The Macro Impact of Micro Shocks: Beyond Hulten's Theorem

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Discussion by Basile Grassi [1]

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- **Roadmap:**

  1. Summary:
     - ★ Hulten's Theorem (1978)
     - ★ Beyond Hulten's Theorem
  2. Comments
     - ★ Comments #1 on the origin of the results
     - ★ Comments #2 on the quantitave exercice

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  - The "fat tail" comes from either Random Growth (Gabaix 2011) or from the I-O Network (Acemoglu et al. 2012)
  - The "fat tail" case is quantitatively (Carvalho and Grassi 2017) and empirically (Di Giovanni et al. 2014, Mangerman 2016) relevant.

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  - minus the change in sales share of other sectors

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- The macro impact of micro shock:

$$\log \frac{C}{\overline{C}} \approx \lambda_i \log(A_i) + \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \left(1 - \frac{1}{\rho_{ij}}\right) \log(A_i)^2 + \lambda_i \frac{d \log \xi}{d \log A_i} \log(A_i)^2$$

now, depends on the *structure of the economy* through the elasticities

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  - Section 8: Calibrate ⇒ BC exhibits Skewness and Kurtosis
    Revisite the effect of the 70s' oil shock

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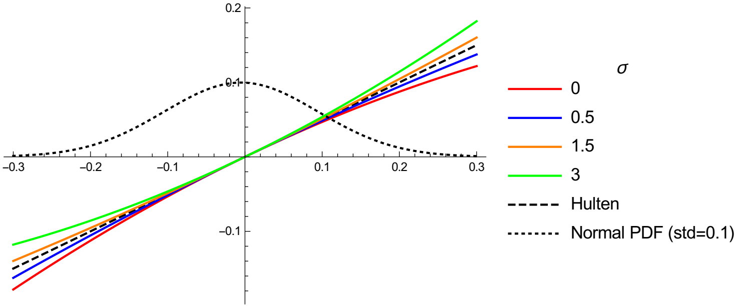
Comment #1: 2^{nd} order $\Rightarrow$ relevant for large shocks

Section 4, Macro Elasticities of Substitution

- Consider the model of Section 4: CES utility, decreasing return to scale, no intermediate inputs



NOTE: x-axis $\log A_i$; y-axis $\log \frac{C}{\overline{C}}$;
σ the *structural* elast. of substitution of utility; span of control $\beta = 0.8$

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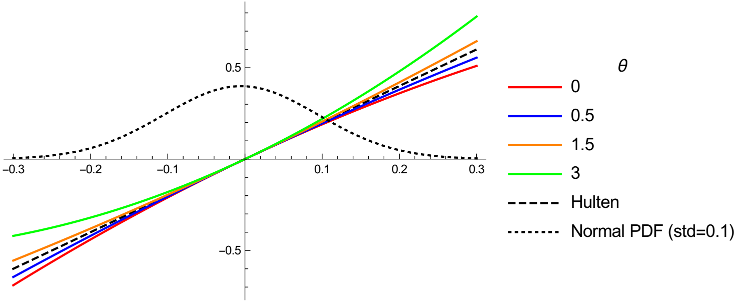
Section 6, Input-Output Multiplier

- Consider the model of Section 6: 1 sector, final good used as intermediate inputs, with a CES production function.



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θ the *structural* elast. of substitution in production; share of intermediate input $a = 1/2$

Comment #1: 2^{nd} order ⇒ relevant for large shocks

- Does deviation from the Hulten's theorem comes from "large" shocks?
- If yes, then it is related to Acemoglu et al. (2017).
- Even if, in your case, the "tail risk" comes from non-linearities.
- It would be nice to use the same asymptotic methodology than in Acemoglu et al. (2017).
- The oil shock experiment in Section 8 is thus very relevant.

Comment #2: Is the calibration it too extreme?

In section 8, you calibrate the structural model.

- Production function is a CES (θ) aggregation of value-added and a composite of intermediate goods X_k with

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- Section 4: Leontieff and some decreasing return to scale $\Rightarrow$ strong magnification of negative shocks.

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- For non-efficient economies, sectoral policy might have macro effects.