Financial Vulnerability and Monetary Policy

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May 2017
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Financial vulnerability: amplification mechanisms in the financial sector

Two questions are hotly debated:

1. Does monetary policy impact the degree of financial vulnerability?

2. Should monetary policy take financial vulnerability into account?
Financial Variables Predict Tail of GDP Distribution

“Vulnerable Growth” by Adrian, Boyarchenko and Giannone (2016)
Motivation

Conditional Distribution of Output Gap Growth

![Graph showing the conditional distribution of output gap growth over time, with lines for the 5th quantile, 95th quantile, conditional mean, and output gap growth.]

- Conditional 5th Quantile
- Conditional 95th Quantile
- Output Gap Growth
- Conditional Mean

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Conditional Mean-Volatility Line for Output Gap Growth

\[ \text{Mean} = 0.67 - 1.15 \text{Volatility} + \varepsilon \]
Inflation Quantiles are More Symmetric

![Graph showing inflation quantiles and changes in core inflation over time. The graph includes lines for the 5th and 95th conditional quantiles, as well as changes in core inflation and the conditional mean.]
Motivation

Mean-Volatility Relation for Inflation

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Modeling Financial Vulnerability: Reduced Form

- Define GDP vulnerability $V_t$ as

$$V_t = -\tau E_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t[dy_t/dt]$$

- New Keynesian IS curve with a risk premium driven by vulnerability

$$dy_t = \gamma^{-1} (i_t - r - \pi_t) dt + d(rp_t)$$

$$d\pi_t = (\beta \pi_t - \kappa y_t) dt$$

$$d(rp_t) = \eta \xi \left( V_t - s_t - \frac{\eta}{2\xi \gamma} \right) dt + \xi (V_t - s_t) dZ_t$$

$$ds_t = -\rho_s (s_t - \bar{s}) dt + \sigma_s dZ_t$$
GDP Distributions under Optimal Policy and Taylor Rules
Overview of Microfounded Non-Linear Model

- Firms are exactly as in basic New Keynesian model
- Households are as in New Keynesian model but
  - Cannot finance firms directly
  - Trade other financial assets (stocks, bonds, deposits) with banks
- Banks
  - Finance firms
  - Trade financial assets among themselves and with households
  - Less risk averse than household
  - Has a preference (risk aversion) shock
  - Subject to Value-at-Risk constraint
- Markets are complete and Modigliani-Miller theorem holds everywhere
1. Financial Markets

Financial Structure and Available Securities

- Single source of risk: Browninan motion $Z_t$

- Real riskless bond in zero net supply

$$dS_{0,t} = S_{0,t}R_t\, dt$$

- Two real stocks in positive net supply with returns

$$\frac{dS_{\text{goods},t}}{S_{\text{goods},t}} = (\mu_{\text{goods},t} - R_t)\, dt + \sigma_{\text{goods},t}\, dZ_t$$

$$\frac{dS_{\text{bank},t}}{S_{\text{bank},t}} = (\mu_{\text{bank},t} - R_t)\, dt + \sigma_{\text{bank},t}\, dZ_t$$

where $\mu_{j,t}$ are excess returns including real dividend flow $D_{j,t}$

- Complete set of Arrow-Debreu securities in zero net supply
Price of Risk and No Arbitrage

▶ A state price density (SPD) is a process with $Q_0 \equiv 1$ and

$$\frac{dQ_t}{Q_t} \equiv -R_t \, dt - \eta_t \, dZ_t$$

such that for all assets $j$

$$S_{j,t} = \frac{1}{Q_t} \mathbb{E}_t \left[ \int_t^\infty Q_s D_{j,s} \, ds \right]$$

where $\eta_t$ is the “market price of risk”
Representative Household

- Household solves

\[
\max_{\{C_t, N_t, \omega_t\}_{t \geq s}} \mathbb{E}_s \left\{ \int_s^\infty e^{-\beta(t-s)} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) dt \right\}
\]

subject to

\[
d (P_t F_t) \leq W_t N_t dt - P_t C_t dt - P_t T_t dt + \omega_t d (P_t S_t)
\]

\[
\omega_{\text{goods},t} = 0
\]

\[
\lim_{t \to \infty} \mathbb{E}_s [Q_t F_t] = 0, \quad F_s \text{ given}
\]
Firms are Standard New Keynesian

- Linear production $Y_t(i) = N_t(i)$
- The FOC for intermediate good producers linearized around deterministic steady state gives the standard New Keynesian Phillips Curve

$$d\pi_t = (\beta\pi_t - \kappa y_t) \, dt$$
The Intermediation Sector Setup

- Each “bank” solves a standard Merton portfolio choice problem augmented by a Value-at-Risk constraint and preference shocks

\[
V(X_s, s) = \max_{\{\theta_t, \delta_t\}_{t \geq s}} \mathbb{E}_s \left[ \int_s^\infty e^{-\beta(t-s)} e^{\zeta_t} \log (\delta_t X_t) \, dt \right]
\]

s.t.

\[
\frac{dX_t}{X_t} = (R_t - \delta_t + \theta_t \mu_t) \, dt + \theta_t \sigma_t dZ_t
\]

\[
\text{VaR}_{\tau, \alpha}(X_t) \leq \alpha V X_t
\]

\[
d\zeta_t = -\frac{1}{2}s_t^2 \, dt - s_t \, dZ_t
\]

\[
ds_t = -\kappa(s_t - \bar{s}) + \sigma_s dZ_t
\]

\[X_s \text{ given}\]
The Banks’ VaR Constraint

- Let $\hat{X}_t$ be projected wealth with fixed portfolio weights from $t$ to $t + \tau$
- $VaR_{\tau,\alpha}(X_t)$ is the $\alpha^{th}$ quantile of the distribution of $\hat{X}_{t+\tau}$ conditional on time-$t$ information
- VaR limit is proportional to $X_t$

$$VaR_{\tau,\alpha}(X_t) \leq X_t a_V \iff g(t, \theta_t, \delta_t, R_t, s_t) \leq \log \frac{1}{1 - a_V} \equiv \text{VaR}$$

where

$$g(t, \theta_t, \delta_t, R_t, s_t) \equiv -\tau \mathbb{E}_t[d \log \hat{X}_t] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(d \log \hat{X}_t)$$
Optimal Portfolio

The optimal portfolio is characterized by

\[
\theta_t = \min \{1, \max \{0, \varphi_t\}\} \theta_{M,t} \\
= \gamma_t^{-1} \theta_{M,t} \\
\delta_t = u(t, \min \{1, \varphi_t\}) f_{M,t} \\
\varphi_t \text{ such that: } g(t, \theta_t, \delta_t, R_t, s_t) = \text{VaR}
\]

with

\[
\theta_{M,t} = \mu_t / \sigma_t^2 - s_t / \sigma_t \\
f_{M,t} = \beta \\
u(t, z) \equiv 1 + \frac{\sqrt{\tau |\theta_{M,t} \sigma_t|}}{\mathcal{N}^{-1}(\alpha)} (1 - z)
\]
Market Clearing Conditions

Intermediate goods

\[ \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t = N_t(i) \]

Final good

\[ C_t = Y_t \]

Labor

\[ \left( \frac{(W_t/P_t)C^{-\gamma}}{C} \right)^{\frac{1}{\xi}} = Y_t(i) \]

Bank stock

\[ \frac{F_t \omega_{bank,t}}{S_{bank,t}} + \frac{X_t \theta_{bank,t}}{S_{bank,t}} = 1 \]

Goods producer stock

\[ \frac{X_t \theta_{goods,t}}{S_{goods,t}} = 1 \]

Zero net supply securities

\[ \frac{F_t \omega_{j,t}}{S_{j,t}} + \frac{X_t \theta_{j,t}}{S_{j,t}} = 0 \text{ for } j \neq bank, goods \]
Equilibrium Conditions

The system of equations

\[
\begin{align*}
  dy_t &= \frac{1}{\gamma} \left( i_t - r - \pi_t + \frac{1}{2} \eta_t^2 \right) \, dt + \frac{\eta_t}{\gamma} \, dZ_t \\
  d\pi_t &= (\beta \pi_t - \kappa y_t) \, dt \\
  i_t &= i \text{(state variables)} \\
  R_t &= i_t - \pi_t \\
  \eta_t &= \eta(\varphi_t, R_t, s_t) \\
  d\varphi_t &= G_\varphi (\varphi_t, R_t, s_t) \, dt + S_\varphi (\varphi_t, R_t, s_t) \, dZ_t \\
  ds_t &= -\kappa (s_t - \bar{s}) + \sigma_s dZ_t 
\end{align*}
\]

fully characterizes the equilibrium
6. Equilibrium Characterization

Characterization with Output Gap Vulnerability

To connect model with the data, introduce

\[ V_t \equiv -\tau \mathbb{E}_t [dy_t/dt] - N^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt) \]

\[ = -\frac{\tau}{\gamma} \left( R_t - r + \frac{1}{2} \eta_t^2 \right) - N^{-1}(\alpha) \frac{\sqrt{\tau} \eta_t}{\gamma} \]

Write \( \eta_t = \eta(\varphi_t, R_t, s_t) \) in terms of \( \varphi_t, V_t, s_t \)
Equilibrium Conditions

The system of equations

\[
\begin{align*}
\frac{dy_t}{dt} &= \frac{1}{\gamma} \left( i_t - r - \pi_t + \frac{1}{2} \eta(\varphi_t, V_t, s_t)^2 \right) dt + \frac{\eta(\varphi_t, V_t, s_t)}{\gamma} dZ_t \\
\frac{d\pi_t}{dt} &= (\beta \pi_t - \kappa y_t) dt \\
i_t &= i \text{(state variables)} \\
R_t &= i_t - \pi_t \\
V_t &= -\tau \mathbb{E}_t[dy_t/dt] - N^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt) \\
d\varphi_t &= G_{\varphi}(\varphi_t, R_t, s_t) dt + S_{\varphi}(\varphi_t, R_t, s_t) dZ_t \\
\frac{ds_t}{dt} &= -\kappa (s_t - \bar{s}) + \sigma_s dZ_t
\end{align*}
\]

fully characterizes the equilibrium
“Second Order” Linear Approximation

- Linearize drift and volatility

\[
\begin{align*}
dy_t &= \left( \frac{1}{\gamma} (R_t - r) + \hat{\eta} \xi \left( V_t - \varphi_t - s_t - \frac{\hat{\eta}}{2\xi\gamma} \right) \right) dt + \\
&\quad + \xi (V_t - \varphi_t - s_t) dZ_t \\
d\pi_t &= (\beta \pi_t - \kappa y_t) dt \\
i_t &= i(\text{state variables}) \\
R_t &= i_t - \pi_t \\
V_t &= -\tau E_t(dy_t/dt) - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt) \\
d\varphi_t &= (\gamma_0 + \gamma_r R_t + \gamma_\varphi \varphi_t + \gamma_s s_t) dt + \\
&\quad + (\psi_0 + \psi_r R_t + \psi_\varphi \varphi_t + \psi_s s_t) dZ_t \\
ds_t &= -\kappa (s_t - \bar{s}) + \sigma_s dZ_t
\end{align*}
\]
Optimal Monetary Policy

- Focus on simpler case with no direct impact of monetary policy on $\varphi$: Mechanism is through general equilibrium (prices of risk) only
- Abstract from Phillips Curve
  - Either fixed prices or long-run equilibrium
  - Straightforward to incorporate into analysis
- General case still linear-quadratic, can be solved in closed form
Optimal Monetary Policy

Central bank solves

\[ L = \min_{\{R_s\}_{s=t}^{\infty}} \mathbb{E}_t \int_t^{\infty} e^{-s\beta} y_s^2 ds \]

subject to

\[ dy_t = \left( \frac{1}{\gamma} (R_t - r) + \hat{\eta} \xi \left( V_t - s_t - \frac{\hat{\eta}}{2\xi\gamma} \right) \right) dt + \xi (V_t - s_t) dZ_t \]

\[ V_t = -\tau \mathbb{E}_t [dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} Vol_t(dy_t/dt) \]

\[ ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t \]
Optimal Monetary Policy

- Plugging

\[ \mathbb{E}_t[dy_t/dt] = \frac{1}{\gamma} (R_t - r) + \hat{\eta} \xi \left( V_t - s_t - \frac{\hat{\eta}}{2\xi \gamma} \right) \]

\[ Vol_t(dy_t/dt) = \xi (V_t - s_t) \]

into

\[ V_t = -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} Vol_t(dy_t/dt) \]

we see that \( V_t \) and \( R_t \) are one-to-one
Optimal Monetary Policy

Output Gap Mean-Volatility Tradeoff

- $R_t$ and $V_t$ are one-to-one, so think of $V_t$ as central bank’s choice
- Eliminating $R_t$, dynamics of the economy are

$$dy_t = \xi \left( M \times V_t + \frac{N^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi (V_t - s_t) dZ_t$$

where

$$M \equiv -\frac{1 + N^{-1}(\alpha) \sqrt{\tau} \xi}{\tau \xi}$$

is the slope of the mean-volatility line for the output gap

$$\mathbb{E}_t [dy_t/dt] = M \times Vol_t (dy_t/dt) - \frac{1}{\tau} s_t$$
Conditional Mean-Volatility Line for Output Gap Growth

Mean = 0.67 - 1.15 Volatility + ε
8. Optimal Monetary Policy

Tradeoff for Monetary Policy

- Mean variance tradeoff since $M < 0$

\[
dy_t = \xi \left( M \times V_t + \frac{N^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi (V_t - s_t) dZ_t
\]

- Changes in $V_t$ move the economy along the mean-vol line
- Shocks $s_t$ shift the line up and down
- Because $M < 0$, we have $\partial V_t / \partial R_t < 0$: Tighter policy reduces vulnerability
- Divine coincidence broken by financial vulnerability
The Optimal Monetary Policy

- Re-introduce Phillips Curve
- Augmented Taylor

\[ i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_v V_t \]

- Can be expressed as flexible inflation targeting

\[ \pi_t = \psi_0 + \psi_y y_t + \psi_v V_t + \psi_s s_t \]

- Coefficients \( \phi \) and \( \psi \) are a function of structural parameters that govern GDP vulnerability
Calibration

- Calibration comes directly from a regression of the conditional mean on the conditional vol of the output gap
- Pick $\beta = 0.01$, $\alpha = 5\%$, $\tau = 1$ and match intercept, slope, standard deviation and AR(1) coefficient of residuals to get

\[
\begin{align*}
\xi &= 0.36 \\
\bar{s} &= -0.67 \\
\sigma_s &= 0.61 \\
\kappa &= 2.14
\end{align*}
\]
Welfare Gains
Our calibration gives answers different from Svensson, Curdia and Woodford

Empirically

- Look at different data – risk premia in financial sector
- Forecast downside risk, not just conditional means

Theoretically

- Different amplification mechanism brought about by VaR constraint
- Not just about crises and tails; tradeoff is always present
Conclusion

- The NK model can be augmented by
  - A financial sector that intermediates subject to a Value-at-Risk constraint
  - Shocks to financial sector

- The “second order” linearization approximation
  - Matches the stylized fact that conditional upper GDP quantiles are constant, while lower GDP quantiles move with financial conditions
  - Mathematically tractable

- Optimal monetary policy always depends on vulnerability
  - Optimal monetary policy conditions on vulnerability
  - Vulnerability responds to monetary policy
  - Magnitudes are potentially large quantitatively