Worker Turnover and Unemployment Insurance

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Because of hearing loss,
I'd appreciate that you speak up when asking questions

Introduction. Question

- Worker turnover is high in the US: average monthly rate is 4.34% in 1994-2007, according to Krusell et al (2011)
- A large number of new matches in the labor market are short-lived.
  - Using SIPP data, we find that over 43% (25%) of newly employed workers return to non-employment within a (second) year
  - Farber (1999) estimates this rate at 50% (33%) using NLSY
  - Anderson & Meyer (1994) finds quarterly permanent separation rate is 34% after a job tenure of 6 months using CWBH

Introduction. Question

- Present and future unemployment risks have mostly been addressed separately in the literature
  - present risks: Hopenhayn & Nicolini (1997), Acemoglu & Shimer (1999), Golosov et al (2013)
  - layoff risks: Blanchard & Tirole (2008), Cahuc & Zylberberg (2008) → (Pigovian) layoff taxes make firms internalize the welfare costs of layoffs
  - Alvarez & Veracierto (2001): trade-off between job-creation and -destruction

→ **Goal**: to study efficiency of worker turnover in an economy where present and future unemployment risks are intertwined

Introduction. Answer

1. Competitive search economy (Peters (1991), Moen (1997)) in which
   - risk-averse workers decide what job to search for and search effort
   - incomplete markets
   - match quality is both an inspection good and an experience good (Jovanovic (1979), Pries & Rogerson (2005))

Introduction. Answer

1. Competitive search economy (Peters (1991), Moen (1997)) in which
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   - incomplete markets
   - match quality is both an inspection good and an experience good (Jovanovic (1979), Pries & Rogerson (2005))
2. Two margins to respond to workers' preferences:
   - job creation margin (Acemoglu and Shimer (1999))
   - match creation/destruction margin

→ worker turnover is **inefficiently high**

Introduction. Answer

1. **Main contribution:** the implementation of planner's solution
   - **unemployment benefits** to insure against these risks
   - **layoff tax must be 0**
     - because it affects vacancy creation in a distortionary way
     - same result in a comp. search version of Blanchard & Tirole (2008)
   - **match creation must be subsidized through wages**
2. Quantitative exercise:
   According to our calibration exercise, the US worker turnover rate falls short of the optimal level (benefits are too high)

Related Literature

- Worker turnover: Hopenhayn and Rogerson (1993), Pries and Rogerson (2005), Farber (1999), Wolff (2004)
- Employment protection: Blanchard and Tirole (2008), Cahuc et al (2008), Alvarez and Veracierto (2001), Hopenhayn and Rogerson (1993), Ljunqvist (2002)
- Learning about match quality: Jovanovic (1979, 1984), Moscarini (2005), Pries (2004), Nagypál (2007)
- Competitive/Directed search:
  - Peters (1991), Moen (1997), Menzio & Shi (2011)
  - risk-averse workers: Acemoglu & Shimer (1999), Jacquet & Tan (2012), Golosov, Maziero & Menzio (2013)
- Mismatch and search: Marimon & Zilibotti (1999), Teulings & Gautier (2004)

Outline

1. Two-period model
2. Infinite horizon model
3. Calibration
4. Policy Analysis

Two-Period Economy

- Mass 1 of risk-averse workers, and a large mass of risk-neutral firms
- Agents start unmatched, and are ex-ante identical
- Agents discount period-two utility at rate β
- Worker's utility: $\upsilon(c) - \phi(s)$, $s \in [0, 1]$
  - υ: increasing and concave, twice-cont. diff., $\upsilon(0) = 0$, $\lim_{c \to 0} \upsilon'(c) = \infty$
  - ϕ: increasing and convex, $\phi(0) = 0$, $\lim_{s \to 1} \phi(s) = \infty$
- Incomplete markets. In particular,
  - workers cannot save nor borrow
  - firms can only pay their own employees
- Frictions:
  - search: workers and firms search for a partner
  - Informational:
    - match quality unknown at the meeting time: *inspection* and *experience* good (Jovanovic (1979), Pries (2004), Pries and Rogerson (2005))
    - search effort is unobserved

Two-Period Economy

- Period 1: Search and Production

- Period 2: Production

Two-Period Economy

- Period 1: Search and Production
  - Search stage: workers and firms search for a partner
    - vacancy-posting at cost k
    - Meeting technology:
      $q(m)$: expected queue length at location m
      $\nu(q(m))$: probability of meeting a job; decreasing in q
      $\eta(q(m))$: probability of meeting a worker; increasing in q
- Period 2: Production

Two-Period Economy

- Period 1: Search and Production
  - Search stage: workers and firms search for a partner
    - vacancy-posting at cost k
    - Meeting technology: $\nu(q)$, $\eta(q)$
  - Production stage:
    - Match quality can be *good* (output y) or *bad* (output 0)
    - It is unobserved at the meeting time
    - A signal $\pi = P(good)$ is drawn from a cdf F
    - Expected output if match is formed $= \pi y$

- Period 2: Production

Two-Period Economy

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    - Expected output if match is formed $= \pi y$
    - If the signal is sufficiently high, a match is formed.
    - O.w., worker stays unemployed for the two periods, produces z
- Period 2: Production

Two-Period Economy

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  - Search stage: workers and firms search for a partner
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    - Match quality can be *good* (output y) or *bad* (output 0)
    - It is unobserved at the meeting time
    - A signal $\pi = P(good)$ is drawn from a cdf F
    - Expected output if match is formed $= \pi y$
    - If the signal is sufficiently high, a match is formed.
    - O.w., worker stays unemployed for the two periods, produces z
- Period 2: Production
  - Actual match quality is observed at the beginning of the period
  - If *good*, output $= y$
  - If *bad*, the match is broken, worker becomes unemployed and produces z

Social Planner Problem

$$\max_{s,q,R,c_u,c_1,c_2} \quad -\phi(s) + s\nu(q)\int_R \Big(\upsilon(c_1) - \upsilon(c_u) + \beta\pi\big(\upsilon(c_2) - \upsilon(c_u)\big)\Big)dF(\pi) + \upsilon(c_u)(1+\beta)$$

s. to $$\phi'(s) = \nu(q)\int_R \Big(\upsilon(c_1) - \upsilon(c_u) + \beta\pi\big(\upsilon(c_2) - \upsilon(c_u)\big)\Big)dF(\pi) \qquad \text{(IC)}$$

$$(z - c_u)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1 - \mathbb{E}(\pi|R))\Big) +$$
$$s\nu(q)(1-F(R))\Big(\mathbb{E}(\pi|R)y - c_1 + \beta(y - c_2)\mathbb{E}(\pi|R)\Big) \geq k\frac{s}{q} \qquad \text{(RC)}$$

where $\mathbb{E}(\pi|R) \equiv \int_R \pi \frac{dF(\pi)}{1-F(R)}$

Proposition

There exists a constrained efficient allocation $(s^p, q^p, R^p, c_u^p, c_1^p, c_2^p)$.

- $z < c_u^p < c^p = c_1^p = c_2^p$.
- R^p *satisfies*

$$\frac{\upsilon(c^p) - \upsilon(c_u^p)}{\upsilon'(c^p)} + y\frac{R(1+\beta)}{1+\beta R} - c^p - z + c_u^p \geq 0, \text{ and } R \geq 0, \text{ with comp. slackness}$$

- $y\frac{R^p(1+\beta)}{1+\beta R^p} < z$

- Tension between consumption smoothing and search incentives
- Tension between utility gains and output costs of sending workers to the unemployment pool
- If workers were risk neutral, $y\frac{R(1+\beta)}{1+\beta R} = z$

Market Economy/Decentralization

- Free entry of firms
- Firms compete for workers, and post a vacancy in a submarket
- Submarket is identified by a contractual offer $x \equiv (\omega, R)$
  - $\omega = (w_1, w_2)$: wage schedule (firms can only pay their own employees)
  - R: reservation probability; match is formed iff signal $\pi \geq R$
- Perfect information about submarkets
- Search is directed: submarkets promising a higher value attract more agents

Value Functions

- Value of a vacancy:

$$V = -k + \max_{x=((w_1,w_2),R)} \eta(q(x)) \int_R \Big(\pi y - w_1 + \beta\pi(y - w_2)\Big) dF(\pi)$$

- Value of Unemployment:

$$\begin{aligned} U &\geq \max_s -\phi(s) + s\nu(q(x)) \int_R \Big(\upsilon(w_1) - \upsilon(z) + \beta\pi\big(\upsilon(w_2) - \upsilon(z)\big)\Big) dF(\pi) \\ &\quad + \upsilon(z)(1+\beta), \\ \text{and} &\quad q(x) \geq 0, \text{ with complementary slackness} \end{aligned}$$

Equilibrium

Definition

A competitive search equilibrium consists of U, $s \in [0,1]$, $Q : X \to \mathbb{R}_+ \bigcup \{\infty\}$, and $x = ((w_1, w_2), R)$ such that

i) Firm's profit maximization and zero-profit condition:
For any contract x',

$$\eta(Q(x')) \int_{R'} \Big(\pi y - w_1' + \beta\pi(y - w_2')\Big) dF(\pi) \leq k,$$

which holds with equality at x.

ii) Worker's optimal search: For any contract x',

$$\begin{aligned} U \quad \geq \quad & \max_s -\phi(s) + s\nu(Q(x')) \int_R \Big(\upsilon(w_1') - \upsilon(z) + \beta\pi\big(\upsilon(w_2') - \upsilon(z)\big)\Big) dF(\pi) \\ & +\upsilon(z)(1+\beta), \\ \text{and} \quad & Q(x') \geq 0, \text{ with complementary slackness} \end{aligned}$$

Equilibrium Characterization

Proposition

1. *There exists a CS equilibrium*
2. *In equilibrium, $z < w = w_1 = w_2$, and the reservation probability R satisfies*

$$\frac{\upsilon(w) - \upsilon(z)}{\upsilon'(w)} + \frac{Ry(1+\beta)}{(1+\beta R)} - w \geq 0,$$

and $R \geq 0$, with complementary slackness

and $Ry(1+\beta) < w(1+\beta R)$

- Firms act both as employers and as insurers of their risk-averse employees (Baily (1974), Azariadis (1975))

Comparative Statics

Let (w_i, q_i, R_i, s_i) denote the equil. tuple of the economy indexed by i.

Lemma

If two economies differ in either

- *workers' utility function* υ, *and* υ_2 *is a concave monotonic transformation of* υ_1, *or*
- *home production, and* $z_2 < z_1$

then

- $w_2 < w_1$
- $q_2 < q_1$
- $R_2 < R_1$ *(unless both are zero)*

$\rightarrow$ *higher job-finding and -termination rate, and lower output per worker*

$\rightarrow$ Two insurance margins:

- job creation (Acemoglu and Shimer (1999))
- match creation (trade-off between providing insurance in the first period and insurance in the second period)

Welfare

Proposition

1. *If the planner is forced to set $c_u = z$, constrained efficiency can be decentralized*
2. *Otherwise, the CS equilibrium is not constrained efficient. (missing markets)*
3. *If $R^p > 0$, then*
   i) $R < R^p$ *and* $q < q^p \Rightarrow$ *worker turnover is too high in equilibrium*

Welfare

Proposition

1. *If the planner is forced to set $c_u = z$, constrained efficiency can be decentralized*
2. *Otherwise, the CS equilibrium is not constrained efficient. (missing markets)*
3. *If $R^p > 0$, then*
   i) *$R < R^p$ and $q < q^p \Rightarrow$ worker turnover is too high in equilibrium*
   ii) *Constrained efficiency is attained if the government sets an unemployment insurance system funded through a lump sum tax, a negative income tax and a zero layoff tax.*

- Acemoglu & Shimer (1999): output-maximizing UI funded by lump sum taxes
- Golosov, Maziero & Menzio (2013): welfare-maximizing UI funded by increasing and regressive income taxes
- Blanchard & Tirole (2008): welfare-maximizing policy consists of a layoff tax equal to unemp. benefits

Understanding the $L = 0$, $\tau < 0$ result

If $R > 0$,

- Regarding match creation,
  Tax-distorted equilibrium R is constrained efficient
  $\Leftrightarrow b = -\tau y \frac{R(1+\beta)}{1+\beta R} - (1-\tau)\beta L \frac{1-R}{1+\beta R}$

Understanding the $L = 0$, $\tau < 0$ result

If $R > 0$,

- Regarding match creation,
  Tax-distorted equilibrium R is constrained efficient
  $\Leftrightarrow b = -\tau y \frac{R(1+\beta)}{1+\beta R} - (1-\tau)\beta L \frac{1-R}{1+\beta R}$

- Regarding job creation,
  the equilibrium condition is

$$\eta(q)(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R)) \left(\left(\frac{\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R}{1+\beta R} \right) y(1+\beta) - L\beta \left(\frac{1-\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{1-R}{1+\beta R} \right) \right) = k$$

  the const. efficiency condition is

$$\eta(q)(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R)) \left(\frac{\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R}{1+\beta R} \right) y(1+\beta) = k$$

$\Rightarrow L = 0$, $\tau < 0$

Understanding the $L = 0$, $\tau < 0$ result

- Key results do not change if match quality is only an experience good. For example, in a competitive search version of Blanchard and Tirole (2008):
  - Tax-distorted equil. R is the constrained efficient level if

$$-\tau R + L(1-\tau) = b$$

  - Equilibrium and efficiency q conditions:

$$(1-\varphi(q))(1-F(R))\left(\mathbb{E}(y|R) - R - \frac{L}{1-F(R)}\right) = \frac{k}{\eta(q)}$$

$$(1-\varphi(q))(1-F(R))\left(\mathbb{E}(y|R) - R\right) = \frac{k}{\eta(q)}$$

$\Rightarrow L = 0$ and $\tau < 0$

subsidizing job creation is necessary, but must be done through wages, which affect workers' search attitudes.

Full Dynamic Model with Adverse Selection
(building on Guerrieri et al (2010))

Dynamic Model with Adverse Selection

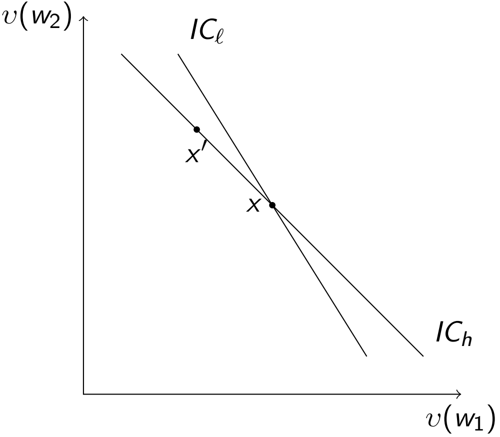
Differences:

- Time is discrete and continues forever.
- Two types of workers: low (ℓ) and high (h)
  - Shares: $\mu_h + \mu_\ell = 1$
  - Types differ in the ability to form good matches
    - The probability of a good match quality is drawn from cdf F_i
    - We assume $F_\ell \lesssim F_h$
- Match quality is learned with probability α every period during the experience phase
- Worker-firm pairs are exogenously terminated with prob. λ
- Submarkets are identified by a contract $x \equiv (\omega, R)$
  - $\omega \equiv (w_1, w_2)$, wages to be paid during 'experiencing' and 'post revelation' phases

Equilibrium Characterization

Proposition

No submarket attracts both types of workers $\Rightarrow$ **Separating equilibrium**



Dynamic Model with Worker Heterogeneity

	LF	OP^A	Baseline	OP^B
Lump sum tax, T	0.000	0.097	0.000	
Income tax, τ	0.000	-0.096	0.019	
Layoff tax, L	0.000	0.000	0.000	
Unemp. benefits, b	0.000	0.243	0.230	
Net benefits, $b - T$	0.000	0.146	0.230	
Endogenous separation rate				
type ℓ	0.028	0.027	0.027	
type h	0.013	0.012	0.011	
average	0.019	0.018	0.017	
Job-finding rate				
type ℓ	0.097	0.055	0.037	
type h	0.131	0.095	0.079	
average	0.114	0.073	0.054	
Wages				
type ℓ	0.791	0.805	0.810	
type h	0.950	0.955	0.956	
average	0.910	0.918	0.922	
Output	0.906	0.896	0.883	
Output per worker	0.942	0.949	0.952	
Separation 1st year	0.502	0.477	0.465	
Turnover (%)	0.923	0.871	0.849	
CEV (%)	-0.476	0.014	0.000	
Transfers to type-ℓ	0.000	0.000	0.006	

Table: Laissez-Faire (LF), baseline (calibrated) and optimal policy (OP) economies. OP^A: Optimal policy with no transfers across worker types. OP^B: OP with transfers not bigger than in the baseline. Transfers to ℓ are defined as $\mu_\ell \left(u^\ell(b-T) - e_1^\ell(\tau w_{1\ell} + T) - e_2^\ell(\tau w_{2\ell} + T)\right)$. *Endogenous separations* are defined as $\alpha(1 - \mathbb{E}_i(\pi|R))$. Job-finding rates are $s_i \nu(q_i)(1 - F_i(R_i))$. *Separation 1st year* refers to the probability of experiencing a $E\not{E}$ transition during the first year following the start of a new job. *Turnover* is the fraction of new hires plus separations over total employment. *CEV* is the consumption equivalent variation with respect to the baseline.

Dynamic Model with Worker Heterogeneity

	LF	OP^A	Baseline	OP^B
Lump sum tax, T	0.000	0.097	0.000	-0.099
Income tax, τ	0.000	-0.096	0.019	0.120
Layoff tax, L	0.000	0.000	0.000	0.004
Unemp. benefits, b	0.000	0.243	0.230	0.085
Net benefits, $b - T$	0.000	0.146	0.230	0.184
Endogenous separation rate				
type ℓ	0.028	0.027	0.027	0.027
type h	0.013	0.012	0.011	0.012
average	0.019	0.018	0.017	0.018
Job-finding rate				
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type ℓ	0.791	0.805	0.810	0.801
type h	0.950	0.955	0.956	0.954
average	0.910	0.918	0.922	0.916
Output	0.906	0.896	0.883	0.894
Output per worker	0.942	0.949	0.952	0.948
Separation 1st year	0.502	0.477	0.465	0.483
Turnover (%)	0.923	0.871	0.849	0.882
CEV (%)	-0.476	0.014	0.000	0.185
Transfers to type-ℓ	0.000	0.000	0.006	0.006

Table: Laissez-Faire (LF), baseline (calibrated) and optimal policy (OP) economies. OP^A: Optimal policy with no transfers across worker types. OP^B: OP with transfers not bigger than in the baseline. *Endogenous separations* are defined as $\alpha(1 - \mathbb{E}_i(\pi|R))$. Job-finding rates are $s_i\nu(q_i)(1 - F_i(R_i))$. *Separation 1st year* refers to the probability of experiencing a *EU* transition during the first year following the start of a new job. *Turnover* is the fraction of hires plus separations over total employment. *CEV* is the consumption equivalent variation with respect to the baseline.

Conclusions

- LF economy: worker turnover is inefficiently high because the two margins (job- and match-creation) are used to insure workers against unemployment.
- Unemployment insurance (UI) increases welfare and induces a better allocation of labor inputs. Matches are more productive and last longer.
- Planner's allocation can be decentralized if UI is financed through lump sum taxes, a negative proportional income tax and a zero layoff tax.
- US economy: worker turnover is below optimal

Full Dynamic Model with Adverse Selection

Dynamic Model with Adverse Selection

- Time is discrete and continues forever.
- Mass one of risk-averse workers, and free entry of risk-neutral firms
- Agents discount future utility at rate β
- Workers
  - Utility: $\upsilon(c) - \phi(s)$
  - Two types: low (ℓ) and high (h)
  - Shares: $\mu_h + \mu_\ell = 1$
  - Types differ in the ability to form good matches
    - The probability of a good match quality is drawn from cdf F_i
    - We assume $F_\ell \lesssim F_h$

Match quality and Production

- Match quality can be *good* (output y) or *bad* (output 0)
- It is unobserved when a worker and a firm meet
- Match quality is both an *inspection* and an *experience* good
  1. *Inspecting:* upon meeting, pairs draw $\pi = Prob_i(good)$ from cdf F_i, for $i = \{\ell, h\}$
  2. *Experiencing:* actual match quality is revealed with prob. α every period $\Rightarrow$ learning is "all or nothing"
- Worker-firm pairs are also subject to exogenous destruction λ

Search

- Workers can be employed or unemployed;
  firms can have a job filled or a vacancy posted.
- Unmatched agents:
  - search for a partner
  - workers derive utility from z, and disutility from s
  - Submarkets are identified by a contract $x \equiv (\omega, R)$
    - R: reservation probability for match formation
    - $\omega \equiv (w_1, w_2)$, a wage schedule (wages to be paid during 'experiencing' and 'post revelation' phases)
  - Firms choose a submarket, and incur cost k when posting a vacancy
  - Workers direct their search to a contract with intensity s
  - Meeting technology as before: $\nu(q(x))$, $\eta(q(x))$

Value Functions

- Unemployment Value

$$(1-\beta)U_i = \upsilon(z) + \max_{x,s}\left\{ -\phi(s) + s\nu(q(x))\beta \int_{\pi \geq R} \left(E_i(x,\pi) - U_i \right) dF_i(\pi) \right\}$$

- Employment Value under contract x and signal π

$$E_i(x,\pi) = \upsilon(w_1)$$

Value Functions

- Unemployment Value

$$(1-\beta)U_i = \upsilon(z) + \max_{x,s}\Big\{ -\phi(s) + s\nu(q(x))\beta \int_{\pi \geq R} \Big(E_i(x,\pi) - U_i\Big) dF_i(\pi)\Big\}$$

- Employment Value under contract x and signal π

$$E_i(x,\pi) = \upsilon(w_1) + \beta\lambda U_i$$

Value Functions

- Unemployment Value

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- Employment Value under contract x and signal π

$$\begin{aligned} E_i(x,\pi) = \upsilon(w_1) &+ \beta\lambda U_i \\ &+ \beta(1-\lambda)\alpha(1-\pi)U_i \end{aligned}$$

Value Functions

- Unemployment Value

$$(1-\beta)U_i = \upsilon(z) + \max_{x,s}\left\{ -\phi(s) + s\nu(q(x))\beta \int_{\pi \geq R} \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi) \right\}$$

- Employment Value under contract x and signal π

$$\begin{aligned} E_i(x,\pi) = \upsilon(w_1) &+ \beta\lambda U_i \\ &+ \beta(1-\lambda)\alpha(1-\pi)U_i \\ &+ \beta(1-\lambda)\alpha\pi \frac{\upsilon(w_2) + \beta\lambda U_i}{1-\beta(1-\lambda)} \end{aligned}$$

Value Functions

- Unemployment Value

$$(1-\beta)U_i = \upsilon(z) + \max_{x,s}\left\{ -\phi(s) + s\nu(q(x))\beta \int_{\pi \geq R} \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi) \right\}$$

- Employment Value under contract x and signal π

$$\begin{aligned} E_i(x,\pi) = \upsilon(w_1) &+ \beta\lambda U_i \\ &+ \beta(1-\lambda)\alpha(1-\pi)U_i \\ &+ \beta(1-\lambda)\alpha\pi\frac{\upsilon(w_2)+\beta\lambda U_i}{1-\beta(1-\lambda)} \\ &+ \beta(1-\lambda)(1-\alpha)E_i(x,\pi) \end{aligned}$$

Value Functions (ctd.)

- Value of a filled vacancy:

$$\begin{aligned} J(x,\pi) = \pi - w_1 &+ \beta\lambda V \\ &+ \beta(1-\lambda)\alpha(1-\pi)V \\ &+ \beta(1-\lambda)\alpha\pi\frac{y - w_2}{1-\beta(1-\lambda)} \\ &+ \beta(1-\lambda)(1-\alpha)J(x,\pi) \end{aligned}$$

- The value of a vacant position:

$$(1-\beta)V = -k + \beta\eta(q(x))\sum_{i=\ell,h}\rho_i(x)\int_{\pi\geq R}(J(x,\pi)-V)\,dF_i(\pi)$$

where $\rho_i(x)$ is the firm's belief of the fraction of type-i workers showing up for contract x

- Free entry $\Rightarrow V = 0$

Equilibrium Definition

Definition

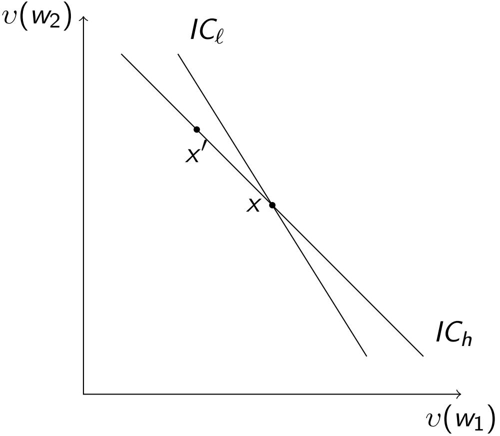
A **steady-state competitive search equilibrium** consists of a distribution G of vacancies in active submarkets with support $\mathcal{X} \subset X$, value functions U_i, E_i and J, search functions $S_i : X \to [0,1]$, a queue length function $Q : X \to \mathcal{R}_+$, and a function $\rho : X \to \Delta^1$ such that

1. Firms maximize profits
2. Workers search optimally
3. Markets clear:
$$\int_{\mathcal{X}} \rho_i(x) \frac{Q(x)}{S_i(x)} dG(x) = u_i \mu_i, \forall i$$

Equilibrium Characterization

Proposition

Assumptions 1 and 2 $\Rightarrow$ *no submarket attracts both types of workers* $\Rightarrow$ **Separating equilibrium**



Equilibrium Characterization

Proposition

If $(G, \mathcal{X}, (U_i, S_i)_i, Q, \rho)$ is an equilibrium with $\mathcal{X} = \{x_\ell, x_h\}$, then (U_i, q_i, x_i, s_i) is a fixed point of $\mathcal{H}_i$.
Conversely, if (U_i, q_i, x_i, s_i) is a fixed point of function $\mathcal{H}_i$ for $i \in \{\ell, h\}$. Then it takes part of an equilibrium allocation, and $Q_i(x_i) = q_i$, $\rho_i(x_i) = 1$

$$\mathcal{H}_\ell(U) \equiv \max_{q,R,w_1,w_2,s} \quad \upsilon(z) - \phi(s) + \beta s \nu(q) \int_R \Big(E_\ell(x,\pi) - U \Big) dF_\ell(\pi) + \beta U$$

$$\text{s. to} \quad \beta \eta(q) \int_R J(x,\pi) dF_\ell(\pi) \geq k$$

Equilibrium Characterization

Proposition

If $(G, \mathcal{X}, (U_i, S_i)_i, Q, \rho)$ *is an equilibrium with* $\mathcal{X} = \{x_\ell, x_h\}$*, then* (U_i, q_i, x_i, s_i) *is a fixed point of* $\mathcal{H}_i$*.*
Conversely, if (U_i, q_i, x_i, s_i) *is a fixed point of function* $\mathcal{H}_i$ *for* $i \in \{\ell, h\}$*.*
Then it takes part of an equilibrium allocation, and $Q_i(x_i) = q_i$*,* $\rho_i(x_i) = 1$

$$
\begin{aligned}
\mathcal{H}_h(U; U_\ell) \equiv \quad & \max_{q,R,w_1,w_2,s} \quad \upsilon(z) - \phi(s) + \beta s \nu(q) \int_R \Big(E_h(x,\pi) - U \Big) dF_h(\pi) + \beta U \\
& \text{s. to} \quad \beta \eta(q) \int_R J(x,\pi) dF_h(\pi) \geq c \\
& \upsilon(z) + \max_s \Big\{ -\phi(s) + \beta s \nu(q) \int_R \Big(E_\ell(x,\pi) - U_\ell \Big) dF_\ell(\pi) \Big\} \leq (1-\beta) U_\ell
\end{aligned}
$$

Equilibrium Characterization

Proposition

If $(G, \mathcal{X}, (U_i, S_i)_i, Q, \rho)$ is an equilibrium with $\mathcal{X} = \{x_\ell, x_h\}$, then (U_i, q_i, x_i, s_i) is a fixed point of $\mathcal{H}_i$.
Conversely, if (U_i, q_i, x_i, s_i) is a fixed point of function $\mathcal{H}_i$ for $i \in \{\ell, h\}$. Then it takes part of an equilibrium allocation, and $Q_i(x_i) = q_i$, $\rho_i(x_i) = 1$

Proposition

There exists a steady-state CS equilibrium. In equilibrium,

- $U_\ell < U_h$
- $w_1^\ell = w_2^\ell$, $w_1^h \leq w_2^h$
- $E_\ell(x, R_\ell) - U_\ell + \upsilon'(w_\ell) J(x, R_\ell) \geq 0$

(Berge + Brouwer)

Calibration

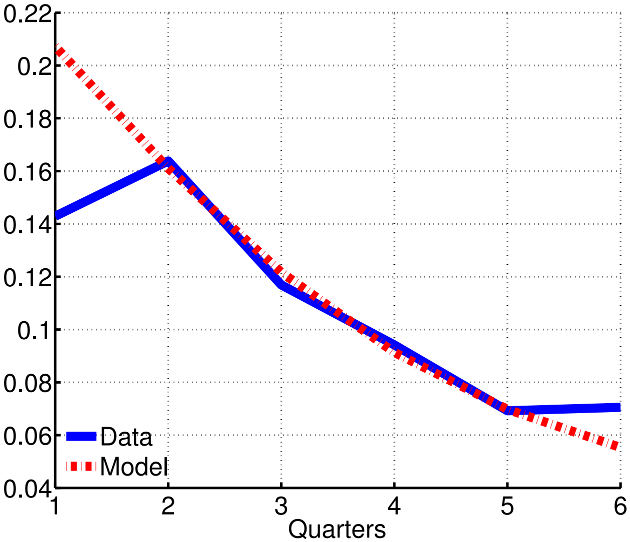
- model period: one week
- preferences: $\upsilon(c) = \frac{c^{1-\theta}}{1-\theta}$, $\phi(s) = \phi_1 \frac{(1-s)^{-\phi_2}-1}{\phi_2} - \phi_1 s$
- meeting function: $\eta(q) = (1 + q^{-\psi})^{-1/\psi}$ (den Haan et al (2000))
- dF_ℓ and dF_h: we follow Pries & Rogerson (2005) and use normal distributions with mean zero and std. dev. σ_i ($i \in \{\ell, h\}$) truncated between $(0, 1)$
- Unemployment insurance: benefits, b, funded through income tax, τ (budget balanced period by period)

Calibration

Parameter	Description	Value	Target
	Exogenously Set Parameters		
β	Discount factor	0.9990	Annual interest rate of 5%
y	Market Productivity	1.0000	(Normalization)
θ	Relative risk aversion coef.	2.0000	
	Jointly Calibrated Parameters		
ϕ_1	scale parameter $\phi(s)$	6.5956	Unemployment rate
ϕ_2	elasticity parameter $\phi(s)$	2.9076	job-finding rate 1st quarter
ψ	Meeting technology parameter	1.0506	vacancies to unemployment ratio
k	Vacancy cost	0.2031	20% avg. quarterly wage per hire
b	Unemployment benefits	0.2330	25% of avg. wage
z	Home production	0.4692	$z + b =$ 71% of avg. productivity
μ	Share of skilled	0.6842	log wage diff. at 0- and 52-week tenure
σ_h	St. dev. of normal dist. F_h	0.1543	
σ_ℓ	St. dev. of normal dist. F_ℓ	1.4775	quarterly $E\not{E}$ transitions
α	Learning speed parameter	0.0345	(2nd-5th quarters)
λ	Exog. job-separation rate	0.0011	

Table: Calibration

Model fit


0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
1
2
3
4
5
6
Quarters
Data
Model

Equilibrium variables in the calibrated economy

	ℓ-**market**	**h-market**
Reservation probability, R	0.1580	0.4787
Queue length, q	0.3333	0.2144
Wages, $w = w_1 = w_2$	0.8250	0.9729
Search intensity, s	0.0849	0.1317
Job-finding rate, $s\nu(q)(1 - F(R))$	0.0201	0.0550
Endogenous separation rate, $\alpha(1 - \mathbb{E}(\pi\|R))$	0.0263	0.0092
Output per worker, $y\big(e_1\mathbb{E}(\pi\|R) + e_2\big)/(1 - u)$	0.9092	0.9887

Table: Equilibrium Variables, Baseline.