Global Sunspots and Asset Prices in a Monetary Economy

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Roger E. A. Farmer

Financial Crises

- This paper presents a calibrated version of the Cass-Shell sunspots paper
- I argue that incomplete participation is a big deal in real world financial markets

Assumptions

- Exchange economy
  - No fundamental uncertainty
  - Complete markets
- Heterogeneous agents
  - Two types that differ in their discount rates

Assumptions

- Nominal assets
  - existence of multiple equilibria
- Incomplete participation
  - allows for sunspots

Main Idea

- Fundamental equilibrium is a highly persistent difference equation
- Incomplete participation due to demographic structure
- Natural market incompleteness

Main Idea

perfect foresight equilibria are solutions to a difference equation

$$m' = F(m,b)$$

m is the equilibrium discount factor

$$b' = G(m,b)$$

b is the real value of government debt

Main Idea

There is one initial condition

$$\delta_0 + \delta_1 b + \delta_2 p_k = a_{1,0}$$

where $a_{1,0}$ are net claims by initial type 1 people

Because debt is nominal, this does not pin down an initial value of p_k

Perfect Foresight Equilibria

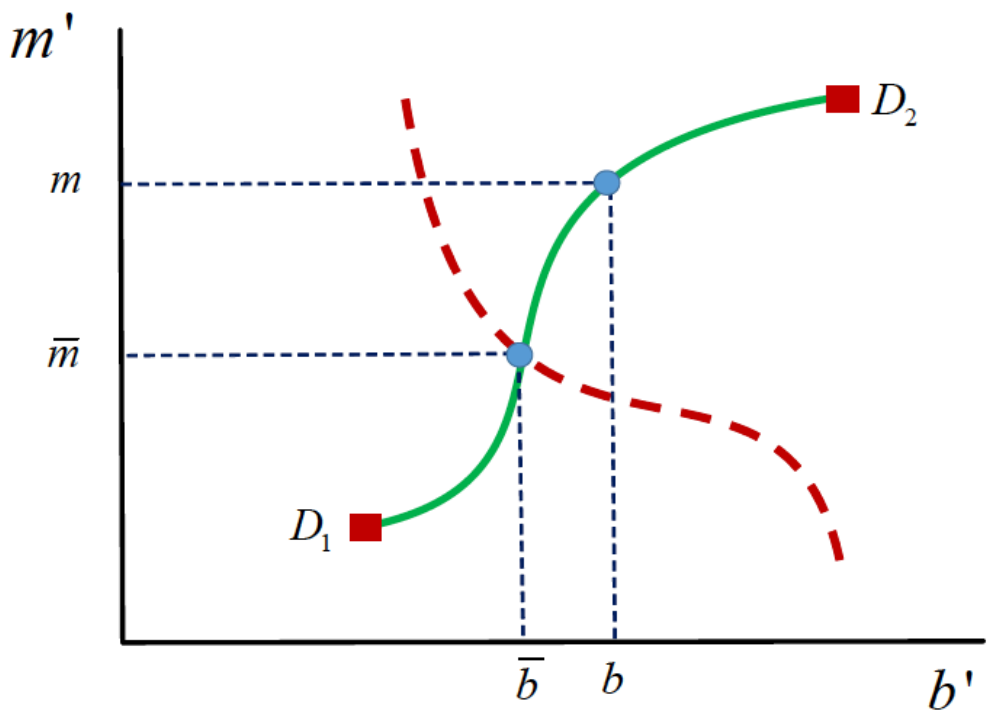


Figure 1: The set of pefect foresight equilibria

Three traded assets

- Government debt

Costs $Q^N D'$ dollars

Pays D' dollars

- Arrow securities

Cost $Q(S')$ apples

Pays 1 apple

- Trees

Cost p_k apples

Pays $\pi\left[p_k(S')+1\right]$

Government budget constraint

$$\frac{D'}{R^N} = D - \tau p$$

In real terms

$$m'b' = b - \tau$$

$$m' = \frac{p'}{pR^N}$$

m' is the pricing kernel

No Ricardian equivalence

$W = p_k - T + b$ Aggregate wealth

$T = p_k \tau$ Tax obligation of current generation

$\tilde{T}$ Tax obligation of future generations

$b = T + \tilde{T}$ Government budget balance

BUT $b \neq T$

Households

$$\mathrm{V}_i\left[W_i\right]=\max_{\{a_i(S')\}}\left\{\log \mathrm{C}_i+\pi\beta_i EV\left[W_i'\left(S'\right)\right]\right\}$$

Value function

$$\sum_{S'}\pi m\left(S'\right)W_i'\left(S'\right)+C\leq a_i\left(S\right)$$

Budget constraint

$$W_i=a_i\left(S\right)$$

Definition of wealth

$$W_{i,0}=p_k\left(S\right)$$

Initial wealth

Solution

$$AC_1 = a_1(S)$$ Type 1 people

$$BC_2 = a_2(S)$$ Type 2 people

$$A = \frac{1}{1-\beta_1\pi} \qquad B = \frac{1}{1-\beta_2\pi}$$

$A > B$ Type 1 more patient

Policy

$$R^N = 1.05$$

Passive monetary policy

$$\frac{D'}{R^N} = D - \tau p$$

$$m'b' = \mathrm{b} - \tau$$

Active fiscal policy

Fiscal theory of the price level DOES NOT hold

Marginal rates of substitution

C_i Aggregate consumption of all type *i* people alive today

$C_i^O(S')$ Aggregate consumption of all type *i* people who are still alive tomorrow

Marginal rates of substitution

$$C_i = \theta_{0,i} + \theta_{1,i}\left[p_k \left(1-\tau\right) + b \right]$$

$$C_i^O\left(S'\right) = \eta_{0,i} + \eta_{1,i} p'_k\left(S'\right)\left(1-\tau\right) + \eta_{2,i} b'\left(S'\right)$$

These equations come from solving individual optimization problems

these are different

Marginal rates of substitution

Using the expressions for consumption

$$m_1\left(b, p_k, b', p'_k\right) = m_2\left(b, p_k, b', p'_k\right)$$

we can equate the marginal rates of substitution state by state… and solve for p'_k

$$p'_k = \psi\left(b, p_k, b'\right)$$

Marginal rates of substitution

We can substitute this into the MRS of either person to obtain an expression for the pricing kernel

$$\phi\left(b, p_k, b'\right) = m_1\left[b, p_k, \mathrm{b}', \psi\left(b, p_k, b'\right)\right]$$

Equilibria

Equilibria are solutions to the equations

$$p'_k = \psi(b, p_k, b')$$

$$b = b'\phi(b, p_k, b') + \tau$$

$$\delta_0 + \delta_1 b + \delta_2 p_k = a_{1,0}$$

There is no initial condition

Change of variables

Change of variables:

$$\left\{ p_k{}', b' \right\} \rightarrow \left\{ b', m' = \phi\left(p_k, b, b' \right) \right\}$$

Computing solutions

Equilibria are solutions to the equations

$$m' - F(m,b) = 0$$

$$b' - G(m,b) = 0$$

Properties of solutions

Unique feasible steady state with positive *b* and non-negative consumption of both types

$$\bar{m} - F\left(\bar{m}, \bar{b}\right) = 0$$

$$\bar{b} - G\left(\bar{m}, \bar{b}\right) = 0$$

The steady state is a saddle

Approximate solution

Search for functions $f(\cdot)$ and $g(\cdot)$ that solve the following functional equation

$$f(m) - F(m, g(m)) = 0$$

$$g[f(m)] - G(m, g(m)) = 0$$

Equilibria

A solution to this difference equation for any initial condition in [D_1, D_2] is an equilibrium

$$m' - f(m) = 0$$

$$b = g(m)$$

The initial price level is indeterminate

Approximate solution

- Method, Chebyshev polynomials of degree 5 with colocation.
- Solution is exact at the colocation points
- Can show that C_1 is increasing (and C_2 is decreasing) in *m*
- Pick the domain of $m[D_1,D_2]$ such that $C_1(D_1)=0$ and $C_2(D_2)=0$

Calibration

Table 1

Parameter Description	Parameter Name	Parameter Value
Discount factor of type 1	β_1	0.98
Discount factor of type 2	β_2	0.90
Survival probability	π	0.98
Fraction of type 1 in the population	μ_1	0.5
Gross nominal interest rate	R^N	1.05
Primary deficit	τ	0.02

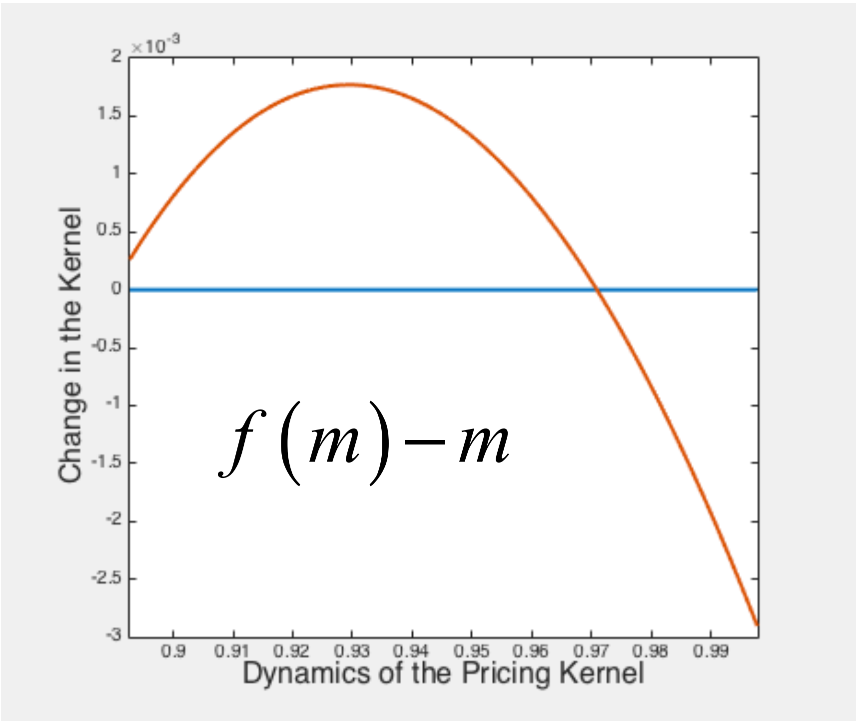
This calibration leads to the steady state values reported in Table 2.

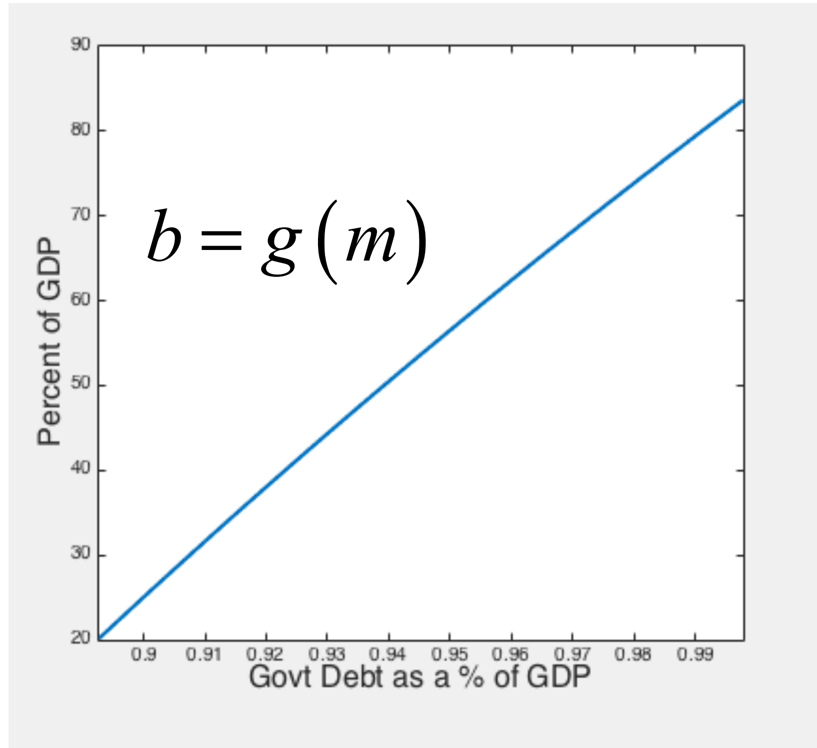
Steady state values

Table 2

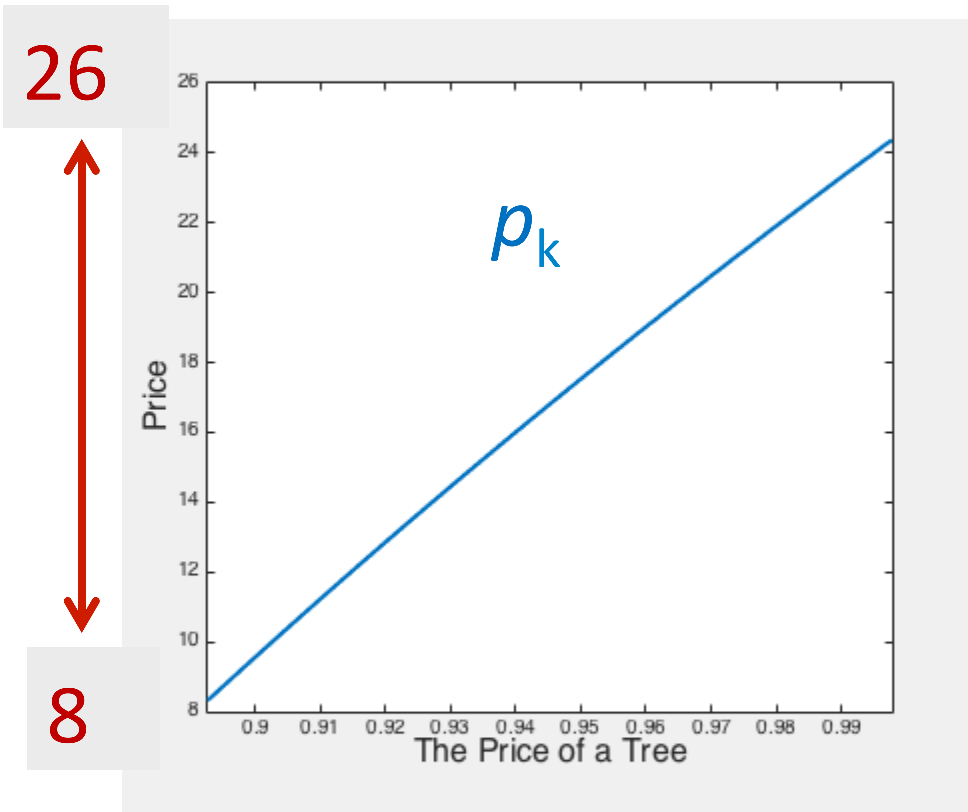
Variable Name	Parameter Name	Parameter Value
Equilibrium discount factor	$\bar{m}$	0.97
Equilibrium government debt	$\bar{b}$	0.69
Equilibrium asset price	$\bar{p}_k$	20.6
Return to a tree	R^{p_k}	1.03
Return to debt	R^b	1.03

Approximate solution





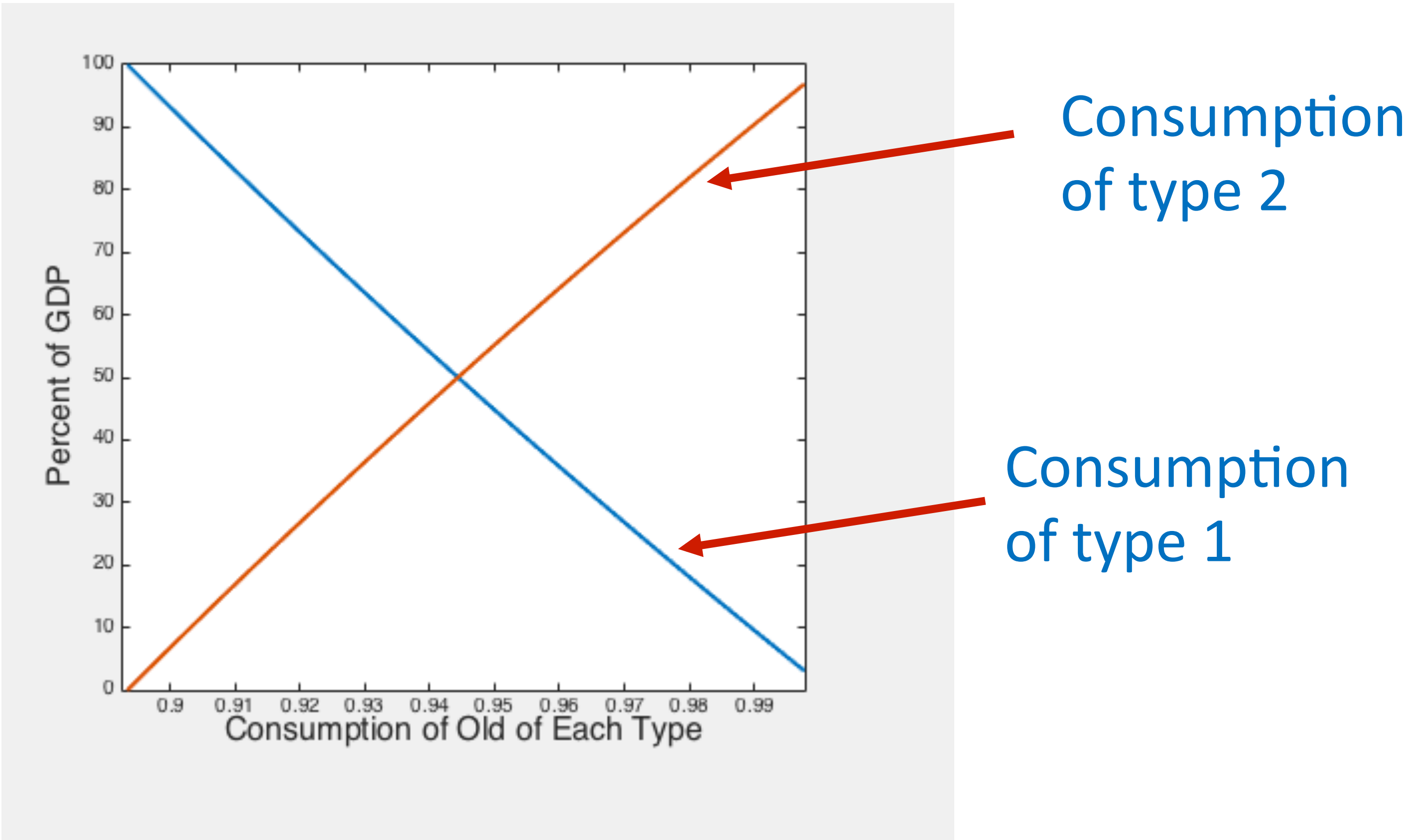
Approximate solution



p_k determines the initial wealth of a newborn

This is how p_k changes with *m*

Approximate solution



Global sunspot equilibria

$$E[m'] - f(m) = 0$$

$$b = g(m)$$

Randomizations across perfect foresight equilibria are also equilibria

Global sunspot equilibria

$$m' : B(\alpha, \beta)$$

I model *m’* as a linear function, *T*, of a Beta distributed random variable

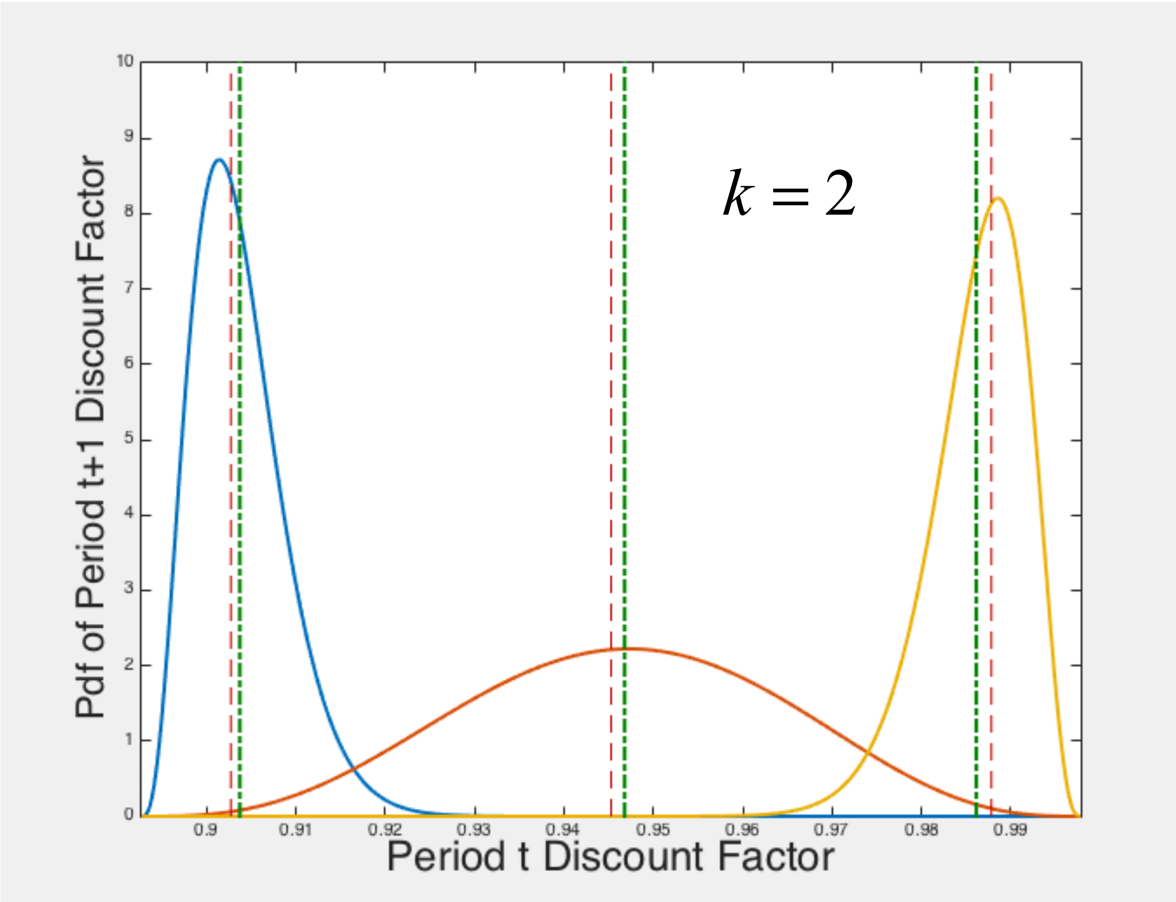
$$T\left(\frac{\alpha}{\alpha+\beta}\right) = f(m) = E[m']$$

There is one degree of freedom in picking *α* and β

Global sunspots

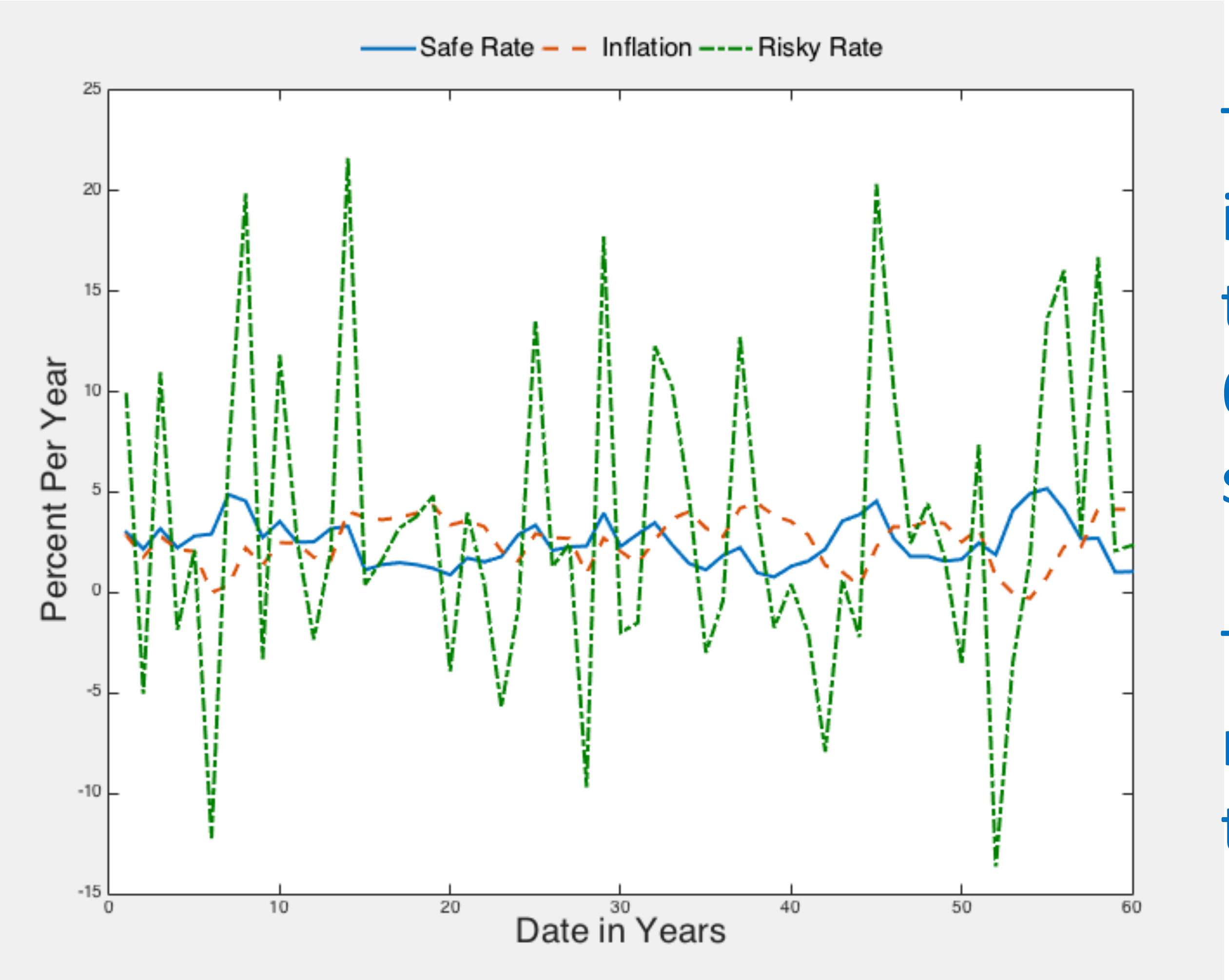
- I define a function of *m* that gives the largest possible variance for *m′* at all points in the domain of *m*
- I chose a constant parameter, *k*, that scales this function
- High *k* means low variance
- Low *k* means high variance

Global sunspot equilibria



I chose k = 2 in my simulations

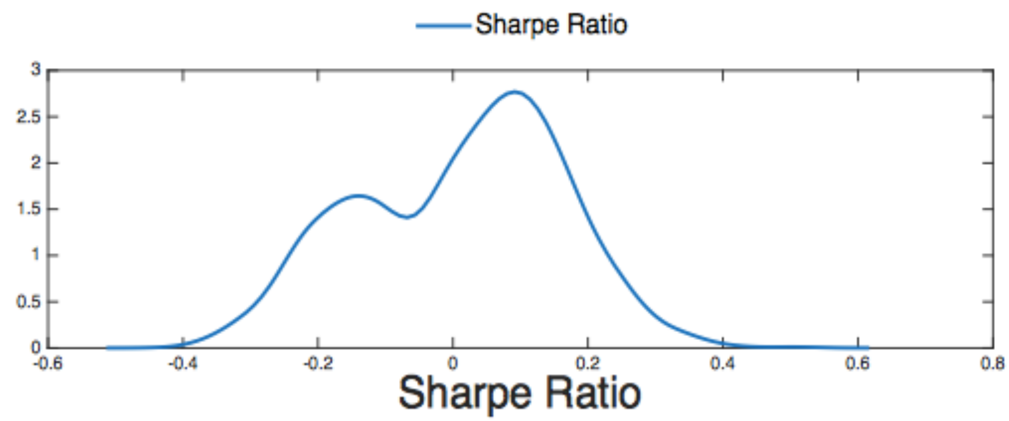
Simulating equilibria



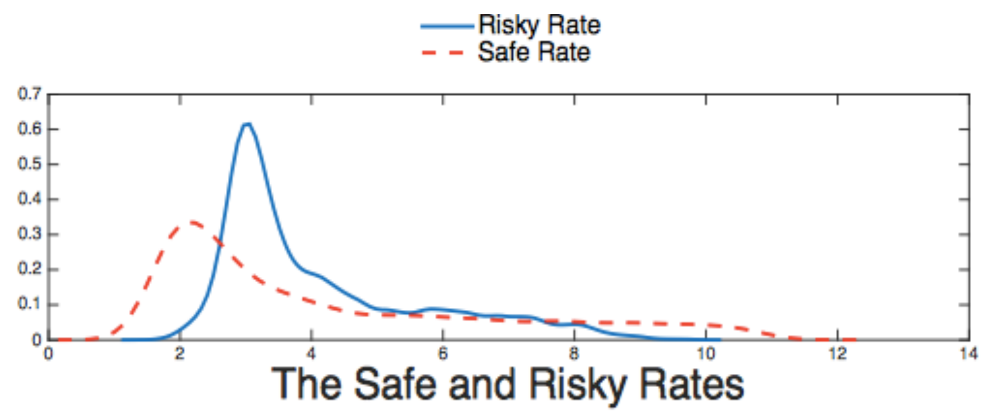
The safe rate, inflation and the risky rate in 60 years of simulated data

The Sharpe ratio is 0.1 for this draw

Simulating equilibria

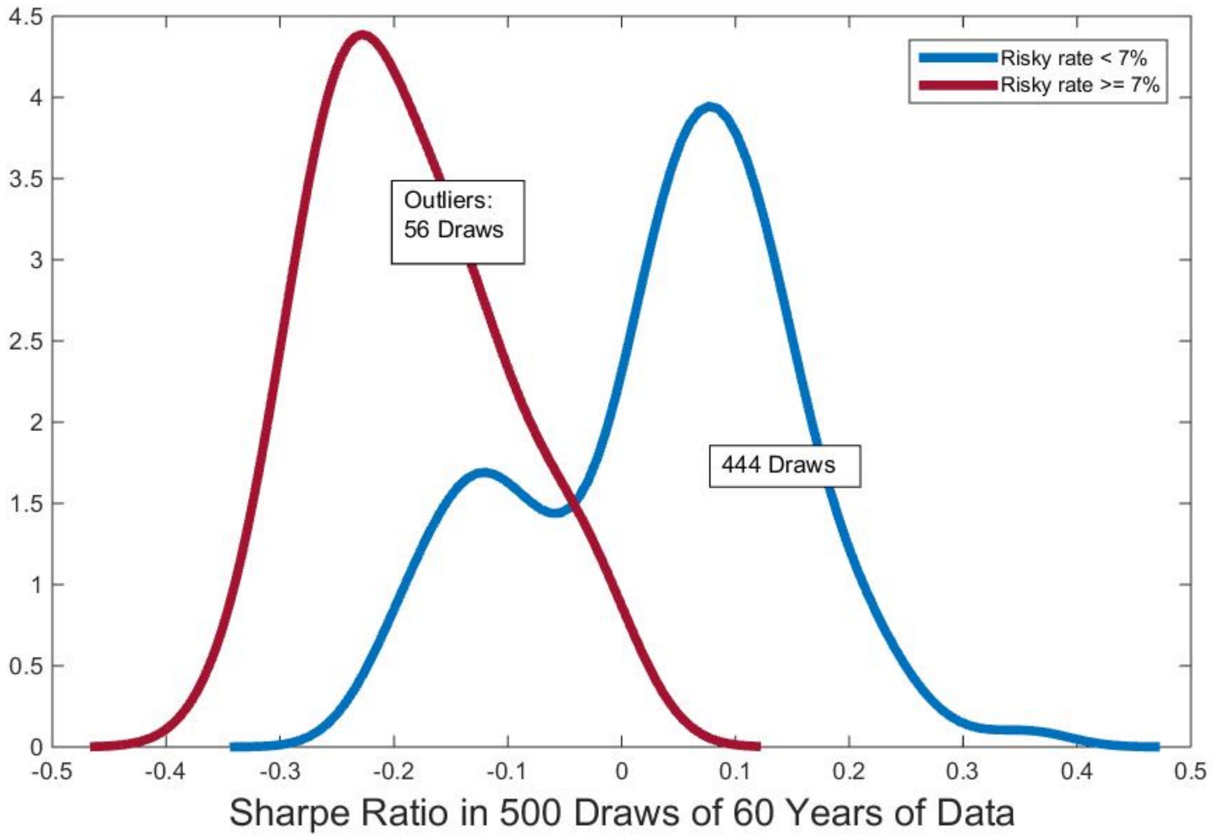


This graph plots the average Sharpe ratio in 6,000 draws of 60 years of data



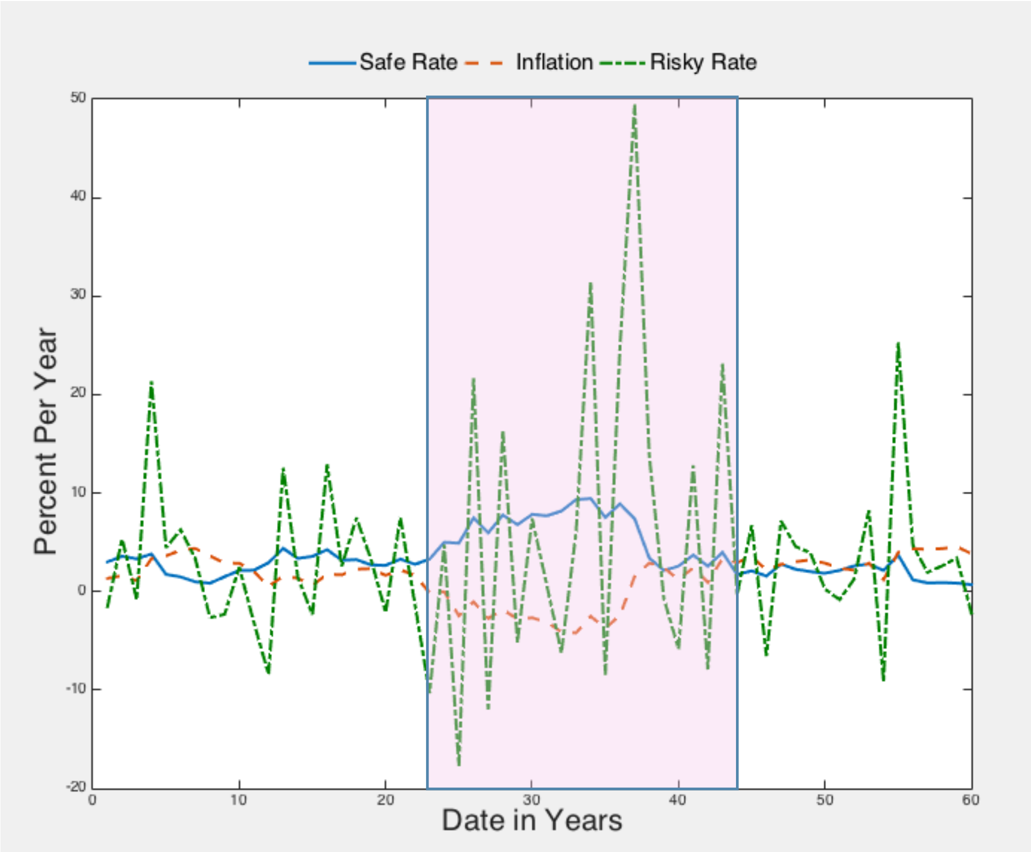
This is the distribution of average safe and risky rates over these 6,000 simulations

Simulating equilibria



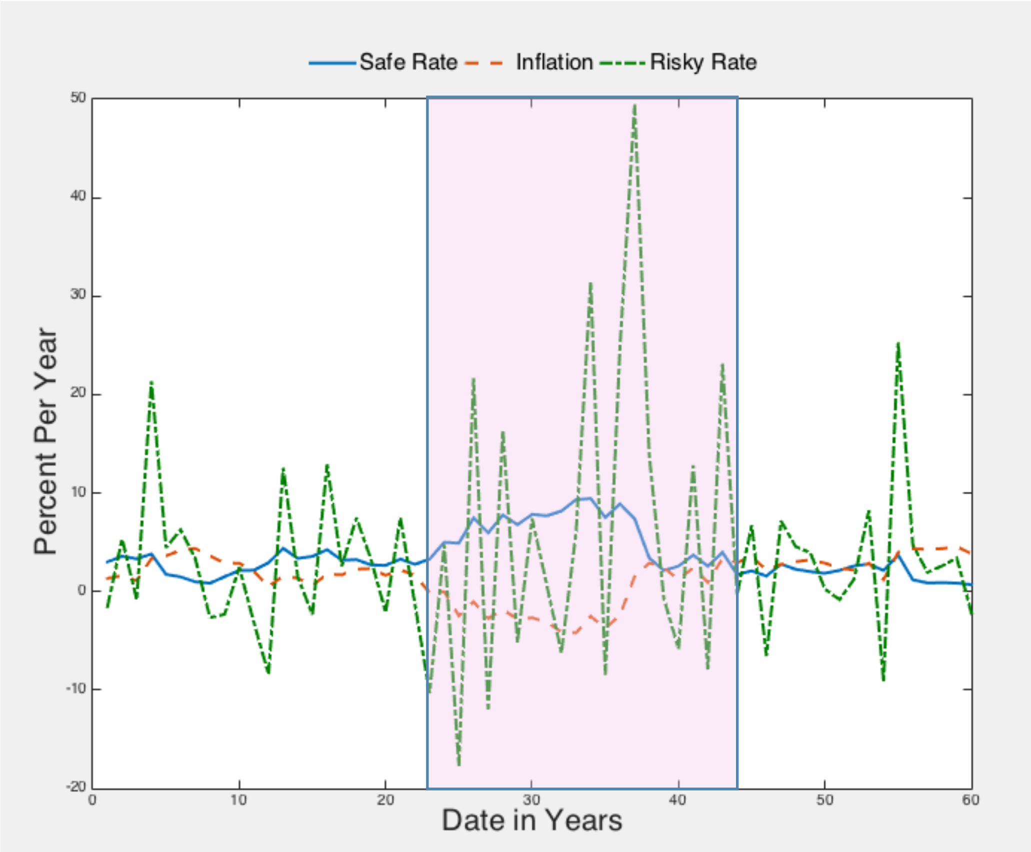
The distribution of Sharpe ratios has a fat left tail

Volatility



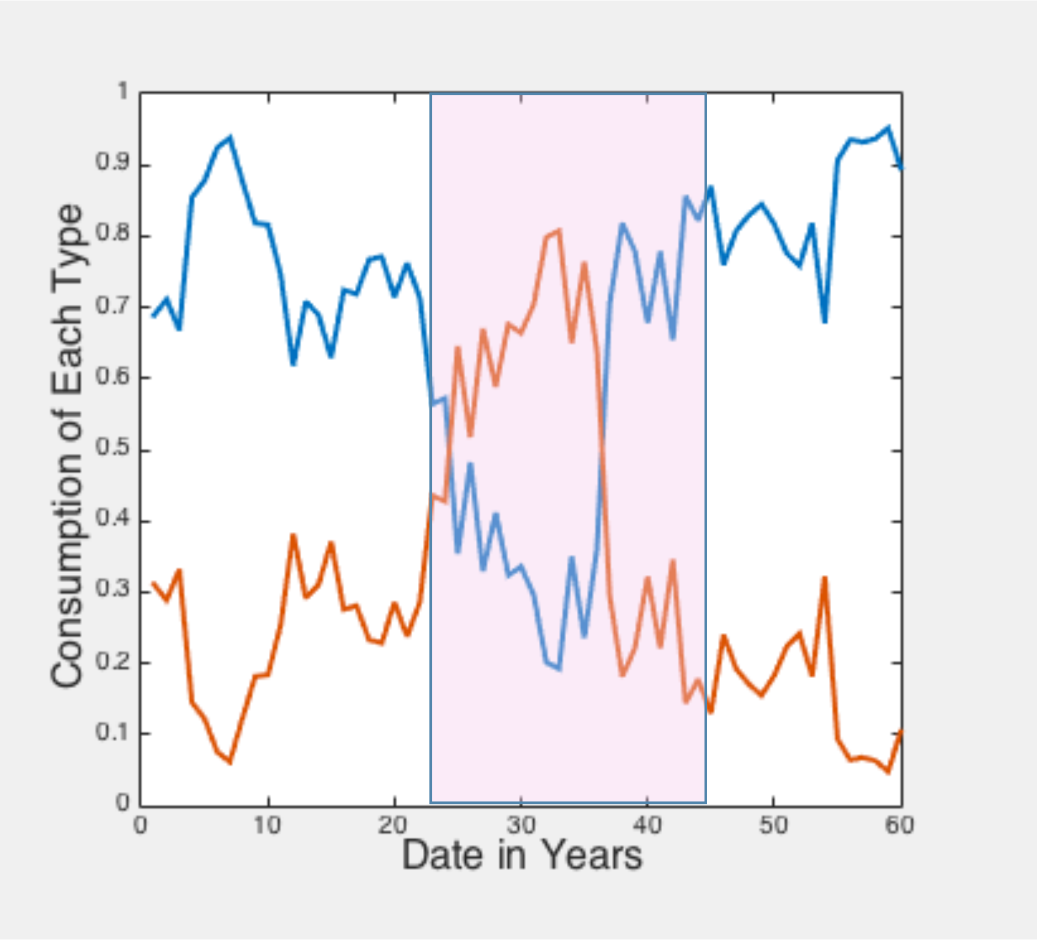
Bursts of volatility are common

Volatility



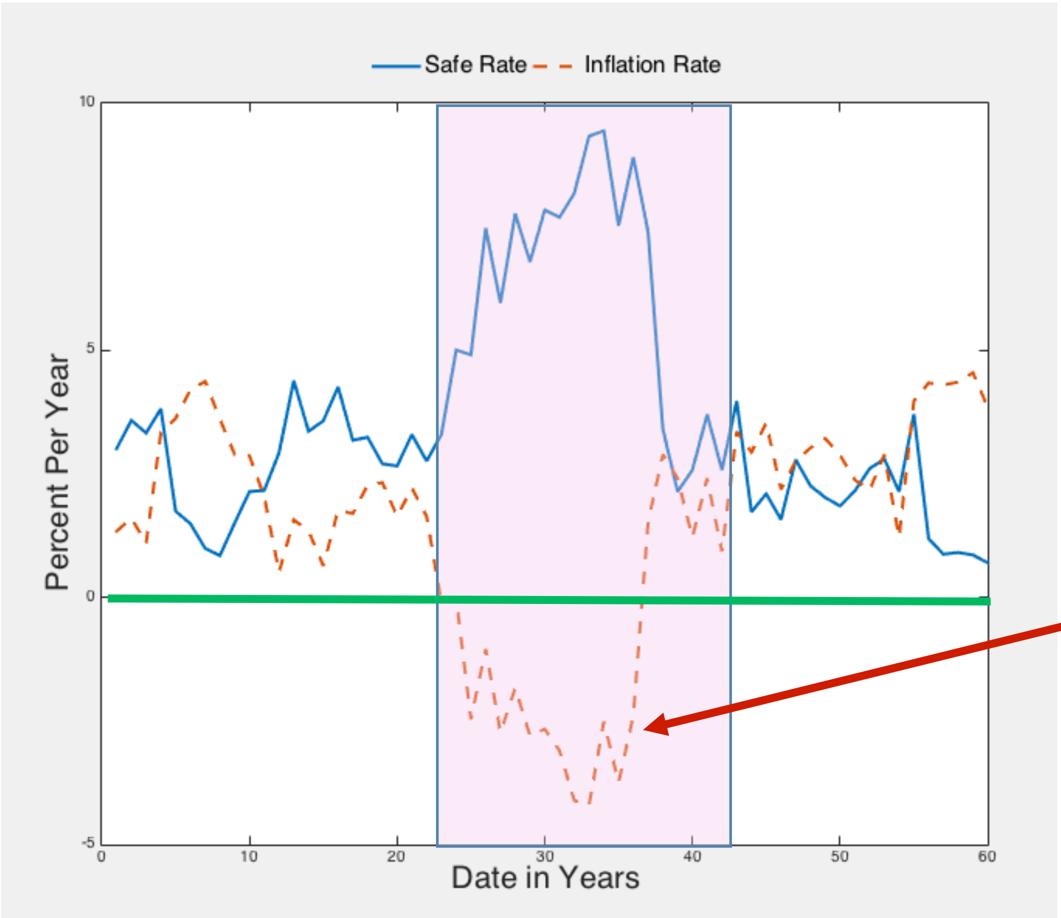
Bursts of volatility are common

Volatility



Volatility is associated with reversals in the pattern of the distribution of consumption across types

Volatility



... this is also a period of deflation that can last for a decade or more

Conclusion

- Excess volatility and the term premium can be explained in a simple and parsimonious model
- Parameters are disciplined by demographics