Moving towards a Single Labour Contract:
Transition vs. Steady-state*

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This version: May, 2015

Abstract

This paper analyzes the optimal design of a single open-ended contract (SOEC) and studies the political economy of moving towards such a SOEC in a dual labour market. We compare two economic environments: one with flexible entry-level jobs and high employment protection at long tenure, and another with a SOEC featuring employment protection levels that increase smoothly with tenure. For illustrative purposes, we specialize the discussion of such choices to Spain. A SOEC has the potential of bringing big time efficiency and welfare gains in a steady-state sense. We also identify winners and losers in the transitional path of such a reform and analyze its political support.

Keywords: Single contract; Employment Protection; Dualism; Labour Market Reform

JEL codes: H29, J33, J65

*We are grateful to Etienne Wasmer for useful comments that helped improve the paper. We also thank seminar participants at Konstanz GSDS-Colloquium and the SaM annual conference in Aix-en-Provence for their suggestions. All errors are our own.
1 Introduction

Employment protection legislation (EPL) has been rationalized on several grounds. These range from strengthening the bargaining power of workers to avoiding moral hazard by employers or improving worker’s employer-sponsored training. Yet, one of the most important reasons for having EPL is to increase job stability of risk-averse workers in order to insure them against dismissals (Pissarides, 2001). In several countries, particularly in Southern Europe, labour markets exhibit a high degree of dualism: workers under open-ended (permanent) contracts enjoy very stringent EPL while those under fixed-term (temporary) contracts enjoy little or even none. In particular, permanent contracts bear mandated severance payment that increase with tenure, typically subject to a cap. This is usually measured in terms of days of wages per years of service (d.w.y.s.), which are lower for dismissals due to fair (economic) reasons than those deemed unfair. In contrast, due to their short-term duration, temporary contracts are hardly ever destroyed although they are subject to a fixed termination cost (again in terms of d.w.y.s.) which is quite lower than permanent workers’ redundancy pay (see Cahuc et al., 2012). As noted by Blanchard and Landier (2002), lacking enough wage flexibility, the large gap in severance costs between these types of workers makes employers reluctant to transform temporary contracts into permanent ones.

Thus, temporary contracts create a “revolving door” with workers rotating between temporary jobs and unemployment. Bentolila et al. (2012) have pointed out that the discontinuity (the so-called “wall”) created by conventional EPL schemes in dual labour markets has negative consequences for unemployment, human capital and innovation since it leads to excessive turnover (Blanchard and Landier, 2002), excessive wage pressure (Bentolila et al., 1994), low investment by firms in employer-sponsored training schemes for temporary workers (Cabrales et al., 2014), and the adoption of mature rather than innovative technologies (Saint-Paul, 2002).

This has triggered a heated debate on redesigning dual employment protection, leading to policy initiatives in Southern Europe defending the suppression of the firing-cost gap once and for all. To achieve this goal, a key policy advice in these proposals is to introduce a single open-ended contract (SOEC hereafter) for new hires, at the same time that most temporary contracts are abolished – the exception being replacement contracts for maternity or sickness/disability leaves. The key feature of SOEC is that it has no ex ante time limit (unlike fixed-term contracts) and that severance payments smoothly increase with seniority (unlike current indefinite term contracts where the same indemnity per year of service applies from the start). In this fashion, a SOEC would provide a sufficiently long entry phase and a smooth rise in protection as job tenure increases. The rationale for the gradually increasing severance pay could be that the longer a worker stays in a given firm, the larger is her/his loss of specific human capital and the psychological costs suffered in case of dismissal – a negative externality that firms should internalize.

However, despite being high on the European political agenda, most SOEC proposals so far have

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1 See Booth and Chatterji (1989, 1998).
3 See Blanchard and Tirole (2003). Further, as argued by García Pérez and Osuna (2014), even if a SOEC implies lower redundancy pay per year of service than current EPL for permanent contracts, the increase in job tenure that it may bring in can lead to a “Laffer curve effect” whereby the total compensation received by a dismissed worker is larger under SOEC than under the current dual EPL system.
been fairly deceptive. As a result, several design and implementation problems need to be worked out before SOEC becomes operative. In this paper, we take a first step towards addressing the following pending issues. First, as mentioned earlier, there is a general agreement that SOEC should feature mandated severance payments that increase smoothly with tenure; but little is known about the exact tenure profile of the contract. Secondly, there is general agreement that SOEC would benefit the functioning of labour markets; but little is known about the magnitude of the allocational and welfare improvements that would result. Thirdly, in the context of dual labour markets, there is suspicion that a non-negligible number of insiders would lose from the policy change and, thus, would oppose the reform; but not much is known about the relevance of this argument, i.e., the political strength of insiders, the size of their welfare losses, and whether an appropriately designed transition towards the new steady-state could limit their losses.

We tackle these questions by developing an equilibrium search and matching model, which we use to investigate the effects of introducing a SOEC in a dual labour market. In our model, risk averse workers demand insurance, a feature that enables us to compute the optimal tenure profile according to some aggregate welfare criterion. More specifically, our model is one where young and older workers coexist and the former become older at a given rate. Both receive severance pay but differ with respect to the use they can make of this compensation. So, while it is assumed that young workers consume it upon reception (say, because of binding credit constraints associated to lower job stability; see Crossley and Low, 2014), older workers are allowed to buy annuities in order to smooth their consumption until retirement. Furthermore, since our focus is on the political economy of the reform introducing SOEC, steady-state comparisons as well as transitional dynamics are considered. Indeed, in our setup, labour market tightness and payroll taxes act are sufficient statistics for the distribution of agents across states of nature, which enables us to compute the model outside the steady-state.

Optimality is defined both in terms of the welfare of a newborn in a steady state. After finding the SOEC that maximizes her welfare, we study welfare across the current population when taking into account the transition from a dual EPL system to SOEC. Steady-state comparisons disregard implementation problems. On the other hand, when we look at transition from the extant regulation to the SOEC, we consider two extreme scenarii: a retroactive vs. a non-retroactive introduction of the SOEC. In so doing, we obtain bounds on the welfare effects of moving towards a SOEC.

For illustrative purposes, the model is calibrated to the Spanish labour market before the Great Recession, when the unemployment rate was similar to the EU average rate, namely about 8%. We choose Spain because it has been often considered as the epitome of a dual labour market (see e.g., Dolado et al., 2002). Yet, the methodology proposed here could be extended to other dual labour markets, like France or Italy. Specifically, once the parameters are calibrated to reproduce targets prior to the crisis, we compute the optimal tenure profile of redundancy pay according to the above-mentioned criterion. A key element in our setup is that unemployment benefits are financed by a payroll tax and that severance pay has in part a layoff tax nature due the existence of red-tape cost associated to litigation procedures, etc. In this fashion, our analysis is related to Blanchard and Tirole’s (2008)’s

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4See Chapter 4 of the OECD 2014, Employment Outlook devoted to this topic.

5In our model, young workers should be interpreted as prime-age workers (workers aged 25 to 54). Correspondingly, older workers are those aged 55 to 64 years old. We use this terminology for simplicity, and in keeping with standard OLG models where young agents are those who work and old agents are those who consume savings and get a pension.
discussion of whether the contribution rate—the ratio between layoff taxes and unemployment (UI) benefits—should be greater than, equal or lower than one, depending on the nature of the deviation from their benchmark model where risk-averse workers can be insured by risk-neutral firms.

Indeed, Blanchard and Tirole (2008) is one the forerunners of this paper. We differ from their analysis in that their focus is essentially normative and does not provide actual figures that would inform specific labour market policies, whereas ours is positive in that we are interested in modelling key features of a notoriously dysfunctional labour market, as is the case of Spain. Further, while their analysis is static, ours involves rich dynamics and considers the transition from dual EPL to SOEC. A second related paper is García Pérez and Osuna (2014) who also look at the effects of moving towards a SOEC in the context of the Spanish labour market. The main differences between our approach and theirs is that: (i) they impose a given tenure profile in a SOEC rather than deriving it, and (ii) workers are risk neutral in their setup whereas they are risk averse and value consumption smoothing in ours. Finally, there is a recent paper by Boeri et al. (2013) which proposes a rationale for mandatory severance pay increasing with tenure on the basis that financing initial investment in training trough wage deferrals is not sustainable if employers cannot commit not to dismiss workers who have invested in training. As before, their model is again one where agents are risk neutral and they do not derive specific tenure profiles.

Our main findings are that when comparing steady states, by removing “revolving doors” in labor market trajectories implied by dual EPL, a SOEC can bring big time efficiency and welfare gains. That is, a SOEC reduces job turnover rates at short tenures, increases outflow rates from unemployment through higher job creation, raises entry wages and flattens out career wage profile. These different effects concur in reducing the payroll tax and produce significant output and welfare gains. Further, along the transition from a dual EPL system to SOEC we identify winners and losers from the reform as well as the fraction of the population supporting it.

The rest of the paper is structured as follows. Section 2 lays out the main ingredients of the model. Section 3 proceeds to calibrate the model to the Spanish labour market prior to the Great Recession. Section 4 present the results of the simulations we carry out involving the optimal tenure profile of SOEC on the basis of several welfare criteria, comparison of steady states and the transition phase when replacing dual EPL by SOEC. Section 5 discusses the trade-off between unemployment insurance and employment protection which is present in our model. Section 6 concludes. An Appendix presents our numerical methodology for computing steady states and transition paths.

2 The model

This section presents our search and matching model. The model is a variant of Mortensen and Pissarides (1994), which we accommodate to: (i) provide a role for insurance, (ii) allow workers to have different tenure at their job and (iii) obtain tractability outside the steady-state.

2.1 Economic environment

Time is discrete and runs forever. The economy may not be in steady-state and thus we need to keep track of calendar time. This is indexed by the subscript $t$. 

2
Workers

The economy is populated by a continuum of risk-averse workers who work and then retire from the labour market. Workers derive utility from consumption $c_t > 0$ according to a constant relative risk aversion (CRRA) utility function:

$$u(c_t) = \frac{c_t^{1-\eta} - 1}{1-\eta}$$

The coefficient of relative risk-aversion, $\eta$, is strictly positive and ensures that $u'(c_t) > 0$ and $u''(c_t) < 0$. A coefficient of one makes this utility function logarithmic.

It is assumed that workers face incomplete asset markets and that there is no storage technology. We preclude access to savings in order to provide an insurance role for employment protection. While this has potential of exacerbating welfare effects, we will also model public insurance coming from unemployment benefits and allow for some form of private insurance (details follow).

Production

Production is carried out by a continuum of firms. A firm is a small production unit with only one job, either filled or vacant. There is a per-period cost $k > 0$ of having a vacant job. Firms enter and leave the market freely and maximize the sum of profit streams discounted by the interest rate $r$, which is exogenous and fixed.

Workers and firms come together via search. They are brought together by a Cobb-Douglas matching function with constant returns-to-scale:

$$m(u_t, v_t) = A u_t^\psi v_t^{1-\psi}$$

where $v_t$ and $u_t$ are the number of vacancies and job-seekers, respectively. $\psi \in (0,1)$ is the elasticity of the number of contact to the number of job-seekers and $A$ is a matching-efficiency parameter. Thus, the vacancy-filling probability for firms, $q(\theta_t) = A \theta_t^{-\psi}$, is decreasing in tightness $\theta_t \equiv v_t/u_t$. Likewise, the job-finding probability for workers is given by $\theta_t q(\theta_t)$, which is increasing in $\theta_t$.

A worker-firm match is characterized by its idiosyncratic productivity $z$. Every worker-firm pair starts at the same productivity level, which is denoted as $z_0$. In subsequent periods, productivity evolves according to a finite Markov process with transition matrix $\Pi = (\pi_{z,z'})$. Fluctuations in productivity may induce the worker-firm pair to destroy the job. Later on in the analysis, we also introduce an exogenous separation shock in order to improve the fit of the model; we defer this element to the calibration section of the paper.

Finally, anticipating on the design of employment protection schemes, we denote by $\tau$ the tenure of a given worker-firm match. In our applications, we impose a cap on tenure at a value $T$. Thus, the law of motion for $\tau$ is: $\tau' = \min\{\tau + 1, T\}$. Observe that, as a result, there are (at least) two state variables for every worker-firm pair: tenure ($\tau$) and productivity ($z$).
Young vs. older workers

The working life span is uncertain, and each period a fraction of newborns enters the labour market to maintain the size of the workforce at a constant unit level. We distinguish young (y) workers from older (o) workers. As in [Castaneda et al., 2003], it is assumed that ageing and retirement occur stochastically: at the end of each period, young workers become older with probability $\gamma$ and older workers retire with probability $\chi$.

There are two key differences between young workers and older workers. First, following job loss, young workers keep searching for new jobs whereas older workers abandon job search until they leave the labour market (a situation we interpret as early retirement). Second, older workers are allowed to buy an annuity from firms upon separation from the job. In so doing, they can increase their consumption until leaving the workforce.

It is appropriate here to comment on these two assumptions. The annuity payment we allow for implies that one needs to keep track of older workers’ employment history after job loss since this capitalizes into the annuity scheme. However, because they stop searching for jobs, the distribution of older unemployed workers across tenure levels in the previous job is irrelevant for the vacancy-posting decision of firms. Conversely, young unemployed workers are homogeneous in that they are prevented from capitalizing their employment history into annuities. Thus, although somewhat extreme, these two assumptions allow us to provide a role for insurance while maintaining feasibility for computations outside the steady-state.

Government-mandated programs

The government runs two labour market programs: unemployment insurance and employment protection schemes. The provision of unemployment insurance is financed by the proceeds of a payroll tax $\kappa_t$. Importantly, we assume that the budget for this program is balanced in every period. The employment protection program, on the other hand, is self-financed (details follow).

The unemployment insurance program consists in providing a constant-level benefit $b$ to the nonemployed. There is no monitoring technology, and therefore older workers can collect $b$ after a job loss, although they stop searching for jobs.

Employment protection is introduced in the form of government-mandated severance payments. As a benchmark, we assume that there are no red-tape costs involved in the dismissal procedure and hence severance payments consist of a transfer from the firm to the worker paid at the time of job separation. This payment is (possibly) a function of tenure $\tau$ and is denoted by the function $\phi(\tau)$. Finally, since severance payments are a pure transfer, the government need not levy a tax to finance their provision.

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6 There is a long-established literature on the difference between two components of severance payments, namely the transfer to the worker and the firing tax capturing costly administrative procedures inherent to the implementation of the policy scheme; see, e.g., [Bertola and Rogerson, 1997]. We focus on the transfer component because of its consumption-smoothing role. Later on in the analysis, we also study the effects of introducing red-tape costs alongside the transfer component (see Section ??).

7 In the event of a separation between a firm and an older worker triggered by the exogenous retirement shock, we assume that severance payments are waived.
Disposable income

Having described the environment, we are now in a position to describe the income workers receive (and consume) in the different states of the labour market. In employment, workers obtain a wage \( w_i(z, \tau) \), where \( i \in \{y, o\} \) is the age of the worker, after bargaining over the surplus of the match (details follow). Notice that the wage can be contracted upon \( z \) and \( \tau \), the age of the worker \( i \), and that it may depend on calendar time \( t \).

In nonemployment, young workers collect unemployment benefits \( b \). As mentioned earlier, lacking access to annuity schemes, they consume the severance package \( \phi(\tau) \) entirely upon separation. Older workers, on the other hand, can buy an annuity upon receipt of the severance pay and collect the proceeds until they leave the workforce. As a result, the total disposable income of older unemployed workers is:

\[
\tilde{b}(\tau) = b + \frac{1}{1 - (1 + r)^{-1/\chi}} \frac{r}{1 + r} \phi(\tau)
\]

namely, the payoff of an actuarially fair annuity associated with the severance payment \( \phi(\tau) \), where \( 1/\chi \) is the expected number of periods until full retirement.

2.2 Bellman equations

We formulate workers’ and firms’ decision problems in recursive form. The value of leaving the workforce is set to zero and we denote by \( U_i^y \) (resp. \( W_i^y \)) the value of being nonemployed (resp. being employed), with \( i \in \{y, o\} \).

While nonemployed, a young worker enjoys a flow income \( b \), remains in the current age category with probability \( 1 - \gamma \) and either finds a job with probability \( \theta_t q(\theta_t) \), or stays nonemployed. Otherwise he/she becomes old with probability \( \gamma \) and the asset value becomes \( U_o^y t+1 (0) \):

\[
U_i^y = u(b) + \frac{1}{1 + r} \left[ (1 - \gamma) \left( \theta_t q(\theta_t) W_{i+1}^y (z_0, 0) + (1 - \theta_t q(\theta_t)) U_i^y t+1 \right) + \gamma U_o^y t+1 (0) \right]
\]

where \( W_{i+1}^y (z_0, 0) \) denotes a young worker’s asset value of being employed at the entry productivity level and no tenure. An old unemployed worker who had tenure \( \tau \) in his/her previous job has flow income \( \tilde{b}(\tau) \) and remains in the labour market with probability \( 1 - \chi \):

\[
U_i^o (\tau) = u \left( \tilde{b}(\tau) \right) + \frac{1 - \chi}{1 + r} U_o^o t+1 (\tau)
\]

While employed at a job with productivity \( z \) and tenure \( \tau \), a young worker consumes the wage \( w_i^y(z, \tau) \) and his/her job is subject to productivity shocks. In the event of job destruction, the value of young workers becomes \( \tilde{U}_i^y (\tau) = U_i^y + u(b + \phi(\tau)) - u(b) \): the worker consumes the severance payment (as a function of previous tenure) in the period immediately after dismissal. Therefore,
Why \( t(z, \tau) \) satisfies:

\[
W_y^y(z, \tau) = u(w_y^y(z, \tau)) + \frac{1}{1+r} \left( (1 - \gamma) \sum_{z'} \pi_{z,z'} \max \left\{ W_{i+1}^y(z', \tau'), U_{i+1}^y(\tau') \right\} \right)
+ \gamma \sum_{z'} \pi_{z,z'} \max \left\{ W_{i+1}^y(z', \tau'), U_{i+1}^o(\tau') \right\}
\]

(6)

The value of employment for older workers, on the other hand, is given by:

\[
W_o^o(z, \tau) = u(w_o^o(z, \tau)) + \frac{1 - \chi}{1+r} \sum_{z'} \pi_{z,z'} \max \left\{ W_{i+1}^o(z', \tau'), U_{i+1}^o(\tau') \right\}
\]

(7)

As for firms, let \( J_i^t \) denote the value of having a filled job, where \( i \in \{y, o\} \) is the age of the worker who is currently employed. Just like the worker, the firm forms expectations over future values of productivity and age. In the event of job destruction, the value of a firm is that of having a vacant position minus the severance package paid to the worker. For a young worker, the severance package is exactly \( \phi(\tau) \), while this is replaced by the expected discounted value of the annuity payment \( \Phi(\tau) \) when the worker is older. Finally, we assume that the value of holding a vacant job is zero in every period \( t \) (free-entry condition). Hence:

\[
J_y^y(z, \tau) = z - (1 + \kappa_t)w_y^y(z, \tau) + \frac{1}{1+r} \left( (1 - \gamma) \sum_{z'} \pi_{z,z'} \max \left\{ J_{i+1}^y(z', \tau'), -\phi(\tau') \right\} \right)
+ \gamma \sum_{z'} \pi_{z,z'} \max \left\{ J_{i+1}^o(z', \tau'), -\Phi(\tau') \right\}
\]

(8)

\[
J_o^o(z, \tau) = z - (1 + \kappa_t)w_o^o(z, \tau) + \frac{1 - \chi}{1+r} \sum_{z'} \pi_{z,z'} \max \left\{ J_{i+1}^o(z', \tau'), -\Phi(\tau') \right\}
\]

(9)

### 2.3 Wage setting

Following much of the literature, we assume that wages are set by Nash bargaining. Let \( \beta \in (0, 1) \) denote the bargaining power of the worker. Wages are given by:

\[
w_y^y(z, \tau) = \arg\max_w \left( W_y^y(z, \tau; w) - O_y^y(\tau) \right)^\beta \left( J_y^y(z, \tau; w) + \phi(\tau) \right)^{1-\beta}
\]

(10)

\[
w_o^o(z, \tau) = \arg\max_w \left( W_o^o(z, \tau; w) - U_o^o(\tau) \right)^\beta \left( J_o^o(z, \tau; w) + \Phi(\tau) \right)^{1-\beta}
\]

(11)
for all \((z, \tau)\). For future reference, it is useful to write the first-order condition associated with the above maximization problems. That is,

\[
(1 - \beta) \frac{1 + \kappa}{J^y(z, \tau) + \phi(\tau)} = \beta \frac{u'(w^y(z, \tau))}{W^y(z, \tau) - U^y(\tau)}
\]

and

\[
(1 - \beta) \frac{1 + \kappa}{J^o(z, \tau) + \Phi(\tau)} = \beta \frac{u'(w^o(z, \tau))}{W^o(z, \tau) - U^o(\tau)}
\]

On the one hand, the numerator in the left-hand side of equations (12) and (13) is the effect for the firm of a marginal reduction in the wage, which increases profit streams by \(1 + \kappa\). On the other hand, the effect of a marginal increase in the wage on the utility of the worker depends on the value of the wage, because of diminishing marginal utility of consumption (right-hand side of the equations). Thus, unlike the canonical search and matching model, our model features nontransferable utilities between agents. This implies that we cannot solve for the joint surplus of the match in order to obtain the wage functions.\(^8\)

### 2.4 Separation decisions

Associated with the maximization of the asset value functions of employment, there are productivity thresholds for separation decisions. Let \(\overline{z}^y_i(\tau)\) (resp. \(\overline{z}^o_i(\tau)\)) denote the productivity cutoff for a match with a young (resp. old) worker with tenure \(\tau\). The threshold \(\overline{z}^i_i(\tau)\), with \(i \in \{y, o\}\), is the value of \(z\) that makes both parties indifferent between keeping the job alive and dissolving the match. Since private bargains are efficient, \(\overline{z}^i_i(\tau)\) can be recovered by using either the value functions of the worker or that of the firm. That is,

\[
W^y_i(\overline{z}^y_i(\tau), \tau) = \bar{U}^y_i(\tau), \quad J^y_i(\overline{z}^y_i(\tau), \tau) = -\phi(\tau)
\]

and

\[
W^o_i(\overline{z}^o_i(\tau), \tau) = U^o_i(\tau), \quad J^o_i(\overline{z}^o_i(\tau), \tau) = -\Phi(\tau)
\]

Due to the nonstandard problem for wages, it is also relevant to define separation decisions in relation to the reservation wage of the worker and that of the firm. Let \(w^i_i(z, \tau)\) denote the lowest possible wage that a worker of age \(i\) and current tenure \(\tau\) would accept in a job with productivity \(\tau\).

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\(^8\)Another implication is that Lazear (1990)’s “bonding critique” is not applicable to our setup. That is, workers and firms cannot fully neutralize severance payments because they differ as to their valuation of a reallocation of payments over time; see Lazear (2014) for a discussion in a similar context. Moreover, in the calibrated version of the model, there is an exogenous separation shock, which hence cannot be contracted upon.
These reservation wages solve:

\[
 u (w^y_y (z, \tau)) = \bar{U}^y_t (\tau) - \frac{1}{1+r} \left( (1 - \gamma) \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ W^y_{t+1} (\tau', \tau'), \bar{U}^y_{t+1} (\tau') \right\} + \gamma \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ W^o_{t+1} (\tau', \tau'), U^o_{t+1} (\tau') \right\} \right) \tag{16}
\]

\[
 u (w^o_o (z, \tau)) = U^o_t (\tau) - \frac{1 - \chi}{1+r} \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ W^o_{t+1} (\tau', \tau'), U^o_{t+1} (\tau') \right\} \tag{17}
\]

Similarly, the highest possible wage that the firm would pay to this worker, \( \bar{w}^y_y (z, \tau) \), is given by

\[
 \bar{w}^y_y (z, \tau) = \frac{1}{1 + \kappa_t} \left[ z + \phi (\tau) + \frac{1}{1+r} \left( (1 - \gamma) \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ J^y_{t+1} (\tau', \tau'), -\phi (\tau') \right\} + \gamma \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ J^o_{t+1} (\tau', \tau'), -\Phi (\tau') \right\} \right) \right] \tag{18}
\]

\[
 \bar{w}^o_o (z, \tau) = \frac{1}{1 + \kappa_t} \left[ z + \Phi (\tau) + \frac{1 - \chi}{1+r} \sum_{\tau'} \pi_{\tau, \tau'} \max \left\{ J^o_{t+1} (\tau', \tau'), -\Phi (\tau') \right\} \right] \tag{19}
\]

A separation occurs when: \( \bar{w}^y_y (z, \tau) < w^o_o (z, \tau) \). Thus, one can determine whether the productivity threshold \( \tau^*_t (\tau) \) is larger than current productivity \( z \) by comparing \( \bar{w}^y_y (z, \tau) \) and \( w^o_o (z, \tau) \). Notice that, in equations (16)–(19), reservation wages depend on calendar time \( t \) only through the outside option of agents and through the payroll tax \( \kappa_t \).

### 2.5 Flow equations

Using labour market tightness \( \theta_t \) and separation decisions \( \tau^*_t (\tau) \) and \( \zeta^*_t (\tau) \), we are in a position to write the flow equations that govern the evolution of population distributions in the labour market. Let \( \lambda^y_t (z, \tau) \) (resp. \( \lambda^o_t (z, \tau) \)) denote the population of young (resp. older) workers employed at a job with current productivity \( z \) and with tenure \( \tau \) at time \( t \). Likewise, let \( \mu^y_t \) (resp. \( \mu^o_t \)) denote the population of young (resp. older) unemployed workers. Note that for older unemployed workers we need to keep track of the tenure variable.

In employment, new hires are given by:

\[
 \lambda^y_{t+1} (z_0, 0) = \theta_t q (\theta_t) (1 - \gamma) \mu^y_t \tag{20}
\]

while employment in on-going jobs (\( \tau' > 0 \)) evolves according to:

\[
 \lambda^y_{t+1} (z', \tau) = \sum_{z} \mathbb{1} \left\{ z' \geq \bar{z}^y_{t+1} (\tau') \right\} \pi_{z, z'} (1 - \gamma) \lambda^y_t (z, \tau) \tag{21}
\]
\[
\lambda_{t+1}(z', \tau') = \sum_{z} \mathbb{1} \left\{ z' \geq z_{t+1}'(\tau') \right\} \pi_{z,z'}(\gamma \lambda_{t+1}'(z, \tau) + (1 - \chi) \lambda_{t+1}^o(z, \tau))
\]

As for the evolution of the nonemployment pool, we have

\[
\mu_{t+1}^y = \Pi^y + (1 - \theta_q(\theta_t)) (1 - \gamma) \mu_{t+1}^y + (1 - \gamma) \sum_{\tau} \sum_{z} \mathbb{1} \left\{ z' < z_{t+1}'(\tau') \right\} \pi_{z,z'} \lambda_{t}^y(z, \tau)
\]

where \(\Pi^y = \chi \frac{y}{x+y}\) is the mass of new entrants in every period\(^9\). Among the old nonemployed with tenure level \(\tau\) at the time of being dismissed from the previous job, the law of motion is:

\[
\lambda_{t+1}^o(\tau) = \gamma \mu_{t+1}^y \mathbb{1} \{ \tau = 0 \} + (1 - \chi) \mu_{t+1}^o(\tau)
\]

\[
+ \sum_{z} \mathbb{1} \left\{ z' < z_{t+1}'(\tau') \right\} \pi_{z,z'} \left( \gamma \lambda_{t}^y(z, \tau) + (1 - \chi) \lambda_{t}^o(z, \tau) \right)
\]

The term with \(\gamma \mu_{t+1}^y \mathbb{1} \{ \tau = 0 \}\) accounts for the fact that a young unemployed worker who becomes old enters the pool of older workers with no tenure accumulated in the previous job.

Finally, given that the size of the workforce is equal to one in every period \(t\), it follows that

\[
\sum_{\tau} \sum_{z} \left( \lambda_{t}^y(z, \tau) + \lambda_{t}^o(z, \tau) \right) + \sum_{\tau} \mu_{t+1}^o(\tau) + \mu_{t}^y = 1
\]

### 2.6 Equilibrium conditions

There are two aggregate quantities which are pinned down by equilibrium conditions, even when the economy is not in steady-state: labour-market tightness \(\theta_t\) and the tax rate \(\kappa_t\).

**Free-entry**

In every period of the model, a free-entry condition dictates that firms exhaust the present discounted value of job creation net of the vacancy-posting cost. This implies that labour market tightness in period \(t\) is given by

\[
\frac{k}{q(\theta_t)} = \frac{1}{1 + r} f_{t+1}^y(z_0, 0)
\]

Notice that the right-hand side of the equation, i.e. the present discounted value of filling a vacant position, depends on calendar time \(t + 1\) only. Using this insight, it follows that the outside options of agents in period \(t\) are fully determined once value functions in period \(t + 1\) are known.

---

\(^9\)That is, with our stochastic life-cycle there are \(\frac{x+y}{x+y}\) older workers in the workforce. A fraction \(\chi\) of them leaves every period, and the same number of individuals enters the labour market to keep the size of the workforce at a constant level.
Balanced budget

Finally, since the government balances the budget of the unemployment insurance system period by period, the payroll tax satisfies

$$\kappa \sum \sum \left( w^y_i (z, \tau) \lambda^y_i (z, \tau) + w^o_i (z, \tau) \lambda^o_i (z, \tau) \right) = b \left( \sum \mu^o_i (\tau) + \mu^y_i \right)$$

(27)

for all $t$. Notice that workers and firms need to know $\kappa$ to set wages, and wages in turn affect the revenues raised by the tax.

2.7 Transition and steady-state

Having described the economic environment and equilibrium conditions, we are in a position to define transition paths and steady-state equilibria. In the sequel, we are typically interested in the transition between two steady-state equilibria which we index by calendar time, say $t_0$ and $t_1 > t_0$. Hence:

**Definition.** A transition path between $t_0$ and $t_1$ is a sequence of value functions $(U^y_t, U^o_t (\tau), W^y_t (z, \tau), W^o_t (z, \tau), J^y_t (z, \tau), J^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, a sequence of wage functions $(w^y_t (z, \tau), w^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, a sequence of rules for separation decisions $(\xi^y_t (\tau), \xi^o_t (\tau))_{t = t_0, \ldots, t_1}$, a time-path for labour market tightness $(\theta_t)_{t = t_0, \ldots, t_1}$ and for the payroll tax $(\kappa_t)_{t = t_0, \ldots, t_1}$, and a sequence of distribution of workers across employment status, productivity levels, tenure and age groups $(\mu^y_t, \mu^o_t (\tau), \lambda^y_t (z, \tau), \lambda^o_t (z, \tau))_{t = t_0, \ldots, t_1}$ such that:

1. Agents optimize: Given $(\theta_t)_{t = t_0, \ldots, t_1}$, $(\kappa_t)_{t = t_0, \ldots, t_1}$ and the sequence of wage functions $(w^y_t (z, \tau), w^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, the value functions $U^y_t, U^o_t (\tau), W^y_t (z, \tau), W^o_t (z, \tau), J^y_t (z, \tau), J^o_t (z, \tau)$ satisfy equations (4) – (9), respectively, in every period $t$.

2. Separation: Given the sequence of value functions $(U^y_t, U^o_t (\tau), W^y_t (z, \tau), W^o_t (z, \tau), J^y_t (z, \tau), J^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, the separation decisions $\xi^y_t (\tau), \xi^o_t (\tau)$ satisfy equations (14) and (15), respectively, in every period $t$.

3. Nash-bargaining: Given $(\theta_t)_{t = t_0, \ldots, t_1}$, $(\kappa_t)_{t = t_0, \ldots, t_1}$ and the sequence of value functions $(U^y_t, U^o_t (\tau), W^y_t (z, \tau), W^o_t (z, \tau), J^y_t (z, \tau), J^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, the wage functions $w^y_t (z, \tau), w^o_t (z, \tau)$ solve equations (12) and (13), respectively, in every period $t$ in matches where $z \geq \xi^y_t (\tau)$ and $i \in \{y, o\}$.

4. Free-entry: Given $(J^y_{t+1} (z_0, 0))_{t = t_0, \ldots, t_1}$, labour market tightness $(\theta_t)_{t = t_0, \ldots, t_1}$ is the solution to equation (26).

5. Balanced budget condition: Given the sequence of wage functions $(w^y_t (z, \tau), w^o_t (z, \tau))_{t = t_0, \ldots, t_1}$ and the sequence of distribution of workers across states of nature $(\mu^y_t, \mu^o_t (\tau), \lambda^y_t (z, \tau), \lambda^o_t (z, \tau))_{t = t_0, \ldots, t_1}$, the solution to equation (27) is the solution to equation (27).

6. Law of motion: Given $(\theta_t)_{t = t_0, \ldots, t_1}$ and the sequence of rules for separation decisions $(\xi^y_t (\tau), \xi^o_t (\tau))_{t = t_0, \ldots, t_1}$, the distribution $\mu^y_t, \mu^o_t (\tau), \lambda^y_t (z, \tau), \lambda^o_t (z, \tau)$ evolves between from $t$ to $t+1$ according to the law of motion described in equations (20) – (25).
When all exogenous features of the economic environment (policy parameters, preferences, etc.) remain constant, and because there is no aggregate shock, the economy reaches a steady-state after a possibly long transition path. We use the following definition:

**Definition.** A steady-state equilibrium is the limit of the sequences of a transition path. In a steady-state, conditions (1) – (5) of the transition path are satisfied. A time-invariant condition replaces condition (6): given \( \theta_t \) and the rules for separation decisions \((\bar{z}_\tau(y), \bar{z}_\tau(o))\), the distribution \( \mu_y, \mu_o(\tau) \), \( \lambda_y(z, \tau) \), \( \lambda_o(z, \tau) \) is invariant for the law of motion described in equations (20) – (25).

Before turning to numerical applications, we remark on a difference between the aggregate quantities of this economy, namely labour-market tightness \( \theta_t \) and the tax rate \( \kappa_t \). Notice that, on the one hand, \( \theta_t \) is a forward-looking variable as per equation (26). Thus, we can proceed backwards from steady-state \( t_1 \) in order to construct the time-path \( (\theta_t)_{t=0,\ldots,t_1} \). On the other hand, the tax rate \( \kappa_t \) depends on wages negotiated in period \( t \) and on the distribution of workers across employment status, productivity levels, tenure and age groups, which is backward-looking due to stock-flow equations. As a result, computing a transition path requires knowledge of the entire sequence \( (\kappa_t)_{t=0,\ldots,t_1} \). Yet, a key feature of our environment is that decisions along the transition path depend on the aggregate state of the economy only through labour-market tightness and the payroll tax. Appendix A presents our numerical methodology for computing transition paths and steady-state equilibria.

### 3 Calibration and steady-state outcomes

This section describes our calibration and characterizes the steady-state of the benchmark economy. We select parameters to reproduce a set of informative data moments for Spain in 2006-2007, i.e. just before the outbreak of the Great Recession.

#### 3.1 Calibration procedure

We need a number of preliminary specification in order to list the parameters of the model. Firstly, we specialize the Markov process for idiosyncratic productivity as follows. We assume that \( z \) takes values in the interval \([0, 1]\) and remains unchanged with probability \( \pi_z \). With complementary probability, \( z \) switches to a value \( z' \) which is drawn from a Normal distribution with mean \( z \) and standard deviation \( \sigma_z \), truncated and normalized to integrate to one over the support of productivity. Next, as indicated in Subsection 2.1, jobs are also subject to an exogenous separation shock; we posit that this shock is realized with per-period probability \( \delta \).

Under these specifications, the model has 14 parameters, namely \( \{r, \eta, \gamma, \chi, T, \psi, \beta, A, k, b, \delta, z_0, \pi_z, \sigma_z\} \). The first seven parameters are set outside the model while the remaining ones are calibrated. Throughout, we interpret a model period as one quarter.

**Parameters set externally**

The first seven rows of Table 1 report parameter values set outside the model. The chosen interest rate is set at \( r = 0.01 \) to yield an annual interest rate of about 4 percent. The coefficient of relative risk
Table 1. Parameter values (one model period is one quarter)

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated externally</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\eta$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ageing probability</td>
<td>$\gamma$</td>
<td>1/120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retirement probability</td>
<td>$\chi$</td>
<td>1/40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cap on tenure</td>
<td>$T$</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching function</td>
<td>$\psi$</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\beta$</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Calibrated internally</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching function</td>
<td>$A$</td>
<td>0.40</td>
<td>Job-finding prob. (%)</td>
<td>40.0</td>
</tr>
<tr>
<td>Vacancy cost</td>
<td>$k$</td>
<td>0.134</td>
<td>Tightness (norm.)</td>
<td>1.00</td>
</tr>
<tr>
<td>Unemployment benefits</td>
<td>$b$</td>
<td>0.258</td>
<td>Replacement rate (%)</td>
<td>58.0</td>
</tr>
<tr>
<td>Exogenous separation</td>
<td>$\delta$</td>
<td>0.006</td>
<td>Fraction of quits (%)</td>
<td>20.0</td>
</tr>
<tr>
<td>Initial productivity</td>
<td>$z_0$</td>
<td>0.143</td>
<td>Job destr. ($\leq$2 years, %)</td>
<td>2.08</td>
</tr>
<tr>
<td>Persistence of productivity</td>
<td>$\pi_z$</td>
<td>0.221</td>
<td>Job destr. ($&gt;2$ years, %)</td>
<td>7.45</td>
</tr>
<tr>
<td>S.d. of productivity draws</td>
<td>$\sigma_z$</td>
<td>0.200</td>
<td>Non-empl. (old, %)</td>
<td>45.0</td>
</tr>
</tbody>
</table>

aversion in (1) is $\eta = 2$, which is a standard value in the literature. The demographic probabilities are set at $\gamma = 1/120$ and $\chi = 1/40$ to match the expected durations of the first (“young”) and second (“old”) phase of a worker’s life cycle. This choice is motivated by our interpretation of young workers as those aged 25–54, and older workers as those aged 55–64.\footnote{Notice that this is also consistent with the observation that workers aged 55–64 account for about 25 percent of the working-age population in Spain.} We set the cap on tenure, $T$, equal to 120 model periods, i.e., 30 years.\footnote{That is, with a deterministic lifecycle, no worker (including the young) would ever reach the maximum tenure level.} Finally, following standard practice in the literature (see Petrongolo and Pissarides [2001]), the unemployment elasticity of the number of matches and worker’s bargaining power are set to $\psi = \beta = 0.5$.

**Calibrated parameters**

The remaining seven rows in Table\footnote{Rebollo-Sanz [2012] reports that quits account for 22 percent of all separations over the years 2000–2007. This was a period of economic expansion in Spain, and job-to-job transitions are notoriously pro-cyclical. For these reasons, we use a more conservative figure of 20 percent.} show the parameters set within the model to match the following moment conditions, most of which are obtained from the Spanish Labour Force Survey for 2006-2007: (1) the average unemployment spell for young workers lasts for 2.5 quarters, i.e., 7.5 months; (2) the replacement rate of unemployment benefits, defined as the ratio between the benefit payment $b$ and the average wage $\tilde{w}$, is 58 percent; (3) the quarterly job destruction rate for temporary jobs is 7.5 percent (García Pérez and Osuna [2014]); (4) the quarterly job destruction rate for permanent jobs is 2.1 percent (García Pérez and Osuna [2014]); (5) the non-employment rate among (male) workers aged 55-64 is 45 percent; (6) quits account for about 20 percent of all separations (Rebollo-Sanz [2012]).\footnote{Rebollo-Sanz (2012) reports that quits account for 22 percent of all separations over the years 2000–2007. This was a period of economic expansion in Spain, and job-to-job transitions are notoriously pro-cyclical. For these reasons, we use a more conservative figure of 20 percent.} To be precise about (5), we consider the non-employment rate of male workers instead of the overall non-employment rate because the latter is driven down by the low participation rates of women aged 55 to 64 reasons, which cannot be explained by the model. As for quits (6), we use this observation
to calibrate $\delta$. That is, $\delta$ puts an upper bound on the number of job separations that could be deterred by enforcing tougher employment protection.\footnote{Following an exogenous separation, we assume that the firms pays the worker the severance package to which she/he is entitled. That is, we do not interpret the $\delta$ shock as a quit decision (it is not a decision). Rather, we use $\delta$ to discipline the elasticity of the separation rate to changes in the employment protection scheme. In sensitivity checks, we also show that our results are robust to the converse situation, i.e. to assuming that severance payments are waived in the event of an exogenous separation.} This upper bound cannot be directly observed in the data, which is why we use a proxy for it. These observations yield a total of six calibration targets. Finally, we follow standard practices and normalize labour market tightness $\theta$ to unity; we use the free-entry condition to pin down the vacancy-posting cost $k$.

**Benchmark severance payments**

The crux of our analysis relates to the severance pay function. We follow\cite{Bentolila2012} and \cite{GarciaPerez2014} in computing a function of job tenure that stands similar to EPL in Spain prior to the reform undertaken in February 2012.\footnote{In this reform, severance pay for unfair dismissals of permanent workers went down from 45 to 33 d.w.y.s. while termination costs to temporary workers went up from 8 to 12 d.w.y.s. (see\cite{GarciaPerez2014} for details). We use the pre-reform EPL scheme since our calibration targets are drawn from pre-2012 data.} As the latter authors do, we specify it as a function of the average annual wage in the labour market, rather than as a function of individual tenure and/or productivity. The reason for this choice is that it makes it easier to solve the model because we do not need any knowledge on the wage profile when specifying $\phi(\tau)$.\footnote{Observe that the average wage is an equilibrium outcome of the model, not a pre-specified parameter. Thus, upon computing a steady-state equilibrium, we add an outside loop to iterate over the average wage used to specify the severance payment function.}

In particular, we use the following pieces of information to compute $\phi(\tau)$. We identify the first two years of employment with those temporary contracts that prevail in the Spanish labour market. These contracts feature termination costs of 8 d.w.y.s., which represents 2.2 percent ($= 8/365$) in terms of average yearly wage. If the worker is not dismissed before the end of this period, we identify the subsequent periods of employment as those regulated by permanent contracts. Workers on permanent jobs are entitled to 45 d.w.y.s. since joining the firm, with a maximum of 3.5 annual wages, under an unfair dismissal which represented most of the dismissals until 2012. For instance, a worker who is employed at the same firm for more than two years and loses her/his job in the third year is entitled to 37 percent ($= 3 \times 45/365$) of average yearly wage.

Using these observations, the severance cost function used in the benchmark economy for workers with tenure $\tau$ (in quarter) at the current firm is computed as follows:

\[
\phi(\tau) = \begin{cases} 
(8/365) \times \bar{w} \times \tau, & 1 \leq \tau \leq 8 \\
(45/365) \times \bar{w} \times \tau, & 9 \leq \tau \leq 113 \\
(45/365) \times \bar{w} \times 113, & \tau > 113
\end{cases}
\] (28)

Figure I depicts this function with tenure (in quarters) in the horizontal axis and a multiple of the average annual wage in the vertical axis.
3.2 Benchmark economy

Table 2 reports a selection of aggregate statistics in our benchmark economy which, whenever possible, are compared to their empirical counterparts. As can be observed in the table, the model fits the moments targeted in our calibration strategy almost exactly. Moreover, most non-targeted moments are reasonably close to the corresponding data values. The simulated unemployment rate among young workers in the benchmark economy is 8.9 percent, while the corresponding value in the data is slightly lower at 7.8 percent. This discrepancy may not be surprising in light of the fact that the Spanish economy was growing at a faster-than-average rate prior to the onset of the Great Recession, a feature that our model cannot capture in the absence of aggregate shocks. The aggregate non-employment rate among all workers in our benchmark economy is 17.2 percent against 17.9 percent in the data. Regarding wage differentials, the model generates a gap of 17.3 percent ($0.5076/0.4327 - 1$) between the average wage of older workers and young workers. This value compares reasonably well with the wage gap observed in the Wage Structure Survey (EES), namely 14.5 percent (€1,522 for younger vs. €1,741 for older workers). Finally, the budget-balancing payroll tax is 12.71 percent in the model, which fares reasonably well against the actual value of 14.95 percent (the latter is computed as the sum of employers’ and workers’ social security contributions that are related to unemployment benefits).

To provide more insight on the equilibrium behavior of worker-firm matches in the model, note that the calibrated productivity process implies that new jobs start at the lower end of the productivity domain ($z_0 = 0.14$). Conditional on not being separated exogenously, worker-firm matches then experience a new productivity draw on average every $1/\pi z = 4.5$ quarters. Recall that a new productivity value $z'$ is drawn from a Normal distribution which is centered around the current productivity $z$. This entails an additional element of persistence while maintaining the possibility to “climb up” or “fall down” the productivity ladder. Figure 2 depicts the job destruction region for young and older work-
ers in our benchmark economy (this is a graphical representation of the productivity cutoffs $\tau(z)$ and $\tau(o)$). First, note that newly-formed matches start at a productivity level that is very close to the corresponding separation threshold. As a result, most matches that face an adverse productivity draw over the first quarters of tenure will be dissolved endogenously. This feature of the model makes new jobs relatively fragile and rationalizes the high job destruction rate at short tenures. By contrast, matches that experience a positive productivity draw move towards the upper region of the productivity domain and thus become less susceptible. These “career jobs” are bound to be converted into permanent jobs and they are characterized by a substantially lower job destruction rate at longer tenures.

![Figure 2. Separation thresholds in the benchmark economy](image)

The plot shows the separation regions for young (blue region) and old workers (yellow region, which includes the blue region). Separation regions are plotted as a function of tenure ($\tau$).

Next, as evidenced in Figure 2 there are characteristic spikes in the job destruction region at $\tau = 8$. These reflect the discontinuous jump (“wall”) in the firing cost schedule if a temporary contract is converted into a permanent contract (Figure 1). Since workers are risk averse, future severance payments are only partially internalized through lower wages. This puts a lower bound on workers’ reservation wages and implies that relatively unproductive matches are destroyed as the temporary contract comes to an end. As can be seen in Figure 2 productivity cutoffs are generally higher for older workers, because they have access to buying an annuity. Finally, the job destruction region is decreasing in tenure as it becomes more costly for a firm to fire a worker.

The wage-tenure profiles for young (upper chart) and older workers (lower chart) are depicted in Figure 3. In both charts, the solid line represents the Nash-bargained wage profile for the initial productivity level $z_0 = 0.14$ by tenure. The dotted and the dashed lines represent wage profiles for matches that are operating at selected higher productivity levels. There are several important observations. Firstly, there is a dip in the wage schedule at the end of the first two years of tenure. The key for this result is the shape of the severance pay function and the fact that wages are renegotiated every
Figure 3. Wage function in the benchmark economy: young (upper) and old workers (lower chart)
The plot shows the wage function in the benchmark economy, in low-productivity (solid line), middle-
productivity (dotted line) and high-productivity (dashed line) matches. Wage functions are plotted as
a function of tenure ($\tau$).
Table 2. Benchmark model economy: Comparison with the data

<table>
<thead>
<tr>
<th>Description</th>
<th>Model</th>
<th>Data</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate, young (%)</td>
<td>8.9</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>Non-employment rate, old (%)</td>
<td>45.2</td>
<td>45.0</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Non-employment rate, all (%)</td>
<td>17.9</td>
<td>17.2</td>
<td></td>
</tr>
<tr>
<td>Replacement ratio of benefits, i.e. $b/\bar{w}$ (%)</td>
<td>58.0</td>
<td>58.0</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Average wage, young</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average wage, old</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative wage (young/old)</td>
<td>0.84</td>
<td>0.87(^{(a)})</td>
<td></td>
</tr>
<tr>
<td>Average productivity, young</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average productivity, old</td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job destruction rate, $\leq 2$ years of tenure (%)</td>
<td>7.5</td>
<td>7.5</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Job destruction rate, $&gt; 2$ years of tenure (%)</td>
<td>2.2</td>
<td>2.1</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Share of quits among separation (%)</td>
<td>18.5</td>
<td>20.0</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Job finding rate (in %)</td>
<td>40.0</td>
<td>40.0</td>
<td>part of calibration</td>
</tr>
<tr>
<td>Payroll tax (in %)</td>
<td>12.71</td>
<td>14.95(^{(b)})</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** \(^{(a)}\) Own calculations, based on data from the Spanish Wage Structure Survey (EES): The average wage for young (resp. older) workers is €1,522 (resp. €1,741). \(^{(b)}\) Own calculations based on estimates of the overall payroll tax in Spain, which finances unemployment benefits and active labor market policies. The overall payroll tax paid in Spain by employers and workers is about 38% of the wage.

period: as the worker-firm match approaches the end of the temporary contract, workers are willing to accept lower wages temporarily in exchange for higher future entitlements to severance payments if their job is converted into a permanent contract. Secondly, the wage curve is rather flat for young workers and steeper for old workers. This reflects the degree to which firm-worker matches are willing to internalize future severance payments through lower initial wages. Young workers consume their severance package instantaneously in the period after a layoff. Thus, their outside option of unemployment increases only very gradually with tenure, because a larger severance payment buys them only a one-time increase in the level of consumption (at diminishing marginal utility). By contrast, the wage schedule is much steeper for older workers. As their tenure increases, a more generous entitlement to severance transfers allows them to buy more valuable annuities. In other words, their wage profile resembles more the shape that one would obtain in a Lazear (1990)-type setting where severance payments are fully neutralized.

4 Numerical experiments

This section contains the main results of the paper: we use our model economy as a laboratory to discuss the effects of moving from a dual labour market, as the one in Spain, towards a SOEC.

In the first subsection below, we present our welfare criterion and define the type of SOEC we consider in the sequel. In the second subsection, we characterize allocations along the transition path towards the SOEC. Finally, we analyze the welfare implications of the SOEC and discuss feasibility of the policy change.
4.1 Designing a SOEC

We select a SOEC according to a welfare criterion suitable for steady-state comparisons. This criterion is based on the metric which we introduce below.

Welfare metric

Let \( V_0 \) denote lifetime utility of a worker at \( t_0 \), and let \( V_1 \) be the lifetime utility of the same worker at \( t_1 \). The comparison of \( V_0 \) and \( V_1 \) measures the welfare effect of the change in labor market situation from \( t_0 \) to \( t_1 \). We express this effect in consumption equivalent units (CEU), which are computed as:

\[
1 + \vartheta = \left( \frac{V_1}{V_0} \right)^{\frac{1}{\eta}}
\]

That is, multiplying lifetime consumption of the worker at \( t_0 \) by \( 1 + \vartheta \) makes her/him indifferent between \( t_0 \) and \( t_1 \). Notice that, since we have normalized the value of retirement to zero, lifetime consumption means consumption during the working life.

From the previous expression, we can measure \( \vartheta \) for each individual of the benchmark economy. We shall use this measurement to characterize the welfare implications of introducing a SOEC for the current generation of workers. Meanwhile, for the purpose of drawing steady-state inferences, there is only one meaningful comparison, namely the change in lifetime consumption for newborn workers. Thus, we shall select a SOEC according to the following criterion:

\[
\{ \phi^*(\tau) \}_{\tau=1}^{\infty} = \text{argmax} \{ U^\gamma \}
\]

Parameterizing the SOEC

To define a SOEC, we explore a relatively simple class of severance payment functions, namely a subset of piecewise linear functions of tenure. Specifically, we consider functions of two parameters: (i) the minimum service tenure for eligibility and (ii) days of wages per year of service (d.w.y.s.), conditional on eligibility. This class of severance payment function is easily interpretable and comparable to existing EPL scheme.

Table 3 reports the welfare change associated with various SOEC: on a given row, we fix the minimum service tenure for eligibility and we increase d.w.y.s. gradually along the columns of the table. As illustrated by the numbers in bold fonts, a SOEC improves on the EPL scheme currently in place in the Spanish labour market, and is also preferred to laissez-faire\(^{17}\). In particular, a SOEC with 2 years of minimum service and a slope of 5 d.w.y.s maximizes the steady-state lifetime utility of a newborn worker. In the sequel, we refer to this combination of parameters as the “optimal SOEC”.

\(^{16}\) \( 1 + \vartheta = \exp \left( \frac{r}{\eta} \right) (V_1 - V_0) \) if \( \eta = 1 \), i.e., when the utility function is logarithmic.

\(^{17}\) With some abuse of terminology, we use the term “laissez-faire” to refer to the economy with no severance payment.
Table 3. Steady-state comparisons of various SOEC

<table>
<thead>
<tr>
<th>Initial eligibility (in months)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.17</td>
<td>4.25</td>
<td>4.17</td>
<td>3.98</td>
<td>3.82</td>
<td>3.35</td>
<td>1.28</td>
<td>-0.83</td>
<td>-3.40</td>
</tr>
<tr>
<td>3</td>
<td>4.17</td>
<td>4.27</td>
<td>4.22</td>
<td>4.07</td>
<td>3.94</td>
<td>3.56</td>
<td>1.79</td>
<td>-0.02</td>
<td>-2.36</td>
</tr>
<tr>
<td>6</td>
<td>4.17</td>
<td>4.28</td>
<td>4.25</td>
<td>4.13</td>
<td>4.02</td>
<td>3.69</td>
<td>2.11</td>
<td>0.48</td>
<td>-1.64</td>
</tr>
<tr>
<td>9</td>
<td>4.17</td>
<td>4.28</td>
<td>4.27</td>
<td>4.16</td>
<td>4.07</td>
<td>3.77</td>
<td>2.34</td>
<td>0.61</td>
<td>-1.13</td>
</tr>
<tr>
<td>12</td>
<td>4.17</td>
<td>4.29</td>
<td>4.28</td>
<td>4.19</td>
<td>4.11</td>
<td>3.84</td>
<td>2.53</td>
<td>0.92</td>
<td>-0.66</td>
</tr>
<tr>
<td>18</td>
<td>4.17</td>
<td>4.29</td>
<td>4.30</td>
<td>4.23</td>
<td>4.17</td>
<td>3.96</td>
<td>2.83</td>
<td>1.43</td>
<td>0.12</td>
</tr>
<tr>
<td>24</td>
<td>4.17</td>
<td>4.29</td>
<td>4.31</td>
<td>4.27</td>
<td>4.21</td>
<td>4.04</td>
<td>3.07</td>
<td>1.87</td>
<td>0.71</td>
</tr>
</tbody>
</table>

NOTE: An entry in the table is the percentage change in lifetime consumption experienced by a newborn worker.

Results

Figure 4 reports a set of outcomes to compare the benchmark economy with the economy featuring the optimal SOEC. These outcomes draw attention on the fact that the optimal SOEC removes those “revolving doors” in labor market trajectories implied by dual EPL.

As shown in the middle chart in Figure 4, a SOEC reduces job turnover rates at short tenures and increases separation slightly at longer tenure. The lower chart in Figure 4 depicts the average wage by tenure across productivities of surviving matches. As can be seen, in both economies, the model generates an upward-sloping wage-tenure profile. This results is driven by a combination of factors described in the previous section. At short tenures, the average wage tends to increase due to a selection effect, because many jobs experience favorable productivity draws and unproductive jobs get destroyed quickly. At longer tenures, the average wage rises further due to an increasing share of jobs occupied by an older worker. Finally, the optimal SOEC flattens out the wage-profile.

Table 4 reports a set of statistics to compare equilibrium allocation and welfare in the benchmark and in the economy with the optimal SOEC.

To get further insights as to the effects of the optimal SOEC, we decompose the overall effects as:

<table>
<thead>
<tr>
<th>Total effect</th>
<th>Remove wall</th>
<th>Adjust slope</th>
<th>Tightness θ</th>
<th>Payroll tax κ</th>
</tr>
</thead>
<tbody>
<tr>
<td>+4.31%</td>
<td>10.3</td>
<td>45.2</td>
<td>21.2</td>
<td>23.3</td>
</tr>
</tbody>
</table>

That is, first we adjust the severance pay function such that the slope in the first 2 years is 45 d.w.y.s. as well (keeping tightness and tax constant): this is the “remove wall” effect. Then we adjust the slope by rotating the severance pay function to the lower slope of the SOEC (again, keeping tightness and tax constant). This yields higher entry wages and a flatter wage-tenure profile. Then we plug in tightness computed from the economy with the optimal SOEC (keeping tax constant), and finally we plug in the payroll tax from the SOEC economy.

4.2 Transition dynamics [TBC]

The steady-state comparison is between the initial and new steady-state, indexed by \( t_0 \) and \( t_1 \), respectively. The more interesting thought experiment is the following: we introduce the severance payment
**Figure 4.** Current vs. optimal SOEC: Steady-state comparison

The upper chart compares the current severance payment scheme (reproduced from Figure 1) and the optimal SOEC. The middle chart shows the probability of job separation conditional on job tenure in the benchmark economy and in the economy with the optimal SOEC. The lower chart reports the average wage conditional on job tenure in the benchmark economy and in the economy with the optimal SOEC.
Table 4. Current vs. optimal SOEC: Steady-state comparison

<table>
<thead>
<tr>
<th>Description</th>
<th>Data</th>
<th>Baseline</th>
<th>Optimal SOEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate, young (%)</td>
<td>7.8</td>
<td>8.9</td>
<td>8.0</td>
</tr>
<tr>
<td>Non-employment rate, old (%)</td>
<td>45.0</td>
<td>45.2</td>
<td>42.6</td>
</tr>
<tr>
<td>Non-employment rate, all (%)</td>
<td>17.2</td>
<td>17.9</td>
<td>16.6</td>
</tr>
<tr>
<td>Average wage, young</td>
<td>–</td>
<td>0.43</td>
<td>0.47</td>
</tr>
<tr>
<td>Average wage, old</td>
<td>–</td>
<td>0.51</td>
<td>0.42</td>
</tr>
<tr>
<td>Average productivity, young</td>
<td>–</td>
<td>0.55</td>
<td>0.56</td>
</tr>
<tr>
<td>Average productivity, old</td>
<td>–</td>
<td>0.73</td>
<td>0.72</td>
</tr>
<tr>
<td>Job destruction rate, less than 2 years of tenure (%)</td>
<td>7.5</td>
<td>7.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Job destruction rate, more than 2 years of tenure (%)</td>
<td>2.1</td>
<td>2.2</td>
<td>2.3</td>
</tr>
<tr>
<td>Job finding rate (in %)</td>
<td>40.0</td>
<td>40.0</td>
<td>45.7</td>
</tr>
<tr>
<td>Labor market tightness, i.e. (v/u)</td>
<td>–</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>Payroll tax (in %)</td>
<td>14.95</td>
<td>12.71</td>
<td>11.25</td>
</tr>
<tr>
<td>Total output (relative to baseline)</td>
<td>–</td>
<td>–</td>
<td>+1.88%</td>
</tr>
<tr>
<td>Welfare of a newborn worker (relative to baseline)</td>
<td>–</td>
<td>–</td>
<td>+4.31%</td>
</tr>
</tbody>
</table>

function that exists in \(t_1\) for every new match that is being formed at time \(t_0\). Agents in matches that already exist at time \(t_0\) now decide whether to remain matched or to separate. Along the transition path, the tax rate adjusts. This affects all matches in the economy, not only those with the new contract.

Figure 5 shows the time path of several labour market variables during the transition towards a SOEC, which is introduced in a non-retroactive manner.

4.3 Welfare analysis [TBC]

We measure average welfare across the current population when taking into account the transition. Specifically,

\[
\sum_{\tau} \sum_{z} \left( \lambda_{t_0}^y (z, \tau) W_{t_0}^y (z, \tau) + \lambda_{t_0}^o (z, \tau) W_{t_0}^o (z, \tau) \right) + \sum_{\tau} \mu_{t_0}^o (\tau) U_{t_0}^o (\tau) + \mu_{t_0}^y U_{t_0}^y
\]

where the distribution is from the benchmark economy, i.e., the time-invariant distribution just before the government enforces the \(t_1\) contract for new hires.

Table 5 reports the welfare effects of moving towards a SOEC for the current generation of workers. In order to gain some insights into the results, columns 2-4 report the welfare change associated with three partial-equilibrium experiments: first we introduce the SOEC, then we adjust labour-market tightness, and finally we adjust the payroll tax.

5 Sensitivity analysis [TBC]

6 Conclusion [TBC]
### Table 5. Current vs. optimal SOEC: Welfare and role of transition dynamics

<table>
<thead>
<tr>
<th></th>
<th>Average welfare gain</th>
<th>Effect of SOEC</th>
<th>Effect of θ</th>
<th>Effect of κ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>A. Non-retroactive reform</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>3.054</td>
<td>1.573</td>
<td>0.624</td>
<td>0.857</td>
</tr>
<tr>
<td>Young workers</td>
<td>3.595</td>
<td>1.861</td>
<td>0.737</td>
<td>0.997</td>
</tr>
<tr>
<td>[0.044, 4.234]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Older workers</td>
<td>0.098</td>
<td>0.0</td>
<td>0.0</td>
<td>0.098</td>
</tr>
<tr>
<td>[-0.337, 0.180]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Retroactive reform</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>1.257</td>
<td>-0.231</td>
<td>0.629</td>
<td>0.859</td>
</tr>
<tr>
<td>Young workers</td>
<td>2.373</td>
<td>0.633</td>
<td>0.744</td>
<td>0.996</td>
</tr>
<tr>
<td>[-1.098, 9.091]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Older workers</td>
<td>-4.846</td>
<td>-4.960</td>
<td>0.0</td>
<td>0.114</td>
</tr>
<tr>
<td>[-8.101, 3.204]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** An entry in the table is the percentage change in lifetime consumption associated with the policy reform (column 1), which we further decompose into three consecutive adjustments: effects of introducing a SOEC (column 2), the resulting change in θ (column 3) and the resulting change in κ (column 4). The numbers in brackets in column 1 are the minimum and maximum welfare change experienced by workers.

### Table 6. Optimal SOEC: Sensitivity analysis

<table>
<thead>
<tr>
<th>Initial eligibility (in months)</th>
<th>Slope (in d.w.y.s.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>24</td>
</tr>
<tr>
<td>Risk aversion: η = 1</td>
<td>24</td>
</tr>
<tr>
<td>Risk aversion: η = 3</td>
<td>24</td>
</tr>
<tr>
<td>UI replacement rate: 50%</td>
<td>24</td>
</tr>
<tr>
<td>UI replacement rate: 65%</td>
<td>laissez-faire</td>
</tr>
<tr>
<td>Red-tape costs: 50%</td>
<td>24</td>
</tr>
<tr>
<td>Quits vs. layoffs</td>
<td>24</td>
</tr>
</tbody>
</table>

**Note:** Each row in the table describes the optimal SOEC obtained under alternative calibration of the economy. Red-tape costs indicate that the worker receives only a fraction (here: 50 percent) of the penalty paid by the firm upon dismissal. Quits vs layoffs indicate that a separation caused by the exogenous separation shock δ is interpreted as a quit rather than a layoff and therefore that the severance pay is waived.
Figure 5. Transition dynamics: A non-retroactive introduction of a SOEC
The charts display the time path of several labour market variables during the transition towards a SOEC introduced in a non-retroactive manner. On the x-axis, time is measured in years relative to the introduction of the SOEC, which occurs in period 0.
References


A Numerical appendix

This appendix details our numerical methodology to compute steady-state equilibria and transition paths of the model economy presented in Section 2.

A.1 Computing steady-states

To indicate that the economy is in steady-state, we drop the time subscript throughout this section. A steady-state is nontrivial to compute because the continuation values in certain labour market states are unknown. Specifically, we need to solve for $U^0$, $W^0(z, T)$, $J^0(z, T)$ and $J^0(z, T)$, as well as for $w^0(z, T)$ and $w^0(z, T)$. The algorithm is as follows:

1. Solve for $W^0(z, T)$, $J^0(z, T)$ and $w^0(z, T)$ using the following steps:
   (a) Set initial guesses $\hat{W}^0(z, T)$, $\hat{J}^0(z, T)$, $\hat{w}^0(z, T)$, where we use $\hat{\cdot}$ to indicate a guess.
   (b) Compute the reservation wage of the worker $w^0(z, T)$ and that of the firm $\bar{w}^0(z, T)$ associated with $\hat{W}^0(z, T)$ and $\hat{J}^0(z, T)$ using equations (17) and (18).
   (c) If $w^0(z, T) \leq \bar{w}^0(z, T)$, then solve for the Nash-bargained wage $w^0$ using the associated first-order condition:

   \[
   \frac{\beta}{1+\kappa_r} \left( z - (1+\kappa)w + \frac{1-\chi}{1+r} \sum_{z'} \pi_{z,z'} \max \left\{ \hat{J}^0(z', T), -\Phi(z') \right\} + \Phi(T) \right) = \frac{1-\beta}{u'(w)} \left( u(w) + \frac{1-\chi}{1+r} \sum_{z'} \pi_{z,z'} \max \left\{ \hat{W}^0(z, T), U^0(T) \right\} \right)
   \]

   and update $\hat{w}^0(z, T)$ using this value (Observe that $U^0(T)$ is completely determined, as per equation (5)). This is a nonlinear equation, which we solve using the bisection method.
   If, on the other hand, $\bar{w}^0(z, T) < w^0(z, T)$, set $\hat{w}^0(z, T) = \frac{1}{2} (\bar{w}^0(z, T) + w^0(z, T))$.
   (d) Update $\hat{W}^0(z, T)$ and $\hat{J}^0(z, T)$ using equations (7) and (9).
   (e) If initial and updated guesses for value functions and wages are close enough, then we are done. Otherwise, go back to step (1a).

2. Solve for $W^0(z, \tau)$, $J^0(z, \tau)$ and $w^0(z, \tau)$ recursively from $\tau = T - 1$. That is:
   (a) Compute the reservation wage of the worker $w^0(z, \tau)$ and that of the firm $\bar{w}^0(z, \tau)$ using equations (17) and (18). Notice that the continuation values only involve $\tau + 1$, which allows to compute $w^0(z, \tau)$ and $\bar{w}^0(z, \tau)$.
   (b) If $w^0(z, \tau) \leq \bar{w}^0(z, \tau)$, then solve for the Nash-bargained wage using the first-order condition (13). The continuation values in this equation depend on $\tau + 1$ only, and the outside option of the worker $U^0(\tau)$ is pre-determined.
   (c) Compute the value functions $W^0(z, \tau)$ and $J^0(z, \tau)$ from equations (7) and (9).

3. Solve for $U^T$, $W^T(z, \tau)$, $J^T(z, \tau)$ and $w^T(z, \tau)$ using the following steps:
(a) Set an initial guess for $\hat{U}_y$.

(b) Solve for $W^y(z, T), J^y(z, T)$ and $w^y(z, T)$ using a methodology similar to step (1), i.e.:

i. Set initial guesses $\hat{W}^y(z, T), \hat{J}^y(z, T)$ and $\hat{w}^y(z, T)$.

ii. Use the analogon of step (1b) to obtain the reservation wage of the worker and the reservation wage of the firm.

iii. Use the analogon of step (1c) to update the wage. Observe that $\hat{U}_y$ is used as the outside option of the worker in the Nash bargain.

iv. Update $\hat{W}^y(z, T)$ and $\hat{J}^y(z, T)$ using equations (6) and (8).

v. Iterate until convergence.

(c) Solve for $W^y(z, \tau), J^y(z, \tau)$ and $w^y(z, \tau)$ recursively from $\tau = T - 1$ using a methodology similar to step (2). Again, observe that knowledge of $\hat{U}_y$ is required to compute the Nash-bargained wage.

(d) Use the Bellman equation of a young unemployed worker to update $\hat{U}_y$. If the initial and the updated guess are close enough, then we are done. Otherwise, go back to step (3a) using the updated $\hat{U}_y$.

The algorithm above builds on the observation that, in a steady-state, the value functions $U^y, W^y(z, T), W^o(z, T), J^y(z, T)$ and $J^o(z, T)$ are the solution to an infinite-horizon problem, whereas the other value functions associated with employment solve a standard finite-period ($T$) problem and $U^o(\tau)$ is completely determined.

A steady-state also features an equilibrium tuple $(\theta, \kappa)$. Thus, the algorithm is nested into outer loops to iterate on $(\theta, \kappa)$: we fix the payroll tax $\kappa$, solve for labour market tightness $\theta$, and then update $\kappa$ until convergence. In the benchmark economy, our calibration procedure allows to skip the loop for $\theta$ (recall that it is normalized to pin down the vacancy creation cost). Finally, the specification of the $\phi(\tau)$ implies an outer loop to iterate on the average wage $\tilde{w}$.

A.2 Computing transition paths

A transition path between $t_0$ and $t_1$ involves a sequence of value functions, wage functions, rules for separation decisions, labour market tightness, the payroll tax, and the distribution of workers across employment status, productivity levels, tenure and age groups. These sequences satisfy a set of conditions presented in Subsection 2.7.

During the transition towards a new steady-state equilibrium, computations at time $t$ are simplified in that all continuation values depend on time $t + 1$. That is, the transition path eliminates the infinite horizon problem that arises in steady-state. As noted in the text, another simplification comes from the fact that the sequence $(\theta)_t=t_0,\ldots,t_1$ can be constructed backwards as value functions are compute along the path. Meanwhile, there is a problem specific to the transition, namely that it requires knowledge of the time-path of $(\kappa)_t=t_0,\ldots,t_1$. Moreover, there is an additional state variable for employed workers and for the old unemployed, $\varepsilon \in \{t_0, t_1\}$, indicating whether their current labour market status pertains to the contract that existed before $t_0$ ($\varepsilon = t_0$) or to the contract that prevails in $t_1$ ($\varepsilon = t_1$).
The structure of our model implies that, instead of storing the sequence for all the objects of the transition path, we need “only” the distribution of agents at $t_0$ and the sequences $(\theta_t)_{t=0,\ldots,t_1}$, $(w^y(z,\tau,\varepsilon), w^o(z,\tau,\varepsilon))_{t=0,\ldots,t_1}$ and $(\pi^y(\tau,\varepsilon), \pi^o(\tau,\varepsilon))_{t=0,\ldots,t_1}$ to check that a path $(k_t)_{t=0,\ldots,t_1}$ is consistent with the equilibrium budget condition.

Our methodology to compute these sequences is as follows:

1. Compute the steady-state of the economy in period $t_1$.

2. Plug the initial severance payment function (that of $\varepsilon = t_0$) into the outside option of agents at time $t_1$. Compute the wage and value functions of being in a match at time $t_1$ with the outside option set by the $\varepsilon = t_0$ contract.

3. Guess a path for the payroll tax $(\hat{k}_t)_{t=0,\ldots,t_1}$.

4. Solve for value functions, wages, separation decisions and labour market tightness recursively from $t_1 - 1$ until $t_0$ as follows:

   (a) Compute labour market tightness consistent with free-entry at time $t$ and store it.

   (b) Compute the value of searching for a new job at time $t$, $U^y_t$. Note that, in every period of the transition path, a young unemployed worker can only find a job with the $\varepsilon = t_1$ contract applying to this job.

   (c) Solve for the wage functions of older and younger workers at time $t$ and store them. Then compute the associated value functions. Finally, compute the separation decisions at time $t$ and store them.

5. Set the initial distribution of agents to the time-invariant distribution that obtains in the steady-state before $t_0$.

6. Using $(\theta_t)_{t=0,\ldots,t_1}$, $(w^y(z,\tau,\varepsilon), w^o(z,\tau,\varepsilon))_{t=0,\ldots,t_1}$ and $(\pi^y(\tau,\varepsilon), \pi^o(\tau,\varepsilon))_{t=0,\ldots,t_1}$ and the stock-flow equations described in Subsection 2.5, compute the evolution of the distribution during the time path. Each period, compute the realized payroll tax $k_t$ implied by the balanced budget condition in order to obtain $(k_t)_{t=0,\ldots,t_1}$.

7. If $(\hat{k}_t)_{t=0,\ldots,t_1}$ and $(k_t)_{t=0,\ldots,t_1}$ are close enough, then we are done. Otherwise, go back to step (3) with a new guess.

To ensure that the payroll tax obtained at the end of the transition path coincide with the steady-state $t_1$ payroll tax, we allow for a very large number of periods between $t_0$ and $t_1$. In our application, we set the number of period to 1,000 (250 years). After 500 periods, the number of workers who are still employed in the $t_0$ contract is less than 0.02 percent.