When Does a Central Bank’s Balance Sheet Require Fiscal Support?

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ESSIM 2015 Tarragona

Disclaimer: The views expressed are ours and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System
Yet another balance sheet simulation?

- Hall and Reis (2013), Carpenter et al. (2013), Greenlaw et al. (2013), Christensen et al. (2013) examine likely scenarios, based on historically normal behavior of interest rates and demand for central bank liabilities.

- None of these papers uses a model where inflation and interest rates are endogenously determined.

- We use a complete, though simplified, economic model to study why a central bank’s balance sheet matters at all and the consequences of a lack of fiscal support for the central bank.

- We show that a large, long-duration central bank balance sheet, together with lack of fiscal support, can impair the central bank’s ability to control inflation.
Contribution

In our model, interest rates, inflation, and seigniorage are endogenous, hence we can answer questions such as:

1 Under what conditions does the central bank need fiscal support?

- The central bank has one quasi-fiscal resource: seigniorage. Seigniorage is quantitatively important (its PDV is very large in our baseline calibration)... but is endogenous.

- CB’s ability to maintain policy commitments without support from the fiscal authority depends on whether seigniorage increases (hedge) or decreases in scenarios where the value of CB’s assets drops.
Can self-fulfilling solvency crises arise when the CB holds (a lot of) long-duration assets?

- If the CB cannot rely on fiscal support → has to rely on seigniorage (→ higher inflation) if the value of its assets were to fall below the value of reserves

- ... an equilibrium may arise where the public’s high interest rates/inflation expectations become self-fulfilling:
  
  - ↑ future interest rates → ↓ value of long-duration assets → balance sheet gap that needs to filled with future seigniorage
How could a central bank be “insolvent”?

- Can’t it always “print money”?
  - CB has of course no problem paying nominal liabilities – but can’t do that and control inflation (e.g., follow the Taylor rule)

- Can’t CB run a Ponzi scheme? That is, continue following Taylor rule and issue an ever-increasing amount of reserves?
  - No. Private sector transversality condition is violated
The model

- Simple, perfect-foresight, non-linear, continuous-time model
  - Flexible prices
    - Exogenous real interest rate $\rho$ and income $Y$
- All uncertainty is revealed at time 0
Households

- Households/private sector maximize

\[ \int_0^\infty e^{-\beta t} \log C \, dt \]

subject to

\[ C(1 + \psi(v)) + \dot{F} + \frac{\dot{V} + \dot{M} + q\dot{B}^P}{P} = \]

\[ Y - \tau + \rho F + r \frac{V}{P} + (\chi + \delta - q\delta) \frac{B^P}{P} \]

- \( F \): storage/foreign assets paying an exogenous real rate \( \rho \)
- \( V \): overnight reserves paying nominal rate \( r \). \( M \): currency \( \rightarrow \psi(v) \)
- \( B^P \): Woodford (2001)-style bond (depreciates at rates \( \delta \), coupon \( \delta + \chi \), duration \( \delta^{-1} \))
- All assets pay same real return
Households

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- All assets pay \underline{same} real return
- Population growth \( n \) and productivity growth \( \gamma \)
Transactions technology

\[ v = \frac{PC}{M}; \quad \psi(v) = \psi_0 e^{-\psi_1/v} \]

- Elasticity of money demand goes to zero as \( r \rightarrow 0 \) (Mulligan and Sala-i-Martin 2000, Alvarez and Lippi 2009).

- Real balances go to zero for \( r > \psi_0 \psi_1 \)
Fiscal policy

• Fiscal Authority budget constraint:

\[ g + (\chi + \delta - \delta q) \frac{B}{P} = \tau + \tau^C + q \frac{\dot{B}}{P} \]

• \( B = B^P + B^C \) (bonds are held either by the public or the CB)

• \( \tau^C \) remittances from CB

• Passive fiscal policy (except for discussion of “fiscal backing”):

\[ \tau = \xi_0 + \xi_1 q \frac{B}{P}, \quad \xi_1 > \rho \]
Conventional and unconventional monetary policy

- Reaction function for interest on reserves $r$:
  \[
  \dot{r} = \theta_r \left( \bar{r} + \theta_\pi \left( \frac{\dot{P}}{P} - \bar{\pi} \right) - r \right), \quad r \geq r
  \]

- Unconventional monetary policy: path for $B^C$ (Treasuries held by CB)

- The central bank budget constraint is:
  \[
  q \frac{\dot{B}^C}{P} - \frac{\dot{V} + \dot{M}}{P} = (\chi + \delta - \delta q) \frac{B^C}{P} - r \frac{V}{P} - \tau^C
  \]
**Equilibrium**

- Fisher eq. + Taylor rule yield a solution for interest rates:

\[ r_t = \int_0^\infty e^{-(\theta \pi - 1) \theta} \theta_s \theta_r (\theta \pi \rho_t + s - (\bar{r} - \theta \pi \bar{\pi})) ds + \kappa e^{(\theta \pi - 1) \theta} \]

- \( \kappa \): This source of multiplicity (Cochrane 2011, Benhabib et al. 1999) is not the only one in this model.
Equilibrium

- Fisher eq. + Taylor rule yield a solution for interest rates:

\[ r_t = \int_0^\infty e^{-(\theta_\pi-1)\theta_r} \theta_r \left( \theta_\pi \rho_{t+s} - (\bar{r} - \theta_\pi \bar{\pi}) \right) ds + \kappa e^{(\theta_\pi-1)\theta_r t} \]

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- \( v_t^2 \psi'(v_t) = r_t, \quad v_t = \frac{P_t C_t}{M_t} \), defines money demand \( \Rightarrow \) seigniorage

- CB determines the size of the balance sheet (\( B^C \)) and \( r \), but its composition between interest-bearing liabilities (\( V \)) and currency (\( M \)) is determined by the private sector
A present value take on CB solvency and remittances

\[
q \frac{B^C_0}{P_0} - \frac{V_0}{P_0} + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt
\]

Mkt value of assts - reserves

PDV seigniorage

\[
= \int_0^\infty \tau^C_t e^{-\int_0^t \rho_s ds} dt
\]

PDV remittances

- if LHS > 0 CB is “solvency”: it can continue to follow the Taylor rule without requiring fiscal support \(\rightarrow\) CB budget constraint irrelevant for equilibrium (like gvt’s with passive fiscal policy)

  - See also Hall and Reis (2013), Bassetto and Messer (2013)...

- As long at \(\tau^C \geq 0\), rule for remittances does not matter for equilibrium
“Backing” vs. “support”

- What we’ve discussed so far is “support” — transfers of resources from the fiscal authority to the central bank.

- The fiscal theory of the price level (FTPL) implies that fiscal “backing”, which is something different (active fiscal policy), can produce a uniquely determined price level.
Parameters

normalization, foreign assets
\[ Y - G = 1 \quad F_0 = 0 \]

discount rate, reversion to st.st., population and productivity growth
\[ \beta = 0.01 \quad \gamma = 0.01 \quad n = 0.0075 \]

monetary policy
\[ \theta_{\pi} = 2 \quad \theta_r = 1 \quad \bar{\pi} = 0.02 \]

Assets (par value), reserves, and currency as of Jan 1 2014
\[ \frac{B^C}{P} = 0.327 \quad \frac{V}{P} = 0.224 \]
\[ \frac{M}{P} = 0.107 \]

bonds: duration and coupon
\[ \delta^{-1} = 6.8 \]

money demand
\[ \psi_0 = 0.63 \quad \psi_1 = 103.14 \]
Money demand and the Laffer curve

Short term interest rates and M/PC

Laffer Curve

- Model
- pre 1959 data
- post 1959 data
Simulations

• Baseline: real rate path ($\rho$) chosen so that $r$ path roughly matches Carpenter et al./current expected path for the FFR

• Scenarios (time 0 “surprise”, perfect foresight afterwards)
  
  1. Exogenous “shocks”
     – Question: Can the CB stick to the Taylor rule (control inflation) without violating solvency?

     • “Higher rates” Carpenter et al. scenario
     • (Exogenous) “inflation scare”
     • Explosive paths (hyperinflations)

  2. Self-fulfilling solvency crises
Baseline and “Higher Rates” scenario

Nominal

Real

Baseline
Higher rates

Baseline
Higher rates
## Balance sheet implications

\[ \frac{qB}{P} - \frac{V}{P}, \quad \text{PDV seigniorage} (1)+(2) \quad q \quad \frac{\bar{B}}{B} \]

| Baseline scenario | 0.126 | 1.139 | 1.264 | 1.08 |

At steady state PDV of seigniorage

\[
\int_{0}^{\infty} \left( \frac{\dot{M}_t}{M_t} + n \right) \frac{M_t}{P_t} e^{-\int_{0}^{t}(\rho_s - n)ds} \, dt
\]

given by \( \frac{(\bar{\pi} + \gamma + n) \bar{m}}{(\beta - n)} \). **Numerator** in our calibration (.0027) is lower than historical average (.0047), but denominator \( \beta - n \) is very small!
### Balance sheet implications

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Other simulation results ※
## Balance sheet implications

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Central bank’s balance sheet

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Self-fulfilling CB solvency crises

• Can multiple equilibria arise in absence of fiscal support? If $\tau_t^C \geq 0$ for all $t$, then the CB must rely on seigniorage if $qB - V < 0$

• Assume that the central bank chooses at time 0 the future inflation target $\bar{\pi}$ (and it can commit to this target from time 0 onward) with the objective of minimizing deviations of inflation from 2%:

$$\min_{\bar{\pi}} \int_0^\infty e^{-\beta t} (\pi_t - 0.02)^2 \, dt$$

subject to the private sector equilibrium conditions, and the constraint $\tau_t^C \geq 0$ all $t$.

• $\bar{\pi} = 0.02$ is one equilibrium (in which the $\tau_t^C \geq 0$ constraint is not binding)
2% Equilibrium

\[ \pi^e = 2\% \]

CB preferences

PDVS(\(\pi\))

\[ V - q(\pi^e)B \]

- Solvency constraint not binding:

\[ \frac{q_0(2\%)B_0^C - V_0}{P_0(2\%)} + PDVS_0(2\%) > 0 \]
Self-fulfilling CB solvency crises

- There may be other equilibria: solutions of

\[
\frac{q_0(\tilde{\pi}) B_0^C}{P_0(\tilde{\pi})} - V_0 + PDVS_0(\tilde{\pi}) = 0
\]

CB cannot deviate as long as \( \hat{\pi} < \tilde{\pi} \) implies a violation of solvency constraint

- Multiplicity here is related to, but very different from, that arising with nominal government debt (Calvo 1988, Corsetti and Dedola 2013, Aguiar et al. 2013)
Equilibrium

\[ \pi^e = \tilde{\pi} \]

\[ V - q(\pi^e)B - q_0(\tilde{\pi})B^C_0 - V_0 + PDVS_0(\tilde{\pi}) = 0 \]

(Solvency constraint binding)

where \( \tilde{\pi} \) solves

- CB preferences

PDVS(\( \pi \))

Del Negro, Sims Central bank's balance sheet
For what level of $B^C$ are self-fulfilling crises possible?

Threshold Balance Sheet Limit

$qB - V$

PDV of Seigniorage

Note: Omitting risk premia!

Alt. calibration: $\beta = .02$ case
For what level of $B^C$ are self-fulfilling crises possible?

- $\kappa < 0$ in $r_t = \text{stable solution} + \kappa e^{\theta_r(\theta_\pi - 1)t}$

Interest Rate Path

Threshold Balance Sheet Limit
fraction of current $B$
Conclusions

- CB solvency would become an issue only under rather extreme scenarios, for current balance sheet size

- These conclusions hinge on the properties of currency demand and seigniorage
  - ... on which there is considerable uncertainty

- Implications: Fiscal support for the CB allows it to pursue its mandate without being concerned about its balance sheet
  - ... and such support seems unlikely to be needed in equilibrium on the basis of our model
Seigniorage and M/PC

Data

Model

M/PCE -- left axis

M/PC -- left axis

Seigniorage -- right axis

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Central bank’s balance sheet

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Inflation Scare

- Fisher equation (nominal premium ↑): \( r_t = \rho_t + \frac{\dot{P}_t}{P_t} + x_t \)

  with \( x_t = x_0 e^{-\chi x t} \), \( x_0 = 0.04 \) and \( \chi x = 0.1 \)

Nominal Short Rate

\[
\begin{align*}
\text{Baseline} & : \quad \theta_\pi = 2 \\
\theta_\pi = 3 & \\
\theta_\pi = 1.05 & \\
\end{align*}
\]
Explosive paths – different $\theta_\pi$’s

- Explosive paths: $r_t = \text{stable solution} + \kappa e^{\theta r (\theta_\pi - 1)t}$
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For what level of $B^C$ are self-fulfilling crises possible?

Threshold Balance
Sheet Limit
fraction of end-of-2013 $B$

$\beta = .02$ Case
$qB - V$

PDV of Seigniorage

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Rule for remittances

- As long at $\tau^C \geq 0$, rule for remittances does not matter for equilibrium
- Two principles: i) remittances cannot be negative, ii) whenever positive, remittances are such that the central bank capital measured at historical costs remains constant over time:

$$\tilde{K} = (\tilde{q}B^C - V - M) e^{nt} = \text{constant}.$$

where $\tilde{q}$ evolves according to

$$\dot{\tilde{q}} = (q - \tilde{q}) \max \left\{ 0, \frac{\dot{B}^C}{B^C} + \delta \right\}$$

yielding

$$\tau^C = \max \left\{ 0, (\chi - \delta(\tilde{q} - 1)) \frac{B^C}{P} \right\}$$

$$+ \left( \tilde{q} - (q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + (\delta + n) \right) \frac{B^C}{P} \right) - r \frac{V}{P} \right\} \mathcal{I}_{\{\tilde{K} \geq \tilde{K}_0\}}$$

Deferred Asset
Paths for remittances