruptions on the secured interbank market

We analyse the impact of disruptions on the secured market by changing the parameter that determines the collateral haircut in the private market, $\widetilde{\eta}$ (where $1 - \widetilde{\eta}$ is the haircut). Figure

5 shows the results of a comparative statics exercise in which $\widetilde{\eta}$ moves from the benchmark pre-crisis value of 0.97 (denoted with a red solid line) to 0.3.

In our calibrated steady state (the red line where $\widetilde{\eta} = 0.97$), the collateral premium is positive and the afternoon constraint binds for unconnected banks. As $\widetilde{\eta}$ falls from 0.97 to 0.8 (the private haircut increases), unconnected banks need to pledge higher amounts of bonds to satisfy the afternoon constraint, for the same level of deposits. The value of bonds for unconnected banks falls. Therefore, they reduce their bond holdings and satisfy the afternoon constraint by increasing the amount of cash brought into the afternoon, and by slightly decreasing deposits they take. The bond price falls, increasing the nominal return to holding bonds. Inflation rises in order to induce households to hold less money, ensuring clearing of the money market. The overall effect is an increase in the real return on bonds, which induces foreigners to increase their holdings. Overall, in this region, unconnected banks reduce somewhat their investment in capital. Connected banks also slightly reduce their investment in capital because the supply of households' deposits decreases.

When $\widetilde{\eta}$ falls below 0.8, it becomes attractive for the unconnected banks to use central banking funding. Although the interest rate cost of central bank funding is higher than the interest rate cost of secured market funding, the central bank haircuts are more favourable relative to those imposed in the private market. Unconnected banks gradually reduce the share of bonds pledged in the private market and increase the share pledged at the central bank. The transition is fast. When $\widetilde{\eta}$ is 0.75, unconnected banks pledge their entire stock of bonds at the central bank. The availability of central bank funding helps to stabilize the economy. As neither the haircut nor the interest charged by the central bank changes, unconnected banks are able to stabilize the amount of deposits they raise and thus their investment in capital.

Overall, a decrease of $\widetilde{\eta}$ from 0.97 to the point where unconnected banks use only central bank funding ($\widetilde{\eta} = 0.75$), generates an output contraction of 0.3 percent. If we run the same comparative statics exercise with $\omega^{\max} = 0.2$, the impact on output increases to 0.8 percent. The reason for the contained macroeconomic impact is that the availability of central bank funding put a floor on the decline in deposits and capital. The impact would be more severe in the absence of the central bank.

6.2.3 Increased risk of a depositor run

During the sovereign debt crisis, some euro area countries faced an increased risk of runs on deposits.[18] We explore the macroeconomic impact of this type of market disruption by increasing the maximum level of expected deposit withdrawals, $\omega^{\max}$, from 0.1 (the value under our benchmark calibration) to 0.6.

At $\omega^{\max} = 0.1$, the economy is in the calibrated steady state described above. As the expected deposit withdrawals, $\omega^{\max}$, increase, unconnected banks become more constrained in the afternoon and the collateral premium - the wedge between the return on capital and the return on bonds - rises sharply for $\omega^{\max}$ below 0.18. Unconnected banks are forced to hold more bonds and cash to satisfy their afternoon constraint. As unconnected banks demand more cash, households reduce their money holdings, which is facilitated by a rise in inflation. For values of $\omega^{\max}$ below 0.18, unconnected banks moderately reduce deposits and capital. For values of $\omega^{\max}$ above 0.18, the run-away constraint, given by (36), becomes slack for unconnected banks. Their deposits start declining, reaching zero when $\omega^{\max} = 0.6$. Because unconnected banks stop raising deposits, connected banks need to hold whatever is supplied by the households. To induce households to rebalance their wealth away from deposits and into cash holdings, inflation declines for $\omega^{\max}$ values above 0.18.

The reduction in the aggregate amount of deposits severely limits productive investment and reduces the capital stock. A doubling of the expected deposit withdrawals compared to the calibrated steady state value, i.e. an increase in $\omega^{\max}$ from 0.1 to 0.2, generates output losses of around 3 percent.

7 Conclusions

We presented a general equilibrium model where banks can finance their liquidity needs in the unsecured or secured interbank markets, and where they have also access to collateralized central bank funding. The model accounts for the reduced ability of banks to access the unsecured market during the financial and sovereign crisis, and their shift to secured market funding. It also accounts for the impaired functioning of the secured market during the sovereign crisis

[18] For instance, the Wall Street Journal reported that on May 15, 2012, at the peak of the sovereign crisis, Greek depositors withdrew 700 million euros (amounting to 0.4 percent of total deposits) from the country's banks on a single day, fueling fears of a bank run.

and the increased reliance on central bank liquidity provision, particularly for banks located in countries with a vulnerable sovereign.

Results from our calibrated model show that disruptions of different segments of the money markets transmit differently to the macroeconomy. In all cases, the macroeconomic impact can be sizeable.

Our model can be used to assess the impact of a range of unconventional monetary policy measures that were undertaken in the euro area since 2008, namely standard interest rate policy, collateral policy (changes in the composition of the collateral pool or in the haircuts applied to specific classes of assets), main refinancing operations (including changes in their maturity), targeted long-term refinancing operations, and asset purchases. We leave this for future work.

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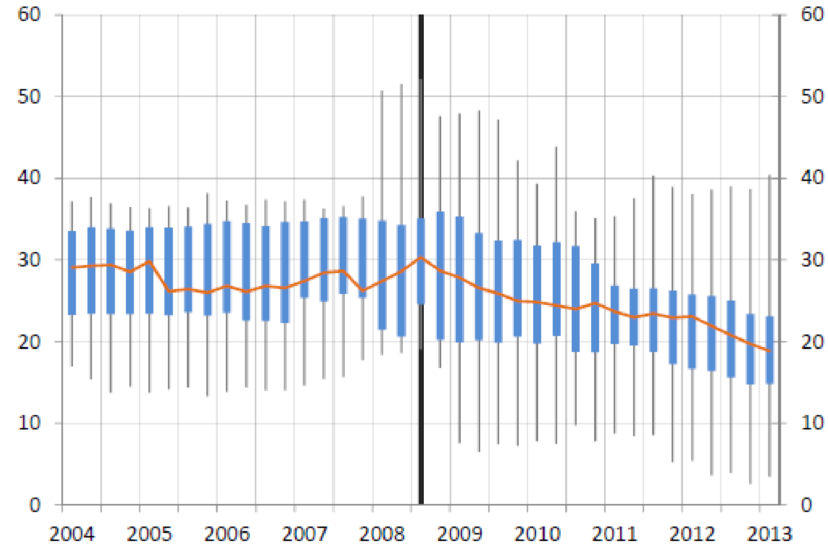


Source: ECB dataset, own calculations

Figure 1: Interbank liabilities in total assets. The grey vertical lines denote the range of observations across euro area countries. The blue vertical lines denote interquartile ranges. The red line denotes the unweighted average ratio.

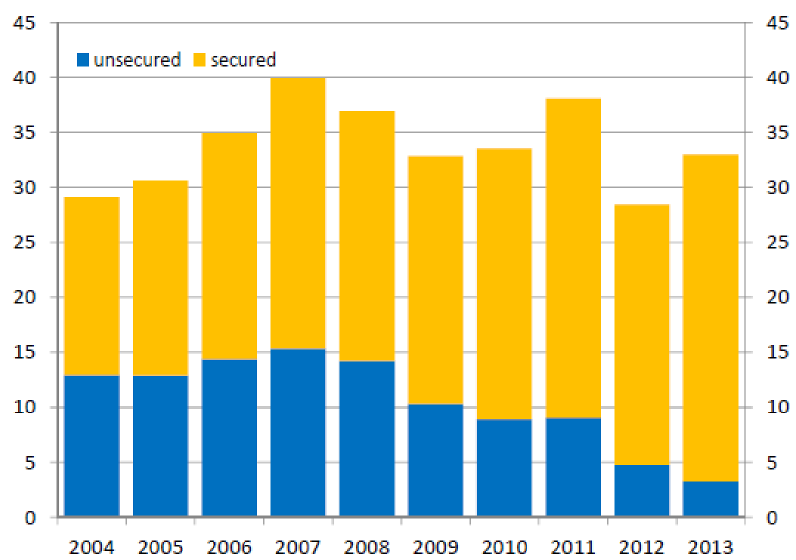
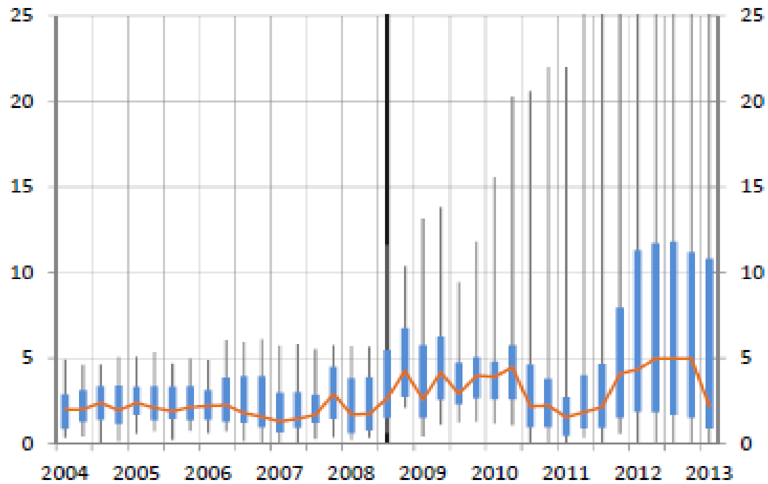


Figure 2: Quarterly turnover in Euro money market. Cumulative quarterly turnove (EUR trillion). Source: Euro Area Money Market Survey. The panel comprises 98 euro area credit institutions.



Source: ECB dataset, own calculations.

Figure 3: Eurosystem funding in total deposit liabilities. The grey vertical lines denote the range of observations across euro area countries. The blue vertical lines denote interquartile ranges. The red line denotes the unweighted average ratio.

Figure 4: Comparative statics with respect to share of connected banks

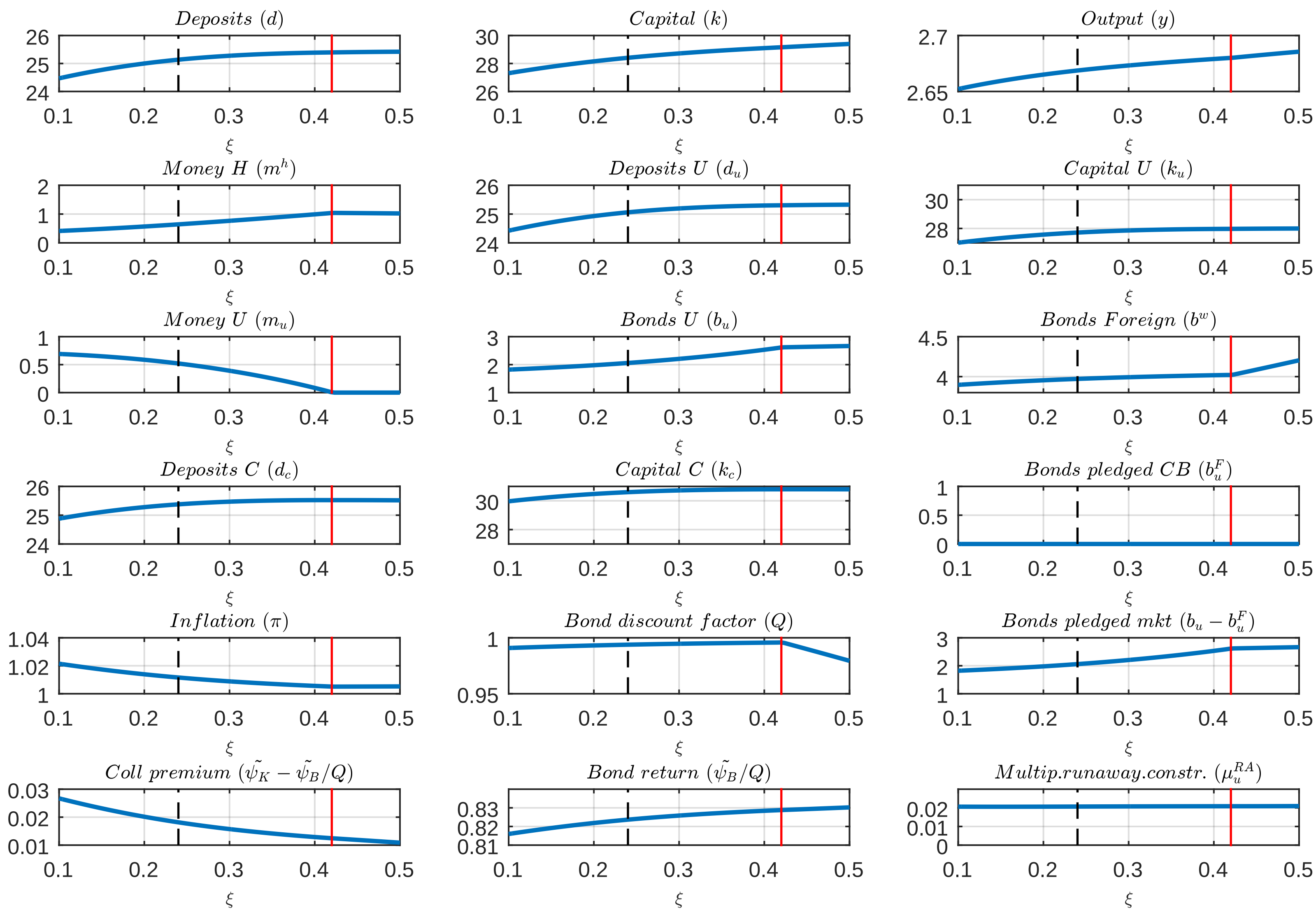


Figure 5: Comparative statics with respect to private haircut on bonds

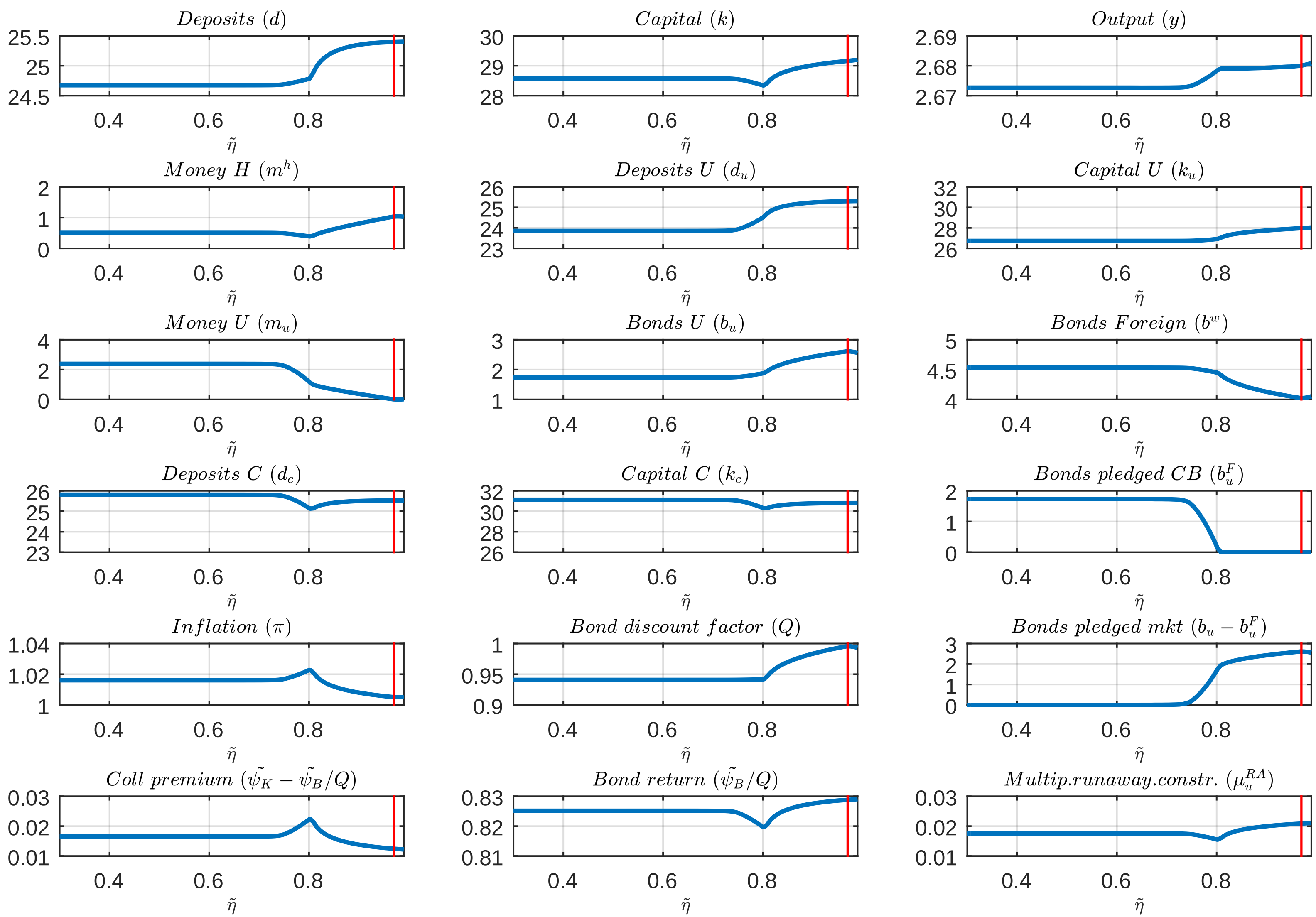


Figure 6: Comparative statics with respect to maximum deposit withdrawal

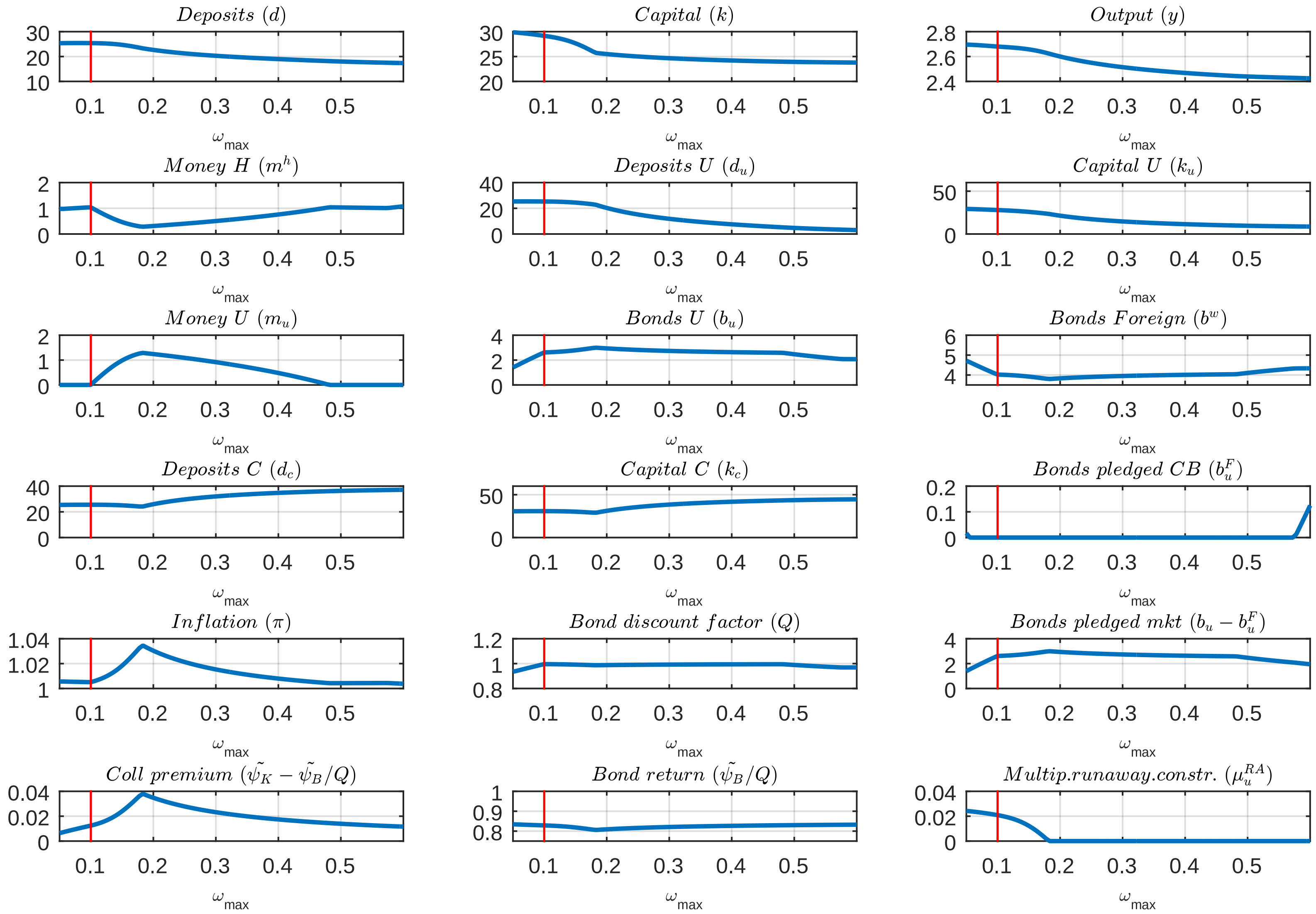


Table 1: ECB vs private haircuts on sovereign bonds

	ECB		Private	
	CQS1-2	CQS3	Germany	Portugal
2010	2.8	7.8	2.7	8.1
2011	2.8	7.8	3.0	10.1
2012	2.8	7.8	3.0	80.0
2013	2.8	7.8	3.0	80.0
2014	2.2	9.4	3.0	80.0

Source: ECB and LCH Clearnet. CQS1-2 refers to sovereign bonds with credit quality 1 and 2, corresponding to a rating AAA to A-. CQS3 refers to bonds with credit quality 3, corresponding to a rating BBB+ to BBB-. Data for private haircuts for 2010 (under the colum 'Germany') refers to an average haircut on bonds from France, Germany and the Netherland.

Table 2: Parameter values

Parameter	Description	Value
θ	Capital share in income	0.33
δ	Capital depreciation rate	0.02
β	Discount rate households	0.995
χ	Coefficient in households' utility	0.006
g	Government spending	0.442
κ^{-1}	Average maturity bonds	9
ϕ	Fraction of net worth paid as dividends	0.031
ξ	Fraction of banks with access to unsecured market	0.42
$\widetilde{\eta}$	Haircut on bonds set by banks	0.97
η	Haircut on bonds set by central bank	0.97
λ	Share of assets bankers can run away with	0.153
$\omega^{\max}$	Max possible withdrawal as share of deposits	0.1
B_C	Bonds held by central bank	1.045
ϱ	Parameter foreign bond demand	0.09

Table 3: Calibration targets

Variable	Data	Model
Debt/GDP	0.57	0.61
Bank leverage	6.00	5.81
Loan spread (annual)	0.021	0.021
Share bonds unconnected banks	0.23	0.23
Share bonds foreign sector	0.64	0.61
Inflation (annual)	0.020	0.021

A The equations characterizing the steady state

We characterize the steady state of the model. For simplicity, we focus on the case when capital is not accepted as collateral at the central bank, $\eta^K = 0$. Recall that we already assume that capital is not accepted as collateral in the private secured market, $\widetilde{\eta}^k = 0$.

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e. $x_t = \frac{X_t}{P_t}$. The steady state is characterized by the following conditions:

1. 4 household equations:

$$R^D = \frac{\pi}{\beta}$$

$$-\frac{u_l(c,l)}{u_c(c,l)} = (1-\tau)\,w$$

$$v_M\left(m^h\right) = u_c(c,n)\left(R^D - 1\right) - \mu_h^M$$

$$c = (1-\tau)\,wl + \left(\frac{1}{\beta} - 1\right)\pi d + (1-\pi)\,m^h + \phi n$$

2. 3 firms' equations:

$$\begin{aligned} y &= \gamma k^{\theta} l^{1-\theta} \\ wl &= (1-\theta)\,y \\ rk &= \theta y \end{aligned}$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$s = Q^F \overline{f} + Q b_C - \overline{m}$$

$$\overline{m} = \left[Q^F - \kappa^F \frac{1}{\pi}\left(1 - Q^F\right)\right]\overline{f} + \left[Q - \kappa \frac{1}{\pi}(1 - Q)\right] b_C$$

plus the value of 3 variables (policy instruments):

$$\eta, Q^F, b_C.$$

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \kappa^F \frac{1}{\pi} \left(1 - Q^F\right) \overline{f} + \kappa \frac{1}{\pi} \left(1 - Q\right) b_C.$$

4. 2 government equations:

$$\begin{aligned} \overline{b} &= \overline{b}^* \\ \tau^* \left(1 - \theta\right) y &= g + \kappa \left(1 - Q\right) \frac{\overline{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi}\right) \overline{b}^* - s. \end{aligned}$$

where g is exogenous.

5. 4 market clearing equations:

$$\overline{f} = f$$

$$\overline{m} = m + m^h$$

$$\overline{b} = b + b_C + b_H + b^W$$

$$y = c + c^W + g + I$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 45 (8+18+7+6+6) bank equations:

8 equations common to c and u banks,

$$\nu = \psi n$$

$$\begin{aligned} n = \; & \max \{0, \; (r + 1 - \delta) \left(\xi k_c + (1 - \xi) k_u\right) \\ & + \left(\xi m_c + (1 - \xi) m_u\right) \frac{1}{\pi} \\ & + \left((1 - \kappa) Q + \kappa\right) \left(\xi b_c + (1 - \xi) b_u\right) \frac{1}{\pi} \\ & - \left(\left(1 - \kappa^F\right) Q^F + \kappa^F\right) \left(\xi f_c + (1 - \xi) f_u\right) \frac{1}{\pi} \\ & - \frac{1}{\beta} \left(\xi d_c + (1 - \xi) d_u\right) \Big\}, \end{aligned}$$

$$\widetilde{v} = \tilde{\psi}_k k + \tilde{\psi}_B b + \tilde{\psi}_M m - \tilde{\psi}_D d - \tilde{\psi}_F f$$

$$\begin{aligned}
\tilde{\psi}_k &= \beta\left(1-\phi\right)\psi\left(r+1-\delta\right)\\
\tilde{\psi}_B &= \beta\left(1-\phi\right)\frac{1}{\pi}\psi\left[\left(1-\kappa\right)Q+\kappa\right]\\
\tilde{\psi}_D &= \beta\left(1-\phi\right)\frac{1}{\beta}\psi\\
\tilde{\psi}_F &= \beta\left(1-\phi\right)\frac{1}{\pi}\psi\left(\left(1-\kappa^F\right)Q^F+\kappa^F\right)\\
\tilde{\psi}_M &= \beta\left(1-\phi\right)\frac{1}{\pi}\psi
\end{aligned}$$

9*2=18 equations for $l = c, u$:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \widetilde{v}_l = \lambda\left(k_l + Qb_l + m_l\right)$$

$$v_l = \phi n + \widetilde{v}_l$$

$$\widetilde{v}_l = \tilde{\psi}_k k_l + \tilde{\psi}_B b_l + \tilde{\psi}_M m_l - \tilde{\psi}_D d_l - \tilde{\psi}_F f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C\left(b_l - b_l^F\right) = 0$$

$$\mu_l^B b_l = 0$$

7 equations for unconnected banks:

$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_k = \mu_u^{BC}+\lambda\mu_u^{RA} \tag{67}$$

$$\left(1+\mu_u^{RA}\right)\frac{\tilde{\psi}_B}{Q}+\mu_u^{B} = \mu_u^{BC}+\lambda\mu_u^{RA}-\mu_u^{C}-\mu_u\widetilde{\eta} \tag{68}$$

$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_M+\mu_u^{M} = \mu_u^{BC}+\lambda\mu_u^{RA}-\mu_u \tag{69}$$

$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_D = \mu_u^{BC}-\omega^{\max}\mu_u \tag{70}$$

$$\left(1+\mu_u^{RA}\right)\frac{\tilde{\psi}_F}{Q^F} = \mu_u^{BC}-\mu_u^{C}\frac{1}{Q^F}+\mu_u^{F}\frac{1}{Q^F} \tag{71}$$

$$\mu_u^{C} = \mu_u^{CC}\eta-\mu_u\widetilde{\eta} \tag{72}$$

$$\mu_u\left[\omega^{\max}d_u-m_u-\widetilde{\eta}Q\left(b_u-b_u^F\right)\right]=0 \tag{73}$$

6 equations for connected banks:

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_k = \mu_c^{BC}+\lambda\mu_c^{RA} \tag{74}$$

$$\left(1+\mu_c^{RA}\right)\frac{\tilde{\psi}_B}{Q}+\mu_c^{B} = \mu_c^{BC}+\lambda\mu_c^{RA}-\mu_c^{C} \tag{75}$$

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_M+\mu_c^{M} = \mu_c^{BC}+\lambda\mu_c^{RA} \tag{76}$$

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_D = \mu_c^{BC} \tag{77}$$

$$\left(1+\mu_c^{RA}\right)\frac{\tilde{\psi}_F}{Q^F} = \mu_c^{BC}-\mu_c^{C}\frac{1}{Q^F}+\mu_c^{F}\frac{1}{Q^F} \tag{78}$$

$$\mu_c^{C} = \mu_c^{CC}\eta \tag{79}$$

6 bank aggregation equations:

$$v=\xi v_c+(1-\xi)\,v_u$$

$$k=\xi k_c+(1-\xi)\,k_u$$

$$d=\xi d_c+(1-\xi)\,d_u$$

$$b=\xi b_c+(1-\xi)\,b_u$$

$$f=\xi f_c+(1-\xi)\,f_u$$

$$m=\xi m_c+(1-\xi)\,m_u.$$

7. 2 rest of the world equations

$$b^W=\varkappa-\frac{1}{\varrho}\log Q_t\pi_t$$

$$Qb^W + c^W = [\kappa + (1-\kappa) Q] \frac{b^W}{\pi}.$$

These are 66 equations (one redundant by the Walras law). The endogenous variables are 65:

$$\left\{ \begin{array}{c} y, k, c, c^W, l, d, n, m^h, b, b^W, f, m, v, \widetilde{v}, \overline{b}, \tau^*, \\ \psi, \tilde{\psi}_k, \tilde{\psi}_B, \tilde{\psi}_M, \tilde{\psi}_D, \tilde{\psi}_F, \mu_u, w, r, Q, R^D, \pi, I, s, \overline{f}, \overline{m} \end{array} \right\}$$

plus

$$\left\{k_l, m_l, f_l, b_l, b_l^F, d_l, v_l, \widetilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\right\},$$

plus the value of the three policy instruments

$$\eta^A, Q^F, b_C,$$

and of the following exogenous variables:

$$g, \xi.$$

The bank first-order conditions can be further simplified as follows. For the unconnected banks, conditions (74)-(79) can be simplified to:

$$\mu_u \left[\omega^{\max} d_u - m_u - \widetilde{\eta} Q \left(b_u - b_u^F\right)\right] = 0 \tag{80}$$

$$\begin{aligned} \omega^{\max}\mu_u &= \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_D\right) - \lambda\mu_u^{RA} && (81) \\ \mu_u^{CC}\eta &= \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^B && (82) \\ \mu_u^M &= \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u && (83) \\ \mu_u^F \frac{1}{Q^F} &= \left(1+\mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) - \omega^{\max}\mu_u + \mu_u^{CC}\frac{1}{Q^F} && (84) \end{aligned}$$

For the connected banks, conditions (74)-(79) can be simplified to,

$$\mu_c^{RA} = \frac{\tilde{\psi}_k - \tilde{\psi}_D}{\lambda - \left(\tilde{\psi}_k - \tilde{\psi}_D\right)} \tag{85}$$

$$\mu_c^{B} = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_c^{CC}\eta \tag{86}$$

$$\mu_c^{M} = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) \tag{87}$$

$$\mu_c^{F}\frac{1}{Q^F} = \left(1 + \mu_c^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_c^{CC}\frac{1}{Q^F} \tag{88}$$

B Proofs

Proof of Proposition 2

By (64), we have $\mu_c^F > 0$ and $f_c = 0$. We first claim that $\mu_c^C = 0$. Intuitively, if a bank l does not borrow from the central bank, it cannot be collateral-constrained at its borrowing from the central bank. Consider the following complementary slackness conditions in the bank problem:

$$\mu_l^C \left(b_l - b_l^F\right) = 0$$

Since $f_l = 0$, we have that $b_l^F = 0$ since $f_l = \eta Q b_l^F$ and $b_l^F \geq 0$. Therefore, the above complementary slackness condition simplifies to

$$\mu_l^C b_l = 0.$$

There are two possibilities: either the bond holdings are positive, $b_l > 0$ or they are zero, $b_l = 0$. In the former case, it follows that $\mu_l^C = 0$, which proves the claim. In the latter case, we have $b_l = 0$, and a bank does not hold any bonds, does not pledge any bonds, and is not constrained by the bond collateral constraint, $\mu_l^C = 0$. Therefore, $\mu_l^C = 0$ holds for a bank that does not borrow from the central bank.

We next show that connected banks do not hold any cash. First-order condition (87) for connected banks implies that whenever

$$\tilde{\psi}_{A,k} > \tilde{\psi}_{A,M}$$

holds, we have $\mu_{A,c}^{M} > 0$ and thus $m_{A,c} = 0$. The above condition is equivalent to

$$\theta \frac{y_A}{(\xi_A k_{A,c} + (1-\xi_A) k_{A,u})} + 1 - \delta > \frac{1}{\pi}.$$

Given (62) and (63), the condition above is always satisfied.

Finally, we show that if the afternoon constraint of unconnected banks binds, $\mu_u > 0$, then a connected bank does not hold any bonds, i.e., $b_c = 0$. Combining (67) with (68), we get

$$\mu_u^B = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^C - \mu_u \tilde{\eta}.$$

Since $\mu_u > 0$, $\mu_u^C \geq 0$ and $\mu_u^B \geq 0$, it follows that

$$\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q} \tag{89}$$

must hold.

Now turning to the connected banks, combine (74) and (75), and use $\mu_c^C = 0$ (since connected banks do not borrow from the central bank), to get

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right)$$

Since $\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q}$, we have that $\mu_c^B > 0$ implying that $b_c = 0$.

Proof of Propositon 3

We first show that if the afternoon constraint is slack, $\mu_u = 0$, then unconnected banks do not borrow from the central bank, $\mu_u^F > 0$ and $f_u = 0$. The claim follows from the first-order condition (84) and condition (64): since $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$ and $\mu_u^{CC} \geq 0$, we have $\mu_u^F > 0$ and $f_u = 0$.

We next show that if $\tilde{\eta} \geq \eta Q^F \omega^{\max}$ holds, then unconnected banks do not borrow from the central bank. We prove the claim by contradiction: suppose $\tilde{\eta} \geq \eta Q^F \omega^{\max}$ holds and yet unconnected banks borrow from the central bank so that $f_u > 0$, $\mu_u^F = 0$ and $b_u^F > 0$. Using (84), we get

$$\omega^{\max} \mu_u = \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u^{CC} \frac{1}{Q^F}.$$

Since $\mu_u^C \geq 0$ we have, by (72), that $\mu_u^{CC} \geq \mu_u \frac{\widetilde{\eta}}{\eta}$. Therefore,

$$\omega^{\max}\mu_u = \left(1+\mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u^{CC}\frac{1}{Q^F} \geq \left(1+\mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_u \frac{\widetilde{\eta}}{\eta}\frac{1}{Q^F}$$

so that

$$\mu_{A,u}\left(\omega^{\max} - \frac{\widetilde{\eta}}{\eta}\frac{1}{Q^F}\right) \geq \left(1+\mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right).$$

The right-hand side of the expression above is positive since $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$ by (64). The left-hand side is non-positive since $\omega^{\max}\eta Q^F \leq \widetilde{\eta}$. A contradiction.

Finally, we show that if $\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}$ holds, then unconnected banks do not borrow in the secured market and instead they borrow only from the central bank and hold money, $m_u > 0$. We prove the claim by contradiction: Suppose that $\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}$ and yet unconnected banks use bonds to borrow from the secured market so that $b_u^F < b_u$ and $\mu_u^C = 0$. Since $b_u > 0$, we have $\mu_u^B = 0$. Since $\mu_u^C = 0$, we have by (72) that $\mu_u^{CC} = \mu_u \frac{\widetilde{\eta}}{\eta}$. Using this to substitute out μ_u^{CC} in (82), we have:

$$\mu_u = \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right)\frac{1}{\widetilde{\eta}}$$

By (83), we have

$$\mu_u^M = \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u \geq 0$$

so that

$$\left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \mu_u.$$

Then, we have that

$$\left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \left(1+\mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right)\frac{1}{\widetilde{\eta}}$$

or, equivalently,

$$\widetilde{\eta} \geq \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}.$$

A contradiction.

Since unconnected banks borrow from the central bank, their afternoon constraint binds, $\mu_u > 0$. Since they do not borrow from the secured market, we have $b_u^F = b_u$. The binding afternoon constraint then implies that

$$m_u = \omega^{\max} d_u > 0.$$

This completes the proof.