

# Term Structure and Real-Time Learning

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Agents' real-time process of learning is fundamental to understand macroeconomic dynamics, moreover:

- agents have limited knowledge about many dimensions of the true DGP (parameter values, state variables, the nature of shocks, ...)
- agents have limited information on observables when forming their expectations (latent variables, real-time information, ...)
- agents have problems processing information in an efficient manner (short vs long sightedness, ...)

These concerns are traditionally ignore by models with Rational Expectations...

There is growing literature analyzing the consequences of deviating from the standard assumption of RE. There are many approaches:

- Rational inattention approach (Sims, 2003; Adam, 2007; Mackowiach and Wiederholt, 2009; ...)
- Sticky information approach (Reis, 2009; ...)
- Imperfect information approach (Svensson and Woodford, 2004; Coenen, Levin and Wieland, 2005; ...)
- Limited information and real-time data (Aruoba, 2004; Pruitt, 2012; Vázquez, María-Dolores and Londoño, 2013; Casares and Vázquez, 2016;...)
- **Adaptive learning (AL) approach** (Orphanides and Williams, 2005; Branch and Evans, 2006; Milani, 2007, 2008, 2011; Eusepi and Preston, 2011; Levine, Pearlman, Perendia and Yang, 2012; Slobodyan and Wouters, 2012a, 2012b; Ormeño and Molnár, 2015;...)

Most estimated AL models typically consider forecasting models based on variables whose observable counterparts are *final revised*

This is problematic: **Learning dynamics are in reality driven by data truly available to agents when forming their expectations in real time**

There are a few exceptions:

- Milani (2011) focuses on real-time data on output and inflation and the forecasts from the SPF recorded in real time when estimating a small-scale DSGE model, but he ignores revised data on macroeconomic variables, which more accurately describe the actual economy
- Slobodyan and Wouters (2017) also used SPF inflation data as observable

This paper deals with the fact of imperfect information by assuming that agents form their expectations using term structure information, which is observed in real time

- We incorporate the term structure of interest rates in an otherwise standard DSGE model. The extended model results in an AL *multi-period* forecasting model
- Agents' form their expectations using term structure information, which is observed in real time
- Survey of Professional Forecasters data is used to discipline Agents' expectations

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The rationale behind the use of term structure information is based on

- From a theoretical perspective: consumption-based asset pricing models show a tight connection between term spreads and the expectation paths of both consumption and inflation
- From an empirical perspective: there is a large empirical literature -among others, Fama (1990), Mishkin (1991), McCallum (1994), Estrella and Mishkin (1997) and Ang, Piazzesi and Wei (2006)- showing evidence of the ability of the term spread to predict the future evolution of both inflation and economic activity

- Multi-period forecasting based on term structure is a key source of aggregate persistence under AL
- The importance of most endogenous sources of aggregate persistence decline dramatically
- Model expectations based on term structure information provides a sound characterization of the consumption growth and inflation forecasts reported in the SPF
- Our extended AL DSGE model outperforms the RE version in terms of likelihood



We build on the Smets and Wouters (2007) model in three directions:

- 1 The model is extended to account for the term structure of interest rates, which is perfectly observable in real time
- 2 Agents' expectations rely only in term structure information available at each period
- 3 Survey of Professional Forecasters data is incorporated to further discipline expectations

## Agents:

- Households derive utility from their consumption relative to their habit and supply differentiated labor in monopolistic competition setting “Calvo-sticky” wages.
- Intermediate firms produce differentiated goods using labor and capital (subject to adjustment costs) in monopolistic competition and they set “Calvo-sticky” prices.
- The final good is produced using intermediate goods by firms under perfect competition
- The monetary authority follows a Taylor-type rule

Following De Graeve et al. (2009) and Vázquez et al. (2013): we extend the DSGE model by explicitly considering the yields associated with alternative bond maturities indexed by  $j$  (i.e.  $j = 1, 2, \dots, n$ ). From the FOC characterizing the optimal decisions of the representative consumer, one can obtain the standard consumption-based asset pricing equations associated with each maturity:

$$E_t \left[ \beta^j \frac{U_C(C_{t+j}, L_{t+j}) \left( \exp(\zeta_t^{\{j\}}) (1 + R_t^{\{j\}}) \right)^j}{U_C(C_t, L_t) \prod_{k=1}^j (1 + \pi_{t+k})} \right] = 1, \text{ for } j = 1, 2, \dots$$

$\zeta_t^{\{j\}}$  can be understood as a *convenience yield term* (Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015) defined as a risk premium associated with the safety and liquidity features of government bonds relative to assets with the same payoff, but without such nice properties

Assuming that utility function is logarithmic in consumption, after some algebra, the (linearized) consumption-based asset pricing equations can be written as

$$\left( \frac{1}{1 - \frac{h}{\bar{\gamma}}} \right) c_t - \left( \frac{\frac{h}{\bar{\gamma}}}{1 - \frac{h}{\bar{\gamma}}} \right) c_{t-1} =$$

$$E_t \left[ \left( \frac{1}{1 - \frac{h}{\bar{\gamma}}} \right) c_{t+j} - \left( \frac{\frac{h}{\bar{\gamma}}}{1 - \frac{h}{\bar{\gamma}}} \right) c_{t+j-1} \right] - \left[ jr_t^{\{j\}} - E_t \sum_{k=1}^j \pi_{t+k} + \zeta_t^{\{j\}} \right], \quad (1)$$

Subtracting the previous expression for  $j = 1$  we obtain the following expression of spread between  $j$ - and period 1

$$r_t^{\{j\}} - r_t^{\{1\}} = \left( \frac{j-1}{j} \right) r_t^{\{1\}} + \frac{1}{j} E_t [c (c_{t+j} - c_{t+1}) + (1-c) (c_{t+j-1} - c_t) + \frac{1}{j} E_t \sum_{k=2}^j \pi_{t+k} - \frac{1}{j} (\xi_t^{\{j\}} - \xi_t^{\{1\}})].$$

It shows that spreads are linked to consumption and inflation expectations in equilibrium, which rationalizes our modeling approach of using term structure information to characterize the formation of agents' expectations in real time.

We further assume that the risk premium shock  $\zeta_t^{\{1\}}$  follows an AR(1) process:

$$\zeta_t^{\{1\}} = \rho^{\{1\}} \zeta_{t-1}^{\{1\}} + \eta_t^{\{1\}},$$

whereas the term premium shocks  $\zeta_t^{\{j\}}$ , for  $j > 1$ , follow AR(1) processes augmented with an additional term that allows for an interaction with the risk premium shock:

$$\zeta_t^{\{j\}} = \rho^{\{j\}} \zeta_{t-1}^{\{j\}} + \rho_{\zeta}^{\{j\}} \eta_t^{\{1\}} + \eta_t^{\{j\}}.$$

The extended model results in an AL *multi-period* forecasting model  
Our model deviates from the two main approaches in the recent literature to AL

- “Euler equation learning” focuses on *short-sighted* agents where their optimal current decisions are based on just one-period-ahead expectations showing up in the standard Euler equations (e.g. Milani, 2007; Slobodyan and Wouters, 2012a,b)
- “Maintained beliefs approach” focuses on *long-sighted* agents taking into account infinite-horizon forecasts driven by their intertemporal decision problem (e.g. Preston, 2005; Eusepi and Preston, 2011; Sinha, 2015; and Sinha, 2016)

## 2. Real-Time learning

The Adaptive Learning literature depending in the information set can be categorized into:

- Minimum State Variable (MSV), followed by Eusepi and Preston (2011) and others (Orphanides and Williams, Milani...) where agents' expectations are based on a function of the state variables of the model
- Euler type Learning, based on small forecasting models formed by endogenous variables such as those in the Euler equation proposed by Slobodyan and Wouters (2012a,b)

We commit to a medium-sighted small forecasting model formed by term-structure data in real-time



- Agents behave as econometricians under AL: they use a linear projection scheme in which the parameters are updated to form their expectations. The forecasting model (or PLM) is defined as follows:

$$E_t y_{t+j} = X_t \beta_{t-1}^{\{j\}},$$

where  $y_{t+j}$  is the vector containing the forward-looking variables of the model,  $X_t$  is the matrix of regressors and  $\beta_t$  is the vector of updating parameters (it includes an intercept)

- $\beta_t$  is further assumed to follow an AR(1) process around  $\bar{\beta}$ , where agents' beliefs are updated through a Kalman filter:

$$\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t,$$

where  $F$  is a diagonal matrix with the learning parameter  $|\rho| \leq 1$  on the main diagonal and  $v_t$  are i.i.d. errors

We consider a specific PLM based on term structure information, which is truly observed when agents form their expectations in real time. As emphasized previously, this is rationalized:

- From a theoretical perspective, by the interaction between term spreads and the expectations of both consumption and inflation implied by the set of optimal conditions (1)
- From an empirical perspective, the use of term structure information in the PLM is further motivated by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997).

*“SPF panelists are quite flexible in their approach to forecasting... They use a combination of models in forming their expectations, rather than just one model. And, they vary their methods with the forecast horizon... the panelist update their projections frequently, suggesting that their projections incorporate the most recent information available on the economy around the survey's deadline.” Stark (2013)*

More precisely, we consider a single term spread for each forecast horizon

$$\left\{ \begin{array}{l} E_t y_{t+1} = \theta_{y,t-1} + \beta_{y,t-1} sp_{t-1}^{\{2\}}, \text{ for } y = i, r^k, q, w \\ E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{y,t-1}^{\{j\}} sp_{t-1}^{\{2\}}, \text{ for } y = c, \pi \text{ and } j = 0, 1, 2, 3 \\ E_t y_{t+j} = \theta_{y,t-1}^{\{4\}} + \beta_{y,t-1}^{\{4\}} sp_{t-1}^{\{4\}}, \text{ for } y = c, \pi \text{ and } j = 4 \end{array} \right. \quad (2)$$

The presence of intercepts  $\theta_{y,t-1}^{\{j\}}$  relaxes the RE assumption of agents having perfect knowledge about a common deterministic growth rate and a constant inflation target assumed in the SW model. Thus, the consideration of a time-varying intercept coefficient allows expectations to trace growth rate shifts in the data as well as changes in the inflation target.

AL is often criticized because it introduces additional degrees of freedom resulting in an arbitrary improvement in model fit, we overcome this by:

- Considering a rather restrictive information set: term structure information observed in real time
- Assuming that deviations in agents' expectations from the (observed) SPF follow an AR(1) process

$$\epsilon_{\pi,t}^{\{j\}} = \rho_{\pi}^{\{j\}} \epsilon_{\pi,t-1}^{\{j\}} + \eta_{\pi,t}^{\{j\}} \text{ and } \epsilon_{\Delta c,t}^{\{j\}} = \rho_{\Delta c}^{\{j\}} \epsilon_{\Delta c,t-1}^{\{j\}} + \eta_{\Delta c,t}^{\{j\}},$$

respectively, for  $j = 1, 2, 3, 4$ .

In contrast to Ormeño and Molnár (2015) We allow for persistent deviations between AL expectations and those reported in the SPF  
The reason is that our extended model uses term structure information to characterize model's expectations, which disciplines them in addition to SPF forecasts

As pointed out in the literature, there is evidence that term structure information is not consistently used by professional forecasters—this is called “the yield spread puzzle” (Rudebusch and Williams, 2009; Lahiri et al., 2013; Stekler and Ye, 2017)

Sample period: 1984:1-2007:4

$$X_t = \begin{bmatrix} dIGDP_t \\ dI\text{CONS}_t \\ dI\text{INV}_t \\ dI\text{WAG}_t \\ dIP_t \\ I\text{Hours}_t \\ \text{FEDFUNDS}_t \\ 1\text{-year yield}_t \\ dI\text{CONS}_t^{e\{j\}} \\ dIP_t^{e\{j\}} \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\pi} \\ \bar{l} \\ \bar{r} \\ \bar{r}^{\{4\}} \\ \bar{\gamma} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ E_t(c_{t+j} - c_{t+j-1}) + \epsilon_{c,t}^{\{j\}} \\ E_t\pi_{t+j} + \epsilon_{\pi,t}^{\{j\}} \end{bmatrix},$$

We present the results in 3 steps, trying to control for the contributions of each extension

Table 1. Selected parameter estimates

	SIW	SIW	SIW-TS	RT-AL	RT-AL	RE
Observables	7	7	8	8	16	16
Sample	1966-2007	1984-2007	1984-2007	1984-2007	1984-2007	1984-2007
Log lik.	-960.22	-424.86	-614.55	-474.92	216.70	186.20
Parameters associated with real rigidities						
$h$ :	0.69	0.83	0.44	0.35	0.31	0.92
habit formation	(0.63,0.75)	(0.78,0.87)	(0.41,0.47)	(0.30,0.40)	(0.21,0.44)	(0.91,0.93)
$\varphi$ : cost of	3.35	6.53	3.63	2.34	1.02	8.88
adjusting capital	(1.88,3.87)	(4.81,8.23)	(3.58,3.69)	(2.19,2.49)	(0.69,1.37)	(8.46,9.50)
$\psi$ : capital	0.51	0.53	0.29	0.21	0.22	0.37
utilization adj. cost	(0.31,0.71)	(0.30,0.76)	(0.25,0.33)	(0.14,0.28)	(0.14,0.29)	(0.31,0.43)
Calvo probabilities						
$\xi_p$ : price	0.65	0.78	0.57	0.62	0.58	0.94
	(0.59,0.69)	(0.74,0.82)	(0.54,0.61)	(0.54,0.69)	(0.51,0.66)	(0.93,0.95)
$\xi_w$ : wage	0.82	0.73	0.35	0.60	0.60	0.75
	(0.77,0.86)	(0.64,0.80)	(0.32,0.37)	(0.52,0.69)	(0.53,0.67)	(0.70,0.81)



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	SIW	SIW-TS
Observables	7	8
Sample	1984-2007	1984-2007
Log lik.	-424.86	-614.55
Parameters associated with real rigidities		
$h$ :	0.83	<b>0.44</b>
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utilization adj. cost	(0.30,0.76)	(0.25,0.33)
Calvo probabilities		
$\xi_p$ : price	0.78	<b>0.57</b>
	(0.74,0.82)	(0.54,0.61)
$\xi_w$ : wage	0.73	<b>0.35</b>
	(0.64,0.80)	(0.32,0.37)

Table 1. Selected parameter estimates

	SIW-TS	RT-AL
Observables	8	8
Sample	1984-2007	1984-2007
Log lik.	-614.55	<b>-474.92</b>
Parameters associated with real rigidities		
$h$ :	0.44	<b>0.35</b>
habit formation	(0.41,0.47)	(0.30,0.40)
$\varphi$ : cost of	3.63	<b>2.34</b>
adjusting capital	(3.58,3.69)	(2.19,2.49)
$\psi$ : capital	0.29	<b>0.21</b>
utilization adj. cost	(0.25,0.33)	(0.14,0.28)
Calvo probabilities		
$\xi_p$ : price	0.57	0.62
	(0.54,0.61)	(0.54,0.69)
$\xi_w$ : wage	0.35	0.60
	(0.32,0.37)	(0.52,0.69)

Table 1. Selected parameter estimates

	RT-AL	RT-AL	RE
Observables	8	16	16
Sample	1984-2007	1984-2007	1984-2007
Log lik.	-474.92	<b>216.70</b>	186.20
Parameters associated with real rigidities			
$h$ :	0.35	<b>0.31</b>	0.92
habit formation	(0.30,0.40)	(0.21,0.44)	(0.91,0.93)
$\varphi$ : cost of	2.34	<b>1.02</b>	8.88
adjusting capital	(2.19,2.49)	(0.69,1.37)	(8.46,9.50)
$\psi$ : capital	0.21	0.22	0.37
utilization adj. cost	(0.14,0.28)	(0.14,0.29)	(0.31,0.43)
Calvo probabilities			
$\xi_p$ : price	0.62	0.58	0.94
	(0.54,0.69)	(0.51,0.66)	(0.93,0.95)
$\xi_w$ : wage	0.60	0.60	0.75
	(0.52,0.69)	(0.53,0.67)	(0.70,0.81 )

- 1 The incorporation of the term structure of interest rates reduces the source of endogenous rigidity
- 2 Small forecasting model consisting of term structure information improves the likelihood and further reduces endogenous sources of persistence
- 3 The AL model with survey data outperforms the RE in terms of likelihood. SPF deviations from model expectations are persistent for inflation and one quarter consumption expectations.

Figure 1. PLM of inflation and consumption expectations

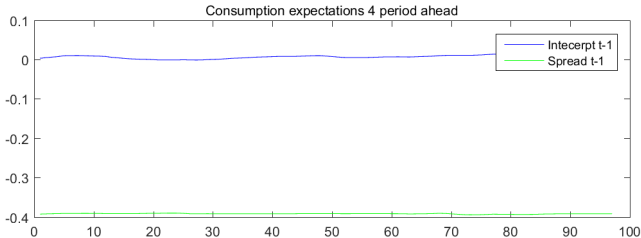
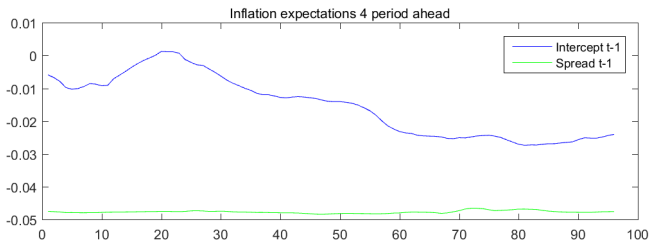


Figure 2. Impulse responses to a term-spread innovation

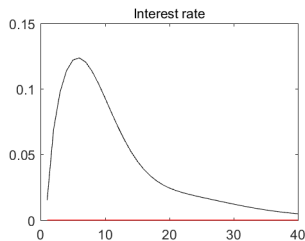
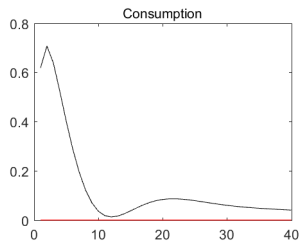
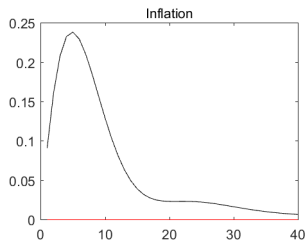
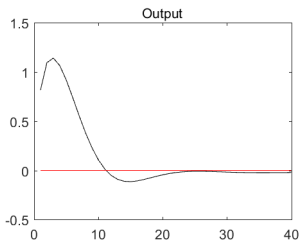


Figure 4. Model's expectations versus SPF's forecasts on inflation and consumption

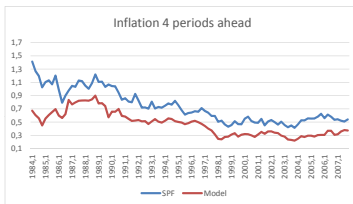
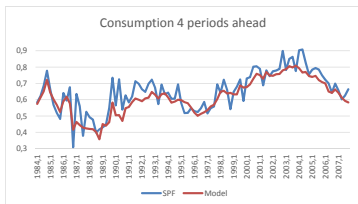


Table 2. Actual and simulated second moments

Actual data	$\Delta c$	$\Delta inv$	$\Delta w$	$\Delta y$	$\pi$
Standard deviation	0.51	1.68	0.62	0.54	0.24
Correlation with $\pi$	-0.30	-0.28	-0.29	-0.29	1
Autocorrelation	0.19	0.51	0.22	0.21	0.69
Simulated data	$\Delta c$	$\Delta inv$	$\Delta w$	$\Delta y$	$\pi$
Standard deviation	0.53	1.61	0.63	0.70	0.26
Correlation with $\pi$	-0.29	-0.26	-0.10	-0.30	1.0
Autocorrelation	0.26	0.70	0.57	0.48	0.97



Table 3. Descriptive statistics of inflation and consumption growth

Mean	$\pi$	$\pi_{t+4}^e$	$\Delta c$	$\Delta c_{t+4}^e$
Data/SPF forecasts	0.63	0.73	0.57	0.65
Model	0.70	0.53	0.53	0.52
Standard deviation	$\pi$	$\pi_{t+4}^e$	$\Delta c$	$\Delta c_{t+4}^e$
Data/SPF forecasts	0.24	0.24	0.51	0.12
Model	0.26	0.16	0.53	0.10
Autocorrelation	$\pi$	$\pi_{t+4}^e$	$\Delta c$	$\Delta c_{t+4}^e$
Data/SPF forecast	0.69	0.96	0.19	0.70
Model	0.97	0.96	0.26	0.78

Table 4. Variance decomposition (long-run)

	$dy$	$dc$	$\pi$
Productivity	0.06	0.10	0.16
Risk premium	65.92	75.59	38.22
Exogenous spending	0.02	0.05	0.01
Investment specific technology	0.19	0.29	0.01
Monetary policy	1.30	1.44	0.52
Price markup	0.09	0.09	0.34
Wage markup	31.99	21.99	60.29
Term spread	0.42	0.45	0.46

We incorporate the information of longer maturity term structure. More precisely, we consider the 3-, 5-, 7- and 10-year TB yields as additional observables in the measurement equation. Moreover, four additional consumption-Euler equations are considered in the estimated model each one associated with each additional yield. For instance, the asset-pricing eq. associated with the 10-year yield is given by

$$\left(\frac{1}{1 - \frac{h}{\bar{\gamma}}}\right) c_t - \left(\frac{\frac{h}{\bar{\gamma}}}{1 - \frac{h}{\bar{\gamma}}}\right) c_{t-1} = E_t \left[ \left(\frac{1}{1 - \frac{h}{\bar{\gamma}}}\right) c_{t+40} - \left(\frac{\frac{h}{\bar{\gamma}}}{1 - \frac{h}{\bar{\gamma}}}\right) c_{t+39} \right] - \left[ 40r_t^{\{40\}} - E_t \sum_{k=1}^{40} \pi_{t+k} + \zeta_t^{\{40\}} \right].$$

When considering longer maturity bonds we end up having a curse of dimensionality problem: there are many more expectation parameters to be identified in the PLM (2) with just a few more observables. We address this issue by defining the following two simple recursive rules:

$$\begin{cases} E_t c_{t+j} = \mu_c E_t c_{t+j-1}, & \text{for } j > 4 \\ E_t \pi_{t+j} = \mu_\pi E_t \pi_{t+j-1}, & \text{for } j > 4 \end{cases} \quad (3)$$

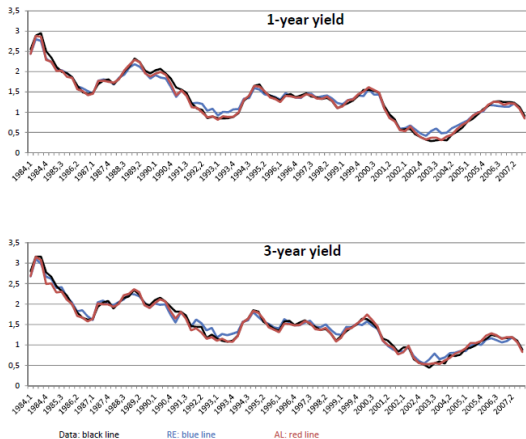
Table 6. Parameter estimates up to 1-year versus *10-year* yield

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
$h$ : habit formation	0.31	(0.21,0.44)	0.92	(0.91,0.93)
	<b>0.37</b>	<b>(0.33,0.41)</b>	<b>0.85</b>	<b>(0.82,0.87)</b>
$\varphi$ : cost of adjusting capital	1.02	(0.69,1.37)	8.88	(8.46,9.50)
	<b>1.19</b>	<b>(1.01,1.44)</b>	<b>7.60</b>	<b>(6.14,9.20)</b>
$\psi$ : capital utilization adjusting cost	0.22	(0.14,0.29)	0.37	(0.31,0.43)
	<b>0.01</b>	<b>(0.00,0.01)</b>	<b>0.81</b>	<b>(0.68,0.91)</b>
$\xi_p$ : price Calvo probability	0.58	(0.51,0.66)	0.94	(0.93,0.95)
	<b>0.56</b>	<b>(0.53,0.59)</b>	<b>0.92</b>	<b>(0.90,0.94)</b>
$\xi_w$ : wage Calvo probability	0.60	(0.53,0.67)	0.75	(0.70,0.81)
	<b>0.57</b>	<b>(0.52,0.63)</b>	<b>0.88</b>	<b>(0.82,0.92)</b>

Table 6. (Continued)

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
$\iota_p$ : price indexation	0.85	(0.73,0.95)	0.11	(0.09,0.13)
	<b>0.26</b>	<b>(0.17,0.31)</b>	<b>0.07</b>	<b>(0.03,0.12)</b>
$\iota_w$ : wage indexation	0.56	(0.39,0.77)	0.21	(0.15,0.27)
	<b>0.43</b>	<b>(0.34,0.52)</b>	<b>0.28</b>	<b>(0.09,0.48)</b>
$\rho_p$ : persistence of price markup shock	0.67	(0.41,0.91)	0.997	(0.994,0.999)
	<b>0.95</b>	<b>(0.92,0.99)</b>	<b>0.03</b>	<b>(0.00,0.06)</b>
$\rho_w$ : persistence of wage markup shock	0.94	(0.91,0.97)	0.83	(0.79,0.89)
	<b>0.99</b>	<b>(0.98,0.99)</b>	<b>0.63</b>	<b>(0.34,0.99)</b>
log data density	216.70		186.20	
	<b>727.80</b>		<b>235.29</b>	
log data density difference	511.10		49.09	

Figure 5. Term structure fitting



We incorporate real time inflation data. Following Casares and Vázquez (2016), we consider the following identity relating inflation revised data,  $\pi_t$ , to both the initial announcement of inflation (i.e. real time inflation),  $\pi_{t,t+1}^r$ , and the final revisions,  $rev_{t,t+S}^\pi$ :

$$\pi_t = \pi_{t,t+1}^r + rev_{t,t+S}^\pi,$$

where  $S$  denotes the number of periods (quarters) of delay for the final release

Many papers (e.g. Aruoba, 2008) have shown that US data revisions of many aggregate time series are not rational forecast errors and might be related to their initial (real-time) announcements. Thus, we assume that

$$\begin{aligned} rev_{t,t+S}^\pi &= b_\pi^r \pi_{t,t+1}^r + \epsilon_{t,t+S}^\pi, \\ \epsilon_{t,t+S}^\pi &= \rho_\pi^r \epsilon_{t-1,t+S-1}^\pi + \eta_{t,t+S}^{\pi r}. \end{aligned}$$



Table 7. Estimates up to 10-year yield *with* (and without) *real-time inflation*

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
$h$ : habit formation	0.37	(0.33,0.41)	0.85	(0.82,0.87)
	<b>0.31</b>	<b>(0.26,0.36)</b>	<b>0.84</b>	<b>(0.81,0.86)</b>
$\varphi$ : cost of adjusting capital	1.19	(1.01,1.44)	7.60	(6.14,9.20)
	<b>1.28</b>	<b>(1.13,1.46)</b>	<b>8.19</b>	<b>(6.48,9.92)</b>
$\psi$ : capital utilization adjusting cost	0.01	(0.00,0.01)	0.81	(0.68,0.91)
	<b>0.22</b>	<b>(0.15,0.28)</b>	<b>0.84</b>	<b>(0.74,0.93)</b>
$\xi_p$ : price Calvo probability	0.56	(0.53,0.59)	0.92	(0.90,0.94)
	<b>0.51</b>	<b>(0.47,0.55)</b>	<b>0.90</b>	<b>(0.88,0.92)</b>
$\xi_w$ : wage Calvo probability	0.57	(0.52,0.63)	0.88	(0.82,0.92)
	<b>0.42</b>	<b>(0.36,0.48)</b>	<b>0.82</b>	<b>(0.77,0.87)</b>

Table 7. (Continued)

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
$\iota_p$ : price indexation	0.26	(0.17,0.31)	0.07	(0.03,0.12)
	<b>0.22</b>	<b>(0.14,0.29)</b>	<b>0.04</b>	<b>(0.01,0.07)</b>
$\iota_w$ : wage indexation	0.43	(0.34,0.52)	0.28	(0.09,0.48)
	<b>0.39</b>	<b>(0.32,0.47)</b>	<b>0.33</b>	<b>(0.13,0.51)</b>
$\rho_p$ : persistence of price markup shock	0.95	(0.92,0.99)	0.03	(0.00,0.06)
	<b>0.94</b>	<b>(0.89,0.98)</b>	<b>0.89</b>	<b>(0.87,0.92)</b>
$\rho_w$ : persistence of wage markup shock	0.99	(0.98,0.99)	0.63	(0.34,0.99)
	<b>0.93</b>	<b>(0.90,0.95)</b>	<b>0.59</b>	<b>(0.44,0.75)</b>
log data density	727.80		235.29	
	<b>698.01</b>		<b>423.01</b>	
log data density difference	-29.79		187.72	

- Term structure of interest rates is incorporated in a DSGE model with AL
- We show that multi-period forecasting based on term structure is a key source of aggregate persistence under AL: the importance of most endogenous sources of aggregate persistence decline dramatically
- Model expectations based on term structure information provides a sound characterization of the consumption growth and inflation forecasts reported in the SPF
- Our extended DSGE model does a good job when reproducing both the yield curve and U.S. business cycle features