Menu Costs, Aggregate Fluctuations and Large Shocks

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*The view expressed are those of the authors, and do not necessarily reflect the official position of the ECB, the Eurosystem or the Central Bank of Hungary
What we do?

▶ Question:

▶ Extent of monetary non-neutrality,

▶ in light of micro-data evidence on price setting
What we do?

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▶ Framework:
  ▶ Menu costs, idiosyncratic shocks, multi-product firms
  ▶ Match frequency, size and dispersion of price changes
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- **Question:**
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  - in light of micro-data evidence on price setting

- **Framework:**
  - Menu costs, idiosyncratic shocks, multi-product firms
  - Match frequency, size and dispersion of price changes

- **Generalize unobserved idiosyncratic shock distribution**
  - Mixture of normals
  - Unsynchronized stochastic volatility
Why we do it?

- Distribution is key
  - Golosov-Lucas (JPE, 2007): Gaussian shocks, near neutrality
  - Midrigan (E, 2011): Poisson shocks, non-neutrality
Why we do it?

- Distribution is key
  - Golosov-Lucas (JPE, 2007): Gaussian shocks, near neutrality
  - Midrigan (E, 2011): Poisson shocks, non-neutrality
- Distribution determines selection (Caplin-Spulber, 1987)
  - Which firms adjust after an aggregate shock
  - High selection in GL
  - Close to random in Midrigan like in Calvo (1983): whoever gets an idiosyncratic shock, adjusts
Preview of results

- Model matches price change size distribution
  - Like Midrigan (2011)
Preview of results

- Model matches price change size distribution
  - Like Midrigan (2011)
- Model predicts near money neutrality
  - In contrast to Midrigan (2011)
Preview of results

- Model matches price change size distribution ▶ SS
  - Like Midrigan (2011)

- Model predicts near money neutrality ▶ IRF
  - In contrast to Midrigan (2011)

- Monetary non-neutrality is not well identified by key steady state moments
Responses to large aggregate shocks facilitate identification

- Fraction of adjusting firms identifies menu costs
Responses to large aggregate shocks facilitate identification

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Responses to VAT changes support our model
GE macro model

- Representative household
  - CES demand with quality shocks ($A_t(i)$),
  - linear labor
GE macro model

- Representative household (HH)
  - CES demand with quality shocks ($A_t(i)$),
  - linear labor

- Heterogeneous firms (Firms)
  - Linear production: $Y_t(i) = L_t(i)/A_t(i)$
  - Idiosyncratic quality shocks $\ln A_t(i) = \ln A_{t-1}(i) + \varepsilon_t(i)$
  - Novel distribution: stochastic volatility: mixed normals

\[
\varepsilon_t(i) = \begin{cases} 
N(0, \lambda^2 \sigma^2) & \text{with probability } p \\
N(0, \sigma^2) & \text{with probability } 1 - p 
\end{cases}
\]

- Menu costs to change prices: $\phi$
- Multi-product firms, correlation $\rho_\varepsilon$, ES: $\gamma$
Equilibrium and solution

- Standard RE equilibrium in the steady state
  - Agents maximize
  - Markets clear
Equilibrium and solution

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- Exogenous, preannounced aggregate policy shocks:
  perfect foresight transition between steady states
Equilibrium and solution

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- Exogenous, preannounced aggregate policy shocks:
  perfect foresight transition between steady states

- Solved numerically with global heterogeneous agent methods
Calibration

- Set some parameters exogenously
  - Correlation of idiosyncratic shocks within firms $\rho_\varepsilon = 0.6$
  - Discount rate: $\beta = 0.96$ yearly
  - Elasticity of substitutions: $\theta = 5, \gamma = 1.1$
  - Trend inflation: $\pi = 4.2\%$
Calibration

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- Calibrate Parameters
  - Menu cost $\phi$
  - Idiosyncratic shock variance $\sigma_\varepsilon$
  - Poisson parameter $\rho$
  - Relative variance parameter $\lambda$
Calibration, cont

- Targeted Moments:
  - frequency and average absolute size of price change
  - kurtosis of the size distribution
  - interquartile range of the absolute size distribution
Calibration, cont

- Targeted Moments
  - frequency and average absolute size of price change
  - kurtosis of the size distribution
  - interquartile range of the absolute size distribution
- Model matches price change distribution
Calibration, cont

- **Target**
  - Targeted Moments
    - frequency and average absolute size of price change
    - kurtosis of the size distribution
    - interquartile range of the absolute size distribution
  
- Model matches price change distribution

- Model predicts near money neutrality
Selection

- High sensitivity to idiosyncratic distribution: why?
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- Pass-through = Intensive margin + Selection
Selection

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  - Measure of adjusters
  - Constant: equals the calibrated frequency
Selection

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- Intensive margin: adjusters change by more
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  - Constant: equals the calibrated frequency
- Selection
  - Response of new adjusters
  - Explains the difference in money neutrality
Selection, cont.

- Selection in single product case (Caballero-Engel, 2007)
  - Multiple of the
  - inaction band width and
  - the density at the thresholds
Selection, cont.

- Selection in single product case (Caballero-Engel, 2007)
  - Multiple of the inaction band width and the density at the thresholds
- Distribution influences both
  - Gaussian: wide inaction band and high mass at thresholds
  - Poisson: narrow inaction band and low mass at thresholds
  - Mixed normal: reinforcing increase in both
Selection, cont.

- In random menu cost ($\rho_{\varepsilon} = 0$):
  - Influence of varying the distribution ($\lambda = 0$ Poisson; $\lambda = 1$ normal; $\lambda = 16\%$ mixed normal)
  - Exercise: vary $\lambda$, keep frequency, size, kurtosis constant
Selection, cont.

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- Standard moments do not identify monetary non-neutrality
  (comp. Alvarez, Bihan, Lippi, 2014)
Why large shocks help identification?

- Fraction of adjusting firms identify the menu cost

- The desired price change distribution reveals itself
Large Shocks

- +5% VAT increase/decrease in Hungary in 2006
  - Government closed the gap between tax rates
  - Processed food sector
  - Gross prices are quoted
  - Easily identifiable cost shocks
Large Shocks

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- Use micro-price data (equivalent Bils-Klenow, 2004)
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- Frequency: 12.6% (steady state)
  - +5% VAT: 62%, −5% VAT: 27%
Large Shocks

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- Use micro-price data (equivalent Bils-Klenow, 2004)
- Frequency: 12.6% (steady state)
  - +5% VAT: 62%, −5% VAT: 27%
- Inflation pass-through \((\pi_t - \bar{\pi})/\Delta \tau_t\):
  - +5% VAT: 99%, −5% VAT: 33%
How our model does?

- **Inflation pass-through**
  - Baseline: matches pass-through, asymmetry
  - Calvo: Small pass-through; no asymmetry
  - Normal shocks: underestimates pass-through, asymmetry
  - Poisson shocks: overestimates pass-through, asymmetry
How our model does?

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- **Frequency effects**
  - Baseline: Matches well
  - Calvo: No frequency effect
  - Normal: Too small
  - Poisson: Too large
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- **Distribution**
Robustness

- Real-effects in various versions
  - Random menu cost ($\rho_{\varepsilon} = 0$)
  - Random menu cost recalibrated for 0 inflation
  - Single product version
  - Baseline version with 2% inflation

Note: Inflation

- Large shocks in the single product version
  - Two-product version: matches price distributions better
  - Different menu cost calibration
  - Similar aggregate implications
  - Random menu cost version fails for large shocks
  - Proportion of free adjusters stays constant with the shock size
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Related literature

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Related literature


- Evidence supporting menu cost assumptions:
  - vs. information frictions: Mankiw-Reis (2002), Woodford (2003), Mackowiak-Wiederholt (2009)
  - vs. fairness: Rotemberg (2005, 2011)
Conclusion

- Menu cost model with mixed normal distribution matches pricing facts well
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- Implies monetary near neutrality
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- In contrast with robust macro-evidence on the real effects of monetary shocks
Conclusion

- Menu cost model with mixed normal distribution matches pricing facts well
- Implies monetary near neutrality
- In contrast with robust macro-evidence on the real effects of monetary shocks
- Need for other frictions: wage-rigidity, real rigidity, information frictions
Thank you!
Steady state distribution of price changes
Impulse responses to a monetary shock
Price changes at the months of tax changes

+5% shock

−5% shock

Data
Mixed
Poisson
Normal
Calibrated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>2.4%</td>
<td>1.6%</td>
<td>5.0%</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>4.3%</td>
<td>4.4%</td>
<td>3.8%</td>
</tr>
<tr>
<td>$p$</td>
<td>91.2%</td>
<td>90.6%</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>8.8%</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
## Matched moments

<table>
<thead>
<tr>
<th>Used in calibration</th>
<th>Data</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Midr. (2011)</td>
</tr>
<tr>
<td>Frequency</td>
<td>12.6%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Size</td>
<td>9.9%</td>
<td>11%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.98</td>
<td>4.02</td>
</tr>
<tr>
<td>Interquartile range</td>
<td>8.13%</td>
<td>8%</td>
</tr>
<tr>
<td>Inflation</td>
<td>4.23%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Simulated effects of large shocks

![Graphs showing pass-through, cumulative real-effect, and frequency distributions for mixed, Poisson, Normal, and Calvo models.](image)
Monthly inflation
Pass-through

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
<th>Calvo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass through</td>
<td>+5%</td>
<td>99%</td>
<td>88%</td>
<td>143%</td>
<td>49%</td>
<td>8.0%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>33%</td>
<td>27%</td>
<td>12%</td>
<td>39%</td>
<td>6.6%</td>
</tr>
</tbody>
</table>
## Frequency

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
<th>Calvo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>+5%</td>
<td>62%</td>
<td>55%</td>
<td>90%</td>
<td>25%</td>
<td>12.6%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>27%</td>
<td>19%</td>
<td>11%</td>
<td>17%</td>
<td>12.6%</td>
</tr>
</tbody>
</table>
### Kurtosis

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>+5%</td>
<td>8.1</td>
<td>13.1</td>
<td>21.3</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>9.2</td>
<td>5.9</td>
<td>3.4</td>
<td>3.4</td>
</tr>
</tbody>
</table>
# Interquartile range

<table>
<thead>
<tr>
<th>Moment</th>
<th>Size</th>
<th>Data</th>
<th>Mixed</th>
<th>Poisson</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interquartile range</td>
<td>+5%</td>
<td>5.9</td>
<td>4.3</td>
<td>2.7</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>5.0</td>
<td>5.8</td>
<td>11.5</td>
<td>6.6</td>
</tr>
</tbody>
</table>
Varying the relative variability $\lambda$

- **Pass-through**
  - Blue: Intensive
  - Red: Selection
  - Green: Extensive

- **Cumulative real effects**

- **Inaction band width**

- **Mass at inactband**
Desired price change distributions and inaction bands

Mixed normal pdf

Normal pdf

Poisson pdf
Varying the relative variability $\lambda$, cont.
<table>
<thead>
<tr>
<th>Pass-through</th>
<th>Dec06-Inc09</th>
<th>Inc06-Inc09</th>
</tr>
</thead>
<tbody>
<tr>
<td># of products</td>
<td>29</td>
<td>73</td>
</tr>
<tr>
<td>Inflation</td>
<td>4.7%</td>
<td>4.7%</td>
</tr>
<tr>
<td>Frequency</td>
<td>12.9%</td>
<td>12.4%</td>
</tr>
<tr>
<td>2006 January, -5%</td>
<td>31.3%</td>
<td></td>
</tr>
<tr>
<td>2006 September, +5%</td>
<td></td>
<td>88%</td>
</tr>
<tr>
<td>2009 July +5%</td>
<td>68.8%</td>
<td>51.1%</td>
</tr>
</tbody>
</table>
Household

- Maximizes utility

\[
\max_{\{C_t(i,g), L_t\}} E \sum_{t=0}^{\infty} \beta^t (\log C_t - \mu L_t),
\]
Household

- Maximizes utility

\[
\max_{\{C_t(i,g), L_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t (\log C_t - \mu L_t),
\]

- subject to

\[
\int_i \sum_g P_t(i, g) C_t(i, g) + B_{t+1}/R_t = B_t + W_t L_t + \tilde{\Pi}_t + T_t,
\]
Household

- Maximizes utility

\[
\max_{\{C_t(i,g),L_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t (\log C_t - \mu L_t),
\]

- subject to

\[
\int \sum_i \sum_g P_t(i,g)C_t(i,g) + B_{t+1}/R_t = B_t + W_t L_t + \tilde{\Pi}_t + T_t,
\]

- CES aggregator:

\[
C_t = \left( \int C_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}}
\]

\[
C_t(i) = \left( \frac{1}{G} \sum_{g=1}^{G} [A_t(i,g)C_t(i,g)]^{(\gamma-1)/\gamma} \right)^{\gamma/(\gamma-1)}
\]
Household, cont.

- **Price indices**

  \[ P_t = \left( \int P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \]

  \[ P_t(i) = \left( \frac{1}{G} \sum_{g=1}^{G} \left[ P_t(i, g)/A_t(i, g) \right]^{1-\gamma} \right)^{1/(1-\gamma)} \]

- **Euler equation**

  \[ \frac{1}{R_t} = \beta E_t \frac{P_tC_t}{P_{t+1}C_{t+1}} \]

- **Product demand**

  \[ C_t(i, g) = A_t(i, g)^{-1} \left( \frac{P_t(i, g)/A_t(i, g)}{P_t(i)} \right)^{-\gamma} \left( \frac{P_t(i)}{P_t} \right)^{-\theta} C_t \]

- **Labor supply**

  \[ \mu C_t = \frac{W_t}{P_t} \]
Firms

- Production function for firm $i$, product $g$:

$$Y_t(i, g) = L_t(i, g)/A_t(i, g),$$
Firms

- Production function for firm $i$, product $g$:

$$Y_t(i, g) = \frac{L_t(i, g)}{A_t(i, g)},$$

- $\ln A_t(i, g) = \ln A_{t-1}(i, g) + \varepsilon_t(i, g)$ idiosyncratic shock, with stochastic volatility

$$\varepsilon_t(i, g) \sim N(0, \delta_t^2), \quad \delta_t^2 = \begin{cases} 
\lambda^2 \sigma^2 & \text{with probability } p \\
\sigma^2 & \text{with probability } 1 - p
\end{cases}$$
Firms

- Production function for firm $i$, product $g$:

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\end{cases}$$

- Golosov-Lucas ($\lambda = 1$), Midrigan ($\lambda = 0$) are special cases
Firms

- Production function for firm $i$, product $g$:

$$Y_t(i, g) = \frac{L_t(i, g)}{A_t(i, g)}$$

- $\ln A_t(i, g) = \ln A_{t-1}(i, g) + \varepsilon_t(i, g)$ idiosyncratic shock, with stochastic volatility

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\sigma^2 & \text{with probability } 1 - p 
\end{cases}$$

- Golosov-Lucas ($\lambda = 1$), Midrigan ($\lambda = 0$) are special cases

- Menu cost $\phi P_t(i)C_t(i)$ to change all prices
Firms, cont.

- Period profit

\[ \tilde{\Pi}_t(i) = \sum_{g=1}^{G} \left[ \frac{1}{1 + \tau_t} P_t(i, g) Y_t(i, g) - W_t L_t(i, g) \right] \]
Firms, cont.

- **Period profit**

\[
\tilde{\Pi}_t(i) = \sum_{g=1}^{G} \left[ \frac{1}{1 + \tau_t} P_t(i, g) Y_t(i, g) - W_t L_t(i, g) \right]
\]

- **Normalized profit (by \( P_t Y_t \)):**

\[
\bar{\Pi}_t(i) = \sum_{g=1}^{G} \left[ \frac{1}{1 + \tau_t} p_t(i, g)^{1-\gamma} - w_t p_t(i, g)^{-\gamma} \right]
\]

\[
\left( \frac{1}{G} \sum_{g=1}^{G} p_t(i, g)^{1-\gamma} \right)^{(\gamma-\theta)/(1-\gamma)}
\]

- **\( p_t(i, g) = \frac{P_t(i, g)}{A_t(i, g)P_t} \) is quality adjusted relative price**

- **\( w_t = W_t/P_t \) is real wage**
Firms’ dynamic program

- Single idiosyncratic state

\[ \mu_{t-1}(i, g) = \frac{P_{t-1}(i, g)}{A_t(i, g)P_{t-1}} = p_{t-1}(i, g)\frac{A_{t-1}}{A_t} \]
Firms’ dynamic program

- Single idiosyncratic state

\[ \mu_{t-1}(i, g) = \frac{P_{t-1}(i, g)}{A_t(i, g)P_{t-1}} = \frac{p_{t-1}(i, g)}{A_{t-1}} \frac{A_{t-1}}{A_t} \]

- Aggregate state variables \( \Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty} \)
Firms’ dynamic program

- Single idiosyncratic state

\[ \mu_{t-1}(i, g) = \frac{P_{t-1}(i, g)}{A_t(i, g)P_{t-1}} = p_{t-1}(i, g)\frac{A_{t-1}}{A_t} \]

- Aggregate state variables \( \Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty} \)

- Two options: change, don’t change
Firms’ dynamic program

- Single idiosyncratic state
  \[ \mu_{t-1}(i, g) = \frac{P_{t-1}(i, g)}{A_t(i, g)P_{t-1}} = p_{t-1}(i, g) \frac{A_{t-1}}{A_t} \]

- Aggregate state variables \( \Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty} \)

- Two options: change, don’t change

- No price-change

\[
V^{NC}(\mu_{t-1}(i), \Omega_t) = \Pi \left( \frac{\mu_{t-1}(i)}{1 + \pi_t}, w_t, \tau_t \right) + \\
\beta E_t V \left( \frac{\mu_{t-1}(i) e^{\varepsilon_{t+1}(i)}}{1 + \pi_t}, \Omega_{t+1} \right),
\]
Firms’ dynamic program, cont.

- Price change

\[
V^C(\Omega_t) = \max_{p_t^*(i)} \{ \Pi(p_t^*(i), w_t, \tau_t) - \phi + \\
\beta E_t V \left( p_t^*(i) \varepsilon_{t+1}(i), \Omega_{t+1} \right) \}.
\]
Firms’ dynamic program, cont.

▶ Price change

\[ V^C(\Omega_t) = \max_{p^*_t(i)} \{ \Pi(p^*_t(i), w_t, \tau_t) - \phi + \beta E_t V \left( p^*_t(i) e^{\varepsilon_{t+1}(i)}, \Omega_{t+1} \right) \}. \]

▶ Value function

\[ V(\mu_{t-1}(i), \Omega_t) = \max_{\{C, NC\}} [V^{NC}(\mu_{t-1}(i), \Omega_t), V^C(\Omega_t)]. \]
Monetary and fiscal policy

- Money supply growth: AR(1) with a drift

\[
\log\left(\frac{M_t}{M_{t-1}}\right) = g_{Mt} = \mu_M + \rho_M g_{Mt-1} + \varepsilon_{Mt}
\]
Monetary and fiscal policy

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- Exogenous value-added tax rate \( \tau_t \)

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- Exogenous value-added tax rate \( \tau_t \)
  \[
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  \]

- Revenues are redistributed lump-sum
  \[
  M_t - M_{t-1} + \frac{\tau_t}{1 + \tau_t} P_t C_t = T_t
  \]
Equilibrium

1. Household maximizes utility subject to budget constraint taking prices, wages as given

2. Firms set nominal prices to maximize their value functions, taking their relative prices and idiosyncratic technology, and the future path of aggregate variables as given.

3. Money supply equals aggregate demand $M_t = P_tC_t$.

4. Money supply growth, taxes follow exogenous path.

5. Market clearing in the goods, bond, labor markets.
Numerical solution: Steady state

► No aggregate uncertainty
Numerical solution: Steady state

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- Aggregate endogenous variables are constant
  - inflation: $\pi_t = \pi = \mu_M / (1 - \rho_M)$, real wage: $w_t$ constant
  - distribution over idiosyncratic state variables $\Gamma$ is time-invariant
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- Iteration in $w$
  1. Guess a value $w_0$ (implies an aggregate supply $Y$)
  2. For $w_i = w_{i-1}$ solve for value and policy functions
  3. Calculate equilibrium quality-adjusted relative price distribution $(\Gamma_i)$
  4. Calculate aggregate demand $(C_t)$
  5. If excess demand, increase $w_{i+1}$; repeat until convergence
Numerical solution: Transitional dynamics

- One time persistent/permanent shock to $g_M, \tau$
  - Shooting
  - Assume new SS reached in $T$ periods
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  - Shooting
  - Assume new SS reached in $T$ periods
  - Iterate on inflation path
    1. Guess inflation path $\{\pi_1, \pi_2, \ldots, \pi_T\}$
    2. Money growth implies an output growth and real wage $\{w_t\}$ path
    3. Calculate value- and policy functions by backward induction
    4. Calculate price distribution path
    5. Obtain resulting inflation path
    6. Do until convergence in inflation paths