Worker Turnover and Unemployment Insurance *

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Abstract

Evidence shows that worker turnover is high and a large number of matches are short-lived in the U.S. This paper studies an economy in which the present and future unemployment risks jobless workers face are intertwined. Workers decide what job search for and the search effort. Risk-averse workers factor in present and future risks in their search decisions when markets are incomplete. In equilibrium, firms respond to worker preferences by creating many low-wage jobs that are easier to get, but typically shorter-lived than the jobs created in a centralized economy where the planner can attenuate the consumption loss of the unemployed by redistributing resources. Therefore, worker turnover is inefficiently high. In addition to a publicly-provided insurance to cover unemployment risks, the implementation of the planner's solution requires a zero layoff tax and subsidizing match formation through wages. According to our calibration, worker turnover is inefficiently low in the U.S. economy because unemployment benefits are excessively high.

Keywords: Worker Turnover, Unemployment Insurance, Competitive Search, Layoff Taxes, Moral Hazard

JEL Codes: J01, J08, J63, J65

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1 Introduction

Worker turnover is high in the U.S. economy. According to Krusell et al. (2011), the 1994-2007 average monthly worker turnover rate amounts to 4.34%. It is partly explained by the fact that a large number of matches are short-lived. In a companion paper, using data from the Survey of Income and Program Participation, we document that over 42% of newly employed workers return to non-employment within a year, whereas the termination rate drops to 25% in the second year. Farber (1999) estimates these rates at 50% and 33%, respectively, using the National Longitudinal Survey of Youth.

These high match termination rates are likely to be taken into account by jobless workers in their search decisions. However, the literature has mostly addressed the present and future unemployment risks separately. Regarding the present risks, a number of papers have studied the optimal design of unemployment insurance. See e.g. Hopenhayn and Nicolini (1997), Shimer and Werning (2008) and, closest to ours, Golosov et al. (2013). Regarding employment protection policies, Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) show that the rationale underlying layoff taxes is to make firms internalize the welfare costs of dismissals,[1] but do not consider the effects of layoff taxes on job creation. Alvarez and Veracierto (2001) conclude that mandated severance payments increase welfare through increasing employment, but its insurance role is minor as reduce private savings.[2]

The goal of this paper is to examine the constrained efficiency of the equilibrium worker turnover in a frictional economy in which present and future unemployment risks are intertwined. To this aim, we construct a two-period competitive search model of the labor market building upon Peters (1991) and Moen (1997). Risk-averse workers decide what sort of job to search for and the search effort. Firms compete for workers by committing to contractual offers, which stipulate a wage schedule and hiring standards. Search is directed in the sense that offers promising a higher employment value attract more applicants. Upon meeting, a firm and a worker must decide whether to form a match. This decision depends on the expected match quality. Following Jovanovic (1979) and Pries and Rogerson (2005), we think of match quality as both an inspection good and an experience good.[3] That is, match quality is unobserved at the meeting time. A firm and a worker observe a signal, which conveys the probability of the match quality being good. An employment relationship starts if the signal is above the hiring standards. Otherwise, workers derive utility from home production. If a match is formed, its actual quality is learned by experiencing it. If the match quality is learned to be bad, then the match is destroyed.

The unemployment risks are consumption risks as consumption differs across employment states in an economy where agents cannot perfectly smooth consumption because markets are incomplete,

[1]This is also the rationale for an experience rate unemployment insurance system. See e.g....

[2]See Ljungqvist (2002) for the different effects of employment protection policies on the employment rate.

[3]The *experience* feature was first introduced by Nelson (1970) meaning that the quality of a match can only be assessed by *experiencing* it. In contrast, Hirshleifer (1973) coined the *inspection* term to point to evaluations carried out prior to the formation of the match.

firms can only make payments to their own employees, and workers cannot self-insure through saving. As a result, risk-averse workers factor in the uninsured risks in their search decisions. There are two margins through which private markets may respond to workers' preferences. First, hiring standards may be set low, and, second, as in Acemoglu and Shimer (1999), a large number of jobs may be created. It is a general equilibrium result that both margins are used: Many low-wage jobs with low hiring standards are posted in equilibrium, despite the low expected quality matches and, hence, the high expected future separation rates. Therefore, firms act both as employers and insurers of their employees as in the implicit contract literature. This is due to the difference in the ability to bear risk between workers and firms. See e.g. Baily (1974) and Azariadis (1975). The insurance motive against present unemployment risks is stronger the more risk-averse workers are, and weaker the larger the workers' outside option is.

We analyze the constrained efficiency of equilibrium worker turnover by comparing with the social planner's solution, i.e. the allocation that maximizes the expected discounted utility of the representative worker subject to the search and information frictions described above. The moral hazard problem ensures consumption differences between employed and unemployed workers, yet the planner redistributes resources from the former to the latter. As a result, the consumption of the unemployed is above their home production, which amounts to the equilibrium consumption level. Therefore, the equilibrium is constrained inefficient as the consumption risks are not efficiently covered in the market economy. Furthermore, the insurance markets offer in equilibrium makes job-creation and job-separation rates inefficiently high.

We show that the planner's allocation can be decentralized if a government sets an unemployment insurance system. Unemployment benefits cover the consumption risks by redistributing resources from the employed to those who fail to find a job (present risks), as in Golosov et al. (2013), but also to those whose match is terminated. Our main result is that the implementation of the planner's alllocation implies a zero layoff tax, a wage subsidy and a lump sum tax. The zero layoff tax contrasts with the Pigovian tax found by e.g. Blanchard and Tirole (2008). The reason is that layoff taxes distort job creation in an inefficient way. In a competitive search setting, contractual offers guide workers' search decisions. Therefore, match creation must be subsidized through wages for firms to set the constrained efficient hiring standards because otherwise firms would run negative expected profits ex-ante. This result is robust within the competitive search framework. In particular, if match quality is an experience, but not an inspection good, such as e.g. in a competitive search version of their model, a Pigovian layoff tax helps to attain efficiency when job-finding rates are assumed as constant (partial equilibrium), but it must be zero when considering them as endogenous instead.

The implementation of such an insurance scheme induces workers to search for higher-wage jobs, which are also longer-lasting. This effect of unemployment benefits on the job composition is similar to the one studied in Acemoglu and Shimer (1999), who show that a public unemployment insurance scheme crowds out the partial private insurance markets offer in their absence, and induces firms to create more capital-intensive, higher-wage jobs. Similarly, in Golosov et al. (2013) there are too few risk-averse workers seeking high-productivity jobs, and unemployment benefits (financed by increasing and regressive income taxes) allow workers to take riskier strategies in their job search. Pries and Rogerson (2005) also find that unemployment benefits lenghten job spells in a Diamond-Mortensen-Pissarides framework with risk-neutral workers. However, their mechanism is different from ours. In their case, the hiring standards are not conceived as an insurance instrument, but are the ones that make the firm-worker pair indifferent between forming a match or not upon meeting. Therefore, higher benefits reduce the surplus, which makes the reservation probability increase.[4]

Finally, we extend this normative exercise to an infinite horizon economy with heterogeneous workers, and calibrate it to match some salient features of the U.S. labor market. Using the calibrated model, we compute the government's optimal policy that maximizes a steady-state utilitarian social welfare function. The main result from this exercise is twofold. First, we confirm the insights from the two-period model: worker turnover is excessively high in the laissez-faire economy, and the optimal unemployment insurance implies zero layoff taxes and reduces turnover. Second, worker turnover and the separation rate within a year after re-employment in the calibrated economy are lower than optimal because unemployment benefits are excessively high.[5]

Two standard assumptions made in the literature on worker turnover are risk-neutral workers and random search. See e.g. Jovanovic (1984) and Moscarini (2005). Pinheiro and Visschers (2014) analyze the allocation of risk-neutral workers to firms in an economy with random search and wage-posting, where firms are heterogeneous in their termination rates and workers take into account wages as well as displacement rates in their offer-acceptance decisions. In the competitive search framework, Menzio and Shi (2011) model an economy with on-the-job search, aggregate shocks, and match quality, but with risk-neutral workers. As a result, they find that the constrained efficient allocation can be decentralized. We deviate from this literature by analyzing the welfare effects of the interaction between risk aversion and directed search on worker turnover.

Our work is also related to the findings in the empirical literature regarding the effects of unemployment insurance on post-unemployment outcomes. In the quantitative policy analysis under the restriction of not exceeding the baseline income-redistribution level across worker types, we find that the optimal policy consists of unemployment benefits 18% lower than those in the baseline. As a result of the differences in benefits, average wage falls by 0,35% and separation rate within a year after re-employment rises by 2.43% relative to the baseline economy. The empirical evidence in this regard is rather mixed. Card et al. (2007), for Austria, and van Ours and Vodopivec (2008), for Slovenia, find that a variation in the generosity of unemployment benefits neither significantly lengthens subsequent employment spells nor increases wages. Instead, Centeno (2004) and Belzil (2001) find that a more generous unemployment insurance does lengthen subsequent job tenure

[4] Output per worker also increases with unemployment benefits in Marimon and Zilibotti (1999), Acemoglu (2001), and Teulings and Gautier (2004). Two key differences with respect to these papers are their assumptions of risk-neutral workers and an exogenous separation rate independent of the match quality.

[5] We abstract from job-specific human capital investments, which would enhance the welfare gains of turnover-reducing unemployment benefits, and might question the result of excessive worker turnover.

using NLSY data for the US and Canadian data, respectively.

The paper is organized as follows. We investigate the efficiency of the worker turnover rate in a two-period economy in Section 2. The infinite horizon model is set in Section 3. The calibration exercise is undertaken in Section 4, and the policy analysis in Section 5. Section **??** discusses assumptions and results. Finally, Section 6 concludes. The data treatment details and the proofs are postponed to the Appendix.

2 A two-period Economy

In this section we investigate worker turnover in a two-period economy. First, we describe the physical environment. Second, we analyze the allocation that a social planner would choose, and then show that it can be decentralized in a market economy with government intervention.

2.1 Setup

The economy lasts for two periods. It is populated by a unit mass of risk-averse workers, and a large continuum of risk-neutral firms. All agents discount period-two utility at rate β. Workers are ex-ante identical, and unemployed at the beginning of period one.

The worker's utility function is additively separable in consumption and search intensity, $v(c)-\phi(s)$. The function v is twice continuously differentiable, increasing and concave in consumption c. We assume that $v(0)=0$ and $\lim_{c\to 0} v'(c)=\infty$. Job-seekers derive disutility $\phi(s)$ if exerting search intensity $s\in[0,1]$. ϕ is an increasing and convex function such that $\phi(0)=0$, $\lim_{s\to 0}\phi'(s)=0$ and $\lim_{s\to 1}\phi'(s)=\infty$.

There is some asymmetry across periods for simplicity. The first period is divided into two stages, search and production, while only production takes place in the second period. Search is a costly activity not only for workers, but also for firms. Let k denote the cost of posting a vacancy. An aggregate technology described below produces bilateral meetings between workers and firms. The decision of forming a match depends on the match quality. This can be either good, in which case the pair produces y units of output, or bad, and then output is 0.

We build upon Jovanovic (1979) and Pries and Rogerson (2005), and model match quality as both an inspection good and an experience good. Match quality is unobservable at the meeting time. However, the parties draw a signal π, which conveys the probability of the match quality being good, from a differentiable cdf F with support the unit interval. Match formation depends on the observed signal. If a match is not formed, the worker stays unemployed for the two periods and produces z units of output at home. Otherwise, the expected productivity in the first period amounts to πy, and match quality is learned at the beginning of the second period. If it turns out to be bad, the match is terminated and the worker produces z at home. Otherwise, the output produced by the match is y.

Search. Workers and firms get together via search. Let $q(m)$ denote the ratio of applicants in search efficiency units to vacancies at location m. We will suppress the dependence of the ratio on the location hereafter unless necessary for readiness. Firms meet workers with probability $\eta(q)$, and workers meet firms with probability $\nu(q)$ per unit of search intensity. Both functions are assumed to be twice continuously differentiable. Furthermore, η is assumed to be increasing in q to capture the intuition that more candidates competing for the available jobs increase the prospects of any given vacancy. Similarly, ν is a decreasing function because more job opportunities for a worker are expected in a tighter labor market. Let φ denote the elasticity of the job-filling rate, which is assumed to be a decreasing function. To guarantee vacancy creation both in the equilibrium and the constrained efficient allocations, first, we impose the following standard limit conditions

$$\lim_{q\to 0}\eta(q) = \lim_{q\to\infty}\nu(q) = 0, \text{ and } \lim_{q\to\infty}\eta(q) = \lim_{q\to 0}\nu(q) = 1,$$

and, second, we assume that the expected discounted output produced at the market net of the vacancy-creation costs exceeds home productivity,

$$y\mathbb{E}(\pi)(1+\beta) - z(1+\beta\mathbb{E}(\pi)) > k \tag{1}$$

2.2 The Planner's Allocation

The social planner dictates workers what to do and chooses the mass of vacancies in order to maximize the expected discounted utility of a representative worker. Since workers are ex-ante identical and firms have access to the same production technology, the planner sends all workers to the same location. The planner's problem is subject to a number of constraints. First, it cannot directly assign workers to vacancies, but it is constrained by the meeting technology described above. Second, it cannot learn the quality of a potential match prior to period two. It observes a signal for a match quality, and decides whether to form a match or not by setting a reservation probability R. Third, the worker's search intensity is unobservable to the planner. The planner recommends an incentive compatible search intensity. Consistent with this recommendation, the planner commits to a reservation probability and a consumption bundle $(c_u, c_1(\pi), c_2(\pi))$. Workers are promised consumption c_u if unemployed in either period, and $c_t(\pi)$ if employed in period t with signal π. That is, the following incentive compatibility condition must hold.

$$\phi'(s) = \nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi) \tag{2}$$

Fourth, the planner's allocation must be feasible. That is, total consumption cannot exceed total output net of vacancy creation costs, and, hence, the following resource constraint must hold.

$$(z-c_u)\Big(\big(1-s\nu(q)(1-F(R))\big)(1+\beta)+s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big) \tag{3}$$
$$+s\nu(q)\int_R\Big(\pi y-c_1(\pi)+\beta\pi\big(y-c_2(\pi)\big)\Big)dF(\pi)\geq k\frac{s}{q}$$

where, to save on notation, we denote the conditional expected probability as

$$\mathbb{E}(\pi|R)\equiv\int_R\pi\frac{dF(\pi)}{1-F(R)}$$

Therefore, the planner's allocation is a solution to the following program:

$$\max_{\substack{s,q,R,c_u\\c_1(\pi),c_2(\pi)}}\Big\{-\phi(s)+s\nu(q)\int_R\Big(v(c_1(\pi))-v(c_u)+\beta\pi\big(v(c_2(\pi))-v(c_u)\big)\Big)dF(\pi)\Big\}+v(c_u)(1+\beta)$$
$$\text{s. t. conditions (2), (3)} \tag{4}$$

The following proposition establishes existence of the planner's solution. There is a tension between the consumption smoothing goal (due to risk aversion of workers) and providing the right search incentives. Therefore, the planner must promise a lower consumption to unemployed workers than to the employed because of the incentive compatibility condition. The tension is optimized by transferring resources produced by the employed to the consumption of the unemployed; that is, the consumption of the unemployed lies above home production. Besides, the constrained efficient allocation ensures perfect consumption smoothing within an employment spell.

Proposition 2.1 *There exists a constrained efficient allocation* $(s^p,q^p,R^p,c_u^p,c_1^p(\pi),c_2^p(\pi))$. *The consumption levels are such that* $z<c_u^p<c^p=c_1^p(\pi)=c_2^p(\pi)$. *The reservation probability* R^p *satisfies*

$$\frac{v(c^p)-v(c_u^p)}{v'(c^p)}+c_u^p-z+y\frac{R(1+\beta)}{1+\beta R}-c^p\geq 0,\ \textit{and}\ R\geq 0,\ \textit{with complementary slackness} \tag{5}$$

Furthermore, $y\frac{R^p(1+\beta)}{1+\beta R^p}<z$, *and*

$$\eta(q^p)(1-F(R^p))(1-\varphi(q^p))\Big(\mathbb{E}(\pi|R^p)-(1+\beta\mathbb{E}(\pi|R^p))\frac{R^p}{1+\beta R^p}\Big)y(1+\beta)=k,\ \textit{if}\ R^p>0 \tag{6}$$

Let us refer to the solution of the equation $yR(1+\beta)=z(1+\beta R)$ as $\hat{R}$. The planner sets a reservation probability such that the marginal matches (i.e. matches with signal R^p) are less productive than workers would be at home, i.e. $R^p<\hat{R}$. This result is not intuitive as one would expect that matches would be formed only if their discounted output exceeded what they can produce separately. This would be exactly the efficiency condition if workers were risk neutral. Because the constrained efficient allocation maximizes net output under risk neutrality, output is maximized at the reservation value $\hat{R}$.

An interpretation of the result $R^p < \hat{R}$ is as follows. There are two types of frictions that separate workers into two different pools. First, luck sorts workers according to whether they meet a firm or not. Unlucky workers fall in the unemployment pool. Second, only matches with a signal above the reservation probability are formed. Bad luck in the signal-drawing stage also sends workers to the unemployment pool. Importantly, consumption levels differ between these two pools due to incentive compatibility. The planner can attenuate these two frictions by creating more vacancies and setting a lower reservation probability, yet these actions are costly. In particular, the planner sets the reservation value R^p to optimize the trade-off displayed in expression (5) between utility gains and output costs of sending workers to employment instead of unemployment. If a risk-averse worker drew signal $\hat{R} - \epsilon$, with ϵ arbitrarily small, then the output costs of sending her to employment would amount to $y\epsilon(1+\beta)$, whereas the utility gains would be discrete. As a result, the planner sets a reservation value such that $R^p y(1+\beta) < z(1+\beta R^p)$, and the marginal employed worker is more productive at home than at the market. Therefore, the efficient worker turnover is higher than the output-maximizing level.

Finally, condition (6) determines the constrained efficient queue length. The three two factors of the left hand side stand for the probability of filling the vacancy given the reservation probability R^p, whereas $1 - \varphi(q^p)$ is the share of the expected output net of the resources necessary to provide the utility difference expressed in condition (5) that is required to cover the vacancy costs of the marginal vacancy.

2.3 Market Economy.

In this section, we determine the equilibrium allocation, and analyze the dependence of the equilibrium variables on some fundamentals. We first describe the additional environment details.

2.3.1 Environment.

There is free entry of firms. That is, firms post vacancies provided that they see an opportunity to obtain profits. There are potentially infinitely many submarkets. Each submarket is identified by a contractual offer $x \equiv (\omega, R)$. Let X denote the set of offers. The first contractual component $\omega = (w_1(\pi), w_2(\pi))$ specifies the wage earned by the employed worker in each period contingent on signal π. The second element R stands for a reservation probability. That is, a match is formed if and only if the signal drawn by a firm-worker pair upon meeting is above this threshold.[6] There is perfect information about contracts. In this economy firms compete for workers. They choose a submarket where to locate its vacancy, and workers direct their search to one submarket to maximize their expected utility. Search is directed in the sense that agents anticipate that submarkets promising a lower expected discounted employment value to applicants will be tighter. Let $q(x)$ denote the

[6]This market structure may be also interpreted as the design of a market maker, who splits the market in submarkets, as analyzed in Moen (1997).

expected number of applicants in search efficiency units per vacancy at submarket x. As before, we will omit the contractual reference in this ratio unless necessary for clarity.

Various forms of market incompleteness are assumed. First, workers have no access to credit markets to smooth consumption across all states of the world. In particular, they cannot save nor borrow. Second, firms can only pay their employees in exchange for labor inputs.

Value Functions. Agents only make decisions at the beginning of the first period when the search activity takes place. A profit-maximizing firm posts a vacancy in submarket x at cost k if it expects to obtain non-negative profits. A vacancy is filled with probability $\eta(q(x))(1-F(R))$. The firm's value is

$$V = -k + \max_{x=((w_1(\pi),w_2(\pi)),R)} \eta(q(x)) \int_R \Big(\pi y - w_1(\pi) + \beta\pi\big(y - w_2(\pi)\big)\Big) dF(\pi) \tag{7}$$

Free entry implies that the expected returns from posting a vacancy must be zero, $V = 0$.

Workers decide which submarket x to search for a job and the search intensity s. They become employed with probability $s\nu(q(x))(1-F(R))$, and remain employed in the second period with probability $\mathbb{E}(\pi|R)$. If employed, workers derive utility from the wages stipulated in the contract. Otherwise, they enjoy their home production. Job-seekers search in submarket x if the expected discounted value promised there coincides with their market value U. That is, the following condition must hold for any submarket $x \in X$.

$$\begin{aligned} U \;&\geq\; \max_s \Big\{ -\phi(s) + s\nu(q(x)) \int_R \Big(v(w_1(\pi)) - v(z) + \beta\pi\big(v(w_2(\pi)) - v(z)\big)\Big) dF(\pi)\Big\} + v(z)(1+\beta), \\ &\text{and } q(x) \geq 0, \text{ with complementary slackness} \end{aligned} \tag{8}$$

2.3.2 Equilibrium

Next, we define the competitive search equilibrium.

Definition 1 *A competitive search equilibrium consists of a market value U, a search intensity $s \in [0,1]$, a queue length function $Q : X \to \mathbb{R}_+ \bigcup \{\infty\}$, and a contract $x = ((w_1(\pi), w_2(\pi)), R) \in X$ such that*

i) Firm's profit maximization and zero-profit condition:

For any contract $x' = ((w_1'(\pi), w_2'(\pi)), R')$,

$$\eta(Q(x')) \int_{R'} \Big(\pi y - w_1'(\pi) + \beta\pi\big(y - w_2'(\pi)\big)\Big) dF(\pi) \leq k,$$

which holds with equality at x.

ii) Worker's optimal search:

For any contract x' and $Q(x')$, condition (8) holds. In particular,

$$U = \max_s \Big\{ -\phi(s) + s\nu(Q(x)) \int_R \Big(v(w_1(\pi)) - v(z) + \beta\pi\big(v(w_2(\pi)) - v(z)\big)\Big) dF(\pi) \Big\} + v(z)(1+\beta)$$

The first condition establishes that firms choose the profit-maximizing contract, and free entry ensures that expected discounted profits are zero in equilibrium. The second condition ensures that workers enter a submarket if the contract maximizes their expected utility. This equilibrium condition also pins down the off-the-equilibrium expectations about the queue length at any submarket. This helps agents make their optimal decisions on the equilibrium path.

The following proposition characterizes the equilibrium allocation. A tuple $(w_1(\pi), w_2(\pi), R, q, s)$ takes part of an equilibrium allocation if and only if it maximizes the worker's expected discounted utility subject to firms obtaining non-negative profits. It also establishes existence of equilibrium.

Proposition 2.2 *A competitive search equilibrium solves the following program, and, conversely, a solution of the program takes part of an equilibrium allocation.*

$$\max_{w_1(\pi), w_2(\pi), R, q, s} \quad \Big\{ -\phi(s) + s\nu(q) \int_R \Big(v(w_1(\pi)) - v(z) + \beta\pi\big(v(w_2(\pi)) - v(z)\big)\Big) dF(\pi) \Big\} \tag{9}$$

$$s.\ to \qquad \eta(q) \int_R \Big(\pi y - w_1(\pi) + \beta\pi\big(y - w_2(\pi)\big)\Big) dF(\pi) \geq k$$

There exists an equilibrium allocation. Furthermore, $z < w_1(\pi) = w_2(\pi) = w$, and, given the wage, the reservation probability R satisfies

$$\frac{v(w) - v(z)}{v'(w)} + y\frac{R(1+\beta)}{1+\beta R} - w \geq 0, \tag{10}$$

$$\textit{and } R \geq 0, \textit{with complementary slackness}$$

In equilibrium, $Ry(1+\beta) < z(1+\beta R) < w(1+\beta R)$, and

$$\eta(q)(1 - F(R))(1 - \varphi(q))\Big(\mathbb{E}(\pi|R) - (1 + \beta\mathbb{E}(\pi|R))\frac{R}{1+\beta R}\Big) y(1+\beta) = k, \ \ \textit{if } R > 0 \tag{11}$$

Because workers are risk averse, the equilibrium contracts offer perfect consumption smoothing while employed. This result is well known from the implicit contract literature, and relies on the different ability of firms and workers to bear risk. unemployed workers consume their home production by assumption. Recall that markets are incomplete, and firms can only make payments to their employees. The equilibrium condition (10) determines the equilibrium reservation value R. The first term of this expression accounts for the worker's discounted adjusted utility in excess of what they derive from home production. The second term refers to the firm's discounted expected profits conditional on the signal being R. All together constitute the adjusted joint value of the job-worker pair. That is, condition (10) states that a match is formed after drawing a signal π if and only if the adjusted joint value of the pair is positive. Notice that $Ry(1+\beta)-w(1+\beta R)<0$ in equilibrium, and, hence, firms make negative profits when the signal drawn falls in a neighborhood of the cutoff value. Given wages and the expected queue length, a higher reservation value would increase firm's profits, but firms find it optimal ex-ante to commit to a low cutoff to attract applicants. In the limit case, if workers were risk neutral, the cutoff R would be such that the expected productivity at the market equalized home productivity. Finally, the necessary condition (11) equates the marginal profits of the marginal firm and its marginal costs.

2.3.3 Comparative Statics.

Next, we perform three comparative static exercises regarding the key primitives of the model.

The first two results stated in Lemma 2.3 can be read as an extension of Proposition 2 in Acemoglu and Shimer (1999) to our economy. First, consider two economies that differ only in how risk-averse workers are. A larger number of jobs with lower wages and a lower cutoff R are created in the economy in which workers are more risk averse. Second, we show that the higher the home productivity z, the higher the reservation probability R and the queue length q.

Our interpretation of these comparative static outcomes is as follows. In an economy with not fully insured unemployment risks, risk-averse job-seekers factor in such risks. They prefer to apply to low-wage jobs inasmuch as they are easier to get. Put differently, risk-neutral firms offer private insurance through two channels. First, a large mass of low-wage jobs are created, similarly to Acemoglu and Shimer (1999). Second, the equilibrium reservation probability is low to increase the employment chances. This second outcome is not obvious as firms trade off providing insurance in the first period against insurance in the second period. The comparative static results imply that the period-two insurance motive is of second order. A plausible explanation is the discounting of period-two utility. The more risk averse workers are, the more important the insurance motive becomes. Similarly, a lower outside option makes workers even less risky in their search strategies, and the insurance role of the equilibrium contracts becomes more important. As a result, matches are of lower quality in expected terms, which leads to poorer post-unemployment outcomes: lower wages and output per worker, and a higher job-separation rate in the second period, $1-\mathbb{E}(\pi|R)$. The overall effects on total output are uncertain because, on one hand, output per worker is lower, and, on the other hand, the unemployment rate is also lower in both periods.

Now, we consider two cumulative distribution functions, F_1 and F_2. We say that F_2 dominates F_1, $F_1 \lesssim F_2$, if the two following assumptions hold.

Assumption 1 *F_1 is less than F_2 in the strict mean residual life order. That is,*

$$\int_{\pi\geq t}\pi\frac{dF_1(\pi)}{1-F_1(t)}<\int_{\pi\geq t}\pi\frac{dF_2(\pi)}{1-F_2(t)},\ \forall t$$

Assumption 2 *F_2 first-order stochastically dominates F_1. That is,*

$$F_1(t) \geq F_2(t), \ \forall t, \ \textit{and strict inequality for some } t$$

We refer to the first assumption as *strict* mean residual life order because it requires a strict inequality instead of weak inequality. This stochastic order implies that the truncated expected probability of the match quality being good is always higher with distribution F_2. The second assumption establishes that good signals are more likely in an economy with cdf F_2.[7] We postpone the discussion about the interest of these assumptions to Section 3.

Lemma 2.3 states that if the signals are drawn from the cdf F_2, then vacancy creation is larger, the reservation probability is lower, and wages and job-finding rates are higher in equilibrium relative to the economy with F_1. As a result, the unemployment rate is lower. We prove that job-separation is lower and output higher with F_2 for the case of a Cobb-Douglas meeting technology, and this also holds for other meeting technologies according to our simulated work.

Certainly, one would expect agents to benefit from the better technology in producing good matches, which turns out into more vacancies available and higher wages. However, it is not intuitive why the reservation probability is lower in the F_2 economy. To understand this result, consider the case in which both reservation probabilities are positive. Notice that the equilibrium condition (10) does not directly depend on the distribution function. The equilibrium cutoff equalizes the joint value to zero, establishing a negative relationship between the equilibrium wages and the reservation probability. Therefore, if wages under F_2 are higher, the associated cutoff must be lower.

Lemma 2.3 *Comparative Statics. Let (w_i, q_i, R_i, s_i) denote the equilibrium tuple of the economy indexed by i.*

1. *If two economies differ in the workers' utility function v, and v_2 is a concave monotonic transformation of v_1, then $w_2 < w_1$, $q_2 < q_1$, and $R_2 < R_1$ (and $R_2 = R_1$ if and only if both are 0).*

2. *If two economies differ in the home production value such that $z_1 < z_2$, then $w_1 < w_2$, $q_1 < q_2$, $R_1 < R_2$, and $s_2 < s_1$ in equilibrium. Further, the job-separation rate is lower and the unemployment rate and the output per worker are higher in both periods, the higher the home production.*

3. *If two economies differ in the cumulative distribution such that $F_1 \lesssim F_2$, then $w_1 < w_2$, $q_2 < q_1$, $R_2 < R_1$ (unless both are zero), and $s_1 < s_2$ in equilibrium. The job-finding probability is higher and, hence, the unemployment rate is lower under F_2. Furthermore, if*

[7] It is worth noticing that, under some additional condition, the mean residual life order implies first order stochastic dominance. See Shaked and Shanthikumar (1994, Ch. 1.D) for further details. As an example, consider a uniform distribution and a perturbation of it, $F_1 \sim U[0,1]$ and $F_2(\pi) = -\epsilon\pi^2/2 + (1+\epsilon/2)\pi$. We have $F_1 \lesssim F_2$ for $\epsilon > 0$.

the meeting technology is Cobb-Douglas, then the job-separation rate is lower and the output per worker and the total output are higher under F_2.

These last comparative static results suggest that if workers were heterogeneous in their ability to form long-lasting matches and the labor market were segmented by types, then the more skilled workers would find jobs sooner and stay employed longer. This would lead to a positive relationship between the average probability of job-separation and the duration of the previous unemployment spell, consistent with the evidence plotted in Figure **??**.

2.4 Implementation

Next, we show that the planner's allocation can be implemented in a market economy, and discuss the role of the government to achieve constrained efficiency in equilibrium.

It is apparent from Propositions 2.1 and 2.2 that the market allocation is not constrained efficient as the equilibrium consumption of the unemployed falls short of the efficient level. To better understand this inefficiency outcome, consider the centralized economy in which the planner is not allowed to promise unemployed workers a consumption level different from z. In this case, constrained efficiency is attained in the market economy. This result implies that the inefficiency in the laissez-faire economy is due to the uninsured unemployment risks, and relies on (at least) two missing markets, as in Golosov et al. (2013). As said above, markets are incomplete, first, because workers cannot sign contracts with third parties to insure themselves against negative search outcomes. Second, and unlike Jacquet and Tan (2012), workers and firms cannot trade applications, but labor. That is, firms can only reward successful applicants while employed. These missing markets prevent the consumption of unemployed workers from exceeding their home production. Firms behave as employers and insurers of their risk-averse workers in equilibrium. This is done by committing to jobs with low and constant wages and low hiring standards to increase the employment chances. As a result, employment and worker turnover are inefficiently high.

Consider now the case with a positive threshold R^p. The following proposition states that the constrained efficient allocation can be decentralized if a unemployment insurance system plus a negative income tax are implemented, funded through a lump sum tax. We now provide the intuition for this result. Consider first a laissez-faire economy. For given wages, the equilibrium reservation value is pinned down by the equilibrium condition (10). That is, a positive R is determined by making the adjusted joint value of the pair be equal to zero. Recall that firms make negative profits ex-post if the signal is within a neighborhood of the cutoff. Furthermore, the expected profits as a function of R, for a given q and w, are hump-shaped, reaching the peak when $J(x, R) = 0$. Therefore, the ex-ante expected-profit-maximizing cutoff is below the ex-post maximizing one because of the competition for workers.

If a government makes transfers to the unemployed, it partly crowds out the private insurance firms offer in the laissez-faire economy and induces firms to increase R and wages as close as possible to the contract that makes $J(x,R)=0$. Job-seekers are now riskier in their search strategies because of the insurance provided by the government. Firms become free-riders of the publicly provided insurance, and commit to an inefficiently high reservation value. A negative income tax provides incentives for firms not only to create vacancies, but to form matches as they benefit from the wage subsidies only after matching. Thus, such subsidies reduce the equilibrium cutoff to the constrained efficient level.

Proposition 2.4 *The competitive search equilibrium is not constrained efficient unless the planner is forced to set $c_u = z$. If $R^p > 0$, then constrained efficiency is attained if a government sets a unemployment insurance system funded through lump sum and negative income taxes, whereas layoff taxes must be zero.*

Our main contribution is that layoff taxes must be zero and match creation must be subsidized in order to attain constrained efficiency in the market economy. It is worth providing an intuition for this result. There are two margins that affect aggregate employment and the risk insurance: firm entry and the reservation probability. Regarding the second margin, the reservation probability R^p is determined by the efficiency condition (5). It is also the equilibrium threshold if condition (10) holds when $c_u^p = z + b - T$ and $c^p = w(1-\tau) - T$, regardless of firm entry. That is, it must be the case that $-\tau R y(1+\beta) - (1-\tau)L\beta(1-R) = b(1+\beta R)$. Therefore, different combinations of income tax rates and layoff taxes make the reservation probability margin be efficiently set in the market economy. In particular, if $\tau = 0$, then $-L\beta(1-R) = b(1+\beta R)$. That is, if $b > 0$, layoff taxes would be negative and the subsidy government sets to increase match creation equals the total unemployment benefits workers would obtain if remained jobless.

Regarding the firm entry margin, the necessary planner and equilibrium conditions with respect to the queue length (6) and (11), respectively, coincide with one another when all taxes are borne by workers. However, when layoff taxes are levied and the threshold is positive, the equilibrium condition (11) becomes

$$\begin{aligned} k \;&=\; \eta(q)(1-F(R))(1-\varphi(q))(1+\beta\mathbb{E}(\pi|R)) \\ &\quad \left(\left(\frac{\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R}{1+\beta R}\right)y(1+\beta) - L\beta\left(\frac{1-\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{1-R}{1+\beta R}\right)\right) \end{aligned}$$

Therefore, layoff taxes must be zero for constrained efficiency to be attained, and, hence, the income tax rate be negative.

2.4.1 Policy Discussion.

We know turn to understand why our policy results differ from Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008). Our model is quite close to theirs in many respects. In particular, there is free entry of firms that commit to a wage-threshold pair to compete for workers. However, there are two key differences with respect to our setting. First, all matches take place, and get destroyed if match quality, which is instantaneously learned, is below the threshold. Therefore, match quality can be interpreted as an experience good, but it is not an inspection good. Second, theirs is a frictionless economy with no unemployment.

To analyze whether the inspection feature of match quality is the key difference, we extend Blanchard and Tirole (2008) static model to a directed search setting. We mostly postpone the details to Appendix 7.2, and briefly summarize the environment and central results here.

Workers are risk averse and firms are risk neutral. There are three stages. In stage 1, a government set a policy, which may consist of unemployment benefits, lump sum taxes, a proportional income tax, and a layoff tax. In the second stage, firms post contracts and workers direct their search. Firms commit to contracts that stipulate a wage and a threshold, (w, R). Matches are heterogeneous in their quality, or productivity, π. All meetings end up in matches. At the beginning of the third stage, match quality is drawn from a cdf F, and is perfectly observed. Separations take place if the productivity draw is below the threshold R. Laid-off workers produce z units of output at home, and consumption takes place.

It is worth investigating the two margins mentioned above separately. Regarding the threshold margin, the necessary efficiency and equilibrium conditions coincide with one another if and only if $-\tau R + L(1-\tau) = b$. Once again, there are multiple combinations of the two fiscal instruments that set this margin efficiently in equilibrium. In particular, if $\tau = 0$, then $L = b$, which has been interpreted as a Pigovian tax that makes firms internalize the social costs of layoffs.[8] When comparing the necessary conditions regarding vacancy creation, we obtain that $L = 0$ and, hence, $\tau < 0$.

Although the work is not shown here, let us say that the zero layoff tax result is also obtained in an economy that only differs from our benchmark in that match quality is not an inspection good and vacancy costs are an increasing function of the reservation probability (i.e. creating better jobs is more costly).

We must conclude that, despite the obvious link between match creation and destruction in our model, the inspection feature is not the underlying force that makes the optimal policy have no layoff taxes. Therefore, search frictions are relevant for this result.

2.4.2 The Role of Savings.

In the competitive search literature, the analysis of the partial insurance that firms offer in private markets has been mostly restricted to economies in which risk-averse workers cannot self-insure through savings.[9] This is for tractability reasons because workers with different asset holdings search for different combinations of unemployment risk and employment values. If saving were allowed, the equilibrium asset distribution would be linked to a contract distribution. Golosov

[8] Blanchard and Tirole (2008) conjecture that layoff taxes should exceed unemployment benefits when search frictions and search incentives are present.

[9] This assumption has also been standard in the implicit contract literature. See e.g. Baily (1974) and Azariadis (1975).

et al. (2013) show the implications for income inequality of risk-averse workers facing uninsured unemployment risks and holding no assets. Lamadon (2014) analyzes the insurance role of long-term contracts and the transmission of firm shocks to wages and employment in an extended version of Menzio and Shi (2011) with risk-averse workers. Rudanko (2009) explains wage rigidity in a business cycle model in which risk-neutral firms smooth workers' consumption through long-term contracts. Her model presents the additional complexities associated with a setting with aggregate shocks. An exception is Acemoglu and Shimer (1999), who do allow workers to save, but reduce the analysis to workers with CARA preferences to avoid the above complications.

Following the literature, we consider the extreme case in which workers cannot insure themselves through precautionary savings, and investigate the insurance role of long-term contracts. In Appendix 7.3 we analyze an extended version of the two-period model in which workers are endowed with asset holdings that can be used to smooth consumption over time, but they are borrowing-constrained. In this setting, firms internalize the worker's ability to transfer resources from period one to period two when designing the profit-maximizing contract, and fully frontload wages. Workers achieve constant consumption across periods regardless of the match-quality-learning outcome conditional on being employed in period one. However, period-one unemployment risks are still not fully insured regardless of the asset holdings workers are endowed with. The central assumption is the worker's inability to borrow against future income. As a result, workers have preferences for securing employment when searching for jobs and firms commit to low hiring standards. Furthermore, if a publicly-provided insurance is implemented, then a negative income tax must be levied in order to attain constrained efficiency as reasoned above. Therefore, the results stated in Proposition 2.4 do not qualitatively change when allowing for precautionary savings, although the insurance motive is obviously quantitatively weaker.

2.4.3 Market Incompleteness and Severance Payments.

The contracting space in our setting is restricted to permit payments only to employees in exchange for labor. Therefore, privately-provided unemployment insurance and, in particular, severance payments are ruled out.[10] In the competitive search literature, severance payments have been often discarded by focusing on equilibria with self-constrained contracts, also referred to as limited commitment equilibria. However, full commitment is assumed in our setting, and, as stated in Proposition 2.2, firms obtain a negative expected discounted value in a neighbourhood of the reservation probability.

Consider the wider contracting environment in which contracts are somewhat more complex and incorporate private unemployment insurance. That is, a firm commits not only to a reservation probability and wages to be paid throughout an employment spell, but also promises benefits or annuities and recommends a search intensity compatible with worker's incentives after separating. We conjecture that the steady-state equilibrium allocation would be constrained efficient in a full

[10] We want to thank Melvyn Coles and Espen Moen for the discussion on severance payments.

dynamic setup, except if new entrants were modelled.

2.4.4 Minimum Wage Policy.

The literature has also considered other policies that reduce worker turnover. In particular, minimum wage policies increase the equilibrium hiring standards and, hence, reduce the insurance firms provide to risk-averse workers in private markets. As a result, the inefficiency resulting from uninsured unemployment risks becomes larger. Our results complement the findings in Pries and Rogerson (2005), who estimate large output losses induced by a minimum wage policy with risk-neutral workers.

3 Infinite Horizon Model

This section extends the previous setting to an infinite horizon economy with worker heterogeneity to quantitatively assess worker turnover in the U.S. economy. Workers heterogeneity is unobserved, and assumed to replicate some facts of the U.S. economy that we document in a companion paper: the probability of transiting from employment to unemployment within the first year after re-employment increases with the duration of the previous unemployment spell, and there is a statistically significant gap in the starting wage between the group of workers who remain employed after one year and those who return to unemployment.

3.1 Setup

Time is discrete. There is a mass one of heterogeneous, infinitely-lived, risk-averse workers and free entry of risk-neutral firms. Firms and workers discount future utility at common rate β. The period utility function of workers, $v(c) - \phi(s)$, satisfies the properties listed in the previous section. Markets are incomplete. In particular, workers cannot insure themselves against income fluctuations through saving and borrowing, and firms can only pay their own employees.

Workers can be either employed or unemployed at the beginning of every period. Employed workers obtain wages and make no decisions. Job-seekers derive utility from home production, z, and disutility from search intensity, s. Firms incur cost k when posting vacancies. Each firm holds a single job. Search frictions and the meeting technology are the same as in Section 2.1. Match quality can be either good, which results in a match productivity y, or bad, and then the match output is zero.

Match quality as both an inspection good and an experience good. The quality of a potential match is unknown at the meeting time. However, a signal π reporting the probability of the match being of good quality is observed. The quality of the match is an inspection good in the sense that it will be formed if the signal is sufficiently good. While firms are ex-ante identical, workers differ by their abilities to form matches of good quality. For simplicity, workers can be either high type (h) or low type (ℓ). The worker's type is unobserved by the recruiting firms. Let $\mu_h = \mu$ denote the mass of type h workers in the economy. The remaining $\mu_\ell = 1 - \mu$ workers are of type ℓ. The signal for a potential match with a type i worker is drawn from a cdf F_i, with $i \in \{\ell, h\}$. We assume that the distribution F_h dominates F_ℓ, $F_\ell \lesssim F_h$, as defined in Section 2.3.3. That is, Assumptions 1 and 2 hold.

The observed output in any given period is equal to the actual output plus some noise, ϵ, which is an iid variable with zero mean. The worker and the firm simultaneously learn about the actual quality of the match by experiencing it. We follow Pries (2004), and assume for simplicity that ϵ is uniformly distributed with support $[-\bar{\epsilon}, \bar{\epsilon}]$. For the match quality not to be learned instantaneously, it is required that $y - \bar{\epsilon} < \bar{\epsilon}$. That is, there is an interval of observed productivities for which the parties cannot infer the type of their match, $[y - \bar{\epsilon}, \bar{\epsilon}]$. This implies that the learning process is of the form "all-or-nothing": a match quality is learned when the observed output falls out of the above interval, which occurs with probability $\alpha \equiv \frac{y}{2\bar{\epsilon}}$. If so, the match quality is revealed to be good with probability π. With the complementary probability, the match is learned to be bad and the job is destroyed. Otherwise, nothing can be inferred and the posterior probability of a good match quality coincides with the prior.

Employment relationships are also subject to exogenous idiosyncratic job-destruction shocks, which arrive with probability λ every period. Notice that otherwise all workers would be employed in the steady state as they would eventually find matches of good quality.

Search Frictions and Contracting Environment. The market is divided into potentially infinitely many submarkets. Each submarket is identified by a contract $x = (\omega, R)$. Let X denote the set of all possible submarkets. Profit-maximizing firms choose a submarket to locate their vacancies, and utility-maximizing workers direct their search to a particular submarket. As in Section 2.3, the contractual terms specify a reservation value, R, such that a match is formed if and only if the signal is above the threshold. Moreover, the contract stipulates a wage schedule. To simplify on notation, we anticipate that equilibrium wages are not contingent on the signal.[11] Firms commit to a two-tier wage schedule $\omega \equiv (w_1, w_2)$. The first wage is paid while the actual match quality remains unknown, and the second wage is paid once it is observed to be good.

The meeting technology is the same as in the two-period economy. Therefore, a worker meets a job with probability $\nu(q(x))$ per unit of search intensity, and a vacant firm meets a worker with probability $\eta(q(x))$ in submarket x. Then, a type i worker becomes employed with probability $s\nu(q(x))\big(1 - F_i(x)\big)$.

Value Functions and State Variables. A type i worker derives utility $v(z) - \phi(s)$ while seeking a job with intensity s. When applying to a contract x, she meets a job with probability $s\nu(q(x))$, observes a signal π, and obtains the discounted value $E_i(x, \pi)$ if the match is formed. Her value

[11] Our analysis in the previous section shows that this reduction of the contracting space is without loss of generality.

function satisfies the following functional equation.

$$(1-\beta)U_i = v(z) + \max_{s,x}\left\{-\phi(s) + \beta s\nu(q(x))\int_R \Big(E_i(x,\pi) - U_i\Big)dF_i(\pi)\right\} \tag{12}$$

The employed worker in a type π match obtains wage w_1. She becomes unemployed if either hit by an exogenous shock, which occurs with probability λ, or if the match is learned to be of bad quality, which happens with probability $(1-\lambda)\alpha(1-\pi)$. Finally, the quality of the match turns out to be good with probability $(1-\lambda)\alpha\pi$ and the wage w_2 is paid to the worker until an exogenous destruction shock hits the pair. Her expected discounted utility is

$$\begin{aligned} E_i(x,\pi) = v(w_1) &+ \beta\lambda U_i \\ &+ \beta(1-\lambda)\alpha\Big((1-\pi)U_i + \pi\frac{v(w_2)+\beta\lambda U_i}{1-\beta(1-\lambda)}\Big) \\ &+ \beta(1-\lambda)(1-\alpha)E_i(x,\pi) \end{aligned} \tag{13}$$

Analogously, the asset value of a filled vacancy with a type π match is

$$\begin{aligned} J(x,\pi) = \pi y - w_1 &+ \beta\lambda V \\ &+ \beta(1-\lambda)\alpha\Big((1-\pi)V + \pi\frac{y-w_2}{1-\beta(1-\lambda)}\Big) \\ &+ \beta(1-\lambda)(1-\alpha)J(x,\pi) \end{aligned} \tag{14}$$

A vacant firm incurs cost k when posting a job offer. The job can be filled by either type h or ℓ workers. Let $\rho_i(x)$ denote the believed proportion of workers of type i in submarket x. Thus, vector $(\rho_\ell(x), \rho_h(x))$ is a point of the simplex Δ^1. The value of a vacant firm is

$$(1-\beta)V = -k + \beta\max_x\left\{\eta(q(x))\sum_i \rho_i(x)\int_R \Big(J(x,\pi) - V\Big)dF_i(\pi)\right\} \tag{15}$$

The expected profits of posting a vacancy must be zero in equilibrium because of free entry. That is, $V=0$. To ensure positive returns to vacancy-posting, we impose the following condition, which is the counterpart of expression (1).

Assumption 3 *There exists a value $r \in [0,1]$ such that*

$$\mathbb{E}_\ell(\pi|r)\big(1-F_\ell(r)\big)y - z\frac{\big(1-\beta(1-\lambda)(1-\alpha\mathbb{E}_\ell(\pi|r))\big)\big(1-F_\ell(r)\big)}{1-\beta(1-\lambda)(1-\alpha)} > \frac{k}{\beta}\big(1-\beta(1-\lambda)\big)$$

3.2 Equilibrium

We turn to define and characterize the steady state competitive search equilibrium in an economy with adverse selection. We build upon Guerrieri et al. (2010).

Definition 2 *A steady-state competitive search equilibrium consists of a distribution G of vacancies in active submarkets with support $\mathcal{X} \subset X$, value functions U_i, $E_i(\cdot,\cdot)$ and $J(\cdot,\cdot)$, a queue length function $Q : X \to \mathcal{R}_+$, search functions $S_i : X \to [0,1]$, and a function $\rho : X \to \Delta^1$ such that*

1. *The value functions U_i, $E_i(\cdot,\cdot)$ and $J(\cdot,\cdot)$ satisfy the functional equations (12)-(14).*

2. *Firms' profit maximization problem and free entry:*

$$\forall x \in X, \;\; \beta\eta(Q(x)) \sum_i \rho_i(x) \int_R J(x,\pi) dF_i(\pi) \leq k,$$

   with equality if $x \in \mathcal{X}$.

3. *Workers' optimal search:*

   The type i worker's value satisfies

$$(1-\beta)U_i = v(z) + \max_{s, x \in \mathcal{X}} \left\{ -\phi(s) + \beta s \nu(Q(x)) \int_R \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi) \right\}$$

   and, $\forall x \in X$,

$$(1-\beta)U_i \geq v(z) - \phi(S_i(x)) + \beta S_i(x) \nu(Q(x)) \int_R \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi),$$

   with equality if $Q(x) > 0$ and $\rho_i(x) > 0$.
   If $\int_R \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi) \leq 0$, then either $Q(x) = 0$ or $\rho_i(x) = 0$.
   Furthermore, $\forall x \in X$,

$$S_i(x) = \operatorname*{argmax}_s \left\{ -\phi(s) + \beta s \nu(Q(x)) \int_R \Big(E_i(x,\pi) - U_i \Big) dF_i(\pi) \right\}$$

4. *Market-clearing condition:*

$$\int_{\mathcal{X}} \rho_i(x) \frac{Q(x)}{S_i(x)} dG(x) = u_i \mu_i, \forall i$$

Let us point out the main differences with respect to the equilibrium definition 1. First, the fourth equilibrium condition implies that adding up job-seekers across submarkets must amount to the total mass of unemployed of each type. Second, as usual, the interest is in the determination of the expectations off-the-equilibrium path that support the equilibrium allocation. Consider the following trembling hand kind of argument. Think of an arbitrarily small measure of firms deviating to submarket $x' \notin \mathcal{X}$. Such firms form rational expectations about the optimal search behavior of workers. If workers were all identical, there would be an outflow of workers from an equilibrium contract $x \in \mathcal{X}$ to submarket x' until the point in which they were indifferent between x and x'.

Such flows would occur even if different types of workers search in the labor market. If type h workers found it optimal to flow in submarket x' at the queue length that made type ℓ workers indifferent, then the ratio $q(x')$ would be determined by the former type and the deviating firms would only receive applications from type h workers, $\rho_h(x') = 1$. This reasoning is captured in the third equilibrium condition.

The following proposition states that it cannot be the case that both types of workers apply to the same jobs in equilibrium. As a result, any equilibrium must be separating.

Proposition 3.1 *There is no open submarket in equilibrium in which firms receive applications from both types of workers.*

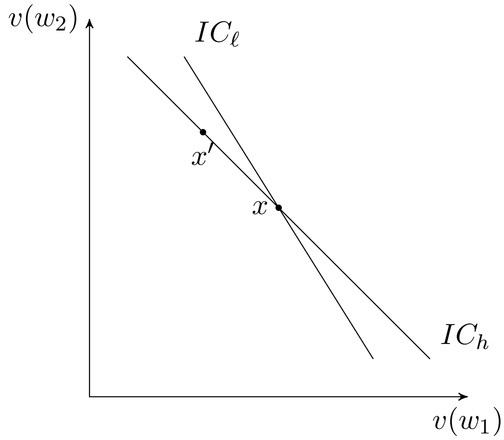


Figure 1: Single-crossing condition.

The intuition behind this result is that firms can always separate types by backloading wages. To be specific, suppose that there existed a submarket x attractive to both types of workers in equilibrium. The expected value promised to type i applicants in this submarket would amount to

$$\int_R \big(E_i(x,\pi) - U_i\big) dF_i(\pi) = \big(1 - F_i(R)\big)\Big(\xi_1 v(w_1) + \xi_2 \mathbb{E}_i(\pi|R) v(w_2)\Big) + \xi_{3,i}(R),$$

where the positive coefficients ξ_1 and ξ_2 are type independent. Figure 3.2 depicts the locus of wage schedules that provide the same expected value to workers, conditional on employment, in the $(v(w_1), v(w_2))-$space. A submarket x that attracted both types of workers would lay in the intersection point of the two lines. Assumption 1 implies that the line of type h workers is flatter. Consider now the alternative submarket x' with a lower first wage and a higher second payment, and the same reservation probability R. A deviation to submarket x' would leave type h workers indifferent, while worsening the value of the type ℓ applicants. As a result, by deviating to contract x', firms would manage to attract only applicants of type h. In other words, Assumption 1 can be read as a single-crossing property.

To see that such a deviation is profitable, we can rewrite the expected profits conditional on matching with a type i worker as

$$\int_R J(x,\pi)dF_i(\pi) = \frac{1-F_i(R)}{1-\beta(1-\lambda)(1-\alpha)}\left(\left(y+(y-w_2)\frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)}\right)\mathbb{E}_i(\pi|R)-w_1\right)$$

Because of Assumption 1, firms anticipate that matches with type h workers last longer in expected terms. However, this does not suffice to ensure that deviating firms make higher profits from screening out type ℓ workers because jobs might be more difficult to fill with high types. Assumption 2 ensures that this is not the case. Therefore, Assumptions 1 and 2 together become a sufficient condition for such deviations to imply a discrete jump in profits, contradicting the equilibrium assumption. Therefore, there cannot be firms receiving applications from both types of workers in equilibrium.[12]

The following proposition states existence of a separating equilibrium, and its proof relies on showing the existence of a fixed point.

Proposition 3.2 *There exists a competitive search equilibrium. Furthermore, $U_\ell < U_h$, and $w_1^\ell = w_2^\ell$ and $w_1^h \leq w_2^h$*

State variables in the separating equilibrium. To save on notation, let $s_i \equiv S_i(x_i)$ and $q_i \equiv Q(x_i)$. The state variables in period t are the unemployment rate, u_t^i, and two employment rates for each type of worker, $e_{1,t}^i$ and $e_{2,t}^i$. Note that the mass of type i workers who are employed at jobs with known quality satisfies $e_{2,t}^i = 1-u_t^i-e_{1,t}^i$. The dynamics of the remaining two variables are determined by the following laws of motion:

$$\begin{aligned}
u_{t+1}^i &= \big(1-s_i\nu(q_i)\big(1-F_i(R_i)\big)\big)u_t^i + \lambda(1-u_t^i) + (1-\lambda)\alpha\big(1-\mathbb{E}_i(\pi|R_i)\big)e_{1,t}^i \\
e_{1,t+1}^i &= (1-\lambda)(1-\alpha)e_{1,t}^i + s_i\nu(q_i)\big(1-F_i(R_i)\big)u_t^i
\end{aligned}$$

The laws of motion are mostly self-explanatory. The mass of unemployed workers of type i increases because of either exogenous separations or the dissolution of matches when the match quality has revealed to be bad, whereas it decreases because of the new matches formed. Likewise, the mass of employed workers increases with the new matches formed, and reduces because of separations due to either exogenous shocks or learning that the match quality is bad.

[12] To discourage type ℓ applications, firms could alternatively increase the reservation probability R and compensate type h workers with a higher wage. This deviating strategy would be profitable if and only if

$$\frac{E_\ell(x,R)-U_\ell}{1-F_\ell(R)}dF_\ell(R) > \frac{E_h(x,R)-U_h}{1-F_h(R)}dF_h(R)$$

However, we would need to make additional assumptions for this inequality to hold.

The steady-state expressions for these two variables are

$$
\begin{aligned}
u^i &= \frac{\lambda}{\lambda + s_i \nu(q_i)\big(1 - F(R_i)\big)\frac{\lambda+\alpha(1-\lambda)\mathbb{E}_i(\pi|R_i)}{\lambda+\alpha(1-\lambda)}} \\
e_1^i &= \frac{s_i \nu(q_i)\big(1 - F(R_i)\big)}{\lambda + \alpha(1-\lambda)} u^i
\end{aligned}
$$

4 Calibration

In this section we calibrate the full dynamic model with a publicly-provided unemployment insurance system to the US economy. The US unemployment insurance system is a joint federal-state program funded through employer payroll taxes. Similarly, we set a government that levies a proportional income tax to balance its budget every period.[13] Therefore, unemployed workers derive utility $v(z+b)$ from consumption, where b stands for benefits, and employed workers obtain utility $v(w(1-\tau))$, where τ denotes the income tax rate. The government budget constraint is

$$
b\left(\mu_\ell u^\ell + \mu_h u^h\right) = \tau \sum_{\substack{i=\ell,h \\ j=1,2}} w_j^i e_j^i \mu_i
$$

We now describe the calibration strategy to parameterize this economy. First, we introduce our choices for functional forms. We assume $v(c) = \ln(c)$ and $\phi(s) = (\phi_1/\phi_2)\left[(1-s)^{-\phi_2} - 1\right] - \phi_1 s$, which satisfy the required properties specified in Section 2.[14] The search cost function is the same one used e.g. by Mitman and Rabinovich (2011). For the meeting technology, we follow den Haan et al. (2000) and set the firm's applicant-finding probability as $\eta(q) = (1 + q^{-\psi})^{-1/\psi}$. The main advantage of this specification is that it is continuously differentiable everywhere and its range is the unit interval. As for the distribution of match qualities F_ℓ and F_h, we follow Pries and Rogerson (2005) and use a zero-mean normal distribution, with standard deviation σ_ℓ and σ_h respectively, truncated between zero and one and re-scaled to form a proper cdf. Given this functional form, we find that $\sigma_h > \sigma_\ell$ is a sufficient condition for Assumptions 1 and 2 to hold.

A period is set to be a week for consistency with SIPP data. Table 1 presents the baseline calibration parameters. We normalize the market marginal productivity of labor to one. The discount factor β is set to be consistent with a 5% annual interest rate. The remaining parameters are jointly calibrated.

The search cost parameters, ϕ_1 and ϕ_2, and the meeting technology parameter, ψ, are directly related to matching probabilities. Therefore, we target the unemployment rate, the monthly job-

[13] Although there are a few states that levy taxes on workers (Alaska, New Jersey, and Pennsylvania), who nominally bears the tax burden is irrelevant for our analysis. We have also considered an insurance system funded by a lump sum tax, and found no significant differences.

[14] We have performed robustness checks with respect to the risk aversion coefficient of CRRA preferences on consumption and the results hold within a reasonable range of values. See Table ?? in the Appendix for an economy with risk aversion coefficient equal to 2.

finding rate, and the vacancy to unemployment ratio. We obtain these three averages using SIPP data for the 1996-2003 period and vacancy information from the Job Openings and Turnover Survey (JOLTS) since December 2010. In a similar spirit to Section **??** regarding attachment to the labor market, we restrict SIPP data to workers who have been employed at least half of their survey time. Although this restriction is somewhat arbitrary, the labor market statistics we obtain appear quite close to the SIPP-based numbers found by Bils et al. (2011) and Fujita and Moscarini (2013) for permanently separated workers after adjusting for marginally attached workers, and also consistent with the CPS-based statistics when treating the marginally attached workers as unemployed and workers on layoff as employed.[15] The resulting unemployment rate is 7.22%, the vacancy to unemployment ratio is 0.3557, and the monthly job-finding rate amounts to 0.1985.

We obtain a value of $\psi = 3.893$ for the meeting technology parameter, which is significantly higher than the 0.407 obtained by Hagedorn and Manovskii (2008) and the 1.27 estimated by den Haan et al. (2000). The primary reason for this difference is due to the fact that the job finding probability for a type i worker in our model is $s_i \nu(q_i)(1 - F_i(R_i))$. Therefore, it includes two other components: search intensity and frictions related to the reservation value and the match-quality distribution.

The cost k firms incur when posting vacancies includes both recruiting and training expenses. Hall and Milgrom (2008) calibrate the former to be 14% of the average quarterly wage per hire. We use information from Abowd and Kramarz (2003), who estimate the recruiting and training costs at 13% and 7% of the average quarterly wage per hire, respectively, using French data. Thus, we calibrate k to match a ratio of the overall costs of vacancy-posting to average quarterly wages per hire of 20%.

The calibration of the flow value of unemployment consists of two parts: home production z and unemployment benefits b. We calibrate these two parameters following Hall and Milgrom (2008). Regarding the ratio of benefits to average wages, Shimer (2005) assumes a rate of 0.40 while Anderson and Meyer (1997) estimate a pre-tax rate of 0.37 from 1960 to the early 1990s. Although the actual replacement rate in terms of past income is 0.60, a large number of unemployed workers are not eligible: Anderson and Meyer (1997) estimate that about 40% of the unemployed file for unemployment benefits. Furthermore, as pointed out by Hornstein et al. (2005), unemployed workers' salaries are below average wages. They report 0.20 as an upper bound for the ratio of benefits to average wages. In addition, a fraction of unemployed in our sample are workers marginally attached to the labor market who collect no benefits. Therefore, we follow the suggestion

[15] For comparison, we look at the CPS statistics for the same time periods. To have a consistent unemployment measure with our data work in Section **??**, we define the category of "unemployment" as formed by the unemployed workers who are looking for job and are not temporarily laid-off plus the marginally attached workers (those in the *Not in the Labor Force* category who want a job). Consistent with our treatment of unemployment, notice that JOLTS does not include as vacant positions those that are open only to recall from layoffs. The average unemployment rate is 7.32%, whereas the tightness amounts to 0.3440. We also compare with the numbers reported by Krusell et al. (2011), after their re-definition of the unemployment concept to include the marginally attached workers. Their 1994-2007 average estimates are 8.3% for the unemployment rate and 0.248 for the job-finding rate. We conclude that our numbers are quite close to theirs after adjusting for not treating workers on layoff as unemployed.

Parameter	Description	Value	Target
	Exogenously Set Parameters		
β	Discount factor	0.999	Annual interest rate of 5%
y	Market Productivity	1	(Normalization)
	Jointly Calibrated Parameters		
ϕ_1	scale parameter $\phi(s)$	0.133	unemployment rate
ϕ_2	elasticity parameter $\phi(s)$	4.028	monthly job-finding rate
ψ	Meeting technology parameter	3.893	vacancies to unemployment ratio
k	Vacancy cost	0.365	20% avg. quarterly wage per hire
b	unemployment benefits	0.230	25% of avg. wage
z	Home production	0.433	$z + b = 71\%$ of avg. productivity
μ	Share of skilled	0.802	log wage diff. at 0- and 52-week tenure
σ_h	St. dev. of normal dist. F_h	0.452	quarterly EU transitions (2nd-5th quarters)
σ_ℓ	St. dev. of normal dist. F_ℓ	0.130	
α	Learning speed parameter	0.038	
λ	Exog. job-separation rate	0.002	

Table 1: Calibration

of Hall and Milgrom (2008), and set b to match a benefit to average wage ratio of 0.25. Second, Hall and Milgrom (2008) also estimate overall consumption of the unemployed as 0.71 of average output. Thus, we set z such that $z + b$ over average output matches 0.71. As said above, the income tax rate τ is determined to balance the government budget every period. We obtain $\tau = 0.0194$, which falls within the range of the so-called *state unemployment insurance tax rate for new employers*, i.e. without the penalizing experience rating feature of the system.[16]

There is no direct evidence of the relative mass of type h workers, μ. To capture the extent of heterogeneity present in the US labor market, we compute the average starting log wages for two groups of workers from our SIPP sample: all workers and those with at least one year of employment tenure. We target the log wage difference between these two groups, 0.0322.[17] Notice that this target is directly related to the degree of heterogeneity because this statistic would be 0 in an economy with homogeneous workers. In our setting instead, the average type in the employment pool leans towards type h as job-tenure increases. Therefore, a higher wage difference will indicate larger differences across types in our economy. Notice that, by targeting starting wages, we abstract from tenure wage premiums.

The four remaining parameters are related to the observed EU flows, σ_ℓ, σ_h, α, and λ. The first three parameters are associated with the match-quality-learning process, and, hence, with endogenous separations, whereas the fourth one captures the separation flows due to exogenous shocks. Moscarini (2003) uses the job tenure profile of the separation rates to calibrate the match-quality-learning rate, whereas Menzio and Shi (2011) use it to calibrate the parameters of the

[16] For further details of the UI system-financing scheme, see the website of the US Department of Labor, http://ows.doleta.gov/unemploy/.

[17] The predicted log wages from a Mincerian regression show a clear upward-sloping trend in the length of an employment spell, but with some ups and downs. To extract a noise-free measure of wage dynamics during the first year of employment, we run a simple linear regression with the predicted starting log wages as the endogenous variable on a constant and a linear trend.

	ℓ-**market**	h-**market**
Reservation probability, R	0.184	0.518
Queue length, q	0.999	0.791
Wages, $w = w_1 = w_2$	0.764	0.955
Search intensity, s	0.262	0.369
Job-finding rate, $s\nu(q)(1 - F(R))$	0.034	0.078
Endogenous separation rate, $\alpha(1 - \mathbb{E}(\pi\|R))$	0.029	0.011
Output per worker, $y\big(e_1\mathbb{E}(\pi\|R) + e_2\big)/(1-u)$	0.843	0.974
Separation 1st year	0.4657	
Turnover (%)	0.8461	

Table 2: Equilibrium Variables, Baseline Model.

Note: *Separation 1st year* refers to the average probability of experiencing a *EU* transition during the first year following the start of a new job. *Turnover* is the fraction of new hires plus separations over total employment.

match quality distribution. Similarly, we use the *EU* transition rates for the second to the fifth quarters after re-employment from our SIPP sample to set the exogenous separation rate and the match-quality-related parameters.

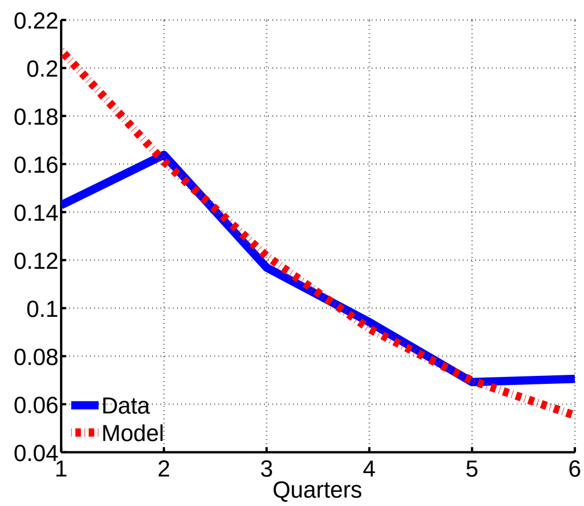


Figure 2: Quarterly *EU* transition rates, data versus model.
Note: The plotted numbers are the probability of transiting to unemployment conditional on being employed at least q quarters. Data refers to predictions from a Probit estimation. The data numbers are from the 1996 and 2001 SIPP panels.

The fit of the model is remarkably good for all targets. In particular, Figure 2 shows the fit with respect to the targeted *EU* transition probabilities by quarter. The model matches these rates almost perfectly for targeted quarters 2 to 5, while the first quarter is overpredicted by around five percentage points and the sixth quarter is underpredicted by around 2 percentage points. Our model is not able to produce a reversion in separations by job tenure, which explains the over/under predictions for the non-targeted quarters 1 and 6.

Table 2 shows equilibrium outcomes from the baseline calibration of the model. As type ℓ

workers have no incentives to search in the type h submarket in the calibrated economy, the equilibrium coincides with the perfect information allocation and workers obtain a constant wage throughout their employment spell. Consistent with our comparative static exercises in Section 2.3.3, the equilibrium search intensity, wages, job-finding rates, and output per worker are higher in the type h submarket, while the queue length and the endogenous separation rate are higher for type ℓ workers. The equilibrium reservation probability is also higher for type h workers, unlike what we obtained in the two-period economy. This difference appears to be due the fact that the continuation value of unemployment is exogenous in the two-period economy, but endogenous in the infinite-horizon setup.

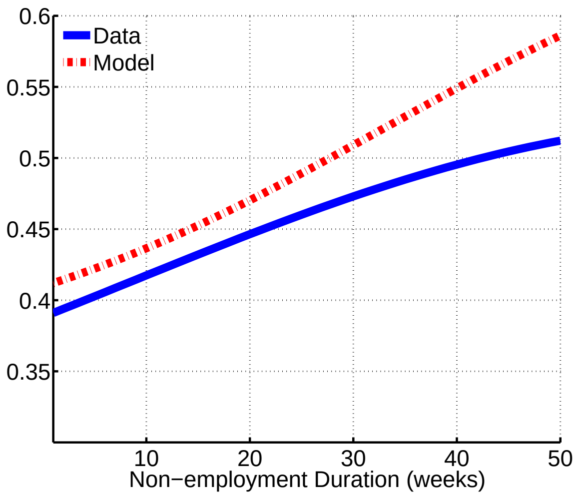
We also report in Table 2 equilibrium worker turnover and separation rate within a year after re-employment. The latter is close to the 44.22% documented in Section **??**. The model weekly worker turnover of 0.85% becomes a monthly 3.63%. The model monthly rate lies in between the 4.34% computed using the numbers obtained by Krusell et al. (2011) and the 2.98% we compute directly from the SIPP data.[18]

Finally, we assess whether the mechanism in the calibrated economy accounts for the upward-sloping trend of the EU transition rates within a year with respect to previous unemployment duration plotted in Figure **??**. We see this as a validation test to our model and calibration strategy since we have not targeted statistics related to the dynamics of the EU transition rates over the length of the previous unemployment spell. We plot the data and model profiles over unemployment duration in the left panel of Figure 4, and the profiles normalized by the value of the first week in the right panel in order to make growth rate comparisons easier. The model replicates quite well the shape of these profiles. The upward-sloping duration profile results from a composition mechanism in our setting. As Lemma 2.3 states for the two-period economy and Table 2 shows for the extended model, type h workers have higher job-finding rates and lower job-separation rates. Therefore, the composition of the pool of unemployed workers varies with the length of the unemployment spell, and the average job-separation rate consequently increases. This remarkable outcome gives us confidence to assess policy implications on worker turnover because variation in the benefit level affects both the expected duration of unemployment and the EU transition rates.

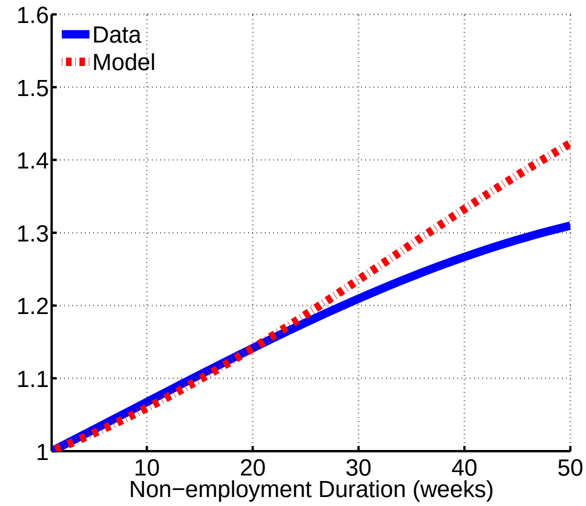
[18] Worker turnover is defined as hires plus separations over total employment. That is,

$$\text{worker turnover} = \frac{u \times ue + e \times eu}{e} = \frac{u}{e} \times ue + eu,$$

where e and u stand for total employment and unemployment, respectively, and ue and eu denote the corresponding transition rates. According to Table 1 in Krusell et al. (2011), the monthly job-finding and -separation rates are 0.248 and 0.021, respectively. The ratio $\frac{u}{e}$ must satisfy the equation $0.083 = \frac{x}{x+1}$ in their case. Recall that they do not adjust for temporary separations and recalls. According to the restrictions on the SIPP data posed above, we obtain an average monthly job-separation rate of 0.0143.



(a) Raw



(b) Normalized

Figure 3: Separation profile by previous unemployment spell.

Note: The figure depicts the data and model probabilities of an EU transition within the first year after re-employment at different lengths of the previous unemployment spell. The data numbers are from the 1996 and 2001 SIPP panels. The predicted probabilities are the Probit-predicted values with all regressors, except for the unemployment duration, evaluated at their sample mean. All variables are normalized by the corresponding value at the first week in the right panel.

5 Policy Analysis

In this section we undertake policy analysis using the calibrated parameters. Notice that welfare gains can be obtained from income redistribution across employed workers of different types.[19] Our aim is to abstract away from such welfare gains, and focus on the ones that result from addressing the uninsured unemployment risks. Furthermore, it is worth underscoring that a publicly-provided insurance crowds out the private insurance firms offer in the market economy.

Government's problem. Building on the results for the two-period economy,[20] we consider a government that chooses the (b, T, τ, L) policy that maximizes the steady state utilitarian social welfare function, SW, taking as given the optimal decisions of the agents, and subject to a period-by-period balanced budget constraint. Notice that the policy instruments cannot be type-contingent because a worker's type is not observable to the government. We will refer to the solution of such a problem as the optimal policy. The social welfare function is defined as

$$SW(b,T,\tau,L) = \sum_{i=\ell,h} \mu_i \left(u^i U_i + e_1^i \int_{R_i} E_i(x_i,\pi) \frac{dF_i(\pi)}{1-F_i(R_i)} + (1-u^i-e_1^i) \frac{v(w_{2i}(1-\tau)-T)+\beta\lambda U_i}{1-\beta(1-\lambda)} \right)$$

[19] Income redistribution across employed workers is limited by the implicit incentive compatibility constraints.

[20] Proposition 2.4 states that a combination of benefits b, lump sum tax T and income tax rate τ can decentralize the constrained efficient allocation

The balanced-budget condition is

$$b\left(\mu_\ell u^\ell + \mu_h u^h\right) = T + \tau \sum_{\substack{i=\ell,h\\ j=1,2}} w_j^i e_j^i \mu_i + L \sum_{i=\ell,h} (1-\lambda)\alpha\big(1 - \mathbb{E}_i(\pi|R_i)\big)e_1^i$$

[To be completed]

6 Conclusions

In this paper we study optimal reallocation of risk-averse workers in a laissez-faire economy with various forms of market incompleteness and search and information frictions. We show that equilibrium is not constrained efficient. We also prove that unemployment insurance yields welfare gains, and improves job composition. In particular, worker turnover decreases and output per worker increases with benefits.

We calibrate our dynamic model to the US economy. The model succeeds in replicating salient features of the US labor market. The main result from our calibration exercise is that worker turnover in the calibrated economy is lower than the one under the optimal policy because unemployment benefits are excessively high.

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7 Appendix

7.1 Proofs.

Proof of Proposition 2.1.

To show that there exists a constrained efficient allocation, let us assume that the consumption functions while employed are indeed constant in π, $c_t(\pi) = c_t$. This is a necessary condition for an interior solution of the planner's problem (4) as shown below, and assuming it now greatly simplifies the proof of existence. The objective function is continuous in all its arguments. Its domain is a non-empty set because of assumption (1). That is, the tuple (q, R, c_u, c_1, c_2, s) such that all consumption levels equal z, $s = 0$, and the reservation probability takes any value, satisfies the two constraints. Furthermore, the domain is a compact set. Therefore, the Bolzano-Weierstrass Theorem applies to ensure the existence of a solution of the maximization problem.

We now characterize the planner's solution. The Lagrangian is

$$\begin{aligned}
\mathcal{L} = \quad & -\phi(s) + s\nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi) + v(c_u)(1+\beta) \\
& +\xi_1\Big(\phi'(s) - \nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi)\Big) \\
& +\xi_2\Big((z - c_u)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1 - \mathbb{E}(\pi|R))\Big) + \\
& +s\nu(q)\int_R \Big(\pi y - c_1(\pi) + \beta\pi\big(y - c_2(\pi)\big)\Big)dF(\pi) - k\frac{s}{q}\Big)
\end{aligned}$$

where ξ_1 and ξ_2 are the multipliers, and $\mathbb{E}(\pi|R) \equiv \displaystyle\int_R \pi\frac{dF(\pi)}{1-F(R)}$ denotes the conditional expected value.

Differentiating the Lagrangian with respect to the variables, we obtain a number of necessary conditions for the constrained efficient allocation. First, the functional derivatives with respect to $c_1(\pi)$ and $c_2(\pi)$, we obtain

$$\begin{aligned}
c_1(\pi) = c_2(\pi) &= c \\
v'(c)(s - \xi_1) &= s\xi_2 \qquad (16)
\end{aligned}$$

Second, the derivative of the Lagrangian with respect to s delivers the following first order condition.

$$\begin{aligned}
\xi_1\phi''(s) &= \xi_2\Big(k/q + \nu(q)(1-F(R))\Big[(z - c_u + c)(1 + \beta\mathbb{E}(\pi|R)) - \mathbb{E}(\pi|R)y(1+\beta)\Big]\Big) \\
\Leftrightarrow \xi_1 s\phi''(s) &= \xi_2(z - c_u), \qquad (17)
\end{aligned}$$

where the last condition comes out after using the second constraint, which must hold with equality. This implies that $\xi_1 < 0$ if and only if $c_u > z$.

Now, differentiating with respect to c_u, using (16), and simplifying, we have

$$\big(v'(c) - v'(c_u)\big)\nu(q)(1-F(R))(1 + \beta\mathbb{E}(\pi|R))(s - \xi_1) + (1+\beta)(v'(c_u) - \xi_2) = 0 \qquad (18)$$

The left hand side is $\frac{\partial\mathcal{L}}{\partial c_u}$. To show that $z < c_u$, it suffices to show that the derivative of the Lagrangain evaluated at $c_u \le z$ is strictly positive. To see this, notice that expression (17) together with $c_u \le z$ implies $\xi_1 \ge 0$, and it also follows that $v'(c) \ge \xi_2$ from equation (16). The inequality $v'(c_u) > v'(c) \ge \xi_2$ follows from the concavity of the utility function and the assumption that $z < c$. Therefore, the derivative evaluated at $c_u \le z$ is strictly positive.

Similarly, to show that $c_u < c$, it suffices to show that the derivate of the Lagrangian for $c_u \ge c$ is strictly negative. This result follows from $\xi_2 > v'(c) > v'(c_u)$ due to expressions (16) and (17).

Next, we look at the necessary condition for the reservation probability. After some manipulations, we obtain

$$\frac{\partial \mathcal{L}}{\partial R} = -\nu(q)(1+\beta R)(s-\xi_1)dF(R)\Big(v(c)-v(c_u)+v'(c)\Big(c_u - z + y\frac{R(1+\beta)}{1+\beta R} - c\Big)\Big) \leq 0,$$
$$\text{and } R \geq 0, \text{ with complementary slackness} \tag{19}$$

The derivative is non-positive if and only if $\frac{v(c)-v(c_u)}{v'(c)} + c_u - z + y\frac{R(1+\beta)}{1+\beta R} - c \geq 0$, which leads to expression (5). Let R be such that $y\frac{R(1+\beta)}{1+\beta R} \geq z$. Notice that $\frac{v(c)-v(c_u)}{v'(c)} + c_u - c > 0$ because of the concavity of the utility function. Therefore, the derivative is strictly negative. That is, such value cannot be the maximizing R because the Lagrangian increases by reducing the reservation value. Therefore, if $R > 0$, then $y\frac{R(1+\beta)}{1+\beta R} < z$. Notice that this inequality also holds if $R = 0$.

The last necessary FOC condition is with respect to the ratio q. After some manipulations, we obtain

$$(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Big(\frac{v(c)-v(c_u)}{v'(c)} - z + c_u + \frac{\mathbb{E}(\pi|R)(1+\beta)}{1+\beta\mathbb{E}(\pi|R)}y - c\Big) = \frac{k}{\eta(q)} \tag{20}$$

Indeed, if $R > 0$, then using condition (19), we can rewrite it as

$$(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Big(\frac{\mathbb{E}(\pi|R)(1+\beta)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R(1+\beta)}{1+\beta R}\Big)y = \frac{k}{\eta(q)}.\| \tag{21}$$

Proof of Proposition 2.2. For the first part of the proof and for the sake of simplicity, we are to impose the necessary condition that the wage variables are constant functions of π. Such first order conditions are derived below.

Let (Q, R, w_1, w_2, s, U) be an equilibrium allocation. We first show that the tuple (q, R, w_1, w_2, s) solves the maximization problem (9), where $q = Q((w_1, w_2), R)$. The proof is by contradiction. Suppose that there exists another tuple (q', R', w_1', w_2', s') such that the constraint holds, and, without loss of generality,

$$\begin{aligned}
&\nu(q')\int_{R'}\Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big)dF(\pi) \\
&> \nu(q)\int_{R}\Big(v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big)dF(\pi) \\
&\geq \nu(Q(x'))\int_{R'}\Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big)dF(\pi),
\end{aligned}$$

where $Q(x')$ denotes the off-the-equilibrium queue length for contract $x' = (w_1', w_2'), R')$. The last inequality results from the definition of equilibrium. It follows from the monotonicity of function ν that $q' < Q(x')$. The first equilibrium condition ensures that

$$\begin{aligned}
k = \quad & \eta(q)\int_R\Big(\pi y(1+\beta) - w_1 - \beta\pi w_2\Big)dF(\pi) \\
&\geq \eta(Q(x'))\int_{R'}\Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big)dF(\pi) \\
&> \eta(q')\int_{R'}\Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big)dF(\pi) \geq k,
\end{aligned}$$

where the last inequality comes from the monotonicity of function η and $q' < Q(x')$. This is a contradiction. Therefore, the equilibrium tuple solves program (9).

Now, let (q, R, w_1, w_2, s) be a solution of the maximization problem (9). We define the equilibrium worker's market value as

$$U = -\phi(s) + s\nu(q)\int_R\Big(v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big)dF(\pi) + v(z)(1+\beta)$$

Likewise, we define the value of the queue length function Q at any submarket x' as the value q' that satisfies

$$U = \max_s -\phi(s) + s\nu(q') \int_{R'} \Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi) + v(z)(1+\beta),$$

if it exists, and $q' = 0$, otherwise.

It remains to show that firms maximize profits. That is, there is no submarket x' such that

$$\eta(Q(x')) \int_{R'} \Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big) dF(\pi) > k$$

Suppose there is such a x'. Then, by the limit conditions and the continuity of function η, there must exist $q' < Q(x')$ such that

$$\eta(q') \int_{R'} \Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big) dF(\pi) = k$$

Since (q', R', w_1', w_2', s') satisfies the constraint of the maximization problem, it follows that

$$\begin{aligned} & \nu(q) \int_{R} \Big(v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big) dF(\pi) \\ \geq\ & \nu(q') \int_{R'} \Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi) \\ >\ & \nu(Q(x')) \int_{R'} \Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi), \end{aligned}$$

which contradicts the definition of $Q(x')$.

Existence of equilibrium follows from the Bolzano-Weierstrass Theorem because the objective function is continuous and the domain is a non-empty compact set because of assumption (1).

We now characterize the equilibrium and allow for wages to be contingent on π. The Lagrangian of the maximization problem is

$$\begin{aligned} \mathcal{L} = \quad & -\phi(s) + s\nu(q) \int_{R} \Big(v(w_1(\pi)) - v(z) + \beta\pi\big(v(w_2(\pi)) - v(z)\big)\Big) dF(\pi) \\ & + \xi\Big(\eta(q) \int_{R} \Big(\pi y(1+\beta) - w_1(\pi) - \beta\pi w_2(\pi)\Big) dF(\pi) - k\Big), \end{aligned}$$

where ξ is the Lagrange multiplier. We differentiate the Lagrangian to obtain the necessary conditions. First, from the functional derivatives with respect to $w_1(\pi)$ and $w_2(\pi)$, we obtain

$$\begin{aligned} w_1(\pi) = w_2(\pi) = w \\ v'(w) = \xi q \end{aligned} \tag{22}$$

Next, differentiating the Lagrangian with respect to R, we obtain, after some manipulations,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial R} \quad = \quad & -\nu(q)(1+\beta R) dF(R) \Big(v(w) - v(z) + v'(w)\Big(y\frac{R(1+\beta)}{1+\beta R} - w\Big)\Big) \leq 0, \\ & \text{and } R \geq 0, \text{ with complementary slackness} \end{aligned} \tag{23}$$

The necessary first order condition with respect to q is, after some manipulations,

$$\eta(q) \frac{1-\varphi(q)}{\varphi(q)} (1 - F(R))(1 + \beta\mathbb{E}(\pi|R)) \frac{v(w) - v(z)}{v'(w)} = k \tag{24}$$

Likewise, the necessary first order condition related to s is

$$\phi'(s) = \nu(q) \int_R \big(v(w) - v(z)\big)\big(1 + \beta\pi\big) dF(\pi) \tag{25}$$

Therefore, the necessary first order conditions (23)-(25) plus the constraint (or zero-profit condition) characterize the equilibrium tuple (q, w, R, s).

Finally, if $R > 0$, it is convenient to combine the last two equilibrium conditions and the zero-profit condition to obtain

$$w\big(1 + \beta\mathbb{E}(\pi|R)\big) = \varphi(q)\mathbb{E}(\pi|R)y(1+\beta) + (1-\varphi(q))y\frac{R(1+\beta)}{1+\beta R}\big(1 + \beta\mathbb{E}(\pi|R)\big) \tag{26}$$

$$\eta(q)(1-\varphi(q))\frac{\mathbb{E}(\pi|R) - R}{1+\beta R}(1 - F(R))y(1+\beta) = k \tag{27}$$

It is worth noticing that expression $\frac{\mathbb{E}(\pi|R)-R}{1+\beta R}(1 - F(R))$ is a decreasing function of R.‖

Proof of Lemma 2.3

1. Consider two economies such that the utility function in the second economy, v_2, is a concave monotonic transformation of its counterpart in the first economy, v_1. That is, $v_2 = g \circ v_1$, for some increasing and concave function g. Let (q_1, R_1, w_1, s_1) and (q_2, R_2, w_2, s_2) be the respective equilibrium vectors. Now, consider the maximization problem (9). As the constraint does not depend on how risk averse workers are, the vector (q_2, R_2, w_2, s_2) belongs to the domain of the program corresponding to the economy with utility function v_1, and vice versa. Therefore,

$$\nu(q_2)\big(v_2(w_2) - v_2(z)\big)\int_{R_2}\big(1+\beta\pi\big)dF(\pi) \geq \nu(q_1)\big(v_2(w_1) - v_2(z)\big)\int_{R_1}\big(1+\beta\pi\big)dF(\pi) \tag{28}$$

$$\nu(q_1)\big(v_1(w_1) - v_1(z)\big)\int_{R_1}\big(1+\beta\pi\big)dF(\pi) \geq \nu(q_2)\big(v_1(w_2) - v_1(z)\big)\int_{R_2}\big(1+\beta\pi\big)dF(\pi)$$

   The proof of $w_2 < w_1$ is identical to the proof of Proposition 2 in Acemoglu and Shimer (1999); hence, we omit it.

   Now, we will show that $q_2 < q_1$ and $R_2 < R_1$. First, suppose that $R_1, R_2 > 0$. We prove by contradiction that $q_2 < q_1$. That is, suppose that the opposite is true: $q_2 \geq q_1$. Then, the equilibrium condition (27) implies that $R_2 \geq R_1$. Since $q_2 \geq q_1$, $R_2 \geq R_1$ and $w_2 < w_1$, expression (28) cannot hold, which is a contradiction. Therefore, $q_2 < q_1$, and $R_2 < R_1$ follows again from condition (27).

   Let us consider the case in which $R_1 = 0 < R_2$. We will show that this case cannot occur in equilibrium. Following the same argument as before, we obtain that $q_2 < q_1$. The necessary equilibrium condition (24) implies

$$\frac{v_2(w_2) - v_2(z)}{v_2'(w_2)} > \frac{v_1(w_1) - v_1(z)}{v_1'(w_1)}$$

   Using this inequality and condition (23), we obtain

$$-y\frac{R_1(1+\beta)}{1+\beta R_1} + w_1 \leq y\frac{R_2(1+\beta)}{1+\beta R_2} + w_2,$$

   and as $w_2 < w_1$, it follows that $R_2 < R_1$, which is a contradiction.

2. Let $z_1 < z_2$, and (q_1, R_1, w_1, s_1) and (q_2, R_2, w_2, s_2) be the respective equilibrium tuples.

   Consider the maximization problem (9). As the constraint does not depend on the value of z, the equilibrium tuple (q_2, w_2, R_2, s_2) satisfies the constraint of the program when $z = z_1$, and so does the vector (q_1, w_1, R_1, s_1) when $z = z_2$. This implies

$$\nu(q_1)\big(v(w_1) - v(z_1)\big) \int_{R_1} (1+\beta\pi)dF(\pi) \quad \geq \quad \nu(q_2)\big(v(w_2) - v(z_1)\big) \int_{R_2} (1+\beta\pi)dF(\pi) \tag{29}$$

$$\nu(q_2)\big(v(w_2) - v(z_2)\big) \int_{R_2} (1+\beta\pi)dF(\pi) \quad \geq \quad \nu(q_1)\big(v(w_1) - v(z_2)\big) \int_{R_1} (1+\beta\pi)dF(\pi) \tag{30}$$

Multiplying these two inequalities and manipulating the outcome, we obtain

$$\begin{aligned} \big(v(w_1) - v(z_1)\big)\big(v(w_2) - v(z_2)\big) &\geq \big(v(w_2) - v(z_1)\big)\big(v(w_1) - v(z_2)\big) \\ &\Leftrightarrow \big(v(z_2) - v(z_1)\big)\big(v(w_2) - v(w_1)\big) \geq 0 \end{aligned} \tag{31}$$

As the utility function is increasing and $z_1 < z_2$, it follows that $w_1 \leq w_2$. Using $z_1 < z_2$ and the inequality (29), it follows from the equilibrium condition (25) that $s_2 \leq s_1$.

Now, we show that $q_1 \leq q_2$ by contradiction. Suppose that $q_1 > q_2$. Then, the monotonicity of function ν and inequality (29) imply that $R_2 > R_1$. We distinguish two cases.

Case 1: $R_1 > 0$. The equilibrium condition (27) and $q_1 > q_2$ imply that $R_2 < R_1$ because its left hand side is a decreasing function in the ratio R. This is a contradiction. Therefore, $q_1 \leq q_2$, and $R_1 \leq R_2$ results again from condition (27).

Case 2: $R_1 = 0$. The steps are analogous, but with a small difference. First, notice that the equilibrium condition (23) implies

$$v(w_1) - v(z_1) - v'(w_1)w_1 \geq 0$$

Now, combining this inequality with equation (24) and the zero-profit condition, we obtain

$$\eta(q_1)(1-\varphi(q_1))\mathbb{E}(\pi|0)y(1+\beta) \leq k = \eta(q_2)(1-\varphi(q_2))\frac{\mathbb{E}(\pi|R_2) - R_2}{1+\beta R_2}(1-F(R_2))y(1+\beta),$$

where the last equality is expression (27). Because $q_1 > q_2$, it must be the case that $R_2 < R_1$. We reached a contradiction. Therefore, $q_1 \leq q_2$. Obviously, in this case, $R_1 = 0 \leq R_2$.

Since the turnover rate amounts to $1 - \mathbb{E}(\pi|R)$ and $R_1 \leq R_2$, it is lower when home production equals z_2 in either case.

3. Next, assume that two economies differ in the signal distribution function, and $F_1 \lesssim F_2$. Let (q_1, R_1, w_1, s_1) and (q_2, R_2, w_2, s_2) be the respective equilibrium tuples.

   First, notice that the equilibrium tuple (q_1, R_1, w_1, s_1) satisfies the constraint of the program when $F = F_2$. This implies

$$\nu(q_2)\big(v(w_2) - v(z)\big) \int_{R_2} (1+\beta\pi)dF_2(\pi) \geq \nu(q_1)\big(v(w_1) - v(z)\big) \int_{R_1} (1+\beta\pi)dF_2(\pi) \tag{32}$$

   We distinguish between several cases.

   Case 1: $R_1, R_2 > 0$. First, we prove by contradiction that $q_2 < q_1$. Suppose instead that $q_1 \leq q_2$. The equilibrium condition (27) implies that

$$\begin{aligned} \frac{\displaystyle\int_{R_1} \pi \frac{dF_1(\pi)}{1-F_1(R_1)} - R_1}{1+\beta R_1}(1-F_1(R_1)) \quad &\geq \quad \frac{\displaystyle\int_{R_2} \pi \frac{dF_2(\pi)}{1-F_2(R_2)} - R_2}{1+\beta R_2}(1-F_2(R_2)) \\ &> \quad \frac{\displaystyle\int_{R_2} \pi \frac{dF_1(\pi)}{1-F_1(R_2)} - R_2}{1+\beta R_2}(1-F_1(R_2)), \end{aligned}$$

where the last inequality results from $F_1 \lesssim F_2$. From comparing the first and the last expressions, we obtain $R_1 < R_2$. This together with the equilibrium equation (23) imply $w_2 < w_1$. Then, it follows from the inequality (32) that

$$\int_{R_1} (1+\beta\pi)dF_2(\pi) < \int_{R_2} (1+\beta\pi)dF_2(\pi) \Leftrightarrow R_2 < R_1,$$

which is a contradiction. Therefore, $q_2 < q_1$. We also proceed by contradiction to prove that $R_2 < R_1$. Suppose the opposite holds, $R_1 \leq R_2$. Then, equation (23) implies $w_2 < w_1$. However, we can rewrite the equilibrium equation (26) as

$$\begin{aligned}
\frac{w_1}{y(1+\beta)} &= \varphi(q_1)\frac{\int_{R_1} \pi \frac{dF_1(\pi)}{1-F_1(R_1)}}{1+\beta\int_{R_1} \pi \frac{dF_1(\pi)}{1-F_1(R_1)}} + (1-\varphi(q_1))\frac{R_1}{1+\beta R_1} \\
&< \varphi(q_2)\frac{\int_{R_1} \pi \frac{dF_1(\pi)}{1-F_1(R_1)}}{1+\beta\int_{R_1} \pi \frac{dF_1(\pi)}{1-F_1(R_1)}} + (1-\varphi(q_2))\frac{R_1}{1+\beta R_1} \\
&< \varphi(q_2)\frac{\int_{R_1} \pi \frac{dF_2(\pi)}{1-F_2(R_1)}}{1+\beta\int_{R_1} \pi \frac{dF_2(\pi)}{1-F_2(R_1)}} + (1-\varphi(q_2))\frac{R_1}{1+\beta R_1} \\
&\leq \varphi(q_2)\frac{\int_{R_2} \pi \frac{dF_2(\pi)}{1-F_2(R_2)}}{1+\beta\int_{R_2} \pi \frac{dF_2(\pi)}{1-F_2(R_2)}} + (1-\varphi(q_2))\frac{R_2}{1+\beta R_2} = \frac{w_2}{y(1+\beta)}
\end{aligned}$$

The first inequality results from the monotonicity with respect to q of the first expression and $q_2 < q_1$, the second inequality comes from $F_1 \lesssim F_2$, and the last one from the monotonicity with respect to R and $R_1 \leq R_2$. We reached $w_1 < w_2$, which is a contradiction. Therefore, $R_2 < R_1$. The equation (23) implies $w_1 < w_2$.

<u>Case 2: $R_2 = 0 < R_1$.</u> The equilibrium condition (23) implies

$$v(w_1) - v(z) - v'(w_1)w_1 < 0 \leq v(w_2) - v(z) - v'(w_2)w_2 \Leftrightarrow w_1 < w_2$$

Suppose now that $q_1 \leq q_2$. Then, condition (24) implies

$$\begin{aligned}
1 - F_1(R_1) + \beta\int_{R_1} \pi dF_1(\pi) &\geq 1 - F_2(R_2) + \beta\int_{R_2} \pi dF_2(\pi) \\
&> 1 - F_1(R_2) + \beta\int_{R_2} \pi dF_1(\pi),
\end{aligned}$$

where the second inequality comes from imposing $F_1 \lesssim F_2$. By monotonicity, we conclude that $R_1 < R_2 = 0$, which is a contradiction. Therefore, $q_2 < q_1$.

<u>Case 3: $R_1 = 0 < R_2$.</u>

We show that this case cannot occur. The equilibrium condition (23) implies

$$v(w_2) - v(z) - v'(w_2)w_2 < 0 \leq v(w_1) - v(z) - v'(w_1)w_1 \Leftrightarrow w_2 < w_1$$

We show by contradiction that $q_2 < q_1$. Suppose the opposite. Then, the inequality (32), together with $q_1 \leq q_2$ and $w_2 < w_1$, implies that $R_2 < R_1 = 0$, which is a contradiction. Therefore, $q_2 < q_1$. Now, combining the equilibrium condition (24) and the zero-profit condition, we obtain

$$\frac{1-\varphi(q)}{\varphi(q)}\frac{v(w)-v(z)}{v'(w)} + w = \frac{\mathbb{E}(\pi|R)(1+\beta)y}{1+\beta\mathbb{E}(\pi|R)} \tag{33}$$

Because the left hand side of this expression is increasing in q and w, it must be the case that

$$\int_0 \pi dF_1(\pi) > \int_{R_2} \pi\frac{dF_2(\pi)}{1-F_2(R_2)} > \int_{R_2} \pi\frac{dF_1(\pi)}{1-F_1(R_2)} > \int_0 \pi dF_1(\pi),$$

where the second inequality is due to $F_1 \lesssim F_2$, and the last inequality results from the monotonicity of the conditional expected value. This is obviously a contradiction. Therefore, this case cannot occur.

Case 4: $R_1 = R_2 = 0.$

Suppose that $q_1 \leq q_2$. Then, the inequality (32) implies that $w_1 \leq w_2$. Because $q_1 \leq q_2$ and $w_1 \leq w_2$, the equilibrium condition (24) implies that

$$\int_0 \pi dF_1(\pi) \geq \int_0 \pi dF_2(\pi),$$

but this cannot be the case as we showed above. It follows that $q_2 < q_1$. Now, suppose that $w_2 \leq w_1$. The equilibrium condition (33) implies

$$\int_0 \pi dF_1(\pi) \geq \int_0 \pi dF_2(\pi)$$

As this inequality does not hold, we conclude that $w_1 < w_2$.

We have obtained $q_2 < q_1$, $w_1 < w_2$ and $R_2 < R_1$ in all cases. When comparing the search intensity, $s_1 < s_2$ results from the convexity of function ϕ and the equilibrium equation (25):

$$\begin{aligned}\phi'(s_1) = \nu(q_1)(v(w_1)-v(z))\int_{R_1} \pi dF_1(\pi) \quad &< \quad \nu(q_2)(v(w_2)-v(z))\int_{R_2} \pi dF_1(\pi) \\ &< \quad \nu(q_2)(v(w_2)-v(z))\int_{R_2} \pi dF_2(\pi) = \phi'(s_2),\end{aligned}$$

where the first inequality comes from the monotonicity of the respective functions, and the second inequality is due to $F_1 \lesssim F_2$. We can similarly show that the job-finding probability is higher in the second economy

$$s_1\nu(q_1)(1-F_1(R_1)) < s_2\nu(q_2)(1-F_2(R_2))$$

Finally, we show that if the meeting technology is Cobb-Douglas, then $\int_{R_1} \pi\frac{dF_1(\pi)}{1-F_1(R_1)} < \int_{R_2} \pi\frac{dF_2(\pi)}{1-F_2(R_2)}$. The proof is by contradiction. Suppose this is not the case. Combining equilibrium conditions (23) and (24), we obtain

$$\begin{aligned}\frac{w_2}{y(1+\beta)} \quad &\leq \quad \varphi\frac{\int_{R_2} \pi\frac{dF_2(\pi)}{1-F_2(R_2)}}{1+\beta\int_{R_2} \pi\frac{dF_2(\pi)}{1-F_2(R_2)}} + (1-\varphi)\frac{R_2}{1+\beta R_2} \\ &< \quad \varphi\frac{\int_{R_1} \pi\frac{dF_1(\pi)}{1-F_1(R_1)}}{1+\beta\int_{R_1} \pi\frac{dF_1(\pi)}{1-F_1(R_1)}} + (1-\varphi)\frac{R_1}{1+\beta R_1} = \frac{w_1}{y(1+\beta)},\end{aligned}$$

which is a contradiction.‖

Proof of Proposition 2.4

First, notice that if the planner is forced to set $c_u = z$, then the planner's problem (4) and the program that characterizes the equilibrium allocation (9) are exactly the same. This is the case because condition (2) is indeed the first order condition (25) of problem (9), and, hence, it must hold at the solution tuple. Therefore, the equilibrium is constrained efficient in this case.

Second, let $(q^p, R^p, c_u^p, c^p, s^p)$ denote the planner's solution to problem (4). We showed in the proof of Proposition 2.1 that $z < c_u^p$. Since unemployed workers just consume their home production in the equilibrium allocation, the planner's solution cannot be decentralized in the laissez-faire economy.

Next, we consider a market economy in which a government implements a lump sum tax T, an income tax τ to the employed workers, a subsidy b to the unemployed, and a layoff tax L in order to decentralize the constrained efficient allocation $(q^p, R^p, c_u^p, c^p, s^p)$. This requires that the following conditions hold in equilibrium:

$$z + b - T = c_u^p \tag{34}$$

$$w(1-\tau) - T = c^p \tag{35}$$

$$\begin{aligned}(T-b)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big)&\\ + Ls\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R)) + (T+\tau w)s\nu(q)(1-F(R))(1+\beta\mathbb{E}(\pi|R)) &= 0,\end{aligned} \tag{36}$$

where the last equation is the intertemporal balanced-budget constraint of the government.

The counterpart of program (9) in this economy with taxes is, after imposing the equilibrium result of $w_1 = w_2$,

$$\begin{aligned}\max_{w,R,q,s} \quad & -\phi(s) + s\nu(q)\int_R \Big(v(w(1-\tau)-T) - v(c_u^p) + \beta\pi\big(v(w(1-\tau)-T) - v(c_u^p)\big)\Big)dF(\pi)\\ \text{s. to} \quad & \eta(q)\int_R \Big(\pi y - w + \beta\big(\pi(y-w) - (1-\pi)L\big)\Big)dF(\pi) \geq k\end{aligned} \tag{37}$$

Notice that we obtain the non-negative profits condition of the above program by combining the intertemporal resource constraint of the planner (3) and the balanced-budget constraint of the government (36). That is, the above problem and the planner's problem (4) are the same. Therefore, if there exists a policy (b, τ, T, L) such that satisfies conditions (34)-(36), then the tax-distorted equilibrium is constrained efficient.

Consider the case of $R^p > 0$. By combining the necessary first order conditions with respect to q and R, we can write the former as

$$\begin{aligned}(1-F(R))(1+\beta\mathbb{E}(\pi|R))\bigg(\Big(\frac{\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R}{1+\beta R}\Big)y(1+\beta) - L\beta\Big(\frac{1-\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{1-R}{1+\beta R}\Big)\bigg) &=\\ = \frac{k}{\eta(q)(1-\varphi(q))}&\end{aligned} \tag{38}$$

This equilibrium condition coincides with the efficiency condition (21) if and only if $L = 0$.

Given q^p and R^p, the equilibrium wage w is determined by the zero-profit condition. Then, the government balanced-budget condition holds at the efficient allocation. Therefore, there are two policy instruments to be determined and two equations to be satisfied, taking into account that b is such that condition (36) holds.

The efficiency condition (19) implies $\frac{v(c^p)-v(c_u^p)}{v'(c^p)} + c_u^p - z + y\frac{R^p(1+\beta)}{1+\beta R^p} - c^p = 0$. Then, plugging conditions (34) and (35) into this expression, and comparing the resulting equation with the tax-distorted equilibrium counterpart of condition (23), we obtain that

$$b = -\tau y \frac{R^p(1+\beta)}{1+\beta R^p} \tag{39}$$

Therefore, $\tau < 0$ if $b > 0$. To isolate the parameter τ, we properly combine equations (34), (35) and (39), and obtain

$$(1-\tau)\bigg(\frac{Ry(1+\beta)}{1+\beta R} - w\bigg) = -\frac{v(c)-v(c_u)}{v'(c)},$$

which is the tax-distorted equilibrium counterpart of condition (23). Then, we can determine the remaining policy parameters from equations (34) and (35).‖

Proof of Proposition 3.1.

To save on notation, let us define, for a given submarket x and type $i \in \{\ell, h\}$,

$$\mathcal{I}_i^w(x) \equiv \int_{\pi \geq R} \Big(E_i(x,\pi) - U_h\Big) dF_i(\pi)$$

The proof is by contradiction. Suppose that there exists an equilibrium in which both types of workers submit applications to the same submarket $x = (\omega, R)$. Let q denote the equilibrium queue length in that submarket. Then,

$$U_i(1-\beta) = v(z) + \max_s \Big\{ -\phi(s) + \beta s \nu(q) \mathcal{I}_i^w(x) \Big\}, \forall i \in \{\ell, h\}$$

Consider now an arbitrarily small mass ζ of firms deviating to submarket x', marginally different from x. The equilibrium expectations on the queue length at submarket x' are determined by what type of workers benefits the most. Let $q_i \equiv q + dq_i$ denote the queue length that makes type i workers indifferent between submarkets x and x'. Then, using the Envelope Theorem, we obtain

$$dq_i \nu'(q) \mathcal{I}_i^w(x) + \nu(q) d\mathcal{I}_i^w(x) = 0 \tag{40}$$

Suppose that the firms deviating to submarket x' offer the same reservation value and a marginally different wage contract which leaves type h workers indifferent. That is, $d\mathcal{I}_h^w(x) = 0$.

The total differential $d\mathcal{I}_i^w(x)$ amounts to

$$\begin{aligned} d\mathcal{I}_i^w(x) &= \int_{\pi \geq R} dE_i(x,\pi) dF_h(\pi) \\ &= \frac{\big(1-F_i(R)\big)}{1-\beta(1-\lambda)(1-\alpha)} \bigg(v'(w_1)dw_1 + \frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)} \mathbb{E}_i(\pi|R) v'(w_2) dw_2 \bigg) \end{aligned} \tag{41}$$

where $\mathbb{E}_i(\pi|R)$ denotes the expected value of the truncated distribution. From $d\mathcal{I}_h^w = 0$, it follows that

$$v'(w_2)dw_2 = -v'(w_1)dw_1 \frac{1-\beta(1-\lambda)}{\beta(1-\lambda)\alpha} \frac{1}{\mathbb{E}_h(\pi|R)},$$

Now, we replace $v'(w_2)dw_2$ in expression (41) for type ℓ workers, and obtain

$$\begin{aligned} d\mathcal{I}_\ell^w(x) &= \frac{1-F_\ell(R)}{1-\beta(1-\lambda)(1-\alpha)} \bigg(v'(w_1)dw_1 + \frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)} \mathbb{E}_\ell(\pi|R) v'(w_2) dw_2 \bigg) = \\ &= v'(w_1)dw_1 \frac{1-F_\ell(R)}{1-\beta(1-\lambda)(1-\alpha)} \bigg(1 - \frac{\mathbb{E}_\ell(\pi|R)}{\mathbb{E}_h(\pi|R)} \bigg) \end{aligned}$$

Assumption 1 tells us that the sign of the total differential $d\mathcal{I}_\ell^w$ is the sign of the differential dw_1. Because $\lim_{c\to 0} v'(c) = \infty$, the after-tax wages must be strictly positive in equilibrium. Therefore, by reducing w_1, the deviating firms ensure that $d\mathcal{I}_\ell^w < 0$ and, hence, $q_\ell < q_h$ because $\frac{dq_i}{dW_i} > 0$ according to expression (40).

The deviating firms end up attracting only type h workers while bearing an arbitrarily small increase in their wage bill. As a result, such a deviation leads to a discrete jump in profits, which is a contradiction. To see why this is the case, let us rewrite the expected profits as

$$\int_{\pi\geq R} J(x,\pi)dF_i(\pi) = \frac{1-F_i(R)}{1-\beta(1-\lambda)(1-\alpha)}\Bigg(\Bigg(y_i + (y_i - w_2)\frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)}\Bigg)\mathbb{E}_i(\pi|R) - w_1\Bigg)$$

Therefore, the discrete jump in profits, i.e. $\int_{\pi\geq R} J(x,\pi)dF_h(\pi) > \int_{\pi\geq R} J(x,\pi)dF_\ell(\pi)$, results from Assumptions 1 and 2.‖

Proof of Proposition 3.2.

We show existence of a separating equilibrium and characterize it. Consider the following functions $\mathcal{H}_i : \mathcal{R}_+ \to \mathcal{R}_+$.

$$\begin{aligned}\mathcal{H}_\ell(U) \equiv \quad & \max_{\substack{q\in[0,\infty],R\in[0,1]\\(w_1,w_2)\in[0,y]^2,s\in[0,1]}} \quad \Bigg\{v(z) - \phi(s) + \beta s\nu(q)\int_R \Big(E_\ell(x,\pi) - U\Big)dF_\ell(\pi)\Bigg\} + \beta U \\ & \text{s. to} \qquad \beta\eta(q)\int_R J(x,\pi)dF_\ell(\pi) \geq k\end{aligned}$$

$$\begin{aligned}\mathcal{H}_h(U;U_\ell) \equiv & \max_{\substack{q\in[0,\infty],R\in[0,1]\\(w_1,w_2)\in[0,y]^2,s\in[0,1]}} \Bigg\{v(z) - \phi(s) + \beta s\nu(q)\int_R \Big(E_h(x,\pi) - U\Big)dF_h(\pi)\Bigg\} + \beta U \\ & \text{s. to} \qquad \beta\eta(q)\int_R J(x,\pi)dF_h(\pi) \geq k \\ & v(z) + \max_s\Bigg\{-\phi(s) + \beta s\nu(q)\int_R \Big(E_\ell(x,\pi) - U_\ell\Big)dF_\ell(\pi)\Bigg\} \leq (1-\beta)U_\ell\end{aligned}$$

As an abuse of language, let us refer to a fixed point of function $\mathcal{H}_i$ as a tuple $(\hat{U}_i, \hat{q}_i, \hat{x}_i, \hat{s}_i)$, where $(\hat{q}_i, \hat{x}_i, \hat{s}_i)$ is the argmax of the maximization problem, given $\hat{U}_i$. Given the value $\hat{U}_i$, the vector $(\hat{q}_i, \hat{x}_i, \hat{s}_i)$ maximizes the expected utility of type i workers subject to firms making non-negative profits. In the case of function $\mathcal{H}_h$, there is an additional constraint: a no-participation condition for type ℓ workers. This additional restriction is necessary to discourage type ℓ workers from applying to type h jobs in a separating allocation. Although an analogous constraint should be written for $\mathcal{H}_\ell$ to ensure the no participation of type h workers, we will show that such a constraint is redundant. The following lemma states that an equilibrium allocation is a fixed point of these two functions, and vice versa.

Lemma 7.1 *If $(G, \mathcal{X}, (U_i, S_i)_i, Q, \rho)$ is an equilibrium allocation with $\mathcal{X} = \{x_\ell, x_h\}$, then (U_i, q_i, x_i, s_i) is a fixed point of $\mathcal{H}_i$, with $s_i \equiv S_i(x_i)$ and $q_i \equiv Q(x_i)$.*

Conversely, if $(\hat{U}_i, \hat{q}_i, \hat{x}_i, \hat{s}_i)$ is a fixed point of function $\mathcal{H}_i$ for $i \in \{\ell, h\}$, then it takes part of an equilibrium allocation, where $Q(\hat{x}_i) = \hat{q}_i$, $S_i(\hat{x}_i) = \hat{s}_i$, and $\rho_i(\hat{x}_i) = 1$.

Proof To save on notation, let us again define, for any given submarket x and worker type $i \in \{\ell, h\}$,

$$\mathcal{I}_i^w(x) \equiv \int_{\pi\geq R}\Big(E_i(x,\pi) - U_h\Big)dF_i(\pi), \text{ and } \mathcal{I}_i^f(x) \equiv \int_{\pi\geq R} J(x,\pi)dF_i(\pi)$$

The proof has two main stages. First, we show that the vectors $(U_i, q_i, x_i, s_i)_i$ of the equilibrium allocation constitute a fixed point of functions $\mathcal{H}_\ell$ and $\mathcal{H}_h$. Obviously, the constraints of both maximization problems hold when evaluated at the equilibrium values. To start with, let $(U_\ell, q_\ell, x_\ell, s_\ell)$ be part of the equilibrium allocation. The third equilibrium condition establishes that

$$(1-\beta)U_\ell - v(z) = -\phi(s_\ell) + \beta s_\ell \nu(q_\ell)\mathcal{I}^w_\ell(x_\ell)$$

This is a proof by contradiction. Assume that the triple (q_ℓ, x_ℓ, s_ℓ) is not a solution of the maximization problem of function $\mathcal{H}_\ell$, given U_ℓ. Then, there must exist a tuple (q', x', s'), with $x' = ((w'_1, w'_2), R')$, such that

$$-\phi(s') + \beta s'\nu(q')\mathcal{I}^w_\ell(x') > -\phi(s_\ell) + \beta s_\ell \nu(q_\ell)\mathcal{I}^w_\ell(x_\ell), \text{ and } \beta\eta(q')\mathcal{I}^f_\ell(x') \geq k$$

From the definition of the off-the-equilibrium expectations, it follows that $q' < Q(x')$. Then,

$$\beta\eta(Q(x'))\sum_i \rho_i(x')\mathcal{I}^f_i(x') > \beta\eta(q')\sum_i \rho_i(x')\mathcal{I}^f_i(x') \geq \beta\eta(q')\mathcal{I}^f_\ell(x') \geq k \tag{42}$$

The second inequality results from Assumptions 1 and 2, which imply that $\mathcal{I}^f_h(x') \geq \mathcal{I}^f_\ell(x')$. Expression (42) implies that firms deviating to submarket x' would make strictly positive expected profits, which contradicts the assumption that $(U_\ell, q_\ell, x_\ell, s_\ell)$ was part of an equilibrium allocation. Therefore, the tuple $(U_\ell, q_\ell, x_\ell, s_\ell)$ is a fixed point of function $\mathcal{H}_\ell$.

The proof for type h agents is analogous. Suppose that the tuple (U_h, q_h, x_h, s_h) takes part of an equilibrium allocation. We show by contradiction that it is a fixed point of function $\mathcal{H}_h$, given U_ℓ. Again, the following equality results from the third equilibrium condition.

$$(1-\beta)U_h - v(z) = -\phi(s_h) + \beta s_h \nu(q_h)\mathcal{I}^w_h(x_h)$$

If the vector $(q_h, x_h.s_h)$ is not a maximizer of the associated maximization problem, given U_ℓ and U_h, there must exist a triple (q', x', s') such that

$$-\phi(s') + \beta s'\nu(q')\mathcal{I}^w_h(x') > -\phi(s_h) + \beta s_h \nu(q_h)\mathcal{I}^w_h(x_h), \quad \beta\eta(q')\mathcal{I}^f_h(x') \geq k,$$
$$\text{and } \max_s \Big\{ -\phi(s) + \beta s\nu(q')\mathcal{I}^w_\ell(x') \Big\} \leq -\phi(s_\ell) + \beta s_\ell \nu(q_\ell)\mathcal{I}^w_\ell(x_\ell)$$

As before, from the first inequality along with the monotonicity of function ν, we obtain that the equilibrium expectations at x' are such that $q' < Q(x')$. Putting this together with the third inequality, we obtain

$$\nu(Q(x'))\mathcal{I}^w_\ell(x') < \nu(q')\mathcal{I}^w_\ell(x') \leq \nu(q_\ell)\mathcal{I}^w_\ell(x_\ell) \Rightarrow \rho_\ell(x') = 0$$

Then, it follows that

$$\beta\eta(Q(x'))\sum_i \rho_i(x')\mathcal{I}^f_i(x') = \beta\eta(Q(x'))\mathcal{I}^f_h(x') > \beta\eta(q')\mathcal{I}^f_h(x') \geq k$$

That is, the expected profits at submarket x' are strictly positive, which contradicts the assumption of x_h taking part of an equilibrium allocation.

Now, we move to the second stage. Let (U_i, q_i, x_i, s_i) be a fixed point of function $\mathcal{H}_i$, for $i \in \{\ell, h\}$. We will show that it takes part of an equilibrium allocation. The proof is by construction. The remaining steady-state equilibrium objects are determined as follows: $\mathcal{X} \equiv \{x_\ell, x_h\}$, $dG(x_i) \equiv \frac{s_i\mu_i u^i}{q_i}$, with u^i defined by expression (16), $Q(x_i) \equiv q_i$ and $\rho_i(x_i) \equiv 1$. We still have to define the off-the-equilibrium beliefs. Let $x = ((w_1, w_2), R)$ be an arbitrary submarket, and let $\widetilde{q}_i(x)$ be defined as

$$\widetilde{q}_i(x) = \begin{cases} q & \text{, s.t. } \max_s \big\{ -\phi(s) + \beta s\nu(q)\mathcal{I}^w_i(x) \big\} = (1-\beta)U_i - v(z), \text{ if it exists} \\ 0 & \text{, otherwise.} \end{cases}$$

Then, we define $Q(x) \equiv \max_i \widetilde{q}_i(x)$, and $\rho_h(x) = \begin{cases} 1 & \text{, if } \widetilde{q}_\ell(x) < \widetilde{q}_h(x) \\ 0 & \text{, otherwise.} \end{cases}$

It follows that type i workers maximize their expected utility at x_i when searching for jobs among open submarkets. It remains to show that the zero-profit condition holds in both submarkets and that firms maximize profits given their expectations. Because of Proposition 3.1, we can focus on firms targeting one type of workers. We only show the case of type h workers as the type ℓ case may be reduced to a particular case of this one. Suppose that firms do not maximize profits at x_h. That is, there exists a submarket $x = ((w_1, w_2), R)$ such that

$$\begin{aligned} \beta\eta(Q(x))\mathcal{I}_h^f(x) &> k, \\ \max_s \big\{ -\phi(s) + \beta s\nu(Q(x))\mathcal{I}_h^w(x) \big\} &= (1-\beta)U_h - v(z), \\ \max_s \big\{ -\phi(s) + \beta s\nu(Q(x))\mathcal{I}_\ell^w(x) \big\} &\leq (1-\beta)U_\ell - v(z) \end{aligned}$$

We then distinguish between two cases.

Case 1: $\max_s \big\{ -\phi(s) + \beta s\nu(Q(x))\mathcal{I}_\ell^w(x) \big\} < (1-\beta)U_\ell - v(z)$.

Then, there must exist $q < Q(x)$ such that $\beta\eta(q)\mathcal{I}_h^f(x) > k$ and $\max_s \big\{ -\phi(s) + \beta s\nu(q)\mathcal{I}_\ell^w(x) \big\} < (1-\beta)U_\ell - v(z)$. From the monotonicity of function ν, it follows that

$$\max_s \big\{ -\phi(s) + \beta s\nu(q)\mathcal{I}_h^w(x) \big\} > (1-\beta)U_h - v(z),$$

which contradicts the assumption that (U_h, q_h, x_h, s_h) is a fixed point of function $\mathcal{H}_h$, given U_ℓ.

Case 2: $\max_s \big\{ -\phi(s) + \beta s\nu(Q(x))\mathcal{I}_\ell^w(x) \big\} = (1-\beta)U_\ell - v(z)$.

We can assume without loss of generality that $w_1 > 0$. Consider the alternative contract $x' = ((w_1', w_2'), R')$ such that $R' = R$, $w_1' = w_1 - \epsilon$, with ϵ arbitrarily small, and w_2' such that type h workers are indifferent between submarkets x and x'. In the proof of Proposition 3.1, we have shown that $\rho_h(x') = 1$, and $\max_s \big\{ -\phi(s) + \beta s\nu(Q(x'))\mathcal{I}_\ell^w(x') \big\} < (1-\beta)U_\ell - v(z)$. Therefore, we are back at the previous case when considering contract x', which leads to a contradiction. Notice that this same reasoning applies to show that the expected profits of firms are zero in equilibrium.‖

We had claimed above that type h workers have no incentives to apply to type ℓ jobs, and, as a result, the maximization problem of function $\mathcal{H}_\ell$ has one constraint less than its counterpart for $\mathcal{H}_h$. To see this, notice that the equilibrium tuple (q_ℓ, x_ℓ, s_ℓ) belongs to the domain of the objective function of the maximization problem of function $\mathcal{H}_h$, yet it is not the maximizer as the vector (q', x_ℓ, s'), with $q' < q_\ell$ and the utility-maximizing intensity s', attains a strictly higher value. This equivalence result is not surprising as the assumptions in Guerrieri et al. (2010) can be properly extended to hold in our dynamic setting. Thus, it can be read as a particular case of their results.

We now show existence of a steady state equilibrium using the Berge Maximum Theorem and the Brower Fixed-point Theorem. We first analyze function $\mathcal{H}_\ell$, and then $\mathcal{H}_h$. Let us refer to the maximization problem associated to function $\mathcal{H}_i$ for some value U as $P_i(U)$.

Let $\mathcal{K} \equiv \big[\frac{v(z)}{1-\beta}, \frac{v(y)}{1-\beta}\big]$. Given some value $U \in \mathcal{K}$, the domain in the maximization problem associated to the value $\mathcal{H}_\ell(U)$ results from the intersection of a finite number of compact sets; hence, the objective function of problem $P_\ell(U)$ is defined on a compact set. Assumption 3 ensures that the domain is a non-empty set. Furthermore, the objective function is a continuous real-valued function. Therefore, there exists a solution to problem $P_\ell(U)$, which is attained within the domain, and, hence, the function $\mathcal{H}_\ell$ is well-defined.

Let $\mathcal{C}$ be a correspondence that assigns the set of maximizers of problem $P_\ell(U)$ to a value $U \in \mathcal{K}$, i.e. $\mathcal{C}(U) \equiv \{p = (q, R, w_1, w_2, s)|\ p \text{ solves } P_\ell(U)\}$. We now show that $\mathcal{C}$ is a compact-valued, continuous correspondence in $\mathcal{K}$. To show upper-hemicontinuity of $\mathcal{C}$ at some value U, consider any sequence $\{U_n\}_n \subset \mathcal{K}$ converging to U and any sequence $\{p_n\}_n$ such that $p_n \in \mathcal{C}(U_n)$ for all n. We need to show that $p \equiv \lim_{n\to\infty} p_n \in \mathcal{C}(U)$. Lower-hemicontinuity requires to show that for any sequence $\{U_n\}_n \subset \mathcal{K}$ converging to U and for any $p \in \mathcal{C}(U)$, there exist a subsequence $\{U_{n_k}\}_k$ and a sequence $\{p_{n_k}\}_k$ such that $p_{n_k} \in \mathcal{C}(U_{n_k})$ and $p \equiv \lim_{k\to\infty} p_{n_k}$. It is easy to see that the continuity of the utility and matching functions and the compactness of set $\mathcal{K}$ ensure that the correspondence $\mathcal{C}$ is both upper- and lower-hemicontinuous, and that $\mathcal{C}(U)$ is a compact set for any value U.

Then, the Maximum Theorem states that $\mathcal{H}_\ell$ is a continuous function in $\mathcal{K}$. Finally, the Brower Fixed-point Theorem applies to ensure existence of a fixed point of function $\mathcal{H}_\ell$.

The proof for the existence of a fixed-point of function $\mathcal{H}_h$ is analogous. We thus only show that the domain of the objective function of problem $P_h(U)$ is non-empty. Notice that the fixed point of function $\mathcal{H}_\ell$ satisfies all the constraints due to Assumptions 1-3.

We conclude that there exists an equilibrium allocation. We now turn to characterize the equilibrium. The Lagrangian of the maximization problem $P_\ell(U)$ is

$$\mathcal{L} = -\phi(s) + \beta s\nu(q)\int_R \big(E_\ell(\pi) - U_\ell\big)dF_\ell(\pi) + \xi\bigg(\eta(q)\int_R J_\ell(\pi)dF_\ell(\pi) - k/\beta\bigg),$$

where ξ is the Lagrange multiplier. We list four of the necessary first order conditions:

$$\begin{gathered}\nu(q)\big(1-F_\ell(R)\big)\big(sv'(w_1)-\xi q\big)\leq 0, w_1\geq 0,\\ \text{and } w_1\nu(q)\big(1-F_\ell(R)\big)\big(sv'(w_1)-\xi q\big)=0\end{gathered}\tag{43}$$

$$\begin{gathered}\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)\big(sv'(w_2)-\xi q\big)\leq 0, w_2\geq 0,\\ \text{and } w_2\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)\big(sv'(w_2)-\xi q\big)=0\end{gathered}\tag{44}$$

$$\begin{gathered}s\nu(q)\big(E_\ell(x,R)-U_\ell+\tfrac{\xi q}{s}J(x,R)\big)dF_\ell(R)\geq 0, R\geq 0,\\ \text{and } RdF_\ell(R)s\nu(q)\big(E_\ell(x,R)-U_\ell+\tfrac{\xi q}{s}J(x,R)\big)=0\end{gathered}\tag{45}$$

$$\begin{gathered}s\nu'(q)\mathcal{I}^w_\ell(x)+\xi\eta'(q)\mathcal{I}^f_\ell(x)\geq 0, q\geq 0\\ \text{and } q\bigg(s\nu'(q)\mathcal{I}^w_\ell(x)+\xi\eta'(q)\mathcal{I}^f_\ell(x)\bigg)=0\end{gathered}\tag{46}$$

where we define, for any given submarket x and worker type $i\in\{\ell,h\}$,

$$\mathcal{I}^w_i(x)\equiv\int_R\bigg(E_i(x,\pi)-U_h\bigg)dF_i(\pi),\ \text{and } \mathcal{I}^f_i(x)\equiv\int_R J(x,\pi)dF_i(\pi)$$

The first two conditions are regarding the two wage levels, the last two refer to the first order conditions regarding the reservation value and the queue length, respectively. As $\lim\limits_{w\to 0} v'(w)=\infty$, we conclude from conditions (43) and (44) that the equilibrium $w_{1\ell}, w_{2\ell}>0$. Condition (46) implies that $q<\infty$, and, hence, the equilibrium conditions (43) and (44) imply that $w_1=w_2$.

The Lagrangian of the maximization problem $P_h(U)$ has one more term:

$$\begin{aligned}\mathcal{L} \;=\;& -\phi(s)+\beta s\nu(q)\int_R\big(E_h(x,\pi)-U_h\big)dF_h(\pi)+\xi_1\bigg(\eta(q)\int_R J_h(x,\pi)dF_h(\pi)-k/\beta\bigg)\\ &-\xi_2\bigg(\max_s\bigg\{-\phi(s)+\beta s\nu(q)\int_R\big(E_\ell(x,\pi)-U_\ell\big)dF_\ell(\pi)\bigg\}+v(z)-(1-\beta)U_\ell\bigg)\end{aligned}$$

The counterparts of conditions (43)-(46) are

$$\nu(q)\big(1-F_h(R)\big)\big(sv'(w_1)-\xi_1 q\big)-\xi_2\beta\hat{s}\nu(q)\big(1-F_\ell(R)\big)v'(w_1)\leq 0, w_1\geq 0,$$
$$\text{and } w_1\nu(q)\bigg(\big(1-F_h(R)\big)\big(sv'(w_1)-\xi_1 q\big)-\xi_2\beta\hat{s}\big(1-F_\ell(R)\big)v'(w_1)\bigg)=0$$
$$\nu(q)\mathbb{E}_h(\pi|R)\big(1-F_h(R)\big)\big(sv'(w_2)-\xi_1 q\big)-\xi_2\beta\hat{s}\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)v'(w_2)\leq 0, w_2\geq 0,$$
$$\text{and } w_2\nu(q)\bigg(\mathbb{E}_h(\pi|R)\big(1-F_h(R)\big)\big(sv'(w_2)-\xi_1 q\big)-\xi_2\beta\hat{s}\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)v'(w_2)\bigg)=0$$
$$s\nu(q)\big(E_h(x,R)-U_h+\xi_1 qJ(x,R)\big)dF_h(R)-\xi_2\beta\hat{s}\nu(q)\big(E_\ell(x,R)-U_\ell\big)dF_\ell(R)\geq 0, R\geq 0,$$
$$\text{and } R\nu(q)\bigg(s\big(E_h(x,R)-U_h+\xi_1 qJ(x,R)\big)dF_h(R)-\xi_2\beta\hat{s}\big(E_\ell(x,R)-U_\ell\big)dF_\ell(R)\bigg)=0$$
$$s\nu'(q)\mathcal{I}_h^w(x)+\xi_1\eta'(q)\mathcal{I}_h^f(x)-\xi_2\beta\hat{s}\nu'(q)\mathcal{I}_\ell^w(x)\geq 0, q\geq 0$$
$$\text{and } q\bigg(s\nu'(q)\mathcal{I}_h^w(x)+\xi_1\eta'(q)\mathcal{I}_h^f(x)-\xi_2\beta\hat{s}\nu'(q)\mathcal{I}_\ell^w(x)\bigg)=0$$

where $\hat{s}$ denotes the optimal search intensity for type ℓ workers. For the same reason as in the previous case, we obtain $w_{1h}, w_{2h}>0$. By manipulating the first two necessary conditions, the inequality $w_{1h}\leq w_{2h}$ follows from Assumption 1 and the concavity of the utility function v. Furthermore, the inequality is strict if and only if the second constraint of problem $P_h(U)$ binds.

Recall that the equilibrium variables related to the type ℓ market satisfy all the constraints of problem $P_h(U_\ell)$ due to Assumptions 1-3. Then, it follows that the market value of type ℓ workers is strictly lower than the one of their type h counterparts, $U_\ell<U_h$.‖

7.2 Extension of Blanchard and Tirole (2008)

In this section we analyze a directed search model à la Blanchard and Tirole (2008). It can be interpreted as a subcase of our benchmark in which match quality is only an experience good.

Set Up This is a one period economy with two stages: search and production. There is a large continuum of risk-neutral firms and a mass one of risk-averse workers. Search is directed. Firms commit to a contract which specifies a wage w and a reservation value R to attract applicants. Upon meeting, a match is automatically formed. Then, match quality $y\in[0,1]$ is learned. If y is below the threshold, then the worker is laid-off and the firm pays a layoff tax L. Unemployed workers derive utility from home production z.

Planner's Economy. The planner's problem is the analog to the one in Section 2. The Lagrangian is

$$\begin{aligned}\mathcal{L} \;=\; & -\phi(s)+s\nu(q)(v(c^e)-v(c^u))(1-F(R))+v(c^u)+\xi_1\big(\phi'(s)-\nu(q)(v(c^e)-v(c^u))(1-F(R))+v(c^u)\big)\\ & +\xi_2\big((z-c^u)(1-s\nu(q)(1-F(R))+s\nu(q)(1-F(R))\bigg(\mathbb{E}(y|R)-c^e\big)-\frac{ks}{q}\bigg)\end{aligned}$$

where $\mathbb{E}(y|R)=\int_R ydF(y)$, and ξ_1 and ξ_2 stand for the Lagrangian multipliers of incentive compatibility condition and the resource constraint, respectively.

The key necessary first-order conditions are

$$\frac{\partial\mathcal{L}}{\partial R} = -dF(R)\nu(q)(s-\xi_1)v'(c^e)\bigg(\frac{v(c^e)-v(c^u)}{v'(c^e)}+c^u-z+R-c\bigg)\leq 0, \text{ and } R\geq 0 \tag{47}$$

$$\frac{\partial\mathcal{L}}{\partial q} = -(1-\varphi(q))(1-F(R))\bigg(\frac{v(c^e)-v(c^u)}{v'(c^e)}+c^u-z+\mathbb{E}(y|R)-c\bigg)+\frac{k}{\eta(q)}=0$$

If $R > 0$, then the second necessary condition can be rewritten as

$$(1 - \varphi(q))(1 - F(R))\Big(\mathbb{E}(y|R) - R\Big) = \frac{k}{\eta(q)} \tag{48}$$

Market Economy. In the market economy, jobless workers obtain unemployment benefits b net of lump sum taxes T, whereas employed workers obtain after-tax wages. The unemployment insurance system is financed through lump sum taxes, a proportional income tax with rate τ, and a layoff tax L.

Similarly to the benchmark case analyzed in Section 2.3, it can be shown that the equilibrium allocation is a solution of the following program

$$\begin{array}{rl} \max & -\phi(s) + s\nu(q)(v(w(1-\tau) - T) - v(z + b - T))(1 - F(R)) \\ \text{s. to} & \eta(q)\big((1 - F(R))(\mathbb{E}(y|R) - w) - LF(R)\big) \geq k \end{array}$$

Let $\mathcal{L}_m$ denote the Lagrangian of this program, $c^e = w(1-\tau) - T$, and $c^u = z + b - T$. Then, the key necessary first-order conditions are

$$\frac{\partial \mathcal{L}_m}{\partial R} = -dF(R)\nu(q)v'(c^e)\left(\frac{v(c^e) - v(c^u)}{v'(c^e)} + (1-\tau)(R - w + L)\right) \leq 0, \text{ and } R \geq 0 \tag{49}$$

$$\frac{\partial \mathcal{L}_m}{\partial q} = -(1 - \varphi(q))(1 - F(R))\left(\frac{v(c^e) - v(c^u)}{v'(c^e)} + (1-\tau)\left(\mathbb{E}(y|R) - w - L\frac{F(R)}{1 - F(R)}\right)\right) + (1-\tau)\frac{k}{\eta(q)} = 0$$

If $R > 0$, then we can rewrite the second condition using the first one as

$$(1 - \varphi(q))(1 - F(R))\left(\mathbb{E}(y|R) - R - \frac{L}{1 - F(R)}\right) = \frac{k}{\eta(q)} \tag{50}$$

Welfare Analysis. The aim is to determine necessary conditions for the planner's allocation to be decentralized. Let us start assuming that firm entry is exogenous or trivial, and pay attention only to the threshold decision. After replacing wages and consumption levels, we obtain that the first order conditions (47) and (49) coincide with one another if and only if $-\tau R + L(1-\tau) = b$. Therefore, $\tau = 0$ and $L = b$ would suffice to decentralize the planner's solution, which is also the result in Blanchard and Tirole (2008).

If firm entry decision is not a trivial one and $R > 0$, then the necessary conditions (48) and (50) must also be the same, which requires $L = 0$, and hence $-\tau R = b$. Notice that this is the analog of the result stated in Proposition 2.4.

7.3 Two-period Model with Savings

Consider a two-period economy that differs from the one set in Section 2 in the following items. All workers are born identical and are endowed with assets a. Workers can save part of their first period wealth to increase consumption in the second period, but cannot borrow. The interest rate r is exogenously determined consistent with the intertemporal resource constraint of the planner. Consistent with our quantitative exercise, we assume that $\beta(1 + r) = 1$.[21] There is lack of commitment on the worker's side. The timing of the events in period one is as follows: agents search for partners, then production and consumption and saving decisions take place.

7.3.1 Planner's Economy

The planner chooses an allocation $(s, q, R, c_u, c_1, c_{2e}, c_{2u})$, where c_{2e} and c_{2u} denote the period-two consumption promised to the workers who are employed in period one contingent on their employment status in

[21] For example, Shimer and Werning (2008) also assume an interest rate equal to the discount rate.

period two. To save on notation and without loss of generality, we will assume that consumption variables are not contingent on signal π. The counterpart of the intertemporal resource constraint (3) is

$$(z-c_u)\big(1-s\nu(q)(1-F(R))\big)(1+\beta)+s\nu(q)(1-F(R))(1-\mathbb{E}(\pi|R))\beta(z-c_{2u})+ \\ s\nu(q)\int_R\Big(\pi y-c_1+\beta\pi\big(y-c_{2e}\big)\Big)dF(\pi)+a\geq k\frac{s}{q} \tag{51}$$

There are two differences with respect to the constraint in Section 2. First, the period-two newly unemployed workers obtain consumption c_{2u}. Second, the total wealth a the economy starts with appears on the output side of the resource constraint as an endowment.

The counterpart of the incentive compatibility condition (2) is

$$\phi'(s)=\nu(q)\int_R\Big(v(c_1)-v(c_u)+\beta\big(\pi v(c_{2e})+(1-\pi)v(c_{2u})-v(c_u)\big)\Big)dF(\pi) \tag{52}$$

The planner's problem becomes

$$\max_{s,q,R,c_u,c_1,c_{2e},c_{2u}} \quad -\phi(s)+s\nu(q)\int_R\Big(v(c_1)-v(c_u)+\beta\big(\pi v(c_{2e})+(1-\pi)v(c_{2u})-v(c_u)\big)\Big)dF(\pi)+v(c_u)(1+\beta)$$
$$\text{s. t. conditions (51) and (52)}$$

Lemma 7.2 *The constrained efficient allocation must satisfy $z+\frac{a}{1+\beta}<c_u^p<c^p=c_1^p=c_{2e}^p=c_{2u}^p$. The reservation probability R^p satisfies*

$$\frac{v(c^p)-v(c_u^p)}{v'(c^p)}(1+\beta)+(c_u^p-c^p)(1+\beta)-z(1+\beta R)+Ry(1+\beta)\geq 0$$
$$\textit{and } R\geq 0,\ \textit{with complementary slackness} \tag{53}$$

Furthermore, $R^py(1+\beta)<z(1+\beta R^p)$. If $R^p>0$, then the constrained efficient queue length q^p satisfies

$$(1-\varphi(q))(1-F(R^p))\big(\mathbb{E}(\pi|R^p)-R^p\big)(y(1+\beta)-\beta z)=\frac{k}{\eta(q)} \tag{54}$$

We obtain the results stated in the lemma by following the same steps as in the proof of Proposition 2.1. As in the economy analyzed in Section 2, the reservation probability is such that a mass of employed workers would be more productive at home. This results from $v(c^p)-v(c_u^p)+v'(c^p)(c_u^p-c^p)>0$ because of the concavity of the utility function.

7.3.2 Market Economy

Consider a general policy. It consists of unemployment benefits b, a lump sum tax and a proportional income tax on workers, T and τ, a lump sum tax on producing firms T_f and layoff taxes, L. For expositional convenience, we first solve the problem of the period-one unemployed worker, which is deterministic. She chooses savings to smooth consumption across periods. That is

$$V^u\equiv\max_{a'}v(a+z+b-T-a')+\beta v(a'(1+r)+z+b-T)$$

Because the objective function is strictly concave, this problem has a unique solution, and consumption is time-invariant and equals $c_u=z+b-T+\frac{a}{1+\beta}$.

Next, we look at the problem of the period-one employed worker with signal π and wages $w_1(\pi)$ and $w_2(\pi)$. She chooses savings taking into account the uncertainty about period-two income. That is

$$V^e(w_1(\pi),w_2(\pi),\pi)\equiv\max_{a'}\quad v(a+w_1(\pi)(1-\tau)-T-a') \\ +\beta\big(\pi v(a'(1+r)+w_2(\pi)(1-\tau)-T)+(1-\pi)v(a'(1+r)+z+b-T)\big)$$

The concavity of the objective function ensures a unique solution, which satisfies the first order condition:

$$v'(c_1(\pi)) = \pi v'(c_{2e}(\pi)) + (1-\pi)v'(c_{2u}(\pi)) \tag{55}$$

The lack of commitment on the worker's side implies that $w_2(\pi)(1-\tau) \geq z+b$ for all π. Using this inequality and equation (55), we obtain that $c_{2e}(\pi) \geq c_1(\pi)$.

Finally, the firm's problem is to maximize expected profits subject to guaranteeing workers their market value U. That is

$$\begin{aligned} \max_{q,R,w_1(\pi),w_2(\pi)} \quad & \eta(q)\int_R \Big(\pi y - w_1(\pi) - T_f + \beta\big(\pi\big(y - w_2(\pi) - T_f\big) - (1-\pi)L\big)\Big)dF(\pi) \qquad (56)\\ \text{s. to} \quad & \max_s\Big\{ -\phi(s) + s\nu(q)\int_R \Big(V^e(w_1(\pi), w_2(\pi), \pi) - V^u\Big)dF(\pi)\Big\} + V^u \geq U \end{aligned}$$

The government is assumed to balance its budget intertemporally, as the planner in the centralized economy. Therefore, its balanced budget constraint is

$$\begin{aligned} & (T-b)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big) \\ +Ls\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R)) + s\nu(q)(1-F(R))(T+\tau w_1 + T_f + \beta\mathbb{E}(\pi|R)(T+\tau w_2+T_f)) \quad & = \quad 0, \end{aligned}$$

Given policy (b, T, τ, L), a tax-distorted competitive search equilibrium consists of consumption streams and wages, a queue length function, a market value, and a reservation probability such that agents behave optimally, and the government budget is balanced. Let us now write the Lagrangian of the firm's problem.

$$\begin{aligned} \mathcal{L} \quad = \quad & \eta(q)\int_R \Big(\pi y - w_1(\pi) - T_f + \beta\big(\pi\big(y - w_2(\pi) - T_f\big) - (1-\pi)L\big)\Big)dF(\pi) \\ & +\xi\Big(-\phi(s) + s\nu(q)\int_R \Big(V^e(w_1(\pi), w_2(\pi), \pi) - V^u\Big)dF(\pi) + V^u - U\Big) \end{aligned}$$

Using the Envelope Theorem, the first order conditions with respect to wages, with complementary slackness, are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_1(\pi)} \quad & = \quad -\nu(q)dF(\pi)\big(q - \xi s(1-\tau)v'(c_1(\pi))\big) \leq 0, w_1(\pi) \geq 0 \\ \frac{\partial \mathcal{L}}{\partial w_2(\pi)} \quad & = \quad -\nu(q)dF(\pi)\beta\pi\big(q - \xi s(1-\tau)v'(c_{2e}(\pi))\big) \leq 0, \text{ and } w_2(\pi)(1-\tau) \geq z+b \\ & \qquad \text{with complementary slackness} \end{aligned}$$

Workers who are employed in period one perfectly smooth consumption in equilibrium. Suppose that this is not the case and $w_2(\pi)(1-\tau) > z+b$ for some π. Then, as reasoned above, the optimality condition (55) together with the concavity of the utility function implies that $c_{2e}(\pi) > c_1(\pi)$. Then, it follows that $0 \geq \frac{\partial \mathcal{L}}{\partial w_1(\pi)} > \frac{\partial \mathcal{L}}{\partial w_2(\pi)}$. Therefore, a wage schedule such that $w_2(\pi)(1-\tau) > z+b$ cannot be offered in equilibrium as firms would make higher profits by reducing the period-two wage. As a result, $w_2(\pi)(1-\tau) = z+b$ for all π. Likewise, from the first order condition with respect to $w_1(\pi)$, we obtain that it is constant in π. Therefore, $w_t(\pi) = w_t$ for $t = 1, 2$. Then, it follows from equation (55) that $c_1(\pi) = c_{2e}(\pi) = c_{2u}(\pi) = c$ for all π. This result is quite intuitive. Firms frontload wages because of the employment uncertainty in period two and the worker's ability to transfer income across periods.

Plugging these results in $\frac{\partial \mathcal{L}}{\partial R}$, we obtain that the first order condition with respect to R and q are

$$\frac{\partial \mathcal{L}}{\partial R} = -\eta(q)dF(R)\Big(Ry(1+\beta) - w_1 - \beta R w_2 - T_f(1+\beta R) - \beta(1-R)L + \frac{v(c)-v(c_u)}{v'(c)(1-\tau)}(1+\beta)\Big) \leq 0$$
$$\text{and } R \geq 0, \text{ with complementary slackness} \quad (57)$$
$$\frac{\partial \mathcal{L}}{\partial q} = 0 \Leftrightarrow (1-\varphi(q))(1-F(R))\big(\mathbb{E}(\pi|R) - R\big)\big(y(1+\beta) - \beta(w_2 + T_f - L)\big) = \frac{k}{\eta(q)} \text{ if } R > 0 \quad (58)$$

Notice that the firm's value at $\pi = R$ is negative, $Ry(1+\beta) - w_1 - \beta R w_2 - T_f(1+\beta R) - \beta(1-R)L < 0$. If $R > 0$, this inequality follows from the first order condition. If $R = 0$, it trivially holds unless $-w_1 - T_f \geq \beta L$. Next, using the budget constraint of the period-one employed and unemployed workers, we can isolate wages as

$$w_1(1-\tau) = c(1+\beta) - (c_u - z - b + T)(1+\beta) - \beta(z+b) + T(1+\beta),$$

We can now substitute out wage w_1 in $\frac{\partial \mathcal{L}}{\partial R}$, and obtain that the optimal reservation probability must satisfy, if positive, the following equation:

$$\frac{v(c)-v(c_u)}{v'(c)}(1+\beta) + (c_u - c)(1+\beta) - (z+b)(1+\beta R) + (1-\tau)\big(Ry(1+\beta) - T_f(1+\beta R) - \beta(1-R)L\big) = 0$$

Consider first a laissez faire economy, i.e. $b = T = \tau = T_f = L = 0$. Notice that this condition coincides with the constrained efficiency condition (53). However, the equilibrium allocation is not constrained efficient. To see this, it suffices to notice that $c_u < c_u^p$. This is the case because unemployed workers cannot borrow against future income.

Next, consider a policy (b, T, τ, T_f, L) such that $c_u^p = z + b - T + \frac{a}{1+\beta}$, $c^p = w_1(1-\tau) - T$, and the government budget constraint holds. Then, the equilibrium and constrained efficiency first order conditions with respect to R coincide with one another if and only if

$$b(1+\beta R) = -\tau R y(1+\beta) - (1-\tau)\big(T_f(1+\beta R) + \beta L(1-R)\big) \quad (59)$$

Notice that this is the same condition (39) we established for efficiency in equilibrium in the economy with no savings if $T_f = 0$. We now compare the constrained efficiency first order condition with respect to q (54) with its equilibrium counterpart (58) under $R^p > 0$. They are the same if and only if $z = w_2 + T_f - L$. The following proposition establishes that the results stated in Proposition 2.4 also hold when workers are allowed to save. We can make $L = 0$, and then determine the fiscal parameters b, τ, T and T_f for the above equations to hold. It can be easily shown that τ and T_f must be negative if $b > 0$. To complete the proof of the decentralization of the constrained efficient allocation, it suffices to replicate the argument in the proof of Proposition 2.4.

Proposition 7.3 *The laissez-faire competitive search equilibrium is not constrained efficient. If $R^p > 0$, then constrained efficiency can be attained in the market economy by the implementation of a unemployment insurance system funded through lump sum taxes on firms and workers, a negative income tax and a zero layoff tax.*

The intuition why layoff taxes are redundant in this setting is because firms frontload wages for their employees to perfectly smooth consumption across periods and across states.