

Integrated Assessment in a Multi-region World with Multiple Energy Sources and Endogenous Technical Change

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 - with regional heterogeneity in climate damages, size and supply of energy,
 - allowing endogenous technical change in energy supply, motivated by recent trends in cost-reductions of green energy and non-conventional fossil fuels like fracking.
- A first attempt, more serious calibration of energy supply and R&D straightforward (we think) but not yet done.

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- Acemoglu, Aghion, Bursztyn and Hemous for directed technical change.

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- 4 Endogenize technology for production fossil fuel and renewable energy.
- 5 Calibration and analysis of how R&D and carbon taxes interact.

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 - 3 climate change.
- Model framework builds on Nordhaus (RICE/DICE), and Golosov et al. (2014).

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$A_{i,t}$ is factor productivity, $L_{i,t}$ is labor, $K_{i,t}$ is the capital stock and $E_{i,t}$ **energy services** (an aggregate of different energy sources/fuels).

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- **Oil supplying region** only sells from its finite oil reserv, extracted at zero cost.

- **Energy services** provided competitively by representative firm in each region:

$$E_{i,t} = \mathcal{E}(e_{1,i,t}, \dots, e_{n,i,t}) = \left(\sum_{k=1}^n \lambda_k (e_{k,t})^\rho \right)^{\frac{1}{\rho}}$$

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- Other energy sources produced in each region at cost $p_{k,i,t}$ and not traded.

- Resource constraint for **oil consuming regions**

$$\begin{aligned}C_{i,t} + K_{i,t+1} &= Y_{i,t} - p_{1,t}e_{1,i,t} - \sum_{k=2}^n p_{k,i,t}e_{k,i,t} + (1 - \delta) K_{i,t} \\ &= \hat{Y}_{i,t} + (1 - \delta) K_{i,t}.\end{aligned}$$

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- for **oil supplying region**

$$\begin{aligned}C_{1,t} &= p_{1,t} \sum_{i=2}^n e_{1,i,t} \\ R_{t+1} &= R_t - \sum_{i=2}^n e_{1,i,t}\end{aligned}$$

where $R_t \geq 0$ is remaining oil in ground.

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- Golosov et al. (2014), building on IPCC, show that a simple depreciation model for the atmospheric excess stock of carbon S_t well replicates advanced carbon circulation models.

$$S_t = \sum_{s=0}^t (1 - d_{t-s}) \sum_i M_{i,t}$$

where

$$1 - d_s = \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^s$$

measures how much of unit of emissions is left s periods after it was emitted.

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- $\gamma_{i,t}$ is **lost flow of GDP per unit of carbon** in atmosphere.

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- Regional competitive energy service provider minimizes cost of energy services

$$\min_{e_{k,i,t}} \sum_{k=1}^n p_{k,i,t} e_{k,i,t} - P_{i,t} \left(\left(\sum_{k=1}^n \lambda_k (e_{k,t})^\rho \right)^{\frac{1}{\rho}} - E_{i,t} \right)$$

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- Simplify by assuming full depreciation $\delta = 1$. Use period length of a decade. Implies constant savings rate at $\frac{\alpha\beta}{1-\nu}$.

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- Equilibrium determined **sequentially** without forward-looking components. Everything but oil price $p_{1,t}$ determined by closed-form expressions. Can solve in excel sheet.

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- **Negligible deviation**, now keep constant savings rate.

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- Focus on amount of global warming and distributional consequences (between Europe, US, China, Africa and Oil producers).

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Calibration

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- **3 sources of energy**, oil (finite supply 400 GtC), coal and renewables. Latter two perfectly elastic at prices $p_{k,i,t}$ (later allow R&D to affect these).

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- Regional damage parameters γ_i ; based on Hassler and Krusell (2012) and Nordhaus. Higher in EU and Africa, lower in China and US.

- Compare global (European only) carbon tax and coal tax. Set to the optimal global tax 56.9 US\$ per ton carbon (Golosov et al. 2014). Increases by 2% per year (follows global GDP).

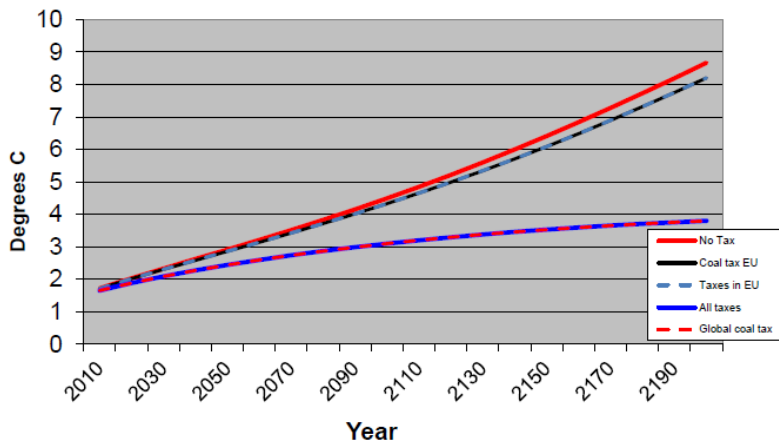
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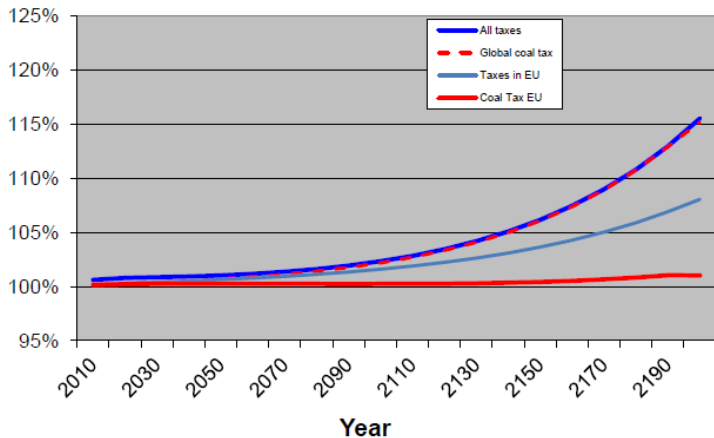
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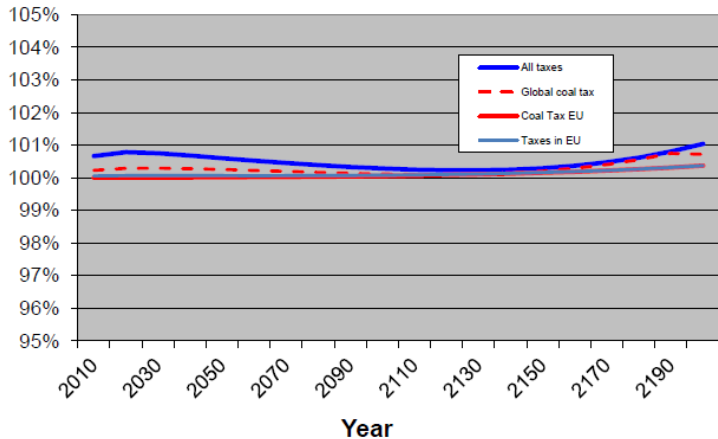
Increase in Global Mean Temp



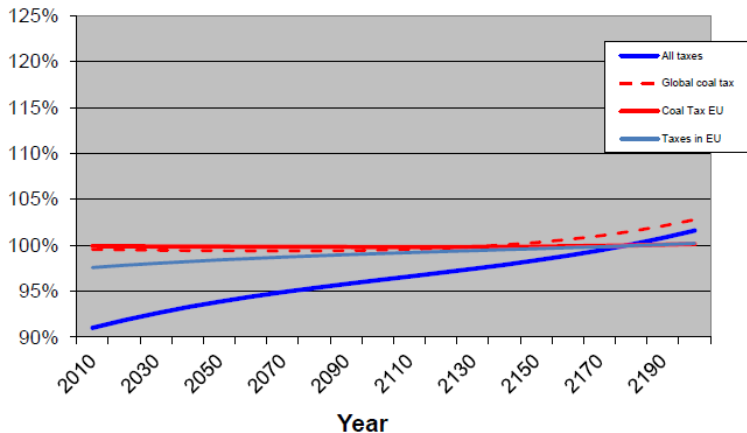
Consumption relative to no tax Europe



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Consumption relative to no tax Oil producers



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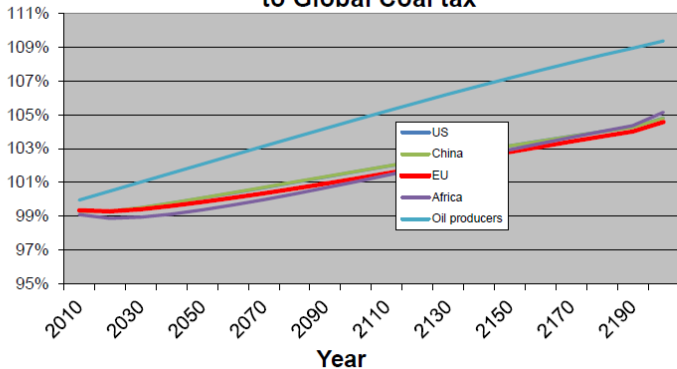
Experiment - tech change

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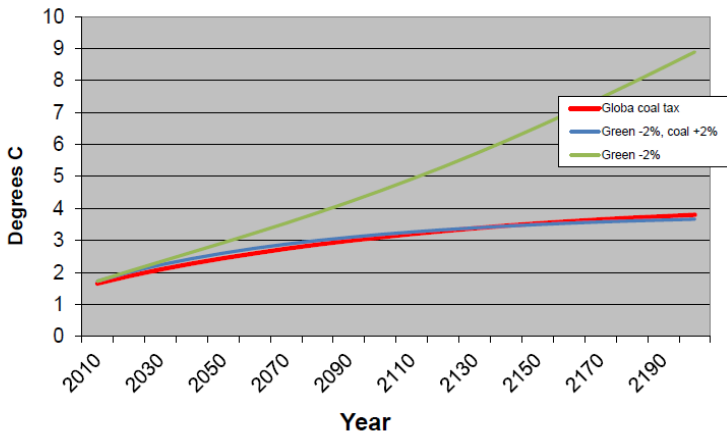
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Consumption Tech Change relative to Global Coal tax



Increase in Global Mean Temp



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- Involves only energy service providers, nothing else is changed in model (including simple recursive structure).

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- Note that direct effect of taxes is to *increase* marginal value of cost reductions (LHS). Also indirect effect in opposite direction coming from $\frac{de_{k,i,t}^*}{d\tau_{k,i,t}} < 0$.

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- Using expression for optimal supply $e_{k,i,t}^*$ (same as before) in $(1 + \tau_{k,i,t}) e_{k,i,t}^* = \frac{\Lambda_{i,t}}{p_{k,i,t}}$ yields

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- R&D completely undo taxes!** (When FOC is satisfied). Intuition, the two effects discussed above cancel each other. Suggest per unit taxes are not neutral.

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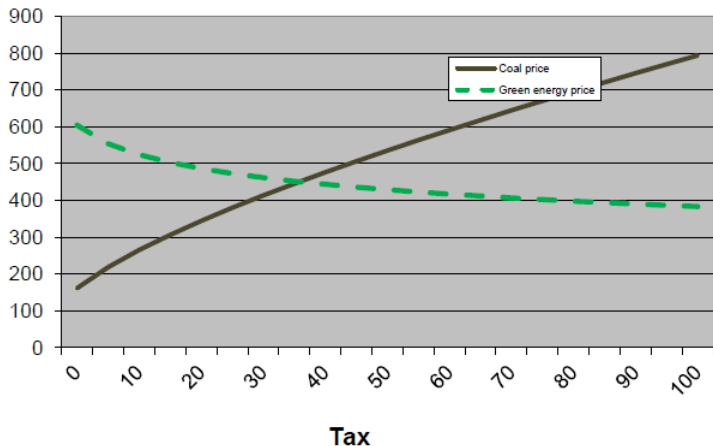
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- Use previous calibration for Europe and assume FOC is satisfied (latter gives $\varepsilon_3 = 0.778$ and $\varepsilon_2 = 0.222$, more difficult to reduce cost of green).

Coal and Green energy prices in interior R&D optimum



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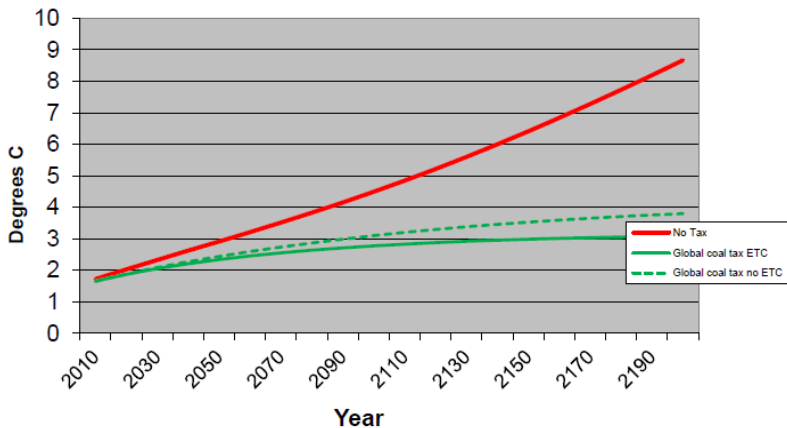
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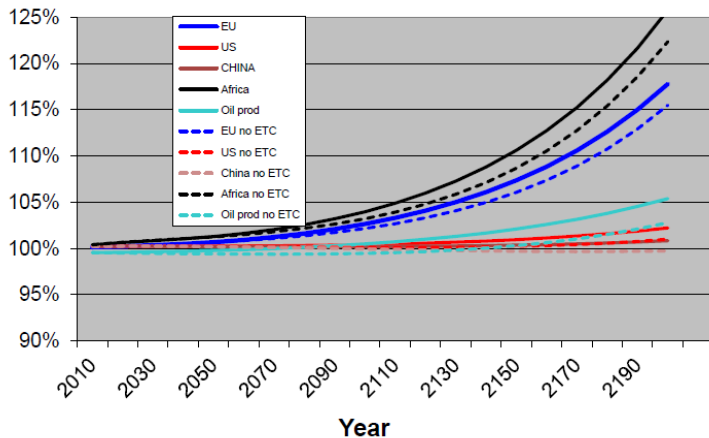
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- Would result also with taxes much smaller than optimal!

Increase in Global Mean Temp



Consumption with coal tax relative to no tax



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