Using Elasticities to Derive Optimal Bankruptcy Exemptions

Eduardo Dávila

NYU Stern

Structural reforms in the wake of recovery: Where do we stand?
Bank of Spain
June 18 2015
Motivation

Question

How large should bankruptcy exemptions be?
Motivation

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- **Exemption**: dollar amount borrower gets to keep if he does not repay
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- **Exemption**: dollar amount borrower gets to keep if he does not repay
- Substantial variation on exemptions across regions/time
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- **Exemption**: dollar amount borrower gets to keep if he does not repay
- Substantial variation on exemptions across regions/time
- This paper
  1. Characterizes the *optimal bankruptcy exemption* and \( \frac{dW}{dm} \)
  - Very generally
Motivation

Question

How large should bankruptcy exemptions be?

- **Exemption**: dollar amount borrower gets to keep if he does not repay

- Substantial variation on exemptions across regions/time

- This paper
  1. Characterizes the **optimal bankruptcy exemption** and $\frac{dW}{dm}$
     - Very generally
  2. As a function of a few measurable **sufficient statistics**
     - Calibrates optimal exemption
Main argument

Risk Averse Borrowers

Borrow from

Risk Neutral Lenders

Key Friction: Debt contract (baseline model)
Incomplete markets (extension)

Second Best:
Bankruptcy exemption provides partial insurance

Marginal Benefit:
More consumption when bankrupt

Marginal Cost:
Higher interest rates

Effects on Borrowing, Default Decision, Labor Supply, Moral Hazard, etc

Optimality do not matter directly
Only through sufficient statistics

Optimal Bankruptcy Exemption

Sufficient statistic logic (CAPM analogy)
Main argument

Risk Averse Borrowers \[\rightarrow\] Borrow from \[\rightarrow\] Risk Neutral Lenders

**First Best:** Full Insurance
Main argument

Risk Averse Borrowers \( \xrightarrow{\text{Borrow from}} \) Risk Neutral Lenders

**First Best:** Full Insurance

Key Friction: Debt contract (baseline model) and incomplete markets (extension).

Second Best: Bankruptcy exemption provides partial insurance.

Marginal Benefit: More consumption when bankrupt.

Marginal Cost: Higher interest rates.

Effects on Borrowing, Default Decision, Labor Supply, Moral Hazard, etc.

Optimality does not matter directly. Only through sufficient statistics.

Optimal Exemption \( m^* \) follows sufficient statistic logic (CAPM analogy).
Main argument

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Exemption ↑

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Marginal Benefit: More consumption when bankrupt
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Optimality: Do not matter directly
Only through sufficient statistics

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Main argument

Risk Averse Borrowers  Borrow from  Risk Neutral Lenders

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Optimal Exemption $m^*$
Main argument

Risk Averse Borrowers  Borrow from  Risk Neutral Lenders

Marginal Benefit:
More consumption when bankrupt

Marginal Cost:
Higher interest rates

Optimal Exemption\[m^*\]

Sufficient statistic logic
(CAPM analogy)
Outline of the talk

1. **Baseline model**
   - Positive analysis
   - Welfare analysis $\Rightarrow \frac{dW}{dm}$ and $m^*$ (**main results**)

2. **Extensions**

3. **Calibration**

4. **Conclusion**
Environment (baseline model)

- Two dates $t = 0, 1$

$\max_{B_0} U(C_0) + \beta E\left[\max\{U(C_{D1}), U(C_{N1})\}\right], C_0 = y_0 + q_0(B_0, m)$

$C_{N1} = y_1 - B_0, C_{D1} = \min\{y_1, m\}$

1. Stochastic endowment $y_1$ (assets), cdf $F(\cdot)$, support on $[y_1, y_1]$

2. Debt contract (key friction)

3. Constant bankruptcy exemption: $m$ dollars

4. Regularity conditions on $F(\cdot)$ and preferences

5. Equilibrium: borrowers internalize $q_0(B_0, m)$

- Risk neutral lenders

- Required return $1 + r^*$, fraction $\delta$

- Zero profit

Eduardo Dávila (NYU Stern) Optimal Bankruptcy Exemptions
Environment (baseline model)

- Two dates $t = 0, 1$
- Risk averse borrowers
Environment (baseline model)

- Two dates $t = 0, 1$
- Risk averse borrowers

- Risk neutral lenders
Environment (baseline model)

- Two dates $t = 0, 1$
- Risk averse borrowers

- Risk neutral lenders
  - Required return $1 + r^*$, fraction $\delta$ deadweight loss in bankruptcy
  - Zero profit
Environment (baseline model)

- Two dates $t = 0, 1$
- Risk averse borrowers

$$\max_{B_0} U(C_0) + \beta \mathbb{E} \left[ \max \{ U(C_1^D), U(C_1^N) \} \right]$$

$$C_0 = y_0 + q_0(B_0, m) B_0 \quad C_1^N = y_1 - B_0 \quad C_1^D = \min \{ y_1, m \}$$

1. Stochastic endowment $y_1$ (assets), cdf $F(\cdot)$, support on $[y_1, \overline{y_1}]$
2. Debt contract (key friction)

- Risk neutral lenders
  - Required return $1 + r^*$, fraction $\delta$ deadweight loss in bankruptcy
  - Zero profit
Environment (baseline model)

- Two dates $t = 0, 1$
- Risk averse borrowers

$$\max_{B_0} U(C_0) + \beta \mathbb{E} \left[ \max \{ U(C^{D}_1), U(C^{N}_1) \} \right],$$

$$C_0 = y_0 + q_0(B_0, m) B_0 \quad C^{N}_1 = y_1 - B_0 \quad C^{D}_1 = \min \{ y_1, m \}$$

1. Stochastic endowment $y_1$ (assets), cdf $F(\cdot)$, support on $[y_1, \bar{y}_1]$
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3. Constant bankruptcy exemption: $m$ dollars

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Environment (baseline model)

- Two dates \( t = 0, 1 \)
- Risk averse borrowers

\[
\max_{B_0} U(C_0) + \beta \mathbb{E} \left[ \max \{ U(C^{D}_1), U(C^{N}_1) \} \right],
\]

\[
C_0 = y_0 + q_0(B_0, m) B_0 \quad C^{N}_1 = y_1 - B_0 \quad C^{D}_1 = \min \{ y_1, m \}
\]

1. Stochastic endowment \( y_1 \) (assets), cdf \( F(\cdot) \), support on \([y_1, \overline{y}_1]\)
2. Debt contract (key friction)
3. Constant bankruptcy exemption: \( m \) dollars
4. Regularity conditions on \( F(\cdot) \) and preferences

- Risk neutral lenders
  - Required return \( 1 + r^* \), fraction \( \delta \) deadweight loss in bankruptcy
  - Zero profit
Environment (baseline model)

- Two dates $t = 0, 1$
- **Risk averse borrowers**

$$\max_{B_0} U(C_0) + \beta \mathbb{E} \left[ \max \{ U(C_{D}^{1}) , U(C_{N}^{1}) \} \right] ,$$

$$C_0 = y_0 + q_0(B_0, m) B_0 \quad C_{N}^{1} = y_1 - B_0 \quad C_{D}^{1} = \min \{ y_1, m \}$$

1. Stochastic endowment $y_1$ (assets), cdf $F(\cdot)$, support on $[y_1, \bar{y_1}]$
2. Debt contract (key friction)
3. Constant bankruptcy exemption: $m$ dollars
4. Regularity conditions on $F(\cdot)$ and preferences
5. Equilibrium: borrowers internalize $q_0(B_0, m)$

- **Risk neutral lenders**
  - Required return $1 + r^*$, fraction $\delta$ deadweight loss in bankruptcy
  - Zero profit
Borrowers’ problem

- Two economic decisions
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- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
  - Period $t = 0$: Borrowing $B_0$
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)

![Diagram showing the relationship between $C_1$, $y_1$, $B_0$, and $y_1$ with the equation $C_1^N = y_1 - B_0$.]
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)

\[
C_1^N = y_1 - B_0
\]

\[
C_1^D = \min\{y_1, m\}
\]
Borrowers’ problem

- Two economic decisions
  - Period \( t = 1 \): Default (given \( B_0 \))
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)

\[
C_1^{\mathcal{N}} = y_1 - B_0
\]

\[
C_1^{\mathcal{D}} = \min\{y_1, m\}
\]

Forced Default

Strategic Default
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
  - Period $t = 0$: Borrowing $B_0$

\[
U'(C_0) \left[ q_0(B_0, m) + \frac{\partial q_0(B_0, m)}{\partial B_0} B_0 \right] = \beta \int_{m+B_0}^{\overline{y_1}} U'(y_1 - B_0) \, dF(y_1)
\]
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
  - Period $t = 0$: Borrowing $B_0$

\[
U'(C_0) \left[ q_0(B_0, m) + \frac{\partial q_0(B_0, m)}{\partial B_0} B_0 \right] = \beta \int_{m+B_0}^{y_1} U'(y_1 - B_0) dF(y_1)
\]

- Marginal Benefit: funds raised at $t = 0$ (accounting for price impact)
- Marginal Cost: repayment at $t = 1$ only if no default
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
  - Period $t = 0$: Borrowing $B_0$

\[
U'(C_0) \left [ q_0(B_0, m) + \frac{\partial q_0(B_0, m)}{\partial B_0} B_0 \right ] = \beta \int_{m+B_0}^{y_1} U'(y_1 - B_0) dF(y_1)
\]

- Marginal Benefit: funds raised at $t = 0$ (accounting for price impact)
- Marginal Cost: repayment at $t = 1$ only if no default
- Characterizes equilibrium borrowing (as a function of $m$)

\[B_0(m)\]
Borrowers’ problem

- Two economic decisions
  - Period $t = 1$: Default (given $B_0$)
  - Period $t = 0$: Borrowing $B_0$

\[
U' (C_0) \left[ q_0 (B_0, m) + \frac{\partial q_0 (B_0, m)}{\partial B_0} B_0 \right] = \beta \int_{m+B_0}^{\bar{y}_1} U' (y_1 - B_0) dF (y_1)
\]

- Marginal Benefit: funds raised at $t = 0$ (accounting for price impact)
- Marginal Cost: repayment at $t = 1$ only if no default
- Characterizes equilibrium borrowing (as a function of $m$)

\[
B_0(m)
\]

\[
\frac{dB_0}{dm} \geq 0 \quad \text{(income, substitution and direct effects)}
\]
Lenders’ interest rate schedule

- Risk neutral pricing

\[ q_0(B_0, m) = \delta \int_m^{m+B_0} \frac{y_1-m}{B_0} dF(y_1) + \int_{m+B_0}^{\bar{y}_1} dF(y_1) \]

\[ 1 + r^* \]

- More borrowing ⇒ Higher interest rates
- Higher exemptions ⇒ Higher interest rates
Lenders’ interest rate schedule

- Risk neutral pricing

\[ q_0(B_0, m) = \frac{\delta \int_m^{m+B_0} \frac{y_1-m}{B_0} dF(y_1) + \int_{m+B_0}^{y_1} dF(y_1)}{1 + r^*} \]

- Properties

\[ \frac{\partial q_0(B_0, m)}{\partial B_0} < 0 \quad \text{and} \quad \frac{\partial q_0(B_0, m)}{\partial m} < 0 \]

- More borrowing ⇒ Higher interest rates
- Higher exemptions ⇒ Higher interest rates
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

### Marginal change in exemption

$$\frac{dW}{dm} = U'(C_0) \frac{\partial q_0}{\partial m} B_0 + \int_{m}^{m+B_0} \beta U'(C_1^D) dF(y_1)$$

- **Marginal Cost:** more expensive borrowing
- **Marginal Benefit:** more consumption when bankrupt

### Intuition
- $\frac{dB_0}{dm}$ and changes in default decision do not appear
- Borrowing and default are done **optimally**
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

$$\frac{dW}{dm} \left\{ \frac{U'(C_0)C_0}{1} \right\} = \frac{-\Lambda \varepsilon \tilde{r},m}{m \frac{C_1^D}{C_0}} + \frac{1}{m \frac{C_1^D}{C_0}}$$

Marginal change in exemption
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

Marginal change in exemption

$$\frac{dW}{dm} \frac{U'(C_0)C_0}{W(C_0)} = -\Lambda \varepsilon \tilde{r},m + \frac{1}{m} \frac{\Pi_m \{ C^D_1 \}}{C_0}$$

Marginal Cost

Marginal Benefit

$$\Lambda \equiv \frac{q_0 B_0}{y_0 + q_0 B_0}$$

Leverage

$$\varepsilon \tilde{r},m \equiv \frac{\partial \log (1 + r)}{\partial m}$$

Interest rate sensitivity
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

Marginal change in exemption

$$\frac{dW}{dm} \frac{U'(C_0)C_0}{-\Lambda\bar{r},m} + \frac{1}{m} \frac{\Pi_m \{C_1^D\}}{C_0}$$

Price-Consumption ratio

$$\Pi_m \frac{\{C_1^D\}}{C_0} \equiv \int_{m}^{m+B_0} \frac{C_1^D}{C_0} \frac{\beta U'(C_1^D)}{U'(C_0)} dF(y_1)$$
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

Marginal change in exemption

\[
\frac{dW}{dm} \frac{U'(C_0)}{C_0} = -\Lambda \epsilon_{\tilde{r},m} + \frac{1}{m} \frac{\Pi_m \{ C_1^D \}}{C_0}
\]

Marginal Cost

Marginal Benefit

\[
\Pi_m \{ C_1^D \} \equiv \int_m^{m+B_0} \frac{C_1^D}{C_0} \frac{\beta U'(C_1^D)}{U'(C_0)} dF(y_1)
\]

Price-Consumption ratio

- Cash Flow and Discount Rate effects
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

**Marginal change in exemption**

$$\frac{dW}{dm} \left( \frac{U'(C_0)}{C_0} \right) = -\Lambda \epsilon \tilde{r}, m + \frac{1}{m} \frac{\Pi_m \left\{ C_1^D \right\}}{C_0}$$

- Marginal Cost
- Marginal Benefit

$$\Pi_m \left\{ \frac{C_1^D}{C_0} \right\} \equiv \int_{m+B_0}^{m+B_0 + B_0} \frac{C_1^D}{C_0} \frac{\beta U'(C_1^D)}{U'(C_0)} dF(y_1)$$

- Price-Consumption ratio

- Cash Flow and Discount Rate effects
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

Marginal change in exemption

$$\frac{dW}{dm} \left| \frac{U'(C_0)C_0}{C_0} \right. = \left. \frac{-\Lambda \varepsilon \tilde{r},m}{m} \right|_{\text{Marginal Cost}} + \left. \frac{1}{m} \frac{\Pi_m \left\{ C_1^D \right\}}{C_0} \right|_{\text{Marginal Benefit}}$$

$$\Pi_m \left\{ C_1^D \right\} \equiv \int_{m}^{m+B_0} \frac{C_1^D}{C_0} \frac{\beta U'(C_1^D)}{U'(C_0)} dF(y_1) \quad \text{Price-Consumption ratio}$$

- Same formula for P/D ratios as in Consumption-Based AP
Main Result 1: Marginal Change (directional test)

- Social welfare $W(m)$ is given by borrowers utility

Marginal change in exemption

$$\frac{dW}{dm} = \frac{-\Lambda \varepsilon \tilde{r}_m}{U'(C_0) C_0} \underbrace{\Pi_m \{C_1^D\} \over C_0}_{Marginal Cost} + \frac{1}{m} \underbrace{\Pi_m \{C_1^D\} \over C_0}_{Marginal Benefit}$$

$$\Pi_m \{C_1^D\} \equiv \int_m^{m+B_0} \frac{C_1^D}{C_0} \frac{\beta U'(C_1^D)}{U'(C_0)} dF(y_1) \quad Price-Consumption\ ratio$$

- CRRA Utility:

$$\Pi_m \{C_1^D\} \equiv \beta \int_m^{m+B_0} \left( \frac{C_1^D}{C_0} \right)^{1-\gamma} dF(y_1)$$
Main Result 2: Optimal Exemption

\[ \frac{dW}{dm} = 0 \text{ (under regularity conditions)} \Rightarrow m^* \]
Main Result 2: Optimal Exemption

- \( \frac{dW}{dm} = 0 \) (under regularity conditions) \( \Rightarrow m^* \)

Optimal exemption

\[
m^* = \frac{\Pi_m \{ C_1^D \}}{C_0} \frac{\Lambda \varepsilon_{\tilde{r},m}}{\Lambda \varepsilon_{\tilde{r},m}}
\]
Main Result 2: Optimal Exemption

• \( \frac{dW}{dm} = 0 \) (under regularity conditions) \( \Rightarrow m^* \)

**Optimal exemption**

\[
m^* = \frac{\Pi_m \{C_1^D\}}{C_0} \frac{\Lambda e^{\tilde{r},m}}{\Lambda e^{\tilde{r},m}}
\]

• Remarks

1. High \( \Lambda e^{\tilde{r},m} \) and Low \( \Pi_m \{C_1^D\} \) \( \Rightarrow \) Low \( m \) (and vice versa)

2. Variables are endogenous and observable
   • Sufficient statistic logic (CAPM analogy)
   • Similar to optimal taxation problems

3. Exact expression (no approximations)
Main Result 2: Optimal Exemption

- \( \frac{dW}{dm} = 0 \) (under regularity conditions) \( \Rightarrow m^* \)

Optimal exemption

\[
m^* = \frac{\Pi_m \{C_1^D\}}{C_0} \frac{C_0}{\Lambda \varepsilon_{\tilde{r},m}}
\]

Remarks

1. High \( \Lambda \varepsilon_{\tilde{r},m} \) and Low \( \frac{\Pi_m \{C_1^D\}}{C_0} \) \( \Rightarrow \) Low \( m \) (and viceversa)
Main Result 2: Optimal Exemption

- \( \frac{dW}{dm} = 0 \) (under regularity conditions) \( \Rightarrow \) \( m^* \)

Optimal exemption

\[
m^* = \frac{\Pi_m \{ C_1^P \} }{C_0 \Lambda \varepsilon_{\tilde{r},m}}
\]

Remarks

1. High \( \Lambda \varepsilon_{\tilde{r},m} \) and Low \( \frac{\Pi_m \{ C_1^P \} }{C_0} \) \( \Rightarrow \) Low \( m \) (and viceversa)
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Main Result 2: Optimal Exemption

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Optimal exemption

\[
m^* = \frac{\Pi_m \{ C_1^D \}}{C_0} \frac{C_0}{\Lambda \varepsilon_{\tilde{r},m}}
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• Remarks

1. High \( \Lambda \varepsilon_{\tilde{r},m} \) and Low \( \frac{\Pi_m \{ C_1^D \}}{C_0} \) \( \Rightarrow \) Low \( m \) (and viceversa)
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   - Sufficient statistic logic (CAPM analogy)
   - Similar to optimal taxation problems
3. Exact expression (no approximations)
Figure: Social welfare

\[ W(m) \]

Exemption \( m \)

Welfare

65 70 75 80 85 90

Welfare

Exemption \( m \)
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
5. Heterogeneous borrowers
Extensions

1. Endogenous labor income and effort choice (moral hazard)

2. Non-pecuniary losses

3. Epstein-Zin utility

4. Multiple contracts

5. Heterogeneous borrowers
   - Social welfare function \( W = \int \lambda(i) W(i) dG(i) \)

\[
\begin{align*}
m^* &= \frac{\mathbb{E}_{G,A} \left[ \Pi_{m,i} \left\{ \frac{C^{D}_{1i}}{C_{0i}} \right\} \right]}{\mathbb{E}_{G,A} \left[ \Lambda_{i \in \tilde{r}_i,m} \right]}
\end{align*}
\]

- \( \mathbb{E}_{G,A}[\cdot] \) are cross sectional averages (for active borrowers)
- **Observed** heterogeneity: exclusion
- **Unobserved** heterogeneity: pooling and exclusion
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
5. Heterogeneous borrowers
6. Price taking borrowers
7. Bankruptcy exemptions contingent on aggregate risk
8. Endogenous income: labor wedges and aggregate demand
9. Dynamics
Calibration

- Baseline model with CRRA

\[ m^* = \frac{\beta \pi_m \left( \frac{C_1}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}} \]
Calibration

- Baseline model with CRRA
- Further assumptions
  - Fully secured collateralized credit $\Rightarrow \varepsilon_{\tilde{r}_{\text{collat}},m} = 0$
  - Borrowers own a house (to use variations in homestead exemptions)

$$m^* = \frac{\beta \pi_m \left( \frac{C_1}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$
Calibration

- Baseline model with CRRA
- Further assumptions
  - Fully secured collateralized credit $\Rightarrow \varepsilon_{\text{collat},m} = 0$
  - Borrowers own a house (to use variations in homestead exemptions)
- Validity of calibration
  - $m^*$ assumes that right hand side variable are constant $\Rightarrow$ approximation error
  - $\frac{dW}{dm}$ more accurate and useful for policymaker (simple test)
Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_D}{C_0}\right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$/$C_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption change</td>
<td>0.9</td>
<td>Filer, Filer 05 PSID</td>
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<tr>
<td>$\Lambda$</td>
<td></td>
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<td>Leverage</td>
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<td>Livshits, MacGee, Tertilt AER07</td>
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<td>$\pi_D$</td>
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<td></td>
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<tr>
<td>Default probability (ch.7)</td>
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<td>$\pi_m</td>
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<td>No-Asset bankruptcy</td>
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<td>$\varepsilon_{\tilde{r},m}$</td>
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<tr>
<td>Credit spread sensitivity</td>
<td>2.5 $\cdot$ 10$^{-7}$</td>
<td>Gropp, Scholz, White QJE97</td>
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<td>$\beta$</td>
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<td>Discount factor</td>
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<td>$\gamma$</td>
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<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>1, 5, 10, 20, 50</td>
<td></td>
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**Calibration**

$$m^* = \frac{\beta \pi_m \left( \frac{C^D_1}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

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Calibration

\[ m^* = \frac{\beta \pi_m \left( \frac{C_D}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}} \]

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- Debt to personal income \( \frac{q_0 B_0}{y_0} = 8.4\% \Rightarrow \Lambda \)
 Calibration

\[
m^* = \frac{\beta \pi_m \left( \frac{C^1}{C^0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}
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\[ \pi_m = \pi_D \times \pi_m|D \left( \text{zero recovery by unsecured creditors} \approx 90\% \right) \]
Calibration

\[ m^* = \frac{\beta \pi_m \left( \frac{C_D^1}{C_0} \right)^{1-\gamma}}{\Lambda \tilde{\epsilon}_{r,m}} \]

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- Increasing \( m \) by $100,000, increases credit spread by 250 basis points
Calibration

\[ m^* = \beta \pi_m \left( \frac{C^D_1}{C^D_0} \right)^{1-\gamma} \Lambda \tilde{\varepsilon}_{r,m} \]

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- Optimal exemption $m^*$ (in dollars):

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Calibration

- Optimal exemption $m^*$ (in dollars):

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  - Average exemption US: $70,000
Calibration

- Optimal exemption $m^*$ (in dollars):

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  - MA,NV homestead exemption: $500,000; NJ none
  - Average exemption US: $70,000
- Size of welfare gains at $70,000 using $\frac{dW}{dm}$ formula ($\$10,000$ change)

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<td>-0.0084%</td>
<td>-0.0027%</td>
<td>0.0089%</td>
<td>0.062%</td>
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</tbody>
</table>
1. **Sufficient Statistics/Public Economics:** Diamond 98, Saez 01, Shimer-Werning 07, Chetty 09, Arkolakis et al 12

2. **General Equilibrium with Incomplete Markets:** Zame 93, Dubey-Geanakoplos-Shubik 05

3. **Quantitative Literature:**
   - **Structural:** Chatterjee-Corbae-Nakajima-Rios-Rull 07, Livshits-MacGee-Tertilt 07,
   - **Microeconometric:** Gross-Souleles 02, Fay-Hurst-White 02, Fan-White 03, Severino-Brown-Coates 14

4. **Security Design:** Ross 76, Allen-Gale 94, Duffie-Rahi 95

5. **Optimal Contracting:** many papers
Conclusion

1. This paper has characterized optimal bankruptcy exemptions
   - As a function of a few **sufficient statistics**
   - For a wide range of environments
   - Sensible calibration (measurement is challenging)

2. New paper on mortgage design (with John Campbell)
   - Optimal recourse
   - ARM vs FRM
Figures: Interest rate schedule

$q_0(B_0, m)$

Price vs. Loan Size $B_0$

Price vs. Exemption $m$
Figures: Borrowers’ choices

$J(B_0, m)$

$B_0(m)$
Extensions

1. Endogenous labor income and effort choice (moral hazard)
Extensions

1. Endogenous labor income and effort choice (moral hazard)
   • Same expression for $m^*$ if disutility of labor is separable (frictionless labor markets)
   • Different characterization of default region: does not matter for $m^*$ (insight applies more generally)
   • Intuition: optimality
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
Extensions

1. Endogenous labor income and effort choice (moral hazard)

2. Non-pecuniary losses

   • Same expression for $m^*$
   • Effects work through spreads in $\varepsilon_{\tilde{r},m}$ and default regions
   • Implicit assumption: no renegotiation
   • Easy to add externalities/internalities
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
   - Same expression for $m^*$

$$
\Pi_m \left\{ \frac{C_1^D}{C_0} \right\} \equiv \left( \frac{Q}{C_0} \right)^{\gamma-\frac{1}{\psi}} \beta \int_{m+B_0} \left( \frac{C_1^D}{C_0} \right)^{1-\gamma} dF(y_1)
$$
- Where $Q$ is the certainty equivalent of $t = 1$ consumption
Extensions

1. Endogenous labor income and effort choice (moral hazard) \( m \) Hazard
2. Non-pecuniary losses Utility loss
3. Epstein-Zin utility Epstein-Zin
   - Same expression for \( m^* \)

\[
\Pi_m \left\{ \frac{C_1^D}{C_0} \right\} \equiv \left( \frac{Q}{C_0} \right)^{\gamma - \frac{1}{\psi}} \beta \int_{m}^{m+B_0} \left( \frac{C_1^D}{C_0} \right)^{1-\gamma} dF(y_1)
\]

- Where \( Q \) is the certainty equivalent of \( t = 1 \) consumption
- Corrected Stochastic Discount Factor
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
Extensions

1. Endogenous labor income and effort choice (moral hazard)  
2. Non-pecuniary losses  
3. Epstein-Zin utility  
4. Multiple contracts
   - Optimal exemption

\[ m^* = \frac{\Pi_m \{ C_1^P \}}{C_0} \frac{\sum_{j=1}^{J} \Lambda_j \varepsilon \tilde{r}_j, m}{\sum_{j=1}^{J} \Lambda_j} \]

\[ \Lambda_j \equiv \frac{q_{0j} B_{0j}}{y_0 + \sum_{j=1}^{J} q_{0j} B_{0j}} \]
Extensions

1. Endogenous labor income and effort choice (moral hazard)
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\[ \Lambda_j \equiv \frac{q_{0j} B_{0j}}{y_0 + \sum_j q_{0j} B_{0j}} \]

- Complete markets as special case \( \Rightarrow m^* = 0 \)
Extensions

1. Endogenous labor income and effort choice (moral hazard)  
2. Non-pecuniary losses  
3. Epstein-Zin utility  
4. Multiple contracts  
5. Heterogeneous borrowers
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
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- Social welfare function \( W = \int \lambda(i) W(i) dG(i) \)
Extensions

1. Endogenous labor income and effort choice (moral hazard)  
2. Non-pecuniary losses  
3. Epstein-Zin utility  
4. Multiple contracts  
5. Heterogeneous borrowers

- Social welfare function \( W = \int \lambda (i) W (i) \, dG (i) \)

\[
m^* = \frac{E_{G, A} \left[ \Pi_{m, i} \left\{ C_{1i}^{D} \right\} \right]}{E_{G, A} \left[ \Lambda_i \varepsilon \tilde{r}_i, m \right]}
\]

- \( E_{G, A} [\cdot] \) are cross sectional averages (for active borrowers)
- **Observed** heterogeneity: exclusion
- **Unobserved** heterogeneity: pooling and exclusion
Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
2. Non-pecuniary losses Utility loss
3. Epstein-Zin utility Epstein-Zin
4. Multiple contracts Multiple
5. Heterogeneous borrowers Heterogeneous
6. Price taking borrowers Price taking
   - Euler equation

\[ U'(C_0) q_0 = \beta \int_{\phi m + B_0}^{y_1} U'(C_{1N}) dF(y_1) \]
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
5. Heterogeneous borrowers
6. Price taking borrowers

- Euler equation

\[ U'(C_0) q_0 = \beta \int_{\phi m + B_0}^{y_1} U'(C_1^N) dF(y_1) \]

- Optimal exemption

\[ m^* = \frac{\Pi_m \{ C_1^D \}}{C_0 \Lambda \tilde{e}_{\tilde{r},m}} \]

where

\[ \varepsilon_{\tilde{r},m} = \frac{d \log(1 + r)}{dm} \]
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
5. Heterogeneous borrowers
6. Price taking borrowers
7. Bankruptcy exemptions contingent on aggregate risk

- Aggregate shocks $\omega \in \Omega$, we can condition exemptions on those

$$m^*(\omega) = \frac{\Pi m(\omega) \{C^D_1\}}{\frac{C_0}{\Lambda \tilde{e}_{\tilde{r}, m(\omega)}}}, \quad \forall \omega$$
Extensions

1. Endogenous labor income and effort choice (moral hazard)  
2. Non-pecuniary losses 
3. Epstein-Zin utility 
4. Multiple contracts 
5. Heterogeneous borrowers 
6. Price taking borrowers 
7. Bankruptcy exemptions contingent on aggregate risk 
8. Endogenous income: labor wedges and aggregate demand

\[ m^* = \frac{\Pi_m \{ C_1^D \}}{C_0} + \frac{\Pi_N \{ \tau(\omega) \frac{dY(\omega)}{dm} \}}{C_0} \]

\[ \Lambda \varepsilon \tilde{r},m \]

- Where \( 1 + \tau(\omega) = \frac{w_1(\omega)}{A} \) is the labor wedge
Extensions

1. Endogenous labor income and effort choice (moral hazard)
2. Non-pecuniary losses
3. Epstein-Zin utility
4. Multiple contracts
5. Heterogeneous borrowers
6. Price taking borrowers
7. Bankruptcy exemptions contingent on aggregate risk
8. Endogenous income: labor wedges and aggregate demand
9. Dynamics

\[ m^* = \frac{\sum_{t=1}^{T} \Pi_{m,t} \{ C_t^D \}}{C_0} \]

\[ \frac{\sum_{t=0}^{T-1} \Pi_{N,t} \{ g_t \Lambda t \epsilon_{\tilde{r},t,m} \}}{\sum_{t=0}^{T-1} \Pi_{N,t} \{ g_t \Lambda t \epsilon_{\tilde{r},t,m} \}} \]

- Simple formula interpreted as \textit{steady state}
- Income process subsumed in sufficient statistics (permanent vs. transitory shocks, health shocks, family shocks, etc...)
Sign of \( \frac{dB_0}{dm} \)

\[
\text{sign} \left( U'' (C_0) \frac{\partial q_0}{\partial m} B_0 \left[ q_0 + \frac{\partial q_0}{\partial B_0} B_0 \right] + U' (C_0) \left[ \frac{\partial q_0}{\partial m} + \frac{\partial^2 q_0}{\partial B_0 \partial m} B_0 \right] \right.

\]

\[
+ \beta U' (m) f (m + B_0) \right)
\]
$m^*$ in (almost) closed form

\[ m^* = \left( \frac{\beta (1 + r^*)}{\Upsilon + \delta (1 - \Upsilon)} \right)^{\frac{1}{\gamma}} C_0 \]

\[ \Upsilon = \frac{f (m^* + B_0) B_0}{F (m^* + B_0) - F (m^*)} \]

- $\Upsilon = 1$ for uniform (measure of curvature)
Moral Hazard/Elastic Labor Supply

• Borrowers’ problem

$$\max_{C_0, \{C_1\}_{y_1}, B_0, \{\xi\}_{y_1}, N_0, \{N_1\}_{y_1}, a} U (C_0, N_0; a) + \beta \mathbb{E}_a [U (C_1, N_1)]$$

s.t. \hspace{1cm} C_0 = y_0 + w_0 N_0 + q_0 B_0

$$C_1^N = y_1 + w_1 N_1^N - B_0; \hspace{0.5cm} C_1^D = \min \{y_1, m\}$$

• Two new optimality conditions

$$\frac{\partial U}{\partial a} (C_0, N_0; a) + \beta \int V (C_1, N_1) f_a (y_1; a) \, dy_1 = 0$$

$$w_0 \frac{\partial U}{\partial C} (C_0, N_0; a) = - \frac{\partial U}{\partial N} (C_0, N_0; a)$$

• Same \(m^*\)
Moral Hazard/Elastic Labor Supply: Default Decision

\[ U(y_1; 0) \quad \tilde{V}(y_1; B^b_0, w_1) \]

\[ U(m) \quad U(\cdot) \quad \tilde{V}(y_1; B^b_0, w_1) \]

\[ y_1 \quad m \quad \tilde{y}_1 \quad \overline{y}_1 \quad \text{Income} \]
Non-Pecuniary Losses

- Assumes that renegotiation is not possible
- Non-pecuniary loss: $U(\phi C)$, where $\phi \in [0, 1]$
- Same $m^*$
- Only default region changes

\[
\text{if } y_1 < \phi m + B_0 \quad \text{Default}
\]
Borrowers Internalize Price Response

• Borrowers optimality condition

\[ U'(C_0) \left[ q_0 + \frac{\partial q_0}{\partial B_0} B_0^b \right] = \beta \int_{y^1}^{y_1} U'(C_1) \, dF(y_1) \]

• Optimal exemption

\[ m^* = \frac{\Pi_m \{ C_1^D \}}{C_0} \frac{\Lambda \left( \varepsilon_{\tilde{r},m} + \varepsilon_{B_0,m} \hat{\varepsilon}_{q_0,B_0} \right)}{\varepsilon_{B_0,m} \equiv \frac{dB_0}{dm}, \hat{\varepsilon}_{q_0,B_0} \equiv \frac{\partial q_0}{\partial B_0^b} B_0} \]

where \( \varepsilon_{B_0,m} \equiv \frac{dB_0}{dm}, \hat{\varepsilon}_{q_0,B_0} \equiv \frac{\partial q_0}{\partial B_0^b} B_0 \)
Epstein-Zin

- Only difference: stochastic discount factor
- $m^*$ doesn't change but

\[
\frac{\Pi_m \{ C^D_1 \}}{C_0} \equiv \left( \frac{Q}{C_0} \right)^{\gamma - \frac{1}{\psi}} \beta \int_{m}^{m+B_0} \left( \frac{C^D_1}{C_0} \right)^{1-\gamma} dF(y_1)
\]

- $Q$ is the certainty equivalent of consumption

\[
Q \equiv \left( \int_{y_1}^{m} (y_1)^{1-\gamma} dF(y_1) + \int_{m}^{m+B_0} (m)^{1-\gamma} dF(y_1) + \int_{m+B_0}^{y_1} (y_1 - B_0) \right)
\]
Multiple Arbitrary Contracts

- Budget constraints

\[ C_0 = y_0 + \sum_{j=1}^{J} q_{0j} (B_{01}, \ldots, B_{0J}, m) B_{0j} \]

\[ C_1^N = y_1 + \sum_{j=1}^{J} \max \{ -z_j(y_1) B_{0j}, 0 \} - \sum_{j=1}^{J} \max \{ z_j(y_1) B_{0j}, 0 \} \]

\[ C_1^D = \min \left\{ y_1 + \sum_{j=1}^{J} \max \{ -z_j(y_1) B_{0j}, 0 \}, m \right\} \]

- Weighted sum of elasticities

\[ m^* = \frac{\prod_{m} \{ C_1^D \}}{C_0} \frac{1}{\sum_{j=1}^{J} \Lambda_j \tilde{r}_j, m} \]
Multiple Arbitrary Contracts

\[ C_1 = \min \{ y_1 + \sum_{j=1}^{J} \max \{-z_j(y_1)B_{0j}, 0\}, m \} \]

\[ C_1^p = \min \{ y_1 + \sum_{j=1}^{J} \max \{-z_j(y_1)B_{0j}, 0\} \} \]

\[ C_1^N = y_1 + \sum_{j=1}^{J} \max \{-z_j(y_1)B_{0j}, 0\} - \sum_{j=1}^{J} \max \{-z_j(y_1)B_{0j}, 0\} \]
Heterogeneous borrowers (observed heterogeneity)

\[ q_{0i}(B_{0i}, m) = \begin{cases} \frac{1}{1+r^*}, & B_{0i} \leq 0 \\ \frac{\delta \int_{m}^{m+B_{0i}} \frac{y_{1i}-m}{B_{0i}} dF_i(y_{1i}) + \int_{m+B_{0i}}^{y_{1i}} dF_i(y_{1i})}{1+r^*}, & B_{0i} > 0 \end{cases} \]

\[ I_A(m) = \{ i | B_{0i}(m) > 0 \}, \quad \text{Active borrowers} \]

\[ I_N(m) = \{ i | B_{0i}(m) = 0 \}, \quad \text{Inactive borrowers} \]
Heterogeneous borrowers (observed heterogeneity)

\[ q_0(B_0, m) \]

\[ J(B_0, m) \]

Back to text
Heterogeneous borrowers (observed heterogeneity)

\[ q_{0i}(B_0, m) = \begin{cases} \frac{1}{1+r^*}, & B_{0i} \leq 0 \\ \int_{I_A(m)} \frac{\tilde{q}_{0i}(B_0, m)}{\int_{I_A(m)} dG(i)} dG(i), & B_{0i} > 0, \end{cases} \]

where

\[ \tilde{q}_{0i}(B_0, m) = \delta \int_{m}^{m+B_0} \frac{y_{1i}-m}{B_0} dF_i(y_{1i}) + \int_{m+B_0}^{y_{1i}} dF_i(y_{1i}) \]

\[ 1 + r^* \]
• Now we have aggregate shocks $\omega \in \Omega$, we can condition the exemption level on those.

• Optimal exemption

\[ m^*(\omega) = \frac{\Pi_{m(\omega)} \{ C_1^P \}}{C_0 \Lambda \varepsilon_{\tilde{r},m(\omega)}}, \quad \forall \omega \]
Dynamics

- Borrowers maximize $\max \mathbb{E} \left[ \sum_{t=0}^{T} \beta^t U(C_t) \right]$
- Recursively $V_{ND,0}(B_{-1}, y_0; m) = \max_{B_0} U(C_0) + \beta \mathbb{E} \left[ \max \left\{ V_{ND,1}(B_0, y_1; m), V_{D,1}(y_1; m) \right\} \right]$
- Optimal exemption

$$m^* = \frac{\sum_{t=1}^{T} \Pi_m \left\{ \frac{C_t^D}{C_0} \right\}}{\sum_{t=0}^{T-1} \Pi_{ND} \left\{ g_t \Lambda_t \bar{\varepsilon}_{\tilde{r}_t}, m \right\}}$$

- Easy to allow for default before bankruptcy, wage garnishments, exclusion period
Evolution filings US

Bankruptcy Filings per Fiscal Year (Ends Sept 30th)

- Personal
- Business

Source: U.S. Courts

http://www.calculatedriskblog.com/
## Parameters

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\gamma = 10$</th>
<th>$\psi = 1.5$</th>
<th>$\beta = 0.96$</th>
<th>$r^* = 4%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endowments</td>
<td>$y_0 = 55$</td>
<td>$\mu = 4.9$</td>
<td>$\sigma = 0.095$</td>
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</tr>
<tr>
<td>Bankruptcy</td>
<td>$\delta = 0.1$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Parameters numerical example
Expression for welfare

\[ W(m) =
\]
\[ U(y_0 + q_0(B_0(m), m)B_0(m)) +
\]
\[ + \beta \left[ \int_{y_1}^{m} U(y_1) dF(y_1) + \int_{m}^{m+B_0(m)} U(m) dF(y_1) + \int_{m+B_0(m)}^{y_1} U(y_1 - B_0(m)) dF(y_1) \right] \]