When Fiscal Consolidation Meets Private Deleveraging

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Abstract

We analyze the interaction between fiscal consolidations and private sector deleveraging in a small open economy undergoing a negative financial shock inside a monetary union. Short-run fiscal multipliers are smaller for large scale and/or faster consolidation programs, but medium-run multipliers are larger. Long term private secured debt plays a critical role in these results. Contractual amortization puts an upper bound on debt repayments, and thus limits the fall in spending capacity produced both by the deleveraging and consolidation processes (*buffering effect*). This gets borrowers into negative equity postponing their access to fresh credit, which makes the recession longer (*duration effect*). The latter effect dominates, making more aggressive consolidation programs costlier in welfare terms. The incidence on the duration of the recession of alternative budgetary instruments also accounts for their differential impact on GDP.

Keywords: fiscal consolidations, long term private debt, financial shock.

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1 Introduction

Since the inception of the crisis, public debt as a percentage of GDP has increased by more than 20 points on average in the OECD countries, up to levels hardly ever seen before in peacetime. In some countries, public debt will end up being multiplied by a factor between 1.5 and 2, similar to what Reinhart and Rogoff (2009) find in their study of past financial crises. Looking ahead, the management of public finances will encounter some additional problems. Bailouts of private firms and social transfers have soared as a consequence of the recession, and unfunded liabilities (commitments made by the government to incur expenses in the future) continue climbing and might require public deficit adjustments at some point in the near future.[1]

Ghosh, Kim, Mendoza, Ostry and Qureshi, (2013) have calculated the *fiscal space*, defined as the difference between current debt ratios and the estimated debt limits, beyond which the government loses access to the market and might be forced to default. They identify just a handful of countries (Australia, Korea, Denmark, Norway and Sweden) that have still the possibility of increasing public debt by a substantial amount should economic conditions worsen. All other advanced economies have much less or no fiscal space left at all. In emerging economies, default is an option, but the problems of debt sustainability in advanced countries are of a different nature and episodes of fiscal stress do not usually lead to defaults but to adjustments in fiscal and monetary policies. Otherwise, when governments do not have credible plans, savers may contemplate the possibility of economies being pushed to their *fiscal limit*, defined as the maximum level of debt that the government is able to service (Bi, 2012).

Is there a maximum amount of debt a country is willing to finance? Or a maximum amount of taxes and/or a minimum public spending the country wants to sustain? If there are no such limits, can the level of debt be unbounded? If these

[1] In 2009, the International Monetary Fund estimated at 35% of GDP the present value of the impact of the crisis on public finances, and a higher value (400% of GDP) for the future commitments associated with aging (39% and 652% in Spain, 37% and 495% for the US or 31% and 276% for France, to give some examples). Cecchetti, Mohanty and Zampolli (2011) estimate between 5 and 10 points of GDP the additional permanent funding needed to meet these obligations in a sustainable manner (maintaining public debt at levels similar to the current ones in developed countries).

limits exist, how close are the advanced economies to them? What are the macroeconomic effects of approaching these limits? The fiscal limit is stochastic and not observable, so that the problems associated with it might not be easily identified in normal times but surface once agents consider that there is a non-negligible (though not necessarily very large) probability that it might be reached. The most obvious consequence of approaching this limit is a sharp increase in financing costs. But this is not the only problem since the uncertainty associated to such limit might endanger other macroeconomic objectives as well (Leeper, 2011).

Although the costs associated to high public debt are difficult to gauge, many authors have argued that beyond the currently observed ratios the negative effect of debt on growth may start being significant.[2] Potent economic growth, structural reforms, inflation and financial repression (low interest rates) are the recipe for successful (effective and painless) reductions in debt to GDP ratios. Unfortunately, the scenario that many countries face nowadays doesn't look very promising on many of these fronts (with the exception of low interest rates). Absent inflation and with the prospects anemic growth in the next years, budgetary adjustments are called for to reduce public debt-to-GDP ratios down to 75% or 60% as targeted by many international institutions.

Issues like the effect of gradualism in fiscal consolidations, or whether these should be based on spending cuts or tax hikes, have been thoroughly discussed in the literature. In the aftermath of the financial crisis, these issues acquire a new dimension since the effectiveness and costs of fiscal retrenchments are likely to be affected by the inability of monetary policy to become more accommodating at the zero lower bound (ZLB) and by the legacy of high private debt. The relevance of the ZLB for

[2] Baum, Checherita-Westphal and Rother (2012) show that the short-run impact of debt on GDP growth is positive and significant, but turns to negative for high debt-to-GDP ratios; additional debt has a negative impact on economic activity and the long-term interest rate is subject to increased pressure when the public debt-to-GDP ratio is above 70%. Cecchetti, Mohanty and Zampolli (2011) estimate that beyond levels of 80-90% of GDP, both public and private debt (particularly debt of non-financial companies) have a significant negative effect on the rate of economic growth in the long term. When public debt is in a range of 85% of GDP, a further 10 percentage point increase reduces trend growth by 0.1 percentage points

fiscal policy has been extensively studied,[3] but the literature has largely ignored the bidirectional links between public and private debt-consolidation processes.[4] The mutual interaction between public and private deleveraging has occupied a central role in policy discussions and in the media, but there is scant academic research that considers how consolidation efforts by the government impinge upon private debt reduction and spending decisions, and conversely how private deleveraging shapes the impact of fiscal consolidations. Placing private and public debt consolidation under the same umbrella may help to better understand the effect of alternative fiscal consolidation strategies and the determinants of the length, depth and costs of private deleveraging. In this paper we develop a framework to analyze this interaction.

To this aim, we build a general equilibrium model of a small open economy in a monetary union. This allows us to retain two features we consider relevant when assessing alternative fiscal strategies in a heavily indebted economy: the lack of an independent monetary policy (with effects similar to those of a binding ZLB but of a more structural nature) and the competitiveness channel. In the model, private debt is long-term and borrowers face collateral constraints. We show that this setup produces a double debt regime. When collateral is sufficiently high, new debt flows are restricted by the value of this collateral; when it is scarce, credit flows freeze and outstanding debt is reduced at the contractual amortization rates, i.e. the economy enters a *slow deleveraging phase*. Large negative shocks may drive the economy into this regime, in which debtors fall into negative equity. As the economy recovers, so does the value of collateral and at some point, the deleveraging process comes to an end and borrowers regain access to new credit. The duration of these two regimes, and the transition in and out of them, are endogenously determined in the model. What is critical for our results is the fact that the determinants of private spending behave differently in these two regimes, thus generating a natural non-linearity in the GDP effects of fiscal shocks.

In this context, we analyze how the size, composition and speed of fiscal consolidations affect the economy, with particular attention to how such consolidations

[3] See Woodford (2011) and the references in the next section.

[4] See Eggertsson and Krugman (2012) and the references in the next section.

interact with private deleveraging. Our main results can be summarized as follows. On the one hand, larger fiscal consolidations imply lower relative output losses in the short run. The reason is that long-term debt contracts, by breaking the link between debt dynamics and collateral values while in a deleveraging phase, cushion the impact of negative fiscal shocks on borrowers' spending capacity for as long as such deleveraging lasts; we label this the 'buffering effect'. On the other hand, larger consolidations also imply higher relative output losses over the medium run, by increasing the length and depth of private deleveraging; we call this the 'duration effect. In spite of this non-linear pattern, the second effect dominates and we find that larger fiscal consolidations imply higher relative output loss over the medium run. Second, front-loaded fiscal consolidations, (of a given size) aimed at reaching the new debt ratio target more quickly, entail higher welfare costs as compared with more gradual adjustments, as the short-run utility costs dominate the medium-run gains in present-discounted terms. And, third, fiscal adjustments based on either expenditure cuts or capital tax hikes prolong the deleveraging phase vis-à-vis those based on consumption or labor income tax hikes.

Our results shed some light on the ongoing debate in policy circles about the appropriate design of fiscal consolidations in terms of their size and speed. Some pundits have advocated large and quick fiscal adjustments as a means of bringing public debt back into a sustainable path, whenever economies approach (or might be in the trajectory of) their fiscal limit. This strategy aims at reducing the period of fiscal pain and getting public finances rapidly back in good shape so that they can play their countercyclical role. According to this view, even if they accentuate the depth of the recession, sharp fiscal adjustments might make it shorter and eventually less painful. Other authors support milder and/or more gradual consolidations when possible, arguing that fiscal multipliers are larger in recessions so that it is wiser to postpone the bulk of the fiscal retrenchment until the economy starts recovering. The presence of long term mortgages in our model reinforces the latter policy option, although it turns the argument around. Smaller consolidations actually increase the (relative) output costs in the short-run, precisely when the recession is more severe, due to the above-mentioned cushioning effect of long-term debt on private spending.

What makes them more benign in relative terms over the medium run is the fact, by favoring a faster recovery in the value of collateral, they shorten the duration of the deleveraging phase. Thus, they bring forward the moment in which borrowers regain access to new credit.

The impact of fiscal consolidation on the length of the deleveraging phase is one of the most powerful transmission channels in our model. Other channels, such as deflationary effects or the ZLB, are of lesser importance. Fiscal rules in our model always render public debt sustainable and we do not allow for an endogenous response of sovereign spreads. Thus, risks to sustainability, which might constitute powerful arguments in favor of fast consolidation programs, are absent from our framework. In that sense our results must be interpreted as uncovering an often neglected channel (namely the endogenous response of private deleveraging) that might be relevant in the design of consolidation packages.

The rest of the paper is organized as follows. We briefly describe the related literature in Section 2. The model and the baseline calibration are presented in Section 3. In Section 4 we analyze the impact of alternative consolidation strategies and in Section 5 we perform a similar exercise against the backdrop of a *credit crunch* scenario. Section 6 concludes.

2 Related literature

There is ample literature assessing the effects of consolidations. A fair reading of the evidence suggests that fiscal consolidations are successful in the medium term, although they might exert non-negligible short term output losses.[5] Also, front-loaded adjustments might be more effective and less costly, and adjustments in public spending, rather than tax hikes, make these consolidations more effective and lasting

[5] There is also a strand of the empirical literature that finds that, under some conditions, fiscal consolidations might even be beneficial in the short run. See, among others, Alesina and Ardagna (2009), Cogan, Taylor, Wieland and Wolters (2013), Giavazzi and Pagano (1990), Hemming, Kell and Mahfouz (2002) and Perotti (1999).

and impose a lesser drag on the economy in the short run.[6] For instance, Kumar, Leigh and Plekhanov (2007) study fiscal consolidations in 24 OECD countries and find they can have positive long-run effects, particularly when the gain in fiscal space is used to cut capital income taxes. However, these long-run gains may not occur if the consolidation involves cuts in public infrastructure spending. Forni, Monteforte and Sessa, (2009) find that tax and spending based consolidations in the Euro area increase GDP and all its components over the long run by around $5\% - 7\%$.

Wieland and Wolters (2013) have simulated the effect of a fiscal consolidation strategy in the U.S. that brings the budget to balance and find also a positive and significant effect on GDP. Almeida, Castro, Mourinho and Maria (2013) simulate the impact on economic activity and welfare of a permanent decline in the ratio of public debt to GDP in a small euro-area economy with no independent monetary policy. They find that spending cuts (in government consumption and transfers to households) combined with a reduction in the labor income tax have positive long-run impacts on economic activity and significantly improve the net foreign asset position; they are also relatively less penalizing than tax hikes for real GDP. As for the timing, the authors conclude that smooth fiscal adjustments reduce the negative impact on output but prolong the period before the economy reaps their full benefits.

Some of these results have been called into question in the aftermath of the financial crisis. For one thing, the capacity of monetary policy to help in smoothing the short run costs of fiscal retrenchments is severely limited by the exhaustion of the conventional stabilizing instruments. At the zero lower bound, fiscal multipliers are higher (Christiano, Eichenbaum and Rebelo, 2011 and Woodford, 2011). Also Eggertsson (2010) finds that, unlike what happens when the ZLB is not binding, the output effect of spending cuts is higher than that of tax rate hikes; Erceg and Linde (2013) extend this result to fiscal adjustments within a monetary union, in which small open economies lose control of monetary policies.[7] In the same vein, the recent IMF Fiscal Monitor finds that government spending cuts have more negative effects

[6]von Hagen, Hallet and Strauch (2001) find that government spending cuts are key in successful fiscal consolidations and point out to the importance of institutional arrangements in pursuing these adjustments.

[7]See also Farhi and Werning (2012) and IMF, Fiscal Monitor, October 2014, c.2

on employment recovery than tax-based consolidations after a protracted recession. But the ZLB and fiscal policy are intertwined in more complex ways. For instance, Erceg and Levine (2014) show that the size of the fiscal multiplier also depends on the incidence of fiscal shocks on the duration of the ZLB regime. Also, Bi, Leeper and Leith (2013) find that the conditions under which uncertain consolidations (about their composition and intensity) may be expansionary in the short run are very demanding, and more so when they are based on spending cuts and the interest rate approaches the ZLB.

Financial crises leave behind a landscape of heavily indebted households and firms, with severe restrictions in their access to new credit. The interaction between private debt and spending decisions is key to understand the effects of fiscal policies. Eggertsson and Krugman (2012) show how fiscal multipliers increase in the presence of high private debt.[8] Despite this, there have been few attempts to analyze jointly the dynamics of private and pubic debt. Batini, Melina and Villa (2015) is one exception. They find that the drag that high private debt imposes on economic growth is more severe than the one caused by public indebtedness. Private and public deleveraging interact in a perverse way and, when possible, the latter should keep the appropriate pace to sustain public finances without inflicting an additional burden on the recovery process. Whereas some authors argue that, despite their stronger impact on GDP, front-loading adjustments might entail lower costs in present value terms (ECB, Monthly Bulletin, 2014), others defend that gradualism is a better option, provided that the size of multipliers changes, so that the (negative) effect of fiscal retrenchments is stronger in the short run than in the medium term.[9] Blanchard and Leigh (2013) advocate modulating fiscal adjustments when possible, to avoid the large fiscal multipliers that are typically associated to recessions.[10] Moreover, such large multipliers can cast a long shadow on output in the presence of hysteresis

[8] Other authors have confirmed this conection in a variety of theoretical and empirical settings. Andrés, Boscá and Ferri (2015) discuss this effect in the context of collateralized debt and labor market frictions. Kaplan, Violante and . Weidner (2014) and Cloyne. and Surico (2014) analyze the relationship between household financial position and spending.

[9] See Fletcher and Sandri (2015) and Corsetti, Kuester Meier and Müller (2010).

[10] This may be particularly true when sovereign spreads are not too high or too sensitive to Debt/GDP ratios (Corsetti, Kuester, Meier, and Müller, 2013).

channels.[11]

To the best of our knowledge, this literature does not allow for the endogenous determination of the duration of the deleveraging period of the private sector. The presence of long term mortgages in our model has a profound effect on the way private spending adjusts to exogenous shocks, fiscal or otherwise. Long-term debt contracts induce non-linearities in the multipliers associated to fiscal retrenchments and, what is more important, establish a theoretical link between the intensity and timing of fiscal adjustments and the duration of private deleveraging processes and, hence, of recessions.

3 Model

We present a general equilibrium model of a small open economy that belongs to a monetary union. The real side of the economy is standard and there are three types of consumers who differ in the intensity with which they discount the future. The fiscal authority collects taxes on households and entrepreneurs, consumes, and issues non-contingent nominal debt. The latter is sold to unconstrained households and to foreigners, and it is stabilized in the long run by means of a fiscal rule.

All variables are in real terms unless otherwise specified, with the consumption goods basket acting as the numeraire. In what follows we describe the basic elements of the model. Appendix A contains the whole set of first order conditions and other equilibrium conditions

3.1 Households

Households obtain utility from consumption goods and from housing units. There are three types of consumers: patient households, impatient households, and (impatient) entrepreneurs. In equilibrium, the latter two borrow from the former and from the rest of the union by issuing long-term nominal debt. In periods in which borrowers

[11] See See Auerbach and Gorodnichenko (2012) and Hernández de Cos and Moral-Benito (2013) on the size of multipliers in difficult times and DeLong and Summers (2012) and Fatás and Summers (2015), reggarding the size of hysteresis effetcs.

are able to receive new credit flows, they do so subject to collateral constraints. Real estate is the only collateralizable asset. We will henceforth refer to impatient and patient households as 'constrained' and 'unconstrained' households, respectively.

There is a representative constrained household and a representative unconstrained household, denoted respectively by superscripts c and u.

3.1.1 Cost minimization

Households consume a basket of home and foreign goods, denoted respectively by subscripts H and F,

$$c_t^x = \left(\omega_H^{1/\varepsilon_H} \left(c_{H,t}^x\right)^{(\varepsilon_H-1)/\varepsilon_H} + (1-\omega_H)^{1/\varepsilon_H} \left(c_{F,t}^x\right)^{(\varepsilon_H-1)/\varepsilon_H}\right)^{\varepsilon_H/(\varepsilon_H-1)}, \tag{1}$$

for $x = c, u$; $c_{H,t}^x$ is a basket of domestic good varieties,

$$c_{H,t}^x = \left(\int_0^1 c_{H,t}^x(z)^{(\varepsilon_p-1)/\varepsilon_p} dz\right)^{\varepsilon_p/(\varepsilon_p-1)}, \tag{2}$$

where $\varepsilon_p > 1$ is the elasticity of substitution across consumption varieties $z \in [0,1]$. Let $P_{H,t}(z)$ denote the price of home good variety z, and $P_{F,t}$ the price of the foreign goods basket. Household $x = c, u$ minimizes nominal consumption expenditure, $\int_0^1 P_{H,t}(z)\, c_{H,t}^x(z)\, dz + P_{F,t} c_{F,t}^x$, subject to (1) and (2). The first order conditions of the static cost minimization problem can be expressed as

$$c_{H,t}^x = \omega_H \left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon_H} c_t^x, \quad c_{F,t}^x = (1-\omega_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\varepsilon_H} c_t^x, \quad c_{H,t}^x(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon_p} c_{H,t}^x, \tag{3}$$

where

$$P_t = \left(\omega_H P_{H,t}^{1-\varepsilon_H} + (1-\omega_H) P_{F,t}^{1-\varepsilon_H}\right)^{1/(1-\varepsilon_H)}, \quad P_{H,t} = \left(\int_0^1 P_{H,t}(z)^{1-\varepsilon_p} dz\right)^{1/(1-\varepsilon_p)}$$

are the *consumer price index* (CPI) and the *producer price index* (PPI), respectively. Nominal spending in domestic goods equals $\int_0^1 P_{H,t}(z)\, c_{H,t}^x(z)\, dz = P_{H,t} c_{H,t}^x$, whereas total nominal consumption spending equals $P_{H,t}c^x_{H,t} + P_{F,t}c^x_{F,t} = P_t c^x_t$.

As noted before, consumption goods are also used as inputs by construction firms and equipment capital producers. The latter are assumed to combine home and foreign goods analogously to households, and similarly for domestic good varieties. This gives rise to investment demand functions analogous to (3).

3.1.2 Unconstrained households

The unconstrained household maximizes

$$E_0 \sum_{t=0}^{\infty} (\beta^u)^t \left\{ \log(c^u_t) + \vartheta \log(h^u_t) - \chi \int_0^1 \frac{n^u_t(i)^{1+\varphi}}{1+\varphi} di \right\}, \tag{4}$$

where $n^u_t(i)$ are labor services of type $i \in [0,1]$ and h^u_t are housing units, subject to the following budget constraint (expressed in units of the consumption goods basket),

$$\begin{aligned}(1+\tau^c_t)c^u_t + d_t + b^{gu}_t + p^h_t\left[h^u_t - (1-\delta_h)h^u_{t-1}\right] &= \frac{R_{t-1}}{\pi_t}\left(d_{t-1} + b^{gu}_{t-1}\right) \\ &\quad + (1-\tau^w_t)\int_0^1 \frac{W_t(i)}{P_t} n^u_t(i)\, di - T_t,\end{aligned}$$

where d_t is the real value of net positions in nominal international debt, and b^{gu}_t is the real value of nominal domestic government debt holdings, R_t is the gross riskless nominal interest rate, δ_h is the depreciation rate of real estate, p^h_t is the real price of real estate, $\pi_t \equiv P_t/P_{t-1}$ is gross CPI inflation, $W_t(i)$ is the nominal wage for labor services of type i, τ^c_t and τ^w_t are tax rates on consumption and labor income, respectively, and T_t are lump-sum taxes. The first order conditions are standard.

3.1.3 Constrained households

The constrained household's preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c^c_t) + \vartheta \log(h_t) - \chi \int_0^1 \frac{n^c_t(i)^{1+\varphi}}{1+\varphi} di \right\}, \tag{5}$$

where $\beta < \beta^u$, i.e. the constrained household is relatively impatient; h_t and $n_t^c(i)$ are housings units owned and type-i labor services supplied by the constrained household, respectively. The household faces the following budget constraint,

$$(1+\tau_t^c)\, c_t^c + \frac{R_{t-1}}{\pi_t} b_{t-1} + p_t^h \left[h_t - (1-\delta_h)\, h_{t-1}\right] = b_t + (1-\tau_t^w) \int_0^1 \frac{W_t(i)}{P_t} n_t^c(i)\, di - T_t,$$

where b_t is the real value of household debt outstanding at the end of period t.

Unlike in most of the literature, which typically assumes short-term (one-period) debt, we assume that debt contracts are *long-term*. In the interest of tractability, we assume that at the beginning of time t the household repays a fraction $1-\gamma$ of all nominal debt outstanding at the end of period $t-1$, regardless of when that debt was issued.[12] This type of perpetual debt is similar to the one proposed by Woodford (2001) as a tractable way of modelling long-term debt. In *real* terms, the outstanding principal of household debt then evolves as follows,

$$b_t = \frac{b_{t-1}}{\pi_t} + b_t^{new} - (1-\gamma)\frac{b_{t-1}}{\pi_t} = b_t^{new} + \gamma\frac{b_{t-1}}{\pi_t}, \tag{6}$$

where b_t^{new} is gross new credit net of *voluntary* amortizations, i.e. amortizations beyond the contractual debt repayment $(1-\gamma)\, b_{t-1}/\pi_t$.

We assume that, in 'normal times' (in a sense to be specified below), household borrowing is subject to collateral constraints, as in Kiyotaki and Moore (1997). Following Iacoviello (2005), outstanding debt b_t cannot exceed a fraction m_t (the 'loan-to-value ratio', which we assume to be exogenously time-varying) of the expected discounted value of the household's residential stock: $b_t \leq m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t$. For brevity, we will refer to such pledgeable value of collateral as *collateral value*. This debt limit, however, is only effective as long as it exceeds $\gamma b_{t-1}/\pi_t$, which we will henceforth refer to as the *contractual amortization path*. If the collateral value falls below such path, lowering b_t to the value of collateral would require lenders not only to reduce gross new credit to zero (its lower bound), but also to *impose* additional

[12] Total debt repayments in each period are then $(1-\gamma) + (R_{t-1} - 1)$ times nominal debt outstanding, i.e. the sum of amortization and interest payments.

amortizations beyond those agreed in the contract (i.e. $b_t^{new} < 0$). Since lenders cannot force borrowers to pay back faster than the contractual amortization rate, the contractual amortization path becomes the effective debt limit. Therefore, long run debt implies the following *asymmetric borrowing constraint*,

$$b_t \leq R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t, \text{ if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t \geq \gamma \frac{b_{t-1}}{\pi_t}, \tag{7}$$

$$b_t \leq \gamma \frac{b_{t-1}}{\pi_t}, \quad \text{if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t < \gamma \frac{b_{t-1}}{\pi_t}. \tag{8}$$

This asymmetry gives rise to a *double debt regime*. In 'normal times' in which collateral values exceed the contractual amortization path, debt is restricted by the former. In this baseline regime, households can receive new credit against their housing collateral, with the constraint that such new credit does not exceed the gap between collateral values and the amortization path.[13] However, in the face of shocks that reduce collateral values sufficiently, the economy switches to an alternative regime, in which new credit dries up and debt is restricted by the contractual amortization path.[14] An important element of our framework is that changes from one regime to the other take place *endogenously*.

3.2 Production

Entrepreneurs produce an intermediate good using labor and consumption goods and sell it to retailers, who transform it into consumption good varieties. Entrepreneurs and retailers conform the consumption goods sector. In addition, construction firms produce real estate, both for residential and commercial use, whereas equipment capital is produced by capital goods producers. Retailers in the consumption-goods sector are characterized by monopolistic competition and nominal rigidities, all other goods markets operate under perfect competition.

[13]Indeed, from (6) and (7) we obtain $b_t^{new} \leq m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t$.

[14]From (6) and (8): $\left(m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t\right) < 0 \Rightarrow b_t^{new} = 0$.

3.2.1 Entrepreneurs

A representative entrepreneur produces an intermediate product and sells it to retailers at a perfectly competitive real (CPI-deflated) price mc_t. The entrepreneur maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \log c_t^e, \tag{9}$$

with the consumption basket c_t^e defined analogously to (1), subject to a budget constraint. Entrepreneurs obtain operating profits from their activities; they are also assumed to own the firms in the other productive sectors of the economy. All operating profits (net of capital depreciation) are taxed at rate τ_t^k; since profits accrue to entrepreneurs and these are the sole owners of productive capital in the model, we will henceforth refer to τ_t^k as the *capital income tax*. The entrepreneur's budget constraint is given by

$$\begin{aligned} \left(1+\tau_t^c\right) c_t^e &= \left(1-\tau_t^k\right)\left(mc_t y_t^e - \frac{W_t}{P_t} n_t^e\right) + b_t^e - \frac{R_{t-1}}{\pi_t} b_{t-1}^e - p_t^h \left[h_t^e - \left(1-\delta_h\right) h_{t-1}^e\right] \\ &\quad - q_t \left[k_t - \left(1-\delta_k\right) k_{t-1}\right] + \tau_t^k \left(\delta_h p_t^h h_{t-1}^e + \delta_k q_t k_{t-1}\right) + \left(1-\tau_t^k\right) \sum_{s=r,h,k} \Pi_t^s, \end{aligned}$$

$$y_t^e = k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{1-\alpha_k-\alpha_h},$$

where y_t^e is output of the intermediate good, k_{t-1} is capital equipment with unit price q_t and a depreciation rate δ_k, h_{t-1}^e is commercial real estate, n_t^e is a basket of labor services, W_t is a nominal wage index, b_t^e is the real value of entrepreneurial debt outstanding at the end of period t, and $\{\Pi_t^s\}_{s=r,h,k}$ are real profits from the retail, construction and equipment goods-producing sectors.

Entrepreneurs' maximization is also subject to an asymmetric borrowing constraint analogous to the one on constrained households,

$$b_t^e \leq R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e, \text{ if } \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t^e \geq \gamma^e \frac{b_{t-1}^e}{\pi_t}, \tag{10}$$

$$b_t^e \leq \gamma^e \frac{b_{t-1}^e}{\pi_t}, \qquad \text{if } \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t^e < \gamma^e \frac{b_{t-1}^e}{\pi_t}, \tag{11}$$

where we allow for a different loan-to-value ratio (m_t^e) and contractual amortization rate $(1-\gamma^e)$ for entrepreneurs.

3.2.2 Retailers

A continuum of monopolistically competitive retailers indexed by $z \in [0,1]$ purchase the intermediate input from entrepreneurs at the real price mc_t, and transform it one for one into final good varieties. Retailers' real marginal cost is thus mc_t. Each retailer z faces a demand curve

$$y_t(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon_p} y_t \equiv y_t^d\left(P_{H,t}(z)\right), \tag{12}$$

where y_t is aggregate demand of the consumption basket (to be derived below). Let $\lambda_t^e \equiv 1/\left[c_t^e\left(1+\tau_t^c\right)\right]$ denote the entrepreneur's marginal utility of real income. Assuming Calvo (1983) price-setting, a retailer that has the chance of setting its nominal price at time t solves

$$\max_{P_{H,t}(z)} E_t \sum_{s=0}^{\infty} \left(\beta\theta_p\right)^s \frac{\lambda_{t+s}^e}{\lambda_t^e}\left(1-\tau_{t+s}^k\right)\left[\frac{P_{H,t}(z)}{P_{t+s}} - mc_{t+s}\right] y_{t+s}^d\left(P_{H,t}(z)\right),$$

where θ_p is the probability of not adjusting the price and τ_p is a tax rate on retailers' revenue. The first-order condition is standard, with all time-t price setters choosing a common optimal price $\tilde{P}_{H,t}$.

3.2.3 Construction and capital producing firms

A representative construction firm maximizes its expected discounted stream of profits, $E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^e}{\lambda_0^e}\left(1-\tau_t^k\right)\Pi_t^h$, where $\Pi_t^h = p_t^h I_t^h - \frac{W_t}{P_t} n_t^h - i_t^h$, subject to the production technology

$$I_t^h = \left(n_t^h\right)^{\omega}\left\{ i_t^h\left[1-\frac{\Phi_h}{2}\left(\frac{i_t^h}{i_{t-1}^h}-1\right)^2\right]\right\}^{1-\omega},$$

where n_t^h are labor services, i_t^h are consumption goods, and I_t^h are new real estate units.[15]

Also, a representative equipment capital producer maximizes its expected discounted stream of profits, $E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^e}{\lambda_0^e} \left(1-\tau_t^k\right) \Pi_t^k$, where $\Pi_t^k = q_t I_t - i_t$, subject to the technology

$$I_t = i_t \left[1 - \frac{\Phi_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right],$$

where i_t are consumption goods, and I_t are new equipment capital goods.

3.3 Wage setting

Both entrepreneurs and construction firms use a basket of labor services offered by constrained and unconstrained households,

$$n_t^s = \left(n_t^{s,c}\right)^{\mu_s} \left(n_t^{s,u}\right)^{1-\mu_s},$$

where $n_t^{s,x}$ are labor services provided by type-x household, $x = c, u$, to each sector $s = e, h$. We assume that both worker types (constrained and unconstrained) earn the same wage. Cost minimization then implies $(1-\mu_s)\, n_t^{s,c} = \mu_s n_t^{s,u}$, for $s = e, h$. From each household type, each sector demands in turn a basket of labor service varieties,

$$n_t^{s,x} = \left(\int_0^1 n_t^{s,x}(i)^{(\varepsilon_w - 1)/\varepsilon_w}\, di\right)^{\varepsilon_w/(\varepsilon_w - 1)},$$

for $x = c, u$ and $s = e, h$, where $\varepsilon_w > 1$ is the elasticity of substitution across labor varieties $i \in [0,1]$. Cost minimization implies $n_t^{s,x}(i) = \left(W_t(i)/W_t\right)^{-\varepsilon_w} n_t^{s,x}$, for $x = c, u$ and $s = e, h$, where $W_t \equiv (\int_0^1 W_t(i)^{1-\varepsilon_w}\, di)^{1/(1-\varepsilon_w)}$ is the nominal wage

[15] We include labor services in the production function of construction firms so as to allow for long-run changes in real estate prices. Without labor in construction ($\omega = 0$), real estate prices are always unity in the long run. More generally, it can be shown that $p_{ss}^h = \left(w_{ss}\right)^{\omega} \omega^{-\omega} \left(1-\omega\right)^{-(1-\omega)}$.

index. Total demand for each variety of labor services is thus

$$n_t^x(i) \equiv n_t^{e,x}(i) + n_t^{h,x}(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} \left(n_t^{e,x} + n_t^{h,x}\right) \equiv n_t^{d,x}(W_t(i)),$$

for $x = c, u$. Total nominal wage income earned by each type-x household equals $\int_0^1 W_t(i)\, n_t^x(i)\, di = W_t n_t^x$, where $n_t^x \equiv n_t^{e,x} + n_t^{h,x}$.

As in Erceg, Henderson and Levin (2000; EHL), nominal wages are set *à la* Calvo (1983). In particular, a union representing all type-i workers maximizes the utility of the households to which such workers belong. Let $\lambda_t^x \equiv 1/\left[c_t^x\left(1 + \tau_t^c\right)\right]$ denote the marginal utility of real income for each household type $x = c, u$. Then a union that has the chance to reset the nominal wage at time t chooses $W_t(i)$ to maximize

$$\sum_{x=c,u} E_t \sum_{s=0}^{\infty} \left(\beta^x \theta_w\right)^s \left[\lambda_{t+s}^x \left(1 - \tau_{t+s}^w\right) \frac{W_t(i)}{P_{t+s}} n_{t+s}^{d,x}(W_t(i)) - \chi \frac{\left(n_{t+s}^{d,x}(W_t(i))\right)^{1+\varphi}}{1+\varphi}\right],$$

where θ_w is the probability of not adjusting the wage and $\beta^c = \beta$. All time-t wage-setters choose a common optimal wage $\tilde{W}_t$; see the first-order condition in the Appendix.

3.4 International linkages

A representative exporter produces a basket of domestic consumption goods: $x_t = \left(\int_0^1 x_t(z)^{(\varepsilon_p - 1)/\varepsilon_p} dz\right)^{\varepsilon_p/(\varepsilon_p - 1)}$, where $x_t(z)$ is demand for each domestic good variety. Cost minimization implies that the exporter's demand for each variety is $x_t(z) = \left(P_{H,t}(z)/P_{H,t}\right)^{-\varepsilon_p} x_t$, and total spending is $\int_0^1 P_{H,t}(z)\, x_t(z)\, dz = P_{H,t} x_t$. The exporter sells the basket x_t in export markets under perfect competition. The zero profit condition implies that the market price of the export basket is exactly $P_{H,t}$. Assuming that foreign consumers' preferences are analogous to those of domestic consumers, foreign demand for the basket of domestic goods is given by

$$x_t = \zeta \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\varepsilon_F} y_{F,t},$$

where $P_{F,t}$ and $y_{F,t}$ are the foreign price level and aggregate demand (both exogenous) and ε_F is the price elasticity of exports. Defining the *terms of trade* $p_t^* \equiv P_{H,t}/P_{F,t}$, the latter evolve according to $p_t^* = p_{t-1}^* \pi_{H,t}/\pi_{F,t}$,where $\pi_{F,t} \equiv P_{F,t}/P_{F,t-1}$ is foreign inflation.

As mentioned before, domestic agents can lend to and borrow from foreigners and other domestic agents at the riskless nominal rate R_t. Following standard practice in the literature, in order to guarantee stationarity of the country's net foreign asset position, we assume that R_t is given by

$$R_t = R^* \exp\left(-\psi \frac{P_t nfa_t}{P_{H,t} gdp_t}\right),$$

for $\psi > 0$, where R^* is the area-wide nominal interest rate (which is assumed to be constant here), and nfa_t and gdp_t are the country's real (CPI-deflated) net foreign asset position and real (PPI-deflated) GDP, both to be defined later.

3.5 Fiscal authority

Real (CPI-deflated) government debt b_t^g evolves as follows,

$$\begin{aligned} b_t^g \;=\; & \frac{R_{t-1}}{\pi_t} b_{t-1}^g + \frac{P_{H,t}}{P_t} g_t - 2T_t - \tau_t^w \frac{W_t}{P_t}\left(n_t^C + n_t^U\right) - \tau_t^c\left(c_t^C + c_t^U + c_t^e\right) \\ & -\tau_t^k\left[mc_t y_t^e - \frac{W_t}{P_t} n_t^e - \left(\delta_h p_t^h h_{t-1}^e + \delta_k q_t k_{t-1}\right) + \sum_{s=r,h,k} \Pi_t^s\right]. \end{aligned} \tag{13}$$

We have assumed full home bias in government consumption, such that its nominal value equals $P_{H,t} g_t$. A fiscal rule ensures stability of government debt. In particular, we will consider rules of the form

$$fi_t = fi_{t-1} + \phi_b\left(b_{t-1}^{gy} - \bar{b}^{gy}\right) + \phi_{\Delta b}\left(b_t^{gy} - b_{t-1}^{gy}\right), \tag{14}$$

where $b_t^{gy} \equiv \frac{P_t b_t^g}{P_{H,t} gdp_t}$ is the government debt-to-GDP ratio, $\bar{b}^{gy}$ is its long-run target, and $fi \in \{g, \tau^w, \tau^c, \tau^k\}$ is the fiscal instrument that is endogenized through the fiscal rule, to be specified in each case below. The other fiscal instruments are held constant at the steady state level $\overline{fi} \in \{\bar{g}, \bar{\tau}^w, \bar{\tau}^c, \bar{\tau}^k\}$.

3.6 Aggregation and market clearing

Each retailer z demands $y_t^d(P_{H,t}(z))$ units of the intermediate input, as given by (12). Total demand for the latter equals $\int_0^1 y_t^d(P_{H,t}(z))\,dz = y_t\Delta_t$, where $\Delta_t \equiv \int_0^1 (P_{H,t}(z)/P_{H,t})^{-\varepsilon_p}\,dz$ denotes relative price dispersion. Market clearing in the intermediate good market thus requires

$$k_{t-1}^{\alpha_k}\left(h_{t-1}^e\right)^{\alpha_h}\left(n_t^e\right)^{1-\alpha_h-\alpha_k} = y_t\Delta_t.$$

As noted before, investment-goods producers and exporters demand the same combination of domestic consumption goods as consumers. Therefore, aggregate demand for the basket of domestic consumption goods is given by,

$$y_t = c_{H,t}^c + c_{H,t}^u + c_{H,t}^e + i_{H,t} + i_{H,t}^h + g_t + x_t. \tag{15}$$

Total demand for real estate must equal total supply,

$$h_t + h_t^u + h_t^e = I_t^h + (1-\delta_h)\left(h_{t-1} + h_{t-1}^u + h_{t-1}^e\right).$$

Total demand for equipment capital must equal total supply

$$k_t = I_t + (1-\delta_k)\,k_{t-1}.$$

Labor market clearing requires

$$n_t^c + n_t^u = n_t^e + n_t^h.$$

Total supply of government debt equals total demand by nationals, i.e. unconstrained households (b_t^{gu}), and by foreigners (b_t^{g*}): $b_t^g = b_t^{gu} + b_t^{g*}$. We may combine all market clearing conditions and budget constraints to obtain the current account identity (which is redundant as a result of Walras' Law),

$$nfa_t = \frac{R_{t-1}}{\pi_t} nfa_{t-1} + \frac{P_{H,t}}{P_t} x_t - \frac{P_{F,t}}{P_t} \left(c^c_{F,t} + c^u_{F,t} + c^e_{F,t} + i_{F,t} + i^h_{F,t} \right),$$

where

$$nfa_t \equiv d_t - b_t - b^e_t - b^{g*}_t = (d_t + b^{gu}_t) - b_t - b^e_t - b^g_t$$

is the real (CPI-deflated) net foreign asset position.[16] We finally define real (PPI-deflated) GDP as

$$\begin{aligned} gdp_t &\equiv y_t + \frac{P_t}{P_{H,t}} (q_t I_t - i_t) + \frac{P_t}{P_{H,t}} \left(p^h_t I^h_t - i^h_t \right) \\ &= \frac{P_t}{P_{H,t}} c^{tot}_t + \frac{P_t}{P_{H,t}} \left(q_t I_t + p^h_t I^h_t \right) + g_t + \left[x_t - \frac{P_{F,t}}{P_{H,t}} \left(c^{tot}_{F,t} + i_{F,t} + i^h_{F,t} \right) \right], \end{aligned}$$

where in the second equality we have used (15) and $z_{H,t} = \frac{P_t}{P_{H,t}} z_t - \frac{P_{F,t}}{P_{H,t}} z_{F,t}$ for $z = c^c, c^u, c^e, i, i^h$, and where $c^{tot}_t \equiv c^c_t + c^u_t + c^e_t$ is total consumption (total consumption imports $c^{tot}_{F,t}$ are defined analogously).

3.7 Calibration and solution method

We calibrate the model to the Spanish economy. As explained in the introduction, we are motivated by the recent experience of developed countries, which are still embarked in a lengthy process of both fiscal consolidation and private sector deleveraging, Spain being a good example thereof. The time period is a quarter. We match the model's steady state to a number of empirical targets in 2007, the year prior to the start of the financial crisis. Our model's steady state should be interpreted as the economy's initial condition for the purpose of our simulation exercises.

The discount factor of the impatient agents is set to $\beta = 0.98$, following Iacoviello

[16] Notice that the distribution of unconstrained households' financial wealth between international and government bonds (d_t, b^{gu}_t) is undeterminate, as only their sum $d_t + b^{gu}_t$ is pinned down in equilibrium. This implies that the distribution of the government debt stock (b^g_t) between domestic and foreign holders (b^{gu}_t, b^{g*}_t) is also undeterminate, although it does not affect equilibrium dynamics either.

(2005). For patient households, we choose $\beta^u = 1.025^{-1/4}$, which is consistent with a steady state nominal interest rate of $R_{ss} = 1.025^{1/4}\pi_{ss} = R^* e^{-\psi(nfa^y_{ss})}$. The union-wide inflation is $\pi_{F,ss} = 1$, which implies $\pi_{H,ss} = \pi_{ss} = 1$ in a stationary equilibrium. The union's nominal interest rate is $R^* = 1.02^{1/4}$ and we set ψ to replicate net foreign assets over GDP in 2007, $nfa^y_{ss} = -79.3\%$. The inverse labor supply elasticity is set to $\varphi = 4$, consistently with a large body of micro evidence. The weight parameter in the consumption basket, ω_H, is set to match gross exports over GDP in 2007 (26.9%). The price elasticity of exports and imports is set to $\varepsilon_F = \varepsilon_H = 1$ (García *et al.*, 2009) and the scale parameter in export demand, ζ, is chosen such that steady-state terms of trade p^*_{ss} are normalized to 1.

The elasticities of substitution across varieties of consumption goods and labor services, ε_p and ε_w, determine the desired markups in product and labor markets, respectively. We set $\varepsilon_p = 7$, consistent with an initial price markup of $\varepsilon_p/(\varepsilon_p - 1) = 1.17$, which is consistent with the findings of Montero and Urtasun (2013) for Spanish firm-level data. To calibrate wage markups we follow Galí (2011) who interprets EHL model of wage-setting in a way that delivers equilibrium unemployment (see Appendix B for details). Targeting an unemployment rate of 8.6% in 2007, we obtain an initial wage markup of $\varepsilon_w/(\varepsilon_w - 1) = 1.43$, which we achieve by setting $\varepsilon_w = 3.31$.

The elasticity of entrepreneurial output with respect to capital and real estate are set to $\alpha_k = 0.11$ and $\alpha_h = 0.21$, which help to replicate the labor share of GDP in 2007 (61.6%) and the share of equipment capital in the total stock of productive capital.[17] Following Iacoviello and Neri (2010) we set $\delta_h = 0.01$, whereas δ_k is set to a standard value of 0.025. The elasticity of construction output with respect to labor ω is chosen to match the construction share of total employment in 2007 (13.4%). We set the weight of utility from housing services, ϑ, to replicate gross household debt over annual GDP (80.2%). The shares of constrained and unconstrained workers in the labor baskets are set to $\mu_h = \mu_e = 1/2$. The scale parameters of convex investment adjustment costs, Φ_h and Φ_k, are chosen such that the dynamics of construction and equipment capital investment in our baseline deleveraging scenario resembles their

[17] The value of equipment capital was estimated at 21.4% of the total value of productive capital in 2007 (using data from BBVA Research).

Table 1: Baseline calibration

Parameter	Value	Description
Preferences		
β^u	0.994	unconstrained household discount factor
β	0.98	constrained household discount factor
φ	4	(inverse) labor supply elasticity
ϑ	0.35	weight on housing utility
ε_p	7	elasticity of subst. across consumption varieties
ε_w	3.31	elasticity of substitution across labor varieties
ω_H	0.65	weight home goods in consumption basket
ε_H	1	elasticity of imports wrt terms of trade
ε_F	1	elasticity of exports wrt terms of trade
ζ	0.83	scale parameter export demand
Technology		
α_h	0.21	elasticity output wrt real estate
α_k	0.11	elasticity output wrt equipment
ω	0.50	elasticity construction wrt labor
δ_h	0.01	depreciation real estate
δ_k	0.025	depreciation equipment
μ_e, μ_h	0.5	share of constr. households in labor baskets
Φ_h	12.0	investment adjustment costs construction
Φ_k	9.9	investment adjustment costs equipment
Price/wage setting		
θ_p	0.67	fraction of non-adjusting prices
θ_w	0.75	fraction of non-adjusting wages
Debt constraints		
$\bar{m}$	0.85	household LTV ratio
$\bar{m}^e$	0.69	entrepreneur LTV ratio
γ	0.98	amortization rate household debt
γ^e	0.97	amortization rate entrepreneurial debt
Fiscal policy		
$\bar{\tau}^c$	0.08	initial tax rate on consumption
$\bar{\tau}^w$	0.16	initial tax rate on labor income
$\bar{\tau}^k$	0.18	initial tax rate on capital income
$\bar{g}$	0.57	initial government spending
$\bar{b}^{gy}/4$	0.80	long-run target for gov't debt-to-annual GDP
$\phi_{b^{gy}}$	0.035	response coefficient in fiscal rule
$\phi_{\Delta b^{gy}}$	0.387	response coefficient in fiscal rule

behavior during the crisis.[18]

The Calvo parameters are set to $\theta_p = 2/3$ and $\theta_w = 3/4$, such that prices and wages are adjusted every 3 and 4 quarters on average, respectively. This is consistent with survey evidence for the Spanish economy (see e.g. Druant et al., 2009).

Regarding the debt contract we set $\bar{m} = 0.85$ for the household's initial loan-to-value ratio, consistently with Spanish evidence on pre-crisis LTV ratios for new mortgages,[19] while the entrepreneurial initial loan-to-value ratio, $\bar{m}^e = 0.69$, is chosen to match the ratio of gross non-financial corporate debt to annual GDP (125.4% in 2007). We calibrate the contractual amortization rates, $1-\gamma$ and $1-\gamma^e$, to replicate the average age of the stock of outstanding mortgage debt prior to the crisis: $1-\gamma = 0.02$ and $1-\gamma^e = 0.03$ per quarter.[20]

As explained above, all fiscal instruments other than the one in the fiscal rule (equation 14) are held constant at some initial values $\{\bar{\tau}^s\}_{s=c,w,k}$ and $\bar{g}$. We calibrate the initial tax rates $\{\bar{\tau}^s\}_{s=c,w,k}$ as in Stähler and Thomas (2012), who calculate pre-crisis average implicit tax rates for different tax figures. We set $\bar{g}$ such that $\bar{g}/gdp_{ss}$ equals the government spending share of GDP in 2007 (18.3%). In the fiscal rule, we set the long-run target for the government debt-to-(quarterly) GDP ratio to $\bar{b}^{gy} = 0.80 \times 4$, consistently with levels reached in Spain only a few years after the start of the crisis and with our focus on scenarios that involve a reduction in government indebtedness towards the EU Treaty target (60% of GDP). The response coefficients in the rule, ϕ_b and $\phi_{\Delta b}$, are calibrated to make the dynamic change in current deficit roughly comparable across consolidation scenarios based on the different fiscal instruments. Table 1 summarizes the calibration.

[18] The accumulated fall in construction and equipment capital investment 8 quarters after the financial shock replicate their accumulated fall 8 quarters after their peak in 2007:Q4 (24.5% and 28% respectively).

[19] See e.g. Masier and Villanueva (2011, Table A1), and Akin et al. (2014, Table A.1).

[20] Under our debt contracts (with a constant fraction of outstanding debt amortized each period), the average age of the debt stock converges in the steady state to $\gamma/(1-\gamma)$ and $\gamma^e/(1-\gamma^e)$ for households and entrepreneurs, respectively. According to calculations by Banco de España, based on data from the Land Registry office and large financial institutions, the average age of outstanding mortgage debt prior to the crisis was close to 12.5 years for households and 8 years for nonfinancial corporations and entrepreneurs. This yields $\gamma = 12.5 \times 4/(12.5 \times 4 + 1) = 0.98$ and $\gamma^e = 8 \times 4/(8 \times 4 + 1) = 0.97$.

Solution algorithm. We assume perfect foresight in all our simulations. We solve for the fully nonlinear equilibrium path, using a variant of the Newton-Raphson algorithm developed by Laffargue (1990), Boucekkine (1995) and Juillard (1996) (LBJ).[21] Our assumption of long-run debt contracts gives rise to two debt regimes for households and entrepreneurs. If collateral values are above the contractual debt amortization paths, then debt levels are restricted by the former, according to equations (7) and (10). If the opposite holds, then new credit flows collapse to zero and debt is restricted by the contractual amortization path (equations 8 and 11). We have therefore modified the LBJ algorithm to allow for endogenous change of debt regime. In particular, the dates at which the regime changes take place (which we will denote by T^{*} for entrepreneurs and T^{**} and households) are solved as equilibrium objects.

4 Effects of a fiscal consolidation

We start our analysis by considering the effects of a fiscal consolidation, abstracting from any other disturbances. This exercise allows us to clarify some important channels of transmission that will play a critical role in the full analysis of fiscal consolidations in a private debt overhang. We pay particular attention to three elements of the model: the fiscal instrument that is used to consolidate (i.e. spending cuts vs tax hikes), the lack of monetary policy response due to the small economy assumption, and, above all, the existence of long-term private debt contracts. We will study first the dynamic effects on GDP and other macroeconomic variables of interest, turning later to the welfare consequences of the shock.

[21] See also Juillard et al. (1998) for an application of the LBJ variant of the Newton-Raphson algorithm.

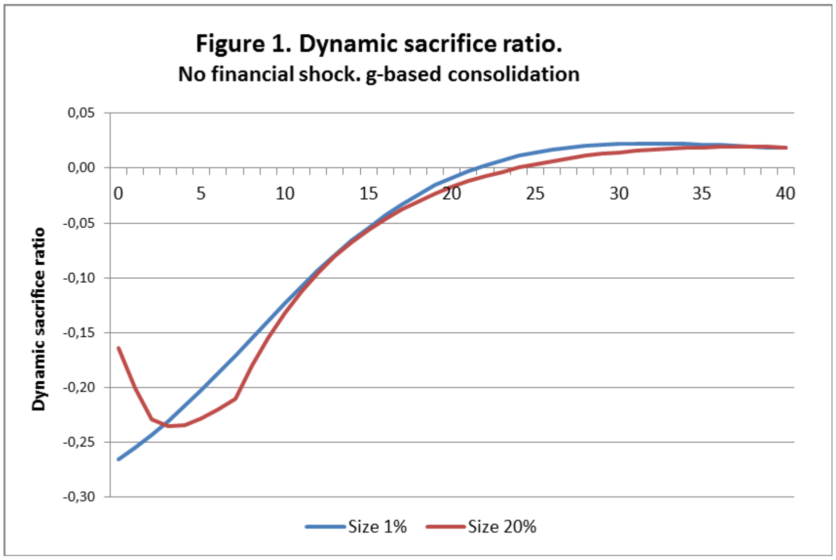
4.1 An intertemporal analysis of the effects of fiscal consolidations

We consider the pure effects of reducing the long-run target for the government debt-to-GDP ratio, $\bar{b}^{gy}$, from its initial value (80% of annual GDP). We will refer to the numerical reduction in such target as the *size* of the fiscal consolidation. The reduction in debt is achieved through the fiscal rule in equation (14). We discuss in detail the effects of a consolidation based on government spending cuts, although we also show the effects of a strategy based on capital income tax hikes to assess the robustness of our results to the adjustment of alternative budgetary instruments. In the rest of the paper, the key outcome variable we will focus on is the *fiscal sacrifice ratio* (Erceg and Lindé, 2013), defined as the change in output (consumption) relative to the size of the fiscal consolidation: $\Delta y_t / \left|\Delta \bar{b}^{gy}\right| \ \left(\Delta c_t / \left|\Delta \bar{b}^{gy}\right|\right)$.[22]

Figure 1 depicts the fiscal sacrifice ratio associated to two different consolidation exercises. The blue line represents the response of output when the fiscal adjustment is designed to reduce the long-run government debt-to-GDP ratio by 1 percentage point (pp), whereas the red line corresponds to a much larger consolidation effort aimed at reducing that ratio by 20 pp. Three features stand out in this figure. First, the fiscal shock lowers GDP in a significant and persistent manner; second, a small scale fiscal adjustment has a stronger short-run (up to $t = 4$) relative effect on output than larger consolidations; and, third, in the medium term this pattern is reversed, as the larger fiscal adjustment gives rise to higher relative output losses.

The sharp and persistent GDP contraction is caused by the rise in the real interest rate and the fall in asset prices and hence in borrowers' collateral values, which makes access to credit more difficult and reduces their spending capacity. The first effect is common to models with a binding zero lower bound (e.g. Erceg and Lindé, 2013). In the absence of monetary accommodation, fiscal multipliers are large. Moreover, they get amplified by the persistent deterioration of borrowers' financial position. These effects, combined with the negative impact on labor income (due both to lower real wages and lower employment) induce a prolonged recession.

[22] In some figures we show a similar ratio referred to consumption: $\Delta c_t / \left|\Delta \bar{b}^{gy}\right|$.



We now turn to the dynamic trade-off between consolidation of different sizes, whereby more severe consolidations produce smaller relative losses in the short-run but larger ones in the medium run. Long-run debt contracts imply that debtors' debt repayments are bounded from above by the contractual amortization rate, which might dampen the impact on their spending capacity of shocks that reduce collateral values relative to outstanding debt. Let us consider constrained households' consumption since the argument applies analogously to entrepreneurs. If the fiscal shock is small, collateral values remain above the contractual amortization path, i.e. *excess collateral* values are positive: $\frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t - \gamma\frac{b_{t-1}}{\pi_t} > 0$. Therefore, borrowers remain in the baseline debt regime, in which they receive new credit subject to the constraint: $b_t \leq \frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t$. In equilibrium, this constraint is binding while in this regime,[23] therefore, the net debt flow for constrained households follows

$$b_t - \frac{R_{t-1}}{\pi_t}b_{t-1} = \frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t - \frac{R_{t-1}}{\pi_t}b_{t-1}.$$

Along this path, the consolidation-driven fall in collateral values, $\frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t$, re-

[23] In particular, we verify that in all our simulations the Kuhn-Tucker multipliers associated to constraints (7) and (10) are strictly positive in all periods in which such constraints are the relevant debt constraint on borrowers.

duces households' net debt flow one for one. This adds to the standard debt-deflation channel operating through the fall in inflation, π_t that increases the real value of the burden of outstanding debt, $\frac{R_{t-1}}{\pi_t}b_{t-1}$. The resulting reduction in net debt flows reduces debtors' spending capacity *ceteris paribus*.

By contrast, if the fiscal consolidation is large enough to drive collateral values below the debt amortization path, then such path becomes the effective debt limit for a number of periods: $b_t \leq \gamma \frac{b_{t-1}}{\pi_t}$. Again, this constraint is binding in equilibrium while this alternative debt regime lasts.[24] Thus, constrained households' net debt flow follows instead

$$b_t - \frac{R_{t-1}}{\pi_t}b_{t-1} = -\left(\frac{R_{t-1} - \gamma}{\pi_t}\right)b_{t-1}.$$

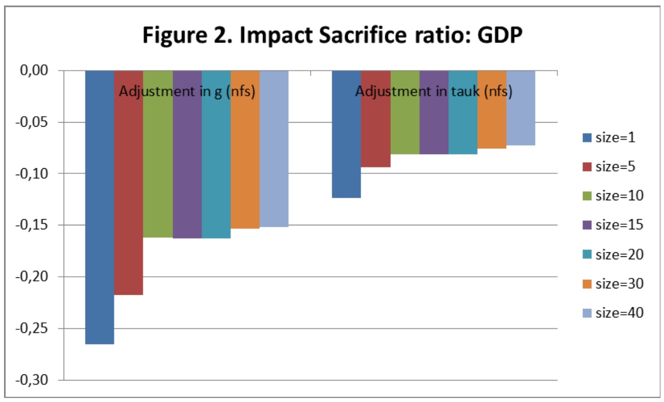
That is *ceteris paribus*, net debt flows and consumption are isolated from collateral values. The only effect of the fiscal consolidation on net debt flows is through the standard debt deflation effect, that in this case is second order since $R_{t-1} - \gamma$ is small. This mechanism, by which long-term debt contracts isolate borrowers spending capacity from the effect of large falls in collateral values, may be referred to as the *'buffering effect'* of long-run debt. Thus, in the short run large consolidations have a *milder* relative effect on output than smaller ones do.

But this is only part of the story. While benefiting from the buffering effect of long-term debt on impact, larger consolidations sharply depress asset prices and collateral values but not so much short run consumption, vis-à-vis smaller ones. That brings indebted consumers into negative equity territory, which in turn requires a prolonged period of spending moderation until positive equity and access to credit are restored. This implies that, if the fiscal consolidation is sufficiently large as to bring the economy on to the deleveraging regime, further increases in the consolidation size will also tend to *prolong* the duration and depth of deleveraging, thus postponing the economic recovery. This '*duration effect*' explains why severe consolidations produce

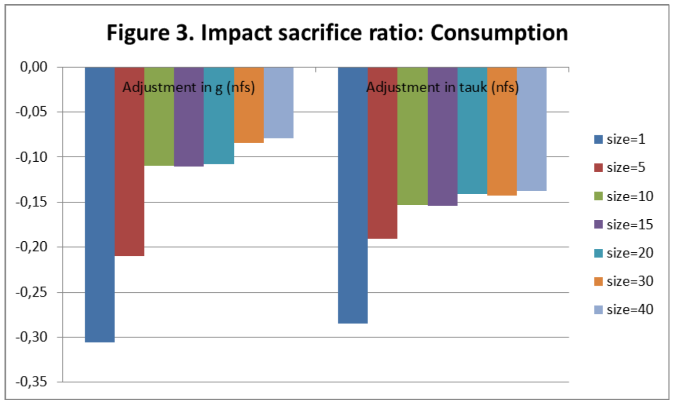
[24] That is, we verify that in all our simulations the Kuhn-Tucker multipliers associated to constraints (8) and (11) are strictly positive in all periods in which such constraints are the relevant debt constraint on borrowers.

larger medium-term relative output losses, as shown in Figure 1.

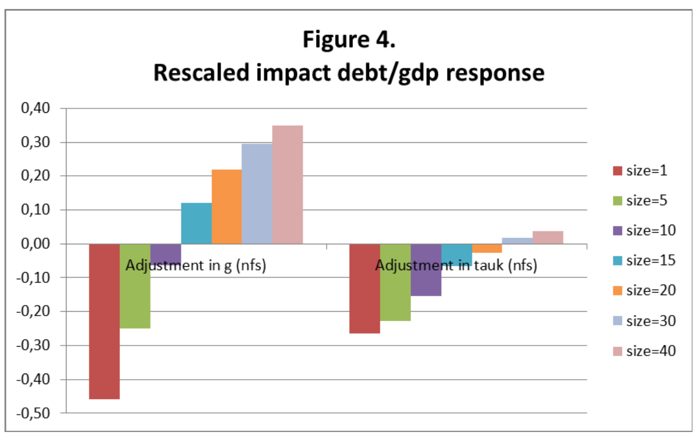
To gain further insight into the costs of fiscal consolidation in the very short-run, Figures 2 and 3 depict the *impact* ($t = 1$) GDP and consumption sacrifice ratios for consolidations of different sizes. For comparison, we also include the same ratios for consolidations implemented through a rise in the capital income tax rate (τ_t^k). In both cases, the impact sacrifice ratios are negative step functions of consolidation size.



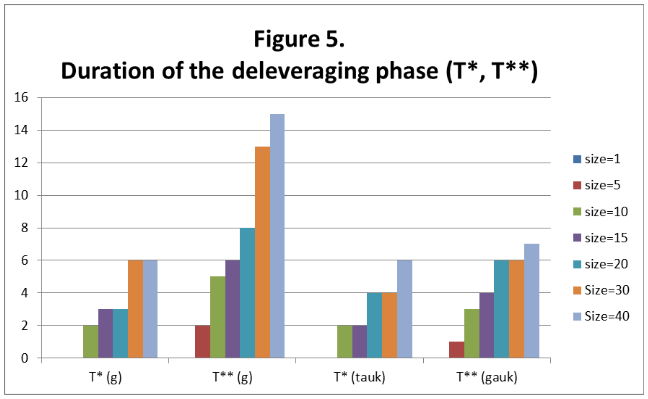
Notice that significantly larger ratios mostly occur when comparing consolidations of relative small size; beyond some threshold (e.g. 10 pp of debt reduction for spending-based consolidations), the impact sacrifice ratios are very similar. As explained before, more intense consolidations have smaller sacrifice ratios in the short run due to the *buffering effect* of long-run debt once the economy enters the deleveraging phase. *Ceteris paribus*, this would tend to make relative output effects decreasing in consolidation size. However, larger fiscal adjustments also postpone the recovery in credit and in economic activity through the 'duration effect' explained above. These anticipated future losses discourage short-run spending by (forward-looking) households and firms. This second effect gains relevance as we increase the size of the fiscal adjustment, thus flattening the step functions in Figures 2 and 3, for consolidation sizes beyond 10 debt-to-GDP pp. Also, these differences among ratios also vanish and even reverse when we look at longer horizons (an effect not shown in the figures), when the *duration effect* dominates the initial *buffering effect*.



The flip side of the consumption pattern lies in the response of private debt for different consolidation sizes. Figure 4 reveals how private deleveraging is more intense (always per unit of fiscal consolidation) on impact for smallish fiscal adjustments, as the latter tend to reduce debt proportionally more than they reduce output. As the size of the consolidation increases and the 'buffering effect' kicks in, the opposite is true, to the point that the private debt ratio actually increases for large enough consolidations.



Finally, Figure 5 displays the effect of consolidation size on the duration of the deleveraging processes. As claimed before, larger consolidations tend to prolong such processes, thus postponing the moment in which borrowers regain access to new credit and hence the economy recovery.

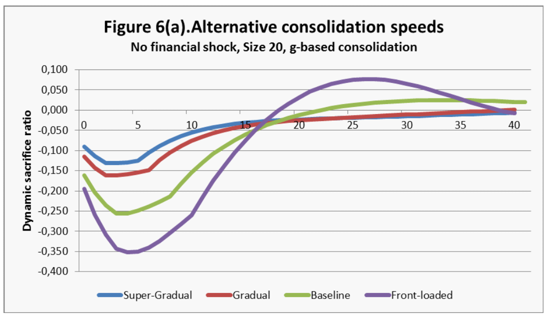


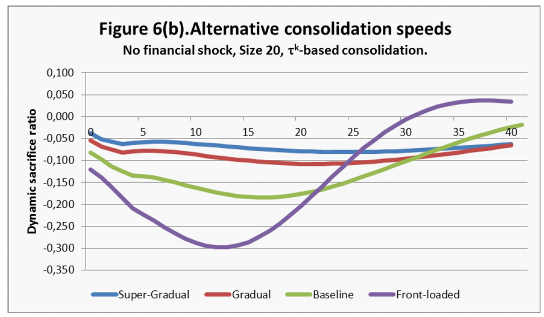
Summing up, fiscal consolidations produce a negative impact on borrowers' assets and income flows, but the evolution of debt in the short run is largely unaffected by the size of the fiscal shock if the latter is big enough. Thus, larger consolidations produce a disproportionately large negative impact on net worth, implying that indebted consumers find themselves with negative equity for a number of periods. Given an initial level of debt, sharp fiscal retrenchments lengthen the deleveraging phase for the private sector until additional spending can be financed with fresh credit. This produces a persistent output loss (relative to the case of a smaller adjustment) and the accumulated medium-term GDP losses are likely to outweigh their short run benefits relative to smaller consolidations.

4.2 Gradualism

Before we proceed to a more comprehensive evaluation of the effect of these alternative fiscal adjustments, it is convenient to look at another dimension of fiscal gradualism that has been much discussed in policy circles: *front loaded* versus *back-loaded* (gradual) fiscal consolidations. So far we have kept constant the **speed** of fiscal consolidation as captured by the (instrument-adjusted) coefficients in the fiscal rule, and focused on differences across consolidation sizes. In this section we fix the consolidation size at 20 percentage points and consider different values of the response coefficient (ϕ_b) to debt deviations from target ($b^{gy}_{t-1} - \bar{b}^{gy}$). In this way we analyze two types of fiscal adjustments: *front-loaded* consolidations for higher values of ϕ_b, and *gradual* consolidations for low values of ϕ_b.[25]

The key question we address is to what extent the gradualism vs. front-loading dimension affects the response of GDP and interacts with the duration of *private-sector deleveraging* (T^{*}, T^{**}). As can be seen in Figure 6, gradualism reduces the short/medium-run costs from fiscal consolidation, but raises the longer-run costs. This holds for both government spending as well as for capital income tax based fiscal retrenchments.





Some authors have defended gradual consolidations, arguing that gradualism helps to avoid large fiscal shocks in the short run, i.e. at the time when the multiplier, under the conditions reflected in our model (indebted agents and zero lower bound), is presumably larger. This argument does not hold in our model. Consistently with our results above, the percentage fall in GDP per unit of change in the instrument (not shown[26]) is decreasing in the size of the shock and hence in the aggressiveness of the consolidation program. Still, front-loaded consolidations are more costly in the short run since we are comparing fiscal adjustments of equal long-run size. For one thing, front-loaded consolidations entail larger fiscal shocks in the early years of the fiscal program; besides, and more importantly, they prolong the duration and the intensity of the deleveraging period. The recession lasts longer in this scenario,

[25] Similar results are obtained if gradualism concerns the smoothing parameter $\rho_{\bar{b}^{gy}}$ in a generalized specification that allows for a time-varying debt ratio target $(\bar{b}_t^{gy})$,

$$\bar{b}_t^{gy} = \rho_{\bar{b}^{gy}} \bar{b}_{t-1}^{gy} + (1 - \rho_{\bar{b}^{gy}}) \bar{b}^{gy}.$$

[26] $\left\{ \left| \frac{\Delta y_t}{\Delta g_t} \right|_{FL} = 0.038, \left| \frac{\Delta y_t}{\Delta g_t} \right|_{BL} = 0.052 \right\}; \left\{ \left| \frac{\Delta y_t}{\Delta \tau_t^k} \right|_{FL} = 0.18, \left| \frac{\Delta y_t}{\Delta \tau_t^k} \right|_{BL} = 0.25 \right\}$

which is reflected in the more intensive fall in private spending of forward looking households and firms. As we can see in Table 2 below, front-loaded consolidation programs delay the exit from the deleveraging phase substantially: relative to a gradual consolidation, it takes 4 additional quarters for households to regain access to new credit in the case of spending-based consolidations, and 2 quarters longer for entrepreneurs when the adjustment rests on capital income taxes.

Milder short run effects of more gradual consolidations are compensated by a higher cost in the long run. Beyond T^* and T^{**}, i.e. once private deleveraging is completed, the fiscal shock continues to affect an economy that has regained access to fresh credit. Slower consolidations postpone the fiscal effort extending the duration of the negative fiscal shock, dragging down the recovery of asset prices and hence of credit flows. In light of this dynamic trade-off, it is useful to summarize the effects of gradualism at different horizons in a single statistic. A natural candidate is social welfare, defined as

$$E_0 \sum_{t=0}^{\infty} \left[(\beta^u)^t U\left(c_t^u, h_t^u, \{n_t^u\}\right) + \beta^t U\left(c_t^c, h_t, \{n_t^c\}\right) + \beta^t \log c_t^e \right],$$

where U is the period utility function of households as introduced in equations (4), (5) and (9). In particular, we compute the *welfare loss* of the fiscal consolidation as the percentage change in permanent consumption that is required to compensate for the effects of the program. Table 2 shows how this welfare loss increases monotonically as we move from more to less gradual fiscal adjustments based on either government spending or capital income taxes. The welfare loss is positively correlated with the duration of the period of private debt adjustment.

Table 2. Gradualism and welfare costs of fiscal consolidation

Instrument:	gov't spending (g_t)	capital income tax (τ_t^k)
Scenario	welf. cost (T^*, T^{**})	welf. cost (T^*, T^{**})
Front-loaded	0.53 (4, 11)	1.19 (4, 5)
Baseline	0.42 (4, 9)	1.07 (3, 5)
Gradual	0.34 (3, 7)	0.88 (2, 5)
Super-gradual	0.30 (3, 6)	0.77 (2, 4)

Note: welfare costs in % of permanent consumption

5 Fiscal consolidation in a financial crisis

We next focus on the empirically more plausible scenario (in light of recent experience of the EMU) in which fiscal consolidation takes place in a context in which private deleveraging is already unfolding as a consequence of a negative financial shock. We subject the model economy to an unexpected, gradual, permanent drop in the LTV ratios of both households and entrepreneurs, m_t and m_t^e respectively. In particular, we assume an autoregressive process for both LTV ratios: $x_t = (1-\rho^x)\,\bar{x} + \rho^x x_{t-1}$, $x = m, m^e$, where we set $\rho^m = \rho^{m^e} = 0.75$. We then simulate an unanticipated fall in the long-run LTV ratios $(\bar{m}, \bar{m}^e)$ of 5 percentage points from their baseline values in Table 1, a conservative choice in the light of recent experience in Spain.[27] This *deleveraging shock* is large enough to get the economy into the "contractual amortization" regime, even in the absence of any fiscal adjustment.

Figure 7 displays the response to the deleveraging shock of collateral values (dashed lines) and the actual equilibrium path of outstanding debt (thick solid lines) of entrepreneurs and households. The deleveraging shock is accompanied by a government spending-based fiscal consolidation, for which we consider two different sizes: 1 pp (red lines), and 20 pp (blue lines). Before these shocks take place ($t = 0$), the

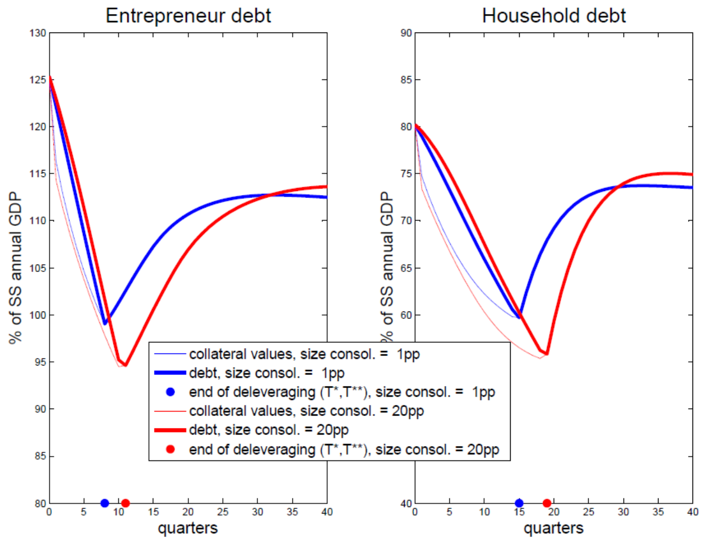
[27] Data from the Spanish Land Registry office shows that average LTV ratios for new mortgages declined by 7.7 percentage points in the 6 years between 2007:Q3 and 2013:Q3.

economy rests in the steady state of the baseline regime, where debt levels equal pledgeable collateral values.[28] The credit crunch shock drives collateral values *below* the contractual amortization paths already on impact ($t = 1$), an effect that is amplified by the fiscal contraction. Therefore, the economy enters the alternative regime in which entrepreneurial and household debt stocks decay at the contractual amortization rates. In this phase, the economy undergoes a gradual and prolonged deleveraging process.

Eventually, collateral values rise again above the contractual amortization path, at which point borrowers are able to regain access to fresh funds (periods T^* and T^{**}). Notice that collateral values and debt both experience a surge at the time of the regime change. This is because real estate becomes again valuable as collateral, which pushes borrowers' demand for real estate up, and hence its price. Thus, T^* and T^{**} also represent the endogenous *duration* of the deleveraging phase for entrepreneurs and households. Deleveraging lasts longer for households than for entrepreneurs, which mainly reflects the slower amortization rate assumed for the former ($1 - \gamma < 1 - \gamma^e$).

[28] Indeed, the fact that constrained households and entrepreneurs are both more impatient than unconstrained households, $\beta < \beta^u$, guarantees that the collateral constraint binds for both agents in the steady state.

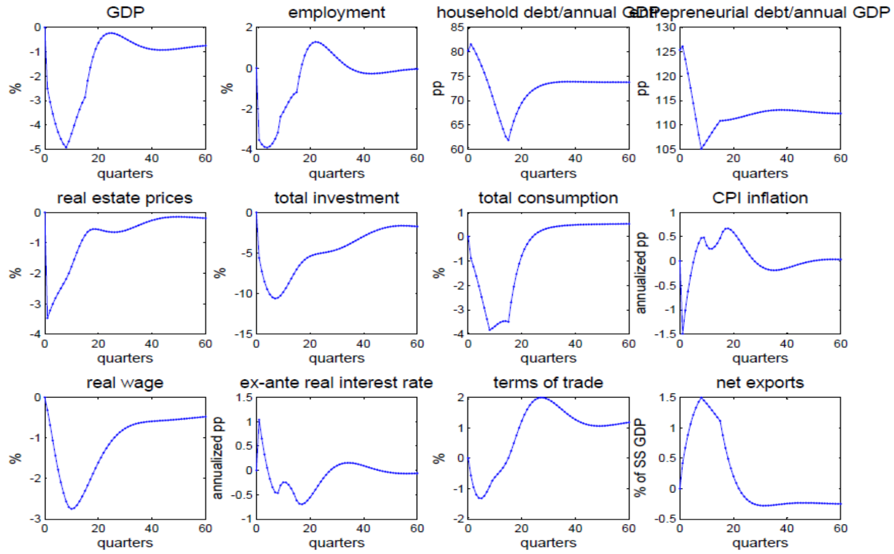
Figure 7: Collateral and debt in a financial crisis



The blue lines in Figure 8 show the economy's response to the deleveraging shock (in the absence of fiscal consolidation). Total consumption declines as a result of the deleveraging process, and then experiences successive recoveries when first entrepreneurs and then households regain access to new loans. The shock has also a negative impact on total investment, driven by lower expenditure in both real estate and equipment capital. Notice that investment recovers from $t = 8$ onwards i.e. before the process of entrepreneurial debt reduction is actually over ($t = 9$). This initial creditless recovery in investment is financed with an increase in borrowers' internal saving.[29]

[29] From $t = 0$ until T^* entrepreneurs reduce their consumption, which in our framework may be interpreted as dividend payments, thus increasing their retained earnings.

Figure 8: Macroeconomic adjustment to a financial crisis

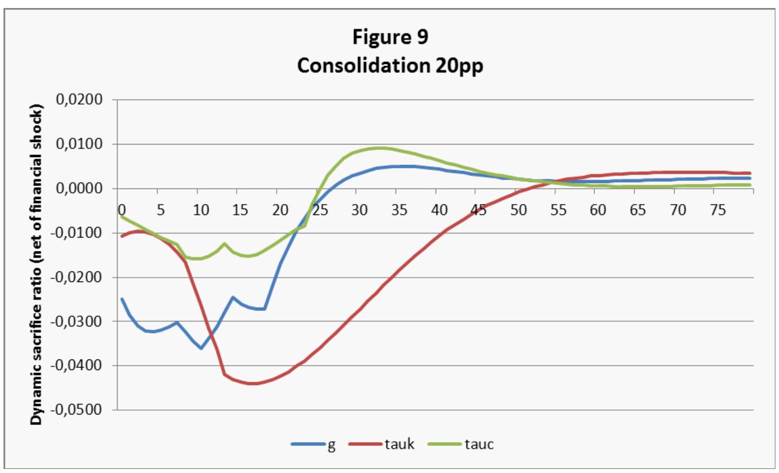


The deflationary process caused by the financial shock leads to a temporary depreciation of the terms of trade, which fosters gross exports. On the other hand, imports fall due to the combined effect of the terms-of-trade depreciation and the severe contraction in domestic demand. Both effects give rise to a substantial improvement in net exports during the deleveraging period. The positive contribution of the external sector, however, is not sufficient to avoid a protracted recession that lasts for 8 quarters. As shown in Andrés, Arce and Thomas (2015), the presence of long-run debt allows the model to replicate well the dynamics of private debt observed during historical deleveraging episodes.

5.1 Alternative instruments

Before proceeding to the evaluation of size and gradualism in fiscal adjustments against the backdrop of the financial contraction, it is worth looking at the comparative effects of a consolidation of a given size (20 pp) based on alternative fiscal instruments: government spending (g_t), consumption tax (τ_t^c), and capital income tax (τ_t^k).[30] Since we have two shocks operating simultaneously, in what follows we calculate the GDP effect of a fiscal consolidation (Δy_t^{fc}) as the difference between the GDP path in the scenario with both deleveraging and fiscal shock ($y_t^{del,fc}$) and that in the deleveraging-shock-only scenario (y_t^{del}): $\Delta y_t^{fc} \equiv y_t^{del,fc} - y_t^{del}$. The *sacrifice ratio* is given by: $\Delta y_t^{fc} / \left|\Delta \bar{b}^{gy}\right|$.

As can be seen in Figure 9, fiscal consolidations deepen the recession caused by the credit contraction.[31] Different instruments give rise to distinct patterns in the impact and medium-run output response. First, consolidations implemented trough adjustments in taxes are less damaging in the short run, than those based on government spending cuts; second, other than the more pronounced fall in GDP following adjustments in g_t, the dynamic pattern of output is very similar to the one following changes in τ_t^c; and, third, the while response of output in the short run is very similar for consolidations based in either τ_t^k or τ_t^c, as the full effect of capital income tax hikes (on investment) gathers momentum, they become the costliest of all.



These patterns can be explained by the evolution of the financial conditions of the agents involved. Eggertsson (2010) and Erceg and Lindé (2013) among others

[30] The effects of consolidations based on labor income taxes (τ_t^w) are relatively similar to those of a consumption tax-based consolidation, and we thus omit them for brevity.

[31] In Figure 9 and subsequent figures, the spikes in the GDP effects reflect fiscally-induced changes in T^* and T^{**} (and hence in the dates at which the consumption spikes associated to the end of deleveraging take place) relative to the baseline no-consolidation scenario.

have explained why government spending cuts might be more damaging than tax hikes, when the monetary policy cannot accommodate the fiscal shock. When the economy is well above the ZLB, tax increases are less deflationary than spending cuts since the negative demand effect common to both is compensated in the case of taxes by increases in the marginal cost. Thus monetary policy accommodation is stronger following spending cuts. By contrast, when the economy is trapped at the *zero lower bound*, the impact of these instruments have the opposite effect on the real rate: spending cuts raise the real rate more than tax hikes do, hence they deliver higher multipliers.

Both g-based and τ^c-based consolidations are similar in shape, since they have a symmetric impact on the duration of deleveraging by households and entrepreneurs: $\Delta T^* = \Delta T^{**}$. Losses caused by the spending cut are bigger because this homothetic effect is nonetheless stronger than in the case of consumption tax rises: $\Delta T^*|_g = \Delta T^{**}|_g = 4$, versus $\Delta T^*|_{\tau^c} = \Delta T^{**}|_{\tau^c} = 1$.

Finally, the instrument whose influence on the dynamics of GDP is truly different is τ^k. This reflects the fact that the adjustment in capital income taxes exerts a disproportionate effect on the deleveraging intensity of entrepreneurs, and hence on investment, as compared with the impact of τ^c and g: $\Delta T^*|_{\tau^k} = 4$, $\Delta T^{**}|_{\tau^k} = 1$. That is, increases in τ_t^k extend entrepreneurs' debt reduction period by four quarters, while that of households barely changes. Thus, while hikes in capital income taxes have mild short-run effects on consumption and output (similar to those of consumption tax increases), by inhibiting the accumulation of collateralizable capital assets (and their market value) they lead to longer and deeper private deleveraging and hence to higher medium-and long-run output losses.

5.2 The size of the fiscal adjustment

We discuss now the output effects of the size of the consolidation program and its time profile under alternative instruments. Figure 10 depicts the *fiscal sacrifice ratio* associated to spending-based consolidations of two different sizes: $\triangledown \bar{b}^{gy} = 1$ pp (blue line), and $\triangledown \bar{b}^{gy} = 20$ pp (red line). Notice first that the main results from the analysis of the fiscal consolidation in isolation are preserved: (i) the fiscal adjustment makes the recession deeper and more persistent (*vis-à-vis* the deleveraging-shock-only scenario), (ii) smaller adjustments have greater relative effects in the short run, and (iii) larger consolidations are more costly in relative terms over the medium run. Again, the last result stems from the fact that larger consolidations *postpone* the end of private deleveraging, as shown previously in Figure 7. Moreover, a quick comparison to Figure 1 reveals that the additional medium-run costs caused by the fiscal consolidation is somewhat higher when the economy is contemporaneously undergoing a private deleveraging process.

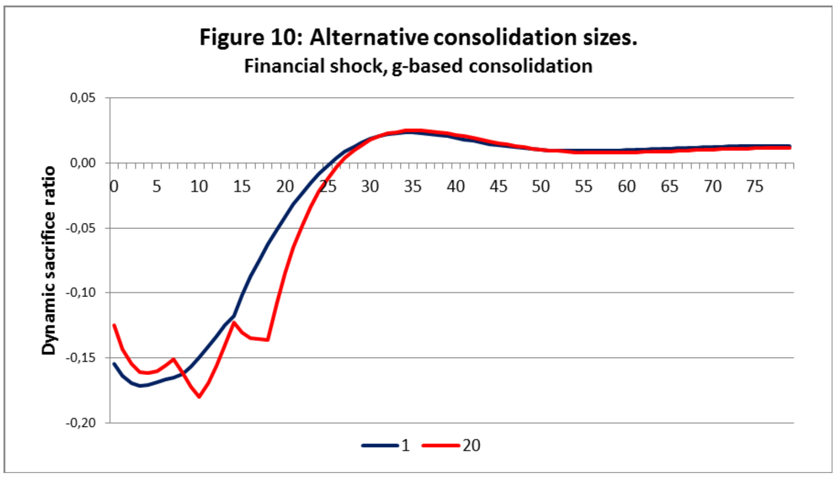
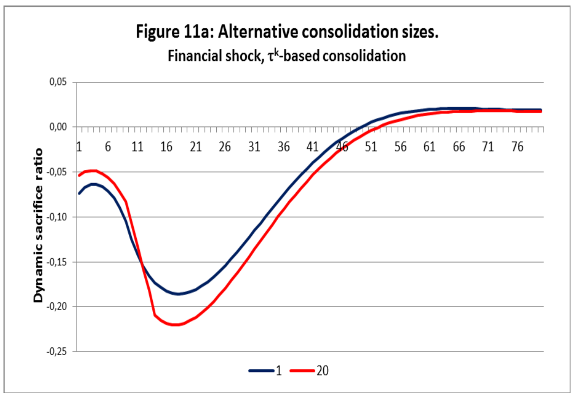
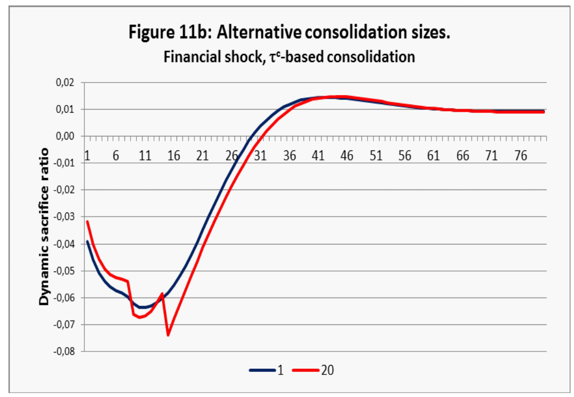


Figure 10: Alternative consolidation sizes.
Financial shock, g-based consolidation

Since the lines in Figure 9 are in differences with respect to the baseline in which there is no fiscal shock, the downward spikes in the red line (size 20) represent the additional GDP loss caused by the increase in the duration of the deleveraging process (T^* and T^{**}). These spikes do not show up in the consolidation of size 1 since the latter is small enough to leave the duration of the deleveraging phase unaffected. On the contrary, the spending cut associated to a target reduction of 20 pp in the debt/GDP ratio delays the exit from deleveraging by four quarters for both entrepreneurs and households. This in turn drags down the recovery of GDP, producing significant and persistent losses over the medium run.

Figure 11 shows the GDP effects of fiscal consolidations based on capital income (Figure 11a) and consumption taxes (Figure 11b). In the figure we appreciate a similar pattern in the comparison across different consolidation sizes: a large consolidation produces (slightly) smaller short-run costs, but persistently higher costs in the medium run.





The economics of these responses is similar to that following an adjustment in government spending. As to the differences between both tax-based consolidations, they reflect their distinct effect on T^* and T^{**}. The downward spikes observed in the case of the consumption tax-based consolidation reflect the fact that T^* and T^{**} are both delayed relative to the no-consolidation scenario. This pattern is very similar to the one observed in the case of g adjustment, although the delay in the deleveraging process is less intense in this case (one quarter here versus one year in the case of g). When consolidations are implemented via adjustments in capital income taxes, we observe a single downward spike associated to the larger consolidation (20 pp). This is due to the fact that the relevant effect of this tax rate, as far as the deleveraging period is concerned, operates mainly via T^*, which is delayed by one year. Thus, as discussed above, it is through lower capital accumulation by entrepreneurs that consolidations based on capital income tax hikes produce persistent output losses.

Finally, we compare the GDP and welfare effects of gradualism in fiscal consolidation. As in the case of no financial shock (Section 4.2), we find that more gradual strategies reduce the output loss in the short/medium run, but tend to increase it in the longer term.[32] We also find that more gradualism reduces the welfare costs of fiscal consolidation, again partly by shortening the duration of the deleveraging phase (reducing T^* and T^{**}). Table 3 summarizes the welfare effects, and the impact on the duration of deleveraging, of consolidation programs that operate at different speeds (as captured by the value of ϕ_b).[33] Welfare losses of fiscal consolidation, relative to the no-consolidation scenario, increase as a larger part of the debt-reduction effort is exerted in the early years of the program.

Table 3. Gradualism and welfare costs of fiscal consolidation in a financial crisis

Instrument:	gov't spending (g_t)	capital income tax (τ_t^k)
Scenario	welf. cost (T^*, T^{**})	welf. cost (T^*, T^{**})
Front-loaded	0.84 (12, 22)	1.50 (14, 14)
Baseline	0.55 (11, 19)	1.26 (13, 14)
Gradual	0.42 (10, 18)	1.00 (11, 14)
Super-gradual	0.36 (10, 17)	0.86 (10, 14)

Note: welfare costs in % of permanent consumption

6 Concluding remarks

We have analyzed the interaction between fiscal consolidation and private sector deleveraging in a small open economy inside a monetary union, thus lacking an independent monetary policy. We have focused on the size, composition and timing of consolidation programs and their interaction with the reduction of private debt that follows a negative financial shock.

We find that very short-run fiscal multipliers are *smaller for large scale* consolidation programs, but medium-run multipliers *increase* with consolidation size. Larger consolidations *delay* the end of deleveraging. Long term private secured (mortgaged)

[32] These results are omitted here for brevity, but they are available upon request.

[33] Again, the results are similar when gradualism is implemented through different speeds in the adjustment of the debt ratio target, $\bar{b}_t^{gy}$. These results are also available upon request.

debt plays a critical role in these results. Contractual amortization limits the speed of the deleveraging process by putting a lower bound to debt repayments since fresh credit cannot be negative. This cushioning effect on private spending after a negative shock is more important, in relative terms, for large consolidation programs thus reducing their associated fiscal sacrifice ratio on impact.

On the flip side, larger consolidations depress the value of collateral and the net worth of borrowers. To the extent that the debt contract prevents a similar downward adjustment in consumption, borrowers' equity plunges into negative territory that takes much longer to be recovered before borrowers have access to fresh credit. This delays the process of debt absorption making the recession longer.

Also front-loaded consolidation programs, that reduce the output loss at the latter stages of the program, delay the recovery, amplifying the short/medium run cost. Despite this trade-off, the welfare-based statistic reveals that more gradual strategies facilitate a faster recovery of private finances and are less costly.

Finally, the lengthening effect on the duration of the recession, along with the lack of nominal interest reaction, explains also the differential GDP effects of fiscal adjustments based on alternative budgetary instruments: public spending, indirect taxes and capital income taxes.

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Appendix

A. Equilibrium conditions

Let $\tilde{p}_t \equiv \tilde{P}_{H,t}/P_{H,t}$, $p_{H,t} \equiv P_{H,t}/P_t$, $w_t \equiv W_t/P_t$, $\tilde{w}_t \equiv \tilde{W}_t/W_t$, $\pi_{wt} \equiv W_t/W_{t-1}$. Equilibrium conditions:

- Unconstrained household budget constraint and first-order conditions (d_t, h_t^u),

$$c_t^u + d_t + p_t^h \left[h_t^u - (1-\delta_h) h_{t-1}^u\right] = \frac{R_{t-1}}{\pi_t} d_{t-1} + (1-\tau_w) w_t n_t^u - T_t, \tag{16}$$

$$\frac{1}{c_t^u} = \beta^u E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^u}, \tag{17}$$

$$\frac{p_t^h}{c_t^u} = \frac{\vartheta}{h_t^u} + \beta^u E_t \frac{(1-\delta_h) p_{t+1}^h}{c_{t+1}^u}. \tag{18}$$

- Constrained household budget constraint, debt constraints, and first-order conditions (b_t, h_t),

$$c_t^c + \frac{R_{t-1}}{\pi_t} b_{t-1} + p_t^h \left[h_t - (1-\delta_h) h_{t-1}\right] = b_t + (1-\tau_w) w_t n_t^c - T_t, \tag{19}$$

$$b_t \leq \begin{cases} R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t \geq \gamma b_{t-1}/\pi_t, \\ \gamma b_{t-1}/\pi_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t < \gamma b_{t-1}/\pi_t, \end{cases} \tag{20}$$

$$\frac{1}{c_t^c} = \beta E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^c} + \xi_t \mathbf{1}(\vartheta_t \geq 0) + \mu_t \mathbf{1}(\vartheta_t < 0) - \beta\gamma E_t \frac{\mu_{t+1}}{\pi_{t+1}} \mathbf{1}(\vartheta_{t+1} < 0), \tag{21}$$

$$\frac{p_t^h}{c_t^c} = \frac{\vartheta}{h_t} + \beta E_t \frac{(1-\delta_h) p_{t+1}^h}{c_{t+1}^c} + \xi_t \mathbf{1}(\vartheta_t \geq 0) \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h, \tag{22}$$

where μ_t is the Lagrange multiplier on constraint (8) in the text, $\mathbf{1}(\cdot)$ is the indicator function and $\vartheta_t \equiv R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t$.

- Entrepreneur budget constraint, debt constraints, and first-order conditions $(b_t^e,\, h_t^e,\, n_t^e,\, k_t)$,

$$\begin{aligned} c_t^e &= mc_t k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{1-\alpha_h-\alpha_k} - w_t n_t^e - p_t^h \left[h_t^e - (1-\delta_h) h_{t-1}^e\right] \\ &\quad + b_t^e - \frac{R_{t-1}}{\pi_t} b_{t-1}^e - q_t \left[k_t - (1-\delta_k) k_{t-1}\right] + \Pi_t^r + \Pi_t^h + \Pi_t^k, \end{aligned} \tag{23}$$

$$b_t^e \leq \begin{cases} R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e, & \text{if } m_t^e R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e \geq \gamma^e b_{t-1}^e / \pi_t, \\ \gamma^e b_{t-1}^e / \pi_t, & \text{if } m_t^e R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e < \gamma^e b_{t-1}^e / \pi_t, \end{cases} \tag{24}$$

$$\frac{1}{c_t^e} = \beta E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^e} + \xi_t^e \mathbf{1}\left(\vartheta_t^e \geq 0\right) + \mu_t^e \mathbf{1}\left(\vartheta_t^e < 0\right) - \beta \gamma^e E_t \frac{\mu_{t+1}^e}{\pi_{t+1}} \mathbf{1}\left(\vartheta_{t+1}^e < 0\right), \tag{25}$$

$$\frac{p_t^h}{c_t^e} = \beta E_t \frac{mc_{t+1} \alpha_h k_t^{\alpha_k} \left(h_t^e\right)^{\alpha_h - 1} \left(n_{t+1}^e\right)^{1-\alpha_h-\alpha_k} + (1-\delta_h) p_{t+1}^h}{c_{t+1}^e} + \xi_t^e \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h \mathbf{1}\left(\vartheta_t^e \geq 0\right), \tag{26}$$

$$w_t = mc_t (1-\alpha_h-\alpha_k) k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{-\alpha_h-\alpha_k}, \tag{27}$$

$$\frac{q_t}{c_t^e} = \beta E_t \frac{mc_{t+1} \alpha_k k_t^{\alpha_k - 1} \left(h_t^e\right)^{\alpha_h} \left(n_{t+1}^e\right)^{1-\alpha_h-\alpha_k} + (1-\delta_k) q_{t+1}}{c_{t+1}^e}, \tag{28}$$

where μ_t^e is the Lagrange multiplier on constraint (8) in the text, and $\vartheta_t^e \equiv R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e - \gamma^e b_{t-1}^e / \pi_t$.

- Retailers' optimal price decision, and aggregate profits,

$$E_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{c_t^e}{c_{t+s}^e} \left[\frac{(1-\tau_p)\tilde{p}_t}{\prod_{j=1}^s \pi_{H,t+j}} p_{H,t+s} - \frac{\varepsilon_p}{\varepsilon_p - 1} mc_{t+s}\right] \left(\frac{\prod_{j=1}^s \pi_{H,t+j}}{\tilde{p}_t}\right)^{\varepsilon_p} y_{t+s} = 0, \tag{29}$$

$$\Pi_t^r = y_t \left((1-\tau_p) p_{H,t} - mc_t \Delta_t\right), \tag{30}$$

- Dynamics of PPI inflation and price dispersion,

$$1 = (1-\theta) \tilde{p}_t^{1-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p - 1}, \tag{31}$$

$$\Delta_t \equiv (1-\theta) \tilde{p}_t^{-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p} \Delta_{t-1}. \tag{32}$$

- Construction firm output, first order conditions (n_t^h, i_t^h), and profits,

$$I_t^h = \left(n_t^h\right)^{\omega} \left\{ i_t^h \left[1 - \frac{\Phi_h}{2} \left(\frac{i_t^h}{i_{t-1}^h} - 1 \right)^2 \right] \right\}^{1-\omega}, \tag{33}$$

$$w_t = p_t^h \omega \left(n_t^h\right)^{\omega-1} \left\{ i_t^h \left[1 - \frac{\Phi_h}{2} \left(\frac{i_t^h}{i_{t-1}^h} - 1 \right)^2 \right] \right\}^{1-\omega}, \tag{34}$$

$$\begin{aligned} 1 \quad = \quad & p_t^h \left(n_t^h\right)^{\omega} (1-\omega) \left\{ i_t^h \left[1 - \frac{\Phi_h}{2} \left(di_t^h\right)^2 \right] \right\}^{-\omega} \left[1 - \frac{\Phi_h}{2} \left(di_t^h\right)^2 - \Phi_h \left(di_t^h\right) \frac{i_t^h}{i_{t-1}^h} \right] \\ & + \beta \frac{\lambda_{t+1}^e}{\lambda_t^e} p_{t+1}^h \left(n_{t+1}^h\right)^{\omega} (1\text{-}\omega) \left\{ i_{t+1}^h \left[1 - \frac{\Phi_h}{2} \left(di_{t+1}^h\right)^2 \right] \right\}^{-\omega} \Phi_h di_{t+1}^h \left(\frac{i_{t+1}^h}{i_t^h} \right)^2 \end{aligned} \tag{35}$$

$$\Pi_t^h = p_t^h I_t^h - w_t n_t^h - i_t^h, \tag{36}$$

for $di_t^h \equiv i_t^h / i_{t-1}^h - 1$.

- Equipment capital producers output, first order condition (i_t), and profits,

$$I_t = i_t \left[1 - \frac{\Phi_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right], \tag{37}$$

$$1 = q_t \left[1 - \frac{\Phi_k}{2} \left(di_t\right)^2 - \Phi_k \left(di_t\right) \frac{i_t}{i_{t-1}} \right] + E_t \frac{\lambda_{t+1}^e}{\lambda_t^e} q_{t+1} \Phi_k di_{t+1} \frac{i_{t+1}^2}{i_t^2}, \tag{38}$$

$$\Pi_t^k = q_t I_t - i_t, \tag{39}$$

for $di_t \equiv i_t / i_{t-1} - 1$.

- Optimal wage decision,

$$\sum_{x=c,u} E_t \sum_{s=0}^{\infty} \left(\beta^x \theta_w\right)^s \left[\frac{(1\text{-}\tau_w)\, \tilde{w}_t}{\prod_{j=1}^{s} \pi_{w,t+j}} \frac{w_{t+s}}{c_{t+s}^x} \text{-} \frac{\varepsilon_w \chi \left(n_{t+s}^x\right)^{\varphi}}{\varepsilon_w\text{-}1} \left(\frac{\tilde{w}_t}{\prod_{j=1}^{s} \pi_{w,t+j}} \right)^{-\varepsilon_w \varphi} \right] \left(\frac{\prod_{j=1}^{s} \pi_{w,t+j}}{\tilde{w}_t} \right)^{\varepsilon_w} n_{t+s}^x = 0, \tag{40}$$

with $\beta^c = \beta$.

- Dynamics of wage inflation and wage dispersion,

$$1 = (1 - \theta_w)\, \tilde{w}_t^{1-\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w - 1}, \tag{41}$$

$$\Delta_t^{w,n} = (1 - \theta_w)\, \tilde{w}_t^{-\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w} \Delta_{t-1}^{w,n}. \tag{42}$$

- Fiscal authority's budget constraint,

$$\tau_w w_t \left(n_t^c + n_t^u\right) + \tau_p p_{H,t} y_t + 2T_t = 0.$$

- Aggregate employment,

$$N_t^c = n_t^c \Delta_t^{w,n}, \tag{43}$$

$$N_t^u = n_t^u \Delta_t^{w,n}, \tag{44}$$

$$N_t = N_t^c + N_t^u, \tag{45}$$

- Export demand,

$$x_t = \zeta \left(p_t^*\right)^{-\varepsilon_F} y_{F,t}. \tag{46}$$

- Intermediate good market clearing,

$$y_t \Delta_t = k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{1-\alpha_h-\alpha_k}, \tag{47}$$

- Labor market clearing,

$$n_t^c + n_t^u = n_t^e + n_t^h. \tag{48}$$

- Consumption goods basket market clearing,

$$y_t = c_{H,t}^c + c_{H,t}^u + c_{H,t}^e + i_{H,t} + i_{H,t}^h + x_t. \tag{49}$$

- Real estate market clearing,

$$h_t + h_t^u + h_t^e = I_t^h + (1 - \delta_h) \left(h_{t-1} + h_{t-1}^u + h_{t-1}^e \right). \tag{50}$$

- Equipment capital market clearing,

$$k_t = (1 - \delta_k) k_{t-1} + I_t. \tag{51}$$

- Real wages,

$$w_t = w_{t-1} \frac{\pi_{wt}}{\pi_t}, \tag{52}$$

- Terms of trade,

$$p_t^* = p_{t-1}^* \frac{\pi_{H,t}}{\pi_{F,t}}. \tag{53}$$

- Relative demand for domestic goods,

$$c_{H,t}^c = \omega_H p_{H,t}^{-\varepsilon_H} c_t^c, \tag{54}$$

$$c_{H,t}^u = \omega_H p_{H,t}^{-\varepsilon_H} c_t^u, \tag{55}$$

$$c_{H,t}^e = \omega_H p_{H,t}^{-\varepsilon_H} c_t^e, \tag{56}$$

$$i_{H,t} = \omega_H p_{H,t}^{-\varepsilon_H} i_t, \tag{57}$$

$$i_{H,t}^h = \omega_H p_{H,t}^{-\varepsilon_H} i_t^h, \tag{58}$$

- Relative demand for constrained/unconstrained household labor,

$$(1 - \mu) n_t^c = \mu n_t^u, \tag{59}$$

  where $\mu \equiv \mu_e = \mu_h$.

- Relative domestic producer prices,

$$p_{H,t} = p_{H,t-1} \frac{\pi_{H,t}}{\pi_t}, \tag{60}$$

- CPI inflation,

$$\pi_t^{1-\varepsilon_H} = \frac{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H}}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H} \pi_{H,t}^{1-\varepsilon_H} + \frac{1-\omega_H}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H}, \tag{61}$$

- Real (PPI-deflated) GDP,

$$gdp_t = y_t + \frac{1}{p_{H,t}} \left(q_t I_t - i_t\right) + \frac{1}{p_{H,t}} \left(p_t^h I_t^h - i_t^h\right), \tag{62}$$

- Gross nominal interest rate,

$$R_t = R^* \exp\left(-\psi \frac{d_t - b_t - b_t^e}{p_{H,t} gdp_t}\right). \tag{63}$$

B. Equilibrium unemployment

Following Galí (2011), we assume that each representative household consists of a unit squared of individuals indexed by $(i, j) \in [0,1] \times [0,1]$, where i represents the variety of labor service provided by the individual and j indexes her disutility from working, given by χj^{φ}. Let $n_t^x(i)$ denote the number of variety-i workers in household $x = c, u$ employed at time t. Total household disutility from working is given by

$$\chi \int_0^1 \int_0^{n_t^x(i)} j^{\varphi} dj di = \chi \int_0^1 \frac{n_t^x(i)^{1+\varphi}}{1+\varphi} di,$$

for $x = c, u$. Given the type-specific wage $W_t(i)$, the number of type-i workers that each household *would like to* send to work is given by

$$\arg\max_{n_t^x(i)} \left\{ \lambda_t^x \frac{W_t(i)}{P_t} n_t^x(i) - \chi \frac{n_t^x(i)^{1+\varphi}}{1+\varphi} \right\} = \left(\frac{\lambda_t^x}{\chi} \frac{W_t(i)}{P_t}\right)^{1/\varphi} \equiv l_t^x(i),$$

for $x = c, u$, where $\lambda_t^x \equiv 1/c_t^x$. Unemployment in the market for type-i labor is just the number of workers willing to work at the going wage minus effective labor demand: $u_t(i) \equiv \sum_{x=c,u} l_t^x(i) - \sum_{x=c,u} n_t^x(i)$ .Let

$$l_t^x \equiv \int_0^1 l_t^x(i)\,di = \left(\frac{\lambda_t^x}{\chi}\frac{W_t}{P_t}\right)^{1/\varphi} \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{1/\varphi} di = \left(\frac{\lambda_t^x}{\chi}\frac{W_t}{P_t}\right)^{1/\varphi} \Delta_t^{w,l},$$

$$N_t^x \equiv \int_0^1 n_t^x(i)\,di = n_t^x \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} di = n_t^x \Delta_t^{w,n},$$

denote total household-specific labor supply and labor demand, respectively, for $x = c, u$, where $\Delta_t^{w,l} \equiv \int_0^1 (W_t(i)/W_t)^{1/\varphi} di$ and $\Delta_t^{w,n} \equiv \int_0^1 (W_t(i)/W_t)^{-\varepsilon_w} di$ are indexes of wage dispersion. Then aggregate unemployment is

$$u_t \equiv \int_0^1 u_t(i)\,di = l_t - N_t.$$

where $l_t \equiv \sum_{x=c,u} l_t^x$ and $N_t \equiv \sum_{x=c,u} N_t^x$ are aggregate labor supply and labor demand, respectively. Finally, the unemployment rate is $u_t^{rate} \equiv u_t/l_t$.