Costly decisions and sequential bargaining

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INTRODUCTION:

Frictions and errors in decision-making

Frictions in DSGE models

Quantitatively realistic DSGE models rely on a large number of **frictions**,

- including **sticky prices**...
- ... and **sticky wages**
- ... and **investment adjustment costs**
- ... and **portfolio adjustment costs**
- ... and **habits in consumption**
- ... and **matching frictions** in the labor market
- ... and **interest rate smoothing** in monetary policy
- ... and various **credit market frictions**,
- ... to name only the most standard ones.

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- But why so many different functional forms?
- Are frictions associated with real resource costs?
- Are frictions policy invariant?

Error-prone decisions in game theory

Game theorists frequently assume **error-prone decision-making**, often modeled as a **mixed strategy** (probability distribution over actions)

- **Trembling-hand equilibrium** (Selten 1975):
  - Small probability of playing some action other than the optimum
- **Quantal response equilibrium** (McKelvey-Palfrey 1995, 1998):
  - Probability of playing any action increases with value of that action
  - **Logit equilibrium**: Special case where the probability is a logit
- **Control cost equilibrium** (*e.g.* Stahl 1990, van Damme 1991 ch. 4)
  - Cost of decision-making increases with the precision of the (mixed) strategy

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- **Control cost equilibrium** (*e.g.* Stahl 1990, van Damme 1991 ch. 4)
  - Cost of decision-making increases with the precision of the (mixed) strategy
  - **Special case:** if precision is measured by **relative entropy**, then probabilities are **logit** (*e.g.* Stahl 1990, Mattson-Weibull 2002, Matejka-McKay 2015)

Decision costs as frictions

Idea: in dynamic contexts requiring **intermittent updating** of controls, **decision costs** might be a good model of **frictions**.

Possible applications:

- **Updating a nominal price** or wage
- **Updating an offer** in a market or an auction or a bargaining game
- **Looking for a match** with a new partner

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- Can distinguish two margins of costly, error-prone decision:
  - **When** to update the control variable
  - **What new value** to set

“Logit price dynamics” (Costain/Nakov 2014)

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Application:

- **Updating nominal prices**

- We distinguished two margins of costly, error-prone decision:
  - Errors in **when to adjust price** cause monetary nonneutrality
  - Errors in **which new price to set** explain retail micro facts

"Assignment of workers to jobs..." (Cheremukhin/Restrepo/Tutino 2012)

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Application:

- **Looking for a match** with a new partner

- "Rational inattention" approach to sorting
  - Results intermediate between random and directed search

This paper: costly decisions in a bargaining game

- Propose and solve a **control cost model** in which decisions are not only costly but also literally **time-consuming**.
  - **Tradeoff: faster decision is less precise**
- Describe decision-making with one or both types of errors:
  - **Which option** to choose
  - **When** to make the decision (*i.e.* how quickly)

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- Nest time-consuming decisions **in extensive form game**
  - **Application: sequential bargaining**
  - Players offer shares of pie
  - Players decide whether or not to accept (or update) offers
- Simulate and characterize bargaining equilibria

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- Simulate and characterize bargaining equilibria
- Ultimate goal: apply this decision framework to **all margins of labor market relationship**, or many margins in DSGE model

OUTLINE

- Introduction
- Model: a one-time, costly decision
- Model: choosing when to make a decision
- Bargaining equilibria

Summary of results

- Model with **decision errors only**
  - One equation determines optimal precision
  - Solution types: **postponement**, **interior solution**, **immediate randomization**
- Model with **decisions errors and timing errors**
  - Additional parameter needed, to measure **speed** as well as **accuracy** of choice
  - Two equations determine optimal precision and optimal decision time
  - **Solution is always interior**, with smooth dependence on parameters

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  - **Solution is always interior**, with smooth dependence on parameters
- Results of bargaining model:
  - Equilibrium similar to Rubinstein (1982), but **nonzero time to agreement**
  - Unlike assumption of Perry/Reny (1993), rejecting by saying "no" is not equivalent to rejecting by making an alternative offer
  - Conjecture supported by preliminary numerical results: **unlike game of Perry/Reny (1993), equilibrium unique if offer space is a continuum**

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Other related literature: costly decisions

- **"Control cost"** models (and/or logit equilibrium) are influential in
  - Experimental games (Anderson-Goeree-Holt, 2002; Magnani-Gorry-Oprea, 2013)
  - Repeated games with reinforcement learning (Baron-Durieu-Haller-Solal 2002)
  - Engineering literature on machine learning (Todorov, 2009)
- **"Sticky information"** models with **fixed cost to fully update information**, as in Mankiw-Reis (2002), Reis (2006), Alvarez-Lippi-Paciello (2011)
- **"Rational inattention"** models with **costly flow of information**, as in Sims (1998, 2003), Mackowiak-Wiederholt (2009), Matejka (2010)
  - Also Woodford (2009), Cheremukhin-Restrepo-Tutino (2012)

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  - Also Woodford (2009), Cheremukhin-Restrepo-Tutino (2012)
- The difference?
  - **"Rational inattention"** assumes a cost function for obtaining information...
  - **"Control costs"** assume a cost function for choice, given information...

Other related literature: bargaining theory under full information

- **Rubinstein (1982)**: exogenous sequence of offers and responses, no other frictions
- Binmore-Rubinstein-Wolinsky (1983), Wolinsky (1987), Hall-Milgrom (2008) clarified how bargains work in search and matching context
- **Perry-Reny (1993)** endogenize sequence of play, assuming offers and responses require fixed quantity of time
- Merlo-Wilson (1995, 1998) study bargaining games with stochastic shocks

CONTROL COSTS IN REAL TIME:

Model 1: solving a single problem

Decision environment

- Decision-maker who must choose between options $i \in \{1, 2, \dots n\}$
  - Could allow for continuum of options instead...
- Must **spend time thinking** in order to evaluate their values V_i, but DM is **impatient**
- **Tradeoff: Choose quickly, or choose accurately?**
  - Rubinstein (2013) presents evidence that slower decisions are more accurate

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- Application in this paper: DM is **player bargaining over a pie**.
  - If no offer is outstanding, can propose a share $s \in \{0, s_2, \dots, s_{n-1}, 1\}$.
  - Can respond to offers received with answer $\in$ $\{accept, reject\}$.

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- How to model this tradeoff?

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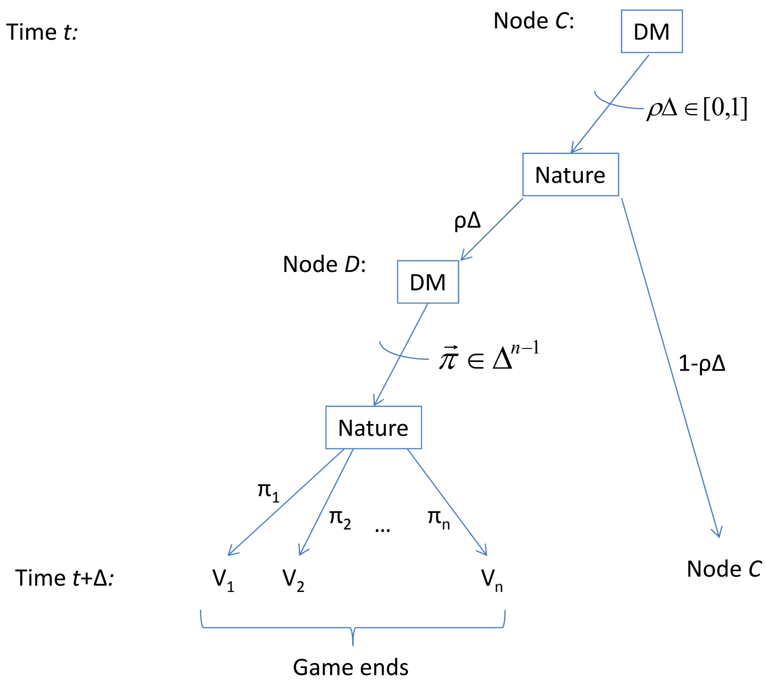
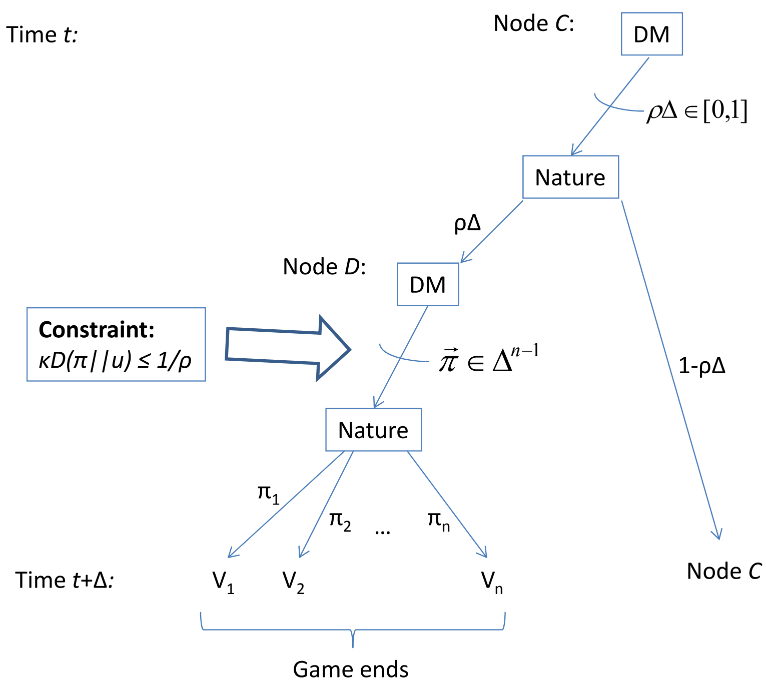
- Assume DM **chooses arrival rate** ρ of decision
- **Precision of choice** is constrained by **expected decision time**

One costly decision in real time



One costly decision in real time: information constraint



Entropy constraint

- Assume **precision is constrained by time spent choosing:**

$$\kappa \mathcal{D}(\vec{\pi}||\vec{u}) \ \leq \ \frac{1}{\rho}$$

  - $\vec{\pi} \equiv (\pi_1, \pi_2, \ldots, \pi_n)$ is probability distribution over actions
  - $\vec{u} \equiv (n^{-1}, n^{-1}, \ldots, n^{-1})$ is uniform distribution
  - $\mathcal{D}(\vec{\pi}||\vec{u})$ is called **relative entropy**, or **Kullback-Leibler divergence**:

$$\mathcal{D}(\vec{\pi}||\vec{u}) \equiv \sum_{j=1}^{n} \pi_j \ln\left(\frac{\pi_j}{n^{-1}}\right) = \ln(n) + \sum_{j=1}^{n} \pi_j \ln \pi_j$$

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    - ★ $\mathcal{D}$ measures precision of the distribution π
    - ★ It normalizes cost of uniform decision to zero.
    - ★ Implies marginal cost of perfect decision is infinite.

Decision problem

- Maximize expected value minus expected costs:

$$W_t = \max_{\rho,\pi_j} (1-\delta\Delta)\left(\rho\Delta\sum_j \pi_j V_j + (1-\rho\Delta)W_{t+\Delta}\right) \quad (1)$$

$$\text{s.t.} \quad \rho\kappa\left(\sum_{j=1}^{n}\pi_j \ln \pi_j + \ln n\right) \leq 1 \qquad \sum_j \pi_j = 1.$$

  - W_t is value of unfinished decision problem
  - V_j is value of choosing alternative j
  - δ is discount rate
  - Δ is time step

First-order conditions for (1)

- FOC π_j:

$$\rho\Delta(1-\delta\Delta)V_j - \lambda_\rho\rho\kappa(1+\ln\pi_j) = \lambda_\pi \quad (2)$$

  - Key point: $\ln\pi_j$ depends linearly on V_j

- FOC ρ:

$$(1-\delta\Delta)\Delta\left(\sum_{i=1}^{n}\pi_i V_i - W_{t+\Delta}\right) = \lambda_\rho\kappa\mathcal{D}(\vec{\pi}||\vec{u}) \quad (3)$$

- Precision constraint:

$$\rho\kappa\mathcal{D}(\vec{\pi}||\vec{u}) = 1 \quad (4)$$

- Probability constraint:

$$\sum_{i=1}^{n}\pi_i = 1 \quad (5)$$

Logit probabilities

- FOC π_i can be rearranged as

$$V_j - \beta^{-1}(1 + \ln \pi_j) \;=\; \frac{\lambda_\pi}{\rho(1-\delta\Delta)}, \tag{6}$$

$$\beta \;\equiv\; \frac{(1-\delta\Delta)\Delta}{\lambda_\rho \kappa} \tag{7}$$

  - Here β is a summary statistic for optimal precision.

- FOC implies probabilities π_j are proportional to $\exp(\beta V_j)$, hence:

$$\pi_j = \frac{\exp(\beta V_j)}{\sum_{i=1}^n \exp(\beta V_i)} \tag{8}$$

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- Define expected payoff of choice:

$$E^\pi V \;\equiv\; \sum_i \pi_i V_i \;=\; \frac{E^u V \exp(\beta V)}{E^u \exp(\beta V)}$$

(Intermediate calculations)

- Logit probabilities:

$$\pi_j = \frac{\exp(\beta V_j)}{\sum_{i=1}^{n} \exp(\beta V_i)} \equiv \frac{\exp(\beta V_j)}{nE^u \exp(\beta V)}$$

- Hence:

$$\pi_i \ln \pi_i = \beta \pi_i V_i - \pi_i \ln(nE^u \exp(\beta V))$$

- Then can calculate **cost function for precision**:

$$\mathcal{D}(\vec{\pi}||\vec{u}) \equiv \sum \pi_i \ln \pi_i + \ln n = \beta E^\pi V - \ln(E^u \exp(\beta V)) \quad (9)$$

- Recall FOC ρ:

$$(1 - \delta\Delta)\Delta \left(E^\pi V - W_{t+\Delta}\right) = \lambda_\rho \kappa \mathcal{D}(\vec{\pi}||\vec{u}) \quad (10)$$

- ... can also be written as

$$\beta \left(E^\pi V - W_{t+\Delta}\right) = \mathcal{D}(\vec{\pi}||\vec{u}) \quad (11)$$

Solving for β (interior solution)

- Can calculate the **cost of precision**:

$$\mathcal{D}(\vec{\pi}||\vec{u}) = \beta E^\pi V - \ln(E^u \exp(\beta V)) \quad (12)$$

- But **FOC for arrival rate** can be written as

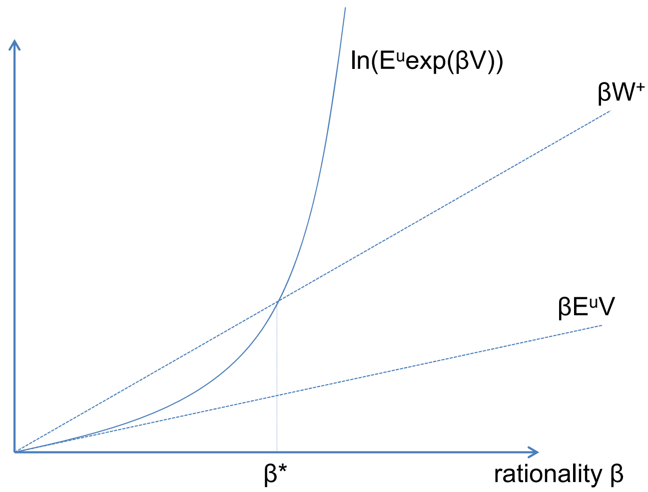
$$\beta \left(E^\pi V - W_{t+\Delta}\right) = \mathcal{D}(\vec{\pi}||\vec{u}) \quad (13)$$

- **By combining (12) and (13), we can solve for** β:

$$\beta W_{t+\Delta} = \ln(E^u \exp(\beta V)) \quad (14)$$

  - Here $\ln(E^u \exp(\beta V))$ is the *cumulant generating function* of a uniform distribution over values V

Solving for β (interior solution)



- $g(\beta; V) \equiv ln(E^u \exp(\beta V))$ is increasing and convex
- $g(0; V) = 0$
- $\frac{\partial g}{\partial \beta}(0; V) = E^u V$
- $\lim_{\beta \to \infty} \frac{\partial g}{\partial \beta}(\beta; V) = \max_i V_i$

Solving for β

Proposition 2. *Decision problem (1) has a unique solution. It takes three possible forms.*

(a.) Postponement: *Suppose* $\max_i V_i \leq W_{t+\Delta}$. *Then it is optimal to postpone solving, setting the arrival rate* $\rho = 0$.

(b.) Interior solution: *Suppose* $E^u V < W_{t+\Delta} < \max_i V_i$, *and* Δ *is sufficiently small. Then (1) is solved by the unique positive* β *that satisfies* $\beta W_{t+\Delta} = \ln(E^u \exp(\beta V))$.

(c.) Immediate solution: *Suppose* $E^u V \geq W_{t+\Delta}$, *or that the arrival probability from (b) exceeds one:* $\rho\Delta \geq 1$. *Then it is optimal to solve (1) in exactly one time step, resulting in a precision level* β_Δ *that satisfies* $\kappa K(\beta_\Delta) = \Delta$ *(or if* $\kappa \ln n \leq \Delta$, *then* $\beta_\Delta = \infty$*).*

- Once β is known, can calculate π_i, $\mathcal{D}$, ρ...

CONTROL COSTS IN REAL TIME:

Model 2: control costs on two margins

General model: errors in the timing decision too

- Our first model allowed for errors in the chosen action, but the **timing of the decision was precisely optimal**
- Next: allow for **errors in timing** too
- Also allow for alternative use of time

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- Time constraint: $h + \mu + \tau \leq 1$, where
  - h is fraction of time spent working
  - τ is fraction of time dedicated to decision
  - τ/ρ is expected total time dedicated to decision (roughly)
  - μ is fraction of time spent **"monitoring" whether decision should be made now**

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  - μ is fraction of time spent **"monitoring" whether decision should be made now**
- Precision of choice is constrained by time spent choosing
- **Precision of timing is constrained by time spent monitoring**

Costs of choosing when to adjust

- We assumed the time cost of the choice probabilities $\vec{\pi}$ was:

$$\kappa_\pi \mathcal{D}(\vec{\pi}||\vec{u}) \;\equiv\; \kappa_\pi \sum_{j=1}^{n} \pi_j \ln\left(\frac{\pi_j}{n^{-1}}\right)$$

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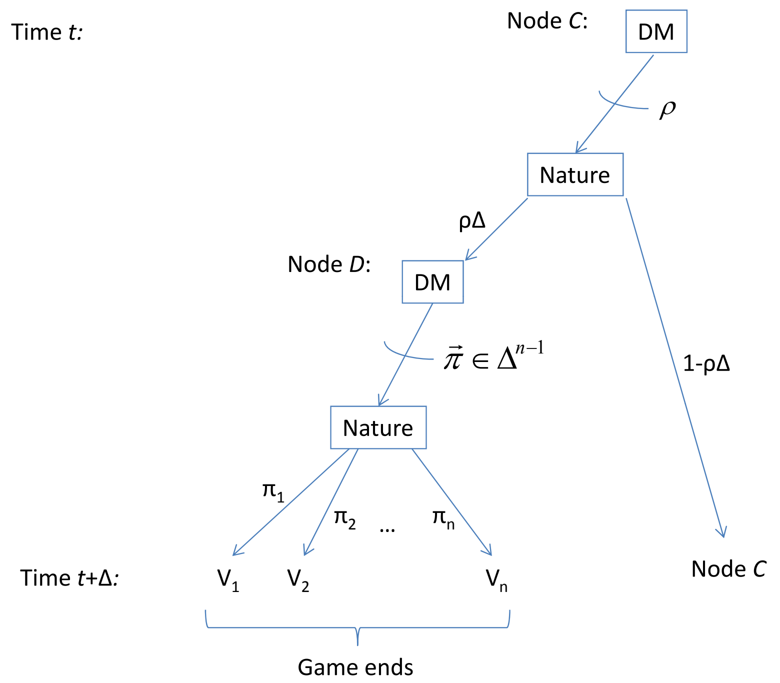
- Next, assume the time cost of the **adjustment hazard** ρ is:

$$\kappa_\rho \mathcal{D}\left(\left(\begin{array}{c}\rho\Delta \\ 1-\rho\Delta\end{array}\right)\middle|\middle|\left(\begin{array}{c}\bar{\rho}\Delta \\ 1-\bar{\rho}\Delta\end{array}\right)\right) \equiv \kappa_\rho\left(\rho\Delta \ln\frac{\rho}{\bar{\rho}} + (1-\rho\Delta)\ln\frac{1-\rho\Delta}{1-\bar{\rho}\Delta}\right)$$

  - This is the **relative entropy** of endogenous adjustment hazard ρ, compared with an exogenous **uniform adjustment hazard** $\bar{\rho}$.
  - It means costs are greater if adjustment probability varies over time.
  - Marginal cost of setting $\rho = 0$ or $\rho = 1$ is infinite.
  - Normalizes cost of **some Calvo model** to zero.

Costly choice with costly timing



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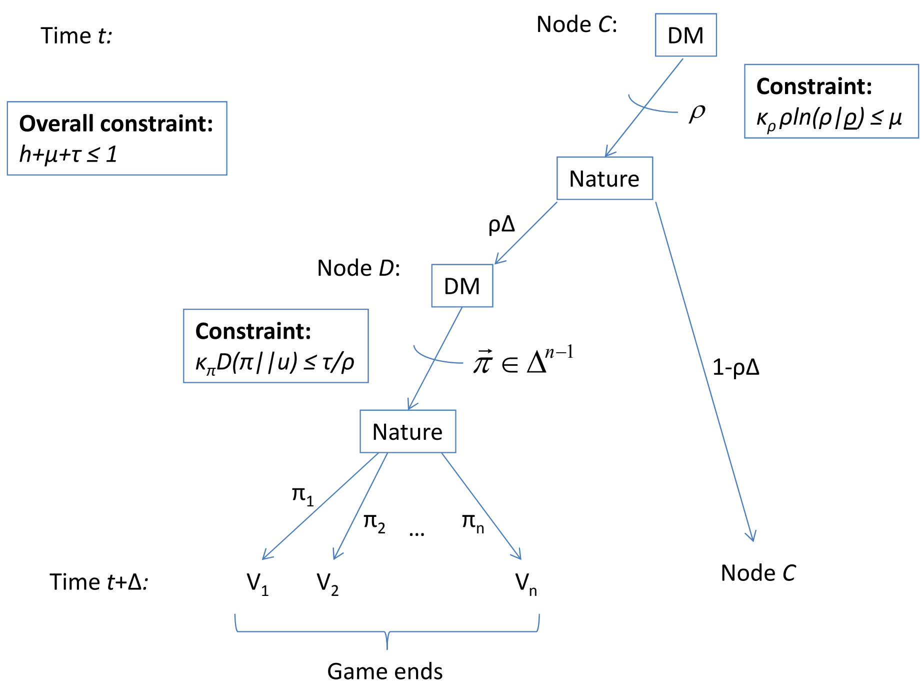


Decision problem

- Maximize expected value minus expected costs:

$$W_t = \max_{\rho,h,\tau,\mu,\pi_j} f(h)\Delta+(1-\delta\Delta)\left(\rho\Delta\sum_j \pi_j V_j + (1-\rho\Delta)W_{t+\Delta}\right) \quad (15)$$

$$\text{s.t.} \quad \rho\kappa_\pi\left(\sum_{j=1}^{n}\pi_j \ln\pi_j + \ln n\right) \leq \tau\,, \qquad \sum_j \pi_j = 1,$$

$$\kappa_\rho\left(\rho\Delta\ln\left(\frac{\rho}{\bar\rho}\right) + (1-\rho\Delta)\ln\left(\frac{1-\rho\Delta}{1-\bar\rho\Delta}\right)\right) \leq \mu\Delta\,, \qquad h+\tau+\mu\leq 1.$$

  - h is fraction of time spent **working**
  - τ is fraction of time spent **deciding** between alternatives V_j
  - μ is fraction of time spent **"monitoring"** whether decision is necessary

First-order conditions for (15)

- Equalize marginal value of time across activities:

$$\lambda_h = f'(h)\Delta = \lambda_\tau = \lambda_\mu\Delta\,,$$

$$\text{that is:} \quad \beta_\pi = \frac{1-\delta\Delta}{\kappa_\pi f'(h)}\,, \qquad \beta_\rho = \frac{1-\delta\Delta}{\kappa_\rho f'(h)}$$

- Probabilities of chosen actions are logit:

$$\pi_i = \frac{\exp(\beta_\pi V_i)}{\sum_{j=1}^{n}\exp(\beta_\pi V_j)}$$

- And probability of deciding is a weighted binary logit:

$$\rho = \frac{\bar\rho\exp(\beta_\rho\tilde V)}{\bar\rho\exp(\beta_\rho\tilde V)+(1-\bar\rho)\exp(\beta_\rho W_{t+\Delta})}, \qquad \tilde V \equiv E^\pi V - \frac{\mathcal{D}_\pi}{\beta_\pi}.$$

  - Given π_i and ρ, can calculate $\tau = \rho\kappa_\pi\mathcal{D}_\pi$ and $\mu = \kappa_\rho\mathcal{D}_\rho$
  - Can then check whether $h+\tau+\mu = 1$

Existence and uniqueness of interior solution

Proposition 8. *Suppose* $f'(h) > 0$ *and* $f''(h) \leq 0$ *for* $h \in [0,1]$, *and* $\min_i V_i < \max_i V_i$. *Then:*

(a.) *Problem (15) is solved by the pair* $h^* \in (0,1)$, $\beta_\pi^* > 0$ *at the unique crossing of the following curves:*

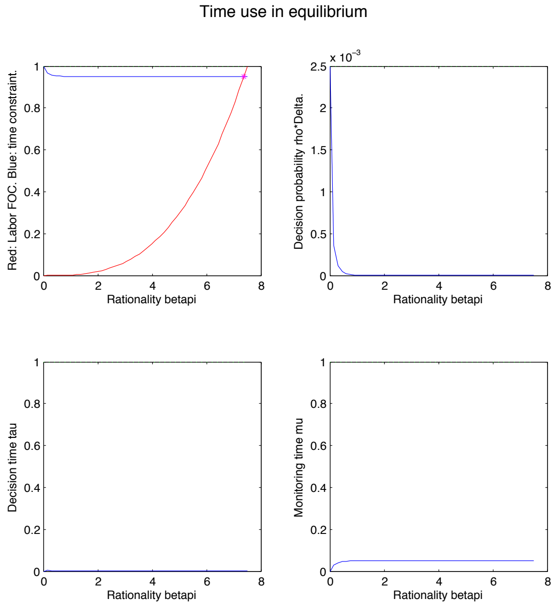
$$1 - h = \rho\kappa_\pi \mathcal{D}_\pi + \kappa_\rho \frac{\mathcal{D}_\rho}{\Delta} \quad (16)$$

$$\beta_\pi = \frac{1 - \delta\Delta}{\kappa_\pi f'(h)} \quad (17)$$

(b.) *The solution* $h^* \in (0,1)$, $\beta_\pi^* > 0$ *varies smoothly with parameters.*

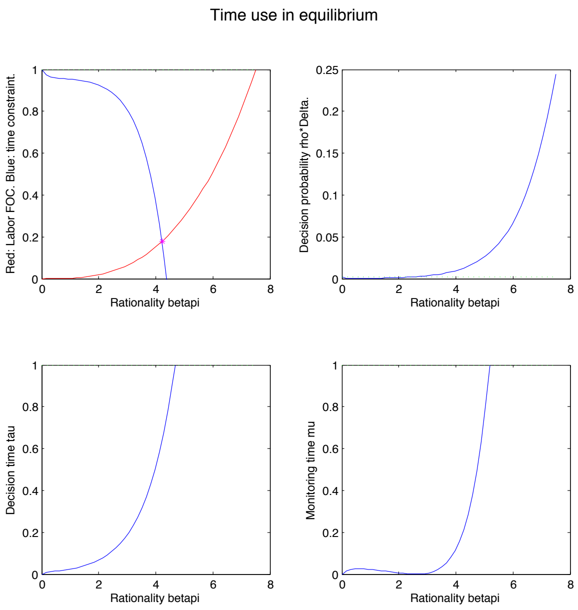
- These curves can be graphed in (β_π, h) space
- Time constraint (16) slopes downward
- The precision FOC (17) slopes up
- Also show that Problem (15) defines a contraction (Prop. 7)

Example: a low-value problem



- $\Delta = 0.005$, $\delta = 0.01$, $\kappa_\pi = \kappa_\rho = 0.1$, $\bar{\rho} = 0.5$
- $A = 2$, $\alpha = 2/3$
- $W_{t+\Delta} = 10$, $\vec{V} = (-20, -18, -16, \ldots, 2, 4, 6)$

Example: a high-value problem



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- $A = 2$, $\alpha = 2/3$
- $W_{t+\Delta} = 10$, $\vec{V} = (-15, -13, -11, \ldots, 7, 9, 11)$

BARGAINING GAMES

Bargaining games with costly decisions

Next, nest the decision model inside a bargaining game.

Motivations:

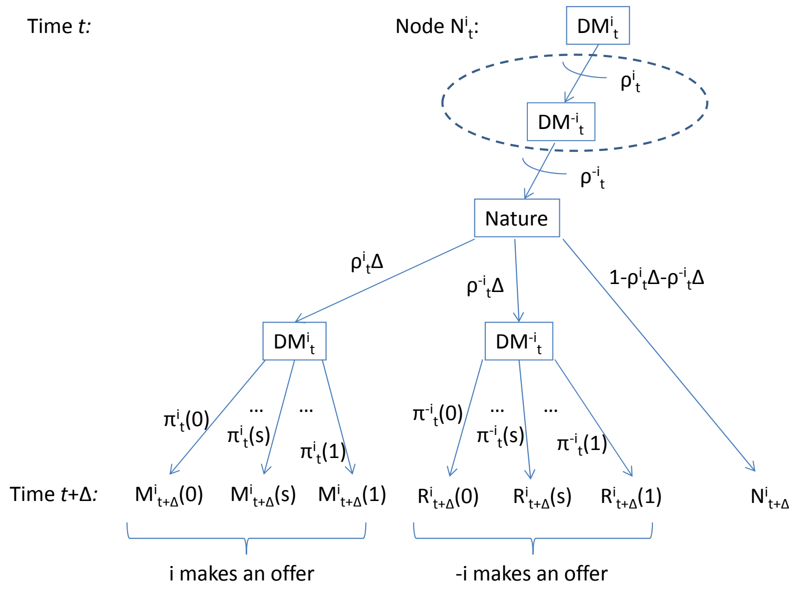
- Tradeoff of **speed vs. accuracy** of choice is especially interesting when two or more decision makers are **interacting**
- **Endogenize negotiation duration** in a bargaining model
  - Rubinstein (1982): Pareto optimal equilibrium reached instantly
  - Perry/Reny (1993): Fixed time cost whenever player makes an offer or a reply
- Useful building block for macro-labor model

Some bargaining protocols

1. **Alternating offers protocol**
   - When no offer is outstanding, either player may make an offer
   - The player who has made an offer cannot withdraw it
   - When an offer is outstanding, the other player may accept it, **or reject by making an alternative offer**
2. **Reject-first protocol**
   - When no offer is outstanding, either player may make an offer
   - The player who has made an offer cannot withdraw it
   - When an offer is outstanding, the other player may accept or reject it (rejection returns the game to a situation with no offer outstanding)
3. **Updateable offers protocol**
   - When no offer is outstanding, either player may make an offer
   - The player who has made an offer may withdraw it (which may or may not require proposing an alternative)
   - When an offer is outstanding, the other player may accept or reject it (which may or may not require proposing an alternative)

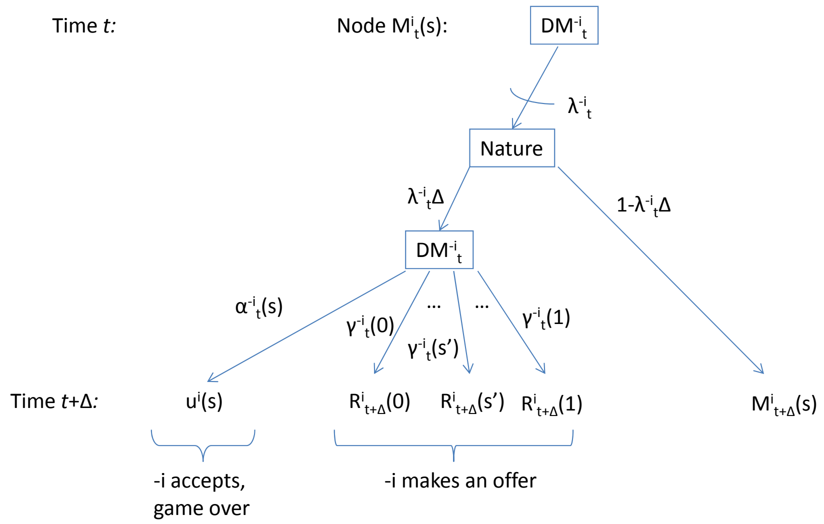
- I will show simulations of (1) and (2).

Game node: no offers outstanding



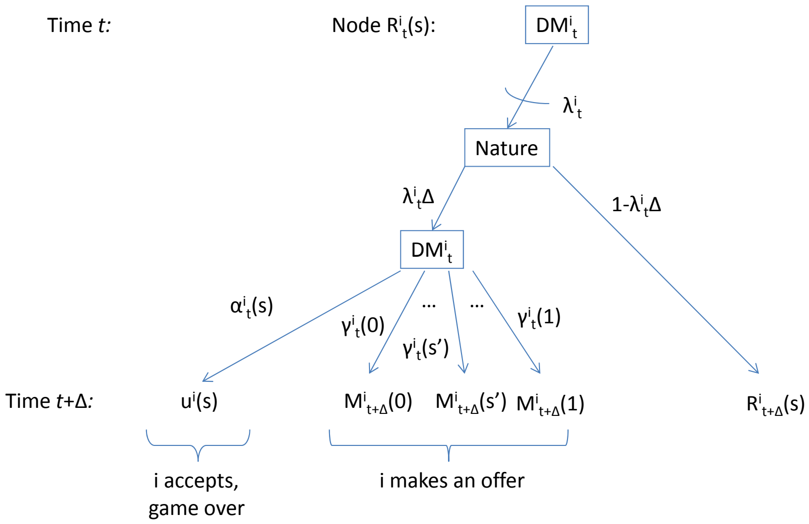
- Players 1 and 2 choose offer arrival rates ρ_t^1 and ρ_t^2 simultaneously
- If player i's offer arrives, its distribution is $\pi_t^i(s)$, where $s \in [0,1]$

My offer outstanding (alternating protocol)



- Offer s of player i is outstanding; $-i$ chooses response rate λ_t^{-i}.
- If player $-i$'s decision arrives, she accepts with probability $\alpha_t^{-i}(s)$
- Or she proposes s' with probability $\gamma_t^{-i}(s'|s)$

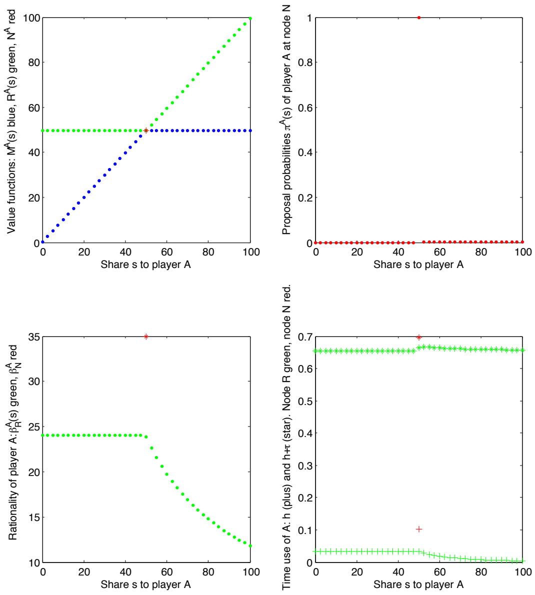
Other's offer outstanding (alternating protocol)



- Offer s of player $-i$ is outstanding; i chooses response rate λ^i_t.
- If player i's decision arrives, he accepts with probability $\alpha^i_t(s)$
- Or he proposes s' with probability $\gamma^i_t(s'|s)$

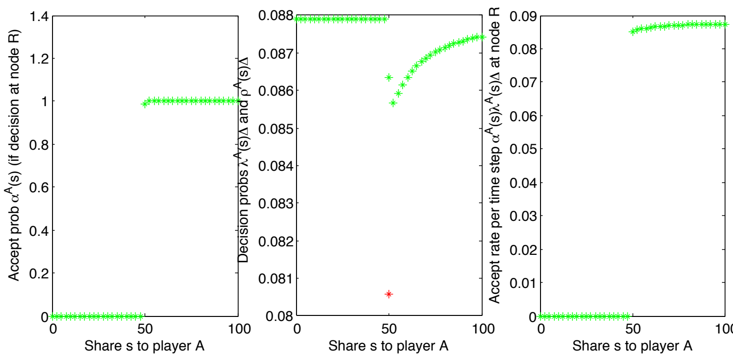
Simulating alternating offers equilibrium

Alternating protocol (equilibrium)



- Game assumes errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $A = 1$, $\alpha = 2/3$, cake=100.
- Offers distribution similar to Rubinstein (1982).

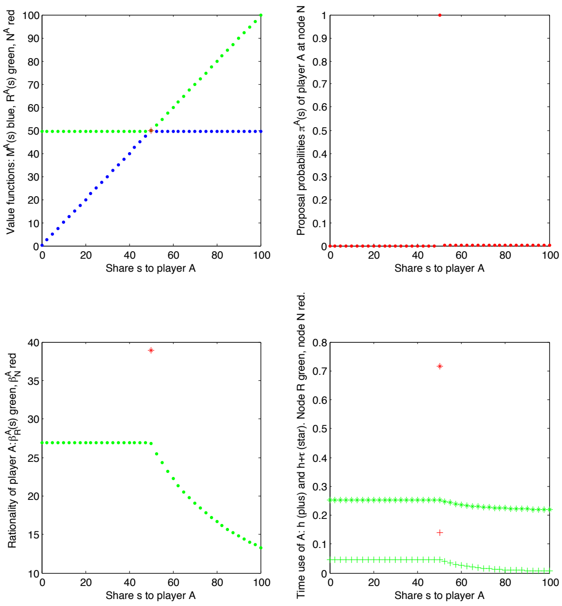
Alternating protocol (acceptance decision)



- Game assumes errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $A = 1$, $\alpha = 2/3$, cake=100.
- Main difference from fully rational model is that **delay occurs** before agreement is reached

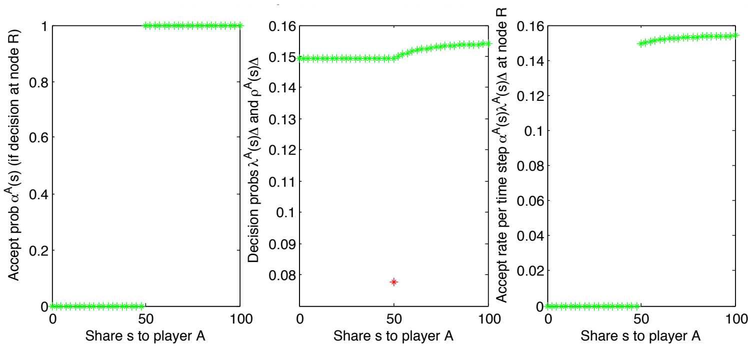
Simulating rejection-first protocol instead

Reject-first protocol (equilibrium)



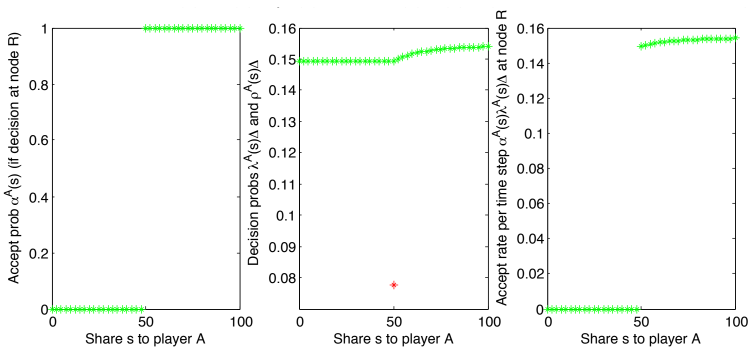
- Game assumes errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $A = 1$, $\alpha = 2/3$, cake=100.
- Offers distribution similar to alternating protocol, and to Rubinstein

Reject-first protocol (acceptance decision)



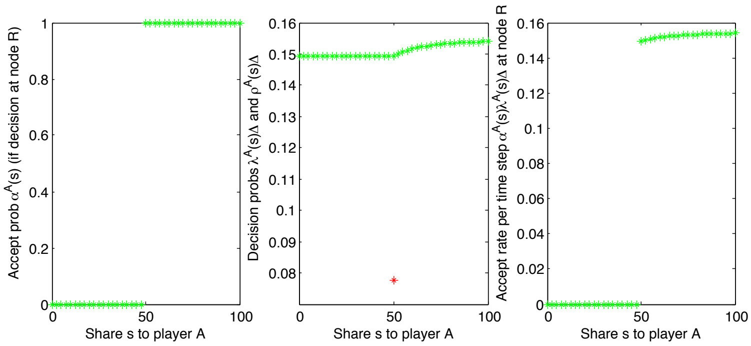
- Notice that agreement is reached **more quickly** when there is an option to **simply reject**, without making an alternative offer

Reject-first protocol (acceptance decision)



- Notice that agreement is reached **more quickly** when there is an option to **simply reject**, without making an alternative offer
- **Entropy constraint works differently in a strategic context**, rather than individual decision context!!

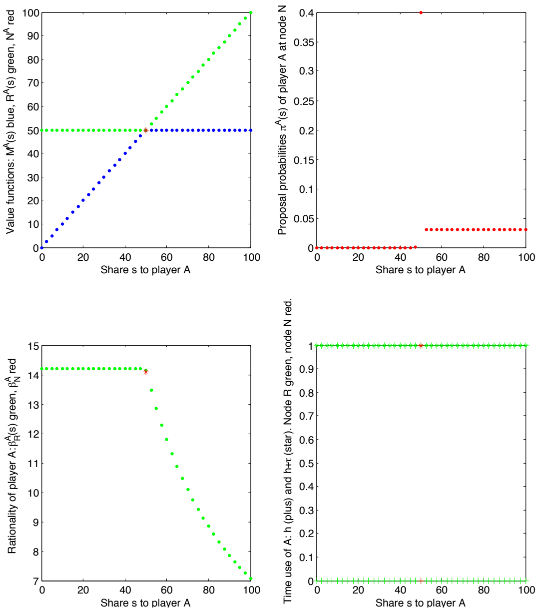
Reject-first protocol (acceptance decision)



- Notice that agreement is reached **more quickly** when there is an option to **simply reject**, without making an alternative offer
- Since this protocol is more efficient, **THIS SPECIFICATION WILL BE TAKEN AS THE BENCHMARK.**

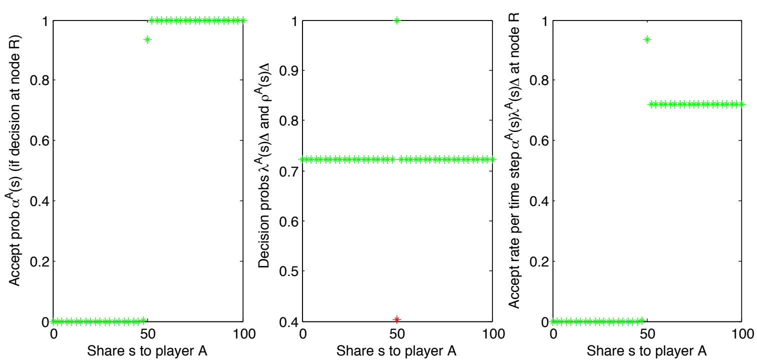
Simulating rejection-first protocol without timing errors

What if there are no errors in timing?



- Rejection-first game with errors in decisions, **but not in timing**
- Parameters: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, cake=100.
- Results very similar to benchmark game with errors in timing.

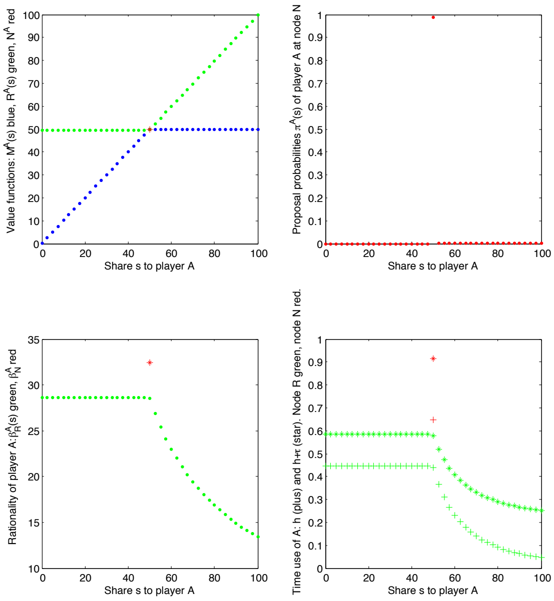
What if there are no errors in timing?



- Results very similar to benchmark game with errors in timing.

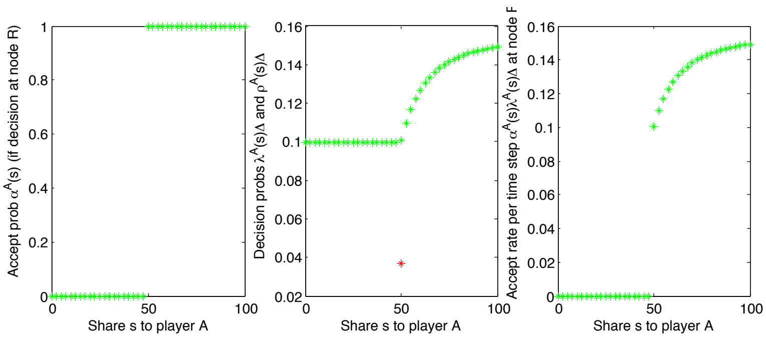
Comparative statics: more valuable time

Comparative statics: $A = 2$



- Rejection-first game with errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $\mathbf{A} = \mathbf{2}$, $\alpha = 2/3$, cake=100.
- Results are again similar to benchmark game... but errors slightly larger.

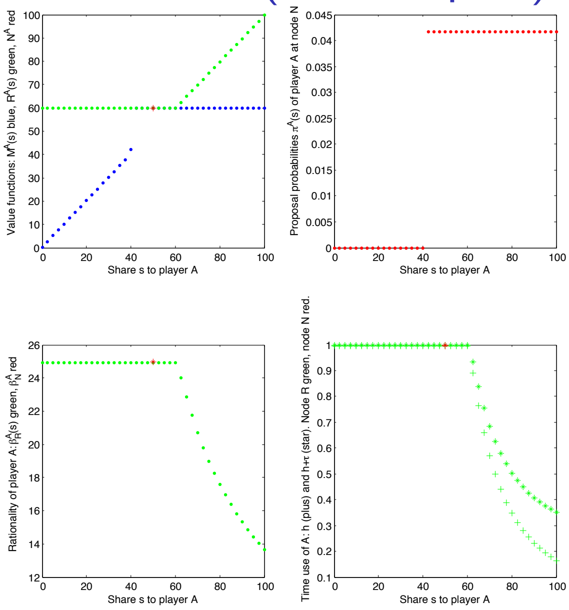
Comparative statics: $A = 2$



- Rejection-first game with errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $\mathbf{A} = \mathbf{2}$, $\alpha = 2/3$, cake=100.
- Main difference from baseline is that **decision arrives more slowly**
- Similar to comparative statics with higher κ or lower $\bar{\rho}$

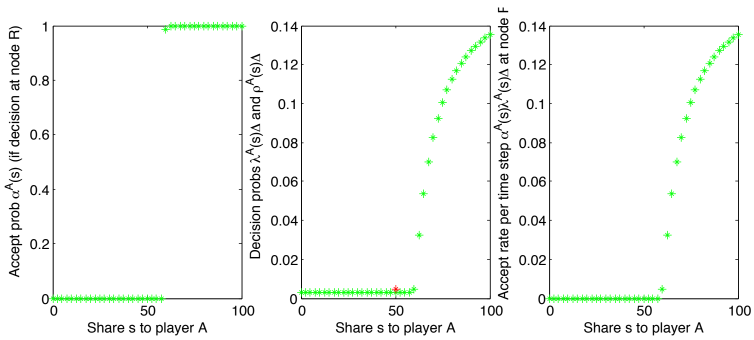
Comparative statics: very valuable time
"sharing a rotten pie"

Comparative statics: $A = 3$ ("rotten pie")



- Rejection-first game with errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $\mathbf{A} = \mathbf{3}$, $\alpha = 2/3$, cake=100.
- Note $cake/2 < A/\delta$: pie is not worth sharing.

Comparative statics: $A = 3$ ("rotten pie")



- Rejection-first game with errors in decisions and in timing
- Params: $\Delta = 0.01$, $\delta = 0.05$, $\kappa = 0.02$, $\bar{\rho} = 0.5$, $\mathbf{A} = \mathbf{3}$, $\alpha = 2/3$, cake=100.
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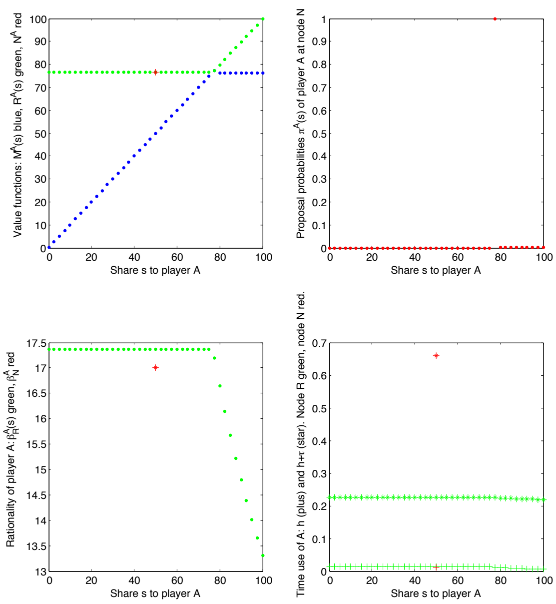
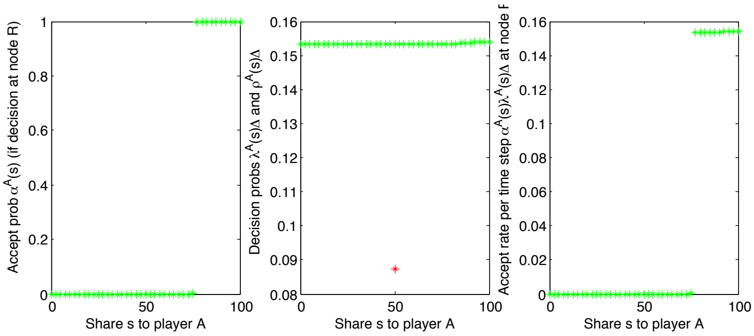
Multiplicity of equilibrium

Multiplicity: work backwards from asymmetric guess



- Specification and parameterization same as benchmark.
- **Asymmetric starting guess:** $N^A = 99$, $N^B = 1$...
- **Converges to** $N^A = 76.7$, $N^B = 22.4$.

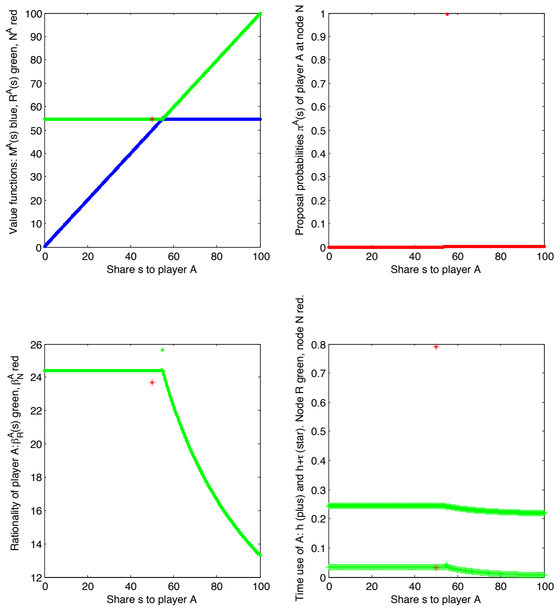
Multiplicity: work backwards from asymmetric guess



- Specification and parameterization same as benchmark.
- **Asymmetric starting guess:** $N^A = 99$**,** $N^B = 1$**...**
- **Converges to** $N^A = 76.7$**,** $N^B = 22.4$.
- Note: Rubinstein (1982) obtains unique equilibrium.
- But Perry/Renny (1993), with fixed decision times, find multiplicity.
- **Is the multiplicity in this example due to nonzero decision time, or just the discrete action space?**

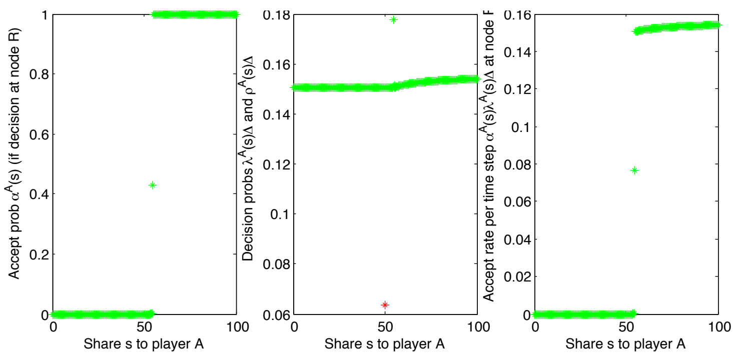
Multiplicity of equilibrium:
What if we start from asymmetric guess but use finer grid of possible offers?

Asymmetric guess and finer grid



- Specification same as benchmark, but step size 0.25 instead of 2.5
- **Asymmetric starting guess:** $N^A = 99$, $N^B = 1$...
- **Converges to** $N^A = 54.8$, $N^B = 44.6$.

Asymmetric guess and finer grid



- Specification same as benchmark, but step size 0.25 instead of 2.5
- **Asymmetric starting guess:** $N^A = 99$, $N^B = 1$...
- **Converges to** $N^A = 54.8$, $N^B = 44.6$.

- **Conjecture: multiplicity appears to result from discrete support only... In contrast to Perry/Reny, it appears unrelated to time-consuming decision.**

CONCLUSIONS

Results: decision model

- Defined control cost setup with **time-consuming** decision costs
- Model with **decision errors only**
  - One equation determines optimal precision
  - Solution types: **postponement**, **interior solution**, **immediate randomization**
- Model with **decisions errors and timing errors**
  - Additional parameter needed, to measure **speed** as well as **accuracy** of choice
  - Two equations determine optimal precision and optimal decision time
  - **Solution is always interior**, with smooth dependence on parameters

Results: bargaining game

- Results of bargaining model:
  - Equilibrium similar to Rubinstein (1982), but **nonzero time to agreement**
  - Unlike assumption of Perry/Reny (1993), rejecting by saying "no" is not equivalent to rejecting by making an alternative offer
  - Conjecture supported by preliminary numerical results: **unlike game of Perry/Reny (1993), equilibrium unique if offer space is a continuum**

EXTENSIONS

Extensions: this paper

- Be more careful about simultaneous offers

- Program decision with support on an interval
- Compute bargaining games with $s \in [0, 1]$ to **check whether equilibrium shares are unique**

- Then explore effects of asymmetric parameters on equilibrium shares
- Do asymmetries in equilibrium arrival rates affect equilibrium shares in ways consistent with predictions of Rubinstein (1982)?

Extensions: future work

- Model all margins of labor market using the same control cost setup:
  - Matching function like Cheremukhin et al
  - Wage bargaining like we did here
  - Continual "monitoring" of option to restart bargaining and/or separate
- Application: when productivity falls permanently, how is adjustment spread between wage changes, unemployment changes, and negotiation time?
- Application: unemployment volatility puzzle
  - Combine Shimer's "mismatch" and endogenous wage rigidity

- Monetary policy paper:
  - Model state-dependent prices and state-dependent wages simultaneously, as in Erceg, Henderson, Levin

THANKS!