Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations

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Motivation

- Asset price bubbles: ubiquitous in the policy debate...

  - key source of macro instability
  - monetary policy: cause and cure

...but absent in modern monetary models

  - no room for bubbles in the New Keynesian model
  - no discussion of possible role of monetary policy

Motivation

- Asset price bubbles: ubiquitous in the policy debate...

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- Present paper: modification of the basic NK model to allow for bubbles
- Key ingredients:

  (i) overlapping generations of finitely-lived agents
  (ii) transitions to inactivity ("retirement")

Related Literature

- *Real* models of rational bubbles: Tirole (1985),..., Martín-Ventura (2012)
- Monetary models with bubbles: Samuelson (1958),..., Asriyan et al. (2016) $\Rightarrow$ flexible prices
- New Keynesian models with overlapping-generations: Piergallini (2006), Nisticò (2012), Del Negro et al. (2015) $\Rightarrow$ no discussion of bubbles
- Monetary policy and bubbles in sticky price models:

  - Bernanke and Gertler (1999, 2001): ad-hoc bubble
  - Galí (2014): 2-period OLG, constant output
  - Present paper:
    - many-period lifetimes
    - variable employment and output
    - nests standard NK model as a limiting case

A New Keynesian Model with Overlapping Generations

- Individual survival rate: γ (Blanchard (1985), Yaari (1965))
- Size of cohort "at birth": $1-\gamma$
- Total population size: 1

- Two types of individuals:

  - "Active": manage own firm, work for others.
  - "Retired": consume financial wealth

- Probability of remaining active: υ (Gertler (1999))
- Labor force (and measure of firms): $\alpha \equiv \frac{1-\gamma}{1-\upsilon\gamma} \in (0,1]$

Consumers

- Complete markets (including annuity contracts)
- Consumer's problem:

$$\max E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t \log C_{t|s}$$

$$\frac{1}{P_t}\int_0^{\alpha} P_t(i) C_{t|s}(i) di + E_t\{\Lambda_{t,t+1} Z_{t+1|s}\} = A_{t|s} \ [+W_t N_{t|s}]$$

$$A_{t|s} = Z_{t|s}/\gamma$$

Firms

- Technology:

$$Y_t(i) = \Gamma^t N_t(i)$$

  where $\Gamma \equiv 1 + g \geq 1$.

- Price-setting à la Calvo

  - incumbent firms: a fraction θ_I keep prices unchanged
  - newly created firms: a fraction θ_N set price equal to P_{t-1}
  - average price rigidity index: $\theta \equiv \upsilon\gamma\theta_I + (1-\upsilon\gamma)\theta_N$

Labor Markets and Inflation

- Wage equation:

$$\mathcal{W}_t = \left(\frac{N_t}{\alpha}\right)^{\varphi}$$

where $\mathcal{W}_t \equiv W_t/\Gamma^t$ and $N_t \equiv \int_0^\alpha N_t(i)di$.

- *Natural* level of output: setting $1/\mathcal{W}_t = \mathcal{M}$

$$Y_t^n = \Gamma^t \mathcal{Y}$$

with $\mathcal{Y} \equiv \alpha \mathcal{M}^{-\frac{1}{\varphi}}$. *Remark*: invariant to bubble size.

- New Keynesian Phillips curve

$$\pi_t = \Phi E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

where $\Phi \equiv \beta\gamma\theta_I/\theta$, $\kappa \equiv \lambda\varphi$, and $\widehat{y}_t \equiv \log(Y_t/Y_t^n)$.

Asset Markets (I)

- Aggregate stock market

$$Q_t^F = \sum_{k=0}^{\infty} (\upsilon\gamma)^k E_t\{\Lambda_{t,t+k} D_{t+k}\}$$

Remark: same discount rate as labor income.

- Bubbly asset

$$Q_t^B(j) = E_t\{\Lambda_{t,t+1} Q_{t+1}^B(j)\}$$

with $Q_t^B(j) \geq 0$ for all t.

- Aggregate bubble:

$$Q_t^B = B_t + U_t$$

where $B_t \equiv \sum_{s=-\infty}^{t-1} Q_{t|s}^B \geq 0$ and $U_t \equiv Q_{t|t}^B \geq 0$

- Equilibrium condition:

$$Q_t^B = E_t\{\Lambda_{t,t+1} B_{t+1}\}$$

Characterization of Equilibria

- Balanced Growth Paths
- Equilibrium Dynamics around a Balanced Growth Path

Remark: key role for consumption function (individual and aggregate) in the determination of equilibria

Balanced Growth Paths

- Consumption function (consumer of age j; normalized by productivity)

(i) active individuals (letting $\Lambda \equiv \frac{1}{1+r}$):

$$\mathcal{C}_j = (1-\beta\gamma)\left[\mathcal{A}_j^a + \frac{1}{1-\Lambda\Gamma\upsilon\gamma}\left(\frac{\mathcal{W}N}{\alpha}\right)\right]$$

(ii) retired individuals

$$\mathcal{C}_j = (1-\beta\gamma)\,\mathcal{A}_j^r$$

- Aggregate consumption function

$$\mathcal{C} = (1-\beta\gamma)\left[\mathcal{Q}^F + \mathcal{Q}^B + \frac{\mathcal{W}N}{1-\Lambda\Gamma\upsilon\gamma}\right]$$

$$= (1-\beta\gamma)\left[\mathcal{Q}^B + \frac{\mathcal{Y}}{1-\Lambda\Gamma\upsilon\gamma}\right]$$

using $\mathcal{Q}^F = \mathcal{D}/(1-\Lambda\Gamma\upsilon\gamma)$ and $\mathcal{Y} = \mathcal{W}N + \mathcal{D}$.

Balanced Growth Paths

- In equilibrium ($\mathcal{C} = \mathcal{Y}$):

$$1 = (1 - \beta\gamma)\left[q^B + \frac{1}{1 - \Lambda\Gamma\upsilon\gamma}\right]$$

where $q^B \equiv \mathcal{Q}^B / \mathcal{Y}$.

- Bubbleless BGP ($q^B = 0$)

$$\Lambda\Gamma\upsilon = \beta$$

or, equivalently,

$$r = (1+\rho)(1+g)\upsilon - 1 \equiv \underline{r}$$

Remark #1: r increasing in υ

Remark #2: $\upsilon < \beta \Leftrightarrow r < g$

Balanced Growth Paths

- Bubbly BGP:

$$q^B = \frac{\gamma(\beta - \Lambda\Gamma\upsilon)}{(1-\beta\gamma)(1-\Lambda\Gamma\upsilon\gamma)} > 0$$

$$u = \left(1 - \frac{1}{\Lambda\Gamma}\right) q^B \geq 0$$

where

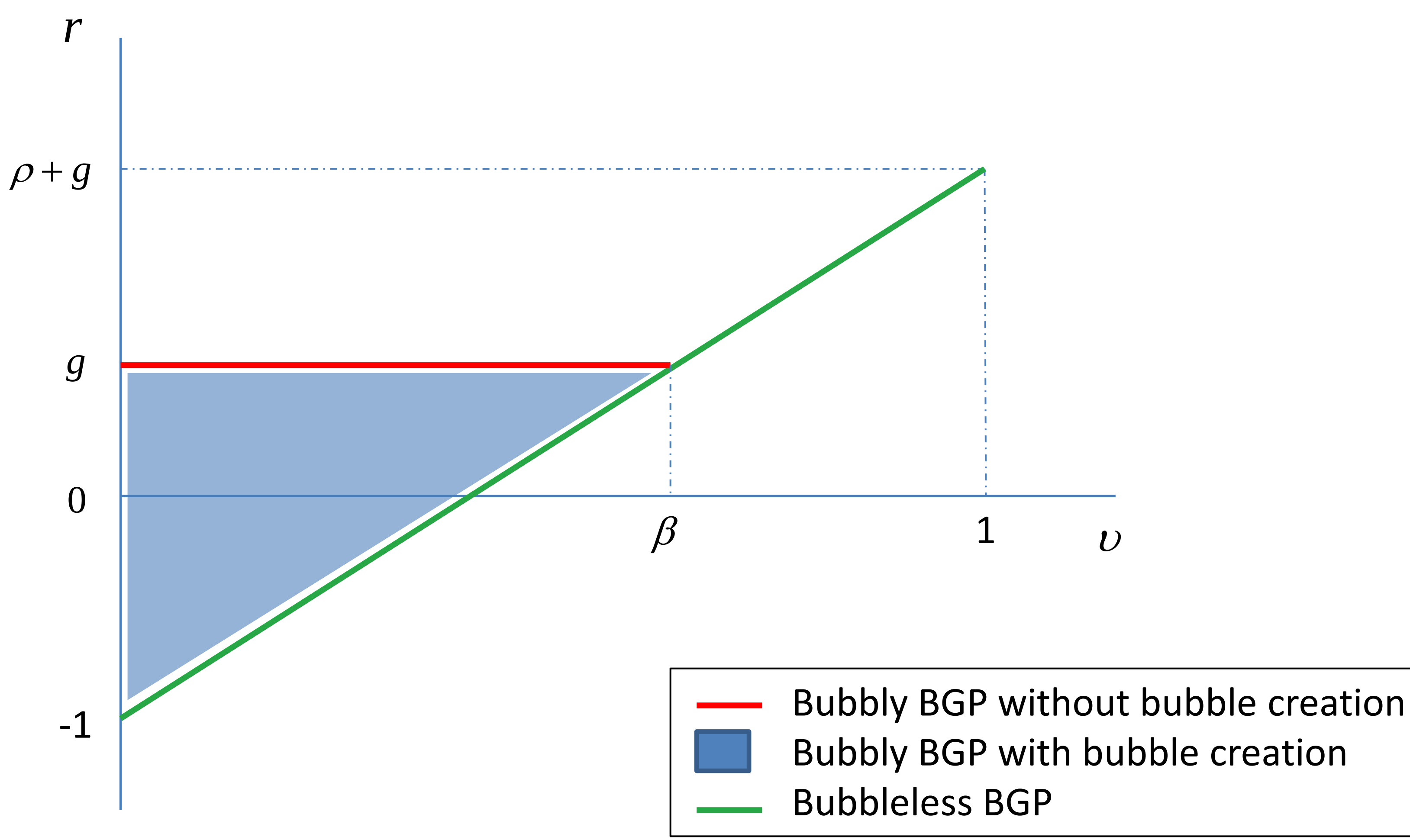
$$\Lambda\Gamma \geq 1 \Leftrightarrow r < g$$

$$\Lambda\Gamma\upsilon < \beta \Leftrightarrow r > \underline{r}$$

- Existence condition:

$$\upsilon < \beta$$

Figure 1. Balanced Growth Paths



Some Numbers

- Life expectancy (at 16): $63 \times 4 = 252$ quarters $\Rightarrow \gamma \simeq 0.996$
- Average employment rate: $0.6 \Rightarrow \upsilon = 0.9973$
- Condition for existence of bubbles: $\beta > 0.9973$
- Average real interest rate (1960-2015): $r = 1.4\% \div 4 = 0.35\%$
- Average growth rate (1960-2015): $g = 1.6\% \div 4 = 0.4\%$

Equilibrium Dynamics (I)

- Goods market clearing:

$$\widehat{y}_t = \widehat{c}_t$$

- Aggregate consumption function:

$$\widehat{c}_t = (1 - \beta\gamma)(\widehat{q}_t^B + \widehat{x}_t)$$

where

$$\widehat{x}_t = \Lambda\Gamma\upsilon\gamma E_t\{\widehat{x}_{t+1}\} + \widehat{y}_t - \frac{\Lambda\Gamma\upsilon\gamma}{1 - \Lambda\Gamma\upsilon\gamma}\left(\widehat{i}_t - E_t\{\pi_{t+1}\}\right)$$

- Aggregate bubble dynamics:

$$\widehat{q}_t^B = \Lambda\Gamma E_t\{\widehat{q}_{t+1}^B\} - q^B\left(\widehat{i}_t - E_t\{\pi_{t+1}\}\right)$$

Equilibrium Dynamics (II)

- New Keynesian Phillips curve

$$\pi_t = \Phi E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

- Monetary Policy

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q \widehat{q}_t^B$$

- *Assumption*: no fundamental shocks, focus on bubble-driven fluctuations

Equilibrium Dynamics: Outline

- Dynamics around the bubbleless BGP

  (a) bubbleless equilibria
  (b) bubbly equilibria

- Dynamics around a bubbly BGP

Bubbleless Equilibria

- Equilibrium dynamics

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - (\widehat{i}_t - E_t\{\pi_{t+1}\})$$
$$\pi_t = \Phi E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$
$$\widehat{i}_t = \phi_\pi \pi_t$$

- Monetary policy and equilibrium determinacy:

$$\phi_\pi > \max\left[1, \frac{1}{\kappa}(\Phi - 1)\right]$$

Thus, if $\theta/\theta_I < \frac{\beta\gamma}{1+\kappa} \Rightarrow \phi_\pi > (\beta\gamma\theta_I/\theta - 1)/\kappa > 1$

$\Rightarrow$"reinforced Taylor principle"

- *Remark*: forward guidance puzzle still present *in equilibrium* despite finitely-lived agents.

Bubble-driven Fluctuations around the Bubbleless BGP

- Assumed bubble process:

$$q_t^B = \begin{cases} \frac{\upsilon}{\beta\delta} q_{t-1}^B + u_t & \text{with probability } \delta \\ u_t & \text{with probability } 1-\delta \end{cases}$$

where $\{u_t\} > 0$ is white noise with mean $\overline{u} \gtrsim 0$.

Remark: it satisfies the relevant bubble condition

$$q_t^B = (\beta/\upsilon) E_t\{q_{t+1}^B - u_{t+1}\}$$

- Equilibrium dynamics:

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - (\widehat{i}_t - E_t\{\pi_{t+1}\}) + \Theta q_t^B$$

$$\pi_t = \Phi E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q \widehat{q}_t^B$$

where $\Theta \equiv (1-\beta\gamma)(1-\upsilon\gamma)/\beta\gamma > 0$.

Bubble-driven Fluctuations around the Bubbleless BGP

- *Remark:* alternative representation of the dynamic IS equation

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - \left(\widehat{i}_t - E_t\{\pi_{t+1}\} - \widehat{r}_t^n\right)$$

where $\widehat{r}_t^n = \Theta q_t^B$ is the *natural* rate of interest.

Bubble-driven Fluctuations around the Bubbleless BGP

- Determinacy condition (for any given bubble):

$$\phi_\pi > \max\left[1, \frac{1}{\kappa}(\Phi - 1)\right]$$

Remark: independent of ϕ_q

- Equilibrium output and inflation (assuming determinacy)

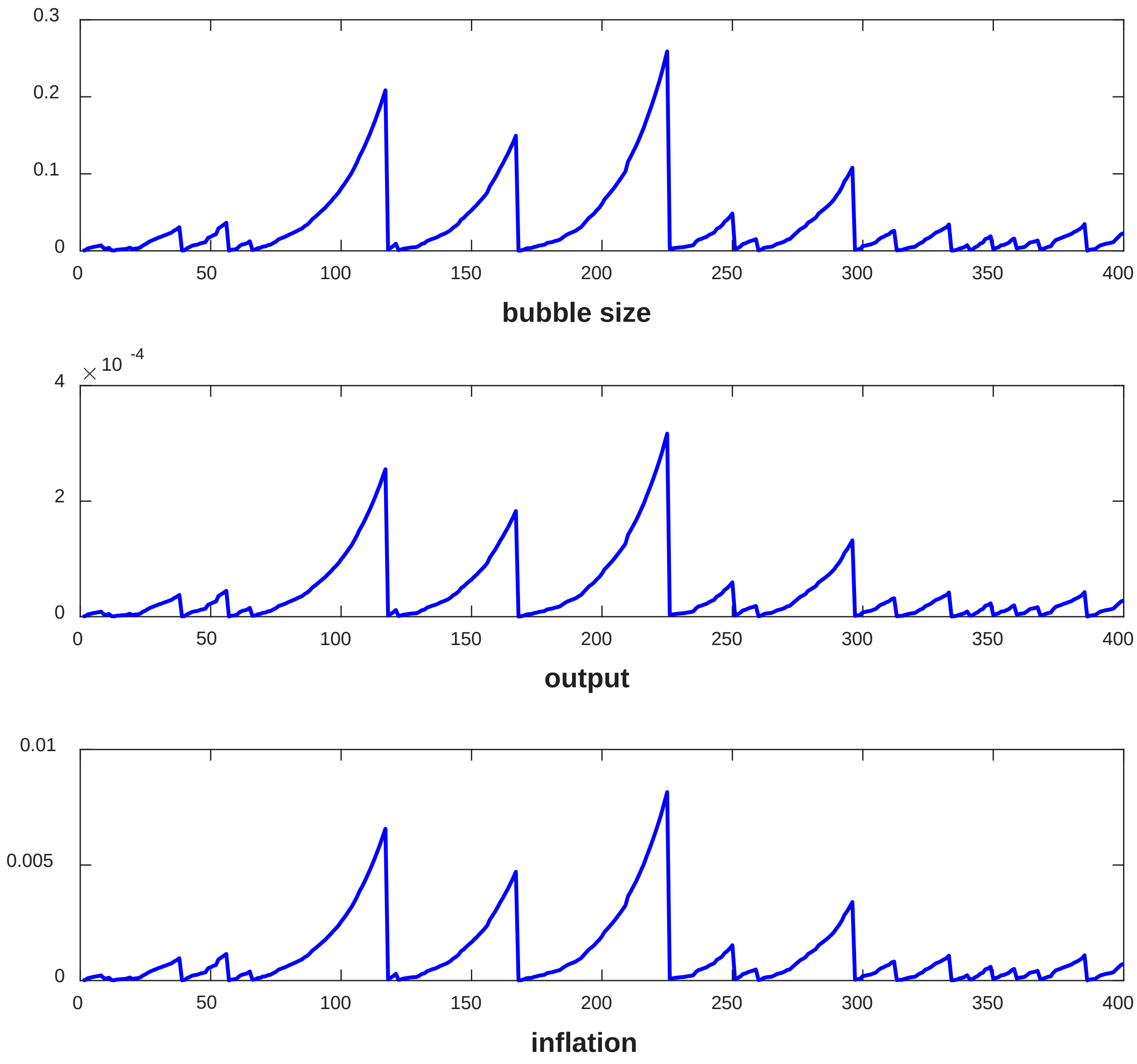
$$\widehat{y}_t = (1 - \upsilon\gamma\theta_I/\theta)\Psi(\Theta - \phi_q)q_t^B$$

$$\pi_t = \kappa\Psi(\Theta - \phi_q)q_t^B$$

where $\Psi \equiv \frac{1}{(1-\upsilon\gamma\theta_I/\theta)(1-\upsilon/\beta)+\kappa(\phi_\pi-\upsilon/\beta)} > 0$.

- Simulated bubble driven fluctuations ($\phi_\pi = 1.5, \phi_q = 0$) (*)

Figure 2. Simulated Bubble-Driven Fluctuations around the Bubbleless BGP



Bubble-driven Fluctuations around the Bubbleless BGP

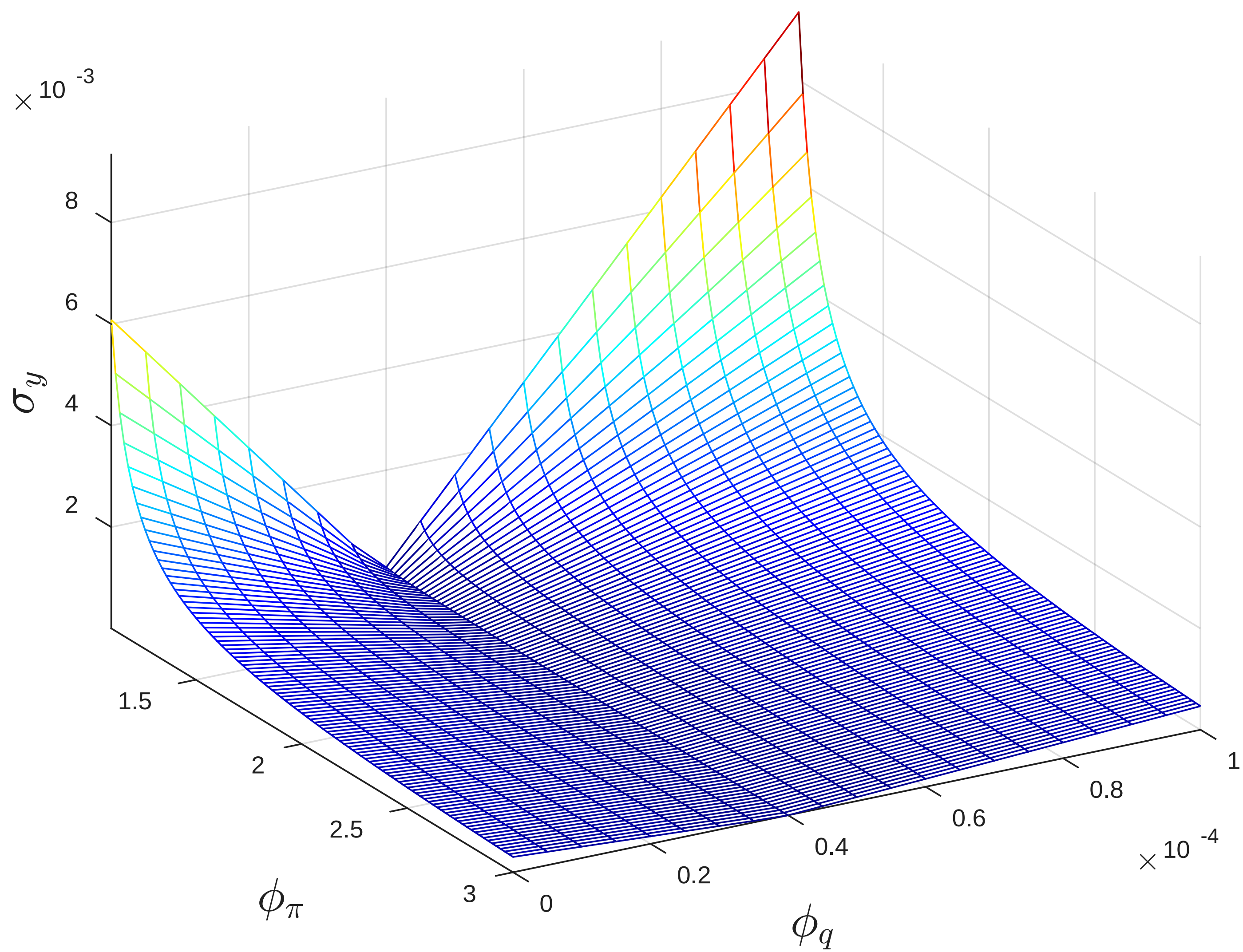
- An *optimal* "leaning against the bubble" monetary policy:

$$\phi_q = \Theta > 0$$

$$\Rightarrow \widehat{y}_t = \pi_t = 0$$

- *Remark*: in the present environment, bubble fluctuations are not affected by "leaning against the bubble" policies
- *Remark*: same outcome can be attained by directly targeting inflation ($\phi_q = 0, \phi_\pi \to +\infty$), with no need to observe the bubble or knowing Θ accurately
- Monetary policy and macro volatility (*)

Figure 3. Bubble-driven Fluctuations: Monetary Policy and Macro Volatility in a Neighborhood of the Bubbleless BGP



Fluctuations around a Bubbly BGP

- Equilibrium dynamics

$$\widehat{y}_t = \frac{\Lambda\Gamma\upsilon}{\beta} E_t\{\widehat{y}_{t+1}\} - \frac{\Upsilon\upsilon}{\beta}(\widehat{i}_t - E_t\{\pi_{t+1}\}) + \Theta\widehat{q}_t^B$$

$$\widehat{q}_t^B = \Lambda\Gamma E_t\{\widehat{q}_{t+1}^B\} - q^B(\widehat{i}_t - E_t\{\pi_{t+1}\})$$

$$\pi_t = \Phi E_t\{\pi_{t+1}\} + \kappa\widehat{y}_t$$

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q q_t^B$$

where $\Theta \equiv \frac{(1-\beta\gamma)(1-\upsilon\gamma)}{\beta\gamma} > 0$ and $\Upsilon \equiv 1 + \frac{(1-\beta\gamma)(\Lambda\Gamma-1)}{1-\Lambda\Gamma\upsilon\gamma} \geq 1$

- *Remark*: $\{\widehat{q}_t^B\}$ no longer independent of monetary policy.

Equilibrium Dynamics around a Bubbly BGP

- An *optimal* "leaning against the bubble" policy

$$\phi_q = \frac{\Theta\beta}{\Upsilon\upsilon} > 0$$

$$\phi_\pi > 1 - \frac{(1-\Phi)(1-\Lambda\Gamma\upsilon/\beta)}{\kappa\Upsilon\upsilon/\beta} \simeq 1$$

then:

$$\Rightarrow \widehat{y}_t = \pi_t = 0$$

but no elimination bubble fluctuations:

$$\Rightarrow \widehat{q}_t^B = (1/\chi)\widehat{q}_{t-1}^B + \xi_t$$

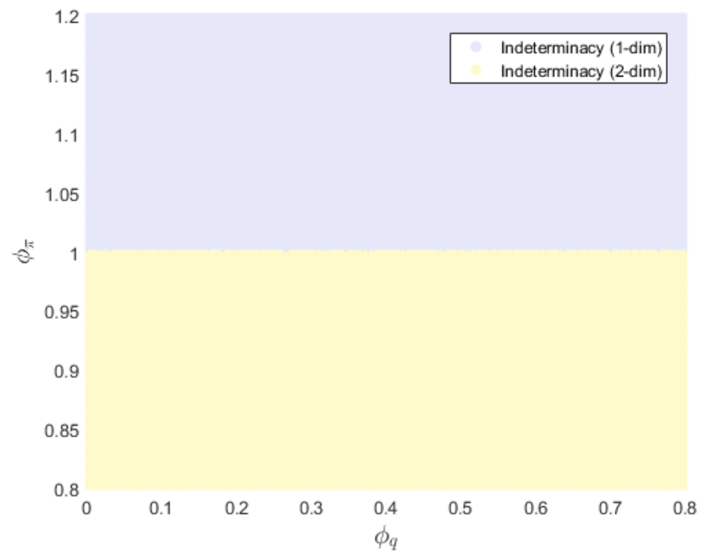
where $\xi_t \equiv \widehat{b}_t - E_{t-1}\{\widehat{b}_t\} + \widehat{u}_t$ and $\chi \equiv \left(\frac{\Lambda\Gamma\upsilon}{\beta}\right)^{\frac{\Lambda\Gamma(1-\beta\gamma)+\gamma(\beta-\Lambda\Gamma\upsilon)}{1-\Lambda\Gamma\upsilon\gamma}} > 1$ if $r < \bar{r}$.

- *Remark:* same outcome with strict inflation targeting policy.

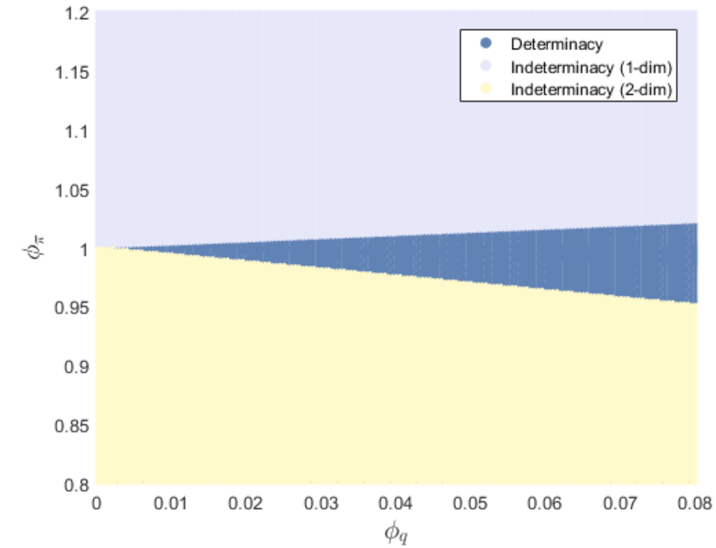
Equilibrium Dynamics around a Bubbly BGP

- Monetary policy and determinacy (*)

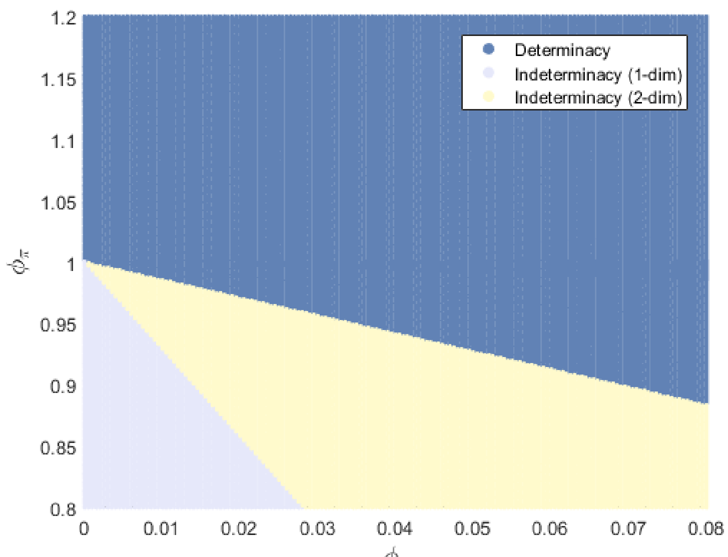
Figure 4.
Determinacy and Indeterminacy Regions around Bubbly BGPs



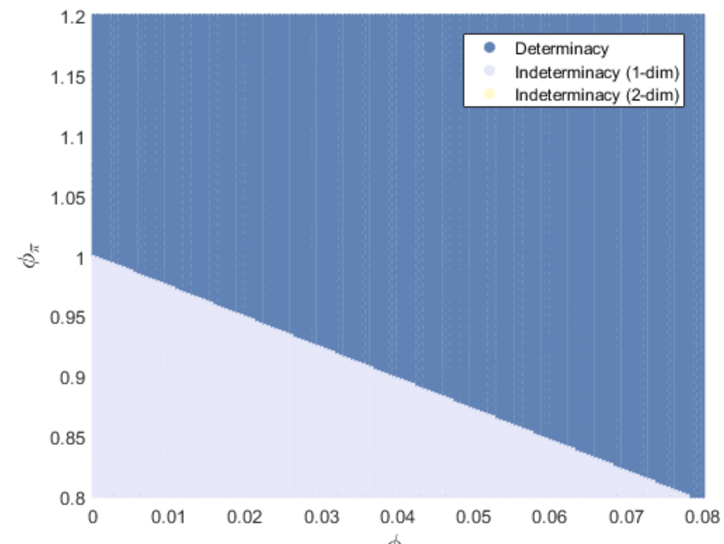
r = 0.00335



r = 0.0035



r = 0.0037

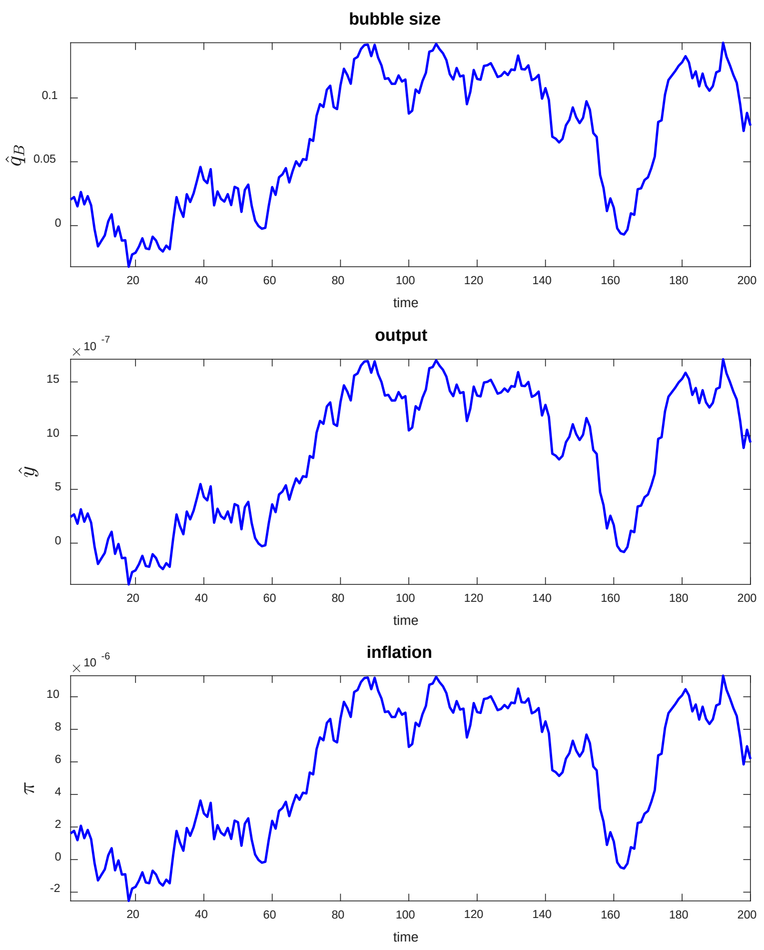


r = 0.004

Equilibrium Dynamics around a Bubbly BGP

- Monetary policy and determinacy (*)
- Simulated bubble-driven fluctuations (*)

Figure 5. Simulated Bubble-driven Fluctuations Around a Bubbly BGP



Equilibrium Dynamics around a Bubbly BGP

- Monetary policy and determinacy
- Simulated bubble-driven fluctuations
- Monetary policy and volatility (under one-dimensional indeterminacy)

  - output
  - bubble

Figure 6.a Bubble-driven Fluctuations: Monetary Policy and Output Volatility in a Neighborhood of a Bubbly BGP

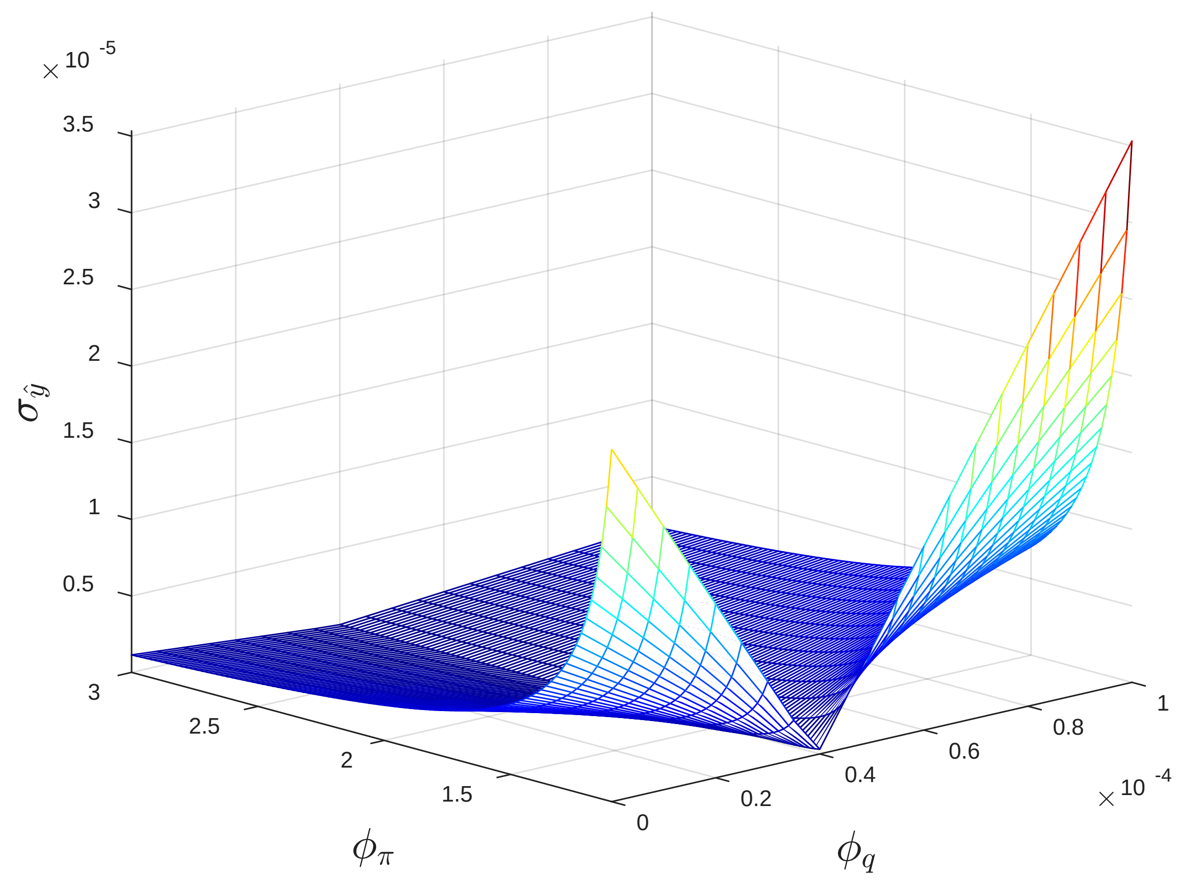
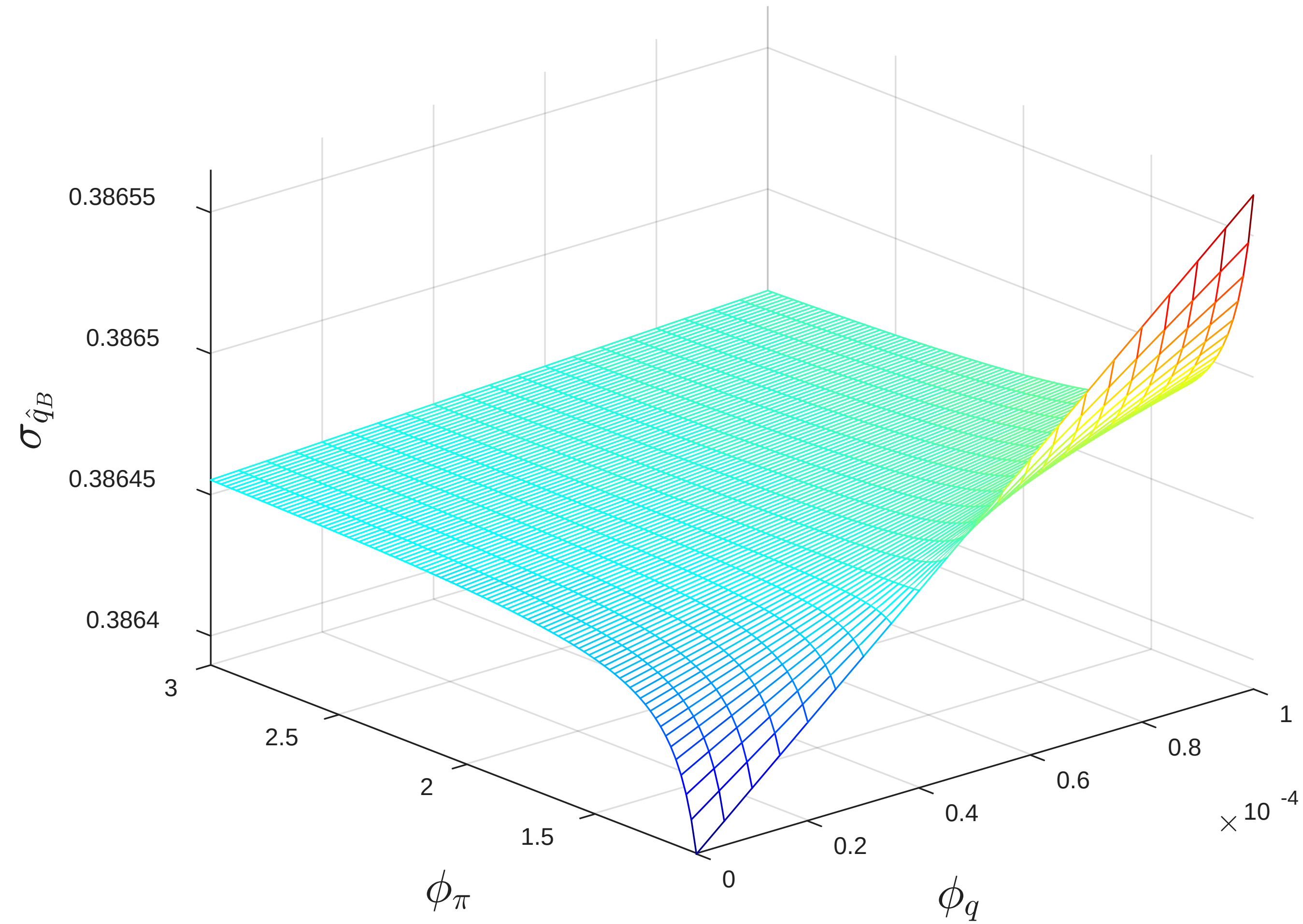


Figure 6.b Bubble-driven Fluctuations: Monetary Policy and Bubble Volatility in a Neighborhood of a Bubbly BGP



Conclusions, Caveats and Possible Extensions

- Bubbly equilibria may exist in the NK model once we depart from the infinitely-lived representative consumer assumption. More likely in an environment of low natural interest rates. Room for bubble-driven fluctuations.
- Under some conditions, need for "reinforced Taylor principle" to guarantee uniqueness.
- No obvious advantages of "leaning against the bubble" policies relative to inflation targeting plus some risks (e.g. may amplify bubble fluctuations)
- Caveats/potential extensions

(i) *Rational* bubbles. But non-rational bubbles can be readily accommodated.
(ii) ZLB has been ignored. Potential interesting interaction with bubbles (e.g. by raising underlying natural rate, bubbles may lower the risk of hitting the ZLB).
(iii) No role for credit supply factors; may be needed to boost "bubble multiplier".