Menu Costs, Aggregate Fluctuations and Large Shocks

Peter Karadi – Adam Reiff

European Central Bank* – Central Bank of Hungary*

ESSIM,

May 2015

*The view expressed are those of the authors, and do not necessarily reflect the official position of the ECB, the Eurosystem or the Central Bank of Hungary

What we do?

- Question:
  - Extent of monetary non-neutrality,
  - in light of micro-data evidence on price setting

What we do?

- Question:
  - Extent of monetary non-neutrality,
  - in light of micro-data evidence on price setting
- Framework:
  - Menu costs, idiosyncratic shocks, multi-product firms
  - Match frequency, size and dispersion of price changes

What we do?

- Question:
  - Extent of monetary non-neutrality,
  - in light of micro-data evidence on price setting
- Framework:
  - Menu costs, idiosyncratic shocks, multi-product firms
  - Match frequency, size and dispersion of price changes
- Generalize unobserved idiosyncratic shock distribution
  - Mixture of normals
  - Unsynchronized stochastic volatility

Why we do it?

- Distribution is key
  - Golosov-Lucas (JPE, 2007): Gaussian shocks, near neutrality
  - Midrigan (E, 2011): Poisson shocks, non-neutrality

Why we do it?

- Distribution is key
  - Golosov-Lucas (JPE, 2007): Gaussian shocks, near neutrality
  - Midrigan (E, 2011): Poisson shocks, non-neutrality
- Distribution determines selection (Caplin-Spulber, 1987)
  - *Which* firms adjust after an aggregate shock
  - High selection in GL
  - Close to random in Midrigan like in Calvo (1983): whoever gets an idiosyncratic shock, adjusts

Preview of results

- Model matches price change size distribution SS
  - Like Midrigan (2011)

Preview of results

- Model matches price change size distribution SS
  - Like Midrigan (2011)
- Model predicts near money neutrality IRF
  - In contrast to Midrigan (2011)

Preview of results

- Model matches price change size distribution SS
  - Like Midrigan (2011)
- Model predicts near money neutrality IRF
  - In contrast to Midrigan (2011)
- Monetary non-neutrality is not well identified by key steady state moments

Preview of results, cont.

- Responses to large aggregate shocks facilitate identification
  - Fraction of adjusting firms identifies menu costs

Preview of results, cont.

- Responses to large aggregate shocks facilitate identification
  - Fraction of adjusting firms identifies menu costs
- Responses to VAT changes support our model Shock

GE macro model

- Representative household HH
  - CES demand with quality shocks $(A_t(i))$,
  - linear labor

GE macro model

- Representative household HH
  - CES demand with quality shocks $(A_t(i))$,
  - linear labor
- Heterogeneous firms Firms
  - Linear production: $Y_t(i) = L_t(i)/A_t(i)$
  - Idiosyncratic quality shocks $\ln A_t(i) = \ln A_{t-1}(i) + \varepsilon_t(i)$
  - Novel distribution: stochastic volatility: mixed normals

$$\varepsilon_t(i) = \begin{cases} N(0, \lambda^2\sigma^2) & \text{with probability } p \\ N(0, \sigma^2) & \text{with probability } 1-p \end{cases}$$

  - Menu costs to change prices: ϕ
  - Multi-product firms, correlation ρ_ε, ES: γ

Equilibrium and solution

- Standard RE equilibrium in the steady state Details
  - Agents maximize
  - Markets clear

Equilibrium and solution

- Standard RE equilibrium in the steady state Details
  - Agents maximize
  - Markets clear
- Exogenous, preannounced aggregate policy shocks Policy: perfect foresight transition between steady states

Equilibrium and solution

- Standard RE equilibrium in the steady state Details
  - Agents maximize
  - Markets clear
- Exogenous, preannounced aggregate policy shocks Policy: perfect foresight transition between steady states
- Solved numerically with global heterogeneous agent methods Solution

Calibration

- Set some parameters exogenously
  - Correlation of idiosyncratic shocks within firms $\rho_\varepsilon = 0.6$
  - Discount rate: $\beta = 0.96$ yearly
  - Elasticity of substitutions: $\theta = 5, \gamma = 1.1$
  - Trend inflation: $\pi = 4.2\%$

Calibration

- Set some parameters exogenously
  - Correlation of idiosyncratic shocks within firms $\rho_\varepsilon = 0.6$
  - Discount rate: $\beta = 0.96$ yearly
  - Elasticity of substitutions: $\theta = 5, \gamma = 1.1$
  - Trend inflation: $\pi = 4.2\%$
- Calibrate Parameters
  - Menu cost ϕ
  - Idiosyncratic shock variance σ_ε
  - Poisson parameter p
  - Relative variance parameter λ

Calibration, cont

- Target Targeted Moments
  - frequency and average absolute size of price change
  - kurtosis of the size distribution
  - interquartile range of the absolute size distribution

Calibration, cont

- Target Targeted Moments
  - frequency and average absolute size of price change
  - kurtosis of the size distribution
  - interquartile range of the absolute size distribution
- Model matches price change distribution SS

Calibration, cont

- Target Targeted Moments
  - frequency and average absolute size of price change
  - kurtosis of the size distribution
  - interquartile range of the absolute size distribution
- Model matches price change distribution SS
- Model predicts near money neutrality IRF

Selection

- High sensitivity to idiosyncratic distribution: why?

Selection

- High sensitivity to idiosyncratic distribution: why?
- Pass-through = Intensive margin + Selection

Selection

- High sensitivity to idiosyncratic distribution: why?
- Pass-through = Intensive margin + Selection
- Intensive margin: adjusters change by more
  - Measure of adjusters
  - Constant: equals the calibrated frequency

Selection

- High sensitivity to idiosyncratic distribution: why?
- Pass-through = Intensive margin + Selection
- Intensive margin: adjusters change by more
  - Measure of adjusters
  - Constant: equals the calibrated frequency
- Selection
  - Response of new adjusters
  - Explains the difference in money neutrality

Selection, cont.

- Selection in single product case (Caballero-Engel, 2007)
  - Multiple of the
  - inaction band width and
  - the density at the thresholds

Selection, cont.

- Selection in single product case (Caballero-Engel, 2007)
  - Multiple of the
  - inaction band width and
  - the density at the thresholds
- Distribution influences both Distributions
  - Gaussian: wide inaction band and high mass at thresholds
  - Poisson: narrow inaction band and low mass at thresholds
  - Mixed normal: reinforcing increase in both

Selection, cont.

- In random menu cost ($\rho_\varepsilon = 0$):
  - Influence of varying the distribution ($\lambda = 0$ Poisson; $\lambda = 1$ normal; $\lambda = 16\%$ mixed normal)
  - Exercise: vary λ, keep frequency, size, kurtosis constant

Pass-through

Selection, cont.

- In random menu cost ($\rho_\varepsilon = 0$):
  - Influence of varying the distribution ($\lambda = 0$ Poisson; $\lambda = 1$ normal; $\lambda = 16\%$ mixed normal)
  - Exercise: vary λ, keep frequency, size, kurtosis constant

    Pass-through
- Standard moments do not identify monetary non-neutrality (comp. Alvarez, Bihan, Lippi, 2014)

Why large shocks help identification?

- Fraction of adjusting firms identify the menu cost

  Large shocks

- The desired price change distribution reveals itself

Large Shocks

- +5% VAT increase/decrease in Hungary in 2006
    - Government closed the gap between tax rates
    - Processed food sector
    - Gross prices are quoted
    - Easily identifiable cost shocks

Large Shocks

- +5% VAT increase/decrease in Hungary in 2006
  - Government closed the gap between tax rates
  - Processed food sector
  - Gross prices are quoted
  - Easily identifiable cost shocks
- Use micro-price data (equivalent Bils-Klenow, 2004)

Large Shocks

- +5% VAT increase/decrease in Hungary in 2006
  - Government closed the gap between tax rates
  - Processed food sector
  - Gross prices are quoted
  - Easily identifiable cost shocks
- Use micro-price data (equivalent Bils-Klenow, 2004)
- Frequency: 12.6% (steady state)
  - +5% VAT: 62%, −5% VAT: 27%

Large Shocks

- +5% VAT increase/decrease in Hungary in 2006
  - Government closed the gap between tax rates
  - Processed food sector
  - Gross prices are quoted
  - Easily identifiable cost shocks
- Use micro-price data (equivalent Bils-Klenow, 2004)
- Frequency: 12.6% (steady state)
  - +5% VAT: 62%, −5% VAT: 27%
- Inflation pass-through $((\pi_t - \bar{\pi})/\Delta\tau_t)$:
  - +5% VAT: 99%, −5% VAT: 33%

How our model does?

- Inflation pass-through Inflation Pass-through
  - Baseline: matches pass-through, asymmetry
  - Calvo: Small pass-through; no asymmetry
  - Normal shocks: underestimates pass-through, asymmetry
  - Poisson shocks: overestimates pass-through, asymmetry

How our model does?

- Inflation pass-through Inflation Pass-through
  - Baseline: matches pass-through, asymmetry
  - Calvo: Small pass-through; no asymmetry
  - Normal shocks: underestimates pass-through, asymmetry
  - Poisson shocks: overestimates pass-through, asymmetry
- Frequency effects Frequency
  - Baseline: Matches well
  - Calvo: No frequency effect
  - Normal: Too small
  - Poisson: Too large

How our model does?

- Inflation pass-through Inflation Pass-through
  - Baseline: matches pass-through, asymmetry
  - Calvo: Small pass-through; no asymmetry
  - Normal shocks: underestimates pass-through, asymmetry
  - Poisson shocks: overestimates pass-through, asymmetry
- Frequency effects Frequency
  - Baseline: Matches well
  - Calvo: No frequency effect
  - Normal: Too small
  - Poisson: Too large
- Distribution Kurtosis IQR

Robustness

- Real-effects in various versions Robustness
  - Random menu cost ($\rho_\varepsilon = 0$)
  - Random menu cost recalibrated for 0 inflation
  - Single product version
  - Baseline version with 2% inflation Inflation

Robustness

- Real-effects in various versions Robustness
  - Random menu cost ($\rho_\varepsilon = 0$)
  - Random menu cost recalibrated for 0 inflation
  - Single product version
  - Baseline version with 2% inflation Inflation
- Large shocks in the single product version Large shocks
  - Two-product version: matches price distributions better
  - Different menu cost calibration
  - Similar aggregate implications

Robustness

- Real-effects in various versions Robustness
  - Random menu cost ($\rho_\varepsilon = 0$)
  - Random menu cost recalibrated for 0 inflation
  - Single product version
  - Baseline version with 2% inflation Inflation
- Large shocks in the single product version Large shocks
  - Two-product version: matches price distributions better
  - Different menu cost calibration
  - Similar aggregate implications
- Random menu cost version fails for large shocks
  - Proportion of free adjusters stays constant with the shock size

Related literature

- Monetary non-neutrality and selection: Caplin-Spulber (1987), Golosov-Lucas (2008), Gertler-Leahy (2008), Midrigan (2011), Vavra (2013), Alvarez, Bihan, Lippi (2014)

Related literature

- Monetary non-neutrality and selection: Caplin-Spulber (1987), Golosov-Lucas (2008), Gertler-Leahy (2008), Midrigan (2011), Vavra (2013), Alvarez, Bihan, Lippi (2014)
- Evidence supporting state-dependent pricing models: Gagnon (2009, et.al. 2012), Costain-Nakov, 2014 Alvarez et.al. (2012)

Related literature

- Monetary non-neutrality and selection: Caplin-Spulber (1987), Golosov-Lucas (2008), Gertler-Leahy (2008), Midrigan (2011), Vavra (2013), Alvarez, Bihan, Lippi (2014)
- Evidence supporting state-dependent pricing models: Gagnon (2009, et.al. 2012), Costain-Nakov, 2014 Alvarez et.al. (2012)
- Evidence supporting menu cost assumptions:
  - vs. information frictions: Mankiw-Reis (2002), Woodford (2003), Mackowiak-Wiederholt (2009)
  - vs. search frictions: Cabral-Fishman (2012), Yang-Ye (2008)
  - vs. fairness: Rotemberg (2005, 2011)

Conclusion

- Menu cost model with mixed normal distribution matches pricing facts well

Conclusion

- Menu cost model with mixed normal distribution matches pricing facts well
- Implies monetary near neutrality

Conclusion

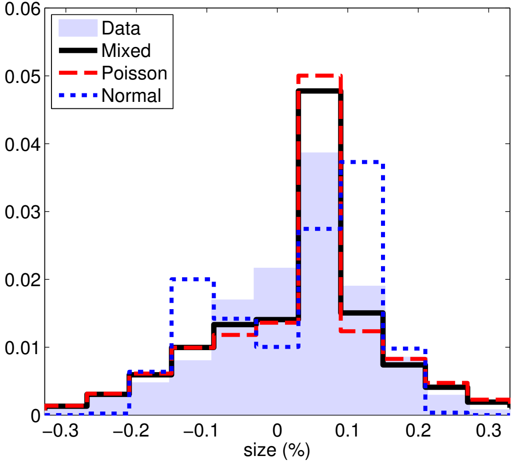
- Menu cost model with mixed normal distribution matches pricing facts well
- Implies monetary near neutrality
- In contrast with robust macro-evidence on the real effects of monetary shocks

Conclusion

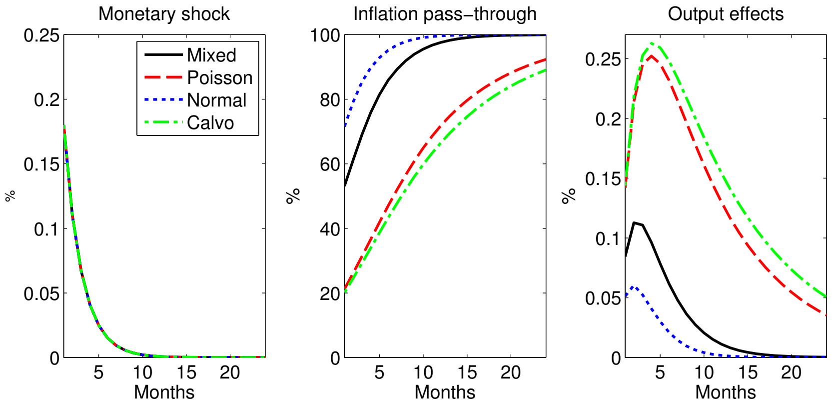
- Menu cost model with mixed normal distribution matches pricing facts well
- Implies monetary near neutrality
- In contrast with robust macro-evidence on the real effects of monetary shocks
- Need for other frictions: wage-rigidity, real rigidity, information frictions

Thank you!

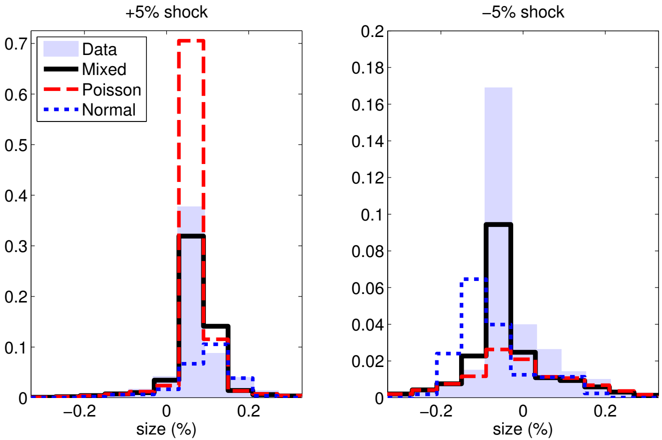
Steady state distribution of price changes



Impulse responses to a monetary shock



Price changes at the months of tax changes



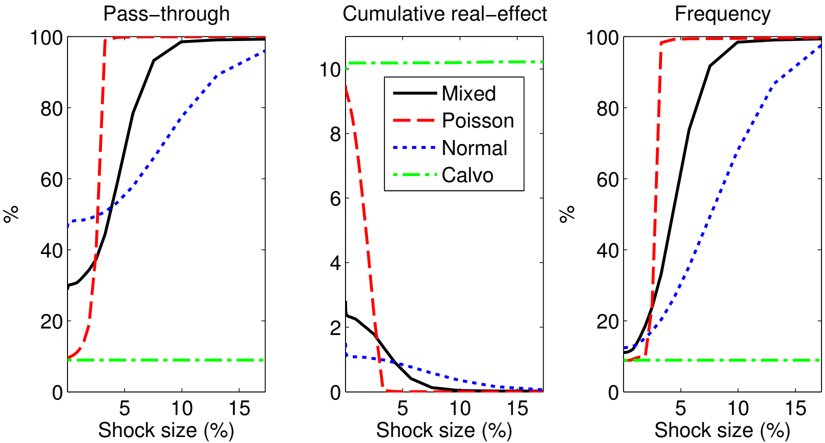
Calibrated parameters

Parameters	Mixed	Poisson	Normal
ϕ	2.4%	1.6%	5.0%
σ_A	4.3%	4.4%	3.8%
p	91.2%	90.6%	0
λ	8.8%	0	1

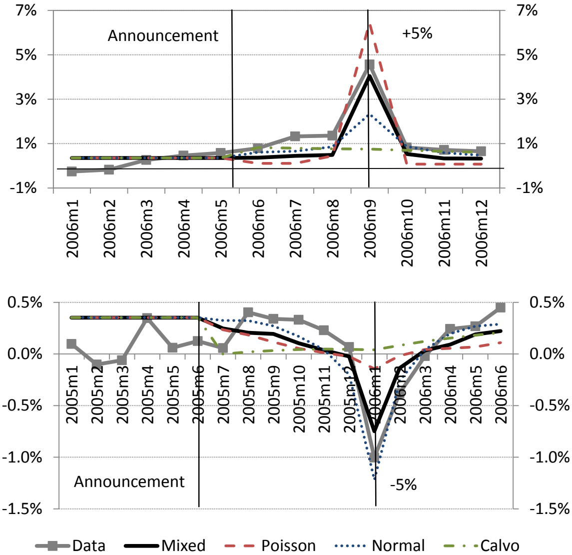
Matched moments

Used in calibration	Data		Models		
	Baseline	Midr. (2011)	Mixed	Poisson	Normal
Frequency	12.6%	11.6%	12.6%	12.6%	12.6%
Size	9.9%	11%	9.9%	9.9%	9.9%
Kurtosis	3.98	4.02	3.98	3.98	*1.97*
Interquartile range	8.13%	8%	8.13%	*9.55%*	*6.3%*
Inflation	4.23%	0%	4.23%	4.23%	4.23%

Simulated effects of large shocks



Monthly inflation



Pass-through

Moment	Size	**Data**	**Mixed**	**Poisson**	**Normal**	**Calvo**
Pass through	+5%	99%	88%	143%	49%	8.0%
	-5%	33%	27%	12%	39%	6.6%

Frequency

Moment	Size	**Data**	**Mixed**	**Poisson**	**Normal**	**Calvo**
Frequency	+5%	62%	55%	90%	25%	12.6%
	-5%	27%	19%	11%	17%	12.6%

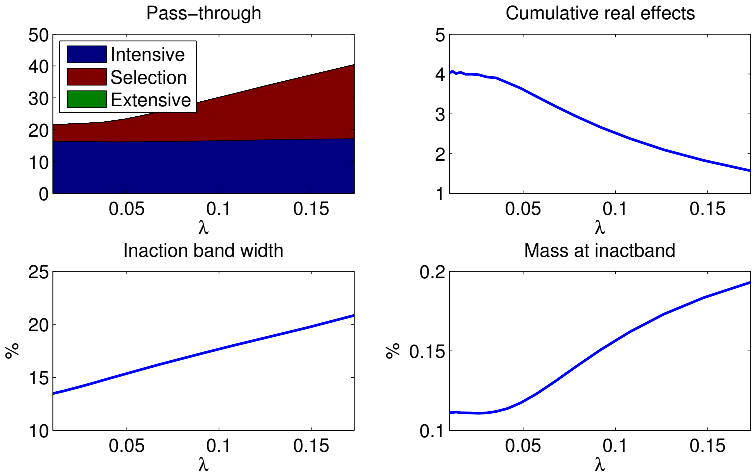
Kurtosis

Moment	Size	**Data**	**Mixed**	**Poisson**	**Normal**
Kurtosis	+5%	8.1	13.1	21.3	5.6
	-5%	9.2	5.9	3.4	3.4

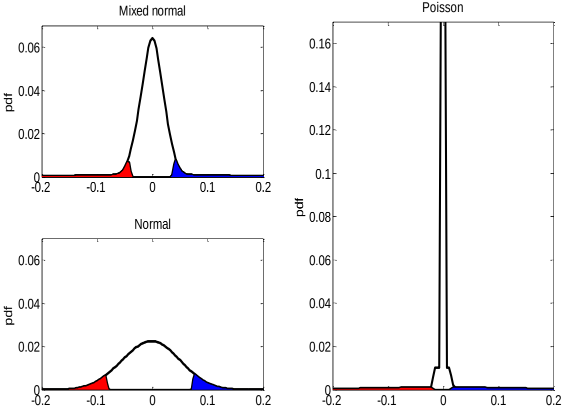
Interquartile range

Moment	Size	**Data**	**Mixed**	**Poisson**	**Normal**
Interquartile range	+5%	5.9	4.3	2.7	6.5
	-5%	5.0	5.8	11.5	6.6

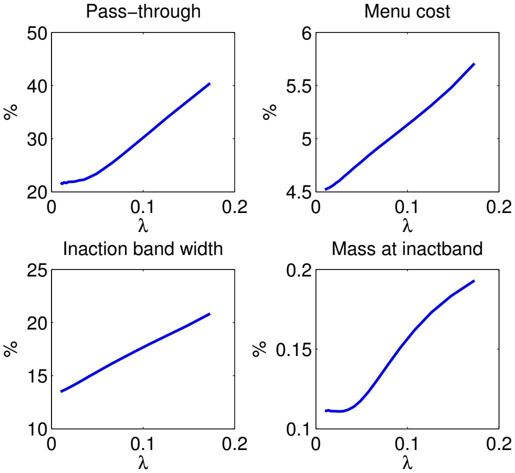
Varying the relative variability λ



Desired price change distributions and inaction bands



Varying the relative variability λ, cont.



Pass-through	**Dec06-Inc09**	**Inc06-Inc09**
# of products	29	73
Inflation	4.7%	4.7%
Frequency	12.9%	12.4%
2006 January, -5%	31.3%	
2006 September, +5%		88%
2009 July +5%	68.8%	51.1%

Household

- Maximizes utility

$$\max_{\{C_t(i,g),L_t\}} E \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu L_t\right),$$

Household

- Maximizes utility

$$\max_{\{C_t(i,g),L_t\}} E \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu L_t\right),$$

- subject to

$$\int_i \sum_g P_t(i,g) C_t(i,g) + B_{t+1}/R_t = B_t + W_t L_t + \tilde{\Pi}_t + T_t,$$

Household

- Maximizes utility

$$\max_{\{C_t(i,g),L_t\}} E \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu L_t\right),$$

- subject to

$$\int_i \sum_g P_t(i,g) C_t(i,g) + B_{t+1}/R_t = B_t + W_t L_t + \tilde{\Pi}_t + T_t,$$

- CES aggregator:

$$C_t = \left(\int C_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

$$C_t(i) = \left(\frac{1}{G} \sum_{g=1}^{G} \left[A_t(i,g) C_t(i,g)\right]^{(\gamma-1)/\gamma}\right)^{\gamma/(\gamma-1)}$$

Household, cont.

- Price indices

$$P_t = \left(\int P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$$

$$P_t(i) = \left(\frac{1}{G}\sum_{g=1}^{G}\left[P_t(i,g)/A_t(i,g)\right]^{1-\gamma}\right)^{1/(1-\gamma)}$$

- Euler equation

$$\frac{1}{R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

- Product demand

$$C_t(i,g) = A_t(i,g)^{-1}\left(\frac{P_t(i,g)/A_t(i,g)}{P_t(i)}\right)^{-\gamma}\left(\frac{P_t(i)}{P_t}\right)^{-\theta} C_t$$

- Labor supply

$$\mu C_t = W_t / P_t$$

Firms

- Production function for firm i, product g:

$$Y_t(i,g) = L_t(i,g)/A_t(i,g),$$

Firms

- Production function for firm i, product g:

$$Y_t(i,g) = L_t(i,g)/A_t(i,g),$$

- $\ln A_t(i,g) = \ln A_{t-1}(i,g) + \varepsilon_t(i,g)$ idiosyncratic shock, with stochastic volatility

$$\varepsilon_t(i,g) \sim N(0,\delta_t^2), \delta_t^2 = \begin{cases} \lambda^2\sigma^2 & \text{with probability } p \\ \sigma^2 & \text{with probability } 1-p \end{cases}$$

Firms

- Production function for firm i, product g:

$$Y_t(i,g) = L_t(i,g)/A_t(i,g),$$

- $\ln A_t(i,g) = \ln A_{t-1}(i,g) + \varepsilon_t(i,g)$ idiosyncratic shock, with stochastic volatility

$$\varepsilon_t(i,g) \sim N(0,\delta_t^2), \delta_t^2 = \begin{cases} \lambda^2\sigma^2 & \text{with probability } p \\ \sigma^2 & \text{with probability } 1-p \end{cases}$$

- Golosov-Lucas ($\lambda = 1$), Midrigan ($\lambda = 0$) are special cases

Firms

- Production function for firm i, product g:

$$Y_t(i,g) = L_t(i,g)/A_t(i,g),$$

- $\ln A_t(i,g) = \ln A_{t-1}(i,g) + \varepsilon_t(i,g)$ idiosyncratic shock, with stochastic volatility

$$\varepsilon_t(i,g) \sim N(0,\delta_t^2), \delta_t^2 = \begin{cases} \lambda^2\sigma^2 & \text{with probability } p \\ \sigma^2 & \text{with probability } 1-p \end{cases}$$

- Golosov-Lucas ($\lambda = 1$), Midrigan ($\lambda = 0$) are special cases
- Menu cost $\phi P_t(i) C_t(i)$ to change all prices

Firms, cont.

- Period profit

$$\tilde{\Pi}_t(i) = \sum_{g=1}^{G} \left[\frac{1}{1+\tau_t} P_t(i,g) Y_t(i,g) - W_t L_t(i,g) \right]$$

Firms, cont.

- Period profit

$$\tilde{\Pi}_t(i) = \sum_{g=1}^{G} \left[\frac{1}{1+\tau_t} P_t(i,g) Y_t(i,g) - W_t L_t(i,g) \right]$$

- Normalized profit (by $P_t Y_t$):

$$\bar{\Pi}_t(i) = \sum_{g=1}^{G} \left[\frac{1}{1+\tau_t} p_t(i,g)^{1-\gamma} - w_t p_t(i,g)^{-\gamma} \right] \left(\frac{1}{G} \sum_{g=1}^{G} p_t(i,g)^{1-\gamma} \right)^{(\gamma-\theta)/(1-\gamma)}$$

  - $p_t(i,g) = \frac{P_t(i,g)}{A_t(i,g) P_t}$ is quality adjusted relative price
  - $w_t = W_t / P_t$ is real wage

Firms' dynamic program

- Single idiosyncratic state

$$\mu_{t-1}(i,g) = \frac{P_{t-1}(i,g)}{A_t(i,g)P_{t-1}} = p_{t-1}(i,g)\frac{A_{t-1}}{A_t}$$

Firms' dynamic program

- Single idiosyncratic state

$$\mu_{t-1}(i,g) = \frac{P_{t-1}(i,g)}{A_t(i,g)P_{t-1}} = p_{t-1}(i,g)\frac{A_{t-1}}{A_t}$$

- Aggregate state variables $\Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty}$

Firms' dynamic program

- Single idiosyncratic state

$$\mu_{t-1}(i,g) = \frac{P_{t-1}(i,g)}{A_t(i,g)P_{t-1}} = p_{t-1}(i,g)\frac{A_{t-1}}{A_t}$$

- Aggregate state variables $\Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty}$
- Two options: change, don't change

Firms' dynamic program

- Single idiosyncratic state

$$\mu_{t-1}(i,g) = \frac{P_{t-1}(i,g)}{A_t(i,g)P_{t-1}} = p_{t-1}(i,g)\frac{A_{t-1}}{A_t}$$

- Aggregate state variables $\Omega_t = \{\tau_{t+i}, M_{t+i}, \Gamma_{t+i}\}_{i=0}^{\infty}$
- Two options: change, don't change
- No price-change

$$V^{NC}\left(\mu_{t-1}(i), \Omega_t\right) = \bar{\Pi}\left(\frac{\mu_{t-1}(i)}{1+\pi_t}, w_t, \tau_t\right) + \beta E_t V\left(\frac{\mu_{t-1}(i)e^{\varepsilon_{t+1}(i)}}{1+\pi_t}, \Omega_{t+1}\right),$$

Firms' dynamic program, cont.

- Price change

$$V^{C}\left(\Omega_t\right) = \max_{\mathbf{p}_t^*(i)} \left\{ \Pi(\mathbf{p}_t^*(i), w_t, \tau_t) - \phi + \beta E_t V\left(\mathbf{p}_t^*(i) e^{\varepsilon_{t+1}(i)}, \Omega_{t+1}\right) \right\}.$$

Firms' dynamic program, cont.

- Price change

$$V^C(\Omega_t) = \max_{\mathbf{p}_t^*(i)} \{\Pi(\mathbf{p}_t^*(i), w_t, \tau_t) - \phi + \beta E_t V\left(\mathbf{p}_t^*(i)e^{\varepsilon_{t+1}(i)}, \Omega_{t+1}\right)\}.$$

- Value function

$$V(\mu_{t-1}(i), \Omega_t) = \max_{\{C,NC\}} \left[V^{NC}(\mu_{t-1}(i), \Omega_t), V^C(\Omega_t)\right].$$

Monetary and fiscal policy

- Money supply growth: AR(1) with a drift

$$\log(M_t/M_{t-1}) = g_{Mt} = \mu_M + \rho_M g_{Mt-1} + \varepsilon_{Mt}$$

Monetary and fiscal policy

- Money supply growth: AR(1) with a drift

$$\log(M_t/M_{t-1}) = g_{Mt} = \mu_M + \rho_M g_{Mt-1} + \varepsilon_{Mt}$$

- Exogenous value-added tax rate τ_t

$$\tau_t = \tau_{t-1} + \varepsilon_{\tau t}$$

Monetary and fiscal policy

- Money supply growth: AR(1) with a drift

$$\log(M_t/M_{t-1}) = g_{Mt} = \mu_M + \rho_M g_{Mt-1} + \varepsilon_{Mt}$$

- Exogenous value-added tax rate τ_t

$$\tau_t = \tau_{t-1} + \varepsilon_{\tau t}$$

- Revenues are redistributed lump-sum

$$M_t - M_{t-1} + \frac{\tau_t}{1+\tau_t} P_t C_t = T_t$$

Equilibrium

1. Household maximizes utility subject to budget constraint taking prices, wages as given
2. Firms set nominal prices to maximize their value functions, taking their relative prices and idiosyncratic technology, and the future path of aggregate variables as given.
3. Money supply equals aggregate demand $M_t = P_t C_t$.
4. Money supply growth, taxes follow exogenous path.
5. Market clearing in the goods, bond, labor markets.

Numerical solution: Steady state

- No aggregate uncertainty

Numerical solution: Steady state

- No aggregate uncertainty
- Aggregate endogenous variables are constant
  - inflation: $\pi_t = \pi = \mu_M/(1-\rho_M)$, real wage: w_t constant
  - distribution over idiosyncratic state variables Γ is time-invariant

Numerical solution: Steady state

- No aggregate uncertainty
- Aggregate endogenous variables are constant
  - inflation: $\pi_t = \pi = \mu_M/(1-\rho_M)$, real wage: w_t constant
  - distribution over idiosyncratic state variables Γ is time-invariant
- Iteration in w
  1. Guess a value w_0 (implies an aggregate supply Y)
  2. For $w_i = w_{i-1}$ solve for value and policy functions
  3. Calculate equilibrium quality-adjusted relative price distribution (Γ_i)
  4. Calculate aggregate demand (C_t)
  5. If excess demand, increase w_{i+1}; repeat until convergence

Numerical solution: Transitional dynamics

- One time persistent/permanent shock to g_M, τ
  - Shooting
  - Assume new SS reached in T periods

Numerical solution: Transitional dynamics

- One time persistent/permanent shock to g_M, τ
  - Shooting
  - Assume new SS reached in T periods
  - Iterate on inflation path
    1. Guess inflation path $\{\pi_1, \pi_2, ..., \pi_T\}$
    2. Money growth implies an output growth and real wage $\{w_t\}$ path
    3. Calculate value- and policy functions by backward induction
    4. Calculate price distribution path
    5. Obtain resulting inflation path
    6. Do until convergence in inflation paths