Resolving the Spanning Puzzle in Macro-Finance Term Structure Models

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Macro-finance term structure models (MTSMs)

Goal of these models

Understand economic forces that drive changes in interest rates

Approach of these models

Jointly model macroeconomy and the yield curve, using absence of arbitrage to price financial assets

Wide variety of models in macroeconomics and finance

- Reduced-form models
  - Time series model for risk factors and flexible stochastic discount factor (SDF)
- Equilibrium (finance) models
  - Endowment economies and micro-founded SDF
- DSGE (macro) models
  - Production economies and micro-founded or ad-hoc SDF

Literature

- Reduced-form MTSMs
  Ang and Piazzesi (2003), Bernanke, Reinhart, Sack (2004), Kim and Wright (2005), Ang, Piazzesi, Wei (2006), Ang, Bekaert, Wei (2008), Campbell, Sunderam, Viceira (2009), Smith and Taylor (2009), Bikbov and Chernov (2010), Ang, Boivin, Dong (2011), Joslin, Le, Singleton (2013a,b), Jardet, Monfort, Pegoraro (2013), Bauer and Rudebusch (2014), Wu and Xia (2014)

- Equilibrium (finance) models
  Wachter (2006), Piazzesi and Schneider (2007), Buraschi and Jiltsov (2007), Gallmeyer et al. (2007), Bekaert et al. (2009), Bansal and Shaliastovich (2007)

- DSGE (macro) models
  Hördahl, Tristani, Vestin (2006), Dewachter and Lyrio (2006), Rudebusch and Wu (2008), Rudebusch and Swanson (2008, 2012), Bekaert, Cho, Moreno (2010), Hördahl and Tristani (2014)

The "Spanning Puzzle"

- MTSMs generally imply *macro spanning*
  - All relevant information about the economy is in the yield curve
  - Macro variation is spanned by (perfectly correlated with) the yield curve
- But regressions show evidence for *unspanned macro information*
  - There is substantial unspanned macro variation
  - And this helps predict future bond returns and macro variables
- Apparent inconsistency between theoretical macro spanning and empirical evidence is puzzling

Serious challenge for entire macro-finance literature

- Kim (2009): *"may undermine the validity of the models that use inflation as a state variable"*
- Gürkaynak and Wright (2012): *"thorny issue with the use of macroeconomic variables in affine term structure models"*
- Duffee (2012a): *"important conceptual difficulty with macro-finance models"*
- Joslin, Priebsch, Singleton (JPS, 2014): *"current generation of MTSMs [...] enforce[s] strong and counterfactual restrictions on how the macroeconomy affects yields"*

Two solutions to spanning puzzle

- JPS develop new type of MTSM
  - Premise: spanned models are invalidated by regression evidence
  - Unspanned MTSM: all macro factors are unspanned
  - *"large step toward bringing MTSMs in line with the historical evidence"*
  - New trend: models with unspanned/hidden factors
    Duffee (2011), Wright (2011), Chernov and Mueller (2012), ...
- Our new solution: Salvage the conventional affine MTSM
  - Spanned models are consistent with the regression evidence when accounting for small measurement error
  - Knife-edge restrictions of unspanned models are rejected
  - Spanned and unspanned models imply the same term premia

Outline

Conventional affine MTSMs

- Economy driven by $\mathcal{N}$ state variables/risk factors X_t
  - $\mathcal{L}$ yield factors in $P_t^{\mathcal{L}}$
  - $\mathcal{M}$ macro factors in M_t
  - $X_t = (P_t^{\mathcal{L}\prime}, M_t')'$, $\mathcal{N} = \mathcal{L} + \mathcal{M}$
- Model specification has three components
  - Gaussian VAR for X_t
    - Either reduced-form specification
    - Or equilibrium solution to structure model
  - Affine short rate specification:

$$r_t = \rho_0 + \rho_P' P_t^{\mathcal{L}} + \rho_M' M_t$$

  - Essentially-affine SDF $\Leftrightarrow$ VAR for X_t under risk-neutral ($\mathbb{Q}$) measure:

$$X_t = \mu^{\mathbb{Q}} + \phi^{\mathbb{Q}} X_{t-1} + \Sigma \varepsilon_t^{\mathbb{Q}}, \quad \varepsilon_t^{\mathbb{Q}} \overset{iid}{\sim} N(0, I_{\mathcal{N}})$$

$\Rightarrow$ Yields affine in risk factors: $Y_t = A + BX_t$

Affine MTSMs generally imply *macro spanning*

- Conditions
  - Yields are affine in risk factors, $Y_t = A + BX_t$
  - Risk factors contain macro variables
  - No knife-edge special cases — B has full rank
- Conditions satisfied in essentially all existing models
- Can invert $\mathcal{N}$ (linear combinations of) model-implied yields to obtain state variables

$$X_t = (B_{\mathcal{N}})^{-1}(Y_t^{(\mathcal{N})} - A_{\mathcal{N}})$$

- Macro factors spanned by $\mathcal{N}$ (linear combinations of) yields

$$M_t = \gamma_0 + \gamma_1 P_t^{\mathcal{N}}$$

- "Theoretical macro spanning"

Testable implications of macro spanning

- R^2 near one in regressions of macro variables on yields
  - Instead, evidence of **unspanned macro variation**
  - Regressions of macro variables on yields have low R^2
  - *"R^2 are on the wrong side of 1/2"* (Duffee, 2013b)
  - Duffee (2013a,b), Joslin, Priebsch, Singleton (2014, JPS)
- Only current yield curve predicts excess bond returns
  - Instead, evidence of **unspanned macro risk**
  - Macro variables help predict excess returns even controlling for information in current yields
  - Cooper and Priestley (2009), Ludvigson and Ng (2009), JPS
- Only current yield curve predicts macro variables
  - Instead, evidence of **unspanned macro forecasts**
  - Macro variables help predict future macro variables even controlling for yields—macro persistence is not fully captured in yields so macro lags matter
  - Duffee (2013a,b)

Unspanned MTSMs

- JPS impose knife-edge restrictions on affine MTSM
  - Short rate does not depend on macro factors

$$r_t = \rho_0 + \rho_P' P_t^{\mathcal{L}} + 0_{\mathcal{M}}' M_t$$

  - Risk-neutral distribution does not depend on macro factors

$$P_t^{\mathcal{L}} = \mu^{\mathbb{Q}} + \phi^{\mathbb{Q}} P_{t-1}^{\mathcal{L}} + 0_{\mathcal{L}\times\mathcal{M}} M_t + \Sigma \varepsilon_t^{\mathbb{Q}}, \quad \varepsilon_t^{\mathbb{Q}} \overset{iid}{\sim} N(0, I_{\mathcal{L}})$$

$\Rightarrow$ Yields do not load on macro factors

$$Y_t = A + B_P P_t^{\mathcal{L}} + 0_{\mathcal{M}\times\mathcal{M}} M_t.$$

$\Rightarrow$ Yields have only $\mathcal{L}$ factors; these do not span macro factors

$$M_t = \gamma_0 + \gamma_P P_t^{\mathcal{L}} + O M_t$$

Outline

Testing for unspanned macro information

- Unspanned macro variation

$$m_t = \beta_0 + \beta_1' P_t^{(3)} + u_t$$

  - How high is the R^2? Evidence against spanning if R^2 is low

- Unspanned macro risk

$$\bar{rx}_{t,t+12} = \beta_0 + \beta_1' P_t^{(3)} + \beta_2 m_t + u_t$$

  - Spanning implies $\beta_2 = 0$
  - Evidence against spanning if m_t has predictive power

- Unspanned macro forecasts

$$m_{t+1} = \beta_0 + \beta_1' P_t^{(3)} + \beta_2 m_t + u_t$$

  - Spanning implies $\beta_2 = 0$
  - Evidence against spanning if m_t has predictive power

Data

- Sample
  - Monthly data, 1985–2007 (same as in JPS)
- Yields
  - Unsmoothed Treasury zero-coupon yields
- Excess bond returns
  - One-year holding period
  - Average across maturities
- Macro variables
  - Measures of economic activity and inflation

Consider ten macro variables for robustness

- Measures of slack
  - Unemp. gap = Unemployment rate - CBO natural rate
  - Output gap = Monthly real GDP - CBO potential GDP
- Measures of underlying inflation
  - INF (used by JPS) = Blue Chip expectations of one-year CPI inflation
  - CPI inflation = Core CPI inflation, year-over-year
  - PCEPI inflation = Core PCEPI inflation, year-over-year
- Measures of growth
  - GRO (used by JPS) = Three-month moving average of Chicago Fed National Activity Index
  - GDP growth (ma3) = Three-month moving average of monthly real GDP growth
  - GDP growth (yoy) = Year-over-year real GDP growth
  - IP growth (ma3) = Three-month moving average of industrial production growth
  - Jobs growth (ma3) = Three-month moving average of payroll employment growth

Macro variables and monetary policy rules

Policy rule: $FFR_t = \beta_0 + \beta_1 g_t + \beta_2 \pi_t + u_t$

	Policy rule		Macro-spanning R^2			
	R^2	partial	joint	level	slope	curvature
Policy factors						
Unemp. gap	0.80	0.59	0.72	0.01	0.67	0.04
Output gap	0.79	0.58	0.57	0.01	0.45	0.10
INF (JPS)	0.75	0.71	0.81	0.74	0.03	0.04
CPI inflation	0.80	0.76	0.81	0.67	0.04	0.10
PCEPI inflation	0.74	0.68	0.77	0.60	0.05	0.12
Non-policy factors						
GRO (JPS)	0.53	0.05	0.28	0.01	0.00	0.27
GDP growth (ma3)	0.52	0.02	0.14	0.01	0.01	0.12
GDP growth (yoy)	0.51	0.01	0.20	0.00	0.00	0.19
IP growth (ma3)	0.60	0.20	0.32	0.14	0.02	0.16
Jobs growth (ma3)	0.61	0.20	0.20	0.04	0.01	0.15

- Fed focuses on certain macro variables when setting the policy rate

Unspanned macro variation

Spanning regression: $m_t = \beta_0 + \beta_1' P_t^{(3)} + u_t$

	Policy rule		Macro-spanning R^2			
	R^2	partial	joint	level	slope	curvature
Policy factors						
Unemp. gap	0.80	0.59	0.72	0.01	0.67	0.04
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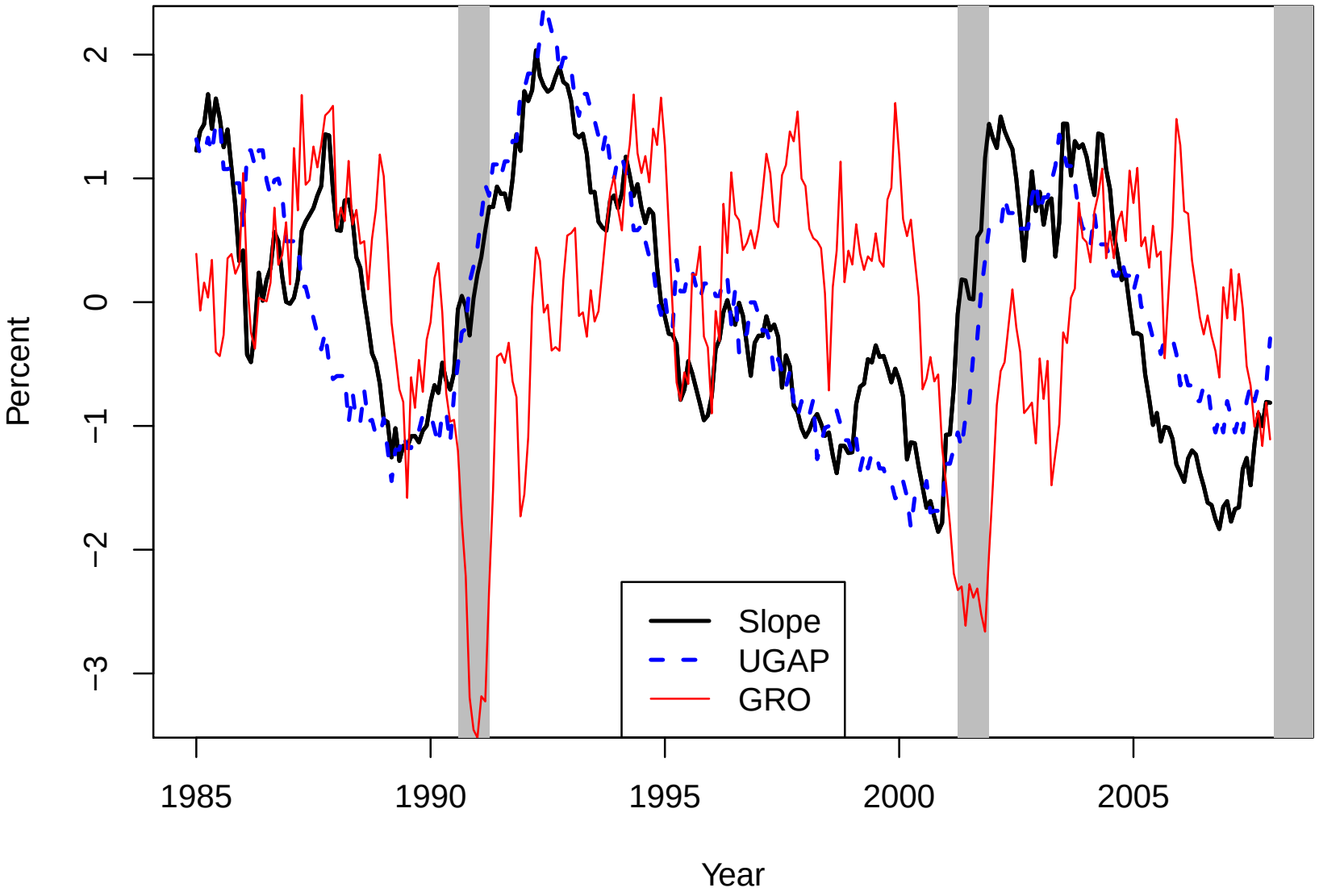
- Monetary policy creates link between some macro variables and yields $\Rightarrow$ policy-based explanation of unspanned macro variation

Not all economic activity measures are unspanned



Unspanned macro risk

Return forecasts: $\bar{rx}_{t,t+12} = \beta_0 + \beta_1' P_t^{(3)} + \beta_2 m_t + u_t$

	Return forecasts			Macro forecasts		
	R^2	t-stat.	RMSE	AC	t-stat.	RMSE
Policy factors						
Unemp. gap	0.20	0.67	1.00	0.98	53.23	0.34
Output gap	0.20	0.73	1.00	0.95	33.21	0.46
INF (JPS)	0.36	4.14	0.89	0.99	42.85	0.34
CPI inflation	0.26	1.43	0.96	0.99	44.06	0.29
PCEPI inflation	0.23	1.67	0.98	0.98	55.32	0.32
Non-policy factors						
GRO (JPS)	0.25	2.75	0.97	0.91	26.55	0.50
GDP growth (ma3)	0.21	2.18	0.99	0.47	5.04	0.92
GDP growth (yoy)	0.20	0.88	1.00	0.77	14.65	0.71
IP growth (ma3)	0.32	3.81	0.92	0.94	31.59	0.42
Jobs growth (ma3)	0.22	1.72	0.98	0.87	22.77	0.53

- Evidence for unspanned macro risk is weak and variable

Unspanned macro forecasts

Macro forecasts: $m_{t+1} = \beta_0 + \beta_1' P_t^{(3)} + \beta_2 m_t + u_t$

	Return forecasts			Macro forecasts		
	R^2	t-stat.	RMSE	AC	t-stat.	RMSE
Policy factors						
Unemp. gap	0.20	0.67	1.00	0.98	53.23	0.34
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- Evidence for unspanned macro forecasts reflects persistence

Outline

Our estimated spanned and unspanned MTSMs

- Risk factors are observable
  - Yield factors P_t: first three PCs of yield curve
  - Macro factors M_t: consider two alternatives:
    - *GRO*, *INF*—just as in JPS
    - Unemp. gap, CPI inflation—more relevant for policy
- Spanned models
  - Canonical form of Joslin, Le, Singleton (2013a)
  - Denote by $SM(3,2)$
- Unspanned models
  - Canonical form of JPS (2014)
  - Denote by $USM(3,2)$
- Estimation with Maximum Likelihood
  - *iid* measurement errors, equal variance for all maturities

Simulation study of spanning implications

- Key open questions
  - How empirically relevant is theoretical spanning in MTSMs?
  - Are MTSMs really inconsistent with regression evidence?
- Investigate regression evidence in simulated vs. actual data
- Consider both spanned and unspanned models
- Experimental design: do the following for 500 replications
  - Simulate risk factors from VAR
  - Obtain model-implied yields using affine loadings
  - Add small *iid* measurement error with SD σ
  - Obtain PCs of simulated yields
  - Run spanning regressions in simulated macro-yields data

Simulation evidence for $USM(3,2)$ – 3 PCs

	Unspanned Macro Variation (R^2)		Unspanned Macro Risk		Unspanned Macro Forecasts (RMSE)	
	GRO	*INF*	*p*-value	RMSE	*GRO*	*INF*
Data	0.279	0.812	0.000	0.857	0.472	0.297
$\sigma = \hat{\sigma}_e^{MLE}$	0.235	0.680	0.123	0.920	0.460	0.347
	(0.115)	(0.152)	(0.222)	(0.057)	(0.055)	(0.054)
$\sigma = 1bp$	0.304	0.708	0.120	0.915	0.487	0.356
	(0.118)	(0.138)	(0.219)	(0.064)	(0.053)	(0.053)
$\sigma = 0$	0.329	0.709	0.120	0.922	0.495	0.358
	(0.112)	(0.140)	(0.209)	(0.057)	(0.049)	(0.050)

Data vs. means (and SDs) across 500 simulations

- Unspanned model matches regression evidence by construction

Simulation evidence for $SM(3,2)$ – 3 PCs

	Unspanned Macro Variation (R^2)		Unspanned Macro Risk		Unspanned Macro Forecasts (RMSE)	
	GRO	*INF*	*p*-value	RMSE	*GRO*	*INF*
Data	0.279	0.812	0.000	0.857	0.472	0.297
$\sigma = \hat{\sigma}_e^{MLE}$	0.206	0.678	0.087	0.910	0.457	0.352
	(0.124)	(0.145)	(0.179)	(0.061)	(0.060)	(0.057)
$\sigma = 1bp$	0.389	0.710	0.118	0.920	0.520	0.372
	(0.159)	(0.151)	(0.214)	(0.061)	(0.061)	(0.067)
$\sigma = 0$	0.447	0.713	0.144	0.924	0.549	0.372
	(0.190)	(0.150)	(0.236)	(0.059)	(0.086)	(0.073)

Data vs. means (and SDs) across 500 simulations

- Spanned model has similar implications as unspanned models if the information set contains only $\mathcal{L} = 3$ yield PCs

Simulation evidence for $SM(3,2)$ – 5 PCs

	Unspanned Macro Variation (R^2)		Unspanned Macro Risk		Unspanned Macro Forecasts (RMSE)	
	GRO	*INF*	*p*-value	RMSE	*GRO*	*INF*
Data	0.379	0.864	0.003	0.876	0.499	0.340
$\sigma = \hat{\sigma}_e^{MLE}$	0.371	0.733	0.101	0.928	0.501	0.379
	(0.114)	(0.124)	(0.191)	(0.051)	(0.051)	(0.056)
$\sigma = 1bp$	0.707	0.863	0.172	0.969	0.660	0.516
	(0.081)	(0.089)	(0.253)	(0.031)	(0.051)	(0.076)
$\sigma = 0$	1.000	1.000	1.000	1.000	1.000	1.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Data vs. means (and SDs) across 500 simulations

- Spanned model generates substantial unspanned macro variation with small measurement errors

The role of measurement errors

- How can small yield measurement errors create substantial unspanned macro information?
  - Measurement errors are tiny relative to yields
  - And get washed out when constructing level/slope/curvature
  - Why do they still introduce a substantial wedge?
- Intuition
  - In spanned models, M_t is spanned by $\mathcal{N}$ yield PCs
  - Low-order PCs $(1, \ldots, \mathcal{L})$ leave unspanned variation (see regression evidence)
  - Higher-order PCs $(\mathcal{L}+1, \ldots, \mathcal{M})$ complete spanning (by construction)
  - But higher-order PCs are small and noisy, and therefore are strongly affected by measurement errors
- Benefits of measurement errors
  - Well-known: Avoid stochastic singularity of factor models
  - New: Make affine MTSMs consistent with regression evidence on unspanned macro information
  - Note: No macro measurement errors needed

Conclusions from simulation study

- Number of yield factors matters
  - Spanned and unspanned models have same implications when using $\mathcal{L}$ PCs of yields in spanning regressions
- Measurement error breaks macro spanning
  - Conventional specification with small yield measurement errors

$\Rightarrow$ *Conventional, spanned macro-finance models are consistent with regression evidence on unspanned macro information*

Outline

Knife-edge unspanned MTSM restrictions

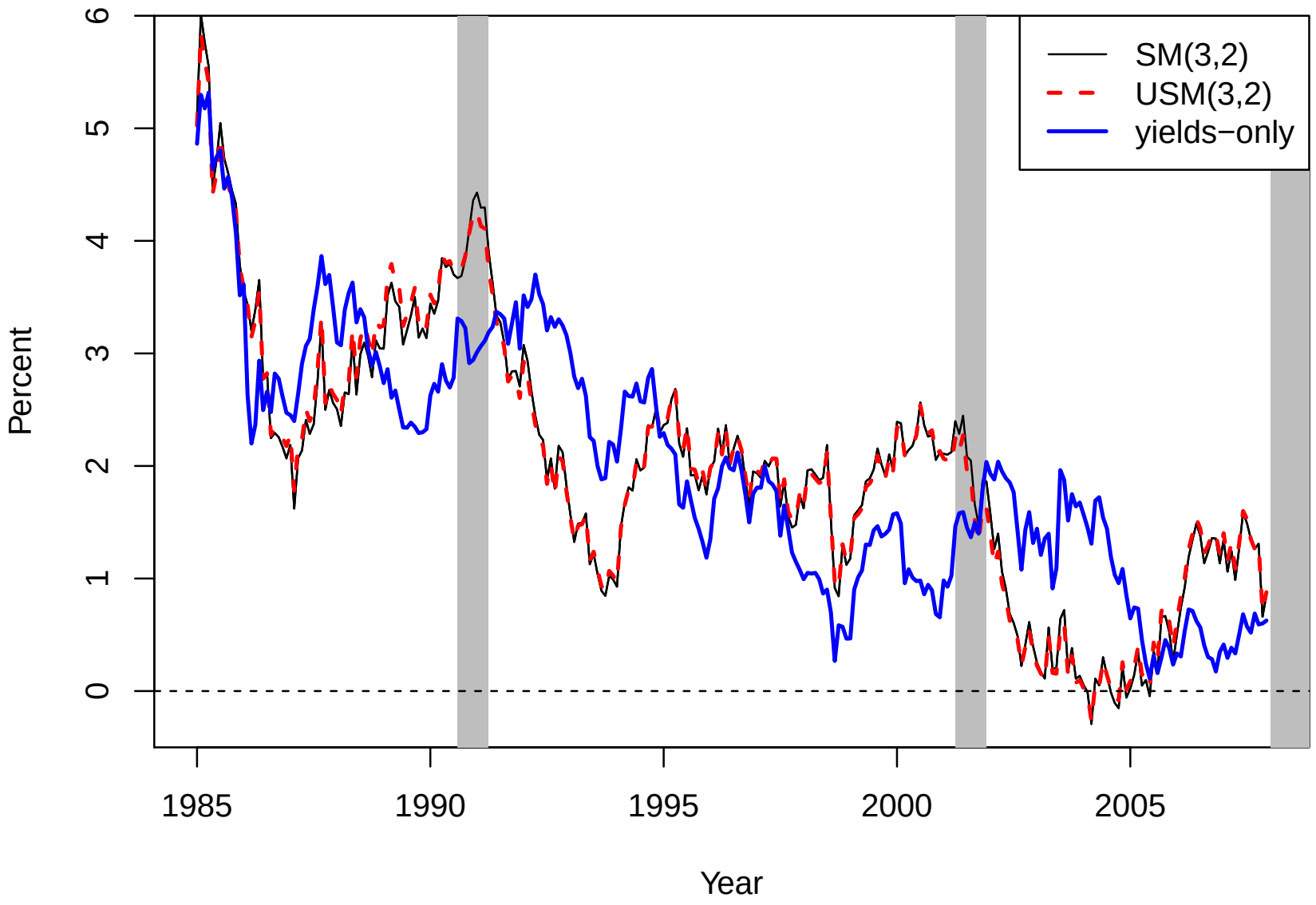
- Unspanned model is special case of affine MTSM
- Restrictions: loadings of yields on macro factors are all zero
  - Yields cannot be inverted to infer macro variables
- Comparable to unspanned stochastic volatility (Collin-Dufresne and Goldstein, 2002)
- "Knife-edge" restrictions?
  - Macro factors must affect expectations and risk premia in opposite directions and with exactly the same magnitude
  - Only in that case will effects on current yields be zero
- We formally test these restrictions in MTSMs

Likelihood-ratio tests of knife-edge restrictions

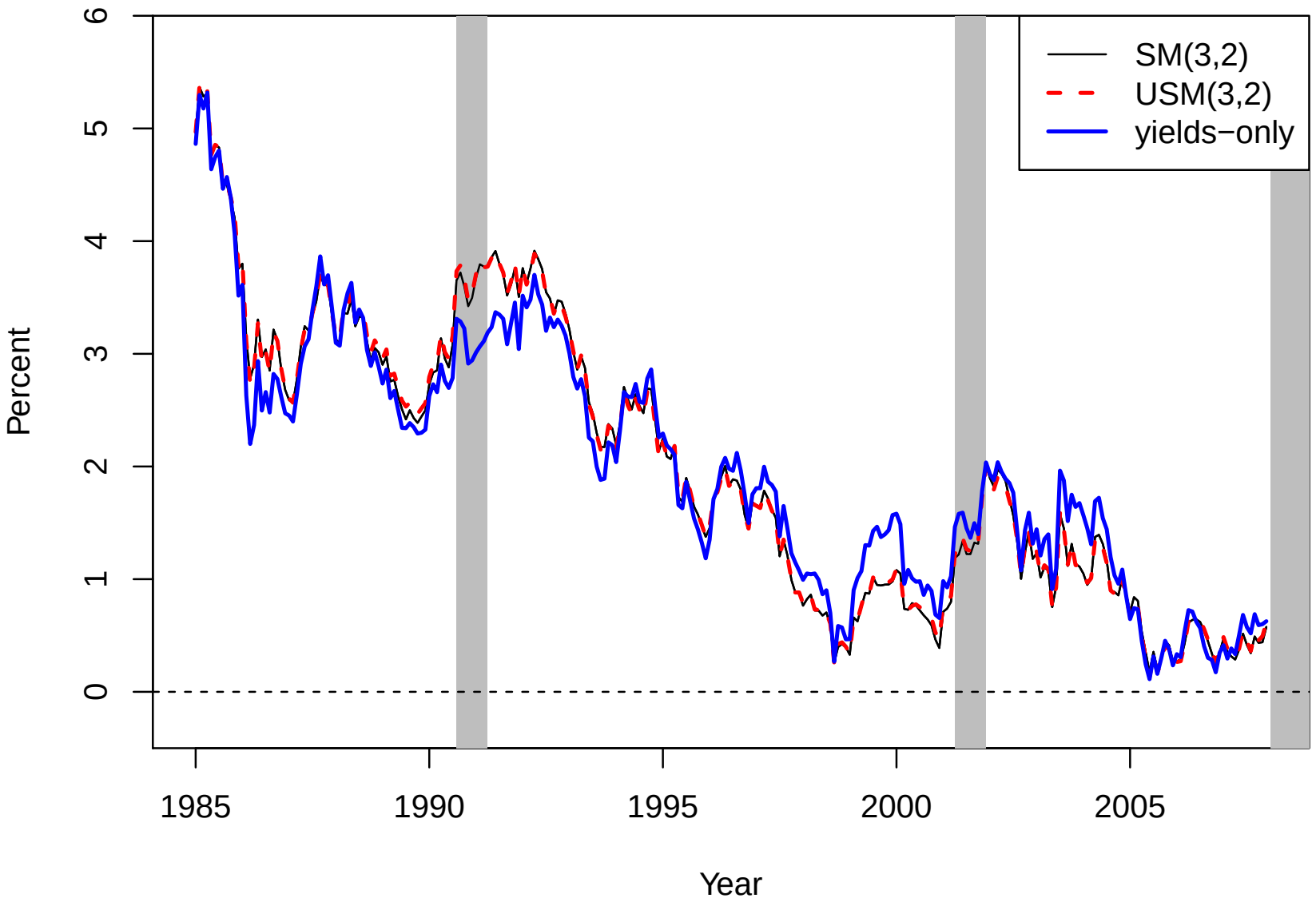
	UGAP, *CPI*	*GRO*, *INF*
Log-L $SM(3,2)$	21,300	22,737
Log-L $USM(3,2)$	21,210	22,439
$\chi^2(14)$	182	595
crit. val.	6.57	6.57

- Exclusion restrictions strongly rejected
- Even stronger rejections for models with one/two yield factors
- Why?
  - Inclusion of macro factors in yield equations improves cross-sectional fit
  - Improvements in fit are statistically significant
  - Are they also *economically* significant? $\rightarrow$ look at term premia

Term premia – models with *GRO*, *INF*



Term premia – models with *UGAP*, *CORECPI*



What does JPS' test of spanning tell us?

- JPS carry out a likelihood-ratio test of spanning
  - Restricted model $\mathcal{M}_{span}$: zero restrictions on VAR feedback matrix – exclude lagged macro variables
  - Rejected with χ^2-statistic of 1,189
- Conclusions to be drawn from this
  - Lags of *GRO* and *INF* help to predict yields/returns
  - Persistence in *GRO* and *INF* not captured by 3 PCs of yields

$\Rightarrow$ This is just the usual regression evidence, repackaged in a different test statistic

Outline

Conclusion

- Evidence on unspanned macro information
  - Policy factors are tightly linked to yield curve
  - Non-policy factors have substantial unspanned variation
  - Unspanned macro risk evidence is weak
  - Strong evidence for unspanned macro forecasts
- Macro spanning of affine MTSMs
  - Has little practical relevance
  - Easily reconciled with regression evidence
  - Conventional measurement error specification does the trick
- Knife-edge restrictions of unspanned models
  - Rejections are strongly statistically significant
  - Inclusion of macro variables slightly improves fit
  - Term premia from spanned and unspanned models are indistinguishable
  - Use of policy factors guards against implausible term premia