Earnings Inequality and the Minimum Wage: Evidence from Brazil[*]

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Abstract

We assess the extent to which a rise in the minimum wage can account for three facts characterizing a large decline in earnings inequality in Brazil from 1996–2012: (i) the decline is more pronounced towards the bottom of the distribution; (ii) one quarter of the decline stems from an increase in relative pay at less productive firms; and (iii) another quarter is attributable to falling pay differences due to worker heterogeneity. To this end, we build an equilibrium search model with heterogeneity in worker ability and firm productivity. The central feature of the model is the presence of spillover effects of the minimum wage on higher earnings ranks due to monopsonistic competition among firms for workers. We estimate the model using indirect inference and find that the rise in the minimum wage explains 70 percent of the decline in the variance of log earnings. Spillover effects of the minimum wage account for more than half of this decline and quantitatively match the three empirical facts. Our results suggest that labor market dynamics can lead to large effects of policy on earnings inequality.

Keywords: Earnings Inequality, Minimum Wage, Brazil, Matched Employer-Employee Data, Monopsony, Worker and Firm Heterogeneity

JEL classification: D33, E24, J08, J31, N36

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1 Introduction

Earnings inequality has become central to recent debates in academic and policy circles.[1] A majority of respondents to an international survey identified government policies as the most frequently cited reason for prevailing inequality levels.[2] In this context, economists seek answers to two important questions: First, what factors drive the evolution of earnings inequality? Second, to what extent can economic policies affect these trends?

To shed light on these questions, we study Brazil as an economy that experienced a rapid decline in earnings inequality between 1996 and 2012. Starting at high initial inequality levels, Brazil saw a 26 log points (or 35 percent) fall in the variance of log earnings over this period. By comparison, in the U.S. the same measure increased by six log points (or 12 percent) during those years.[3] Concurrent with Brazil's remarkable inequality decline, the country's minimum wage rose by 119 percent in real terms. Yet, given the ongoing debate in the literature about consequences of the minimum wage, the extent to which these two trends are related is far from clear.[4]

The main contribution of this paper is to quantify the effect of the rise in the minimum wage on Brazil's inequality evolution. To this end, we use administrative matched employer-employee data to review and extend key empirical patterns that we document in detail in Alvarez, Engbom, and Moser (2015). Specifically, the three facts characterizing Brazil's earnings inequality decline that we address in the current paper are: (i) the decline is more pronounced towards the bottom of the distribution; (ii) one quarter of the decline stems from an increase in relative pay at less productive firms; and (iii) another quarter of the decline is attributable to falling pay differences due to worker heterogeneity, largely driven by decreasing returns to education and age. Hence, any candidate explanation for Brazil's inequality evolution needs to generate compression in the earnings distribution driven from the bottom with changes in the returns to firm and worker characteristics playing a prominent role.[5]

To assess the extent to which the rise in the minimum wage can account for these facts, we

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[1]Examples of recent research concerned with earnings inequality include Atkinson and Bourguignon, eds (2015) for an overview of academic work, OECD (2015) for policy issues in a number of developed countries, and IMF (2015); World Bank (2013) for policy relating to emerging markets and developing economies.

[2]See Pew Research Center (2014).

[3]Inequality measures are defined over labor income for workers of age 18–64 using the *March Current Population Survey (CPS)* for the U.S.; and the *Relação Anual de Informações Sociais (RAIS)* for Brazil. See Appendix A.1 for details.

[4]See Flinn (2010) for a selective survey of the literature.

[5]In contrast, we show that the underlying distributions of firm and worker characteristics, notably firm productivity and workers' educational attainment, became more dispersed over the period.

build a model of frictional wage dispersion based on the canonical search framework by Burdett and Mortensen (1998). Motivated by our empirical findings, we extend this framework in a tractable way to include heterogeneity in both worker ability and firm productivity. The key feature of this environment is that the minimum wage indirectly affects higher parts of the earnings distribution. Because firms compete for workers on wages, higher productivity firms increase their equilibrium wage offers above the new minimum wage in order to poach and retain workers. Therefore, while the minimum wage has a direct impact on the least productive workers and firms in the economy, its effects will slowly fade out towards the top of the earnings distribution. These spillover effects open the door to the minimum wage qualitatively accounting for the three facts on Brazil's inequality decline.

We find that the minimum wage is also quantitatively successful at explaining the documented facts on the inequality evolution. To this end, we estimate key model parameters guiding labor mobility as well as heterogeneity in worker ability and firm productivity using indirect inference on the Brazilian microdata from 1996–2000. We then use the estimated model to simulate the effects of the observed minimum wage increase. The main result of this experiment is that 70 percent of the observed decline in the variance of log earnings are accounted for by the rise in the minimum wage. More than half of this decline is due to indirect effects of the minimum wage. In line with our empirical findings, the model generates significant compression up to the top decile of the earnings distribution. A sizable share of the overall inequality decline is due to a weaker productivity-pay gradient across firms, with the model generating a drop of 4.3 log points in the variance of log earnings due to this channel, relative to 5.0 log points in the data. Furthermore, the model predicts a fall in the dispersion of worker pay heterogeneity explaining an additional 6.2 log points fall in the variance of log earnings, compared to 5.6 log points in the data.[6] Together, these results suggest that the minimum wage was an important driver behind Brazil's inequality decline.

A central feature of the model is the presence of spillover effects of the minimum wage on higher earnings ranks. Their source is the upwards-sloping labor supply curve faced by monopsonistic firms under search frictions, creating a trade-off in wage setting between firm size and profitability. In equilibrium, more productive firms offer higher wages and workers gradually

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[6]In the model, all of the decline in worker-specific pay is due to convergence in the returns to worker types. Also in the data, we find that all of the decline in worker heterogeneity explained by observable worker characteristics (age, education, and occupation) is due to decreasing returns to these characteristics.

climb up a job ladder by moving to better-paying employers. Since the rates of poaching and re-taining workers depend on a firm's rank in the wage offer distribution within each labor market segment, the minimum wage indirectly affects equilibrium wage posting strategies of all firms in the market. As the competitive pressure in response to a rise in the minimum wage is weaker for firms further up in the productivity distribution, the resulting productivity-pay gradient across firms is lower and earnings are less dispersed. Analogously, since lower ability worker are more likely and more intensely affected, the minimum wage also leads to compression in the relative rents captured by different worker types. We provide empirical evidence for the mechanism by identifying a job ladder across firms in the Brazilian microdata and show that, in line with the model predictions, this job ladder has become flatter as the minimum wage increased over time.

Our model highlights the redistributive effects of the minimum wage. In the presence of search frictions, firms generate monopsony rents because they generally pay workers below their marginal product. Through its direct and indirect effects on firms' wage posting decisions, the minimum wage transfers some of these rents towards workers. While individuals who remain employed at a higher minimum wage benefit unambiguously, not everyone gains from the policy change. The lowest productivity firms stop recruiting from low ability markets or exit altogether for high enough levels of the minimum wage. Similarly, the lowest ability workers first relocate to more productive firms before eventually being forced into unemployment. Evaluating these channels quantitatively, however, we find small displacement effects of the minimum wage for both firms and workers.

As a validation of the minimum wage mechanism, we show that it explains salient sectoral and regional trends in the Brazilian household survey data (*Pesquisa Nacional por Amostra de Domicílios*, or *PNAD*). Exploiting the universal coverage of the survey data, we document that the earnings inequality declined markedly among formal sector workers in Brazil and to a lesser degree among workers in the informal economy. To the extent that labor regulations like the minimum wage are more strictly enforced in the formal sector, this finding lends additional support to the minimum wage hypothesis. Furthermore, we confirm that sectors and regions that started out at lower aver-age earnings levels experienced more pronounced declines in inequality, in line with the minimum wage having disproportionate effects on those parts of the economy where it is most binding.

We also confirm a key prediction of the model for the effect of the minimum wage on the allo-cation of workers across firms in the microdata. In the model, the minimum wage renders matches

between the lowest productivity firms and the lowest ability workers infeasible because the surplus generated from such matches falls short of the minimum wage. Consequently, the policy induces negative sorting between workers and firms towards the bottom of the firm productivity distribution. In support of this mechanism, we document a negative correlation between firm effect and worker effect ranks estimated using our earlier two-way fixed effects decomposition.[7] Consistent with our model prediction, we show that this negative sorting pattern becomes more pronounced as the minimum wage increases over time.

A general insight emanating from our analysis is that labor market dynamics can propagate effects of policy on the earnings distribution. Our quantitative analysis attributes more than half of the overall decline in the variance of log earnings to spillover effects of the minimum wage. Consequently, only considering the direct effect of a rise in the minimum wage would significantly understate its impact on earnings inequality. In our framework, the effects of the minimum wage are propagated through monopsonistic competition among firms for workers due to on-the-job mobility.[8] While our analysis focuses on one particular policy and economic environment, we conjecture that similar quantitative results would obtain in a broader class of models featuring spillover effects in wage setting, and considering a range of other labor market policies such as unemployment insurance, employment protection legislation, and non-discrimination laws.

Related literature. Our work relates to three strands of the literature within the broad realm of understanding inequality in labor markets. The first strand is concerned with decomposing the determinants of earnings inequality into components relating to workers, firms, and other factors, and using this decomposition to understand changes in the earnings distribution over time. The seminal work in this area is that of Abowd, Kramarz, and Margolis (1999, henceforth AKM) who propose a two-way fixed effects framework controlling for unobserved worker and firm heterogeneity. They find that firms explain a significant share of earnings inequality levels in French linked employer-employee data (but do not study changes over time).[9] In later work,

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[7]Overall, we find significant positive sorting between workers and firms. Speaking to the weakness of the AKM estimation framework pointed out by Bonhomme et al. (2015), we verify that the negative correlation at the bottom is robust to restricting our sample to large firms and frequent worker switchers.

[8]More precisely, our model features interdependence in firms' wage offers whenever workers receive outside job offers while employed with strictly positive but finite rate (probability less than one) in a continuous (discrete) time setup.

[9]In a later paper, Abowd et al. (2002) note that the approximate estimates obtained in AKM, particularly firm effects, were contaminated due to the approximation method used at the time. However, the authors note that their original conclusion, namely that person effects account for most wage variation in France, remains unchanged.

Card et al. (2013) apply the same methodology to Germany and argue that firms explain a quarter to a third of the overall rise in earnings inequality in Germany. Card et al. (2015) use a static AKM framework to investigate the degree of differential sorting and rent sharing between male and female workers in Portugal. Alvarez, Engbom, and Moser (2015) applies this methodology to understand Brazil's decline in inequality between 1988 and 2012, and find that falling inequality between firms in pay is an important component of this decline. Although not within an AKM framework, Barth et al. (2014) and Song et al. (2015) argue that changes in pay across firms were important in understanding the increase in wage dispersion in the U.S. during the last decades.

Second, our theoretical framework is closely related to the literature using search models to study wage dispersion. While work in this area goes back to at least McCall (1970), a more recent class of search models pioneered by Burdett (1978) and further developed by Burdett and Mortensen (1998) lays the foundation for our analysis of the effects of the minimum wage in a job ladder environment. A rich body of follow-up work has used versions of this model to jointly study wage dispersion and labor dynamics (van den Berg and Ridder, 1998; Bontemps et al., 1999, 2000; Mortensen, 2000, 2003; Postel-Vinay and Robin, 2002; Cahuc et al., 2006; Jolivet et al., 2006; de Araujo, 2014). To this literature we contribute a tractable model of the minimum wage with heterogeneity in both firm productivities and worker abilities, an environment that previous research highlighted as important but difficult to study. In related work, Hornstein et al. (2011) note that several search models struggle to generate the observed amount of wage dispersion in the data. Their argument is that on-the-job search is crucial for these models to generate realistic levels of frictional wage dispersion. Our complementary insight is that also the effects of policy, such as the minimum wage, can be amplified in such models.

We also connect our structural search model to empirical studies of wage determination. Whereas several empirical studies document significant dispersion in pay across firms using the original AKM methodology, few studies have provided a formal justification for this framework.[10] Providing such a microfoundation is important since other papers have stressed that sorting models of labor markets may lead the AKM framework to produce misleading results (Lentz and Mortensen, 2010; Eeckhout and Kircher, 2011; Lopes de Melo, 2013; Bonhomme et al., 2015). We bridge these two literatures by contributing a tractable model of frictional wage dispersion with heterogeneity in both worker ability and firm productivity that maps directly into the AKM framework. We

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[10]A notable recent exception is Burdett et al. (2014).

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characterize this mapping and show that the AKM regression framework recovers the underlying structural parameters of the model from the data.

Finally, our focus on the effects of the minimum wage on the earnings distribution complements a long-standing debate in the literature on how the minimum wage affects labor market outcomes. A salient debate in this literature revolves around the employment consequences of the minimum wage (Card and Krueger, 1994; Neumark and Wascher, 1994; Manning, 2005), with mixed findings but pointing in the direction of small negative employment effects. DiNardo et al. (1996), Lee (1999), Card and DiNardo (2002) argue that a decline in the federal minimum wage in the U.S. in the 1980's explains a significant amount of the increase in earnings inequality during that time. Going against this previous literature's conclusions, Autor et al. (2008) and Autor et al. (2015) argue that nonmarket factors such as the decline in the minimum wage contributed little to the dynamics of U.S. earnings inequality. We contribute to this debate by showing that the predictions of a structural model of the minimum wage are quantitatively consistent with sizable effects throughout large parts of the earnings distribution. Bárány (2015) studies a model with complementarity between skill groups in the production technology and endogenous educational investment. Like in our framework, the minimum wage in that model has spillover effects on higher income groups, but for orthogonal reasons. Harasztosi and Lindner (2015) study an increase in the minimum wage in Hungary in 2001 that is of similar size as the one experienced in Brazil 1996–2012, and find that it pushed up wages with only a small negative impact on employment. In comparison to their work, our focus is on the effects of the minimum wage on earnings inequality. Komatsu and Menezes Filho (2015) argue empirically that increases in the minimum wage can explain all of the reduction in earnings inequality in Brazil between 2002 and 2013. Complementary to the focus of his paper, our structural model allows us to separately identify the direct and indirect effects of the minimum wage on Brazil's earnings distribution.

Outline. The rest of the paper is structured as follows. Section 2 introduces the three datasets we use to study the evolution of earnings inequality and the role of the minimum wage. Section 3 presents key facts on the decline of earnings inequality in Brazil building on Alvarez, Engbom, and Moser (2015). Section 4 provides an institutional history of the minimum wage in Brazil. In Section 5, we describe our structural model of frictional wage dispersion and characterize the effects of a rise in the minimum wage on workers and firms in the economy. Section 6 describes

our estimation strategy and the main policy experiment identifying the effects of the observed rise in the minimum wage on the earnings distribution. Section 7 presents quantitative results on the effect of the minimum wage on earnings inequality, on compression throughout the income distribution, and on the productivity-pay gradient across firms. In Section 8, we provide empirical evidence for effects of the minimum wage on the earnings distribution, discuss our modeling assumptions, and discuss welfare implications of the minimum wage in our framework. Finally, Section 9 concludes by putting into context the paper's main findings.

2 Data

Our analysis combines data from three separate sources: The first dataset are the Brazilian Household surveys *Pesquisa Nacional por Amostra de Domicílios (PNAD)*, which contain a representative sample of households covering all of Brazil, including workers in the formal and informal sectors. Our second data source consists of an administrative linked employer-employee dataset called *Relação Anual de Informações (RAIS)*, containing annual information from 1996–2012 on earnings and demographic characteristics of formal sector workers as reported by employers. The third dataset is the *Pesquisa Industrial Anual Empresa (PIA)*, which contains information on the revenue and cost structure of large firms in Brazil's mining and manufacturing sectors from 1996–2012, and which we merge with the worker-level data contained in RAIS. The following subsections describe each of the three datasets in detail.[11]

2.1 Household survey data (PNAD)

The PNAD household surveys consist of a double-stratified sampling scheme by region and municipality, interviewing a representative of households in Brazil. The survey asks the household head to respond on behalf of all family members and report a rich set of demographic and employment-related questions. In particular, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify individuals as working in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the U.S. March Current Population Survey (CPS). We keep only observations that satisfy our selection criteria and

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[11]Appendix A.2 also contains summary statistics for PNAD, RAIS, and PIA at a period frequency.

have non-missing observations for labor income, whose variable definition we harmonize across years.[12]. Table 1 presents basic summary statistics on the PNAD data.

Table 1. PNAD summary statistics

	(1)	Log earnings		(4)
		(2)	(3)	
	# Workers	Mean	Std. dev.	Formal share
1996	60,176	6.81	0.98	0.65
1997	64,204	6.79	1.00	0.64
1998	63,016	6.78	0.97	0.64
1999	64,328	6.72	0.95	0.63
2000	-	-	-	-
2001	70,558	6.68	0.95	0.63
2002	72,273	6.66	0.93	0.63
2003	71,959	6.59	0.93	0.64
2004	75,617	6.61	0.91	0.64
2005	78,064	6.64	0.90	0.65
2006	78,627	6.71	0.89	0.66
2007	76,616	6.76	0.87	0.68
2008	76,571	6.80	0.85	0.69
2009	77,037	6.83	0.84	0.70
2010	-	-	-	-
2011	67,884	6.93	0.80	0.73
2012	69,297	6.98	0.80	0.73

Notes: All statistics are for adult male workers of age 18–64. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period. The The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period. Surveys are not available in years 2000 and 2010.

2.2 Linked employer-employee data (RAIS)

The RAIS is constructed from a mandatory survey filled annually by all formally registered firms in Brazil. The data collection is administered by the Brazilian Ministry of Labor and Employment, which kindly provided the data for the purposes of this research under a confidentiality agreement. Data collection was initiated in 1986 within a nationally representative set of regions, reaching complete coverage of all employees at tax-registered establishments across all sectors of the economy in 1994. It is common practice for businesses to hire a specialized accountant to help with the completion of the RAIS survey to avoid fines levied on late, incomplete, or inaccurate reports. The data contain a unique, completely anonymized, time-invariant person identifier, which

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[12]Standardized cleaning procedures are adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at http://www.econ.puc-rio.br/datazoom/english/index.html.

allows us to follow workers over time. It also contains unique, completely anonymized time-invariant establishment and firm IDs that we use to link multiple workers to firms and follow firms over time. We follow our previous work in conducting all analyses at the firm-level.

The dataset contains information on average gross monthly labor earnings including regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit-sharing agreements as well as start and end month of the job. The measure of income adjusts for labor supply by dividing annual earnings by the number of months worked at the job. A worker might have multiple spells in a year if he or she switched employer during the year or worked multiple jobs. We restrict attention to a unique observation per worker-year by choosing the highest-paying among all longest employment spells in any given year. In addition, we observe the age, gender, educational level, and occupation[13] of each worker. On the firm side, we also use sub-sector categories from IBGE, the national statistical institute.[14] Our firm size measure is the number of full-time equivalent workers during the reference year.

We exclude individual observations that have either firm IDs or worker IDs reported as invalid as well as data points with missing earnings, dates of employment, educational attainment or age. Together, these basic cleaning procedures drop less than 1% of the original population, indicative of the high quality of the administrative dataset.

Table 2 provides key summary statistics for the RAIS data for six periods spanning 1988-92, 1992-96, 1996-2000, 2000-2004, 2004-08, and 2008-12.[15] All numbers reported in the table are for adult male workers of age 18 to 64. We make the selection based on gender and age to be consistent with our previous work.[16] The group of adult males represents 55% of the total dataset in 2000 and their average earnings, educational attainment, and age are largely representative of the overall population.

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[13]We use occupations from the pre-2003 Classificação Brasileira de Ocupações (CBO) at the two-digit level.

[14]Both the industry and occupation classification systems changed during the period we study. We use conversion tables provided IBGE to standardize classification between different years and choose categories for both occupations and sectors coarse enough in order to avoid potential biases arising from mechanical changes in the classification system over time.

[15]To calculate the standard deviation, we demean the data by year before we pool the years within a subperiod.

[16]Extensive labor supply decisions correlated with schooling choice or the timing of retirement could bias our estimates if we were to include these population subgroups. For similar reasons, we also exclude women to avoid biases caused by job switching decisions motivated by maternal leaves and other motherhood-related labor market movements.

Table 2. RAIS summary statistics

	(1)	(2)	Log earnings	
			(3)	(4)
	# Workers	# Firms	Mean	Std. dev.
1996	18.05	0.98	1.32	0.87
1997	18.31	1.06	1.32	0.85
1998	18.65	1.12	1.28	0.85
1999	18.54	1.18	1.25	0.84
2000	19.15	1.22	1.20	0.83
2001	20.45	1.30	1.12	0.83
2002	21.22	1.37	1.06	0.81
2003	21.70	1.42	0.99	0.79
2004	22.78	1.48	0.98	0.78
2005	23.96	1.54	0.94	0.77
2006	25.14	1.61	0.86	0.75
2007	26.58	1.68	0.83	0.74
2008	28.45	1.76	0.83	0.73
2009	29.17	1.84	0.80	0.73
2010	31.01	1.95	0.78	0.71
2011	32.38	2.05	0.81	0.71
2012	33.23	2.13	0.78	0.70

Notes: All statistics are for male workers age 18–64.Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adultmale workers. The standard deviation is calculatedby first demeaning variables by year and then poolingthe years within a sub-period.

2.3 Firm characteristics data (PIA)

The PIA dataset contains data on firm characteristics from 1996 to 2012. It is constructed from annual surveys filled by firms in the manufacturing and mining sector and collected by the Brazilian Statistics and Geography Institute (*Instituto Brasileiro de Geografia e Estatística*, or *IBGE*), with whom we have signed a confidentiality agreement. This survey is mandatory for all firms with either more than 30 employees or more than $300,000 in revenues. As with RAIS, completion of the survey is mandatory and non-compliance is subject to a fine by national authorities. Each firm has a unique, anonymized identifier, which we use to link firm characteristics data from PIA data to worker-level outcomes in the RAIS data. Each firm has a unique, completely anonymized identifier which we use to link the PIA dataset with employee data from RAIS.

The PIA dataset includes a breakdown of operational and non-operational revenues, costs, investment and capital sales, number of employees and payroll. All nominal values are converted to real values using the CPI index provided by the IBGE. Instead of the measure of firm size in the PIA, we prefer our measure of full-time-equivalent employees constructed from the RAIS as

it accounts for workers only employed during part of the year. We define operational costs as the cost of raw materials, intermediate inputs, electricity and other utilities, and net revenues as the gross sales value due to operational and non-operational firm activities net of any returns, cancellations, and corrected for changes in inventory.[17]. We finally construct value added as the difference between net revenues and intermediate inputs, and value added per worker as value added divided by full-time equivalent workers. This is our main measure of firm productivity.[18]

Table 3 shows key summary statistics for the RAIS data for the four periods for which we have firm financial data in the PIA: 1996-2000, 2000-2004, 2004-08, and 2008-12. All results are weighted by the number of full-time equivalent workers employed by the firm.

Table 3. PIA Summary Statistics

	(1)	Log revenues per worker		Log value added per worker	
		(2)	(3)	(4)	(5)
	# Firm-years	Mean	S.d.	Mean	S.d.
1996	21,840	11.83	1.00	11.15	1.08
1997	21,022	11.86	1.03	11.15	1.11
1998	22,096	11.88	1.09	11.17	1.19
1999	22,771	12.01	1.16	11.29	1.27
2000	22,751	12.00	1.19	11.22	1.30
2001	24,920	12.01	1.24	11.23	1.32
2002	26,418	12.02	1.30	11.26	1.39
2003	27,853	11.96	1.31	11.18	1.37
2004	28,708	12.00	1.32	11.21	1.35
2005	30,628	11.94	1.30	11.16	1.33
2006	31,962	11.94	1.28	11.18	1.32
2007	31,808	11.97	1.28	11.21	1.31
2008	33,349	12.01	1.27	11.26	1.30
2009	34,200	12.01	1.23	11.31	1.27
2010	34,650	12.03	1.22	11.32	1.25
2011	36,773	12.06	1.20	11.34	1.23
2012	37,858	12.07	1.18	11.36	1.20

Notes: Sample includes all firms covered by the PIA dataset in the mining and manufacturing sectors. The number of firm-years and number of unique firms are reported in thousands. Allmeans and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within asub-period.

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[17]We have explored alternative revenue definitions such as only restricting attention to operational revenues or excluding certain types of non-operational revenues. Such robustness checks yield very similar results to what we report below.

[18]We have also constructed alternative measures of firm productivity by cleaning value added per worker off industry-year effects and some measures of worker skill.

3 Facts about Brazil's inequality decline

The goal of the current project is to quantify the contribution of a rise in Brazil's minimum wage towards earnings inequality dynamics during 1996–2012. To provide context for our study, we present in this section some key facts on the evolution of earnings inequality in Brazil over this period. All statistics are computed for the population of male formal sector workers of age 18–64 using the RAIS data, but similar trends hold for the overall worker population.[19]

Our motivating observation is that earnings inequality has declined rapidly in Brazil. Figure 1 plots the evolution of the variance of log earnings of adult males in the formal sector between 1996 and 2012. The data show a steady decline in the variance of log earnings by 26 log points or 35 percent, from 0.76 to 0.49, over the period. To put this evolution into context, the variance of log earnings for adult male workers increased by six log points in the U.S. over the same period.[20]

Figure 1. Decline in the variance of log earnings in Brazil, 1996–2012
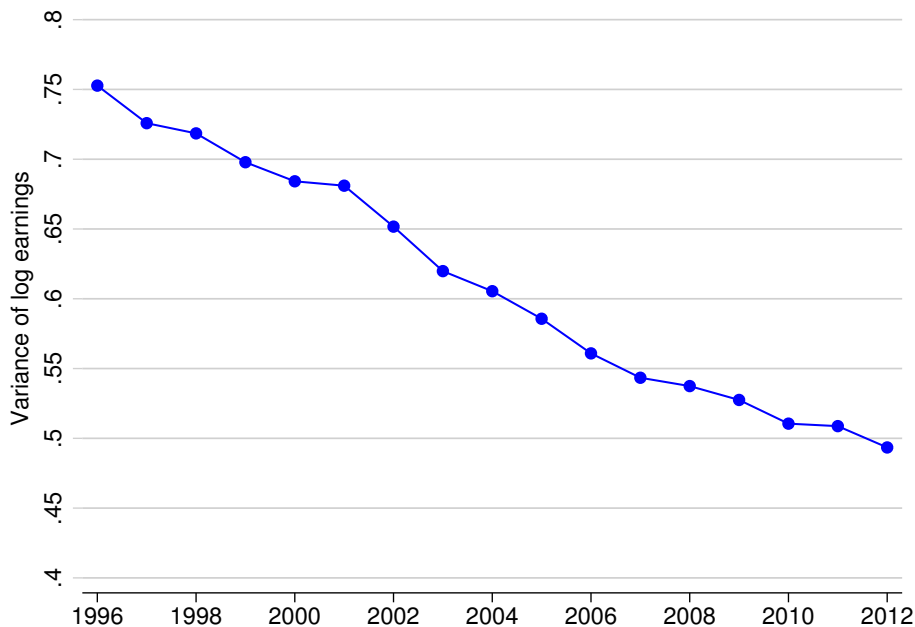
We now present three facts characterizing Brazil's inequality evolution between 1996 and 2012.

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[19]In Section 8, we explore inequality trends in Brazil's formal versus informal sectors using the PNAD household survey data. See Alvarez, Engbom, and Moser (2015) for a detailed exploration of inequality trends by population subgroups.

[20]Calculations for the U.S. are based on the *March Current Population Survey (CPS)*, but evidence from other datasets, including alternative survey data used in Heathcote et al. (2010), or administrative tax returns data from Kopczuk and Saez (2010), point towards similar magnitudes. Thus, by any measure, the decline of inequality in Brazil can be considered large.

Fact 1. *The inequality decline is more pronounced towards the bottom of the earnings distribution.*

While Brazil's inequality decline was characterized by widespread compression throughout the earnings distribution, there was more pronounced catch-up from the bottom. For illustration purposes, we present two particularly prominent measures of top- and bottom-inequality, namely the log 90–50 and log 50–10 percentile ratios of the earnings distribution. Figure 2 plots these inequality measures based on the universe of adult male workers in the RAIS data. There was a significant decline in both measures, but the log 50–10 percentile ratio declined significantly more over the period. Specifically, the log 50–10 percentile ratio declined by 38 log points while the log 90–50 percentile fell declined by 19 log points at the same time.[21] Hence, while there was rapid compression of the earnings distribution relative to the top percentiles, this catch-up was more pronounced among the lowest earnings groups.

Figure 2. Normalized evolution of earnings percentile ratios measuring top and bottom inequality, 1996–2012
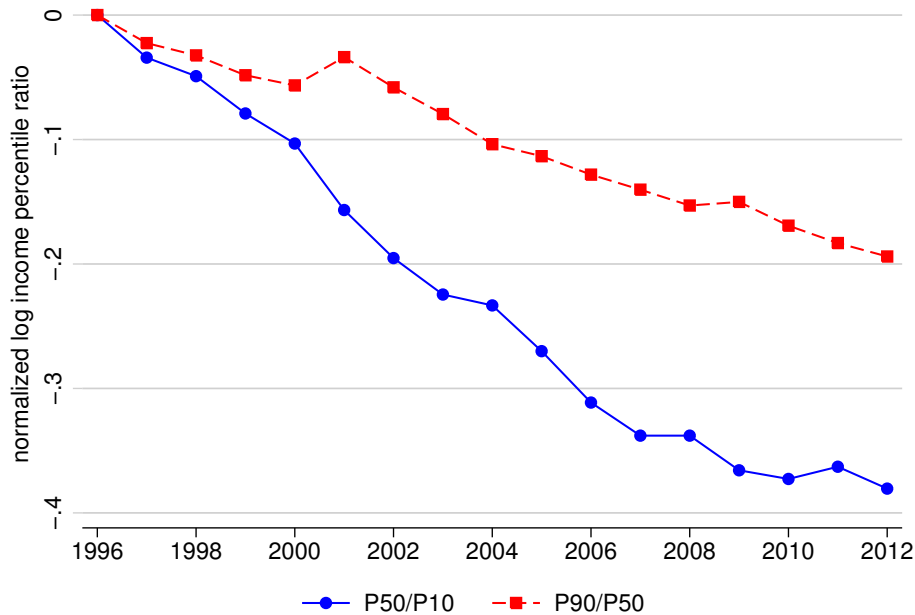
Before proceeding with our characterization of Brazil's inequality decline, we present a de-composition of the overall inequality evolution into one component capturing differences in firm

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[21]Fact 4 in Appendix A.3 shows that the bottom-driven inequality decline is apparent more broadly using other percentile ratios of the earnings distribution. Specifically, we find that there was compression up to, but not above, the 90th percentile of the earnings distribution. Furthermore, we show that that all percentiles of the earnings distribution experienced rapid real earnings growth over this period.

pay and another component capturing worker heterogeneity. To motivate this analysis, which was first presented in Alvarez, Engbom, and Moser (2015), we note that most initial earnings inequality and most of the decline are due to dispersion of raw earnings between firms in the data.[22] To distinguish between inherent firm pay differences and the sorting of heterogeneous workers across firms, we follow an estimation methodology pioneered by AKM.[23] Specifically, we estimate a two-way fixed effect regression of log monthly earnings on a large set of worker effects, firm effects and year dummies in five-year sub-periods:

$$\log \left(y_{ijt}\right) = \alpha_i^p + \alpha_{J(i,t)}^p + \gamma_t + \varepsilon_{it} \tag{1}$$

for $t \in p = \{t_1, \ldots, t_5\}$ and where α_i^p denotes the individual fixed effect of worker i in period p, $\alpha_{J(i,t)}^p$ denotes the firm effect of the employer of worker i at year t, Y_t is a year dummy, and ε_{it} is an error term that satisfies the strict exogeneity condition $\mathbb{E}\left[\varepsilon_{it}\mid i, t, J\left(i, t\right)\right] = 0$.

Table 4 presents results from the above regression. In particular, we compute and report the variance of the predicted value due to each component from the AKM framework in equation (1). The variance of firm effects falls from 17 log points in 1996–2000 to eight log points in 2008–2012, which constitutes 45 percent of the overall inequality decline over the period. Similarly, the variance of worker effects falls from 36 log points in 1996–2000 to 31 log points in 2008–2012, making up 24 percent of the overall decline.

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[22]Details of the between- and within-firm analysis are summarized in Fact 5 of Appendix A.3.

[23]To test the validity of this framework, Figure 22 in Appendix A.4 plots the changes in estimated firm effects for workers switching firms by quartile of estimated firm effect before and after the switch. Alvarez, Engbom, and Moser (2015) discuss a range of additional checks and conclude that the assumptions underlying AKM holds in Brazil during this time.

Table 4. AKM variance decomposition between periods

	(1)	(2)	(3)
	1996-2000	2008-2012	Change
Variance of total earnings	0.72 (100%)	0.52 (100%)	-0.20 (100%)
Variance of worker effects	0.36 (50%)	0.31 (60%)	-0.05 (24%)
Variance of firm effects	0.17 (23%)	0.08 (15%)	-0.09 (45%)
Covariance	0.14 (20%)	0.10 (20%)	-0.04 (22%)
Variance of residual	0.06 (8%)	0.04 (7%)	-0.02 (10%)
# worker years	90.2	151.0	
R^2	0.92	0.93	

Note: Cells contain variance (share) explained by each component. Year dummies are omitted but account for a negligible share of the overall variation. The "Covariance" term reports two times the covariance between worker effects and firm effects from the AKM estimation. Number of worker years is in millions.

With this statistical decomposition in mind, we now continue our characterization of Brazil's inequality decline.

Fact 2. *One quarter of the overall decline in the variance of log earnings is attributable to an increase in relative pay at less productive firms.*

We now move on to investigating the drivers behind the fall in the variance of firm effects, which declined by 17 log points in 1996–2000 to 8 log points in 2008–2012, constituting 45 percent of the overall decline in the variance of log earnings over the period. In explaining the compression in firm-specific pay components, we consider two potential explanations.

The first possibility is that, to the extent that firm characteristics such as productivity matters for pay (Blanchflower et al., 1996; Abowd et al., 1999; Margolis and Salvanes, 2001), firms could have become more equal in such underlying characteristics. Figure 3, however, shows that dispersion in worker-weighted firm productivity as measured by value added per worker increased slightly over this period. Qualitatively similar trends are observed for the raw productivity measure shown in the figure and alternative cleaned measures of productivity that control for worker composition and industry, discussed in detail in Appendix A.6. Hence, there is no evidence in favor of a decline in dispersion of underlying firm characteristics.[24]

The second possibility we investigate is that the degree to which firm productivity passes through to workers' wages could have declined, leading firms with given productivity differences

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[24]We discuss in more detail the trends in other firm characteristics, including firm size and export intensity, in Alvarez, Engbom, and Moser (2015).

Figure 3. Evolution of dispersion of various productivity measures
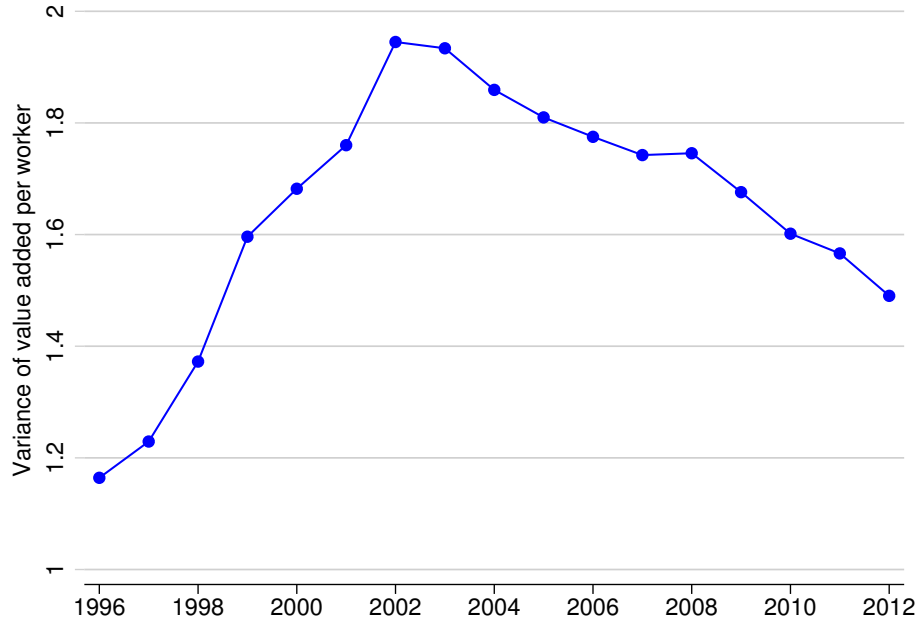
to pay their workers more equally over time. Henceforth, we will refer to the relationship between firm productivity and worker pay as the "productivity-pay gradient." Using the estimated firm effects from the AKM regression above, we evaluate this possibility by regressing the estimated firm effects on average value added per worker in each five-year subperiod. Consider a given subperiod and let a_j be the estimated firm component of pay and let VA_j denote average log value added per worker during the subperiod. For each subperiod, we regress[25]

$$a_j = \zeta_0 + \zeta_1 VA_j + \varepsilon_j$$

Notice that all regressions are run with sub-period averages of all variables. Based on the above regression, we compute the variance due to value added per worker as

$$Var\left(\hat{a}_j\right) = \left(\hat{\zeta}_1\right)^2 Var\left(VA_j\right)$$

Table 5 shows that firm productivity explains a significant amount of variation in pay across firms and that the regression coefficient between firm effects and firm productivity dropped from

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[25]We have also considered versions including a range of other firm characteristics as well as subsector controls, but since these are not of first order importance we omit the results here.

0.258 to 0.147 over the period we study.[26]

To evaluate the importance of a declining pass through from firm productivity to pay for the overall decline in earnings, we compute the variance of the predicted value from this regression for each subperiod. This fell by five log points, namely from ten to five log points, between 1996–2000 and 2008–2012. As the variance of the underlying productivity distribution did not fall during this time, we conclude that a weakening firm productivity-pay gradient accounts for approximately all of the explained decline in the variance of firm effects, thereby explaining over 25 percent of the decline in the overall variance of log earnings in Brazil over the period.

Table 5. Regression of firm pay component on productivity

	(1)	(2)	(3)
	1996–2000	2008–2012	Change
Value added p.w.	0.258	0.147	-0.111
Constant	-2.883	-1.599	1.284
Explained variance	0.10	0.05	-0.05
—due to returns			-0.05
—due to composition			0.01
# worker years	16.6	26.3	
R^2	0.583	0.465	

Note: Dependent variable is AKM estimate of firm effect on wages. Independent variable is log value added per worker. Explained variance holds R^2 fixed in 1996–2000. Number of worker years is in millions.

Fact 3. *Another quarter is due to declining differences in pay for unobserved worker characteristics.*

As we saw in Table 4, another five log points or 24 percent of the decline in the variance of log earnings are due to compression in estimated worker effects in the AKM framework. Analogously to our firm-level analysis, we regress the estimated worker effects, a_i, on five age bin dummies, four education dummies[27]:

$$a_i = \text{age}_i + \text{edu}_i + \varepsilon_i$$

Based on the above regression estimates, we predict the variance due to each of age and education. As in our firm-level analysis, we thereby distinguish between changes in the predicted variance

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[26]We do not present standard errors adjusted for the fact that the left hand side of this regression contains estimation error. Yet given the large sample size, we expect such adjustments to still yield strongly statistically significant estimates.

[27]We have also examined versions of this regression with additional occupation controls, as well as with age and education interacted and additional sector controls, but neither of these alternatives changes the estimated results significantly.

due to changes in the fundamental distribution of worker characteristics versus changes in the returns to such characteristics.

Table 6 shows the result from this regression of estimated worker effects on age and education. We see that the estimated coefficients on both age and education uniformly declined over time. Furthermore, the explained variance of worker effects attributable to these worker observable characteristics declines by 3.1 log points over the period, all of which is driven by decreasing returns to the characteristics rather than due to changes in the underlying composition of workers.

Table 6. Regression of estimated worker effects on worker characteristics

	(1)	(2)	(3)
	1988–1992	2008–2012	Change
Age groups			
25–29	0.20	0.16	-0.04
30–39	0.39	0.30	-0.09
40–49	0.52	0.42	-0.10
50–64	0.48	0.51	-0.03
Education groups			
Middle school	0.21	0.11	-0.10
High school	0.61	0.27	-0.34
College or more	1.21	1.10	-0.11
Explained variance	0.11	0.08	-0.03
—due to returns			-0.03
—due to composition			0.01
# worker years	90.2	151.0	
R^2	0.34	0.37	

Note: Dependent variable is the estimated worker effect a_i. Number of workers in millions. Education estimates are relative to "less than middle school (<7 years)" category. Age estimates are relative to "age 18–24" category. Number of worker years is in millions.

It is important to note that our analysis of the explained decline in worker effects as well as the share explained by lower returns to these characteristics is limited to *observable* worker characteristics. It is well known since Mincer (1958) that observable worker characteristics only explain a fraction of the variation in earnings and we confirm this for the Brazilian case in Alvarez, Engbom, and Moser (2015). One may naturally suspect that the returns to unmeasured ability or other pay relevant characteristics, which have been argued to explain a large share of overall earnings dispersion (Abowd et al., 1999; Card et al., 2013; Alvarez, Engbom, and Moser , 2015), have declined in tandem with the returns to observable characteristics such as age and education,

which we are able to explicitly investigate above. In this case, our results on the share of the decline in the variance of log earnings due to weaker returns on observable worker characteristics should be interpreted as a lower bound on the true decline explained by returns to both observable and unobservable worker characteristics.

Summary. Summarizing Facts 1–3 above, Brazil experienced a large decline in earnings inequality between 1996–2012, which was bottom-driven, and a large part of which was due to a weaker productivity-pay gradient across firms as well as a a decline in the returns to worker characteristics. Together, these facts suggest that Brazil's inequality decline was due to changes in pay policies rather than changes in economic fundamentals on either the worker- or the firm-side, and that these changes were particularly salient towards the bottom of the earnings distribution. The goal of the remainder of this paper is to build a model to rationalize these findings.

4   The minimum wage in Brazil

The three facts from Section 3 that characterize Brazil's earnings inequality decline between 1996 and 2012 highlight the importance of changes in the returns to worker and firm characteristics, rather than changes in their underlying distributions. Our results thus point towards one subset of explanations for Brazil's inequality decline, which we term changes "wage setting policies." This insight by itself rules out a host of competing explanations for the sharp fall in earnings dispersion[28]. Thus, we are led to search for changes within the set of wage setting policies that can help rationalize Brazil's inequality decline over the period.

In the current paper, we ask whether changes in economic policy can explain the large decline in earnings inequality in general, and the three facts we discussed in Section 3 in particular. Our analysis focuses on evaluating the effects of one particular policy, namely the minimum wage. Before proceeding to evaluate its effects on the earnings distribution, we now provide some institutional context and a description of the evolution of the minimum wage in Brazil.

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[28]Among the set of other explanations, we explore and rule out changes in the worker composition between the formal and informal sectors, regional earnings differences, sectoral composition, and labor mobility patterns. Details of our empirical investigation into each of these candidate explanations are contained in Alvarez, Engbom, and Moser (2015).

4.1 History

The minimum wage in Brazil is primarily a federal institution with only minor adjustments for regional price level differences. It was institutionalized as Decree-Law No. 2162 in 1940 and consolidated in 1943 under new labor laws (*Consolidação das Leis do Trabalho*, or *CLT*).[29] While the minimum wage was initially region-specific and not automatically adjusted to inflation or even legally enforced, it underwent several reforms under different political regimes between the 1940s and 1984, when it was unified across regions.

Leading up to and during Brazil's hyperinflationary period from 1980–1994, the minimum wage was adjusted first annually and later at monthly intervals according to a formula based on realized productivity growth and inflation as well as expected future inflation. Yet, due to forecasting errors in the price level during these turbulent times, the minimum wage lost over a third in real value. Following several failed stabilization plans, the *Plano Real* in 1994 stabilized the monetary system by pegging the local currency to the U.S. dollar (it was allowed to float again starting in 1999).[30] With the monetary stabilization of 1994 and the new president Fernando Henrique Cardoso of the centrist Brazilian Social Democracy Party taking office in 1995, the minimum wage became a renewed policy focus.

Nowadays, the minimum wage acts as a floor on monthly earnings for every formally registered worker. The Brazilian Ministry of Labor (*Ministério do Trabalho e Emprego,* or *MTE*) heads a compliance unit, which audits businesses through on-site visits and surveying local employees. Yet, according to official reports as well as information from our household and administrative data sources, compliance is less than perfect. While overall compliance is thought to be good in the formal sector, the minimum wage is plausibly less binding in Brazil's sizable informal economy.[31]

4.2 Evolution of the minimum wage

Between 1988 and 1996, the real minimum wage declined and experienced significant volatility as a result of hyperinflation. Then between January 1996 and December 2012, the real minimum

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[29]The original law was based in parts on Mussolini's Carta del Lavoro in Italy.

[30]See Garcia et al. (2014) for a comprehensive summary of Brazil's inflation experience and the effects of the various stabilization plans.

[31]We will return to the distinction between the effects of the minimum wage on Brazil's formal and informal sectors in Section 8 of the current paper.

wage grew by a total of 119 percent, reaching a value of 622 Brazilian Reais (410 PPP-adjusted U.S. dollars) per month by the end of the period. To put these numbers into context, the minimum wage as a fraction of median earnings increased from around 34 percent in 1996 to 60 percent in 2012. Over the same period, average labor productivity in manufacturing and mining increased by 16.6 percent; hence the ratio of the minimum wage to average labor productivity increased by 56.3 percent over this period.

Figure 4 summarizes the evolution of the variance of log earnings (in blue) and also annual averages of the real minimum wage (in red) between 1988 and 2012.[32] Suggestive of the minimum wage being related to the evolution of earnings inequality, we see that the two time series approximately mirror each other, with the correlation between the two being -0.82 in levels and -0.55 in Hodrick-Prescott (HP) filter cycles over the period.

Figure 4. Evolution of earnings inequality versus the minimum wage, 1988–2012
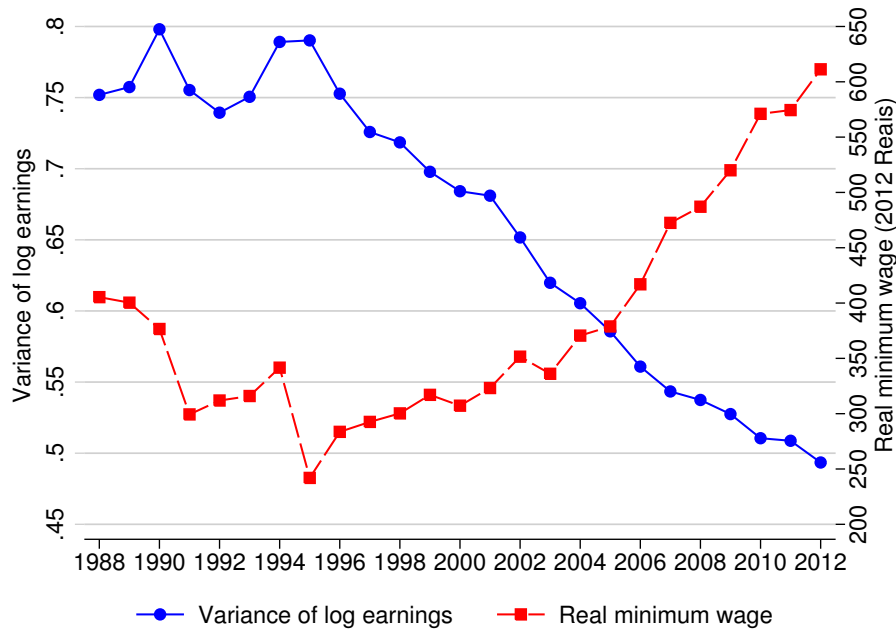
4.3 Evaluating effects of the minimum wage through a structural model

While the joint evolution of the minimum wage and earnings inequality between 1988 and 2012 suggest that the two trends might be related, it remains an open question whether their relationship is causal. Importantly, one may note that the direct effect of the minimum wage is bounded

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[32]For comparison, Appendix A.5 also shows 3-month running averages of the real minimum wage over the period.

above by the fraction of workers between the original and the new level of the minimum wage.[33] In spite of the large increase in real levels, a back-of-the-envelope calculations shows that these direct effects fall short of explaining the massive decline in earnings inequality over the period, as documented in the beginning of Section 3. Furthermore, the direct effects of the minimum wage could hardly speak to either the global compression of the earnings distribution documented in Fact 1, nor could they quantitatively explain the documented drop in the productivity-pay gradient across firms as well as the lower returns to worker characteristics noted in Facts 2–3.

A simplistic view of the minimum wage would thus conclude that its effects are limited to a small population subgroup and its effects have difficulty explaining the three facts from Section 3. Hence, in order for the minimum wage to be a promising candidate explanation, its effects need to extend beyond the direct impact on workers earning exactly the minimum wage.

Contrary to this simplistic view, a strand of the labor economics literature has suggested that the minimum wage might lead to spillover effects further up in the earnings distribution. Theories of such indirect effects of the minimum wage go back to at least Burdett and Mortensen, 1998. In a frictional labor market, monopsonistic competition among firms for workers would lead a minimum wage hike to affect wages of workers strictly above the minimum wage. In such an environment, is possible that an increase in the minimum wage ripples through the earnings distribution through such equilibrium effects. Recently, Autor et al. (2015) took up this debate empirically in arguing that the magnitude of such spillover effects is indistinguishable from measurement error in the data in the case of the U.S. labor market. To take up this debate and evaluate the importance of such channels for the case of Brazil, we proceed to build and estimate a structural equilibrium model of the Brazilian labor market. We then proceed to use the estimated model to quantify the degree of spillover effects from minimum wage increases over the period.

5   Model

We build an equilibrium search model in the spirit of Burdett and Mortensen (1998) that matches some of the key characteristics of the Brazilian labor market. In particular, our model reproduces

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[33]An even more critical view would suggest that the share of people affected by the minimum wage is restricted to the share of workers working exactly at the new minimum wage, after the increase. For this to be true, one would need to rationalize disproportionately fast productivity growth among workers with the lowest earnings, which we view as broadly incompatible with widely held beliefs that technical change over this period was characterized as high skill-biased.

the fact that identical workers are paid significantly different depending on where they work. Wage dispersion of this kind arises naturally in our model as a result of labor market frictions impeding the reallocation of workers across firms, thus giving firms monopsony power over workers. Our model also predicts that workers gradually move to better paying employers by climbing a "job ladder," which we show is a prevalent feature of the Brazilian labor market. We use our model to evaluate the impact of an increase in the minimum wage on the earnings distribution.

5.1 Environment

Time is continuous and the economy is in steady state. A unit mass of heterogeneous workers and a unit mass of heterogeneous firms meet in a frictional labor market. We describe the two groups of agents in sequence.

Workers. Workers are infinitely-lived, discount a stream of consumption at rate ρ, and differ in their permanent ability level θ. For expositional purposes, we present an economy with a continuum of types $\theta \in \Theta = [\theta_0, \overline{\theta}]$, each with mass m_θ.[34]

Workers engage in undirected search for jobs in labor markets segmented by worker ability (van den Berg and Ridder, 1998), both from non-employment and while employed at a given firm. The assumption that labor markets are separated by worker ability buys us analytical tractability, but we think it also captures a stylized feature of real-world labor markets. For instance, one would expect that a Ph.D. economist will not compete with a high school dropout for the same job, even within firms.

Because of frictions in the labor market, workers do not instantly find a job. Let λ^u denote the probability that a non-employed worker receives a job offer and λ^e the probability that an employed worker receives a job offer. A job offer is an opportunity to work for a wage w drawn from distribution $F_\theta(w)$ with support $[\underline{w}_\theta, \overline{w}_\theta]$. Although a worker treats this distribution as exogenous, it will be determined endogenously in equilibrium through competition among firms for employees. A job is terminated exogenously with probability δ, upon which the worker re-enters non-employment, which gives flow utility b_θ.

Let W_θ denote the value function of a non-employed worker of ability θ and $V_\theta(w)$ the value function of the same worker employed at wage w. These value functions satisfy the Bellman

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[34]Alternatively, all our results could be stated in terms of an integer number N of worker types.

equations

$$\rho W_\theta = b_\theta + \lambda^u \int_{\underline{w}_\theta}^{\overline{w}_\theta} \max \{V_\theta(w) - W_\theta, 0\} \, dF_\theta(w)$$

and

$$\rho V_\theta (w) = w + \lambda^e \int_w^{\overline{w}_\theta} \left[V_\theta (w') - V_\theta (w) \right] dF_\theta (w') + \delta \left[W_\theta - V_\theta (w) \right]$$

The value function V_θ is strictly increasing in w, and hence the optimal strategy of a non-employed worker is characterized by a reservation wage ϕ_θ. A non-employed worker accepts all wage offers above ϕ_θ and rejects offers below it. Following Burdett and Mortensen (1998), one can show that the reservation wage ϕ_θ is implicitly defined by

$$\phi_\theta = b_\theta + (\lambda^u - \lambda^e) \int_{\phi_\theta}^{\overline{w}_\theta} \frac{1 - F_\theta (w)}{\rho + \delta + \lambda^e (1 - F_\theta (w))} dw$$

The lowest wage at which a worker of type θ can be employed is thus given by

$$\underline{w}_\theta \left(w^{min} \right) = \max \left\{ \phi_\theta, w^{min} \right\}$$

We refer to $w^{min} > \phi_\theta$ as a binding minimum wage in market θ.

Firms. Firms are characterized by a constant productivity level $p \sim \Gamma (\cdot)$, which is continuous over full support $P = [p_0, \overline{p}] \subseteq \mathbb{R}_{++}$. Firms produce output by combining workers of different ability levels using a linear production technology. Letting l_θ denote the number of employees from market θ, flow output of a firm with productivity p is

$$y \left(p, \{l_\theta\}_{\theta \in \Theta} \right) = p \int_{\theta \in \Theta} \theta l_\theta d\theta$$

A firm attracts workers by posting market-specific wages, w_θ, in the markets it decides to be active in. In equilibrium the wage a firm posts determines the steady state amount of workers it attracts in that market, $l_\theta = l_\theta(w_\theta)$. Its firm size is determined as the sum of workers employed in each market, $l = \int_{\theta \in \Theta} l_\theta d\theta$. In choosing what wage to post, a firm trades off two forces: on the one hand, it attracts and retains a greater mass of workers and consequently produces more output

by offering a higher wage. On the other hand, a higher wage reduces its profits per employed worker.

Because workers of different ability are perfect substitutes, the firm maximizes profits in each labor market separately. Conditional on choosing to recruit workers from market θ, the problem faced by a firm with productivity p is to post a wage w_θ to maximize steady-state profits:

$$\max_{w_\theta \geq w^{min}} (p\theta - w_\theta) \, l_\theta \, (w_\theta)$$

where the mass of workers employed at the firm, $l_\theta \, (w_\theta)$, is an equilibrium object that we characterize below.

5.2 Equilibrium definition

A key equilibrium object in this economy is the distribution of wages across workers. Let $G_\theta \, (w, t; w^{min})$ denote the wage distribution in market θ at time t and let $u \, (t)$ denote the share of workers that are unemployed. Because of on-the-job search, the realized wage distribution in the economy differs from the offer distribution $F_\theta(w; w^{min})$. The following Kolmogorov forward equation describes worker dynamics in each submarket for $w \geq \underline{w}_\theta \, (w^{min})$:

$$\frac{\partial G_\theta \, (w, t; w^{min})}{\partial t} = -\delta G_\theta \, (w, t; w^{min}) - \lambda^e \left(1 - F_\theta \, (w, t; w^{min})\right) G_\theta \, (w, t; w^{min}) + \lambda^u \frac{u \, (t)}{1 - u \, (t)} F_\theta \, (w, t; w^{min})$$

In steady state, the unemployment rate is $u = \frac{\delta}{\delta + \lambda^u}$ and the realized wage distribution $G_\theta \, (w; w^{min})$ and wage offer distribution $F_\theta \, (w; w^{min})$ are related as

$$G_\theta \, (w; w^{min}) = \frac{F_\theta \, (w; w^{min})}{1 + \kappa^e \, (1 - F_\theta \, (w; w^{min}))}$$

where $\kappa^e \equiv \frac{\lambda^e}{\delta}$. A direct adaptation of the equilibrium characterization in Burdett and Mortensen (1998) shows that the realized wage distribution $G_\theta \, (\cdot)$ first-order stochastically dominates the wage offer distribution $F_\theta \, (\cdot)$, and that both cumulative distribution functions are continuous and strictly increasing. The equilibrium mass of workers employed at a firm that posts wage w is given by

$$l_\theta \, (w; w^{min}) = m_\theta \, (1 - u) \frac{dG_\theta \, (w; w^{min})}{dF_\theta \, (w; w^{min})}$$

We are now ready to define an equilibrium of our economy.

Definition 1. A *search equilibrium with segmented labor markets* is defined as a set

$$\left\{ w^{min}, \phi_\theta, u_\theta, l_\theta \left(w; w^{min} \right), F_\theta \left(w; w^{min} \right), G_\theta \left(w; w^{min} \right) \right\}$$

for each $p \in [p_0, \overline{p}]$ and $\theta \in \Theta = \{\theta_1, \ldots, \theta_N\}$ such that:

1. The firm productivity distribution $\Gamma_\theta(p)$ is truncated below at the threshold given by $\underline{p}\left(\theta; w^{min} \right) = \max\left\{ \frac{\phi_\theta}{\theta}, \frac{w^{min}}{\theta}, p_0 \right\}$.

2. The worker ability distribution $H(\theta)$ is truncated below at $\underline{\theta}\left(w^{min} \right) = \frac{w^{min}}{\overline{p}}$.

3. Workers accept any higher-paid job while employed and any job whose wage exceeds their reservation value while unemployed.

4. Taking as given $F_\theta\left(\theta; w^{min} \right)$ and $G_\theta\left(\theta; w^{min} \right)$, firms choose which markets θ to recruit from and make wage offers $w_\theta\left(p; w^{min} \right)$ to maximize steady-state profits.

5. The aggregate unemployment rate $u = \sum_{\theta \in \Theta} u_\theta m_\theta$ and firm sizes $l(\cdot) = \sum_{\theta \in \Theta} l_\theta\left(\cdot; w^{min} \right) m_\theta$ are consistent with the wage offer distributions $F_\theta(\cdot)$, realized wage distributions $G_\theta(\cdot)$, and labor market frictions $(\delta, \lambda^u, \lambda^e)$.

5.3 Equilibrium characterization

Before turning to our main results on the effects of the minimum wage on earnings inequality, we characterize the solution to the general problem of a firm choosing which labor markets to recruit from and what wages $w\left(p, \theta; w^{min} \right)$ to post in each market. Lacking any conclusive data on the type-specific value of unemployment, we assume that the value of unemployment equals output at the lowest productivity firm, so that[35]

$$\underline{p}\left(\theta; w^{min} \right) = \max\left\{ p_0, \frac{w^{min}}{\theta} \right\}.$$

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[35]We note three things with respect to this assumption and the critique raised by Hornstein et al. (2011). Firstly, as noted by these authors, models with on-the-job search are less susceptible to the issues they raise, since it reduces the option value of staying unemployed. Secondly, in our model the option value of remaining unemployed depends on wage dispersion conditional on worker ability, not overall wage variation. We document later that in the data a significant amount of wage dispersion is due to worker heterogeneity. Thirdly, recent papers introducing on-the-job accumulation of human capital show that this produces significant levels of wage dispersion while maintaining realistic assumptions on the value of the outside option (see e.g. Burdett et al., 2014). Although our model does not contain this element, we believe that it could be introduced without changing any of the insights our model provides with regards to the minimum wage.

Lemma 1. *Firms' optimal wage posting decisions are:*

1. *A firm with productivity p posts wages in all labor markets θ that satisfy*

$$\theta \geq \frac{w^{min}}{p}$$

2. *The unique equilibrium wage offered by a firm with productivity $p \geq \underline{p}\left(\theta; w^{min}\right)$ to workers of ability θ is given by:*

$$w\left(p, \theta; w^{min}\right) = \theta p - \theta \int_{\underline{p}(\theta; w^{min})}^{p} \left[\frac{1 + \kappa^e\left(1 - \Gamma_\theta\left(p; w^{min}\right)\right)}{1 + \kappa^e\left(1 - \Gamma_\theta\left(x; w^{min}\right)\right)}\right]^2 dx \tag{2}$$

where

$$\Gamma_\theta(p; w^{min}) = \frac{\Gamma(p) - \Gamma\left[\underline{p}\left(\theta; w^{min}\right)\right]}{1 - \Gamma\left[\underline{p}\left(\theta; w^{min}\right)\right]}$$

The mapping in equation (2) is strictly increasing in p.

Proof. All proofs are contained in Appendix B. □

Lemma 1 extends the equilibrium wage characterization from Burdett and Mortensen (1998) to our model. Part 1 states that firms recruit only from markets in which they can make positive profits. Part 2 shows that the solution to firms' trade-off between monopsony profits and firm size yields an optimal wage policy as a function of firm productivity p, the labor mobility parameter κ^e, and the offer distribution of other employers in that labor market.

Since the mapping from firm productivity to wages is strictly increasing, the equilibrium wage offer distribution in each market satisfies

$$F_\theta\left[w\left(p, \theta; w^{min}\right)\right] = \Gamma_\theta\left(p\right)$$

5.4 Theoretical effects of the minimum wage

Combining our above results, earnings can be written as

$$\log w\left(p, \theta; w^{min}\right) = \log \theta + \log\left(p - \int_{\max\left\{p_0, \frac{w^{min}}{\theta}\right\}}^{p} \left[\frac{1 - \Gamma\left(\max\left\{p_0, \frac{w^{min}}{\theta}\right\}\right) + \kappa^e\left(1 - \Gamma\left(p\right)\right)}{1 - \Gamma\left(\max\left\{p_0, \frac{w^{min}}{\theta}\right\}\right) + \kappa^e\left(1 - \Gamma\left(x\right)\right)}\right]^2 dx\right) \tag{3}$$

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Based on this, the variance of log earnings can be decomposed into two sources: differences in average earnings across θ markets and differences in pay within θ markets. In markets where the minimum wage is not binding, this decomposition is particularly straightforward because the second term in (3) is independent of worker ability. As a result, in these markets a firm pays workers of different ability the same multiple of their underlying worker ability, and log earnings are additively separable into a worker and firm effect. Across-markets variance is hence in the non-binding case given by the variance of the underlying distribution of worker ability, and the within variance is identical in each market and determined by the distribution of underlying firm productivities through the mapping from productivity to the firm component of pay.

With a binding minimum wage, the second term in equation (3) depends on the minimum wage relative to worker ability. This in general will lead to a minimum wage affecting both the expected value and the variance of the second term, and hence it will have an effect on both inequality across and within markets.[36] We now characterize this in further detail.

First, a minimum wage increases earnings at all firms in affected markets. If the minimum wage is such that it binds in some markets but not all, this will lead to compression between low and high ability workers. Our first proposition states this formally,

Proposition 1 (Greater impact at the bottom). *Suppose the minimum wage is initially non-binding. As the minimum wage is gradually raised, it boosts pay of low ability workers relative to high ability workers.*

Proof. See Appendix B. □

Secondly, an increase in the minimum wage reduces the return to worker ability within the set of markets where the minimum wage is binding. This also contributes to lower across-market inequality. We state this result formally in Proposition 2 under the assumption that firm productivity is uniformly distributed:

Proposition 2 (Lower returns to worker ability). *Suppose $p \sim U(p_0, \overline{p})$. A minimum wage reduces the worker ability-pay gradient in all markets affected by the minimum wage:*

$$\partial \left[\frac{\partial w(p, \theta_i; w^{min})}{\partial \theta} \right] / \partial w^{min} < 0, \quad \forall \theta_i < \frac{w^{min}}{p_0}$$

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[36]A minimum wage also cuts off some low productivity firms from some markets and possibly some low ability workers completely. However, it is not clear that the by cutting off the lowest ability individuals and the lowest productivity firms, overall inequality among remaining workers and remaining firms will be reduced, as this depends on the underlying distribution of worker ability and firm productivity. We explore this issue quantitatively in Section 8.

Proof. See Appendix B □

Both proposition 1 and 2 lead to earnings compression across worker ability types, which Fact **??** of our empirical documentation established to be a pervasive feature of the overall decline in earnings inequality in Brazil during this time.

Thirdly, a minimum wage reduces within-market inequality in markets affected by the minimum wage. This is because although a minimum wage increases pay at all firms in markets affected by it, it disproportionately increases compensation at lower productivity firms. We again state this formally under the assumption that firm productivity is uniformly distributed:

Proposition 3 (Lower productivity-pay gradient). *Suppose $p \sim U(p_0, \overline{p})$. A minimum wage reduces the firm productivity-pay gradient in all markets affected by the minimum wage:*

$$\partial \left[\frac{\partial w(p, \theta_i; w^{min})}{\partial p} \right] / \partial w^{min} < 0, \quad \forall \theta_i < \frac{w^{min}}{p_0}$$

Proof. See Appendix B. □

Proposition 3 states equivalently that the minimum wage leads to a flattening of the job ladder in markets affected by it. Hence it speaks to Fact **??** of our empirical documentation, which showed that the firm productivity-pay gradient fell as the minimum was raised in Brazil.

We summarize the effects of an increase in the minimum wage as follows: First, the minimum wage reduces differences in pay between workers of different abilities. Second, earnings inequality falls within markets affected by the minimum wage due to a weaker productivity-pay gradient across firms. As we will see in the quantitative section, these two channels hold more generally in simulations and add up to produce bottom-driven earnings compression that reaches far up in the earnings distribution, in line with our empirical Fact **??**.

6   Estimating the model

In order to quantitatively evaluate the importance of the minimum wage for earnings inequality, we proceed to estimate the model on the initial 1996–2000 subperiod. Subsequently, the next section uses the estimated model to quantify the impact on inequality of a an increase in the minimum wage of the same magnitude as observed in Brazil over this period of time.

6.1 Estimation strategy

First stage. We first use panel information on worker flows together with non-parametric estimates of conditional earnings distributions and estimates of the size of the formal sector labor force in order to estimate the three labor market frictions parameters in our model: the separation rate, δ, the job offer arrival rate from non-employment, λ^u, and the job offer arrival rate from employment, λ^e. As will become clear, these parameters do not depend on the remaining parameters of the model, and hence to simplify the second stage we can pre-calibrate them.

We use a 10% monthly panel from the RAIS to calculate the fraction of entrants, leavers and job-to-job switchers in every year 1996–2012. We also use non-parametric estimates of the overall distribution of firm effects as well as non-parametric estimates of the distribution of firm effects among entrants to the formal sector for each of these years from the RAIS, and data on the relative size of the formal sector labor force among prime age males from the PNAD household survey for each of these years. Since we are unable to distinguish flows from formal sector employment into unemployment, informal employment, or out of the labor force, we can only classify workers as leaving formal sector employment. We label such transitions as employment to non-employment. Figure 5 plots the average monthly fraction of formal sector entrants, leavers and job-to-job switchers in each year 1996–2012. We note that each series remains fairly stable over this period of time in Brazil.

Figure 5. Average monthly fraction of leavers, entrants and job-to-job movers from the formal sector, 1996–2012
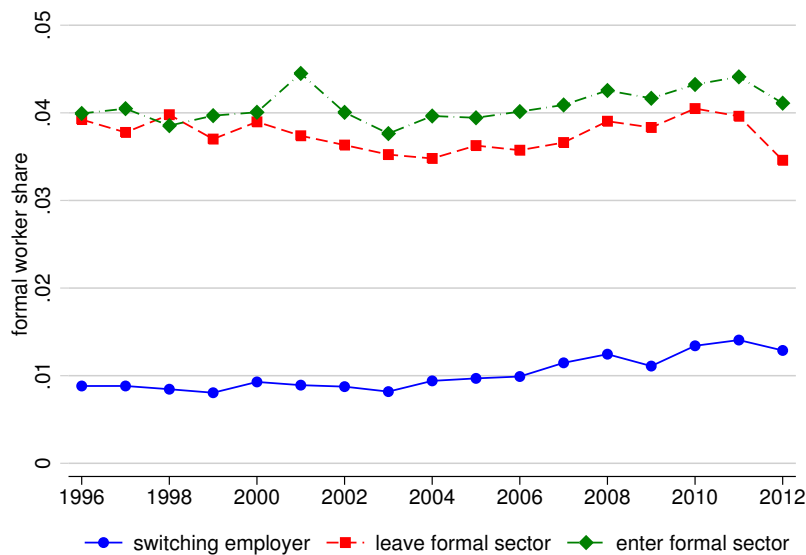
The separation hazard, δ, can be directly inferred from observed flows out of the formal sector. The offer arrival rate from non-employment, λ^u, can be inferred from the fraction of newly employed workers in the formal sector (%entrants) as well as information of the fraction of the total population of prime age males who work in the formal sector (%formal) through:

$$\lambda^u = \frac{\%\text{formal}}{1 - \%\text{formal}} \times \%\text{entrants}$$

where we imposed our assumption that non-employed workers accept the first job offer they receive.

Finally, the job offer arrival rate on the job, λ^e, cannot be directly inferred from observed job-to-job flows, since an employed worker only accepts offers paying more than his current employer. Our model, however, suggests that by governing the speed through which workers move up the job ladder, λ^e is intricately linked to the difference between the distribution G and the wage offer distribution F,

$$G_\theta(w) = \frac{F_\theta(w)}{1 + \kappa^e(1 - F_\theta(w))}$$

where $\kappa^e = \lambda^e/\delta$ is the relative probability of getting an offer versus being separated. Although this is based on the distribution of earnings within a θ market, we note that the estimated firm effects perfectly rank firms in each market and that the rank of firms is the relevant notion of the job ladder in our model. Thus we can estimate κ^e non-parametrically using kernel density approximations[37] of the empirical distribution of firm effects, $\hat{\mathcal{G}}(fe)$, as well as the job offer distribution, $\hat{\mathcal{F}}(fe)$, where the latter is approximated using people that just entered the formal sector. The nonparametric estimate of κ^e is then

$$\hat{\kappa}^e = \frac{\hat{\mathcal{F}}(fe) - \hat{\mathcal{G}}(fe)}{(1 - \hat{\mathcal{F}}(fe))\hat{\mathcal{G}}(fe)}$$

Using our earlier estimate of δ, we can back out the implied value for λ^e.[38]

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[37]We use an Epanechnikov kernel with bandwidth 0.04 and 90 bins although we tried alternative kernel, bandwidth, and bin number choices without significant effects on our estimation results.

[38]In Appendix C, we discuss alternative ways of estimating κ^e, all producing similar estimates.

Second stage. We estimate the remaining parameters of our model using indirect inference. Thus we solve and simulate the model for a large set of potential parameter values in order to minimize the distance between model generated output and their data equivalents.

The model is estimated fully parametrically

$$\log \theta \sim \mathcal{N}\left(0, \sigma_\theta^2\right), \quad \log p \sim \mathcal{N}\left(0, \sigma_p^2\right)$$

This gives two parameters to estimate: the standard deviation of worker ability, σ_θ, and the standard deviation of firm productivity, σ_p. Additionally, we need an estimate of the minimum wage. In order for this to make sense within the model, we need a numeraire of the economy. We chose log median earnings as the numeraire, and express the minimum wage relative to that. Several other choices of numeraire are possible, though, including average earnings or average labor productivity. The former yields very similar results as the median. Average value added per worker is less attractive to us because we only have data on value added for the PIA subsample, and it is plausible that average value added per worker is higher in the subsample of large manufacturing and mining firms, leading us to underestimate the bindingness of the minimum wage (additionally, we worry about measurement error affecting the level of average value added per worker in the data). We do, however, use information on the *growth* in average value added per worker to estimate the growth in real, productivity adjusted minimum wages between 1996–2000 and 2008–2012, which is an important input in our policy experiment in Section 7.

The choice of what moments to target is important. In the discussion below of our choice of target moments, the reader should keep in mind that the model is jointly identified and hence it should be viewed as a single moment being particularly informative of one parameter. As we noted earlier, absent a binding minimum wage, log earnings in our model perfectly separates into a worker and a firm component

$$\underbrace{\log w_{ijt}}_{\text{earnings}} = \underbrace{\log \theta_i}_{\text{worker effect}} + \underbrace{\log r(p_j)}_{\text{firm effect}} + \underbrace{\varepsilon_{ijt}}_{\text{error}}$$

where $r(p_j)$ is the firm component of pay as specified in the wage equation (3). Thus, underlying worker productivity could be directly inferred from an AKM regression and underlying firm productivity inferred from the estimated firm effect by inverting the mapping between firm pro-

ductivity and the firm component of pay. Although in the presence of a binding minimum wage log earnings do not perfectly separate into a worker and a firm component, we think that AKM still captures some of the key dimensions of our model. Hence, we view it provides sufficient information to identify several of the underlying structural parameters of our model. We have conducted several exercises to check the uniqueness of the optimum reached by our estimation algorithm, and it appears unique. We thus use AKM as an auxiliary model and target the variance of estimated AKM worker effects, $Var(\alpha_i)$, for the variance of underlying worker productivity.

From AKM as an auxiliary model we also use the variance of estimated AKM firm effects, $Var(\alpha_j)$, for the variance of underlying firm productivity. Absent a minimum wage, we can find an algebraic expression for how firm productivity maps into pay of workers of that firm. Importantly this is monotonically increasing in firm productivity, implying that the firm component of pay perfectly informs the underlying distribution of firm productivity. In all of our simulations with a binding minimum wage, this monotonicity is preserved and AKM firm effects identify underlying firm productivity.

Our third and final target is the ratio of the log minimum wage to log median earnings, mM. Targeting the minimum wage in our estimated model serves as a normalization as we picked the numeraire to be the expected earnings of a median worker at a median firm by making the parametric assumptions on worker and firm effects above.

We now define the distance criterion for our indirect inference as part of the simulated method of moments. Let S^D denote the statistic of interest, S, in the data and S^M that in the model— formally we estimate the parameters $(\sigma_\theta, \sigma_p, w^{min})$ by assigning the values that minimize the un-weighted[39] sum of squared percentage deviations between model moments and data moments:

$$\left(\hat{\sigma}_\theta, \hat{\sigma}_p, w^{min} \right) = \text{argmin}_{\sigma_\theta, \sigma_p, w^{min}} \left\{ \left[\frac{Var\left(\alpha_i^D\right) - Var\left(\alpha_i^M\right)}{Var\left(\alpha_i^D\right)} \right]^2 + \left[\frac{Var\left(\alpha_j^D\right) - Var\left(\alpha_j^M\right)}{Var\left(\alpha_j^D\right)} \right]^2 + \left[\frac{mM^D - mM^M}{mM^D} \right]^2 \right\}$$

Further details of the estimation procedure can be found in Appendix C.

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[39] We opted for uniform weights on the distance criteria because convergence was very smooth and we did not want to build in any ex-ante restrictions on the relative importance of worker effects versus firm effects in the estimation procedure.

6.2 Parameter estimates and model fit

Before entering the estimation procedure, we fix the discount rate ρ to match an average annual interest rate of 12 percent. Subsequently, we estimate the labor market frictions parameters. We find that δ is 3.8 percent at the monthly level, λ^u is 7.1% and λ^e is 5.3%. From an international perspective, we note that δ is higher than most estimates from the U.S. (and thus also than most European countries). However, our estimate is based on all workers leaving the formal sector, regardless of the destination. If we were able to condition on staying in the labor force, we suspect that our estimate would be lower. λ^u is lower than estimates from European markets (and hence substantially lower than the U.S.). However, it is again using all workers not in formal sector employment, and we suspect that if we were able to calculate the unemployment to employment hazard rate it would be substantially higher. Finally, our estimate of λ^e is on the lower side compared to most European markets (and hence again substantially lower than the U.S.). Our robustness section establishes that our results are not sensitive to a wide range of these underlying labor market friction parameters. Table 7 summarizes our estimates:

Table 7. Monthly model parameters

Description	Parameter	Value	Target
Discount rate	ρ	0.009	Annual interest rate of 12%
Exogenous separation rate	δ	0.038	Fraction of formal sector leavers
Job finding rate from unemployment	λ^u	0.071	Fraction of entrants and size of formal sector
Labor market friction parameter	λ^e	0.053	Firm effect distribution, firm effect offer distribution and δ

Table 8 presents estimates of the variance of underlying worker ability and firm productivity as well as the minimum wage. The model fits the data well. Our estimates imply that heterogeneity in worker ability exceeds variation in firm productivity, but by less than the difference in the variance of the estimated AKM worker and firm effects. The reason is that at the top and the bottom of the firm productivity distribution, little between firm competition for workers imply that an increase in productivity translates to a very small increase in wages.

Table 8. Parameter estimates and model fit

Description	Parameter	Estimate	Target moment, 1996–2000	Data	Model
Variance of worker ability	σ_θ^2	0.700	Variance of AKM worker effects	0.347	0.347
Variance of firm productivity	σ_p^2	0.523	Variance of AKM firm effects	0.167	0.168
Minimum wage	w^{min}	0.189	Minimum-to-median log ratio	34%	34%

To investigate whether the underlying parameters are well identified in the model, we evaluated how the difference between model generated moments and the corresponding moments in the data changed when changing one parameter at a time away from the optimum. The distance increases monotonically. Moreover, given the small set of parameters to estimate, we conducted an extensive search for an optimum over a wide grid of values. Although this does not guarantee global uniqueness of the minimum, all the robustness exercises we have conducted indicate that it is.

6.3   Policy experiment in the model

We now turn to evaluating the impact on income inequality in the model of an increase in the minimum wage of the same magnitude as that in the data. To do so, we first compute average growth in productivity adjusted real minimum wages between 1996–2000 and 2008–2012. The average real minimum wage (in 2012 values) is 384 Reais in 1996–2000 and 701 in 2008–2012, implying an 60.2 log point growth in real minimum wages. Average log value added per worker grows by a total of 15.4 log points between 1996–2000 and 2008–2012. Thus, we estimate that the real, productivity adjusted minimum wage grew by 44.7 log points between 1996–2000 and 2008–2012. Given this data target, we reestimate our model for the 2008–2012 period by changing the minimum wage to hit a 44.7 log point growth in the minimum wage relative to average log labor productivity, while holding all other parameters fixed at their 1996–2000 estimated values.[40] This implies a hike in the minimum wage from 0.189 to 0.315 or roughly 67 percent. We evaluate the implications for income inequality of imposing this higher minimum wage through the lens of our model.

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[40]For robustness, we also explored alternative targets for the increase in the minimum wage, including the growth rate of the minimum wage relative to productivity growth in Brazil's services, commerce, and construction sectors (for which we have firm-level productivity data); or relative to growth in aggregate output per capita from national accounts. These alternative targets imply similar increases in the minimum wage and therefore lead to comparable results.

7 Quantitative results

7.1 Effect of minimum wage on earnings distribution

In this section, we evaluate the effects of the minimum wage on the earnings distribution in our estimated model. Figure 6 shows how the overall distribution of income changes in the model as we increase the minimum wage while holding everything else constant at the 1996–2000 values. We note that the model underestimates the overall variance of wages in the data by 25 log points in the initial period, because we do not calibrate it to match the error component as well as age, education and year effects in the data. Yet the magnitude of the fall in the overall variance of log earnings is comparable to the data: the variance of log earnings falls by 14.1 log points in the model or 70 percent of the fall in the data.

Figure 6. Effect of minimum wage on earnings distribution in model vs. data

(a) 1996–2000   (b) 2008–2012
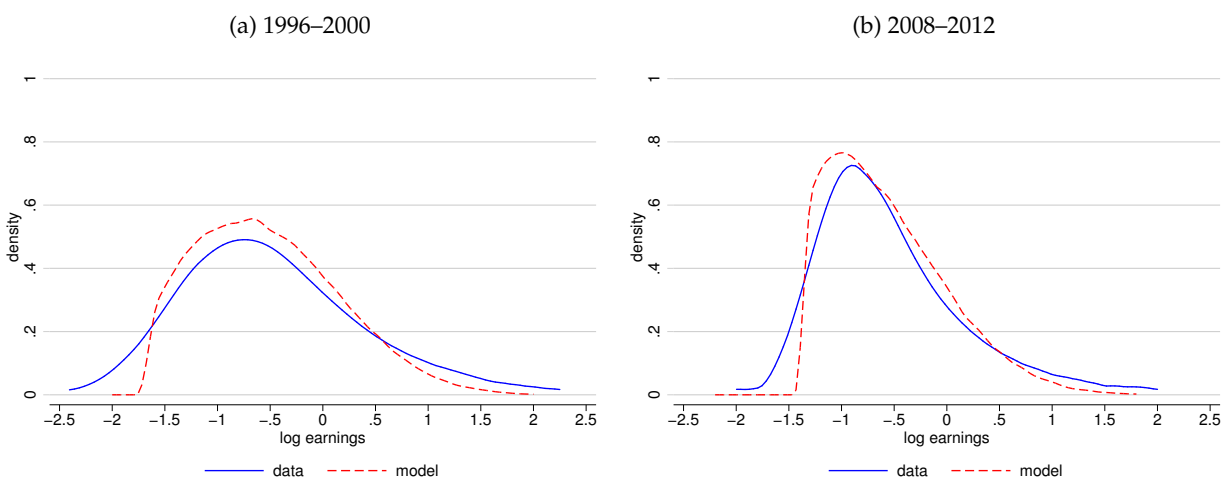
The increase in the minimum wage induces a significant compression in both estimated worker and firm effects, as can be seen in Table 9. The model generates a 6.2 log point fall in the variance of person effects, a 4.3 log point compression in the variance of firm effects and a 1.8 log point fall in the covariance between them. The corresponding numbers in the data are 5.4 log points, 9.0 log points and 4.4 log points, respectively. Thus like in the data, firm effects account for an outsized share of the inequality decline: the variance of person effects falls by 18 percent and the variance of firm effects falls by 26 percent in the model (versus 16 percent and 54 percent in the data, respectively). Thus the model slightly overpredicts the compression in person effects observed

in the data. However, given significant evidence from other countries that technological change over the last two decades has increased the return to ability (so called *skill-biased technical change*), we find it plausible that other forces served to increase the dispersion in person effects over this period in the data. Moreover, although the model cannot account for the positive covariance between firm and worker effects in the data, it is able to replicate almost 80 percent of the decline in the covariance in the data.

Table 9. Variance of earnings in data versus model

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Variance of total earnings	0.72	0.46	0.52	0.32	-0.20	-0.14	70%
Variance of worker effects	0.36	0.35	0.31	0.29	-0.05	-0.06	110%
Variance of firm effects	0.17	0.17	0.08	0.13	-0.09	-0.04	48%
Covariance worker-firm	0.14	-0.03	0.10	-0.05	-0.04	-0.02	78%
Variance of residual	0.06	0.00	0.04	0.00	-0.02	0.00	0%

The model does a remarkably good job at generating compression in the earnings distribution up to very high percentiles. As can be seen in Table 10 the model predicts consistently more than 64% of compression in the selected percentile ratios, and approximately the full amount higher up in the distribution. For instance, the 50–10 log ratio compresses by 31 log points in the data versus 22 log points in the model (or 71 percent) whereas the log 90-50 ratio compresses by 13 log points in the model versus 12 log points in the data (or 92 percent). Matching the data, the model displays a declining amount of compression in the top of the distribution, as shown by the decreasing marginal increase in the log ratios up to the 90th percentile. For instance, the log 95-50 ratio falls by 2 log points less than the log 90-50 ratio in the model. Hence the model is successful at replicating Fact **??** from our empirical section.

Table 10. Compression in log percentile ratios of earnings distribution

| | 1996–2000 | | 2008–2012 | | Change | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
	Data	Model	Data	Model	Data	Model	% Explained
P50-P05	1.06	0.90	0.62	0.62	-0.44	-0.28	64%
P50-P10	0.86	0.77	0.55	0.55	-0.31	-0.22	71%
P50-P25	0.48	0.46	0.33	0.35	-0.15	-0.11	73%
P75-P50	0.60	0.52	0.50	0.44	-0.10	-0.08	80%
P90-P50	1.30	1.01	1.17	0.89	-0.13	-0.12	92%
P95-P50	1.76	1.30	1.65	1.17	-0.11	-0.13	118%

Turning now to a decomposition of the overall inequality decline in our model relative to the data, Figure 27 in Appendix D shows the model distributions of firm effects and workers effects from the model before and after the minimum wage change. Both distributions experience particularly pronounced compression in their shape at the bottom, consistent with our empirical findings.

The results from inspecting the firm and worker components from the AKM decomposition of earnings in our model is broadly consistent with our empirical findings. As predicted by our theory, the increase in the minimum wage compresses pay across θ markets by disproportionately boosting pay of low productivity workers. This is due to the two channels highlighted in our theoretical section: average firm productivity increases for low productivity workers and average pay increases among continuously active firms. Figure 7 demonstrates the quantitative importance of this channel by plotting estimated AKM firm effects from our model simulated data against firm productivity, and AKM worker effects against worker ability in the model for the 1996–2000 and 2008–2012 periods. Average pay clearly compresses across markets, and this is person effects against underlying worker ability and estimated firm effects against underlying firm productivity in the model for the two subperiods.

Figure 7. Gradient effects of the minimum wage

(a) Effect on firm productivity-earnings gradient

(b) Effect on worker ability-earnings gradient
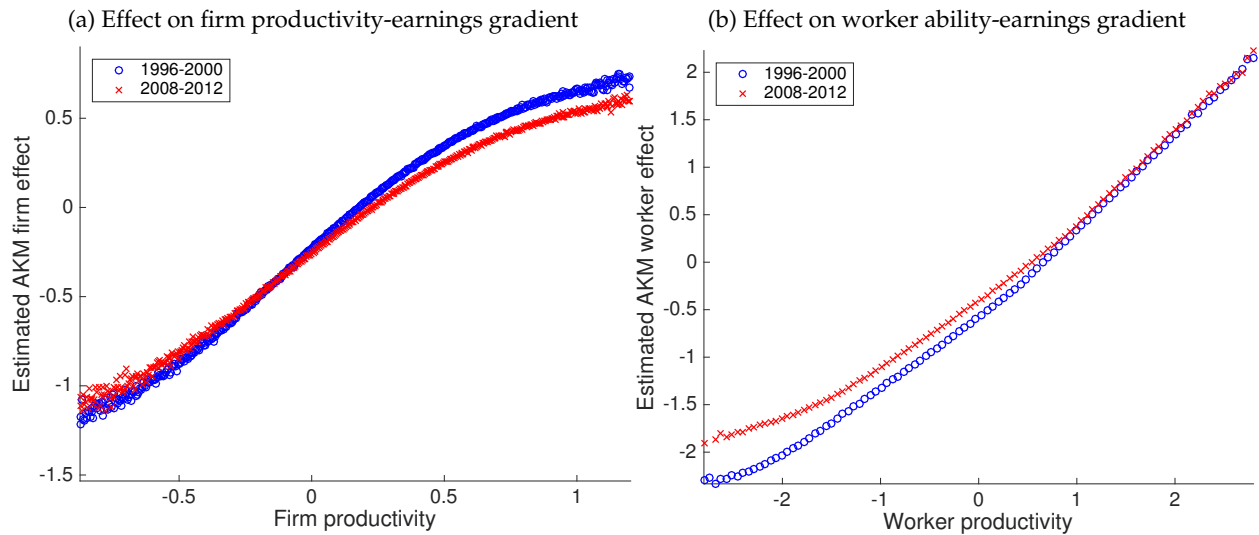
Both gradients notably fall. In fact the fall in the gradient explains all of the fall in firm effects in the model, because the minimum wage increase is not high enough to significantly reduce the variance of firm productivity in the model (in fact it increases marginally as a result of reallocation of workers among continuously active firms).

To further investigate this, Table 11 presents results from regressing estimated firm effects on firm productivity in the model and in the data. We note that these regressions have a much higher explanatory power in the model relative to the data, likely due to measurement error in productivity in the data. However, the fall in the variance of firm effects attributable to a change in the coefficient is similar in the data and in the model: 5.0 log points versus 4.3 log points.

Thus, we conclude that the model cannot account for the decline in the variance of firm effects not explained by productivity, but that it can explain up to 90 percent of the fall due to a weaker productivity-pay gradient. Furthermore, as can be seen in Table 11, all of the decline in the variance of person effects in the model is due to the weaker worker ability-pay gradient, because the magnitude of the minimum wage increase is not sufficiently large to make some workers unemployable. Although we unfortunately cannot decompose the change in the variance of person effects in the data into a change in underlying characteristics versus returns to these characteristics, we think that a reasonable first pass would be to assume that the underlying distribution of worker abilities did not change much during these 17 years in Brazil (recall that we control for changes in education and age).

Table 11. Effect of minimum wage on productivity-earnings gradient

	1996–2000		2008–2012		Change in predicted variance		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Value added p.w.	0.257	1.088	0.141	0.934	-0.050	-0.043	86%
Constant	-2.883	-0.236	-1.599	-0.254			
# worker years	15.5	0.5	23.9	0.5			
R^2	0.583	0.976	0.465	0.973			

Note: Dependent variable is estimated firm effect from AKM regression, independent variable is average log value added per worker within a subperiod. Number of observation is in millions.

7.2 The importance of indirect effects of the minimum wage

In order to evaluate the importance of the equilibrium mechanism that we emphasize in this paper for wage compression, we consider a scenario in which counterfactually there is no impact on earnings above the new minimum wage. To calculate the amount of inequality associated with such a scenario, we hence assume that everyone earning below the new minimum wage gets earnings equal to the minimum wage, but nothing happens to wages above the new threshold.[41] Figure 8 illustrates the direct and indirect effects of the minimum wage using actual distributions from our simulation exercise. The variance of log earnings explained by the direct versus indirect effects of the minimum wage are summarized in Table 12. More than half of the overall decline in the variance of log earnings is explained by the minimum wage raising earnings of workers earning above the new minimum wage. Thus, we conclude that modeling the equilibrium effect of raising the minimum wage is crucial for understanding its impact on inequality.

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[41] An alternative scenario would have been to simply cut off workers below the new minimum wage, but as this yields similar conclusions we do not report that here.

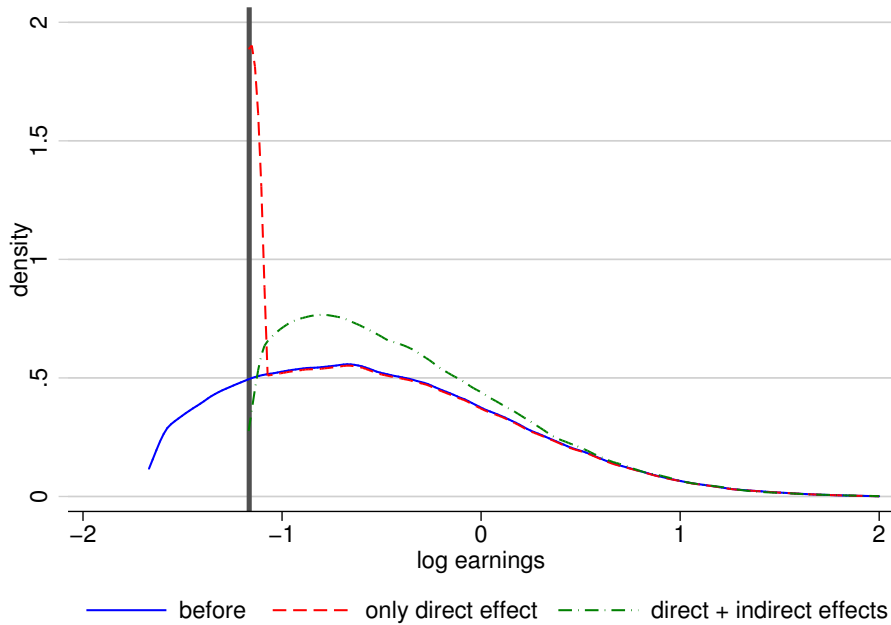Figure 8. Direct and indirect effects of minimum wage on earnings distribution

Table 12. Direct and indirect effects of the minimum wage on variance of log earnings

	Data			Model		
	(1)	(2)	(3)	(4)	(5)	(6)
	Variance of		% of total	Variance of		% of total
	log earnings	Change	change	log earnings	Change	change
Before minimum wage raise, 1996–2000	0.721	-	-	0.458	-	-
Only direct effect	0.612	-0.109	54.2%	0.391	-0.067	47.5%
Direct and indirect effects, 2008–2012	0.520	0.201	100.0%	0.317	-0.141	100.0%

7.3 Effects on unemployment

Consistent with empirical observations in Brazil over this time, our model generates very little increase in unemployment.[42] The increase in unemployment predicted by our model is on the order of a tenth of percentage point (from a starting point of 6.9 percent unemployment).

It is important to note, though, that our model *has the potential* to generate significant unemployment. We explore this point in more detail in Appendix D.

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[42]It is important to highlight that a higher minimum wage might affect the job finding rates, which is a channel that is absent from our model. We note, though, that there is little change in job finding rates in Brazil over this period.

Table 13. Effects of the minimum wage on unemployment

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Unemployment rate	0.067	0.067	0.068	0.068	0.001	0.001	79%

7.4 Effects on the firm distribution

Finally, we note that the higher minimum wage leads a set of the least productive firms to exit, thus raising average productivity and the average firm size in the economy. The findings are summarized in Table 14. Specifically, while seven percent of all firms are forced to exit because they can no longer operate profitably at the new minimum wage, the re-allocation of workers across remaining firms raises aggregate TFP by four log points and average firm size by two log points. While some of these effects are due to the least productive firms exiting, much of the positive effects on aggregate productivity stem from the re-allocation of workers to more productive firms.

Table 14. Effects of the minimum wage on firm distribution

|	(1)	(2)	(3)
	1996–2000	2008–2012	Change
Share of active firms	1.00	0.95	-0.05
Average productivity (TFP)	0.79	0.83	0.04
Average number of employees	18.1	18.3	0.2

8 Discussion

8.1 Empirical evidence for effects of the minimum wage on earnings inequality

More pronounced inequality decline among formal sector workers. If the minimum wage played an important role in the decline of earnings inequality in Brazil from 1996–2012, then we would expect the magnitude of the decline to vary with the degree to which the legal minimum wage is enforced. Hence, the inequality decline should be less pronounced in Brazil's informal sector. The latter constitutes a significant share of overall employment but, due to imperfect monitoring of economic activity in Brazil's shadow economy, is less subject to laws and labor regulations. Figure 9 confirms this hypothesis in the Brazilian PNAD household data. Our analysis

shows that most of the decline in earnings inequality over the period is due to developments in the formal sector.

Figure 9. More pronounced decline of earnings inequality in formal sector, 1996–2012
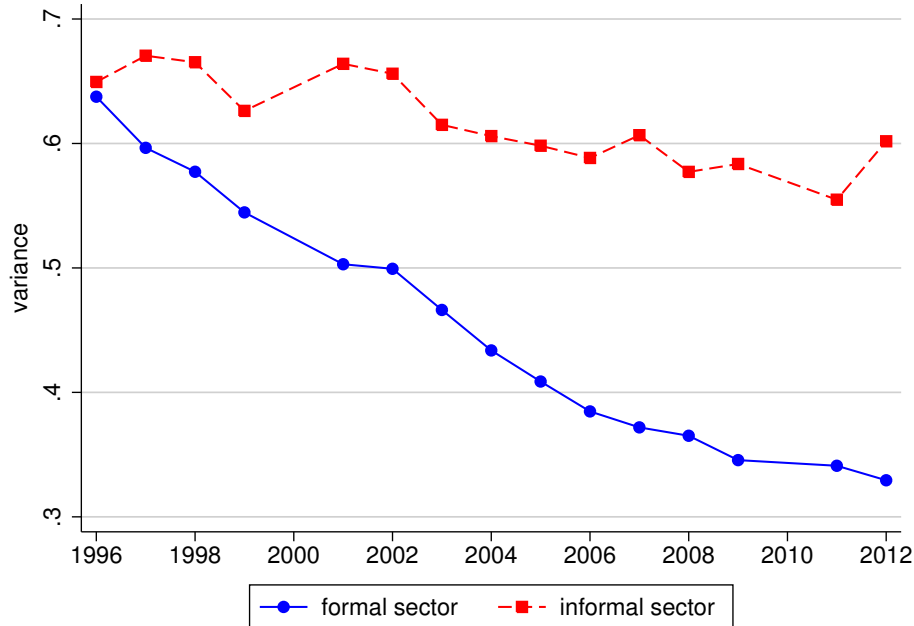

Greater impact on low-income regions. While the minimum wage is enacted at the federal level, not all parts of the economy are affected equally. For instance, sectors with higher initial average earnings may be less affected by the rise in the minimum wage only with a delay. If the minimum wage was an important driver behind the decline of earnings inequality, we would expect that inequality within geographic regions of the country more affected by the minimum wage would experience more pronounced inequality declines. To pursue this hypothesis, we use the PNAD household survey data[43] and sort the five big geo-economic regions of Brazil (North, Northeast, South, Southeast, Centre-West) into two groups[44] by their average per capita income level over the period. Figure 10 plots the variance of log labor earnings in different regions of the country, grouped by two income levels. We see that inequality declined more rapidly within regions that started out at lower average incomes. Specifically, the variance of log earnings was just marginally higher in high-income regions than other parts of the country in 1996, yet by 2012 a 6 log points

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[43]Our confidentiality agreement with the Brazilian Ministry of Labor does not permit us to disseminate results from analysis using regional identifiers in the administrative RAIS data.

[44]Very similar results obtain when using finer regional units such as states or municipalities.

gap had arisen between the two groups. This is again consistent with a story of the minimum wage affecting these regions differentially.

Figure 10. More pronounced decline of earnings inequality in initially low-income regions, 1996–2012
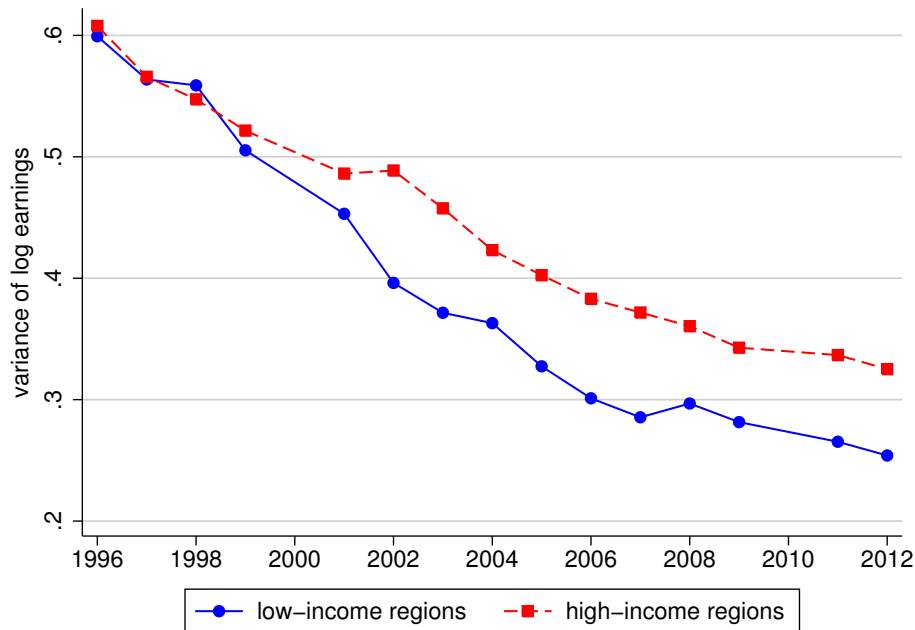
Greater impact on low-income sectors. In a similar vein, using data from the PNAD household surveys, Figure 11 plots the variance of log labor earnings for the set of adult males across sectors classified as low- or high-income in 1996. Among others, low-paying sectors include agriculture, services, and commerce; while high-paying sectors include manufacturing, mining, and public administration. Initially, the variance of log earnings was 13 log points higher in the high-income sectors. Yet, by the end of the period that difference had widened to almost 20 log points. The fact that inequality is nowadays significantly lower within low-income sectors and that inequality in those sectors declined more quickly over the period is consistent with stories of the minimum wage affecting those sectors disproportionately more than high-paying sectors.

Figure 11. More pronounced decline of earnings inequality in initially low-income sectors, 1996–2012
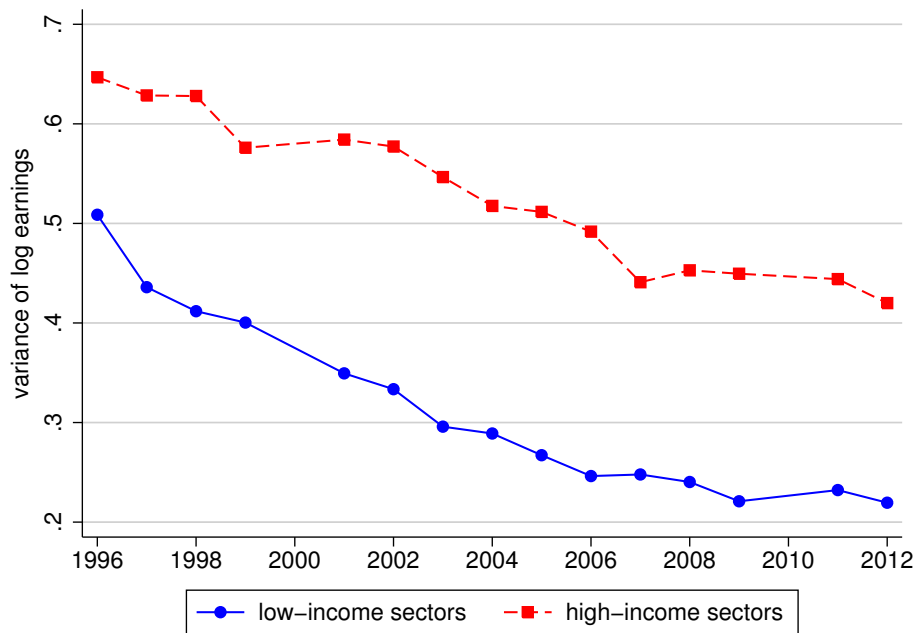
Summary of empirical evidence. In our earlier empirical analysis, we established that Brazil's inequality decline from 1996–2012 features particularly pronounced compression at the bottom of the earnings distribution, and the U-shaped evolution of the real minimum wage since 1988 mirrored that of earnings inequality. In further support of the minimum wage playing an important role in the evolution of inequality in Brazil over this period, we presented three additional pieces of evidence: First, the inequality decline was more pronounced in Brazil's formal sector relative to the informal sector where labor regulations like the minimum are plausibly harder to enforce. Second, inequality started to decline later in initially higher-paying sectors such as manufacturing and mining, consistent with a rising minimum wage affecting these sectors with a delay. Third, regions that started out at higher average income levels experienced less of a decline in earnings inequality. Together, these facts support the hypothesis of a causal relationship between the rise in the minimum wage and the decline of earnings inequality over the period, which we implicitly adapted in our theoretical framework.

8.2 Job ladder in firm effects

The key ingredient of our model is the ability of workers to receive job offers while currently employed. The possibility of on-the-job mobility circumvents the Diamond (1971) paradox by inducing firms to compete for workers. This competition among firms leads to spill-over effects of an institutional wage floor: some workers will be affected directly and, if the surplus they generate at a given employer is positive net of the new minimum wage, will relocate to the new minimum wage level; but their wages will on average increase further because firms will want to recruit the mass of workers close to the new minimum wage. These ripple effects in wage setting will slowly fade out as we move up the firm productivity distribution. Such competition of firms for workers will result in workers moving up the firm pay ranks during employment spells—a dynamic commonly referred to in the literature as a "job ladder," though in our model such a ladder is really between firms, so we will call it a "firm ladder" henceforth.[45]

Existing work on job ladder models has highlighted their success in capturing key labor market characteristics (Mortensen, 2003). Naturally, testing for the presence of a firm ladder is essential for our proposed mechanism to work and lead to large effects of the minimum wage on the earnings distribution. We present four pieces of evidence in favor of a firm ladder, which we quantify using firm effects from the AKM estimation as the empirical counterpart to our model's firm ladder:

1. The firm effects distribution of stayers first-order stochastically dominates that of previously non-employed workers; see Figure 12.

2. Job-to-job transitions on are associated with an average increase in firm effects of 5%, equivalent to 5 percentile ranks of the firm effects distribution; see Table 15.

3. Workers move up in firm effects more quickly towards the bottom; see Figure 13.

4. Worker turnover is lower at employers with higher firm effects.; see Figure 14.

Together, these facts support our firm ladder view of the Brazilian labor market.

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[45]Partly due to data limitations, the previous literature has focused on various alternative manifestations of a job ladder, including firm size and average wages at a firm. We argue that our choice of firm effects is an intuitively appealing counterpart of the job ladder in our context.

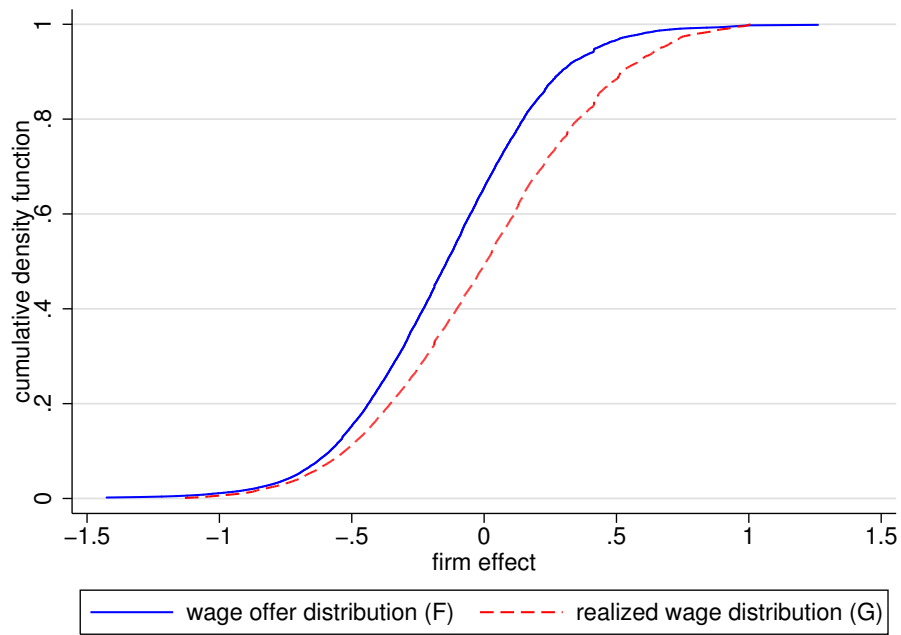Figure 12. Job ladder fact 1: realized wage distribution FOSDs wage offer distribution, 1996

Table 15. Job ladder fact 2: Large positive gains in firm effects from switching employers, 1996–2012

Change in firm effect from switching employer	Average value, 1996–2012
Absolute change	3.2
Percentile rank change	6.0

Figure 13. Job ladder fact 3: Greater gains in firm effects for workers at lower firm effects employers, 1996
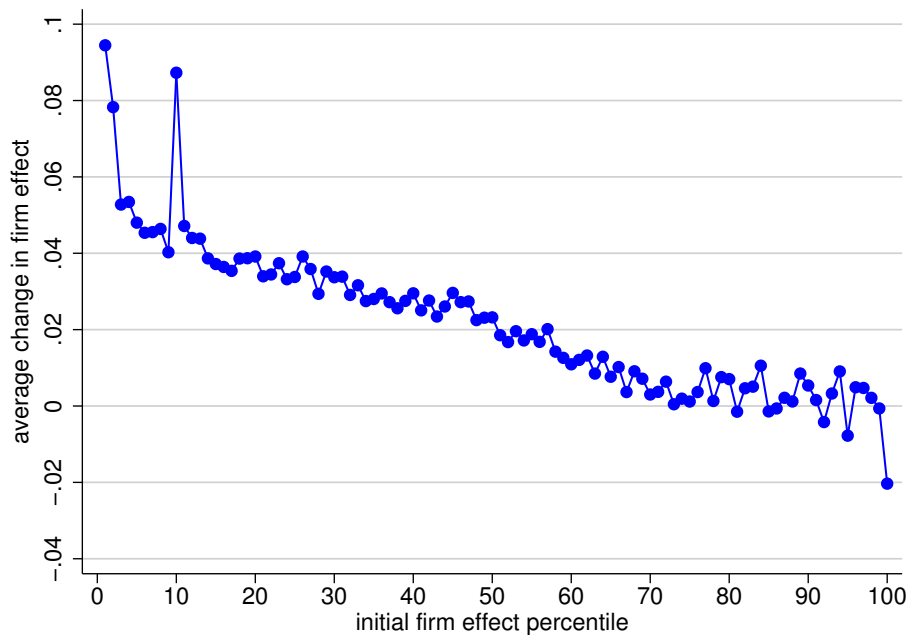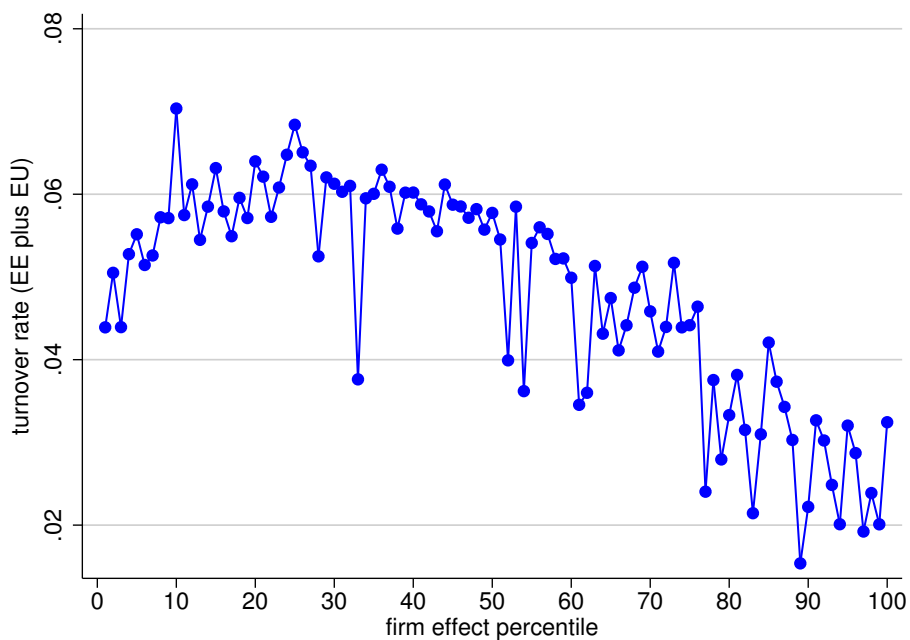

Figure 14. Job ladder fact 4: Lower turnover rates at higher fixed effect firms, 1996

While the previous facts lend support to the Brazilian labor market being characterized by a job ladder, a corollary of our theory is that as the minimum wage increases over time the rungs

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of the job ladder should become compressed. We verify this central prediction of our model by measuring the gains in earnings form switching firms, as given by the difference in estimated AKM firm effects between the source and the target firm, and confirming that these gains are indeed decreasing in magnitude over time.[46] In line with our model predictions, we find strong support for both claims, as shown in Figure 15. Specifically, between 1996 and 2012 the average gain in firm effects from switching employers declines by 1.1 log points (or 28 percent; see blue solid line) in the overall worker population and by 3.8 log points (or 36 percent; see red dashed line) among recent labor market entrants.

Figure 15. Decline of gains in firm effects from switching employers by worker group, 1996–2012
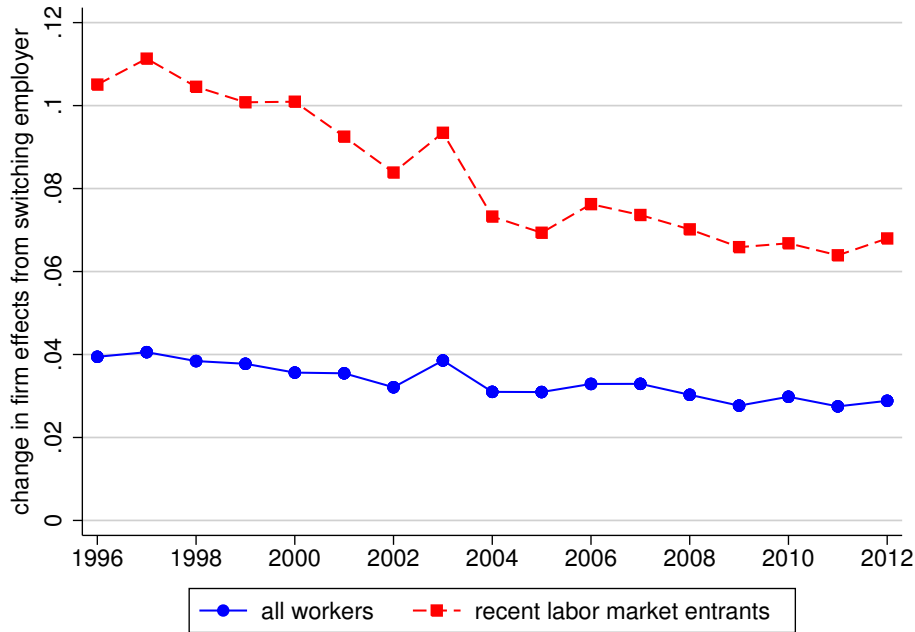
8.3 Sorting pattern induced at low productivity firms

The way we modeled the minimum wage has direct implications for the sorting pattern of heterogeneous workers across firms of different productivity. Specifically, a natural prediction of our model is that matches between low ability workers and low productivity firms eventually become infeasible as the minimum wage gradually increases. In this case, low productivity firms recruit from a subset of labor markets above an ability cutoff satisfying $\theta \geq w^{min}/p$. Consequently, we

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[46]Furthermore, in line with our model predictions, we verify that the relative gain in firm effects, as measured by the ranks climbed by transitioning, does not decrease over time.

would expect the average worker quality to be *higher* at low productivity firms. While this is a very straight-forward prediction of our model[47], to the best of our knowledge no previous work has tested for such policy-induced sorting patterns.

To test for changes in the sorting pattern towards the bottom of the firm effects distribution, we first rank worker effects and firm effects within AKM subperiods. We then compute the average worker effects percentile for a given firm effects percentile. Figure 16 plots the results of this exercise for the bottom half of the firm effects distribution. We confirm the presence of a negative sorting pattern between workers and firms among the lowest-paying employers, and note that this pattern is becoming more pronounced between 1996 and 2012, consistent with the minimum wage becoming more binding over this period. We view this result as corroborating evidence for our specific model mechanism by which the minimum wage affects labor market outcomes in Brazil.[48]

Figure 16. Sorting between worker and firm effects, bottom half of firm effects distribution

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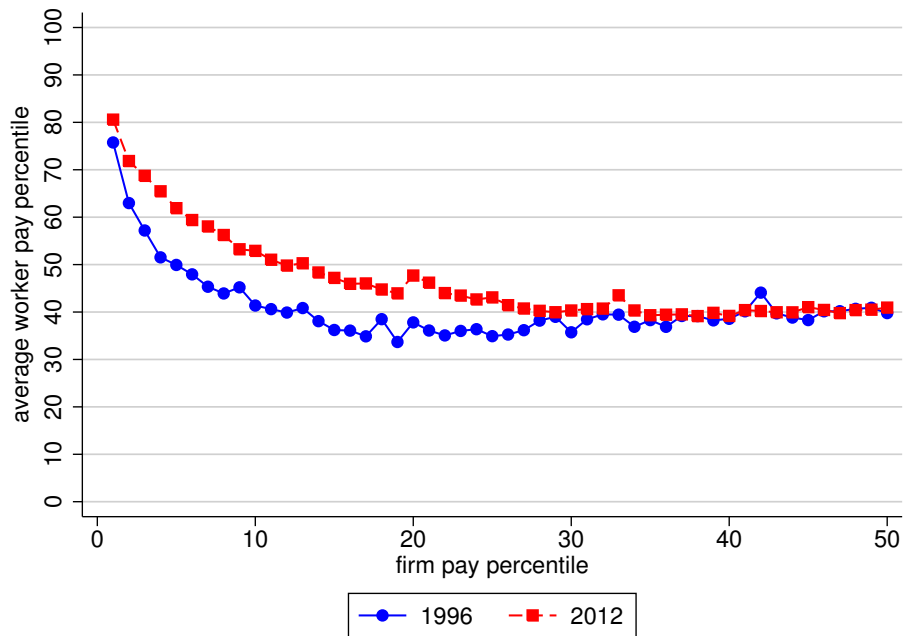
[47]A similar effect would obtain in a broad class of other models featuring production functions that are log-linear in firm productivity and worker ability.

[48]Figure 28 in Appendix D.2 shows the sorting pattern of workers across firms for the complete distribution of firm effects, which exhibits positive sorting overall, particularly towards the top of the firm effects distribution. We view the positive sorting pattern as plausibly induced by a mechanism outside of our benchmark model, although were we to allow for heterogeneous job arrival rates, as discussed in Section 8.4, then our model would also be able to qualitatively replicate this pattern, even absent other technological adaptations.

8.4 Discussion of modeling assumptions

While our extended job ladder model is plausibly also going to be successful in these dimensions, the parsimony of the model also begs the question if our specific modeling choices are necessary to obtain similar qualitative and quantitative results. To this end, we argue that our model is a specific instance of a broad class of model in which a similar economic mechanism leads to spillover effects of the minimum wage. The key ingredient that unites these models is the competition among firms for workers that arises in the presence of on-the-job arrival of job opportunities, a salient feature of many search models also highlighted by Hornstein et al. (2011). With this in mind, we proceed to discussing how various ingredients featured in our model could be changed without fundamentally changing our conclusions.

Wage posting. Conducting a survey among a representative sample of U.S. workers, Hall and Krueger (2012) present evidence in support of the prevalence of job-to-job mobility in general and of the importance of wage posting in particular. They find that 94 percent of blue-collar workers and two thirds of their overall sample did not negotiate their wage upon entering their last employment spell. Consistent with the wage posting assumption, a large share of these workers reported knowing the exact wage at the prospective employer before making the job transition. They also find that wage bargaining becomes more prevalent among senior employees and "knowledge workers."

While a systematic study of wage setting policies is lacking for the Brazilian case, we think that the wage posting assumption is reasonable for two reasons. First, to the extent that significantly lower education levels in Brazil are associated with jobs that resemble more closely the blue collar jobs found in the U.S., the wage posting assumption appears more appropriate. Second, Brazilian wage contract laws and regulations imposed by central bargaining institutions leave limited scope for individual negotiation of worker pay. For example, Brazilian labor law (*Consolidação das Leis do Trabalho*, or *CLT*) precludes changing individual workers' wages without adjusting accordingly the wages of other workers employed at the same firm.

While the assumption of wage posting appears less restrictive in light of this institutional background, we also believe that a similar mechanism would be at work in models where wages are bargained upon starting the employment relationship, and possibly throughout tenure.[49] In such

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[49] Examples of such model environments have been developed by Postel-Vinay and Robin (2002) and Cahuc et al.

a model, the minimum wage would have a direct effect on the lowest productivity firms as well as an indirect effect on firms higher up the productivity distribution by boosting workers' outside option in the bargaining game.

Informal sector.   Recent work by Meghir et al. (2015) explicitly incorporates an informal sector into an otherwise standard job ladder model. Introducing the informal sector into the model has the advantage of being able to speak to worker transitions between the two sectors and hence to competition of firms for workers in the two sectors. While these considerations are of great interest to studying the Brazilian economy, which features a sizable informal sector, we view our abstraction from the informal sector as putting a lower bound on the degree of competition among firms for workers, and hence on the degree of spill-over effects in wage setting across firms. In this sense, our results can be viewed as providing a lower bound on the degree of compression due to the minimum wage.

On the other hand, incorporating the informal sector may provide an important way for firms to substitute between workers in the two sectors. For example, one may predict that a large increase in the minimum wage in the presence of a shadow economy that is not subject to such labor market regulations would lead to a sizable shift from formal to informal activity. Contrary to this hypothesis, we find that the informal sector in Brazil shrank over the period we study, comprising 36% of all prime-age male employees in 1996 but only 26% in 2012. Hence, we conclude that such additional considerations would not detract from our current analysis, but could be modeled in parallel to our analysis.
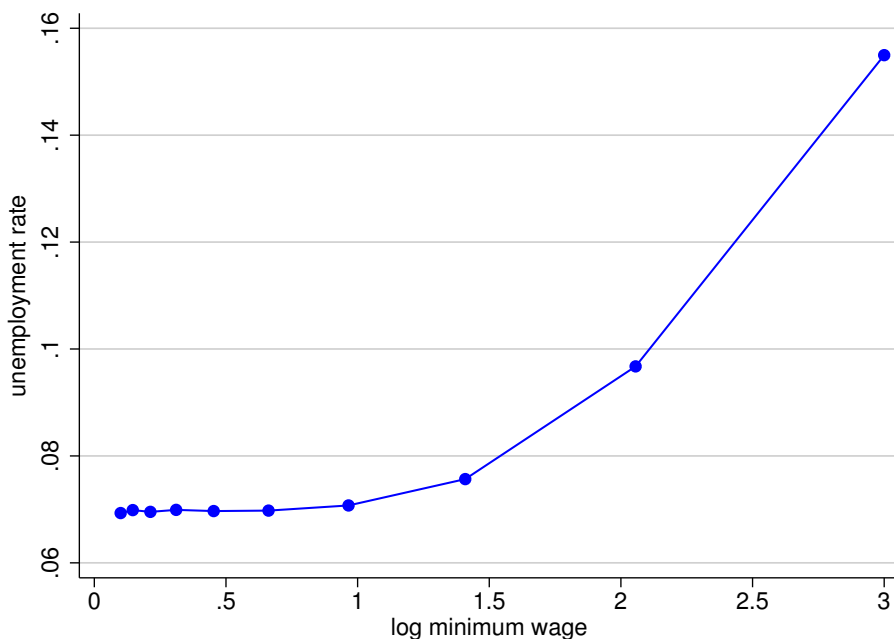
Unemployment and endogenous vacancy creation.   While our model quantitatively generates little unemployment in response to the observed minimum wage increase, the model does generate significant unemployment for larger minimum wage increases. The mechanism for this is that as the minimum wage continues to increase, an entire labor market segment θ is cut off from work activity as soon as even the most productive firm no longer finds it profitable to recruit from this market, that is as soon as the minimum wage crosses the threshold $w^{min} = \theta\overline{p}$. Figure 17 plots the unemployment rate in response to the minimum wage. In the region of the graph corresponding to the 2012 level of the minimum wage, around 0.897 on the horizontal axis, the unemployment

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(2006). In their model, a worker's starting wage equals the value of the productive match in their previous employment, or the value of unemployment, respectively.

rate is little affected by increases in the minimum wage. However, the unemployment rate would more than double and display a steep gradient if the minimum wage were to be increased by an additional 200 log points.

Figure 17. Steady-state equilibrium unemployment vs. minimum wage level

A feature absent from our model specification is endogeneity in firms' vacancy creation (Mortensen, 2000, 2003). Allowing firms to respond in their extensive margin recruiting decisions would plausibly lead to a greater unemployment response to the minimum wage. While the empirical evidence on the employment effects of the minimum wage is mixed and pointing towards zero or small positive effects[50], we cannot rule out negative employment effects of the minimum wage in Brazil. Yet, the Brazilian unemployment rate has fallen from 6.5% to 5.5% from 1996 to 2012, the same time period during which also the informal sector shrank, indicating that such effects could not have been of first-order importance.

Allowing search friction parameters to differ by worker type. One may think that the search friction parameters, which we here restrict to be the same for the entire worker population, may differ systematically across worker groups. To allow for this possibility, our model could be read-

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[50]See Card and Krueger (1994), Neumark and Wascher (1994), and a large follow-up literature for an important part of the debate.

ily extended to feature worker type-specific mobility parameters $\{\delta_\theta, \lambda_\theta^u, \lambda_\theta^e\}_{\theta \in \Theta}$. Such a model has important implications for the observed pattern of sorting of worker types across firms, potentially rationalizing the positive sorting pattern emerging from the AKM analysis. We caution, however, that the same estimation procedure by which we identify the current model parameters guiding worker and firm heterogeneity would no longer be unbiased in the AKM estimation. This is because the strict exogeneity condition, $\mathbb{E}\left[\varepsilon_{it} | i, t, J(i,t)\right] = 0$, required for unbiased identification of worker and firm effects in the AKM framework would no longer be satisfied. However, estimating differences in worker flow rates across worker types from the data, the sign and extent of this bias could be estimated in large samples, which we view as a promising extension for future work.

8.5 Welfare evaluation

Search frictions generate monopsonistic firm rents, which a rise in the minimum wage transfers to workers. While workers who remain employed at the new minimum wage benefit, not everyone gains from the minimum wage increase. The lowest productivity firms stop recruiting low ability workers or exit altogether, while the lowest ability workers are forced into unemployment. In an extension of the model with firm owner-managers, we also account for the loss of rents by monopsony shareholders. Evaluating these channels quantitatively, however, we find small displacement effects of the minimum wage, which are more than offset by allocative efficiency gains from inducing workers to relocate to more productive employers. Nevertheless, the distributional effects of the minimum wage are a significant factor in policy considerations.

Yet other channels through which the minimum wage may effect welfare in the economy are absent from our model. Notably, two channels not present in our benchmark framework would lead counteract the welfare gains of the minimum wage described in our previous analysis.

The first channel is a feedback of the minimum to firms' endogenous vacancy creation. If creating job openings comes at a fixed cost to firms and since the minimum wage will reduce the rent that monopsonist firms can appropriate by posting a job opening, this additional channel would lead to reduced vacancy creation and thus a slow-down in the reallocation of workers across firms in response to the minimum wage increase. An example of such general equilibrium effects on firms' vacancy creation decision is contained in Mortensen (2000) and its application to our framework is left for future exploration.

The second channel is the possibility of pass-through of the minimum wage to consumers through firms' pricing decisions. This channel, which is currently absent from our model to the assumption of a linear production function, would diminish the welfare gains to low-income workers by raising the price of final goods consumption. However, since firms employ a mix of workers and not all are affected (to the same extent) by the minimum wage, one would expect the pass-through to prices to only partially offset the welfare gains to workers with the lowest level of earnings. On the other hand, the pass-through into consumption prices would pose an added effect on high income workers, reducing their welfare relative to the economy with a lower minimum wage.

9 Conclusion

In this paper, we analyzed sources of earnings inequality dynamics in general and the role of the minimum wage specifically. The starting point of our investigation were three key facts about Brazil, which experienced a rapid decline in earnings inequality between 1996 and 2012. Brazil's overall decline in earnings inequality was driven from the bottom. We find that one quarter of this decline stems from a weaker degree of pass-through from firm productivity to wages, and another quarter of the decline is attributable to falling pay differences due to unobserved worker characteristics.

To investigate the contribution of the minimum wage to these facts, we built a search model in the spirit of Burdett and Mortensen (1998), extended with heterogeneous firms and workers. The key feature of the model were spillover effects of the minimum wage due to monopsonistic competition among firms for workers. We characterize the equilibrium of this model and showed that the minimum accounts qualitatively for our documented facts.

Estimating the model on Brazilian microdata, we are also successful in explaining a large share of the overall inequality decline and quantitatively accounting for the three facts. Consistent with the observed compression of earnings, a large share of the inequality decline in our model is due to indirect effects of the minimum wage, resulting in a lower productivity-pay gradient across firms and lower returns to worker ability.

While the minimum wage may affect many other outcomes of interest (Card and Krueger, 1994; Manning, 2005; Harasztosi and Lindner, 2015), we have focused our analysis on the effects

of the minimum wage on the earnings distribution. Although the key mechanism in our model is a general one and relies only on the inter-dependence between firms' wage offers, a key question is to what extent the Brazilian experience carries over to other economies such as the United States, where policy makers currently debate an increase in the minimum wage from 7.25 to 15.00 dollars. Our analysis sheds new light on one aspect of this question and suggests that the effects on earnings inequality will depend crucially on the structural parameters guiding the between-firm competition among firms for employees in those markets. Assessing the strength of this channel for other economies as well as for alternative policies including unemployment insurance, employment protection legislation, and non-discrimination laws would shed further light on the degree to which labor market dynamics can amplify the effects of policy on earnings inequality.

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Appendix

A Empirics

A.1 Inequality trends in Brazilian and U.S. household survey data

To put the magnitude of Brazil's inequality decline into context, Figure 18 plots the evolution of a common inequality measure, the variance of log earnings, from 1996–2012. Data for Brazil come from the largest national household survey, the *Pesquisa Nacional por Amostra de Domicílios (PNAD)*. Data for the U.S. are based on the *March Current Population Survey (CPS)*. In both datasets, earnings inequality is computed over log earnings for male and female labor market participants of age 18–64. The income concept is taken to be labor earnings in the week preceding the survey, and the top and bottom 1% of all observations are dropped to control for outliers.

Figure 18 shows that while the variance of log earnings in the Brazilian household survey dropped by 27 log points from 1996 to 2012, it rose by six log points in the U.S. household data over the same period. Thus, Brazil's inequality decline is of a relatively large magnitude, both within the Brazilian context and in the comparison with the U.S. experience.

Figure 18. Evolution of variance of log earnings in Brazil and the U.S., 1996–2012
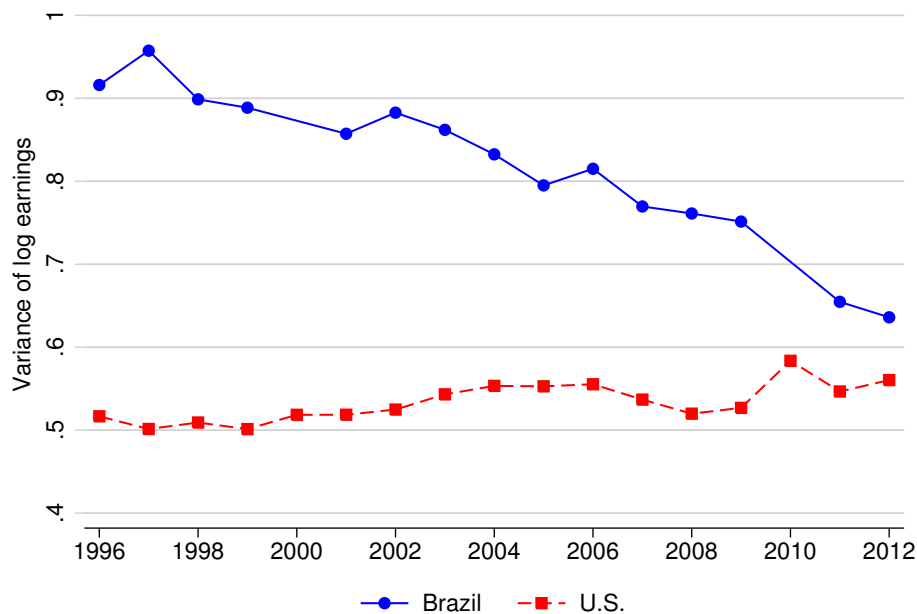
A.2 Dataset descriptions

Table 16. PNAD summary statistics, by period

		Log earnings		
	(1)	(2)	(3)	(4)
	# Workers	Mean	Std. dev.	Formal share
1996–2000	251,724	6.78	2.75	0.64
2000–2004	290,407	6.63	2.94	0.63
2004–2008	385,495	6.71	2.67	0.67
2008–2012	290,789	6.89	2.00	0.72

Notes: All statistics are for male workers age 18–64 and pooled within 5-year periods. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers.Means are computed by period. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period. Surveys are not available in years 2000 and 2010.

Table 17. RAIS summary statistics, by period

			Log earnings	
	(1)	(2)	(3)	(4)
	# Worker-years	# Workers	Mean	Std. dev.
1996-2000	92.7	28.8	1.27	0.85
2000-2004	105.3	32.5	1.07	0.80
2004-2008	126.9	37.3	0.88	0.75
2008-2012	154.2	43.9	0.80	0.72

Notes: The number of worker-years and number of unique workers are reported in millions. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period.The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

Table 18. PIA summary statistics, by period

|			Log revenues per worker		Log value added per worker	
	(1)	(2)	(3)	(4)	(5)	(6)
	# Firm-years	# Unique firms	Mean	S.d.	Mean	S.d.
1996-2000	110,480	34,768	11.85	1.04	11.15	1.13
2000-2004	130,650	40,916	11.98	1.23	11.19	1.32
2004-2008	156,455	48,771	12.02	1.32	11.22	1.34
2008-2012	176,830	55,784	12.06	1.27	11.30	1.31

Notes: Population includes all firms covered by the PIA dataset in the mining and manufacturing sectors. All means and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

A.3 Additional facts about Brazil's inequality decline

Fact 4. *The inequality decline featured compression up to the 90th percentile of the earnings distribution. Yet all parts of the distribution experienced earnings growth between 1996 and 2012.*

Figure 19 plots the evolution of normalized (to zero in 1996) log percentile ratios, all relative to the median of the earnings distribution. There was pronounced catch-up throughout most of the earnings distribution, but more rapidly between the median and the bottom percentiles, as seen by the drop of the bottom percentile ratios relative those at the top. In fact, we see that above the 90th percentile there was little or no compression, evidenced by the log percentile ratio lines coinciding in the graph.

Figure 19. Normalized evolution of earnings percentile ratios, 1996–2012
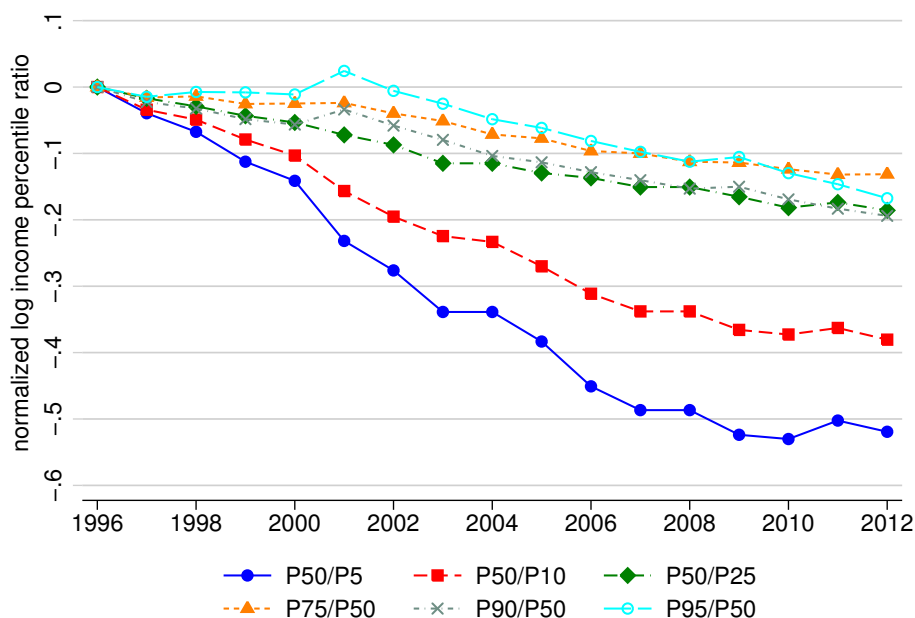
While there was compression throughout most of the earnings distribution, all workers experienced rapid earnings growth over the period. The left panel of Figure 20 plots various percentile ratios of the raw earnings distribution with all ratios being relative to the 90th percentile and normalized to zero in 1996, using the RAIS data. Note that since these are inequality measures (the lower percentile is always in the denominator), a declining line implies lower inequality or, more concretely, compression relative to the 90th percentile. The figure shows that there was a compression up to the 90th percentile of the earnings distribution, with lower income groups growing

monotonically faster in relative terms. The right panel of the figure shows that this compression happened at the same time that all income percentiles experienced growth in real income relative to their 1996 starting point.

Figure 20. Normalized evolution of earnings percentiles, 1996–2012
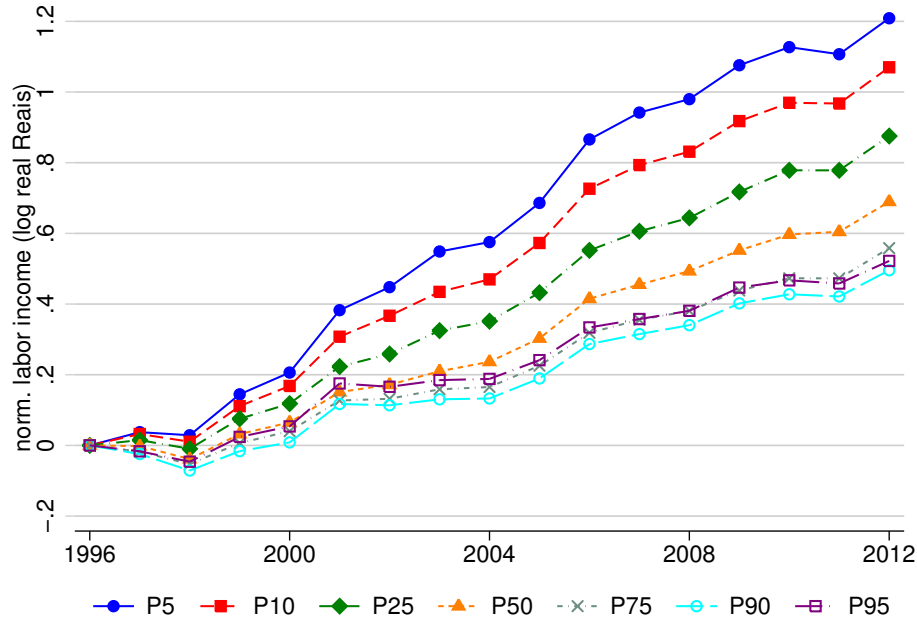
Fact 5. *Most initial earnings inequality and most of its decline are between firms.*

Following a growing literature highlighting the importance of firms in wage setting, we investigate this potential explanation using the employer ID contained in the RAIS data.

Let y_{ijt} denote earnings of worker i employed by firm j in year t, then:

$$y_{ijt} = \underbrace{\overline{y_t}}_{\text{economy average}} + \underbrace{\left(\overline{y}_t^j - \overline{y_t}\right)}_{\text{employer deviaion}} + \underbrace{\left(y_{ijt} - \overline{y}_t^j\right)}_{\text{worker deviation}}$$

Re-arranging and taking variances on both sides we get

$$Var\left(y_{ijt} - \overline{y_t}\right) = Var\left(\overline{y}_t^j - \overline{y_t}\right) + Var\left(y_{ijt} - \overline{y}_t^j\right) + \underbrace{2Cov\left(\overline{y}_t^j - \overline{y_t}, y_{ijt} - \overline{y}_t^j\right)}_{=0}$$

Simplifying, we have

$$
Var\left(y_{ijt}\right) \;=\; \underbrace{Var\left(\bar{y}_t^j\right)}_{\text{between firms}} \;+\; \underbrace{\overline{Var\left(y_{ijt}\,\middle|\, i \in j\right)}}_{\text{within firms}}
$$

Figure 21 plots the results of this decomposition, showing that most initial inequality and most of the decline are in earnings differences across firms.

Figure 21. Between-firm vs. within-firm inequality
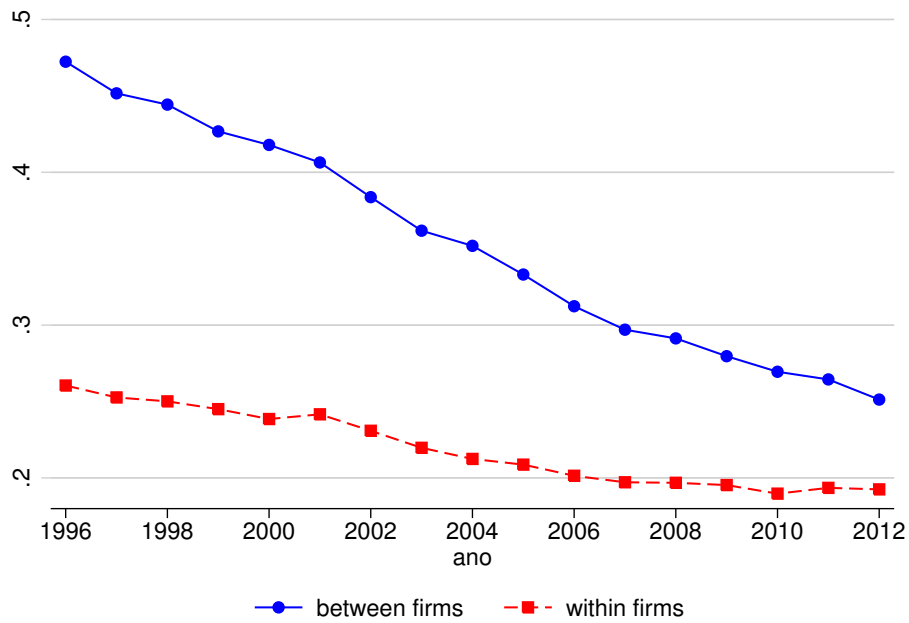
A.4 Robustness checks for AKM framework.

Figure 22. Event study graph for switchers between estimated AKM firm effects quartiles

(a) 1996–2000

(b) 2008–2012

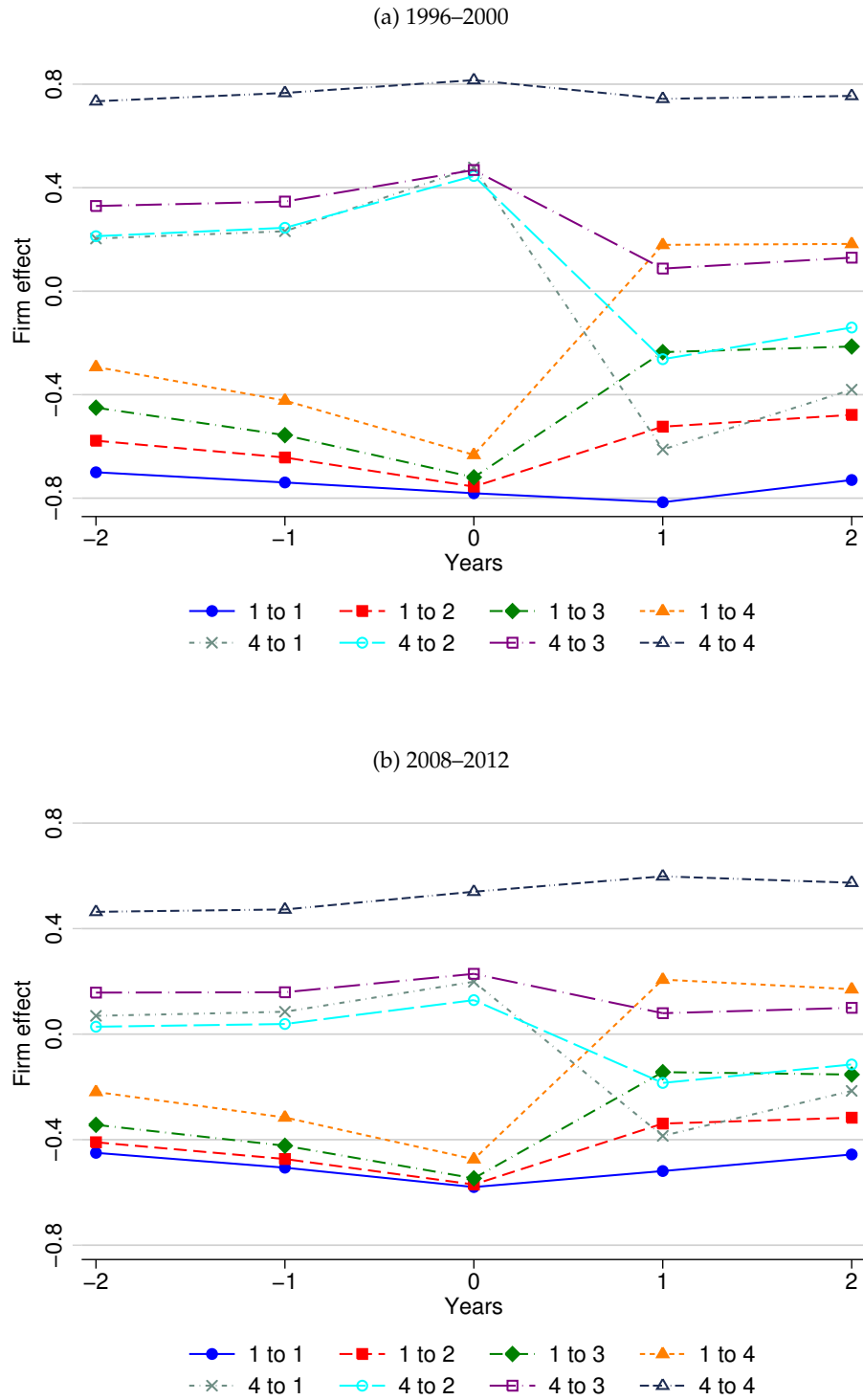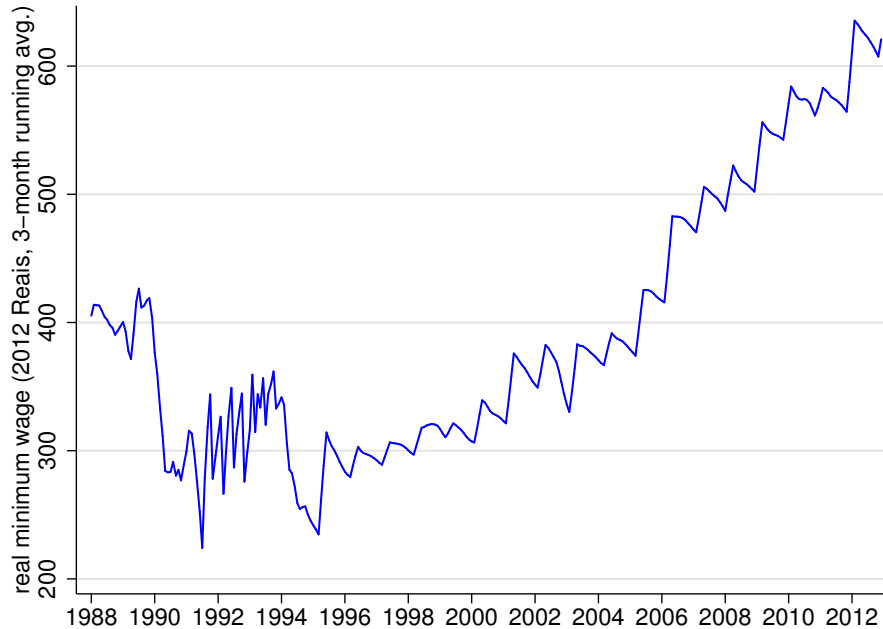## A.5 Evolution of the real minimum wage in Brazil

Figure 23. Evolution of the real minimum wage in Brazil, 3-month running averages (Source: IPEA)

A.6 Distributions of cleaned productivity measures

Fact 2 of Section 3 argued that there was an increase in productivity dispersion across firms as measured by the variance of employee-weighted value added per worker. But, similar to the reason why we opted for the AKM framework on the wage side, a concern with this statement is that differences in the composition of heterogeneous across firms may hinder inference about underlying firm productivity, which is often regarded as an important pay-relevant firm character-istic (Blanchflower et al., 1996; Abowd et al., 1999; Margolis and Salvanes, 2001). To address this concern, we clean the raw productivity measure in the PIA data, namely reported value added per worker, in three alternative ways: first, controlling for only observable worker demographics including age and education; second, controlling for worker demographics and the estimated un-observable worker characteristics obtained from the AKM wage regression; and third, controlling for observable demographics and industry (Bartelsman et al., 2013). The following figures com-pare the raw productivity measure and its three cleaned versions in the cross-section (Figure 24)

and in the time-series (Figure 25).

A noteworthy feature of the cross-sectional comparison in Figure 24 is that the cleaned productivity measures are more concentrated relative to the raw measure, consistent with our previous finding that there is positive sorting of workers across firms along both observable and unobservable dimensions. Furthermore, the various cleaned productivity measures show comparable levels of dispersion and a similar shape overall.

Figure 25 shows that the time series evolution is also qualitatively the same across all productivity measures: while the various cleaning procedures reduce the overall dispersion, we still find that the variance of each measure is increasing between 1996 and 2012. For example, the increase in the variance of raw productivity is 0.35 log points (or 30 percent) between 1996 and 2012, while the increase in the variance of productivity cleaned by only worker demographics is 0.15 log points (or 23 percent) over the same period.

Figure 24. Cross-sectional comparison of various productivity measures in 2004
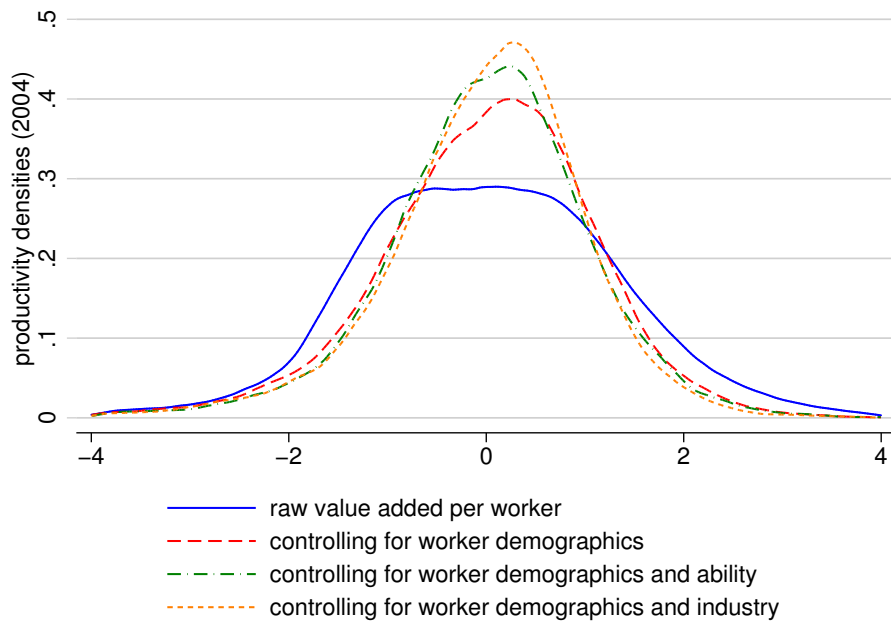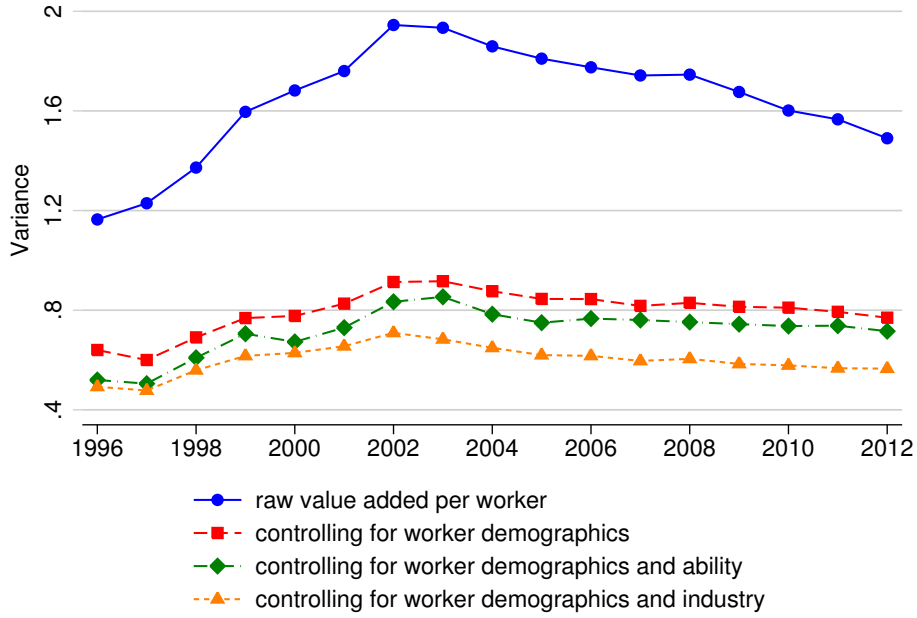
Figure 25. Time-series comparison of variance of of various productivity measures

B Proofs

Proof of Lemma 1

We proceed in order:

1. Because of the minimum wage, workers from markets with $\theta < w^{min}/p$ can not be hired at positive profits by a firm with productivity p. Conversely, since workers from markets with $\theta \geq w^{min}/p$ produce positive profits when working at firm p, that firm will want to attract as many workers as possible from that group.

2. The proof from Burdett and Mortensen (1998) applies to each of our submarkets. The equilibrium wage mapping follows from firms' profit maximization and applying the envelope theorem. The equilibrium wage offered by a firm of productivity p in labor market θ satisfies

$$w\left(p, \theta; w^{min}\right) = \arg\max_{w \geq w^{min}} \frac{p - w}{\left[1 + \kappa_e(1 - F_\theta(w))\right]^2}$$

By use of the envelope theorem, this implicit relation defines a unique mapping from pro-

ductivities to wages given by

$$w\left(p, \theta; w^{min}\right) = p - [1 + \kappa_e(1 - F_\theta(p))]^2 \int_{w^M}^{p} \frac{1}{[1 + \kappa_e(1 - F_\theta(x))]^2} dx$$

To prove that $w(p)$ is strictly increasing in p. Specifically, for any two productivity levels $p_1 > p_2$ profit maximization yields:

$$(p_1 - w_1) \, l\,(w_1) > (p_1 - w_2) \, l\,(w_2) > (p_2 - w_2) \, l\,(w_2) > (p_2 - w_1) \, l\,(w_1)$$

Subtracting the last from the first term, and subtracting the third from the second term, we get:

$$
\begin{aligned}
(p_1 - w_1) \, l\,(w_1) - (p_2 - w_1) \, l\,(w_1) \quad &> \quad (p_1 - w_2) \, l\,(w_2) - (p_2 - w_2) \, l\,(w_2) \\
\Rightarrow \quad l\,(w_1) \quad &> \quad l\,(w_2) \\
\Rightarrow \quad w_1 \quad &> \quad w_2
\end{aligned}
$$

where the last line is a consequence of the fact that $l\,(\cdot)$ is strictly increasing in w:

$$l\,(w; \theta) = m_\theta \,(1 - u_\theta) \frac{dG_\theta\,(w)}{dF_\theta\,(w)} = (1 - u_\theta)\, m_\theta \frac{1 + \kappa^e}{[1 + \kappa^e\,(1 - F_\theta\,(w))]^2}$$

Uniqueness of equilibrium in pure strategies and the inverted mapping from wages into productivity follows the proof in Bontemps et al. (1999, 2000). This concludes the proof of Lemma 1.

Proof of Proposition 1

Without a binding minimum wage, the piece rate paid by a firm p is given by

$$r(p, \theta; w^{min}) = p - \int_{p_0}^{p} \left[\frac{1 + \kappa^e\,(1 - \Gamma\,(p))}{1 + \kappa^e\,(1 - \Gamma\,(x))}\right]^2 dx$$

which is independent of θ.

Suppose the minimum wage is binding, then

$$\frac{\partial r(p, \theta; w^{min})}{\partial w^{min}} = \left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(p)\right)}{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma\left(\frac{w^{min}}{\theta}\right)\right)}\right]^2$$

$$- \int_{\frac{w^{min}}{\theta}}^{p} 2\frac{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(p)\right)}{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(x)\right)} \times \left[-\gamma\left(\frac{w^{min}}{\theta}\right)\frac{1}{\theta}\right]$$

$$\times \frac{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(x)\right) - \left[1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(p)\right)\right]}{\left[1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(x)\right)\right]^2} dx$$

$$= \frac{1}{\theta}\left[\frac{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(p)\right)}{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma\left(\frac{w^{min}}{\theta}\right)\right)}\right]^2$$

$$+ \frac{2\kappa^e\gamma\left(\frac{w^{min}}{\theta}\right)}{\theta} \int_{\frac{w^{min}}{\theta}}^{p} \frac{1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(p)\right)}{\left[1 - \Gamma\left(\frac{w^{min}}{\theta}\right) + \kappa^e\left(1 - \Gamma(x)\right)\right]^3}\left(\Gamma(p) - \Gamma(x)\right) dx$$

Clearly, both terms in the above expression are positive. This concludes the proof of Proposition 1.

Proof of Propositions 2 and 3

Assume that $p \sim U\left(\underline{p}, \overline{p}\right)$. Then we can write the piece rate $\tilde{w}$ offered by a firm with productivity p in market θ as

$$\frac{\tilde{w}\left(p, \theta; w^{min}\right)}{\theta} = p - \int_{\underline{p}(\theta; w^{min})}^{p} \left[\frac{1 + \kappa^e\left(1 - F_\theta(p)\right)}{1 + \kappa^e\left(1 - F_\theta(x)\right)}\right]^2 dx$$

$$= p - \int_{\underline{p}(\theta; w^{min})}^{p} \left[\frac{1 + \kappa^e\left(\frac{\overline{p} - p}{\overline{p} - \underline{p}(\theta; w^{min})}\right)}{1 + \kappa^e\left(\frac{\overline{p} - x}{\overline{p} - \underline{p}(\theta; w^{min})}\right)}\right]^2 dx$$

From here, we consider two cases.

Case 1. $\theta \leq \frac{w^{min}}{\tilde{b}}$ In this first case, for markets affected by the minimum wage, we can write:

$$\frac{\tilde{w}\left(p,\theta;w^{min}\right)}{\theta} = p - \int_{w^{min}/\theta}^{p} \left[\frac{1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)}{1+\kappa^e\left(\frac{\bar{p}-x}{\bar{p}-w^{min}/\theta}\right)}\right]^2 dx$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)\right]^2 \int_{w^{min}/\theta}^{p} \frac{dx}{\left[1+\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)\bar{p}-\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)x\right]^2}$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)(\bar{p}-x)\right]}\right\}_{x=w^{min}/\theta}^{p}$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)\right]^2$$

$$\times \left\{\frac{1}{\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)(\bar{p}-p)\right]} - \frac{1}{\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-w^{min}/\theta}\right)(\bar{p}-w^{min}/\theta)\right]}\right\}$$

$$= p - \left\{\frac{1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)}{\frac{\kappa^e}{\bar{p}-w^{min}/\theta}} - \frac{\left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-w^{min}/\theta}\right)\right]^2}{\frac{\kappa^e}{\bar{p}-w^{min}/\theta}(1+\kappa^e)}\right\}$$

$$= p - \left(\frac{\bar{p}-\frac{w^{min}}{\theta}+\kappa^e(\bar{p}-p)}{1+\kappa^e}\right)\left(\frac{p-\frac{w^{min}}{\theta}}{\bar{p}-\frac{w^{min}}{\theta}}\right)$$

Case 2. $\theta \leq \frac{w^{min}}{\tilde{b}}$ In this second case, for markets affected by the minimum wage, we have:

$$\frac{\tilde{w}\left(p,\theta;w^{min}\right)}{\theta} = p - \int_{\tilde{b}}^{p} \left[\frac{1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)}{1+\kappa^e\left(\frac{\bar{p}-x}{\bar{p}-\tilde{b}}\right)}\right]^2 dx$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)\right]^2 \int_{\tilde{b}}^{p} \frac{dx}{\left[1+\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)\bar{p}-\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)x\right]^2}$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)(\bar{p}-x)\right]}\right\}_{x=\tilde{b}}^{p}$$

$$= p - \left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)(\bar{p}-p)\right]} - \frac{1}{\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\bar{p}-\tilde{b}}\right)(\bar{p}-\tilde{b})\right]}\right\}$$

$$= p - \left\{\frac{1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)}{\frac{\kappa^e}{\bar{p}-\tilde{b}}} - \frac{\left[1+\kappa^e\left(\frac{\bar{p}-p}{\bar{p}-\tilde{b}}\right)\right]^2}{\frac{\kappa^e}{\bar{p}-\tilde{b}}(1+\kappa^e)}\right\}$$

$$= p - \left(\frac{\bar{p}-\tilde{b}+\kappa^e(\bar{p}-p)}{1+\kappa^e}\right)\left(\frac{p-\tilde{b}}{\bar{p}-\tilde{b}}\right)$$

Thus, we can write the wages offered at any firm in the economy with a minimum wage as

follows:

$$
w\left(p,\theta;w^{min}\right) = \begin{cases} p\theta - \theta\left(\dfrac{\overline{p}-\frac{w^{min}}{\theta}+\kappa^e\left(\overline{p}-p\right)}{1+\kappa^e}\right)\left(\dfrac{p-\frac{w^{min}}{\theta}}{\overline{p}-\frac{w^{min}}{\theta}}\right) & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\[4mm] p\theta - \theta\left(\dfrac{\overline{p}-\tilde{b}+\kappa^e\left(\overline{p}-p\right)}{1+\kappa^e}\right)\left(\dfrac{p-\tilde{b}}{\overline{p}-\tilde{b}}\right) & \text{otherwise} \end{cases}
$$

Taking derivatives of this expression:

$$
\frac{\partial w\left(p,\theta;w^{min}\right)}{\partial p} = \begin{cases} \dfrac{2\theta\kappa^e\left(p-\frac{w^{min}}{\theta}\right)}{(1+\kappa^e)\left(\overline{p}-\frac{w^{min}}{\theta}\right)} > 0 & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\[4mm] \dfrac{2\theta\kappa^e\left(p-\tilde{b}\right)}{(1+\kappa^e)\left(\overline{p}-\tilde{b}\right)} > 0 & \text{otherwise} \end{cases}
$$

$$
\partial\left[\frac{\partial w\left(p,\theta;w^{min}\right)}{\partial p}\right] / \partial w^{min} = \begin{cases} -\dfrac{2\kappa^e\left(\overline{p}-p\right)}{(1+\kappa^e)\left(\overline{p}-\frac{w^{min}}{\theta}\right)^2} < 0 & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\[4mm] 0 & \text{otherwise} \end{cases}
$$

To prove the second part of the proposition, consider two worker types θ_i and θ_j with $\theta_i > \theta_j$ and a firm p active in both markets. Suppose a binding minimum wage is imposed and consider the difference in the firm component of pay between firm p to the two types of workers

$$
r\left(p,\theta_j;w^{min}\right) - r\left(p,\theta_i;w^{min}\right)
$$
$$
= \int_{\frac{w^{min}}{\theta_i}}^{p}\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 dx - \int_{\frac{w^{min}}{\theta_j}}^{p}\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 dx
$$
$$
> \int_{\frac{w^{min}}{\theta_j}}^{p}\left\{\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 - \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2\right\} dx
$$

It is hence sufficient to show that for $x \in \left[\frac{w^{min}}{\theta_j}, p\right]$:

$$
\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 \geq \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2
$$
$$
\iff \Gamma\left(\frac{w^{min}}{\theta_j}\right)\left[\Gamma\left(p\right)-\Gamma(x)\right] \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right)\left[\Gamma\left(p\right)-\Gamma(x)\right]
$$

For $x = p$ this inequality is clearly satisfied. For any $x < p$, since by assumption $\theta_i > \theta_j$ it follows that $\Gamma\left(\frac{w^{min}}{\theta_j}\right) \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right)$ by virtue of Γ being a CDF.

To prove the final part of the proposition, note that

$$\mathbb{E}_{\theta_i}\left(p; w^{min}\right) = \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right)$$

Clearly,

$$\theta_i > \theta_j \implies \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right) \leq \mathbb{E}\left(p \mid p \geq \max\left\{\frac{w^{min}}{\theta_j}, p_0\right\}\right)$$

This concludes the proof of Propositions 2 and 3.

C   Estimation

C.1   Details of estimation procedure

First stage.   In the pre-stage of our estimation procedure, we use panel information on job du-
ration and worker flows together with non-parametric estimates of conditional earnings distribu-
tions in order to infer the key labor frictions parameter. This key parameter is $\kappa^e = \lambda^e/\delta$, the ratio
of the on-the-job offer arrival rate to the exogenous separation rate. While all later parameters will
depend on the estimated degree of search frictions embodied in κ^e, the latter parameter is deter-
mined only by the relative ranks of firms and information on worker job mobility. This allows us
to separately estimate κ^e before proceeding to the main stage of our estimation procedure.

In connecting κ^e from the model to the data, it turns out that the parameter is over-identified
and can thus be estimated off different sets of empirical moments. Following the literature (Jolivet
et al., 2006), we estimate κ^e in three alternative ways:

1. Using a model prediction of the following linear relationship between the average duration
   of employment at a given wage and the cumulative distribution of wages up to that point:

   $$\bar{d}\left(w\right) = \frac{1}{\delta}\left(\frac{1}{1 + \kappa^e} + \frac{\kappa^e}{1 + \kappa^e}G\left(w\right)\right)$$

   By means of a linear regression of $\bar{d}\left(w\right)$ on $G\left(w\right)$ we can then recover the coefficient of
   interest as

   $$\hat{\kappa}^e_{duration} = \frac{\hat{a}_1}{\hat{a}_0}$$

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where hats denote estimates from an ordinary least squares regression and

$$a_0 = \frac{1}{\delta} \frac{1}{1 + \kappa^e}$$

$$a_1 = \frac{1}{\delta} \frac{\kappa^e}{1 + \kappa^e}$$

2. Purely non-parametrically using the model-implied relationship between the wage offer distribution $F(w)$ and the realized wage distribution $G(w)$. While the latter can be estimated directly using a kernel density approximation[51] of the empirical wage distribution, $\hat{G}(w)$, the former must be inferred from the wage distribution of workers just hired out of unemployment, $F^0(w)$. The nonparametric estimate of the search parameter is then

$$\hat{\kappa}^e_{nonparametric} = \frac{\hat{F}^0(w) - \hat{G}(w)}{\left(1 - \hat{F}^0(w)\right) \hat{G}(w)}$$

3. Using a nonlinear least squares estimate of the relation between nonparametric estimates of the wage distribution of workers just recruited from either unemployment or another firm, $G_m(w)$, relative to the overall realized wage distribution, $G(w)$:

$$\hat{G}_m(w) = \frac{\log\left(1 + \hat{\kappa}^e_{nonlinear} \hat{G}(w)\right)}{\log\left(1 + \hat{\kappa}^e_{nonlinear}\right)}$$

It is worth highlighting that all three estimation strategies above use different dimensions of the RAIS data to identify the key parameter κ^e, including a mix of cross-sectional and longitudinal moments. If the model is well specified these different estimation strategies should yield similar results.

For identification of the key search parameter as well as other basic parameters relating to labor mobility, we use the variables dating workers' dates of accession and separation in the RAIS data in order to convert the dataset to a monthly panel. From this large panel, we draw a 10% random sample of worker IDs, which we use to construct all subsequent labor flow statistics.

Results of the three identification procedures are summarized in Figure 26.

---

[51] In practice, we use an Epanechnikov kernel with bandwidth 0.04 and 90 bins although we tried alternative kernel, bandwidth, and bin number choices without significant effects on our estimation results.

Figure 26. Estimation results for key search friction parameter κ^e
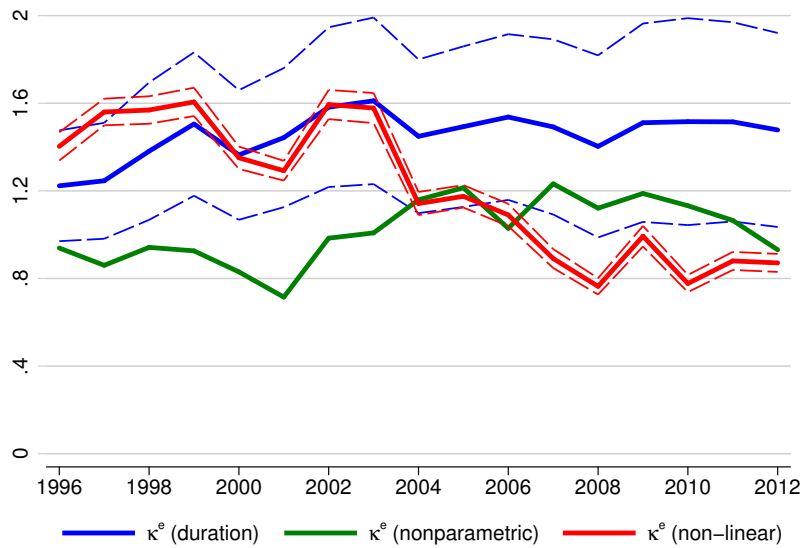
Note: Solid lines denote annual point estimates. Dashed lines denote 95% confidence intervals.

Second stage. In the second stage of our estimation procedure, we take as given the estimate of κ^e from the previous section. We then use a full simulated method of moments procedure to infer distributions of worker ability and firm productivity to match our empirical estimates of worker and firm effect estimates from the AKM decomposition for the period 1996–2000. Remaining details of the estimation procedure are as described in Section 6.

D Quantitative results

D.1 Distributions of worker effects and firm effects

Figure 27. Effect of the minimum wage on AKM estimates in the model

(a) Worker effects
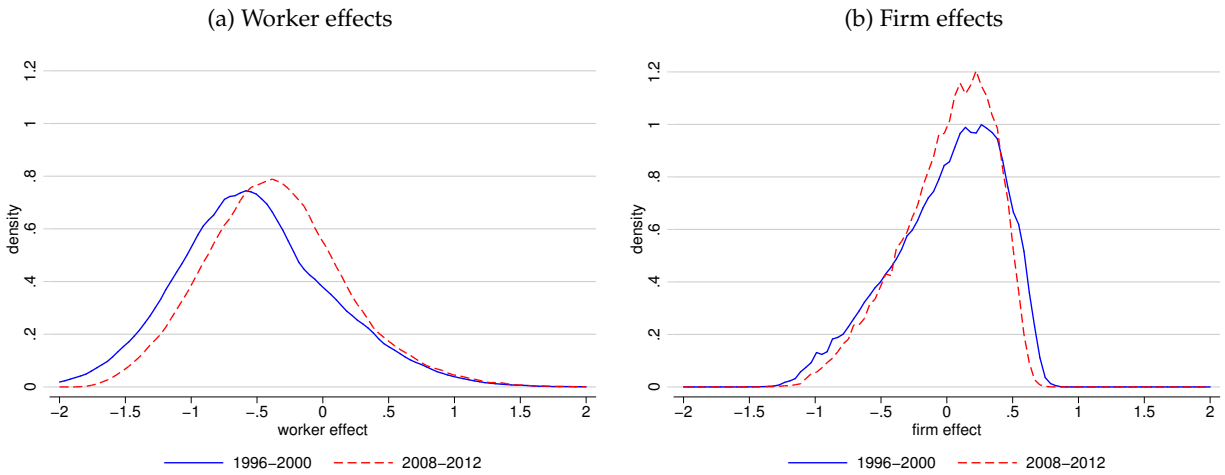
(b) Firm effects

D.2 Sorting pattern throughout the firm effects distribution

Figure 28. Sorting pattern between worker and firm effects, complete firm effects distribution