

# Earnings Inequality and the Minimum Wage: Evidence from Brazil

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# This project

Shed light on drivers of **earnings inequality** by studying Brazil

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**Question:** To what extent can the rise in minimum wage explain Brazil's inequality decline?

# What we do

1. Decompose evolution of earnings inequality in Brazil
2. Build a search model with heterogeneous firms and workers
3. Quantify effects of increase in minimum wage

# Data

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1. Administrative linked employer-employee data (RAIS)
  - Universe of formal sector workers from 1988–2012
  - Restriction to male workers age 18–64
  - Earnings = average monthly labor income in employment
2. Administrative firm characteristics data (PIA)
  - All Manufacturing & Mining (M&M) firms with  $\geq 30$  employees or  $\geq \$300,000$  revenues from 1996–2012
  - Value added p.w. = (revenues - operating costs) / effective hours
3. Publicly available household survey data (PNAD)
  - Geography and informal sector

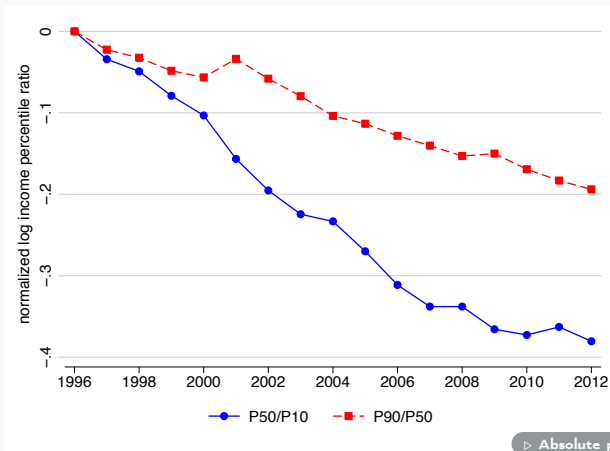


# Facts

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# Fact 1: Compression throughout earnings distribution

- Compression up to 90th percentile, more pronounced at bottom



# Decomposition into firm and worker components

- Most initial inequality and the decline are between firms [▶ Graph](#)

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# Decomposition into firm and worker components

- Most initial inequality and the decline are between firms ▶ Graph
- How to tell apart changes among firms vs. workers?
- Two-way fixed effects model (AKM 1999):

$$\log(y_{it}) = \alpha_i + \alpha_{J(i,t)} + \gamma_t + \varepsilon_{it}$$

where  $\alpha_i$  are worker effects,  $\alpha_{J(i,t)}$  are firm effects,  $\gamma_t$  are year dummies, and  $\varepsilon_{it}$  is an error term

- Estimate this by OLS in overlapping 5-year periods
- Restriction to largest connected set

# Decomposition into firm and worker components

**Table 1:** Variance decomposition into components from AKM estimation

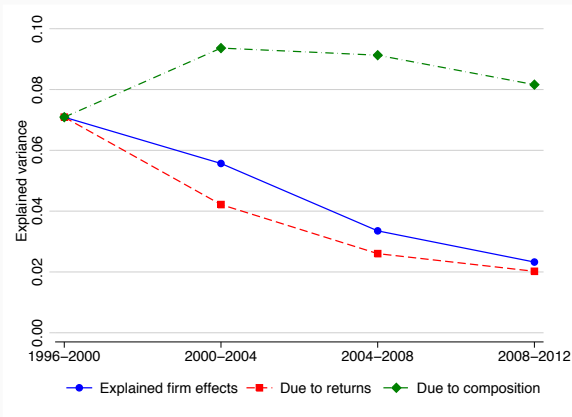
	(1)	(2)	(3)
	1996–2000	2008–2012	Change
Total variance of log earnings	0.72 (100%)	0.52 (100%)	-0.20 (100%)
Variance of firm effects	0.17 (24%)	0.08 (15%)	-0.09 (45%)
Variance of individual effects	0.35 (49%)	0.29 (57%)	-0.06 (28%)
Covariance	0.14 (19%)	0.11 (21%)	-0.03 (16%)
Variance of residual	0.06 (7%)	0.04 (7%)	-0.02 (10%)
# worker years	90.2	123.7	
$R^2$	0.92	0.93	

Note: Cells contain variance level (share) explained by each component.

## Fact 2: Lower pass-through from firm productivity to pay

- Firm productivity explains 50% of variation in firm pay premia
- And >50% of compression in firm pay premia

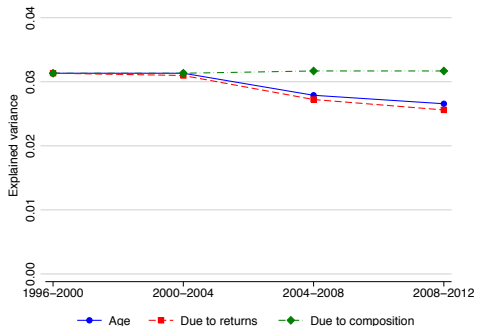
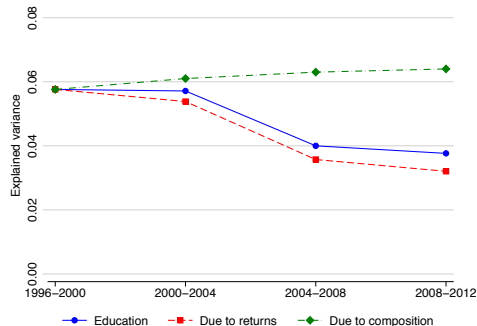
⇒ All due to rapid fall in pass-through from productivity to pay



## Fact 3: Lower returns to worker ability

- Worker observables explain 35-45% of variation in worker component
- And close to 50% of the declining dispersion

⇒ All due to rapid fall in return to education and age





# What do we learn about Brazil's inequality decline?

- Key insight:
  - In spite of greater underlying inequality...
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# What do we learn about Brazil's inequality decline?

- **Key insight:**
  - In spite of greater underlying inequality...
  - ...changes in “wage policies” drove the decline
- **Salient change in “wage policy”:** **rise of minimum wage**
  - 119% real growth
  - Minimum-to-median earnings from 34% to 60%

# Model Summary

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# Model fundamentals

- Extension of Burdett-Mortensen (1998) equilibrium search model
- Heterogeneous worker abilities and firm productivities
- Workers search in frictional labor markets:
  - Search for jobs from unemployment
  - Search for better jobs while employed
- Firms post wages to maximize profits:
  - Profit per workers vs. number of employees
- **Key feature:** optimal wage depends on wages offered by other firms  
⇒ spill-over effects of minimum wage

## Theoretical results:

1. More productive firms pay more for any worker
2. More able workers are paid more within any firm
3. Minimum wage reduces pass-through from productivity to pay as well as return to worker ability

# Estimation

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# Quantitative experiment

- Estimate the model to fit data moments in 1996–2000 period
- Productivity-adjusted real minimum wage growth of 44.7 log points
- Holding all else constant, evaluate impact on earnings distribution

# Estimation part 1

- One key parameter:  $\kappa^e = \lambda^e / \delta =$  speed of climbing firm ladder
- Similar estimates and time trends for  $\kappa^e$  across methods [▷ Details](#)
- Calibrate or fix other parameters

**Table 2:** Monthly model parameters

Description	Parameter	Value
Discount rate	$\rho$	0.009
Exogenous separation rate	$\delta$	0.030
Job finding rate from unemployment	$\lambda^u$	0.200
Labor market friction parameter	$\kappa^e$	1.101



### Method of simulated moments / indirect inference (Smith 1993):

- Solve and simulate the model for a range of parameter values
- Apply AKM framework as auxiliary model on simulated data [▶ Details](#)
- Find model parameters that minimize distance between AKM components in model versus data

# Effects of the Minimum Wage

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# Inequality decomposition in model vs. data

**Table 3:** AKM decomposition of variance of log earnings

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
<b>Variance of earnings</b>	0.72	0.46	0.52	0.32	-0.20	-0.14	70%
<b>Firm effects</b>	0.17	0.17	0.08	0.13	-0.09	-0.04	48%
<b>Worker effects</b>	0.35	0.35	0.29	0.29	-0.06	-0.06	110%
<b>Covariance</b>	0.14	-0.06	0.11	-0.10	-0.03	-0.04	118%
<b>Residual</b>	0.06	0.00	0.04	0.00	-0.02	0.00	0%

## Explaining Facts 1–3

Model predicts:

1. Largest effect at the bottom, yet significant compression far up the distribution ✓ **Fact 1**

**Table 4:** Percentile ratios of earnings in data vs. model

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
P50-P05	1.06	0.90	0.62	0.62	-0.44	-0.28	64%
P50-P10	0.86	0.77	0.55	0.55	-0.31	-0.22	71%
P50-P25	0.48	0.46	0.33	0.35	-0.15	-0.11	73%
P75-P50	0.60	0.52	0.50	0.44	-0.10	-0.08	80%
P90-P50	1.30	1.01	1.17	0.89	-0.13	-0.12	92%
P95-P50	1.76	1.30	1.65	1.17	-0.11	-0.13	118%

Model predicts:

1. Largest effect at the bottom, yet significant compression far up the distribution ✓ Fact 1
2. All compression in firm effects due to lower pass-through from productivity ✓ Fact 2

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2. All compression in firm effects due to lower pass-through from productivity ✓ Fact 2
3. All compression in worker effects driven by fall in returns to worker ability ✓ Fact 3

# Empirical Evidence

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# Further support for the model

- We find empirical evidence in support of:
  1. The minimum wage story: different exposure by region and sector  
[▷ Details](#)
  2. The model key ingredient: job ladder view of the labor market  
[▷ Details](#)
  3. The model mechanism: minimum wage effect on worker composition  
[▷ Details](#)



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  1. Fall in inequality throughout the distribution
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  1. Fall in inequality throughout the distribution
  2. Compression in firm component due to lower pass-through from productivity to pay
  3. Compression in worker component due to lower return to worker ability
- Minimum wage was a significant contributor to the decline in earnings inequality (up to 70%)

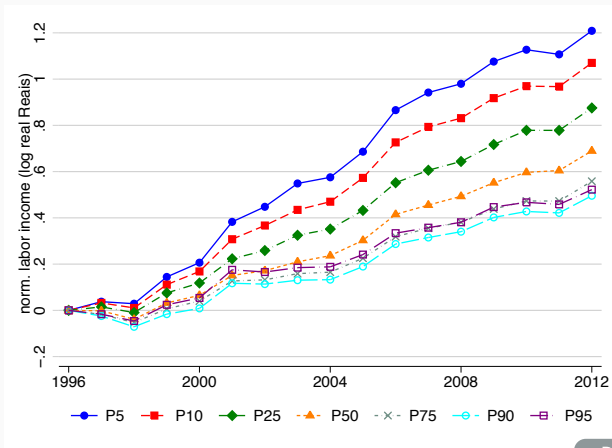
# Backup

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# Absolute earnings growth across percentiles

- All percentiles experienced real earnings growth from 1996–2012
- Fastest growth among bottom 75 percentiles



▷ Back to Fact 1

▷ Back to minimum wage

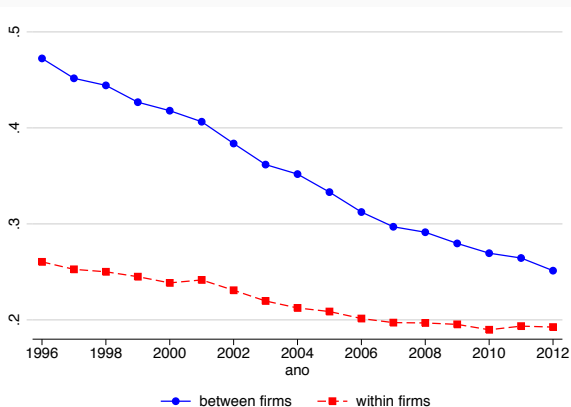
# Much initial inequality and decline was between firms

- Recent work stresses firms as drivers of inequality dynamics
- Let  $y_{ijt}$  denote log earnings of worker  $i$  at firm  $j$  in year  $t$ , then:

▷ Derivation

$$\text{Var}(y_{ijt}) = \underbrace{\text{Var}(\bar{y}_t^j)}_{\text{between firms}} + \overbrace{\text{Var}(y_{ijt} \mid i \in j)}^{\text{within firms}}$$

▷ Percentiles



▷ Back

# Between and within firms: derivation

- Let  $y_{ijt}$  denote earnings of worker  $i$  employed by firm  $j$  in year  $t$ , then:

$$y_{ijt} = \underbrace{\bar{y}_t}_{\text{economy average}} + \underbrace{(\bar{y}_t^j - \bar{y}_t)}_{\text{employer deviation}} + \underbrace{(y_{ijt} - \bar{y}_t^j)}_{\text{worker deviation}}$$

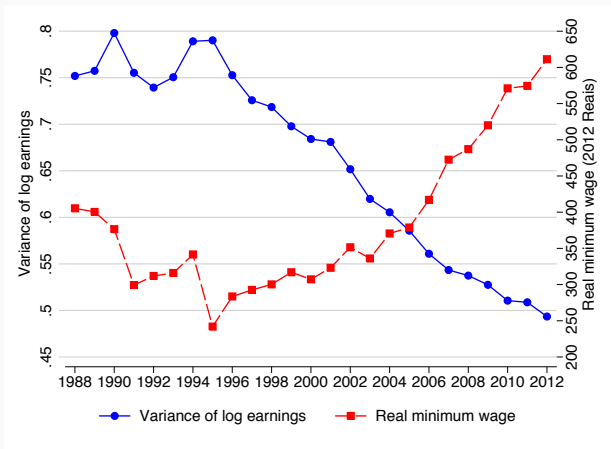
- Re-arranging and taking variances on both sides we get

$$\text{Var}(y_{ijt} - \bar{y}_t) = \text{Var}(\bar{y}_t^j - \bar{y}_t) + \text{Var}(y_{ijt} - \bar{y}_t^j) + \underbrace{2\text{Cov}(\bar{y}_t^j - \bar{y}_t, y_{ijt} - \bar{y}_t^j)}_{=0}$$

- Simplifying, we have

$$\text{Var}(y_{ijt}) = \underbrace{\text{Var}(\bar{y}_t^j)}_{\text{between firms}} + \underbrace{\text{Var}(y_{ijt} \mid i \in j)}_{\text{within firms}}$$

# Minimum wage evolution mirrors earnings inequality



[▶ Back to minimum wage](#)

# Can the minimum wage explain Brazil's inequality decline?

- Rapid rise in federal real minimum wage from 1996–2012:
  - 119% growth, reaching BRL622 (USD-PPP410) per month
  - Minimum-to-median earnings from 34% to 60%
- Suggestive evidence on link b/w minimum wage and inequality:
  1. Minimum wage mirrors earnings inequality from 1988–2012
    - ▷ Mirror image
  2. Faster wage growth at the bottom
    - ▷ Absolute percentiles
- But <5% of workers earning exactly minimum wage throughout
  - Qualitative challenge: compression throughout distribution
  - Quantitative challenge: magnitude of decline & worker/firm channels
- **Potential solution:** indirect “spill-over” effects of minimum wage

# Model: workers' problem

- Value of unemployment:

$$\rho W_\theta = b_\theta + \lambda_\theta^u \int \max\{V_\theta(w) - W_\theta, 0\} dF_\theta(w)$$

- Value of employment of type  $\theta$  at current wage  $w$ :

$$\rho V_\theta(w) = w + \lambda_\theta^e \int_w [V_\theta(w') - V_\theta(w)] dF_\theta(w') + \delta_\theta [W_\theta - V_\theta(w)]$$

- Worker types' reservation wage:

$$w_\theta^R = b_\theta + (\lambda_\theta^u - \lambda_\theta^e) \int_{w_\theta^R} \frac{1 - F_\theta(w)}{\phi + \delta_\theta + \lambda_\theta^e(1 - F_\theta(w))} dw$$

## Model: firms' problem

- In each active market  $\theta$ , a firm with productivity  $p$  solves:

$$\max_{w_\theta \geq w^{min}} (p\theta - w_\theta) l_\theta(w_\theta)$$

- Equilibrium firm size:

$$l_\theta(w) = (1 - u_\theta) m_\theta \frac{dG_\theta(w)}{dF_\theta(w)} = (1 - u_\theta) m_\theta \frac{1 + \kappa^e}{[1 + \kappa^e (1 - F_\theta(w))]^2}$$
$$l(w) = \int l_\theta(w) d\theta$$

# Equilibrium with segmented labor markets

A **search equilibrium with segmented labor markets** is a set

$$\{w^{min}, \phi_\theta, u_\theta, l_\theta(w), F_\theta(w), G_\theta(w)\}$$

for each  $\theta \in \Theta = \{\theta_1, \dots, \theta_N\}$  such that:

1. Productivity  $\Gamma_\theta(p)$  is truncated at  $\underline{p}(\theta; w^{min}) = \max\left\{\frac{\phi_\theta}{\theta}, \frac{w^{min}}{\theta}, p_0\right\}$ .
2. Ability distribution  $H(\theta)$  is truncated at  $\underline{\theta}(w^{min}) = \frac{w^{min}}{\bar{p}}$ .
3. Workers accept any higher-paid job while employed and any job whose wage exceeds their reservation value  $\phi_\theta$  while unemployed.
4. Firms choose which markets  $\theta$  to recruit from and offer wage schedule  $\{w_\theta(p)\}_\theta$  to maximize profits.
5. The unemployment rate  $u = \int u_\theta dH(\theta)$  and firm sizes  $l(\cdot) = \int l(\cdot; \theta) dH(\theta)$  are consistent with  $F_\theta(\cdot)$ ,  $G_\theta(\cdot)$ , and  $(\delta, \lambda^u, \lambda^e)$ .



# Characterizing equilibrium firm decisions

## Lemma 1

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2. Unique equilibrium wage posted:

$$w(p, \theta; w^{min}) = \theta p - \theta \int_{\underline{p}(\theta; w^{min})}^p \left[ \frac{1 + \frac{\lambda^e}{\delta} (1 - \Gamma_\theta(p; w^{min}))}{1 + \frac{\lambda^e}{\delta} (1 - \Gamma_\theta(x; w^{min}))} \right]^2 dx$$

where

$$\Gamma_\theta(p; w^{min}) = \frac{\Gamma(p) - \Gamma(\underline{p}(\theta; w^{min}))}{1 - \Gamma(\underline{p}(\theta; w^{min}))}$$

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3. More productive firms post higher wages:

$$\partial w(p, \theta; w^{min}) / \partial p > 0$$

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4. Higher ability workers are offered higher wages:

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## Estimation part 1

- Key labor parameter  $\kappa^e$  over-identified in data relative to model:

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$$\bar{d}_\theta(w) = \underbrace{\frac{1}{\delta(1+\kappa^e)}}_{\equiv \beta_0} + \underbrace{\frac{\kappa^e}{\delta(1+\kappa^e)}}_{\beta_1} G_\theta(w) \iff \hat{\kappa}_{duration}^e = \frac{\hat{\beta}_1^{OLS}}{\hat{\beta}_0^{OLS}}$$

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2. **Nonparametric:** Relation between job offer distribution  $F_\theta$  and realized wage distribution  $G_\theta(w)$

$$F_\theta(w) = \frac{(1+\kappa^e)G_\theta(w)}{1+\kappa^e G_\theta(w)} \iff \hat{\kappa}_{nonparametric}^e = \frac{\hat{F}_\theta(w) - \hat{G}_\theta(w)}{(1 - \hat{F}_\theta(w)) \hat{G}_\theta(w)}$$

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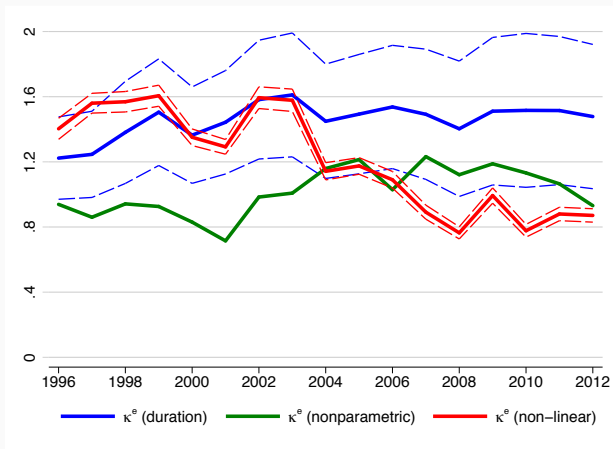
3. **Nonlinear:** From distribution of wages of recently hired workers  $G_{m,\theta}(w)$

$$G_{m,\theta}(w) = \frac{\log(1+\kappa^e G_\theta(w))}{\log(1+\kappa^e)} \iff \hat{\kappa}_{nonlinear}^e \text{ using NLLS}$$



# Estimation part 1

Figure 1: Different estimates of labor mobility parameter  $\kappa^e$



# Mapping from model into AKM decomposition

## Proposition 1

Without binding minimum wage, workers' earnings are given by

$$\log w(p, \theta) = \underbrace{\log \theta}_{\text{"worker effect"}} + \underbrace{\log \tilde{w}(p)}_{\text{"firm effect"}}$$

where

$$\tilde{w}(p) = p - \int_{p_0}^p \left[ \frac{1 + \kappa^e (1 - F(x))}{1 + \kappa^e (1 - F(p))} \right]^2 dx$$

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### Key insight:

- Exact mapping of model into AKM framework
- Minimum wage distorts mapping  $w(p, \theta)$ , but retains monotonicity

## Estimation part 2

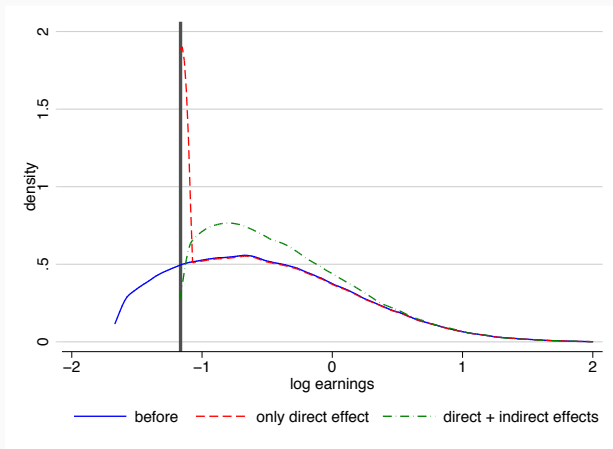
- Sparse parameterization of worker and firm heterogeneity:

$$\log(\theta) \sim \mathcal{N}(0, \sigma_\theta^2), \quad \log(p) \sim \mathcal{N}(0, \sigma_p^2)$$

- Three model parameters:  $\sigma_\theta$ ,  $\sigma_p$ , and minimum wage (numeraire)
- Three data targets:
  1. Variance of AKM worker effects
  2. Variance of AKM firm effects
  3. Minimum-to-median wage ratio

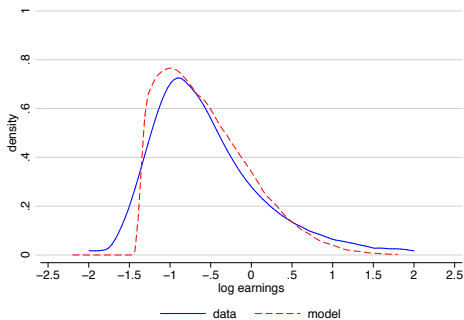
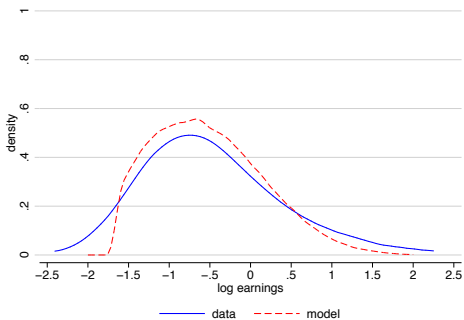
# Direct vs. indirect effects

**Figure 2:** Illustration of direct and indirect effects of minimum wage



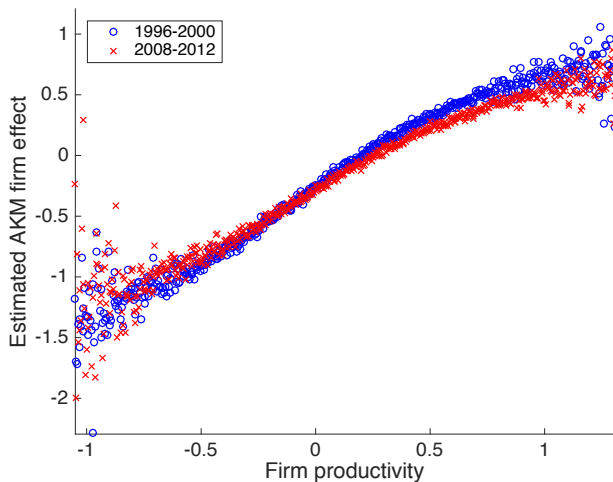
# Explaining Fact 1: Bottom-driven inequality decline

Figure 3: Earnings distributions in 1996–2000 (left) and 2008–2012 (right)



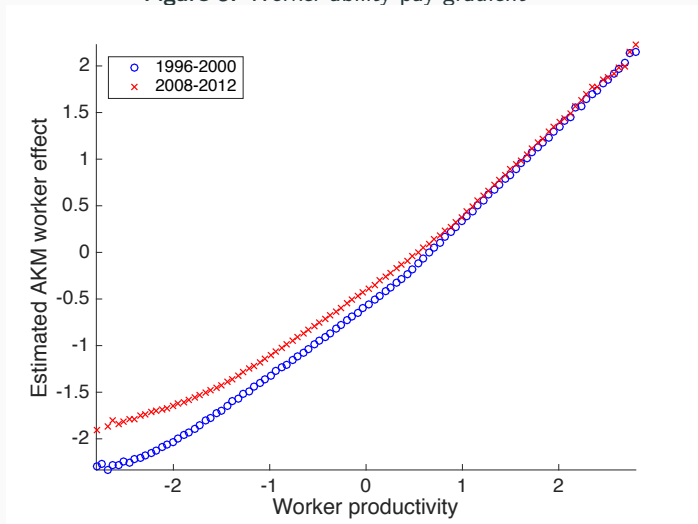
## Fact 2: Illustration

Figure 4: Firm productivity-pay gradient



## Fact 3: Illustration

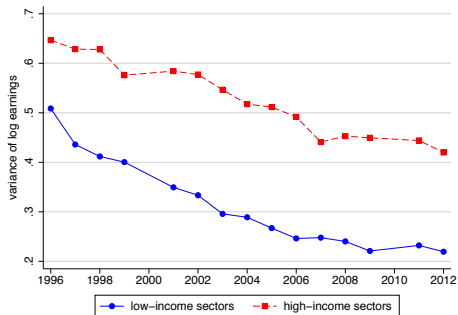
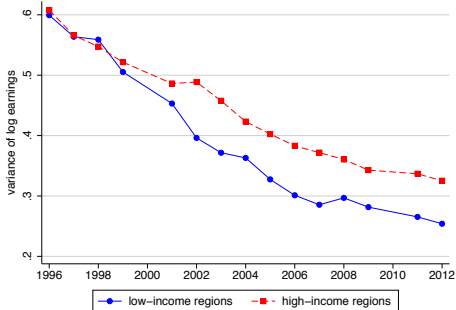
Figure 5: Worker ability-pay gradient





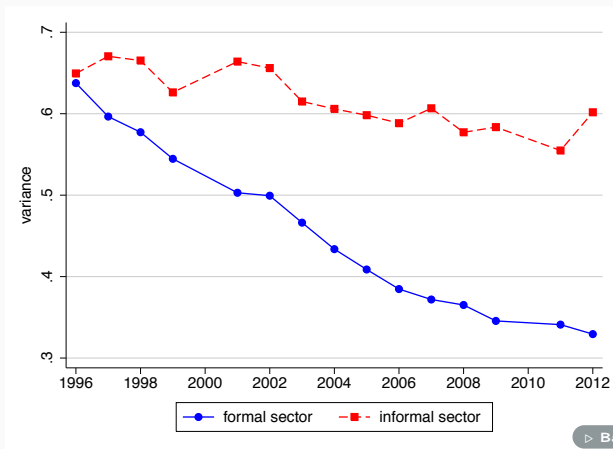
# Evidence in support of minimum wage #1

- More pronounced decline of earnings inequality in:
  - initially low-income regions
  - initially low-income sectors



## Evidence in support of minimum wage #2

- More pronounced decline of earnings inequality in formal sector
- Consistent w/ enforcement of labor regulation in formal sector

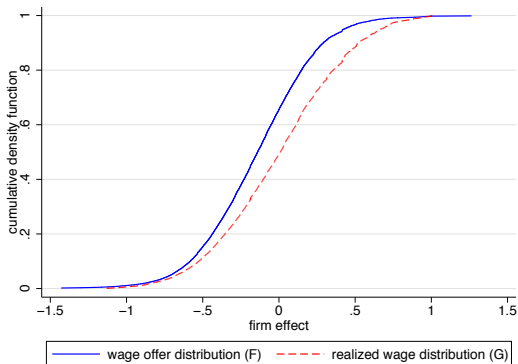


▷ Back to decline

▷ Back to evidence

# Evidence in support of job ladder #1

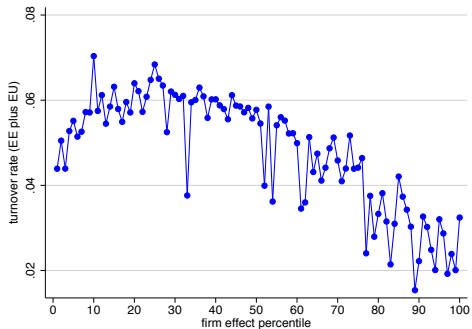
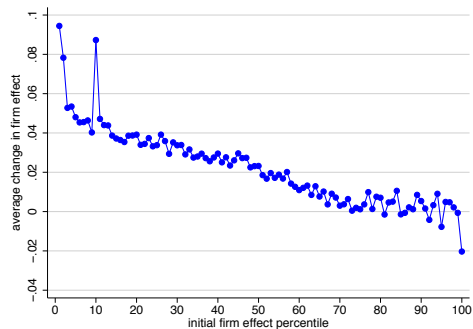
- Workers climb up firm ranks
  - Realized wage distribution FOSDs wage offer distribution
  - Employer transitions associated with positive change in firm effect



Change in firm effect from switching employer	Average value, 1996–2012
Absolute change	6.8
Percentile rank change	6.0

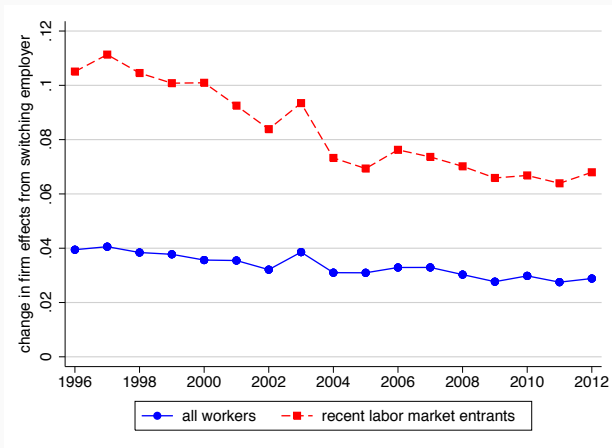
# Evidence in support of job ladder #2

- Further evidence in support of job ladder:
  - Gains from switching decline in previous firm pay percentile
  - Turnover rate lower for higher-paying firms



# Evidence in support of job ladder #3

- Job ladder becomes flatter as minimum wage increases
  - Particularly pronounced for new labor market entrants



# Evidence in support of model mechanism

- Confirm key model prediction:
  - Minimum wage cuts off lowest-paying firms from lowest-paid workers
  - Degree of negative sorting becomes stronger as minimum wage rises

