Aggregate Demand Externalities in a Global Liquidity Trap

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Abstract

A recent literature has suggested that macroprudential policies can act as second-best stabilization tools when monetary policy is constrained by the zero lower bound. In this paper we show that, once their international dimension is taken into account, macroprudential policies can backfire. We provide a tractable multi-country framework of an imperfectly financially integrated world, in which equilibrium interest rates are low and monetary policy is occasionally constrained by the zero lower bound. Idiosyncratic shocks generate capital flows and asymmetric liquidity traps across countries. Due to a domestic aggregate demand externality, it is optimal for governments to implement countercyclical macroprudential policies, taxing borrowing in good times, as a precaution against the risk of a future liquidity trap triggered by a negative shock. The key insight of the paper is that this policy is inefficient from a global perspective, because it depresses global rates and deepens the recession in the countries currently stuck in a liquidity trap. This international aggregate demand externality points toward the need for international cooperation in the design of financial market interventions. Indeed, under the cooperative optimal financial policy countries internalize the fact that a stronger demand for borrowing and consumption from countries at full employment sustains global rates, reducing the recession in liquidity trap economies.

JEL Codes: E32, E44, E52, F41, F42.

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1 Introduction

The current state of the global economy is characterized by exceptionally low interest rates. In recent years, in fact, nominal rates have hit the zero lower bound in most advanced economies, including the US, the Euro area and Japan (Figure 1, left panel). Interestingly, all these liquidity trap episodes have started with some turmoil on financial markets, and have been accompanied by debt deleveraging (Figure 1, right panel). The link between deleveraging and liquidity traps has been formalized by Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011). Tight access to credit, these authors argue, depresses aggregate demand, pushing down the natural interest rate. If the underlying interest rate is low enough, a period of debt deleveraging will then be associated with a liquidity trap and an economic slump.

Motivated by these facts, a recent literature has suggested that, in a low interest rate environment, governments should actively intervene on the financial markets by implementing countercyclical macroprudential policies (Farhi and Werning, 2016; Korinek and Simsek, 2016). Limiting debt accumulation during a boom, the argument goes, will reduce the drop in aggregate demand and the recession in the event of a deleveraging episode. The need for government intervention arises due to an aggregate demand externality, caused by the fact that atomistic agents do not internalize the impact of their financial decisions on aggregate spending and income. A benevolent government should then tax debt in periods of abundant access to credit, as a precaution against the recessionary liquidity trap that might arise following a negative financial shock.

In this paper we argue that, once their international dimension is taken into account, these policy prescriptions can actually backfire. The intuition is simple. The implementation of macroprudential policies by countries experiencing an economic boom leads to an increase in the global supply of savings or, equivalently, to a fall in the global demand for consumption. In turn, weaker consumption demand generates a drop in interest rates throughout the world, exacerbating the recession in countries currently stuck in a liquidity trap. As a result of these international aggregate demand externalities, global welfare might be higher under financial laissez faire, compared to a world in which countries intervene unilaterally on the credit markets. Our analysis thus points toward the importance of cooperation in the design of macroprudential policies in a low interest rate world.

To formalize these insights, we develop a tractable framework of an imperfectly financially integrated world, in which equilibrium interest rates are low and monetary policy is occasionally constrained by the zero lower bound. The model is simple enough so that many insights can be derived analytically, but still sufficiently rich to perform a quantitative analysis. We study a world composed of a continuum of small open economies inhabited by infinitely lived agents. Countries are hit by uninsurable idiosyncratic shocks. Because of this feature, there is heterogeneity in the demand and supply of savings across countries, and foreign borrowing and lending emerge naturally. For most of the paper, we study a stationary equilibrium in which the cross-country distribution of net foreign assets is constant. Of course, due to the idiosyncratic shocks, individual countries

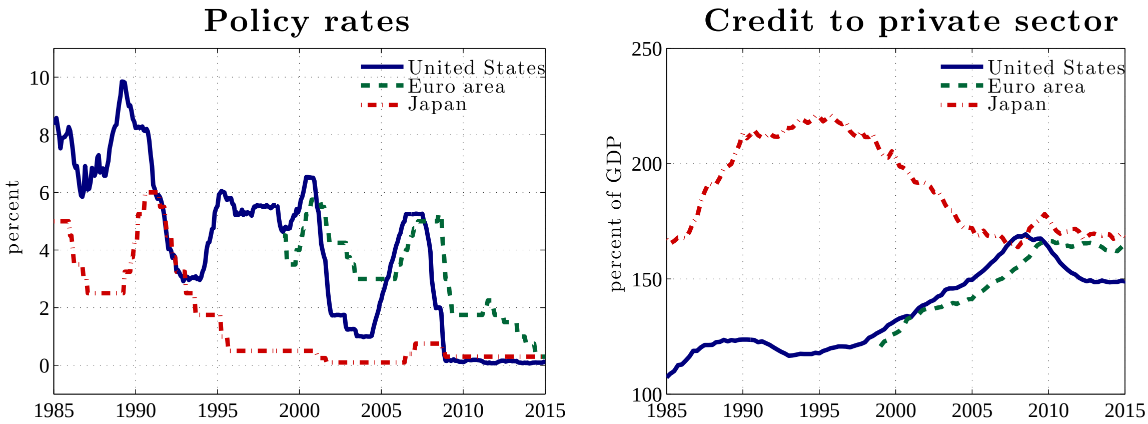

Figure 1: Policy rates and credit to the private non-financial sector. Note: the left panel shows the exceptionally low interest rates characterizing the post-2008 period. Both panels show the emergence of liquidity traps during periods of debt deleveraging by the private sector. See Appendix *D* for data sources.

experience fluctuations in their foreign asset position and in economic activity over time. Aside from standard productivity shocks, we consider "deleveraging" shocks, which tighten a country's access to credit and generate sudden stops in capital inflows. The presence of uninsurable risk against these shocks gives rise to a demand for precautionary savings. In turn, precautionary savings, coupled with a limited supply of assets arising from frictions on the credit markets, depress global interest rates.

Due to the presence of nominal rigidities, monetary policy plays an active role in stabilizing the economy. In fact, when a country experiences a fall in aggregate demand triggered by a negative shock, the domestic interest rate has to fall to keep the economy at full employment. The zero lower bound, however, might prevent monetary policy from fully offsetting the impact of negative shocks on the economy. Indeed, if global rates are sufficiently low, the world can be stuck in a *global liquidity trap*. This is a situation in which a significant fraction of the world economy experiences a liquidity trap with unemployment. Importantly, during a global liquidity trap not all countries need to be constrained by the zero lower bound and experience a recession. Moreover, even among those countries stuck in a liquidity trap there is asymmetry in terms of the severity of the recession. The model thus captures situations such as the asymmetric recovery that has characterized advanced countries in the aftermath of the 2008 financial crisis (Figure 2). Interestingly, a global liquidity trap can persist for an arbitrarily long time, in line with the notion of secular stagnation described by Hansen (1939) and Summers (2016).[1]

Against this background, we show that in good times governments have an incentive to subsidize private savings, or tax borrowing, as a precaution against the risk of a future liquidity trap triggered by a negative shock. This is due to the same domestic aggregate demand externality described by Farhi and Werning (2016) and Korinek and Simsek (2016). In fact, governments perceive that private agents save too little in times of robust economic performance, because they do

[1]Both authors refer to a state of secular stagnation as characterized by low global interest rates, and by countries undergoing long-lasting liquidity traps, followed by fragile recoveries.

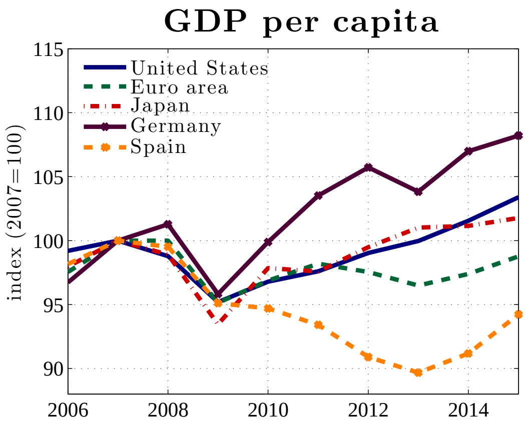

Figure 2: Real gross domestic product per capita. Note: the figure highlights the relatively fast recoveries from the 2009 recession experienced by the US and Japan, and the slow recovery in the Euro area. The figure also shows the heterogeneity between fast-recovering core Euro area countries, captured by Germany, and the stagnation experienced by peripheral Euro area countries, captured by Spain. See Appendix *D* for data sources.

not internalize the impact that their saving decisions will have on aggregate employment and income in the event of a future liquidity trap. Hence, in the absence of international cooperation, governments in countries operating at full employment implement policies to increase savings and current account surpluses beyond what private agents would choose in a laissez faire equilibrium.

The key insight of the paper is that this state of affair is inefficient from a global perspective. By stimulating savings and current account surpluses, governments in countries undergoing a period of robust economic performance increase the global supply of savings, depressing interest rates around the world. This, in turn, aggravates the recession in those countries stuck in a liquidity trap. For instance, switching from a regime of financial laissez faire to one in which governments implement non-cooperative financial policy might exacerbate a global liquidity trap, and lead to a fall in global output and welfare. Moreover, non-cooperative financial market interventions can open the door to global liquidity traps purely driven by pessimistic expectations about the future.

Crucially, since individual countries are small, when acting non-ooperatively governments do not internalize the impact of their financial policies on interest rates and employment in the rest of the world. This is an *international aggregate demand externality* that calls for international cooperation to be corrected. We show that in an uncooperative equilibrium governments in countries at full employment subsidize savings, or tax borrowing, at an inefficiently high rate, compared to what would happen if countries cooperate to maximize global welfare. In fact, under the cooperative optimal policy countries internalize the fact that a stronger demand for borrowing and consumption from countries at full employment sustains global rates, reducing the recession in liquidity trap economies. Indeed, in some cases, in a cooperative equilibrium countries in good times might even impose a tax on savings, in order to stimulate global demand for consumption. Hence, our results point toward the need for international cooperation in the design of financial market interventions when the world experiences a global liquidity trap.

Related literature. This paper is related to three literatures. First, the paper contributes to the emerging literature on secular stagnation in open economies (Caballero et al., 2015; Eggertsson et al., 2016). As in this literature, we study a world trapped in a global liquidity trap. This is a persistent, or even permanent, state of affairs in which global rates are extraordinarily low and countries are frequently constrained by the zero lower bound. Both Caballero et al. (2015) and Eggertsson et al. (2016) study two-country overlapping generations models, in which interest rates are low because of a global shortage of safe assets. Instead, we study economies inhabited by infinitely lived agents, in line with most literature on monetary economics. Moreover, a distinctive feature of our framework is that the shortage of safe assets driving down global rates emerges from countries' demand for precautionary savings against idiosyncratic risk. Finally, while both Caballero et al. (2015) and Eggertsson et al. (2016) present insightful discussions about the international spillovers arising in a global liquidity trap, we are, to the best of our knowledge, the first to derive the optimal cooperative and uncooperative financial policies in a secular stagnating world, as well as to quantify the gains from international cooperation.

The paper is also related to the literature on deleveraging and liquidity traps. As already discussed, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) show that in closed economies deleveraging generates a drop in aggregate demand that can give rise to a recessionary liquidity trap. Building on these positive contributions, Farhi and Werning (2016) and Korinek and Simsek (2016) derive the optimal financial market interventions in closed or small open economies at risk of a liquidity trap following a deleveraging shock.[2] Benigno and Romei (2014) and Fornaro (2012) study deleveraging and liquidity traps in open economies. Both works consider only temporary liquidity traps driven by a one-time global deleveraging shock, and do not focus on optimal financial policy. We contribute to this literature by showing that, aside from domestic aggregate demand externalities, a global liquidity trap is characterized by international aggregate demand externalities, which require international cooperation to be corrected.

Third, our paper is related to the vast literature on international policy cooperation. For instance, Obstfeld and Rogoff (2002) and Benigno and Benigno (2003, 2006) study international monetary policy cooperation in models with nominal rigidities. In these frameworks, the gains from cooperation arise because individual countries have an incentive to manipulate their terms of trade at the expenses of the rest of the world. Similarly, Acharya and Bengui (2016) show that terms of trade externalities create gains from international policy cooperation during a temporary liquidity trap. In our framework, terms of trade are constant and independent of government policy, and hence terms of trade externalities are absent. Thus, our results show that aggregate demand externalities can, on their own, create gains from international policy cooperation during a global liquidity trap. Sergeyev (2016) studies optimal monetary and financial policy in a monetary union, and shows that gains from international cooperation arise because individual countries do not internalize the impact of liquidity creation by the domestic banking sector on the rest of the world. His analysis, however, abstracts from the zero lower bound, which is the source of gains

[2] Farhi and Werning (2012a,b, 2014) and Schmitt-Grohé and Uribe (2015) study optimal financial market interventions when the constraint on monetary policy is due to fixed exchange rates.

from cooperation in our framework.

The rest of the paper is composed by five sections. Section 2 presents a simple baseline framework of an imperfectly financially integrated world with nominal rigidities. Section 3 shows how, in absence of financial market interventions, the world can fall in a global liquidity trap. In Section 4, we use the baseline model to shed light on the optimal cooperative and uncooperative financial policies during a global liquidity trap. Section 5 provides a quantitative analysis based on an extended version of the model. Section 6 concludes.

2 Baseline model

In this section we present a stylized model that delivers transparently the key message of the paper. As we will show in Section 5, the intuitions from this simple model carry through to the extended framework that we use for numerical analysis.

We consider a world composed of a continuum of measure one of small open economies indexed by $i \in \{0, 1\}$. Each economy can be thought of as a country. Time is discrete and indexed by $t \in \{0, 1, ...\}$, and there is perfect foresight.

2.1 Households

Each country is populated by a continuum of measure one of identical infinitely lived households. The lifetime utility of the representative household in a generic country i is

$$\sum_{t=0}^{\infty} \beta^t \log(C_{i,t}), \tag{1}$$

where $C_{i,t}$ denotes consumption and $0 < \beta < 1$ is the subjective discount factor. Consumption is a Cobb-Douglas aggregate of a tradable good $C_{i,t}^T$ and a non-tradable good $C_{i,t}^N$, so that $C_{i,t} = (C_{i,t}^T)^{\omega}(C_{i,t}^N)^{1-\omega}$ where $0 < \omega < 1$.

Each household is endowed with one unit of labor. There is no disutility from working, and so households supply inelastically their unit of labor on the labor market. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_{i,t} < 1$ units of labor. Hence, when $L_{i,t} = 1$ the economy operates at full employment, while when $L_{i,t} < 1$ there is involuntary unemployment, and the economy operates below capacity.

Households can trade in one-period real and nominal bonds. Real bonds are denominated in units of the tradable consumption good and pay the gross interest rate R_t. The interest rate on real bonds is common across countries, and R_t can be interpreted as the *world interest rate.* Nominal bonds are denominated in units of the domestic currency and pay the gross nominal interest rate $R_{i,t}^n$. $R_{i,t}^n$ is the interest rate controlled by the central bank, and thus can be thought of as the domestic policy rate. Notice that, since there is no uncertainty, enriching the menu of assets available to the households would not change the results. In fact, we restrict attention to these two bonds purely to simplify the exposition.

The household budget constraint in terms of the domestic currency is

$$P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N + P_{i,t}^T B_{i,t+1} + B_{i,t+1}^n = W_{i,t} L_{i,t} + P_{i,t}^T Y_{i,t}^T + P_{i,t}^T R_{t-1} B_{i,t} + R_{i,t-1}^n B_{i,t}^n. \quad (2)$$

The left-hand side of this expression represents the household's expenditure. $P_{i,t}^T$ and $P_{i,t}^N$ denote respectively the price of a unit of tradable and non-tradable good in terms of country i currency. Hence, $P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N$ is the total nominal expenditure in consumption. $B_{i,t+1}$ and $B_{i,t+1}^n$ denote respectively the purchase of real and nominal bonds made by the household at time t. If $B_{i,t+1} < 0$ or $B_{i,t+1}^n < 0$ the household is holding a debt.

The right-hand side captures the household's income. $W_{i,t}$ denotes the nominal wage, and hence $W_{i,t} L_{i,t}$ is the household's labor income. Labor is immobile across countries and so wages are country-specific. $Y_{i,t}^T$ is an endowment of tradable goods received by the household. Changes in $Y_{i,t}^T$ can be interpreted as movements in the quantity of tradable goods available in the economy, or as shocks to the country's terms of trade. $P_{i,t}^T R_{t-1} B_{i,t}$ and $R_{i,t-1}^n B_{i,t}^n$ represent the gross returns on investment in bonds made at time $t-1$.

There is a limit to the amount of debt that a household can take. In particular, the end-of-period bond position has to satisfy

$$B_{i,t+1} + \frac{B_{i,t+1}^n}{P_{i,t}^T} \geq -\kappa_{i,t}, \quad (3)$$

where $\kappa_{i,t} > 0$. In words, the maximum amount of debt that a household can take is equal to $\kappa_{i,t}$ units of tradable goods.

The household's optimization problem consists in choosing a sequence $\{C_{i,t}^T, C_{i,t}^N, B_{i,t+1}, B_{i,t+1}^n\}_t$ to maximize lifetime utility (1), subject to the budget constraint (2) and the borrowing limit (3), taking initial wealth $P_0^T R_{-1} B_{i,0} + R_{i,-1}^n B_{i,0}^n$, a sequence for income $\{W_{i,t} L_{i,t} + P_{i,t}^T Y_{i,t}^T\}_t$, and prices $\{R_t, R_{i,t}^n, P_{i,t}^T, P_{i,t}^N\}_t$ as given. The household's first-order conditions can be written as

$$\frac{\omega}{C_{i,t}^T} = R_t \frac{\beta\omega}{C_{i,t+1}^T} + \mu_{i,t} \quad (4)$$

$$\frac{\omega}{C_{i,t}^T} = \frac{R_{i,t}^n P_{i,t}^T}{P_{i,t+1}^T} \frac{\beta\omega}{C_{i,t+1}^T} + \mu_{i,t} \quad (5)$$

$$B_{i,t+1} + \frac{B_{i,t+1}^n}{P_{i,t}^T} \geq -\kappa_{i,t} \quad \text{with equality if} \quad \mu_{i,t} > 0 \quad (6)$$

$$C_{i,t}^N = \frac{1-\omega}{\omega} \frac{P_{i,t}^T}{P_{i,t}^N} C_{i,t}^T, \quad (7)$$

where $\mu_{i,t}$ is the nonnegative Lagrange multiplier associated with the borrowing constraint. Equations (4) and (5) are the Euler equations for, respectively, real and nominal bonds. Equation (6) is the complementary slackness condition associated with the borrowing constraint. Equation (7) determines the optimal allocation of consumption expenditure between tradable and non-tradable goods. Naturally, demand for non-tradables is decreasing in their relative price $P^N_{i,t}/P^T_{i,t}$. Moreover, demand for non-tradables is increasing in $C^T_{i,t}$, due to households' desire to consume a balanced basket between tradable and non-tradable goods.

2.2 Exchange rates, interest rates and aggregate demand

Before moving on, it is useful to illustrate the channels through which the policy rate and the world interest rate affect demand for non-tradable goods. Let us start by establishing a link between demand for non-tradables and the exchange rate. Since the law of one price holds for the tradable good we have that[3]

$$P^T_{i,t} = S_{i,t} P^T_t, \tag{8}$$

where $P^T_t \equiv \exp\left(\int_0^1 \log P^T_{j,t} dj\right)$ is the average world price of tradables, while $S_{i,t}$ is the effective nominal exchange rate of country i, defined so that an increase in $S_{i,t}$ corresponds to a nominal depreciation. Equations (7) and (8) jointly imply that, keeping $P^N_{i,t}$ and P^T_t constant, a nominal exchange rate depreciation increases demand for the non-tradable good. Intuitively, when the exchange rate depreciates the relative price of non-tradables falls, inducing households to switch expenditure away from tradable goods and toward non-tradable goods.

We now relate the exchange rate to the policy and the world interest rates. Combining (4) and (5) gives a no arbitrage condition between real and nominal bonds

$$R^n_{i,t} = R_t \frac{P^T_{i,t+1}}{P^T_{i,t}}. \tag{9}$$

This is a standard uncovered interest parity condition, equating the nominal interest rate to the real interest rate multiplied by expected inflation. Since real bonds are denominated in units of the tradable good, the relevant inflation rate is tradable price inflation. Combining this expression with (8) gives

$$R^n_{i,t} = R_t \frac{S_{i,t+1}}{S_{i,t}} \frac{P^T_{t+1}}{P^T_t}.$$

Taking everything else as given, this expression implies that a drop in $R^n_{i,t}$ produces a rise in $S_{i,t}$. In words, a fall in the policy rate leads to a nominal depreciation, which induces households to switch expenditure out of tradable goods and toward non-tradables. Through this channel, a cut in the policy rate boosts demand for non-tradable goods. Conversely, a fall in the world interest rate R_t generates a nominal exchange rate appreciation which, due to its expenditure switching effect, depresses demand for non-tradables.

To capture these effects more compactly, it is useful to combine (7) and (9) into a single

[3] To derive this expression, consider that by the law of one price it must be that $P^T_{i,t} = S^j_{i,t} P^T_{j,t}$. for any i and j, where $S^j_{i,t}$ is defined as the nominal exchange rate between country i's and j's currencies, that is the units of country i's currency needed to buy one unit of country j's currency. Taking logs and integrating across j gives $P^T_{i,t} = S_{i,t} P^T_t$, where $S_{i,t} \equiv \exp\left(\int_0^1 \log S^j_{i,t} dj\right)$ and $P^T_t \equiv \exp\left(\int_0^1 \log P^T_{j,t} dj\right)$.

aggregate demand (AD) equation

$$C_{i,t}^N = \frac{R_t \pi_{i,t+1}}{R_{i,t}^n} \frac{C_{i,t}^T}{C_{i,t+1}^T} C_{i,t+1}^N, \tag{AD}$$

where $\pi_{i,t} \equiv P_{i,t}^N / P_{i,t-1}^N$. This expression is essentially an open-economy version of the New-Keynesian aggregate demand block. As in the standard closed-economy New-Keynesian model, demand for non-tradable consumption is decreasing in the real interest rate $R_{i,t}^n / \pi_{i,t+1}$ and increasing in future non-tradable consumption $C_{i,t+1}^N$. In addition, changes in the consumption of tradable goods act as demand shifters. As already explained, a higher current consumption of tradable goods increases the current demand for non-tradables. Instead, a higher future consumption of tradables induces households to postpone their non-tradable consumption, thus depressing current demand for non-tradable goods. Finally, due to the expenditure switching effect just discussed, a lower world interest rate is associated with lower demand for non-tradable consumption.

2.3 Firms and nominal rigidities

Non-traded output $Y_{i,t}^N$ is produced by a large number of competitive firms. Labor is the only factor of production, and the production function is $Y_{i,t}^N = L_{i,t}$. Profits are given by $P_{i,t}^N Y_{i,t}^N - W_{i,t} L_{i,t}$, and the zero profit condition implies that in equilibrium $P_{i,t}^N = W_{i,t}$.

We introduce nominal rigidities by assuming that nominal wages are subject to the downward rigidity constraint

$$W_{i,t} \geq \gamma W_{i,t-1},$$

where $\gamma > 0$. This formulation captures in a simple way the presence of frictions to the downward adjustment of nominal wages, which might prevent the labor market from clearing. In fact, equilibrium on the labor market is captured by the condition

$$L_{i,t} \leq 1, \quad W_{i,t} \geq \gamma W_{i,t-1} \quad \text{with complementary slackness.} \tag{10}$$

This condition implies that unemployment arises only if the constraint on wage adjustment binds.[4]

2.4 Monetary policy

We describe monetary policy in terms of targeting rules. In particular, in our baseline model we consider central banks that target inflation of the domestically produced good. More formally, the objective of the central bank is to set $\pi_{i,t} = \bar{\pi}$. Throughout the paper we focus on the case

[4]This form of wage rigidity gives rise to a non-linear wage Phillips curve. For values of wage inflation lower than γ the relationship between wage inflation and employment is vertical. Instead, in presence of unemployment the wage Phillips curve becomes horizontal. It would be easy to allow for an upward-sloped wage Phillips curve. For instance, one could assume that

$$W_{i,t} \geq \tilde{\gamma}(L_{i,t}) W_{i,t-1},$$

where $\tilde{\gamma}'(\cdot) \geq 0$, to capture a setting in which wages are more downwardly flexible the lower employment. For simplicity, in our baseline model we focus on the special case $\tilde{\gamma}'(\cdot) = 0$, but our results readily extend to the more general case $\tilde{\gamma}'(\cdot) \geq 0$.

$\bar{\pi} > \gamma$, so that when the inflation target is attained the economy operates at full employment ($\pi_{i,t} = \bar{\pi} \rightarrow L_{i,t} = 1$). Hence, monetary policy faces no conflict between stabilizing inflation and attaining full employment, thus mimicking the divine coincidence typical of the baseline New Keynesian model (Blanchard and Galí, 2007).[5]

The central bank runs monetary policy by setting the nominal interest rate $R^n_{i,t}$, subject to the zero lower bound constraint $R^n_{i,t} \geq 1$.[6] Monetary policy can then be captured by the following monetary policy (MP) rule[7]

$$R^n_{i,t} = \begin{cases} \geq 1 & \text{if } Y^N_{i,t} = 1, \pi_{i,t} = \bar{\pi} \\ = 1 & \text{if } Y^N_{i,t} < 1, \pi_{i,t} = \gamma, \end{cases} \tag{MP}$$

where we have used (10) and the equilibrium relationships $W_{i,t} = P^N_{i,t}$ and $L_{i,t} = Y^N_{i,t}$. The (MP) equation captures the fact that unemployment ($Y^N_{i,t} < 1$) arises only if the central bank is constrained by the zero lower bound ($R^n_{i,t} = 1$).

2.5 Market clearing and definition of competitive equilibrium

Since households inside a country are identical, we can interpret equilibrium quantities as either household or country specific. For instance, the end-of-period net foreign asset position of country i is equal to the end-of-period holdings of bonds of the representative household, $NFA_{i,t} = B_{i,t+1} + B^n_{i,t+1}/P^T_{i,t}$. Throughout, we focus on equilibria in which nominal bonds are in zero net supply, so that

$$B^n_{i,t} = 0, \tag{11}$$

for all i and t. This implies that the net foreign asset position of a country is exactly equal to its investment in real bonds, i.e. $NFA_{i,t} = B_{i,t+1}$.

Market clearing for the non-tradable consumption good requires that in every country consumption is equal to production

$$C^N_{i,t} = Y^N_{i,t}. \tag{12}$$

[5] Since only the non-tradable good is produced, we are in practice assuming that the central bank follows a policy of producer price inflation targeting. This is a common assumption in the open economy monetary literature. Another option is to consider a central bank that targets consumer price inflation. We have experimented with this possibility, and found that the results are robust to this alternative monetary policy target. The analysis is available upon request.

[6] We provide in appendix C some possible microfoundations for this constraint. In practice, the lower bound on the nominal interest rate is likely to be slightly negative. In this paper, with a slight abuse of language, we will refer the the lower bound on $R^n_{i,t}$ as the zero lower bound. It should be clear, though, that conceptually it makes no difference between a small positive or a small negative lower bound.

[7] One could think of the central bank as setting $R^n_{i,t}$ according to the rule

$$R^n_{i,t} = \max\left(\bar{R}^n_{i,t}\left(\frac{\pi_{i,t}}{\bar{\pi}}\right)^{\phi_\pi}, 1\right),$$

where $\bar{R}^n_{i,t}$ is the value of $R^n_{i,t}$ consistent with $\pi_{i,t} = \bar{\pi}$. In the baseline model we focus on the limit $\phi_\pi \rightarrow \infty$. This means that the inflation target can be missed only if the zero lower bound constraint binds.

Instead, market clearing for the tradable consumption good requires

$$C_{i,t}^{T} = Y_{i,t}^{T} + R_{t-1}B_{i,t} - B_{i,t+1}. \tag{13}$$

This expression can be rearranged to obtain the law of motion for the stock of net foreign assets owned by country i, i.e. the current account

$$NFA_{i,t} - NFA_{i,t-1} = CA_{i,t} = Y_{i,t}^{T} - C_{i,t}^{T} + B_{i,t}\left(R_{t-1} - 1\right).$$

As usual, the current account is given by the sum of net exports, $Y_{i,t}^{T} - C_{i,t}^{T}$, and net interest payments on the stock of net foreign assets owned by the country at the start of the period, $B_{i,t}(R_{t-1} - 1)$.

Finally, in every period the world consumption of the tradable good has to be equal to world production, $\int_0^1 C_{i,t}^{T}\,\mathrm{d}i = \int_0^1 Y_{i,t}^{T}\,\mathrm{d}i$. This equilibrium condition implies that bonds are in zero net supply at the world level

$$\int_0^1 B_{i,t+1}\,\mathrm{d}i = 0. \tag{14}$$

We are now ready to define a competitive equilibrium.

Definition 1 ***Competitive equilibrium.*** *A competitive equilibrium is a path of real allocations* $\{C_{i,t}^{T}, C_{i,t}^{N}, Y_{i,t}^{N}, B_{i,t+1}, B_{i,t+1}^{n}, \mu_{i,t}\}_{i,t}$, *inflation rates* $\{\pi_{i,t}\}_{i,t}$, *policy rates* $\{R_{i,t}^{n}\}_{i,t}$ *and world interest rate* $\{R_t\}_t$, *satisfying* (4), (6), (11), (12), (13), (14), (AD) *and* (MP) *given a path of endowments* $\{Y_{i,t}^{T}\}_{i,t}$, *a path for the borrowing limits* $\{\kappa_{i,t}\}_{i,t}$, *and initial conditions* $\{B_{i,0}\}_i$.

2.6 Some useful simplifying assumptions

We now make some simplifying assumptions that allow us to solve analytically the baseline model. We will relax these assumptions in Section 5, where we perform a numerical analysis.

We want to consider a world in which the global supply of saving instruments is limited, and in which borrowing constraints are tight. The simplest way to formalize this idea is to focus on the limit $\kappa_{i,t} = \kappa \to 0$ for all i and t, so that households cannot take any (significant amount of) debt. This corresponds to a zero liquidity economy, in the spirit of Werning (2015). Later on, in Section 5, we will relax this assumption and allow households to take some debt.

We also focus on a specific process for the tradable endowment. We consider a case in which there are two possible realizations of the tradable endowment: high (Y_h^T) and low (Y_l^T) with $Y_l^T < Y_h^T$. We assume that half of the countries receives Y_h^T in even periods and Y_l^T in odd periods. Symmetrically, the other half receives Y_l^T during even periods and Y_h^T during odd periods. From now on, we will say that a country with $Y_{i,t}^T = Y_h^T$ is in the high state, while a country with $Y_{i,t}^T = Y_l^T$ is in the low state. This endowment process captures in a tractable way an environment in which countries are hit by asymmetric shocks.

Finally, we are interested in studying stationary equilibria in which the world interest rate and the net foreign asset distribution are constant. As we will see, this requires that the initial bond position satisfies $B_{i,0} \approx 0$ for every country i, which we assume throughout our analysis of the baseline model. Moreover, we focus on equilibria in which all the countries with the same endowment shock behave symmetrically. Hence, with a slight abuse of notation, we will sometime omit the i subscripts, and denote with a h (l) subscript variables pertaining to countries in the high (low) state.

3 Equilibrium under financial laissez faire

Before introducing governments' interventions on the financial markets, we characterize the equilibrium under financial laissez faire. This will serve as a benchmark against which to contrast the equilibrium with financial policy. We start by solving for the behavior of a single small open economy, taking the world interest rate as given. We then turn to the global equilibrium, in which the world interest rate is endogenously determined.

3.1 Small open economy

To streamline the exposition, we impose some restrictions on the world interest rate. We will later show that these restrictions emerge naturally in general equilibrium.

Assumption 1 *The world interest rate is constant* ($R_t = R$ *for all* t) *and satisfies* $\beta R < 1$.

Solving for the path of tradable consumption is straightforward. From period 0 on, the economy enters a stationary equilibrium in which households purchase $B_{h,t+1} = B_h \geq 0$ bonds in the high state, while the borrowing constraint binds in the low state, so that $B_{l,t+1} = B_l = 0$.[8] The Euler equation (4) in the high state then implies

$$\frac{1}{C_h^T} \geq \beta R \frac{1}{C_l^T}, \tag{15}$$

where we have removed the time subscripts to simplify the notation. Combining this expression with the resource constraint (13) and using $B_l = 0$ gives the optimal demand for bonds in the high state

$$B_h = \max\left\{\frac{\beta}{1+\beta}\left(Y_h^T - \frac{Y_l^T}{\beta R}\right), 0\right\}, \tag{16}$$

From this expression it is then easy to solve for C_l^T and C_h^T using

$$C_h^T = Y_h^T - B_h \tag{17}$$

$$C_l^T = Y_l^T + RB_h. \tag{18}$$

Notice that, since $\beta R < 1$ and $Y_h^T > Y_l^T$, the competitive equilibrium is such that $C_h^T > C_l^T$. Hence, fluctuations in the endowment translate into fluctuations in the consumption of tradable

[8] To be precise, we should write $B_h \geq -\kappa \approx 0$ and $B_l = -\kappa \approx 0$. To streamline the exposition, however, we slightly abuse the notation and describe the case $\kappa = 0$.

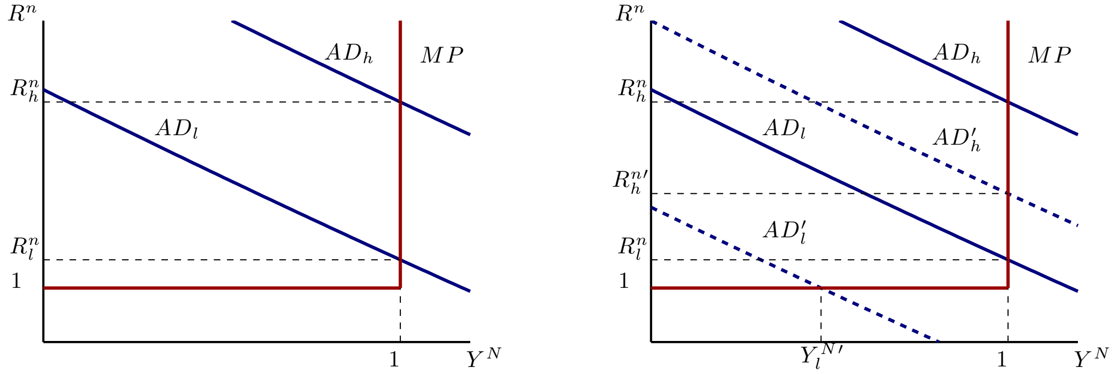

Figure 3: Aggregate demand and employment. Left panel: equilibrium on market for non-tradables. Right panel: high R (solid lines) vs. low R (dashed lines).

goods.

We now turn to the market for non-tradable goods. Equilibrium on this market is reached at the intersection of the (AD) and (MP) equations, which we rewrite here for convenience

$$Y^N_{i,t} = \frac{R\pi_{i,t+1}}{R^n_{i,t}} \frac{C^T_{i,t}}{C^T_{i,t+1}} Y^N_{i,t+1}. \tag{AD}$$

$$R^n_{i,t} = \begin{cases} \geq 1 & \text{if } Y^N_{i,t} = 1, \pi_{i,t} = \bar{\pi} \\ = 1 & \text{if } Y^N_{i,t} < 1, \pi_{i,t} = \gamma, \end{cases} \tag{MP}$$

where we have used the equilibrium condition $C^N_{i,t} = Y^N_{i,t}$. Let us start by taking a partial equilibrium approach, i.e. by deriving the equilibrium holding future variables constant. Figure 3 shows the AD and MP curves in the $R^n_{i,t} - Y^N_{i,t}$ space. The AD curve captures the negative relationship between aggregate demand and the policy rate, while the L-shape of the MP curve captures the aggressive response of the central bank to unemployment. In the left panel, we have drawn two AD curves. The AD$_h$ curve refers to demand in the high state, while AD$_l$ captures demand in the low state. The diagram shows that changes in tradable consumption act as demand shifters, so that aggregate demand is lower in the low state compared to the high state. Hence, when the economy transitions from the high to the low state the central bank decreases the policy rate to sustain aggregate demand.

The right panel of the figure shows how the equilibrium is affected by changes in the world interest rate R. The solid lines capture a world in which R is high. In this case, aggregate demand is sufficiently strong for the economy to operate at full employment in both states. Instead, the dashed lines refer to a low R world. In this case, in the low state aggregate demand is so weak that monetary policy is constrained by the zero lower bound and the economy experiences unemployment.

It turns out that the insights of the partial equilibrium analysis extend to the general equilibrium. We summarize these results in the following proposition.

Assumption 2 *The parameter γ and the world interest rate R are such that $R\gamma > 1$.*

Proposition 1 ***Small open economy under financial laissez faire.*** *There exists a threshold R^*, such that if $R \geq R^*$ then $Y_h^N = Y_l^N = 1$, otherwise $Y_h^N = 1$ and $Y_l^N = R\bar{\pi}\max\left(\beta R, Y_l^T/Y_h^T\right) < 1$. R^* solves $R^*\bar{\pi}\max\left(\beta R^*, Y_l^T/Y_h^T\right) = 1$.*

Proof. See Appendix *B*.1. ■

Proposition 1 states that there exists a threshold R^* for the world interest rate, such that if $R \geq R^*$ the economy always operates at full employment. Instead, if $R < R^*$ aggregate demand in the high state is strong enough to guarantee full employment, while in the low state aggregate demand is sufficiently weak so that monetary policy is constrained by the zero lower bound and unemployment arises. The role of assumption 2 is to guarantee that demand in the high state is always strong enough so that $Y_h^N = 1$. While in principle one could imagine a case in which liquidity traps have infinite duration, here we restrict attention to the, more traditional, case in which liquidity traps are temporary.

3.2 Global equilibrium

We now solve for the global equilibrium. In a global equilibrium the world bond market has to clear, so that (14) holds. Bonds are supplied by countries in the low state. These countries are against the borrowing constraint, and hence the supply of bonds is $-B_l = 0$.[9] Demand for bonds comes from countries in the high state and is given by B_h. In general equilibrium it must then be that $B_h = -B_l = 0$. Hence, the equilibrium allocation of tradable consumption corresponds to the financial autarky one, implying that $C_h^T = Y_h^T$ and $C_l^T = Y_l^T$.

The equilibrium world interest rate must be such that countries in the high state do not want to borrow or save.[10] The value of the equilibrium world interest rate can then be found by substituting $C_h^T = Y_h^T$ and $C_l^T = Y_l^T$ in (15) holding with equality

$$R = \frac{Y_l^T}{\beta Y_h^T} \equiv R^{lf}, \tag{19}$$

where the superscript lf stands for financial *laissez faire*. Expression (19) relates the world interest rate to the fundamentals of the economy. Naturally, a higher discount factor β leads to a higher demand for bonds by saving countries, and thus to a lower world interest rate. Moreover, the world interest rate is decreasing in Y_h^T/Y_l^T, because a higher distance between the two realizations of the endowment increases the desire to save to smooth consumption for countries in the high state. Notice that the equilibrium interest rate satisfies $\beta R < 1$, consistent with assumption 1. We collect these results in the following lemma.

[9] To be precise, since $\kappa \approx 0$, the supply of bonds is positive, albeit infinitesimally small. We write, with a slight abuse of notation, $-B_l = 0$ to streamline the exposition.

[10] In fact, if the borrowing constraint were to bind in the high state, all the countries in the world would want to borrow (albeit an infinitesimally small amount) preventing equilibrium from being reached.

Lemma 1 ***Global equilibrium under financial laissez faire.*** *In a global equilibrium* $C^T_{h,t} = Y^T_h$, $C^T_{l,t} = Y^T_l$. *Under financial laissez faire the equilibrium world interest rate is* $R_t = Y^T_l/(\beta Y^T_h) \equiv R^{lf} < 1/\beta$ *for all* t.

Depending on fundamentals, R^{lf} might be greater or smaller than R^*, the threshold world interest rate below which the zero lower bound binds for countries in the low state.[11] We think of the case $R^{lf} < R^*$ as capturing a world trapped in a global liquidity trap. In such a world, global aggregate demand is weak and countries hit by negative shocks experience liquidity traps with unemployment. Interestingly, this state of affair can persist for an arbitrarily long period of time, as long as global forces imply that $R^{lf} < R^*$. In this sense, the model captures in a simple way the salient features of a world undergoing a period of secular stagnation, in which interest rates are low and liquidity traps frequent (Summers, 2016).

4 Aggregate demand externalities and financial policy

Since there is no disutility from working, unemployment in our model is inefficient. Hence, governments have an incentive to implement policies that limit the incidence of liquidity traps on employment. For instance, a large literature has emphasized how raising expected inflation can mitigate the inefficiencies due to the zero lower bound. However, a robust conclusion of this literature is that, in presence of inflation costs, circumventing the zero lower bound by raising inflation expectations is not an option when the central bank lacks commitment (Eggertsson and Woodford, 2003).

In this paper we take a different route and consider the role of financial policies, broadly defined as policies that affect the country's saving and borrowing decisions, in stabilizing aggregate demand and employment. In particular, we will endow governments with the power to choose directly the country's net foreign asset position and the path of tradable consumption, as long as these do not violate the resource constraint (13) and the borrowing limit (3).

Definition 2 ***Equilibrium with financial policy.*** *An equilibrium with financial policy is a path of real allocations* $\{C^T_{i,t}, C^N_{i,t}, Y^N_{i,t}, B_{i,t+1}, B^n_{i,t+1}, \mu_{i,t}\}_{i,t}$, *inflation rates* $\{\pi_{i,t}\}_{i,t}$, *policy rates* $\{R^n_{i,t}\}_{i,t}$ *and world interest rate* $\{R_t\}_t$, *satisfying* (3), (11), (12), (13), (14), (*AD*) *and* (*MP*) *given a path of endowments* $\{Y^T_{i,t}\}_{i,t}$, *a path for the borrowing limits* $\{\kappa_{i,t}\}_{i,t}$, *and initial conditions* $\{B_{i,0}\}_i$.

An equilibrium with financial policy has to satisfy all the conditions of a competitive equilibrium, except for the Euler equation (4). Hence, even in presence of financial policy the market for non-tradable goods clears competitively, so that the (AD) and (MP) equations have to hold.[12] We will later discuss which instruments governments need to implement the allocation under financial market interventions as part of a competitive equilibrium.

[11] Precisely, $R^{lf} < R^*$ if $\bar{\pi} < \beta(Y^T_h/Y^T_l)^2$, otherwise $R^{lf} \geq R^*$.

[12] Notice that to derive that (AD) equation we have used the no arbitrage condition between real and nominal bonds. Hence, we are assuming that in an equilibrium with financial policy households decide how to allocate their savings between the two bonds. This assumption captures a world with high degree of capital mobility, in which it is difficult for governments to discriminate between domestic and foreign assets.

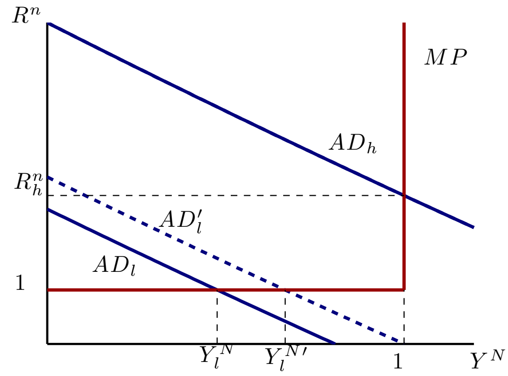

Figure 4: Response to increase in B_h. Solid lines refer to low B_h, dashed lines refer to high B_h.

4.1 Non-cooperative financial policy

We are interested in contrasting the equilibrium reached when governments intervene on the financial markets in absence of cooperation, against a case in which financial policies are designed cooperatively. We begin with the former case.

Let us start by considering a single small open economy. To understand why a government might choose to intervene on the financial markets to stabilize employment, consider that, combining (AD) and (MP), output in the low state under financial laissez faire can be written as

$$Y_l^N = \min\left(R\bar{\pi}C_l^T/C_h^T, 1\right), \tag{20}$$

where we have used the fact that Proposition 1 implies $Y_h^N = 1$, $\pi_h = \bar{\pi}$. Now consider a case in which the zero lower bounds binds, so $Y_l^N < 1$ and output in the low state is demand determined. Imagine that the government implements a policy that leads to an increase in households' savings while the economy is booming, so that B_h increases. On the one hand, higher savings in the high state depress C_h^T. On the other hand, households now enter the low state with higher wealth and, since they are borrowing constrained, increase spending so that C_l^T rises. The net result is that C_l^T/C_h^T increases, boosting demand for non-tradables in the low state. In turn, since the central bank is constrained by the zero lower bound, higher demand for non-tradables leads to higher output and employment. Graphically, as illustrated by Figure 4, an increase in B_h makes the AD$_l$ curve shift right to AD$_l'$, and generates a rise in Y_l^N.[13] Notice that atomistic households, which take aggregate demand and employment as given, do not internalize the impact of tradable consumption decisions on production of non-tradable goods. As we will see, the presence of this *domestic aggregate demand externality* might lead governments to implement financial policies to influence private agents' saving and borrowing decisions.

How does a government optimally intervene on the financial markets? We address this question by taking the perspective of a domestic government, or planner, that designs financial policy to maximize domestic households' welfare. Importantly, since each country is infinitesimally small,

[13] One can show that in our baseline model, as long as $Y_l^N < 1$, changes in C_l^T/C_h^T do not alter aggregate demand in the high state, and hence the AD$_h$ curve does not move after the increase in B_h.

the domestic government takes the world interest rate R as given, and does not internalize the impact of its actions on the rest of the world. Because of this, we refer to the domestic planning allocation as the non-cooperative financial policy.

As it turns out, the planning allocation might differ depending on whether the government operates under commitment or discretion. Rather than considering both cases, throughout the paper we restrict attention to a government that lacks commitment, and hence takes future variables as given. We do this for two reasons. First, the existing literature has shown that if the government operates under commitment monetary policy alone can mitigate substantially the inefficiencies due to the zero lower bound. This suggests that alternative policies, such as financial market interventions, are most useful when the government operates under discretion. Second, the planning problem under discretion highlights transparently the macroprudential role of financial policy in stabilizing the economy.

Formally, we define the period t problem of the domestic planner in a generic country i as

$$\max_{C_{i,t}^T, Y_{i,t}^N, B_{i,t+1}} \sum_{t=0}^{\infty} \beta^t \left(\omega \log C_{i,t}^T + (1-\omega) \log Y_{i,t}^N\right) \tag{21}$$

subject to

$$C_{i,t}^T = Y_{i,t}^T - B_{i,t+1} + RB_{i,t} \tag{22}$$

$$B_{i,t+1} \geq -\kappa_{i,t} \tag{23}$$

$$Y_{i,t}^N \leq 1 \tag{24}$$

$$Y_{i,t}^N \leq R\mathcal{P}(B_{i,t+1}, Y_{i,t+1}^T) \frac{C_{i,t}^T}{\mathcal{C}_T(B_{i,t+1}, Y_{i,t+1}^T)} \mathcal{Y}_N(B_{i,t+1}, Y_{i,t+1}^T). \tag{25}$$

The resource constraints are captured by (22) and (24). (23) implies that the government is subject to the same borrowing constraint imposed by the markets on individual households.[14] Instead, constraint (25), which is obtained by combining the (AD) and (MP) equations, encapsulates the requirement that consumption of non-tradable goods is constrained by private sector's demand. The functions $\mathcal{P}(B_{i,t+1}, Y_{i,t+1}^T)$, $\mathcal{C}_T(B_{i,t+1}, Y_{i,t+1}^T)$ and $\mathcal{Y}_N(B_{i,t+1}, Y_{i,t+1}^T)$ determine respectively inflation, consumption of tradable goods and production of non tradable goods in period $t+1$ as a function of the country's stock of net foreign assets ($B_{i,t+1}$) and the endowment of tradables ($Y_{i,t+1}^T$) at the beginning of next period. Since the current planner cannot make credible commitments about its future actions, these variables are not into its direct control. However, the current planner can still influence these quantities through its choice of net foreign assets. In what follows, we focus on equilibria in which these functions are differentiable, and in which $\mathcal{C}_{B,T}(B_{i,t+1}, Y_{i,t+1}) \geq 0$.

The first order conditions are

$$\bar{\lambda}_{i,t} = \frac{\omega}{C_{i,t}^T} + \bar{\upsilon}_{i,t} \frac{Y_{i,t}^N}{C_{i,t}^T} \tag{26}$$

[14] To write this constraint we have used the equilibrium condition $B_{i,t+1}^n = 0$. It is straightforward to show that allowing the government to set $B_{i,t+1}^n$ optimally would not change any of the results.

$$\frac{1-\omega}{Y_{i,t}^N} = \bar{\nu}_{i,t} + \bar{\upsilon}_{i,t} \tag{27}$$

$$\begin{aligned}\bar{\lambda}_{i,t} &= \beta R \bar{\lambda}_{i,t+1} + \bar{\mu}_{i,t} \\ &+ \bar{\upsilon}_{i,t} Y_{i,t}^N \left[\frac{\mathcal{Y}_{B,N}(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{Y}_N(B_{i,t+1}, Y_{i,t+1}^T)} + \frac{\mathcal{P}_B(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{P}(B_{i,t+1}, Y_{i,t+1}^T)} - \frac{\mathcal{C}_{B,T}(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{C}_T(B_{i,t+1}, Y_{i,t+1}^T)} \right]\end{aligned} \tag{28}$$

$$B_{i,t+1} \geq -\kappa_{i,t} \quad \text{with equality if} \quad \bar{\mu}_{i,t} > 0 \tag{29}$$

$$Y_{i,t}^N \leq 1 \quad \text{with equality if} \quad \bar{\nu}_{i,t} > 0 \tag{30}$$

$$Y_{i,t}^N \leq R\pi_{t+1} \frac{C_{i,t}^T}{C_{i,t+1}^T} Y_{i,t+1}^N \quad \text{with equality if} \quad \bar{\upsilon}_{i,t} > 0, \tag{31}$$

where $\bar{\lambda}_{i,t}, \bar{\mu}_{i,t}, \bar{\nu}_{i,t}, \bar{\upsilon}_{i,t}$ denote respectively the nonnegative Lagrange multipliers on constraints (22), (23), (24) and (25). In addition, $\mathcal{P}_B(B_{i,t+1}, Y_{i,t+1}^T)$, $\mathcal{Y}_{B,N}(B_{i,t+1}, Y_{i,t+1}^T)$ and $\mathcal{C}_{B,t}(B_{i,t+1}, Y_{i,t+1}^T)$ are the partial derivatives of $\mathcal{P}(B_{i,t+1}, Y_{i,t+1}^T)$, $\mathcal{Y}_N(B_{i,t+1}, Y_{i,t+1}^T)$ and $\mathcal{C}_T(B_{i,t+1}, Y_{i,t+1}^T)$, respectively, with respect to B_{t+1}.

To understand why the planner allocation can differ from the competitive equilibrium, it is useful to combine (26) and (28) to obtain

$$\begin{aligned}\frac{1}{C_{i,t}^T}\left(\omega + \bar{\upsilon}_{i,t} Y_{i,t}^N\right) &= \frac{\beta R}{C_{i,t+1}^T}\left(\omega + \bar{\upsilon}_{i,t+1} Y_{i,t+1}^N\right) + \bar{\mu}_{i,t} \\ &+ \bar{\upsilon}_{i,t} Y_{i,t}^N \left[\frac{\mathcal{Y}_{B,N}(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{Y}_N(B_{i,t+1}, Y_{i,t+1}^T)} + \frac{\mathcal{P}_B(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{P}(B_{i,t+1}, Y_{i,t+1}^T)} - \frac{\mathcal{C}_{B,T}(B_{i,t+1}, Y_{i,t+1}^T)}{\mathcal{C}_T(B_{i,t+1}, Y_{i,t+1}^T)} \right].\end{aligned} \tag{32}$$

This is the planner's Euler equation. Comparing this expression with the households' Euler equation (4), it is easy to see that the marginal benefit from a rise in $C_{i,t}^T$ perceived by the planner differs from households' whenever $\bar{\upsilon}_{i,t} > 0$ in any period t, that is when the zero lower bound constraint binds. This happens because, contrary to atomistic households, the government internalizes the aggregate demand externalities that financial decisions have on the domestic economy. For instance, consider a case in which the borrowing constraint does not bind $\bar{\mu}_{i,t} = 0$, the economy operates at full employment in the present $\bar{\upsilon}_{i,t} = 0$, but the zero lower bound binds next period $\bar{\upsilon}_{i,t+1} > 0$. In this case, savings are higher in the planning allocation compared to the financial laissez faire equilibrium. The planner, in fact, internalizes that increasing savings in the present leads to higher aggregate demand and output next period.

The next proposition characterizes the non-cooperative financial policy in a small open economy as a function of the world interest rate.

Proposition 2 ***Small open economy allocation under non-cooperative financial policy.*** *Consider stationary solutions to the non-cooperative planning problem. Define* $R^{**} = \omega Y_l^T/(\beta Y_h^T)$ *and* $\tilde{R} \equiv (\omega/(\bar{\pi}\beta))^{1/2}$*. The planning allocation is such that* $Y_h^N = 1$*,* $B_l = 0$ *and*

$$
\begin{cases}
B_h = 0, Y_l^N = R\bar{\pi}Y_l^T/Y_h^T < 1 & \text{if} \quad R < R^{**} \\
B_h = \frac{\beta}{\omega+\beta}\left(Y_h^T - \frac{\omega Y_l^T}{\beta R}\right), Y_l^N = R^2\bar{\pi}\beta/\omega < 1 & \text{if } R^{**} \leq R < \tilde{R} \\
B_h = \frac{Y_h^T - R\bar{\pi}Y_l^T}{1+R^2\bar{\pi}}, Y_l^N = 1 & \text{if } \tilde{R} \leq R < R^{*} \\
B_h = \max\left\{\frac{\beta}{1+\beta}\left(Y_h^T - \frac{Y_l^T}{\beta R}\right), 0\right\}, Y_l^N = 1 & \text{if } R^{*} \leq R.
\end{cases}
\tag{33}
$$

Moreover, $\bar{\mu}_h > 0$ if $R < R^{**}$ or $R^* \leq R < Y_l^T/(Y_h^T\beta)$, otherwise $\bar{\mu}_h = 0$.

Proof. See Appendix B.2. ■

Corollary 3 *Consider a small open economy facing the world interest rate R. If $R^{**} < R < R^*$ both Y_l^N and B_h are higher under the non-cooperative financial policy than under financial laissez faire, otherwise the two allocations coincide.*

Corollary 3 provides two results. First, if $R \geq R^*$, so that the zero lower bound never binds, the planner chooses the same path for tradable consumption and bonds that households would choose in absence of financial regulation. This result highlights the fact that in our simple model there is no incentive for the domestic government to intervene on the financial markets if monetary policy is not constrained by the zero lower bound.

Second, when the zero lower bound binds in the low state ($R < R^*$), the government intervenes on the financial markets to stabilize output. These interventions have a macroprudential flavor. In fact, the government boosts savings in the high state, when the economy is booming, to mitigate the recession occurring when the economy transitions to the low state.[15] In our financially integrated world, moreover, domestic financial policies have a natural counterpart in terms of international capital flows. Indeed, higher domestic savings manifest themselves in an improvement in the current account. Hence, the government financial policy can be interpreted as improving the current account during booms, so that the economy enters the liquidity trap with a higher stock of external wealth.

Before moving on, it is useful to spend some words on the instruments that a government needs to decentralize the planning allocation. One possibility is to give to the government the power to impose a borrowing limit tighter than the market one, so that (3) is replaced by

$$
B_{i,t+1} + \frac{B^n_{i,t+1}}{P^T_{i,t}} \geq -\min\left\{\kappa_{i,t}, \kappa^g_{i,t}\right\},
$$

where $\kappa^g_{i,t}$ is the borrowing limit set by the government. By setting $\kappa^g_{h,t} < \kappa_{i,t}$ appropriately, the government can replicate the planning allocation as part of a competitive equilibrium. Alternatively, the planning allocation could also be decentralized by means of a subsidy on savings/tax on borrowing. The government would then increase the subsidy in the high state, to foster households' savings. We refer the reader to Farhi and Werning (2016) and Korinek and Simsek (2016)

[15] More precisely, macroprudential policies are most often defined as policies that reduce debt during booms. As we show in Section 5, this is exactly what happens in our full model.

for a detailed discussion of the instruments that the government needs to implement the planning allocation in the competitive equilibrium.

This section extends the results of the literature on aggregate demand externalities and financial market interventions to our setting. Due to the presence of domestic aggregate demand externalities, financial policies act as a complement for monetary policy when the monetary authority is constrained by the zero lower bound. While this point is well understood, little is known about the global implications of these financial market interventions. We tackle this issue in the next section.

4.2 Global equilibrium under the non-cooperative financial policy

We now characterize the global equilibrium when all the countries implement the financial policy described by Proposition 2. Our key result is that, once general equilibrium effects are taken into account, uncoordinated government interventions on the financial markets can backfire by exacerbating the global liquidity trap.

Start by considering that, since countries cannot issue (any significant amount of) debt, in a global equilibrium all the countries must hold (approximately) zero bonds. It follows that, just as in the competitive equilibrium, the allocation of tradable consumption corresponds to the autarky one ($C_l^T = Y_l^T$, $C_h^T = Y_h^T$). Hence, in equilibrium governments' efforts to alter the path of tradable consumption are ineffective.

This does not, however, mean that financial market interventions do not have an effect on other variables or on welfare. The following proposition characterizes the impact of the non-cooperative financial policy, when fundamentals are such that the equilibrium under financial laissez faire corresponds to a global liquidity trap.

Proposition 3 ***Global equilibrium with non-cooperative financial policy.*** *Suppose that $R^{lf} < R^*$. Then $R^{nc} < R^{lf}$, where R^{nc} is the equilibrium interest rate under the non-cooperative financial policy. Moreover, switching from financial laissez faire to non-coordinated financial market interventions lead to lower output and welfare for every country.*

Proof. See Appendix B.3. ∎

Proposition 3 provides a striking result: non-cooperative financial market interventions exacerbate the global liquidity trap, and have a negative impact on global output and welfare. Perhaps the best way to gain intuition about this result is through a diagram. The left panel of Figure 5 displays the demand B_h and supply $-B_l$ of bonds as a function of the world interest rate R. The solid line B_h^{nc} corresponds to the demand for bonds when governments intervene on the financial markets, while the dashed line B_h^{lf} displays the demand for bonds under financial laissez faire. Notice that for $R^{**} < R < R^*$ demand for bonds under the non-cooperative policy is higher than under financial laissez faire. Indeed, this is the range of R for which governments in high-state countries intervene to boost savings. The supply of bonds, instead, does not depend on whether

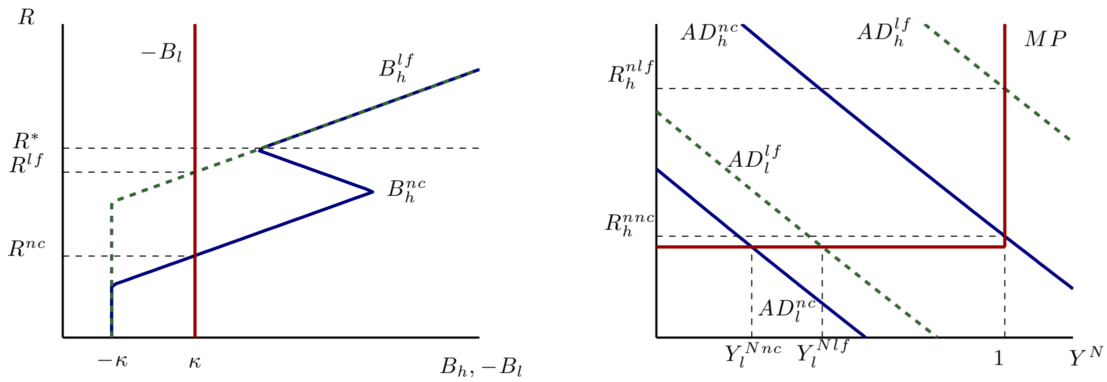

Figure 5: Impact of non-cooperative financial policy in a global liquidity trap.

governments intervene on the financial markets. In fact, in both cases countries in the low state end up being borrowing constrained, and the supply of bonds is $-B_l = \kappa$.

The equilibrium world interest rate is found at the intersection of the B_h and $-B_l$ curves. The diagram shows that $R^{nc} < R^{lf}$, meaning that the world interest rate is lower under non-cooperative financial market policies compared to financial laissez faire. To gain intuition, consider a world with no financial regulation. Now imagine that governments in countries in the high state start to intervene on the financial markets to stimulate savings. This generates an increase in the global demand for bonds. World bonds supply, however, is fixed because countries in the low state are borrowing constrained. To restore equilibrium the interest rate has to fall, so as to bring back the demand for bonds to its equilibrium value of zero.

We now describe how the market for non-tradable goods adjusts to the fall in the world interest rate caused by the implementation of non-cooperative financial policies. As shown by the right panel of Figure 5, a lower world interest rate depresses demand for non-tradable consumption across the whole world, and deepens the recession in countries experiencing a liquidity trap. Through this channel, financial market interventions in booming countries exacerbate the liquidity trap in the rest of the world, leading to a world with lower non-tradable output, and welfare, compared to the unregulated economy.[16]

Importantly, governments in booming countries do not internalize the negative spillovers that their financial policies impose on global demand and on production in countries currently experiencing a liquidity trap. The presence of these *international aggregate demand externalities* suggests that there is a role for international coordination in the design of financial policies. It is then interesting to understand how a global planner that aims at maximizing global welfare chooses financial market interventions optimally.

Before doing that, however, we consider the impact of non-cooperative financial market interventions when fundamentals are such that $R^{lf} \geq R^*$. This corresponds to a case in which, under

[16]To see why welfare is lower under the non-cooperative policy compared to financial laissez faire, consider that financial policies do not affect the equilibrium path of tradable consumption. It follows that their impact on welfare is fully captured by the drop in non-tradable output and consumption.

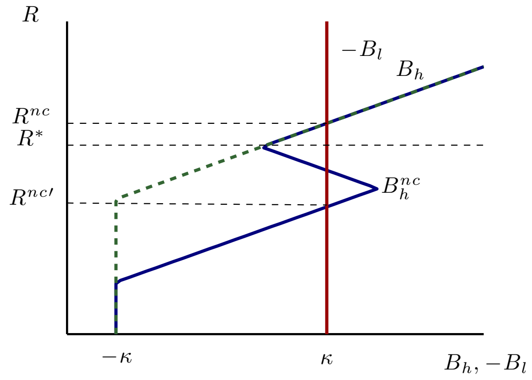

Figure 6: Multiple equilibria with non-cooperative financial policy.

financial laissez faire, the world interest rate is sufficiently high so that the zero lower bound never binds.

Proposition 4 ***Multiple equilibria with non-cooperative financial policy.*** *Suppose that $R^{lf} \geq R^*$. Then $R^{nc} = R^{lf}$ is an equilibrium under the non-cooperative financial policy. This equilibrium is isomorphic to the laissez faire one. However, if $R^{**} < R^*$, there exists at least another equilibrium under the non-cooperative financial policy with associated world interest rate $R^{nc\prime} < R^*$. This equilibrium features lower output and welfare than the laissez faire one.*

Proof. See Appendix *B*.4. ■

One might be tempted to conclude that if $R^{lf} \geq R^*$ then governments will not intervene on the financial markets, and the equilibrium under the non-cooperative policy will coincide with the financial laissez faire one. Indeed, Proposition 4 states that this is a possibility. However, Proposition 4 also states that there might be other equilibria, characterized by financial market interventions and associated with global liquidity traps. Hence, the fact that fundamentals are sufficiently good to rule out a global liquidity trap under financial laissez faire, does not exclude the possibility of a global liquidity trap when governments intervene on the financial markets in absence of international coordination. This result is illustrated by Figure 6, which shows that multiple intersections between the B_h and $-B_l$ curves are possible under uncoordinated financial market interventions.

To gain intuition about this result, consider that governments' actions depend on their expectations about the path of the world interest rate. This happens because the zero lower bound binds only if the world interest rate is sufficiently low. For instance, consider a case in which governments expect that the world interest rate will be always larger than R^*. In this case, governments expect that the zero lower bound will never bind, and hence do not intervene on the financial markets. Since we are focusing on the case $R^{lf} \geq R^*$, in absence of policy interventions the zero lower bound will indeed never bind, confirming the initial expectations. But now think of a case in which governments anticipate that the world interest rate will be always smaller than R^*, so that the zero lower bound is expected to bind in the low state. Then, governments subsidize savings in an attempt to reduce future unemployment. These interventions increase the global supply of savings above its value under financial laissez faire, putting downward pressure on the world interest rate. If $R^{**} < R^{*}$ holds, the drop in the interest rate is sufficiently large so that $R < R^{*}$, validating governments' initial expectations. Thus, in absence of international cooperation, expectations of a future global liquidity trap might generate a global liquidity trap in the present.

We have seen that non-cooperative financial market interventions, while being desirable from the point of view of a single country, can lead to perverse outcomes once their general equilibrium effects are taken into account. First, unilateral implementation of financial regulation during a global liquidity trap generates a drop in output and welfare. Second, uncoordinated interventions on the financial markets open the door to global liquidity traps purely driven by pessimistic expectations. Since all these general equilibrium effects are mediated by the world interest rate, which countries take as given, domestic governments do not internalize the side effects of their financial policies on the rest of the world. We will now show that international cooperation can lead to an efficient outcome, by ensuring that countries internalize the international aggregate demand externalities arising from financial market interventions.

4.3 Cooperative financial policy

We now take the perspective of a global planner that coordinates financial market interventions across countries. Specifically, the global planner selects the equilibrium with financial policy that maximizes the unweighted sum of countries' welfare. Crucially, the global planner internalizes the impact of financial policy on the world interest rate. Hence, the global planner understands that in general equilibrium financial market interventions do not alter the path of tradable consumption, which will always be such that $C_{i,t}^{T} = Y_{i,t}^{T}$ for every i and t. However, the global planner takes into account the impact of financial regulation on aggregate demand for non-tradables and output.

Proposition 5 ***Cooperative financial policy.*** *Consider a global planner that solves*

$$\max_{Y_{i,t}^{N}, R_t} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\omega \log Y_{i,t}^{T} + (1-\omega) \log Y_{i,t}^{N}\right) di$$

subject to

$$Y_{i,t}^{N} \leq 1$$

$$Y_{i,t}^{N} \leq R_t \pi_{t+1} \frac{Y_{i,t}^{T}}{Y_{i,t+1}^{T}} Y_{i,t+1}^{N},$$

taking $Y_{i,t+1}^{N}$ and π_{t+1} as given. The solution is such that $R_t \geq R^{*}$ and $Y_{i,t}^{N} = 1$ for all i and t.

Proof. See Appendix B.5. ■

Corollary 4 *Suppose that the global equilibrium with non-cooperative financial policy corresponds to a global liquidity trap. Then, switching to the cooperative financial policy leads to an increase in the world interest rate, output and welfare.*

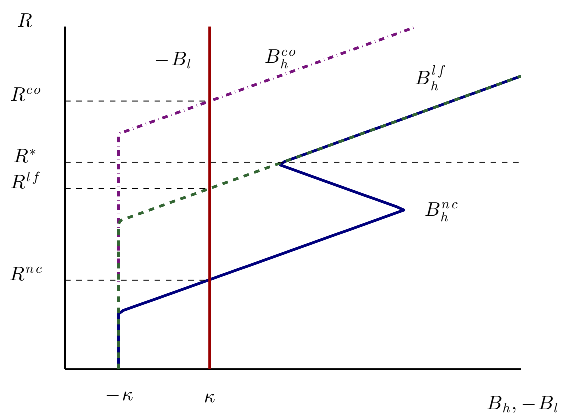

Figure 7: Global equilibrium under cooperative financial policy.

Corollary 4 contains the key insight of the paper: letting governments implement non-cooperative financial policies during a global liquidity trap is inefficient. Indeed, in absence of coordination the supply of savings in booming countries is inefficiently high, because domestic governments do not internalize the negative demand spillovers that their financial policies impose on countries stuck in a liquidity trap. Hence, switching from the uncooperative to the cooperative financial policy during a global liquidity trap entails a reduction in the supply of savings by booming countries. In turn, this leads to a rise in the world interest rate, which sustains global demand for consumption, output and welfare. This result points toward the need to coordinate internationally financial market interventions during a global liquidity trap.

Proposition 5 states some additional results, more specific to the simplified baseline model. In particular, it states that under the cooperative financial policy every country operates at full employment. Intuitively, by intervening on the financial markets, and thus manipulating the demand and supply of bonds, the planner can effectively set the world interest rate R_t. In a global equilibrium, since the allocation of tradable consumption corresponds to the autarky one, changes in R_t affect welfare only through their impact on $Y_{i,t}^N$. It is then easy to see that the global planner will always design financial regulation so that $R_t \geq R^*$. In fact, by doing that, the planner fully eliminates the distortions due to the zero lower bound.[17] Figure 7 illustrates this result by showing that the cooperative financial policy reduces the supply of savings by booming countries, captured by the B_h^{co} line, until the equilibrium world interest rate is sufficiently high to restore full employment ($R^{co} \geq R^*$).[18]

Summarizing, we have shown that the combination of borrowing constraint and shocks to the endowment can give rise to a low interest rate world, in which countries frequently experience liquidity traps and unemployment due to weak aggregate demand. In this world, due to domestic aggregate demand externalities, governments have an incentive to subsidize savings in good times

[17] In Appendix ?? we consider a version of the baseline model in which borrowing is allowed. In that version of the model changes in the subsidy have an impact on the path of tradable consumption in general equilibrium. In the appendix, we discuss the trade-offs faced by the global social planner, and show that it might not be optimal for the global planner to restore full employment.

[18] To implement the allocation under cooperative financial policy as part of a competitive equilibrium, the planner could impose a cap on savings, or impose a subsidy on consumption, in high-state countries.

as a precaution against the risk of a future liquidity trap triggered by a negative shock. However, this policy, by decreasing world demand for consumption and the world interest rate, imposes negative externalities on the countries currently experiencing a liquidity trap. This result points toward the need for cooperation when implementing macroprudential policies in order to correct for aggregate demand externalities.

So far we have drawn conclusions based on an admittedly stylized model. While this model is useful to derive intuition, one might wonder whether these results are driven by some of the specific assumptions that we have made. In what follows, we consider a more realistic framework and show that our conclusions hold true even in a more general setting.

5 Quantitative analysis

TO BE WRITTEN

6 Conclusion

In this paper we have studied optimal credit market interventions during a persistent global liquidity trap. When acting uncooperatively, governments implement macroprudential policies to correct for domestic aggregate demand externalities. The key result of the paper is that this behavior generates international aggregate demand externalities. This happens because when taxing borrowing, governments in countries operating at full employment depress the world interest rate, deepening the recession in countries currently experiencing a liquidity trap. This result points toward the need for international coordination in designing credit market policies in a low interest rate world.

This paper is a first step toward understanding whether gains from cooperation arise in a low interest rate and financially integrated world. A natural future step in this research program would be to analyze the need for coordination arising from other policies. A natural candidate would be fiscal policy. What are the international spillovers from fiscal interventions arising during a global liquidity trap? Is there a need for international coordination when designing public debt policy? We believe that our framework, which strikes a balance between tractability and the ability to deliver quantitative results, represents an excellent laboratory to address these questions.

Appendix

A Additional lemmas

Lemma 2 *Suppose that the market for non-tradable goods clears competitively, so that the (AD) and (MP) equations hold. Then there cannot be a stationary equilibrium with $Y^N_{i,t} < 1$ for all t. Moreover if $C^T_h \geq C^T_l$ then $Y^N_h = 1$.*

Proof. The proof is by contradiction. Suppose that $Y^N_{i,t} < 1$ for all t. (MP) then implies $R^n_{i,t} = 1$ and $\pi_{i,t} = \gamma$ for all t. In a stationary equilibrium the AD equation in the low and high state can then be written as

$$Y^N_h = R\gamma \frac{C^T_h}{C^T_l} Y^N_l \tag{A.1}$$

$$Y^N_l = R\gamma \frac{C^T_l}{C^T_h} Y^N_h. \tag{A.2}$$

Combining these equations gives $1 = (R\gamma)^2$. This contradicts Assumption (2), which states $R\gamma > 1$. Hence, in a stationary equilibrium it must be that $\max\{Y^N_h, Y^N_l\} = 1$.

We now prove that if $C^T_h \geq C^T_l$ then $Y^N_h = 1$. Suppose that this is not the case and $Y^N_h < 1$. Then (MP) implies $R^N_h = 1$, $Y^N_l = 1$ and $\pi_l = \bar{\pi}$. We can then write the (AD) equation in the high state as

$$Y^N_h = R\bar{\pi}\frac{C^T_h}{C^T_l}. \tag{A.3}$$

Assumption (2), $\bar{\pi} \geq \gamma$ and $C^T_h \geq C^T_l$ imply that the right-hand side is larger than one. Hence, $Y^N_h > 1$. We have found a contradiction, and so $C^T_h \geq C^T_l$ implies $Y^N_h = 1$. ■

Lemma 3 *Assume that $\bar{Y}^N_h$, $\bar{\upsilon}_h$ and $\bar{\mu}_l$ are part of a stationary solution in the equilibrium under financial regulation. Then $\bar{Y}^N_h = 1$, $\bar{\upsilon}_h = 0$ and $\bar{\mu}_l > 0$.*

Proof.

Throughout the proof, we will denote with a $^-$ the value of the corresponding variable in the equilibrium under non-cooperative financial policy.

We start by showing that $\bar{Y}^N_h = 1$. Suppose, to reach a contradiction, that with $\bar{Y}^N_h < 1$. Applying Lemma 2, we can set $\bar{Y}^N_l = 1$. Thus, (25) in the high state implies $1 > R\bar{\pi}\bar{C}^T_h/\bar{C}^T_l$. Since $1 < R\gamma \leq R\bar{\pi}$[19] the following inequality needs to hold $\bar{C}^T_l > \bar{C}^T_h$. In our framework, this is possible only if $\bar{B}_h > 0$ ($\bar{\mu}_h = 0$).[20] Combining (27) and (32) in the high state and rearranging gives

$$\frac{\bar{C}^T_l}{\bar{C}^T_h} = \beta R(1-\bar{\nu}_l) - (1-\omega)Y^N_h \frac{\mathcal{C}_{B,T}(B_h, Y^T_l)}{\mathcal{C}_T(B_h, Y^T_l)} \tag{A.4}$$

Since $\beta R < 1$, $\bar{\nu}_l \geq 0$ and $\mathcal{C}_{B,T}(.) \geq 0$, the right-hand side of this equation is always smaller than 1,

[19] We use Assumption (2) together with $\bar{\pi} \geq \gamma$.
[20] We use the following conditions $Y^T_h > Y^T_l$ and $B_l \geq 0$.

implying $\bar{C}_l^T < \bar{C}_h^T$. This is clearly a contradiction. Thus, in the equilibrium under non-cooperative financial policy $\bar{Y}_h^N = 1$ and $\pi_h = \bar{\pi}$.

We turn to prove that $\bar{\upsilon}_h = 0$. Again, to reach a contradiction, assume $\bar{\upsilon}_h > 0$. Since we proved that $\bar{Y}_h^N = 1$, we can use (25) to write

$$1 = R\pi_l \frac{\bar{C}_h^T}{\bar{C}_l^T} Y_l^N. \tag{A.5}$$

First, suppose that $\bar{\upsilon}_l > 0$. Equation (25) implies $\bar{Y}_l^N = R\bar{\pi}\bar{C}_l^T/\bar{C}_l^T$. Using these two conditions equation (A.5) reduces to $1 = R^2\bar{\pi}\pi_l$. We get $\pi_l \geq \gamma$, that contradicts our assumption $R\gamma > 1$. Therefore, it does not exist an equilibrium under non-cooperative financial policy characterized by $\bar{\upsilon}_l > 0$ and $\bar{\upsilon}_h > 0$.

Let us now turn to analyze the case $\bar{\upsilon}_l = 0$ and $\bar{\upsilon}_h > 0$. We set $\bar{Y}_l^N = 1$ and we use (25) to obtain $1 \leq R\bar{\pi}\bar{C}_l^T/\bar{C}_h^T$. Moreover, by using (A.5), we get $\bar{C}_l^T/\bar{C}_h^T = R\bar{\pi}$. Jointly, these two conditions imply $\bar{C}_h^T \leq \bar{C}_l^T$. This is possible only if $\bar{\mu}_h = 0$. Note that if $\bar{\upsilon}_l = \bar{\mu}_h = 0$ and $\bar{Y}_h^N = 1$, we can write equation (32) in the high state as

$$\frac{\omega + \bar{\upsilon}_h}{\bar{C}_h^T} = \beta R \frac{\omega}{\bar{C}_l^T} - \bar{\upsilon}_h \frac{\mathcal{C}_{B,T}(B_h, Y_l^T)}{\mathcal{C}_T(B_h, Y_l^T)}. \tag{A.6}$$

Since $\beta R < 1$ and $\mathcal{C}_T(.) \geq 0$, this expression implies $\bar{C}_l^T < \bar{C}_h^T$. This contradicts our previous finding. Thus, an equilibrium under non-cooperative financial policy is characterized by $\bar{\upsilon}_h = 0$.

We are left to prove that $\bar{\mu}_l > 0$. Suppose that it exists an equilibrium under non-cooperative financial policy characterized by $\bar{\mu}_l = 0$. Thus the Euler equation (32) in the low state implies

$$\frac{\omega + \bar{\upsilon}_l \bar{Y}_l^N}{\bar{C}_l^T} = \beta R \frac{\omega}{\bar{C}_h^T} - \bar{\upsilon}_l \frac{\mathcal{C}_{B,T}(B_l, Y_h)}{\mathcal{C}_T(B_l, Y_h^T)}. \tag{A.7}$$

Since $\beta R < 1$ and $\mathcal{C}_{B,T}(.) \geq 0$, the following condition needs to hold $\bar{C}_h^T < \bar{C}_l^T$. This means that $\bar{\mu}_h = 0$. Then the Euler equation (32) in the high state is

$$\frac{\omega}{\bar{C}_h^T} = \beta R \frac{\omega + \bar{\upsilon}_l \bar{Y}_l^N}{\bar{C}_l^T}. \tag{A.8}$$

By combining (A.7) and (A.8), we find $1 = (\beta R)^2$. This contradicts the assumption $\beta R < 1$. Thus, the equilibrium under non-cooperative financial policy features $\bar{\mu}_l > 0$. ■

B Proofs

B.1 Proof of Proposition 1

Proposition 1 ***Small open economy under financial laissez faire.*** *There exists a threshold R^*, such that if $R \geq R^*$ then $Y_h^N = Y_l^N = 1$, otherwise $Y_h^N = 1$ and $Y_l^N = R\bar{\pi}\max\left(\beta R, Y_l^T/Y_h^T\right) < 1$. R^* solves $R^*\bar{\pi}\max\left(\beta R^*, Y_l^T/Y_h^T\right) = 1$.*

Proof. Since the competitive equilibrium is stationary and satisfies $C_h^T > C_l^T$ Lemma 2 applies. Hence, $Y_h^N = 1$ and $\pi_h = \bar{\pi}$. Using these conditions the AD equation in the low state can be written as

$$Y_l^N = \frac{R\bar{\pi}}{R_l^n}\frac{C_l^T}{C_h^T} = \frac{R\bar{\pi}}{R_l^n}\max\left\{\beta R, \frac{Y_l^T}{Y_h^T}\right\},$$

where the second equality makes use of (16), (17) and (18). Define R^* as the solution to $1 = R^*\bar{\pi}\max\left\{\beta R^*, Y_l^T/Y_h^T\right\}$. Combining the expression above with the MP equation, gives that if $R \geq R^*$, then $Y_l^N = 1$ and $R_l^n \geq 1$, otherwise $R_l^n = 1$ and $Y_l^N = R\bar{\pi}\max\left(\beta R, Y_l^T/Y_h^T\right) < 1$. ■

B.2 Proof of Proposition 2

Proposition 2 ***Small open economy allocation with non-cooperative financial policy.*** *Consider stationary solutions to the non-cooperative planning problem. Define $R^{**} = \omega Y_l^T/(\beta Y_h^T)$ and $\tilde{R} \equiv (\omega/(\bar{\pi}\beta))^{1/2}$. The planning allocation is such that $Y_h^N = 1$, $B_l = 0$ and*

$$\begin{cases} B_h = 0, Y_l^N = R\bar{\pi}Y_l^T/Y_h^T < 1 & \text{if} \quad R < R^{**} \\ B_h = \frac{\beta}{\omega+\beta}\left(Y_h^T - \frac{\omega Y_l^T}{\beta R}\right), Y_l^N = R^2\bar{\pi}\beta/\omega < 1 & \text{if } R^{**} \leq R < \tilde{R} \\ B_h = \frac{Y_h^T - R\bar{\pi}Y_l^T}{1+R^2\bar{\pi}}, Y_l^N = 1 & \text{if } \tilde{R} \leq R < R^* \\ B_h = \max\left\{\frac{\beta}{1+\beta}\left(Y_h^T - \frac{Y_l^T}{\beta R}\right), 0\right\}, Y_l^N = 1 & \text{if } R^* \leq R. \end{cases}$$

Moreover, $\bar{\mu}_h > 0$ if $R < R^{**}$ or $R^* \leq R < Y_l^T/(Y_h^T\beta)$, otherwise $\bar{\mu}_h = 0$.

Proof. In this proof we solve for the equilibrium under non-cooperative financial policy as a function of R. Throughout the proof, we will denote with a $\bar{}$ the value of the corresponding variable in the equilibrium under non-cooperative financial policy, while a $\hat{}$ will denote the value of a variable in the financial laissez faire.

We start by considering the case $R \geq R^*$. Our goal is to show that in this case the equilibrium under financial policy and the one under financial laissez faire coincide. Let us start by guessing that $\bar{\upsilon}_{i,t}^{sp} = 0$ for all t. In this case, the Euler equation in an economy regulated by non-cooperative financial policy (32) is identical to (4), the households' Euler equation in the financial laissez faire. It follows that $\bar{C}_{i,t}^T = \hat{C}_{i,t}^T$ and $\bar{B}_{i,t+1} = \hat{B}_{i,t+1}$ for all t. Since $R \geq R^*$, Proposition 1 implies $\hat{C}_{i,t}^N = 1$ for all t. Moreover, following the steps of the proof of Proposition 1, it is easy to check that $\bar{C}_{i,t}^N = \hat{C}_{i,t}^N = 1$ for all t. This implies that it is possible to set $\bar{\nu}_{i,t}^{sp} > 0$ for all t, and that we can set $\bar{\upsilon}_{i,t}^{sp} = 0$ for all t without violating the optimality condition (27). This verifies our initial guess $\bar{\upsilon}_{i,t}^{sp} = 0$ for all t, and it proves that if $R \geq R^*$ the equilibrium under non-cooperative financial policy and the one under financial laissez faire coincide.

We now turn to the case $R < R^*$. Using Lemma 3, we can show that the equilibrium under non-cooperative financial policy is characterized by $\bar{C}_h^N = 1$, $\bar{\upsilon}_h = 0$ and $\bar{\mu}_l > 0$. The next step of the proof establishes that if $R < R^*$ then $\bar{\upsilon}_l > 0$. Suppose the contrary, meaning that $R < R^*$ and $\upsilon_l = 0$. Then (32) is identical to (4) and the equilibrium under non-cooperative financial policy and the one under financial laissez faire coincide. Note that $R < R^*$ implies $\hat{C}_l^N < 1$.[21] Therefore, by using condition (27) we get $\bar{v}_l > 0$ contradicting our initial assumption. This proves that if $R < R^*$ then $\bar{v}_l > 0$.

For future reference note that when $R < R^*$ the optimality condition (25) in the low state implies

$$\bar{C}_l^N = R\bar{\pi}\bar{C}_l^T/\bar{C}_h^T. \tag{B.1}$$

We are left to prove that when $R < R^*$ if $\bar{\mu}_h = 0$ then $\bar{B}_h > \hat{B}_h$, while if $\bar{\mu}_h > 0$ then $\bar{B}_h = \hat{B}_h = 0$. Consider the Euler equations in the non-cooperative financially regulated economy and in the financial laissez faire economy, respectively, in the high state

$$\frac{\omega}{\bar{C}_h^T} = \frac{\beta R}{\bar{C}_l^T}(\omega + \bar{v}_l\bar{C}_l^N) + \bar{\mu}_h \tag{B.2}$$

$$\frac{\omega}{\hat{C}_h^T} = \frac{\beta R}{\hat{C}_l^T}\omega + \hat{\mu}_h. \tag{B.3}$$

Notice that when $\bar{\mu}_l > 0$ and $\hat{\mu}_l > 0$ then $\bar{B}_l = \hat{B}_l = 0$. Now suppose that $\bar{\mu}_h = 0$ and $\hat{\mu}_h = 0$. Then, since $\bar{v}_l > 0$, the equations above imply that $\bar{C}_l^T/\bar{C}_h^T > \hat{C}_l^T/\hat{C}_h^T$. Clearly, this is also true if $\bar{\mu}_h = 0$ and $\hat{\mu}_h > 0$.[22] But using the resource constraint and $\bar{B}_l = \hat{B}_l = 0$ it is easy to see that to have $\bar{C}_l^T/\bar{C}_h^T > \hat{C}_l^T/\hat{C}_h^T$ it must be that $\bar{B}_h > \hat{B}_h$. This proves that if $\bar{\mu}_h = 0$ then $\bar{B}_h > \hat{B}_h$. Let us now turn to the case $\bar{\mu}_h > 0$. In this case, since $\bar{\mu}_l > 0$, equation ($B.2$) implies that $Y_l^T/Y_h^T > \beta R$. Now suppose that $\hat{\mu}_h = 0$. Then ($B.3$) implies that $\hat{C}_l^T/\hat{C}_h^T = \beta R$. These two expressions jointly imply $Y_l^T/Y_h^T > \hat{C}_l^T/\hat{C}_h^T$. However, $\hat{\mu}_l > 0$ implies $\hat{C}_l^T \geq Y_l^T$, while $\hat{\mu}_l > \hat{\mu}_h = 0$ imply $\hat{C}_h^T \leq Y_h^T$. We have found a contradiction, which means that $\bar{\mu}_h > 0$ implies $\hat{\mu}_h > 0$. This means that if $\bar{\mu}_h > 0$ then $\bar{B}_h = \hat{B}_h = 0$.

In the last part of the proof we solve for $\bar{C}_l^N$ and $\bar{B}_h$ as a function of $R < R^*$. We start by deriving conditions under which $\bar{C}_l^N = 1$. Note that if $R < R^*$ there cannot be an equilibrium with $\bar{C}_l^N = 1$ and $\bar{\mu}_h > 0$. [23] Setting $\bar{\mu}_h = 0$, we can write (32) in the high state as

$$\frac{\omega}{\bar{C}_h^T} = \frac{\beta R}{\bar{C}_l^T}(\omega + \bar{v}_l\bar{C}_l^N) = \frac{\beta R}{\bar{C}_l^T}(1 - \bar{\nu}_l), \tag{B.4}$$

where the second equality makes use of $\bar{C}_l^N = 1$ and (27). Moreover, equation ($B.1$) implies

$$1 = R\bar{\pi}\bar{C}_l^T/\bar{C}_h^T. \tag{B.5}$$

Combining ($B.4$) and ($B.5$) gives

$$\omega = R^2\bar{\pi}\beta(1 - \bar{\nu}_l). \tag{B.6}$$

[21] We can use Proposition 1 to prove it.

[22] In fact, $\hat{\mu}_h > 0$, in conjunction with $\hat{\mu}_l > 0$, implies $\hat{C}_l^T/\hat{C}_h^T = Y_l^T/Y_h^T$. However, $\bar{\mu}_h = 0$ and $\bar{\mu}_l > 0$ imply $\bar{C}_l^T/\bar{C}_h^T > Y_l^T/Y_h^T$.

[23] To see this point, assume that $\bar{\mu}_h > 0$ and $\bar{C}_l^N = 1$. Consider that $R < R^*$ implies $R\bar{\pi}\hat{C}_l^T/\hat{C}_h^T < 1$. Moreover, we can use equation ($B.1$) to write $R\bar{\pi}Y_l^T/Y_h^T = 1$. Hence, $\hat{C}_l^T/\hat{C}_h^T > Y_l^T/Y_h^T$ which is not feasible. We have found a contradiction. Thus, if $R < R^*$ and $\bar{C}_l^N = 1$, it must be that $\bar{\mu}_h = 0$.

Since we are free to set $\bar{\nu}_l$ to any non-negative number, the expression above implies that in order for $\bar{C}_l^N = 1$ to be a solution it must be that $R \geq (\omega/(\bar{\pi}\beta))^{1/2} \equiv \tilde{R}$. To solve for $\bar{B}_h$ we can use (17), (18) and (B.5) to write

$$\bar{B}_h = \frac{Y_h^T - R\bar{\pi}Y_l^T}{1 + R^2\bar{\pi}}. \tag{B.7}$$

It is straightforward to prove that $R < R^*$ implies $Y_h^T > R\bar{\pi}Y_l^T$, and hence $\bar{B}_h > 0$.[24] We can then conclude that for $\tilde{R} \leq R < R^*$ the equilibrium under non-cooperative financial policy features $\bar{C}_l^N = 1 > \hat{C}_l^N$, while $\bar{B}_h$ is given by (B.7).

Finally we characterize the equilibria where $\bar{C}_l^N < 1$. We set $\bar{\nu}_l = 0$ and we use (27) to obtain $\bar{v}_l = (1-\omega)\bar{C}_l^N$. Suppose that the equilibrium is such that $\bar{\mu}_h = 0$. Plugging this condition in the Euler equation for the high state gives

$$\frac{\bar{C}_l^T}{\bar{C}_h^T} = \frac{\beta R}{\omega}. \tag{B.8}$$

By combining the expression above with (B.1) we can write $\bar{C}_l^N$ as a function of exogenous parameters, meaning

$$\bar{C}_l^N = \bar{\pi}\frac{\beta R^2}{\omega}. \tag{B.9}$$

This is a solution if $\bar{B}_h \geq 0$. To solve for $\bar{B}_h$ we can use (17), (18) and (B.8) to write

$$\bar{B}_h = \frac{\beta}{\omega + \beta}\left(Y_h^T - \frac{\omega Y_l^T}{\beta R}\right). \tag{B.10}$$

Hence in the interval $\omega Y_l^T/(\beta Y_h^T) \equiv R^{**} < R < \min\left(R^*, \tilde{R}\right)$, $\bar{C}_l^N = \bar{\pi}\beta R^2/\omega > \bar{\pi}R\max\{\beta R, Y_l^T/Y_h^T\}$ $= \hat{C}_l^N$, while $\bar{B}_h$ is given by (B.10) . Instead if $R \leq R^{**}$ then $\bar{\mu}_h > 0$ and the equilibrium under non-cooperative financial policy and the equilibrium under financial laissez faire coincide. ■

B.3 Proof of Proposition 3

Proposition 3 ***Global equilibrium with non-cooperative financial policy.*** *Suppose that $R^{lf} < R^*$. Then $R^{nc} < R^{lf}$, where R^{nc} is the equilibrium interest rate under the non-cooperative financial policy. Moreover, switching from financial laissez faire to non-coordinated financial market interventions lead to lower output and welfare for every country.*

Proof. Define the function $\mathcal{B}_h(R)$, as the planner's demand for bonds in the high state (33). A solution has to feature $\mathcal{B}_h(R^{nc}) = 0$ and $\bar{\mu}_h = 0$. We start by observing that $R^{nc} \geq R^{**}$. This follows from Proposition 2 which states that if $R < R^{**}$ then $\bar{\mu}_h > 0$.

We now show that if $R^{lf} < R^*$ then there exist a unique $R^{nc} < R^{lf}$. Clearly $R^{nc} \geq R^*$ cannot be a solution. In fact, for $R \geq R^*$ the demand for bonds coincide under financial laissez faire and under non-cooperative financial policy, and so $\mathcal{B}(R) > 0$. Moreover, $\tilde{R} \leq R^{nc} < R^*$ cannot be a

[24] Since $R < R^*$, it is sufficient to prove that $\bar{\pi}R^* \leq Y_h^T/Y_l^T$. Consider that R^* solves $1 = R^*\bar{\pi}\max\left\{\beta R^*, Y_l^T/Y_h^T\right\}$. Suppose that $\beta R^* > Y_l^T/Y_h^T$, meaning $R^* = (\beta\bar{\pi})^{-1/2}$. Then, $(\beta/\bar{\pi})^{1/2} = \frac{1}{\bar{\pi}R^*} > Y_l^T/Y_h^T$. Assume instead $\beta R^* \leq Y_l^T/Y_h^T$, it follows that $\bar{\pi}R^* = Y_h^T/Y_l^T$. Thus, since $R^* \leq \frac{Y_h^T}{Y_l^T\bar{\pi}}$, all $R < R^* < \frac{Y_h^T}{Y_l^T\bar{\pi}}$.

solution. Consider that $\mathcal{B}(R^*) > 0$, and that over the range $\tilde{R} \leq R^{nc} < R^*$ we have $\mathcal{B}'(R) > 0$. This implies that there cannot be an $\tilde{R} \leq R < R^*$ such that $\mathcal{B}(R) = 0$. It must then be that $R^{**} \leq R^{nc} < R^*$. Note R^{nc} is such that $\mathcal{B}(R^{nc}) = 0$, thus we have $R^{nc} = \omega Y_l^T/(\beta Y_h^T) = R^{**}$. Moreover since $\omega < 1$, we obtain $R^{nc} < R^{lf}$.

We now show that Y_l^N and welfare are lower under the non-cooperative interventions compared to laissez faire. Independently of whether governments intervene on the credit markets $C_l^T = Y_l^T, C_h^T = Y_h^T, Y_h^N = 1$ and $\pi_h = \bar{\pi}$. Using (20), we can then write output in the low state as

$$Y_l^N = \min\left(R\bar{\pi}Y_l^T/Y_h^T, 1\right).$$

Since $R^{nc} < R^{lf}$ it immediately follows that Y_l^N is lower under the non-cooperative financial policy equilibrium than under laissez faire equilibrium. Since the impact on welfare of credit market interventions is fully determined by Y_l^N, it follows that welfare is lower under non-cooperative financial policy than under financial laissez faire. ■

B.4 Proof of Proposition 4

Proposition 4 ***Multiple equilibria with non-cooperative financial policy.*** *Suppose that $R^{lf} \geq R^*$. Then $R^{nc} = R^{lf}$ is an equilibrium under the non-cooperative financial policy. This equilibrium is isomorphic to the laissez faire one. However, if $R^{**} < R^*$, there exists at least another equilibrium under the non-cooperative financial policy with associated world interest rate $R^{nc\prime} < R^*$. This equilibrium features lower output and welfare than the laissez faire one.*

Proof. A solution must be such that $\mathcal{B}_h(R^{nc}) = 0$ and $\hat{\mu}_h = 0$. Notice that $R^{nc} = R^{lf}$ is a solution. This is the case because $R^{lf} \geq R^*$ implies that at the solution the demand for bonds in the competitive and uncooperative equilibrium coincide. If $R^{**} \geq R^*$, this is the unique solution, because the demand for bonds are identical for any value of R. Now assume that $R^{**} < R^*$. Then there exists a second solution $R^{nc\prime} = R^{**}$, because $\mathcal{B}_h(R^{**}) = 0$. Moreover, since $R^{**} < R^*$ this second solution corresponds to a global liquidity trap. The welfare statement can be proved following the steps in the proof to Proposition 4. ■

B.5 Proof of Proposition 5

Proposition 5 ***Cooperative financial policy.*** *Consider a global planner that solves*

$$\max_{Y_{i,t}^N, R_t} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\omega \log Y_{i,t}^T + (1-\omega)\log Y_{i,t}^N\right) di$$

subject to

$$Y_{i,t}^N \leq 1$$

$$Y_{i,t}^N \leq R_t \pi_{t+1} \frac{Y_{i,t}^T}{Y_{i,t+1}^T} Y_{i,t+1}^N,$$

taking $Y^N_{i,t+1}$ and π_{t+1} as given. The solution is such that $R_t \geq R^*$ and $Y^N_{i,t} = 1$ for all i and t.

Proof. Suppose that the planner sets $R_t \geq R^*$ for all t. Then, following the steps of the proof to Proposition 1, one can check that $Y^N_{i,t} = 1$ for all i and t. Since the objective function is increasing in $Y^N_{i,t}$ and since the resource constraint implies $Y^N_{i,t} \leq 1$ we have found a solution to the planning problem. ■

C Microfoundations for the zero lower bound constraint

In this appendix we provide some possible microfoundations for the zero lower bound constraint assumed in the main text. First, let us introduce an asset, called money, that pays a private return equal to zero in nominal terms.[25] Money is issued exclusively by the government, so that the stock of money held by any private agent cannot be negative. Moreover, we assume that the money issued by the domestic government can be held only by domestic agents.

We modify the borrowing limit (3) to

$$B_{i,t+1} + \frac{B^n_{i,t+1}}{P^T_{i,t}} + \frac{M_{i,t+1}}{P^T_{i,t}} \geq -\kappa_{i,t},$$

where $M_{i,t+1}$ is the stock of money held by the representative household in country i at the end of period t. The optimality condition for money holdings can be written as

$$\frac{1}{C^T_{i,t}} = \frac{P^T_{i,t}}{P^T_{i,t+1}} \frac{\beta}{C^T_{i,t+1}} + \mu_{i,t} + \mu^M_{i,t},$$

where $\mu^M_{i,t} \geq 0$ is the Lagrange multiplier on the non-negativity constraint for private money holdings, divided by $P^T_{i,t}$. Combining this equation with (5) gives

$$\left(R^n_{i,t} - 1\right) \frac{\beta}{C^T_{i,t+1}} = \mu^M_{i,t}.$$

Since $\mu^M_{i,t} \geq 0$, this expression implies that $R^n_{i,t} \geq 1$. Moreover, if $R^n_{i,t} > 1$, then agents choose to hold no money. If instead $R^n_{i,t} = 1$, agents are indifferent between holding money and bonds. We resolve this indeterminacy by assuming that the aggregate stock of money is infinitesimally small for any country and period.

D Data sources

- Policy rate, US: Board of Governors of the Federal Reserve System (US), Effective Federal Funds Rate.

[25] Here we focus on the role of money as a saving vehicle, and abstract from other possible uses. More formally, we place ourselves in the cashless limit, in which the holdings of money for purposes other that saving are infinitesimally small.

- Policy rate, Euro area and Japan: International Monetary Fund, Discount Rate.
- Credit to private non-financial sector: Bank for International Settlements, Total Credit to Private Non-Financial Sector, Adjusted for Breaks.
- GDP per capita: World Development Indicators, Constant GDP per capita.

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