Using Elasticities to Derive Optimal Bankruptcy Exemptions

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NYU Stern

Structural reforms in the wake of recovery: Where do we stand?
Bank of Spain
June 18 2015

Motivation

Question

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  1. Characterizes the **optimal bankruptcy exemption** and $\frac{dW}{dm}$
     - Very generally

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- Substantial variation on exemptions across regions/time
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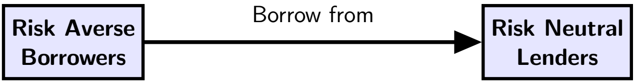
  1. Characterizes the **optimal bankruptcy exemption** and $\frac{dW}{dm}$
     - Very generally
  2. As a function of a few measurable **sufficient statistics**
     - Calibrates optimal exemption

Main argument



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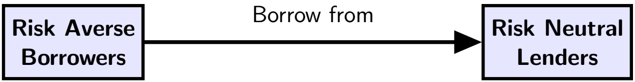

First Best: Full Insurance

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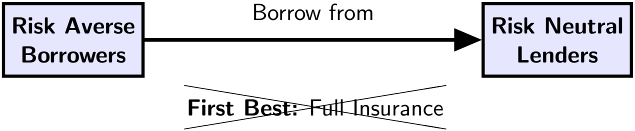

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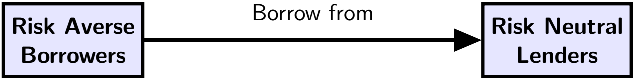

Key Friction: **Debt contract** (baseline model)
Incomplete markets (extension)

Second Best: Bankruptcy exemption provides partial insurance

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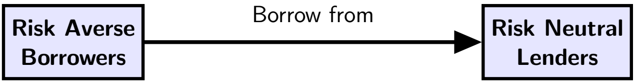

Exemption ↑

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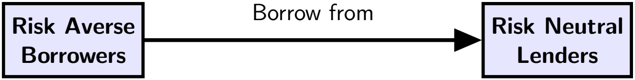

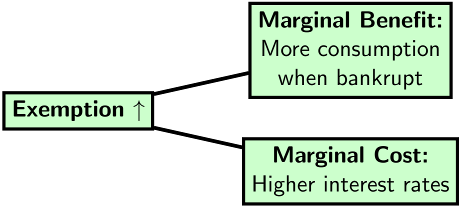

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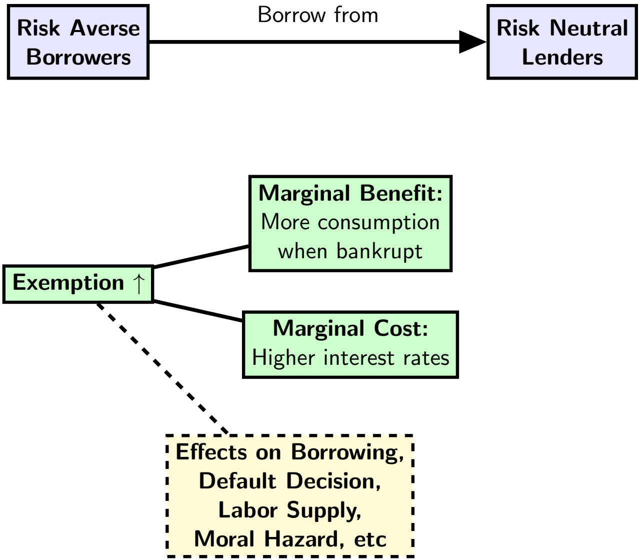

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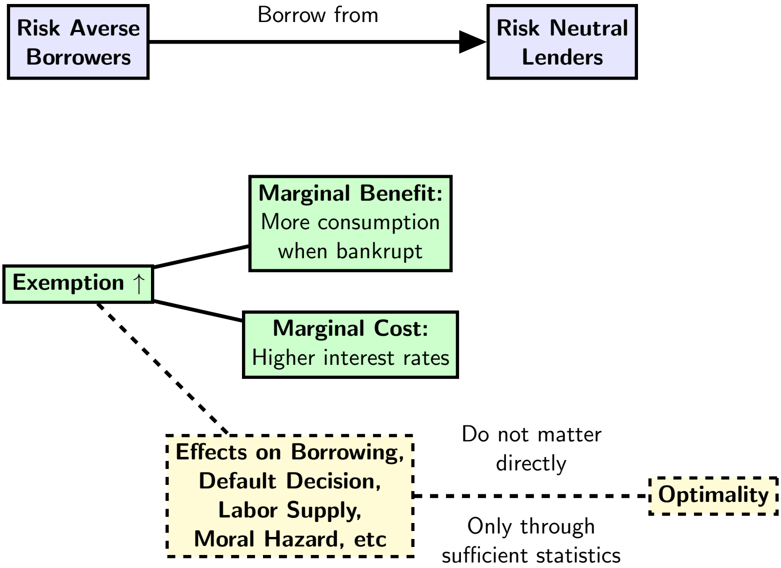

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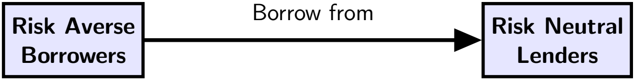

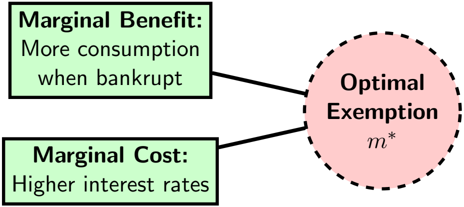

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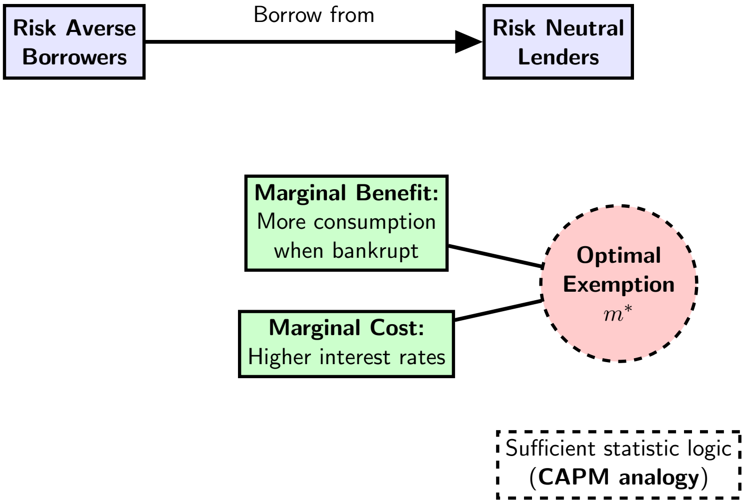

Outline of the talk

1. **Baseline model**
   - Positive analysis
   - Welfare analysis $\Rightarrow \frac{dW}{dm}$ and m^* **(main results)**
2. **Extensions**
3. **Calibration**
4. **Conclusion**

Environment (baseline model)

- Two dates $t = 0, 1$

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$$\max_{B_0} U(C_0) + \beta \mathbb{E}\left[\max\left\{U\left(C_1^{\mathcal{D}}\right), U\left(C_1^{\mathcal{N}}\right)\right\}\right],$$

$$C_0 = y_0 + q_0(B_0, m) B_0 \qquad \begin{aligned} C_1^{\mathcal{N}} &= y_1 - B_0 \\ C_1^{\mathcal{D}} &= \min\{y_1, m\} \end{aligned}$$

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4. Regularity conditions on $F(\cdot)$ and preferences
5. Equilibrium: borrowers internalize $q_0(B_0, m)$

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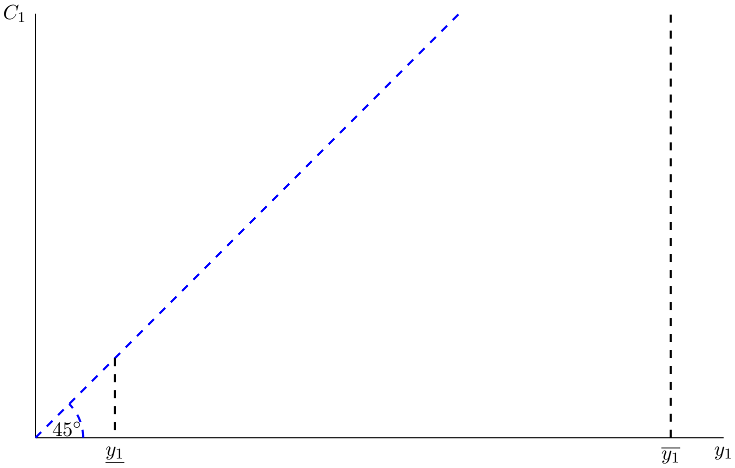

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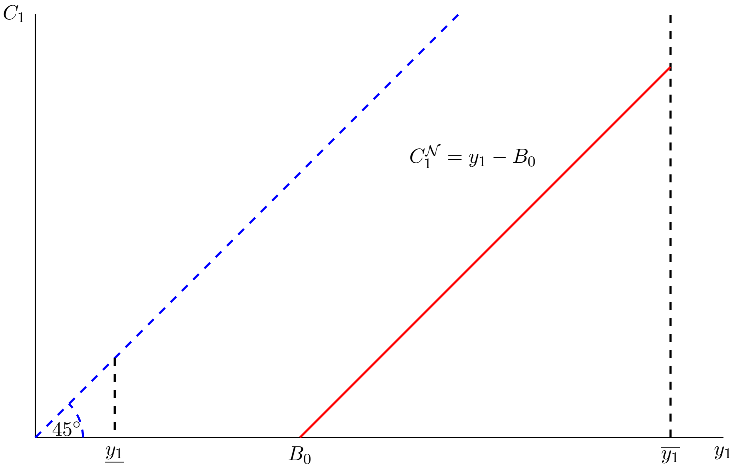

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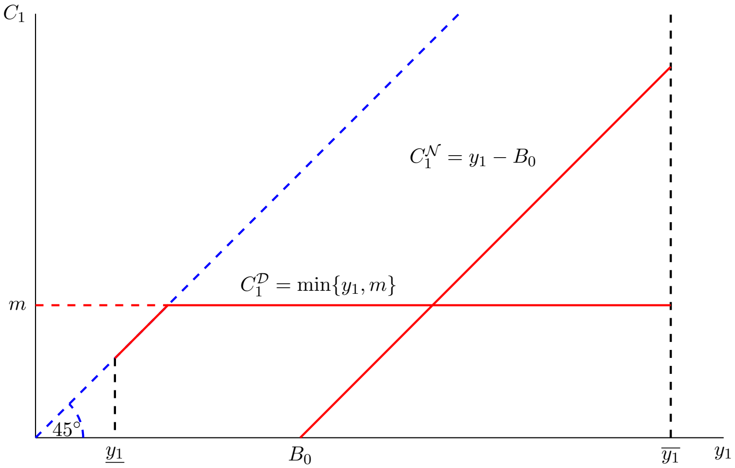

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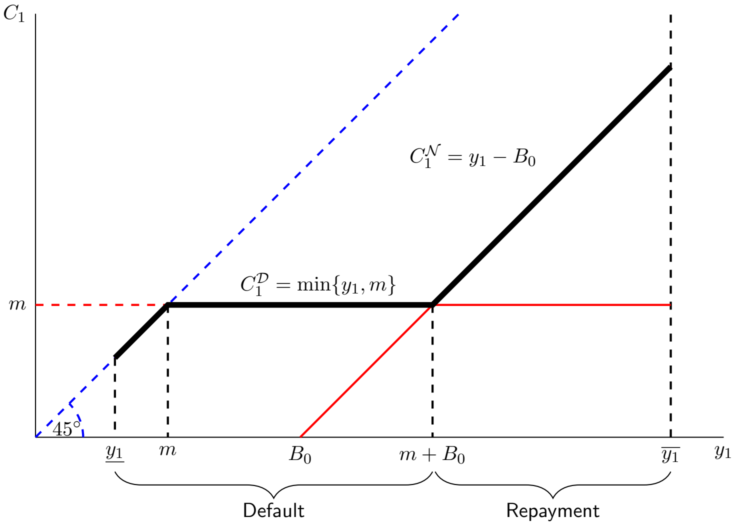

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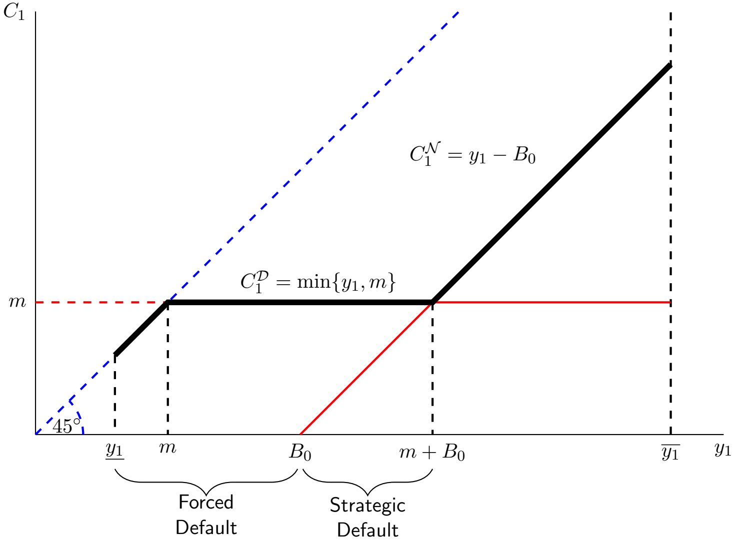

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$$\frac{dB_0}{dm} \gtreqless 0 \quad \text{(income, substitution and direct effects)}$$

Lenders' interest rate schedule

- Risk neutral pricing

$$q_0(B_0, m) = \frac{\delta \int_m^{m+B_0} \frac{y_1 - m}{B_0} dF(y_1) + \int_{m+B_0}^{\overline{y_1}} dF(y_1)}{1 + r^*}$$

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- Properties

$$\frac{\partial q_0(B_0, m)}{\partial B_0} < 0 \qquad \text{and} \qquad \frac{\partial q_0(B_0, m)}{\partial m} < 0$$

- More borrowing ⇒ Higher interest rates
- Higher exemptions ⇒ Higher interest rates

Main Result 1: Marginal Change (directional test)

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Marginal change in exemption

$$\frac{dW}{dm} = \underbrace{U'(C_0)\frac{\partial q_0}{\partial m}B_0}_{\text{Marginal Cost: more expensive borrowing}} + \underbrace{\int_m^{m+B_0} \beta U'\left(C_1^{\mathcal{D}}\right) dF(y_1)}_{\text{Marginal Benefit: more consumption when bankrupt}}$$

- **Intuition**
  - $\frac{dB_0}{dm}$ and changes in default decision do not appear
  - Borrowing and default are done **optimally**

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Marginal change in exemption

$$\frac{\frac{dW}{dm}}{U'\left(C_0\right)C_0} = \underbrace{-\Lambda\varepsilon_{\tilde{r},m}}_{\text{Marginal Cost}} + \underbrace{\frac{1}{m}\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0}}_{\text{Marginal Benefit}}$$

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$$\Lambda \equiv \frac{q_0 B_0}{y_0 + q_0 B_0} \qquad \text{Leverage}$$

$$\varepsilon_{\tilde{r},m} \equiv \frac{\partial \log(1+r)}{\partial m} \qquad \text{Interest rate sensitivity}$$

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- Same formula for P/D ratios as in Consumption-Based AP

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- CRRA Utility:

$$\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0} = \beta\int_m^{m+B_0}\left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma}dF(y_1)$$

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     - Similar to optimal taxation problems

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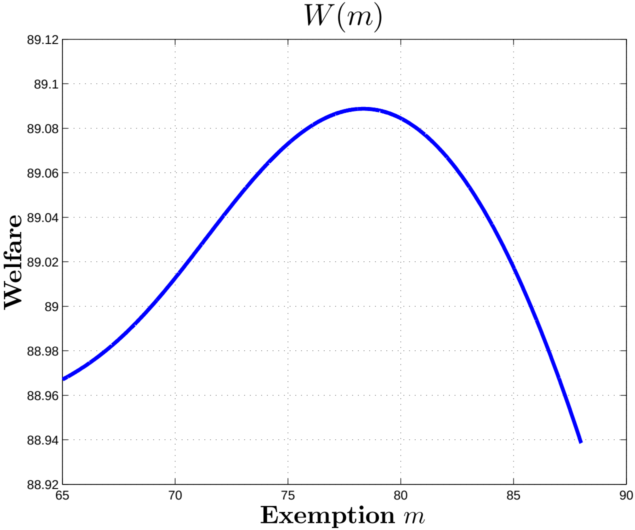
     - Similar to optimal taxation problems
  3. Exact expression (no approximations)

Figure: Social welfare



Extensions

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2. Non-pecuniary losses Utility loss
3. Epstein-Zin utility Epstein-Zin
4. Multiple contracts Multiple
5. Heterogeneous borrowers Heterogeneous

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5. Heterogeneous borrowers Heterogeneous
   - Social welfare function $W = \int \lambda(i) W(i) dG(i)$

$$m^* = \frac{\mathbb{E}_{G,A}\left[\frac{\Pi_{m,i}\left\{C_{1i}^{\mathcal{D}}\right\}}{C_{0i}}\right]}{\mathbb{E}_{G,A}\left[\Lambda_i \varepsilon_{\tilde{r}_i,m}\right]}$$

   - $\mathbb{E}_{G,A}[\cdot]$ are cross sectional averages (for active borrowers)
   - **Observed** heterogeneity: exclusion
   - **Unobserved** heterogeneity: pooling and exclusion

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6. Price taking borrowers Price taking
7. Bankruptcy exemptions contingent on aggregate risk Aggregate Risk
8. Endogenous income: labor wedges and aggregate demand Agg. demand
9. Dynamics Dynamics

Calibration

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  - Fully secured collateralized credit $\Rightarrow \varepsilon_{\tilde{r}_{\text{collat}},m} = 0$
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  - Borrowers own a house (to use variations in homestead exemptions)
- Validity of calibration
  - m^* assumes that right hand side variable are constant $\Rightarrow$ approximation error
  - $\frac{dW}{dm}$ more accurate and useful for policymaker (**simple test**)

Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

Parameter/Variable		Value	Source
$\frac{C_1^{\mathcal{D}}}{C_0}$	Consumption change	0.9	Filer, Filer 05 PSID
Λ	Leverage	0.0775	Livshits, MacGee, Tertilt AER07
π_D	Default probability (ch.7)	0.008	"
$\pi_{m\|D}$	No-Asset bankruptcy	0.1	Lupita ABILawReview12
$\varepsilon_{\tilde{r},m}$	Credit spread sensitivity	$2.5 \cdot 10^{-7}$	Gropp, Scholz, White QJE97
β	Discount factor	0.96	
γ	Risk aversion	$1, 5, 10, 20, 50$	

Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

Parameter/Variable		Value	Source
$\frac{C_1^{\mathcal{D}}}{C_0}$	Consumption change	0.9	Filer, Filer 05 PSID
Λ	Leverage	0.0775	Livshits, MacGee, Tertilt AER07
π_D	Default probability (ch.7)	0.008	"
$\pi_{m\|D}$	No-Asset bankruptcy	0.1	Lupita ABILawReview12
$\varepsilon_{\tilde{r},m}$	Credit spread sensitivity	$2.5 \cdot 10^{-7}$	Gropp, Scholz, White QJE97
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β	Discount factor	0.96	
γ	Risk aversion	$1, 5, 10, 20, 50$	

- Debt to personal income $\frac{q_0 B_0}{y_0} = 8.4\% \Rightarrow \Lambda$

Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

Parameter/Variable		Value	Source
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γ	Risk aversion	$1, 5, 10, 20, 50$	

- $\pi_m = \underbrace{\pi_D}_{0.008} \times \underbrace{\pi_m | D}_{0.1}$ (zero recovery by unsecured creditors $\approx 90\%$)

Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0} \right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

Parameter/Variable		Value	Source
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β	Discount factor	0.96	
γ	Risk aversion	$1, 5, 10, 20, 50$	

- Increasing m by \$100,000, increases credit spread by 250 basis points

Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

Parameter/Variable		Value	Source
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Calibration

$$m^* = \frac{\beta \pi_m \left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma}}{\Lambda \varepsilon_{\tilde{r},m}}$$

- Optimal exemption m^* (in dollars):

	$\gamma = 1$ (log)	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$	$\gamma = 50$
m^*	39,638	60,416	**102,314**	293,435	6,922,079

Calibration

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- References
  - MA,NV homestead exemption: $\$500,000$; NJ none
  - Average exemption US: $\$70,000$

Calibration

$$\frac{\frac{dW}{dm}}{U'(C_0)\,C_0} = -\Lambda\varepsilon_{\tilde{r},m} + \frac{1}{m}\beta\pi_m\left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma}$$

- Optimal exemption m^* (in dollars):

	$\gamma = 1$ (log)	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$	$\gamma = 50$
m^*	39,638	60,416	**102,314**	293,435	6,922,079

- References
  - MA,NV homestead exemption: $\$500,000$; NJ none
  - Average exemption US: $\$70,000$
- Size of welfare gains at $\$70,000$ using $\frac{dW}{dm}$ formula ($\$10,000$ change)

	$\gamma = 1$ (log)	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$	$\gamma = 50$
$\frac{\frac{dW}{dm}}{U'(C_0)C_0}$	-0.0084%	-0.0027%	**0.0089%**	0.062%	1.897%

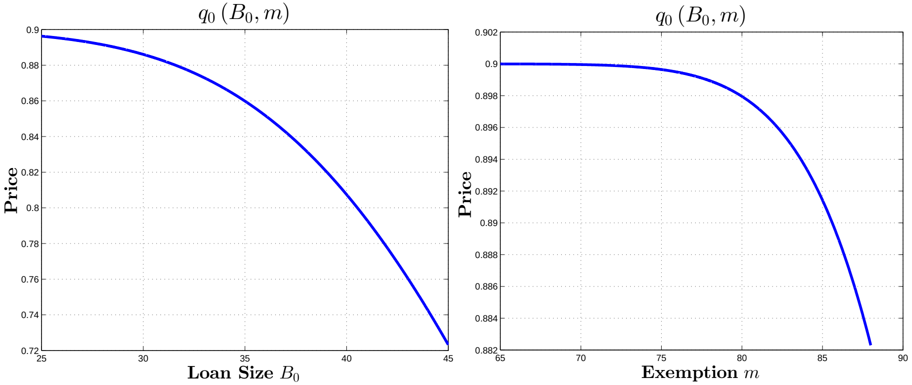
Literature

1. **Sufficient Statistics/Public Economics:** Diamond 98, Saez 01, Shimer-Werning 07, Chetty 09, Arkolakis et al 12
2. **General Equilibrium with Incomplete Markets:** Zame 93, Dubey-Geanakoplos-Shubik 05
3. **Quantitative Literature:**
   - **Structural:** Chatterjee-Corbae-Nakajima-Rios-Rull 07, Livshits-MacGee-Tertilt 07,
   - **Microeconometric**: Gross-Souleles 02, Fay-Hurst-White 02, Fan-White 03, Severino-Brown-Coates 14
4. **Security Design:** Ross 76, Allen-Gale 94, Duffie-Rahi 95
5. **Optimal Contracting:** many papers

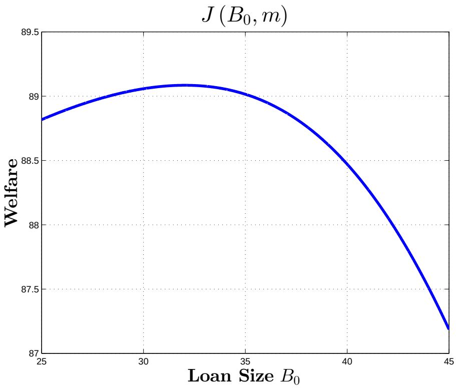
Conclusion

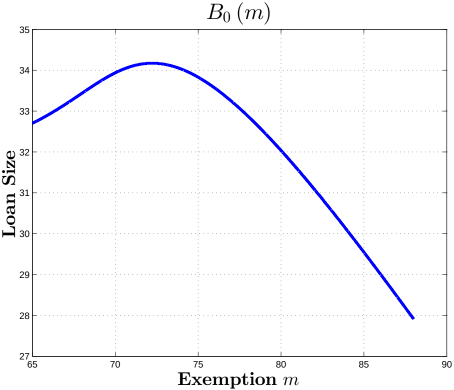
1. This paper has characterized optimal bankruptcy exemptions
   - As a function of a few **sufficient statistics**
   - For a wide range of environments
   - Sensible calibration (measurement is challenging)
2. New paper on mortgage design (with John Campbell)
   - Optimal recourse
   - ARM vs FRM

Figures: Interest rate schedule



Figures: Borrowers' choices





Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
   - Same expression for m^* if disutility of labor is separable (frictionless labor markets)
   - Different characterization of default region: does not matter for m^* (insight applies more generally)
   - Intuition: optimality

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
2. Non-pecuniary losses Utility loss

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
2. Non-pecuniary losses Utility loss
   - Same expression for m^*
   - Effects work through spreads in $\varepsilon_{\tilde{r},m}$ and default regions
   - Implicit assumption: no renegotiation
   - Easy to add externalities/internalities

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
2. Non-pecuniary losses Utility loss
3. Epstein-Zin utility Epstein-Zin

Extensions

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    - Same expression for m^*

$$\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0} \equiv \left(\frac{Q}{C_0}\right)^{\gamma-\frac{1}{\psi}} \beta \int_m^{m+B_0} \left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma} dF\left(y_1\right)$$

    - Where Q is the certainty equivalent of $t=1$ consumption

Extensions

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2. Non-pecuniary losses Utility loss
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    - Where Q is the certainty equivalent of $t = 1$ consumption
    - Corrected Stochastic Discount Factor

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
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4. Multiple contracts Multiple

Extensions

1. Endogenous labor income and effort choice (moral hazard) M. Hazard
2. Non-pecuniary losses Utility loss
3. Epstein-Zin utility Epstein-Zin
4. Multiple contracts Multiple
    - Optimal exemption

$$m^* = \frac{\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\sum_{j=1}^{J} \Lambda_j \varepsilon_{\tilde{r}_j,m}}$$

$$\Lambda_j \equiv \frac{q_{0j} B_{0j}}{y_0 + \sum_{j=1}^{J} q_{0j} B_{0j}}$$

Extensions

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$$m^* = \frac{\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\sum_{j=1}^{J} \Lambda_j \varepsilon_{\tilde{r}_j,m}}$$

$$\Lambda_j \equiv \frac{q_{0j} B_{0j}}{y_0 + \sum_{j=1}^{J} q_{0j} B_{0j}}$$

   - Complete markets as special case $\Rightarrow m^* = 0$

Extensions

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2. Non-pecuniary losses Utility loss
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4. Multiple contracts Multiple
5. Heterogeneous borrowers Heterogeneous

Extensions

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    - Social welfare function $W = \int \lambda(i) W(i) dG(i)$

Extensions

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   - Social welfare function $W = \int \lambda(i) W(i) dG(i)$

$$m^* = \frac{\mathbb{E}_{G,A}\left[\frac{\Pi_{m,i}\left\{C_{1i}^{\mathcal{D}}\right\}}{C_{0i}}\right]}{\mathbb{E}_{G,A}\left[\Lambda_i \varepsilon_{\tilde{r}_i,m}\right]}$$

   - $\mathbb{E}_{G,A}[\cdot]$ are cross sectional averages (for active borrowers)
   - **Observed** heterogeneity: exclusion
   - **Unobserved** heterogeneity: pooling and exclusion

Extensions

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6. Price taking borrowers Price taking
    - Euler equation

$$U'(C_0)\, q_0 = \beta \int_{\phi m + B_0}^{\overline{y_1}} U'\left(C_1^{\mathcal{N}}\right) dF(y_1)$$

Extensions

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    - Optimal exemption

$$m^* = \frac{\frac{\Pi_m \left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\Lambda \varepsilon_{\tilde{r},m}} \qquad \text{where} \qquad \varepsilon_{\tilde{r},m} = \frac{d \log(1+r)}{dm}$$

    - Full GE effect

Extensions

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4. Multiple contracts Multiple
5. Heterogeneous borrowers Heterogeneous
6. Price taking borrowers Price taking
7. Bankruptcy exemptions contingent on aggregate risk Aggregate Risk
   - Aggregate shocks $\omega \in \Omega$, we can condition exemptions on those

$$m^*(\omega) = \frac{\frac{\Pi_{m(\omega)}\left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\Lambda \varepsilon_{\tilde{r},m(\omega)}}, \quad \forall \omega$$

Extensions

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7. Bankruptcy exemptions contingent on aggregate risk Aggregate Risk
8. Endogenous income: labor wedges and aggregate demand Agg. demand

$$m^* = \frac{\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0} + \frac{\Pi_{\mathcal{N}}\left\{\tau(\omega)\frac{dY(\omega)}{\frac{dm}{m}}\right\}}{C_0}}{\Lambda\varepsilon_{\tilde{r},m}}$$

- Where $1+\tau(\omega) = \frac{w_1(\omega)}{A}$ is the labor wedge

Extensions

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4. Multiple contracts Multiple
5. Heterogeneous borrowers Heterogeneous
6. Price taking borrowers Price taking
7. Bankruptcy exemptions contingent on aggregate risk Aggregate Risk
8. Endogenous income: labor wedges and aggregate demand Agg. demand
9. Dynamics Dynamics

$$m^* = \frac{\sum_{t=1}^{T} \frac{\Pi_{m,t}\left\{C_t^{\mathcal{D}}\right\}}{C_0}}{\sum_{t=0}^{T-1} \Pi_{\mathcal{N},t}\left\{g_t \Lambda_t \varepsilon_{\tilde{r}_t,m}\right\}}$$

- Simple formula interpreted as **steady state**
- Income process subsumed in sufficient statistics (permanent vs. transitory shocks, health shocks, family shocks, etc...)

Sign of $\frac{dB_0}{dm}$

$$\text{sign}\left(U''(C_0)\frac{\partial q_0}{\partial m}B_0\left[q_0+\frac{\partial q_0}{\partial B_0}B_0\right]+U'(C_0)\left[\frac{\partial q_0}{\partial m}+\frac{\partial^2 q_0}{\partial B_0 \partial m}B_0\right]\right.$$
$$\left.+\beta U'(m)f(m+B_0)\right)$$

Back to text

m^* in (almost) closed form

$$m^* = \left(\frac{\beta\left(1+r^*\right)}{\Upsilon+\delta\left(1-\Upsilon\right)}\right)^{\frac{1}{\gamma}} C_0$$

$$\Upsilon = \frac{f\left(m^*+B_0\right)B_0}{F\left(m^*+B_0\right)-F\left(m^*\right)}$$

- $\Upsilon = 1$ for uniform (measure of curvature)

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Moral Hazard/Elastic Labor Supply

- Borrowers' problem

$$\max_{C_0,\{C_1\}_{y_1},B_0,\{\xi\}_{y_1},N_0,\{N_1\}_{y_1},a} U\left(C_0,N_0;a\right)+\beta\mathbb{E}_a\left[U\left(C_1,N_1\right)\right]$$

$$\text{s.t.} \qquad C_0=y_0+w_0N_0+q_0B_0$$

$$C_1^{\mathcal{N}}=y_1+w_1N_1^{\mathcal{N}}-B_0; \quad C_1^{\mathcal{D}}=\min\left\{y_1,m\right\}$$

- Two new optimality conditions

$$\frac{\partial U}{\partial a}\left(C_0,N_0;a\right)+\beta\int V\left(C_1,N_1\right)f_a\left(y_1;a\right)dy_1=0$$

$$w_0\frac{\partial U}{\partial C}\left(C_0,N_0;a\right)=-\frac{\partial U}{\partial N}\left(C_0,N_0;a\right)$$

- Same m^*

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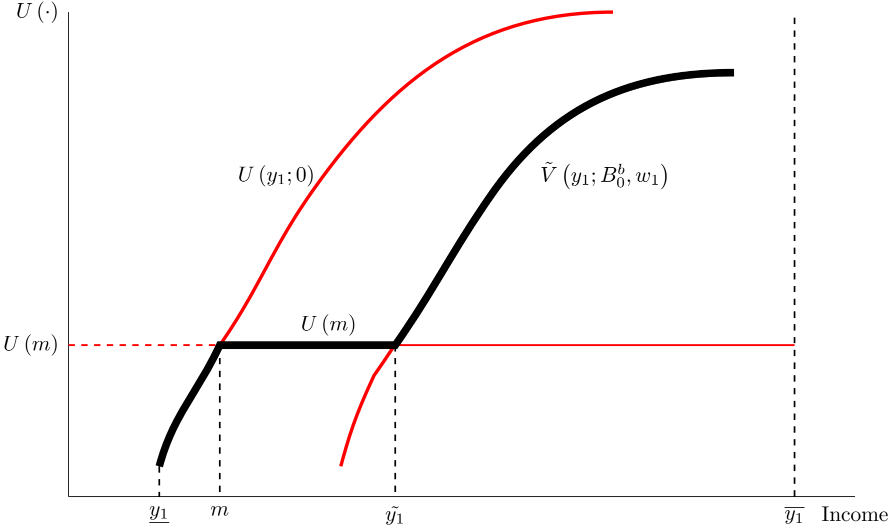
Moral Hazard/Elastic Labor Supply: Default Decision



Non-Pecuniary Losses

- Assumes that renegotiation is not possible
- Non-pecuniary loss: $U(\phi C)$, where $\phi \in [0,1]$
- Same m^*
- Only default region changes

$$\text{if } y_1 < \phi m + B_0 \quad \text{Default}$$

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Borrowers Internalize Price Response

- Borrowers optimality condition

$$U'(C_0)\left[q_0 + \frac{\partial q_0^l}{\partial B_0} B_0^b\right] = \beta \int_{\phi m + B_0^b}^{\overline{y_1}} U'\left(C_1^{\mathcal{N}}\right) dF(y_1)$$

- Optimal exemption

$$m^* = \frac{\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\Lambda\left(\varepsilon_{\tilde{r},m} + \varepsilon_{B_0,m}\hat{\varepsilon}_{q_0^l,B_0}\right)}$$

where $\varepsilon_{B_0,m} \equiv \frac{\frac{dB_0}{B0}}{dm}$, $\hat{\varepsilon}_{q_0^l,B_0} \equiv \frac{\frac{\partial q_0^l}{q_0}}{\frac{\partial B_0^b}{B_0}}$

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Epstein-Zin

- Only difference: stochastic discount factor
- m^* doesn't change but

$$\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0} \equiv \left(\frac{Q}{C_0}\right)^{\gamma-\frac{1}{\psi}} \beta \int_m^{m+B_0} \left(\frac{C_1^{\mathcal{D}}}{C_0}\right)^{1-\gamma} dF\left(y_1\right)$$

- Q is the certainty equivalent of consumption

$$Q \equiv \left(\int_{\underline{y_1}}^{m} \left(y_1\right)^{1-\gamma} dF\left(y_1\right) + \int_m^{m+B_0^b} (m)^{1-\gamma} dF\left(y_1\right) + \int_{m+B_0}^{\overline{y_1}} \left(y_1 - B_0 \right.$$

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Multiple Arbitrary Contracts

- Budget constraints

$$C_0 = y_0 + \sum_{j=1}^{J} q_{0j}\left(B_{01,\ldots}B_{0J}, m\right) B_{0j}$$

$$C_1^{\mathcal{N}} = y_1 + \sum_{j=1}^{J} \max\left\{-z_j\left(y_1\right) B_{0j}, 0\right\} - \sum_{j=1}^{J} \max\left\{z_j\left(y_1\right) B_{0j}, 0\right\}$$

$$C_1^{\mathcal{D}} = \min\left\{y_1 + \sum_{j=1}^{J} \max\left\{-z_j\left(y_1\right) B_{0j}, 0\right\}, m\right\}$$

- Weighted sum of elasticities

$$m^* = \frac{\frac{\Pi_m\left\{C_1^{\mathcal{D}}\right\}}{C_0}}{\sum_{j=1}^{J} \Lambda_j \varepsilon_{\tilde{r}_j, m}}$$

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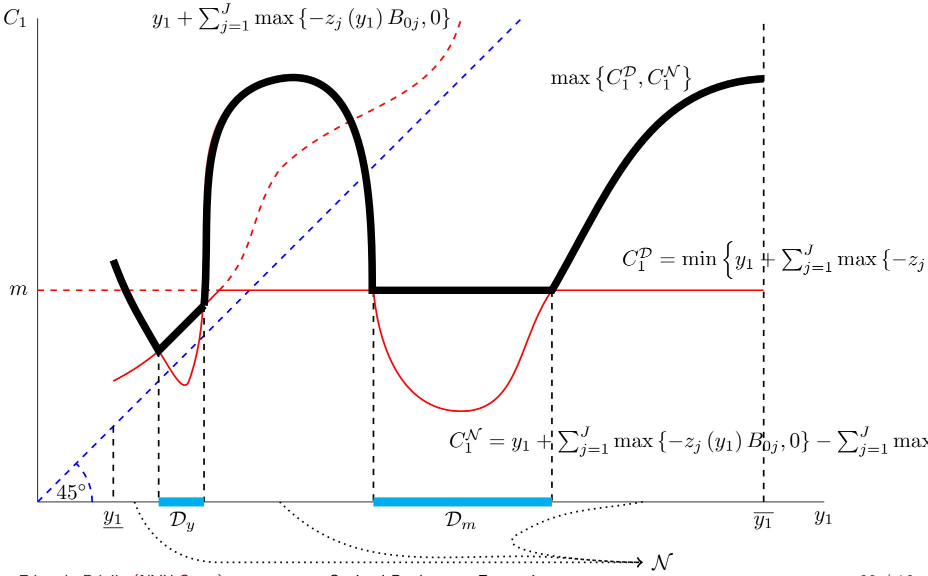
Multiple Arbitrary Contracts



Heterogeneous borrowers (observed heterogeneity)

$$q_{0i}\left(B_{0i},m\right)=\begin{cases}\frac{1}{1+r^{*}}, & B_{0i}\leq 0\\ \frac{\delta\int_{m}^{m+B_{0i}}\frac{y_{1i}-m}{B_{0i}}dF_i(y_{1i})+\int_{m+B_{0i}}^{\overline{y_{1i}}}dF_i(y_{1i})}{1+r^{*}}, & B_{0i}>0\end{cases}$$

$$I_A\left(m\right)=\left\{i|B_{0i}\left(m\right)>0\right\},\quad \text{Active borrowers}$$

$$I_N\left(m\right)=\left\{i|B_{0i}\left(m\right)=0\right\},\quad \text{Inactive borrowers}$$

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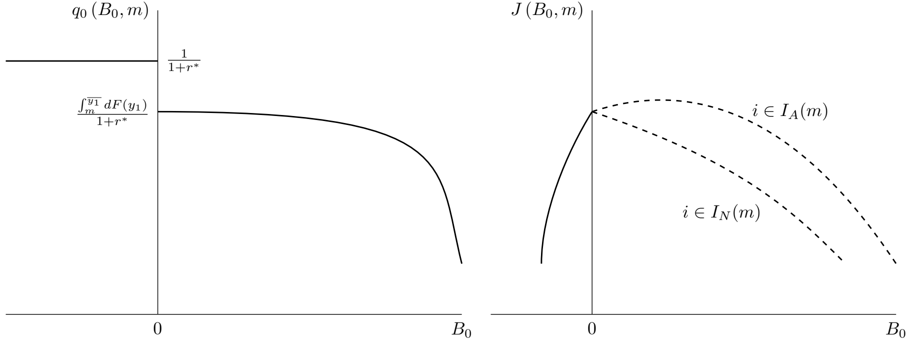
Heterogeneous borrowers (observed heterogeneity)



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Heterogeneous borrowers (observed heterogeneity)

$$q_{0i}\left(B_0,m\right)=\begin{cases}\frac{1}{1+r^*}, & B_{0i}\leq 0\\ \int_{I_A(m)}\frac{\tilde{q}_{0i}(B_0,m)}{\int_{I_A(m)}dG(i)}dG\left(i\right), & B_{0i}>0,\end{cases}$$

where

$$\tilde{q}_{0i}\left(B_0,m\right)=\frac{\delta\int_m^{m+B_0}\frac{y_{1i}-m}{B_0}dF_i\left(y_{1i}\right)+\int_{m+B_0}^{\overline{y_{1i}}}dF_i\left(y_{1i}\right)}{1+r^*}$$

Back to text

Aggregate Risk

- Now we have aggregate shocks $\omega \in \Omega$, we can condition the exemption level on those
- Optimal exemption

$$m^{*}\left(\omega\right)=\frac{\frac{\Pi_{m(\omega)}\left\{C_{1}^{\mathcal{D}}\right\}}{C_{0}}}{\Lambda\varepsilon_{\tilde{r},m(\omega)}},\quad\forall\omega$$

Back to text

Dynamics

- Borrowers maximize $\max \mathbb{E}\left[\sum_{t=0}^{T} \beta^t U\left(C_t\right)\right]$
- Recursively $V_{ND,0}\left(B_{-1}, y_0; m\right) =$
  $\max_{B_0} U\left(C_0\right) + \beta \mathbb{E}\left[\max\left\{V_{ND,1}\left(B_0, y_1; m\right), V_{D,1}\left(y_1; m\right)\right\}\right]$
- Optimal exemption

$$m^* = \frac{\sum_{t=1}^{T} \frac{\Pi_m\left\{C_t^{\mathcal{D}}\right\}}{C_0}}{\sum_{t=0}^{T-1} \Pi_{ND}\left\{g_t \Lambda_t \varepsilon_{\tilde{r}_t, m}\right\}}$$

- Easy to allow for default before bankruptcy, wage garnishments, exclusion period

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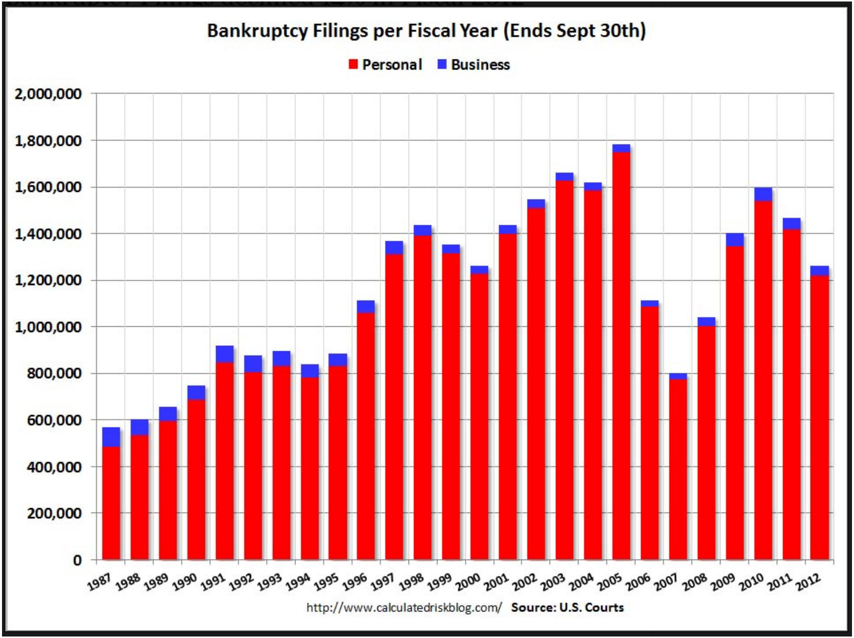
Evolution filings US



Parameters

Preferences	$\gamma = 10$	$\psi = 1.5$	$\beta = 0.96$	$r^* = 4\%$
Endowments	$y_0 = 55$	$\mu = 4.9$	$\sigma = 0.095$	
Bankruptcy	$\delta = 0.1$			

Table 4: Parameters numerical example

Expression for welfare

$$W(m) =$$

$$U\left(y_0 + q_0\left(B_0(m), m\right) B_0(m)\right) +$$

$$+\beta\left[\int_{\underline{y_1}}^{m} U(y_1)\, dF(y_1) + \int_{m}^{m+B_0(m)} U(m)\, dF(y_1) + \int_{m+B_0(m)}^{\overline{y_1}} U\left(y_1 - B_0\right.\right.$$