

# The Fundamental Surplus in Matching Models

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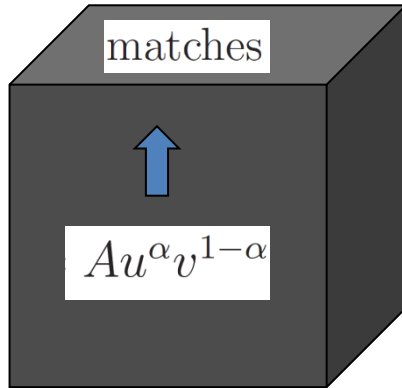
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Following Petrongolo and Pissarides (2001) Looking into the Black Box: A Survey of the Matching Function



A key equilibrium variable

$$\theta = \frac{v}{u} \quad (\text{market tightness})$$

We investigate what is common across

- matching models that can generate productivity-driven business cycles
- matching models that can explain the outbreak of high European unemployment as coming from productivity changes



The **fundamental surplus fraction** is small.

**Fundamental surplus fraction** in a matching model of type  $j$

$$\frac{y - x^j}{y}$$

$x^j$  is the part of a job's output  $y$  that the "invisible hand" cannot divert to vacancy creation, such as the value of leisure, unemployment compensation, annuitized values of training costs and layoff costs, worker's ability to exploit a firm's cost of delay in particular bargaining protocols, etc.

We investigate what is common across

- matching models that can generate productivity-driven business cycles
- matching models that can explain the outbreak of high European unemployment as coming from productivity changes



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# Outline

- Steady-state comparative statics and stochastic simulations of the elasticity of market tightness with respect to productivity
  - Revisit the Shimer (2005) critique
  - Steady-state closed-form solutions for a variety of matching models
- Connections between models of welfare state dynamics and models of business cycle fluctuations

Mortensen and Pissarides (1999) and Hagedorn and Manovskii (2008)

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Illustrations:

- Sticky wage (Hall 2005)
  - Alternating-offer wage bargaining (Hall and Milgrom 2008)
  - Technology–policy interaction behind European unemployment (Hornstein, Krusell and Violante 2007)
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- Mistaken imputations of the sources of a high elasticity of market tightness but also observations that foreshadowed the fundamental surplus fraction

## Match surplus

## Fundamental surplus

Measure of

the value of a match in excess of parties' outside values

an upper bound on resources that the 'invisible hand' can allocate to vacancy creation

Relationship to the elasticity of market tightness

in general, none

key determinant

(inconsequentially) expressed as

a capitalized value

a flow value

Steady-state comparative statics in a matching model of type  $j$   
(with exogenous separation) imply an  
**elasticity of market tightness with respect to productivity**

$$\eta_{\theta,y}^j = \Upsilon^j \frac{y}{y - x^j} \quad x^j \in [0, y)$$

where both multiplicative factors are bounded from below by unity

- $\Upsilon^j$  is also bounded from above due to a consensus about reasonable parameter values
- the inverse of the fundamental surplus fraction,  $\frac{y}{y - x^j}$ , has no such agreed upon upper bound.

**A high elasticity = A small fundamental surplus fraction**

In a basic matching model with Nash wage bargaining,  $x = z$  (the value of leisure)

$$\eta_{\theta,y} = \Upsilon^{\text{Nash}} \frac{y}{y - z}$$

$\Upsilon^{\text{Nash}}$  is a function of the separation rate ( $s$ ), workers' bargaining power ( $\phi$ ), discount rate ( $\beta = (1+r)^{-1}$ ), matching function parameters including the elasticity of matching with respect to unemployment ( $\alpha$ ), and the endogenous market tightness ( $\theta$ ). **Its upper bound is  $1/\alpha$ .**

**Shimer (2005):** *the textbook search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude. In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility.*

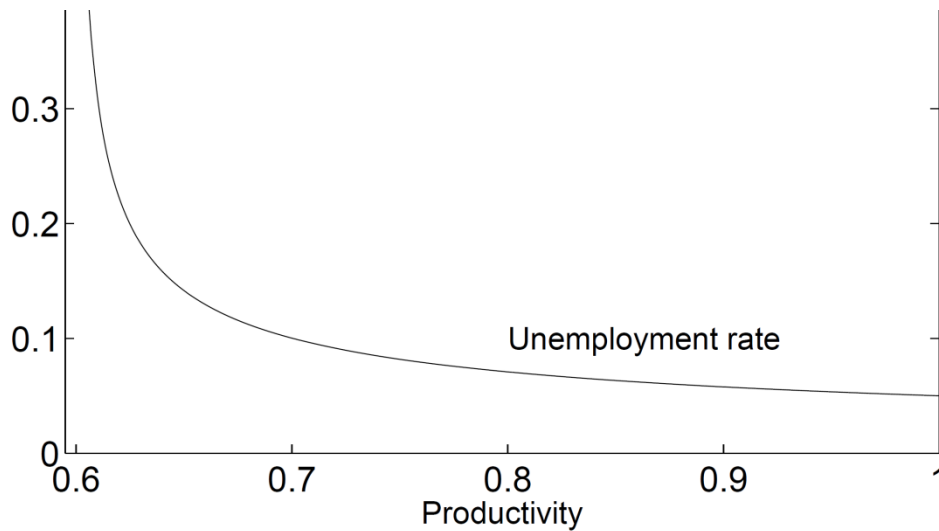
$[\Upsilon^{\text{Nash}}]$  is 1.03. It would take implausible parameter values [for  $\Upsilon^{\text{Nash}}$ ] to exceed 2. This implies that unless the value of leisure is close to labor productivity, the v-u ratio is likely to be unresponsive to changes in the labor productivity.

Business cycle context	Elasticity	Key variables
Nash bargaining (Hagedorn and Manovskii 2008)	$\Upsilon^{\text{Nash}} \frac{y}{y - z}$	$z$ , value of leisure
Sticky wage (Hall 2005)	$\Upsilon^{\text{sticky}} \frac{y}{y - \hat{w}}$	$\hat{w}$ , sticky wage
... and a financial accelerator (Wasmer and Weil 2004)	$\Upsilon^{\text{sticky}} \frac{y}{y - \hat{w} - a}$	$a$ , annuitized value of credit search costs
Alternating-offer bargaining (Hall and Milgrom 2008)	$\Upsilon^{\text{alternating}} \frac{y}{y - z - \beta(1 - s)\gamma}$	$\gamma$ , firm's cost of delay in bargaining

### Welfare state context <sup>†</sup>

Unemployment insurance	$\Upsilon^{\text{Nash}} \frac{y}{y - z - b}$	$b$ , unemployment benefit
Layoff costs	$\Upsilon^{\text{Nash}} \frac{y}{y - z - \beta s \tau}$	$\tau$ , layoff tax

<sup>†</sup> Examples include a widened earnings distribution in Mortensen and Pissarides (1999), higher capital-embodied technological change in Hornstein, Krusell and Violante (2007), and shocks to human capital in Ljungqvist and Sargent (2007).



### ***Mortensen and Pissarides (1999)***

Heterogeneous workers in type-specific matching functions.

All workers have  $z = 0.6$  but are permanently differentiated by productivity  $y \in (0.6, 1]$ ;  $\phi = 0.5$

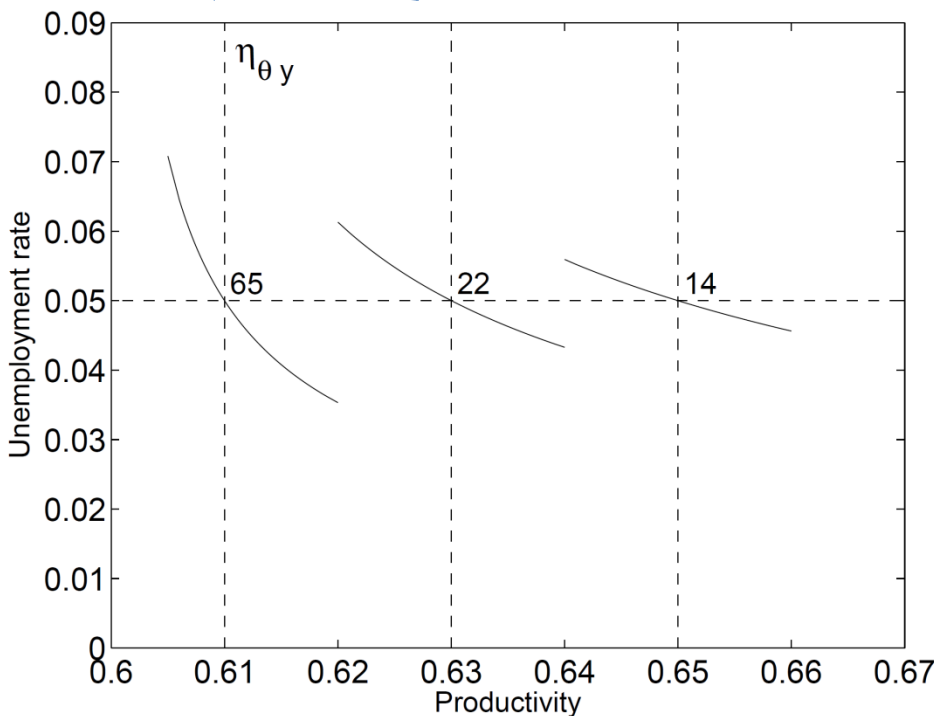
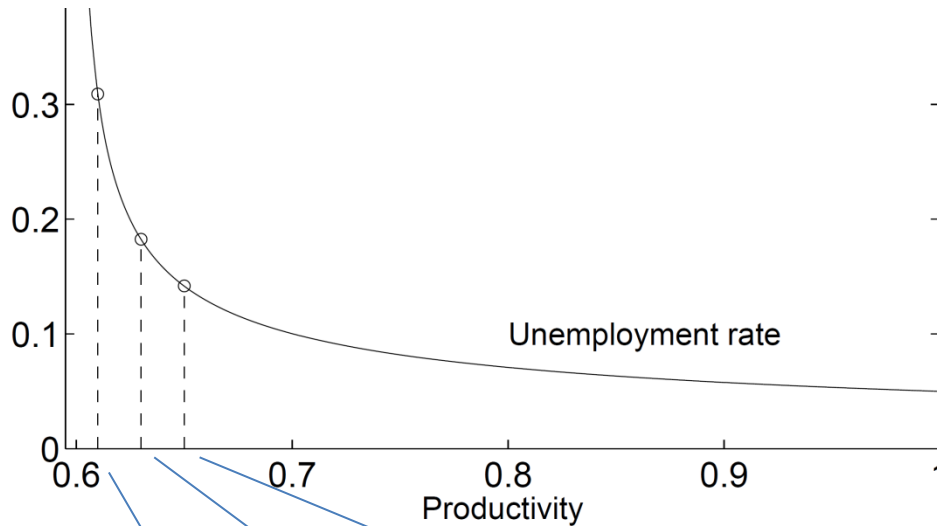
$$\eta_{\theta,y} = \Upsilon^{\text{Nash}} \frac{y}{y - z}$$

**Mortensen and Pissarides (1999):** the relationship between the unemployment rate and worker productivity is *much more convex* in the ‘European’ case than in the ‘US’. As a consequence, persistent skill-biased shocks, appropriately modelled as *an increase in mean preserving spread of the distribution of productivity across workers*, induces larger increases in unemployment in economies characterised by higher UI replacement ratios and EP implicit tax rates.

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### *Hagedorn and Manovskii (2008)*

Homogeneous workers in a single matching function.

Given a common productivity

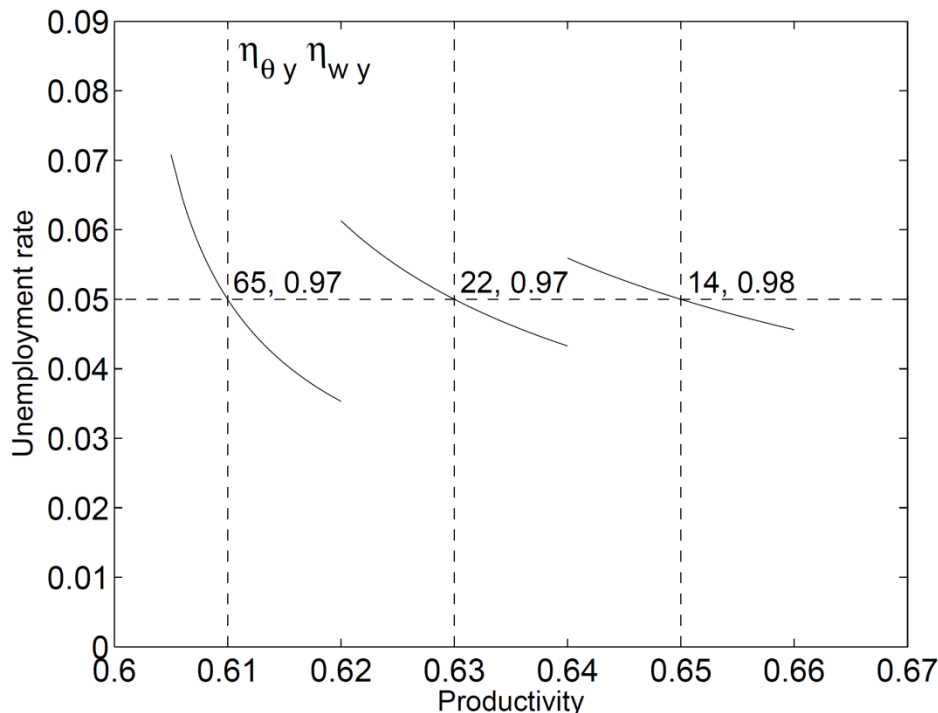
$$y \in \{0.61, 0.63, 0.65\}$$

unemployment is set to be 5 %.

Rogerson and Shimer (Handbook of Labor 2011) emphasize that : wages are rigid. An early example is Hagedorn and Manovskii (2008), although it is worth noting that the authors do not interpret their paper as one with wage rigidities. They calibrate the Nash bargaining parameter using information that wages move less than one-for-one with productivity, which gives them a small value for the workers' bargaining power  $\phi$ . This significantly amplifies productivity shocks relative to the baseline search model.

Recall  
 $\phi = 0.5$

Hagedorn and Manovskii (2008): . The elasticity of wages does not matter per se. the volatility of labor market tightness is almost independent of  $\phi$  and is determined only by the level of  $z$ .



$$\eta_{\theta,y} = \Upsilon^{\text{Nash}} \frac{y}{y - z}$$

**Hagedorn and Manovskii (2008)**

Homogeneous workers in a single matching function.

Given a common productivity

$$y \in \{0.61, 0.63, 0.65\} ,$$

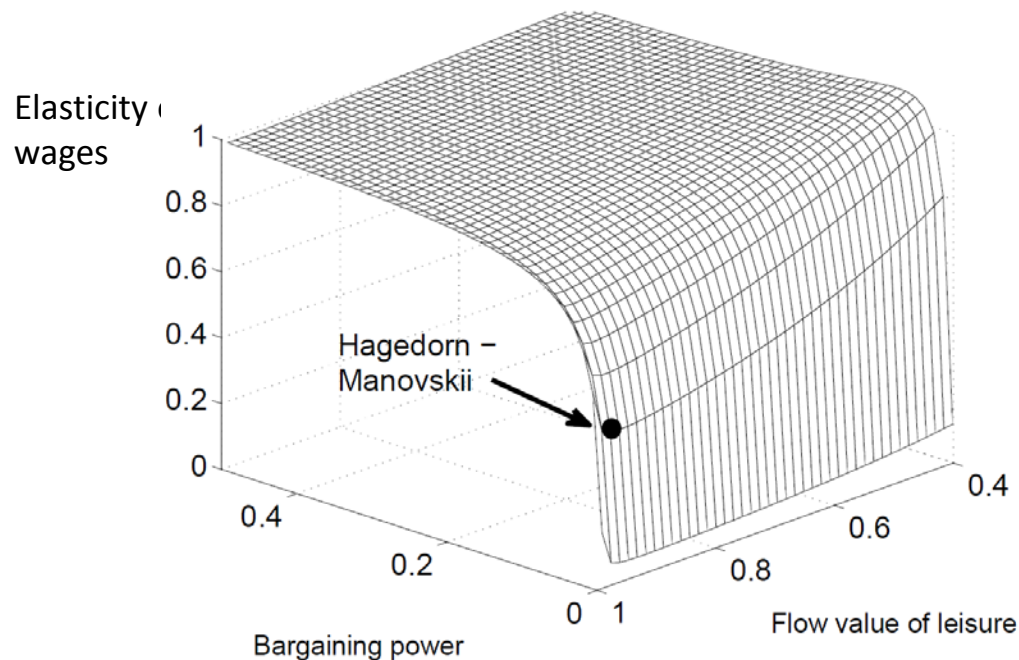
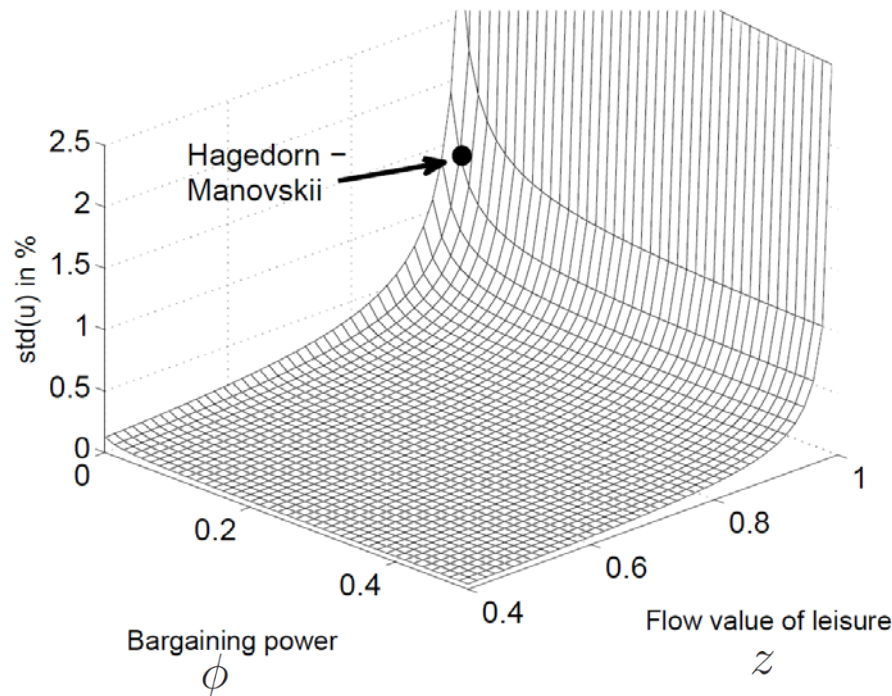
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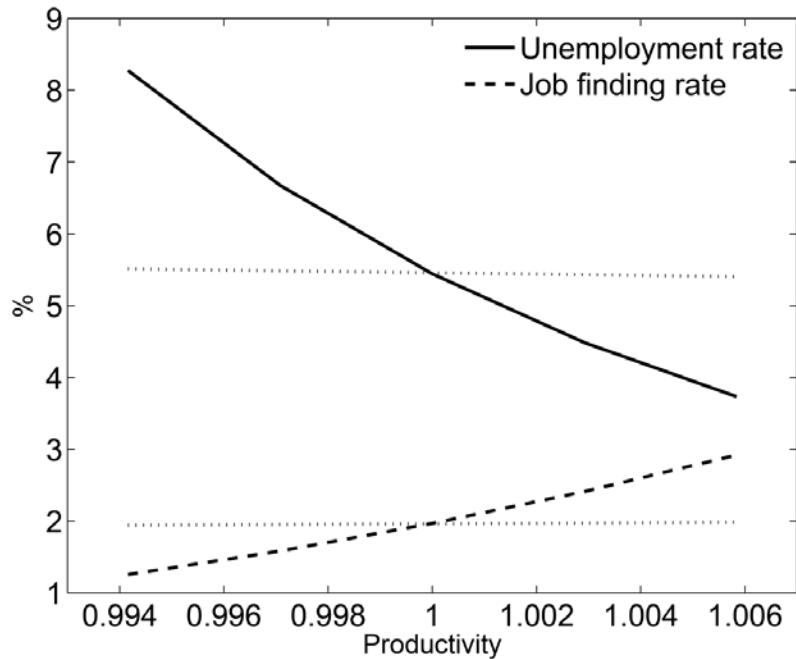
Our version of  
Hagedorn and Manovskii's (2008)  
calibration strategy:

High value of leisure,  $z = 0.960$ ,  
implies a small fundamental  
surplus fraction, i.e., volatile  
unemployment.

A low workers' bargaining power,  
 $\phi = 0.0135$ , yields a low elasticity  
of wages.

Hall's (2005) stochastic  
environment with mean  
productivity equal to one.



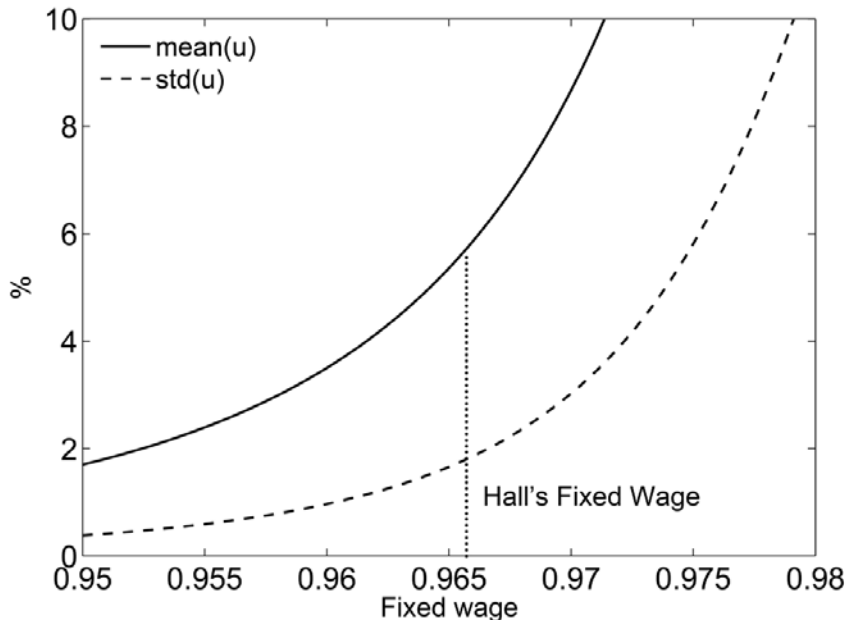


## Hall (2005): Sticky wage

set the fixed wage to the Nash-bargain wage for the median state

The resulting model makes recruiting effort, job-finding rates, and unemployment remarkably sensitive to changes in determinants. A small decline in productivity results in a slump in the labor market.

$$\gamma^{\text{sticky}} \frac{y}{y - \hat{w}}$$



Fundamental surplus = firms' profits

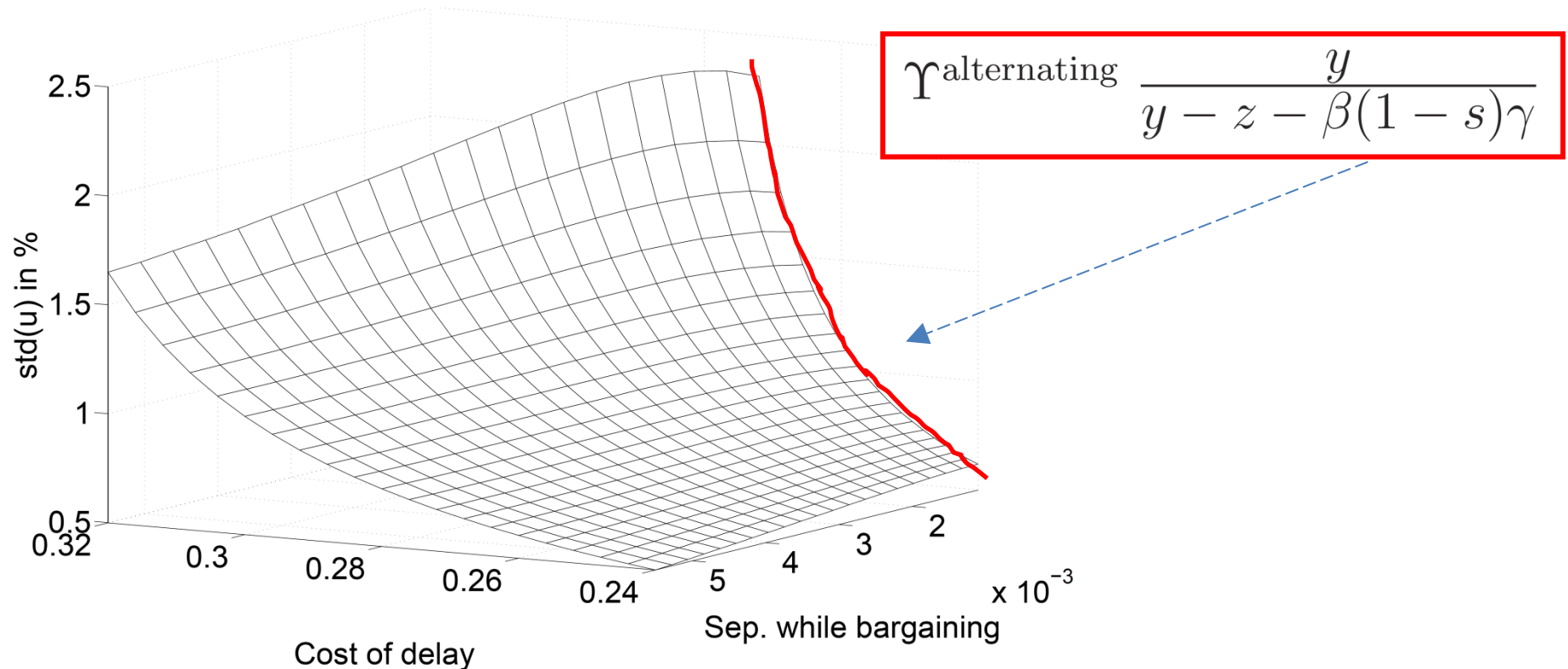
Hall's (2005) stochastic environment with mean productivity equal to one.

## Hall and Milgrom (2008): Alternating-offer wage bargaining

- Bargaining theory from Binmore, Rubinstein and Wolinsky
- Threats are to extend bargaining (rather than go for outside values).
- Firms incur a cost of delay ( $\gamma$ ) when no agreement has been reached.

Hall and Milgrom (2008): the limited influence of unemployment on the wage results in large fluctuations in unemployment under plausible movements in the driving force.

The sum of  $z$  and  $\gamma$  is 0.98, not very different from the value of  $z$  by itself in our version of Hagedorn and Manovskii's calibration,



## Hornstein, Krusell and Violante (2007): Vintage capital model of European unemployment

<u>Government policies</u>	<u>Europe</u>	<u>U.S.</u>	
unemployment benefit ( $b$ )	75%	10%	replacement rate in 1970
layoff tax ( $\tau$ )	1	0	year of average wages in 1970
Income / payroll taxes	.24 / .21	.17 / .08	

Postulating different exogenous separation rates, unemployment rates are 4% in both Europe and the U.S. in 1970.

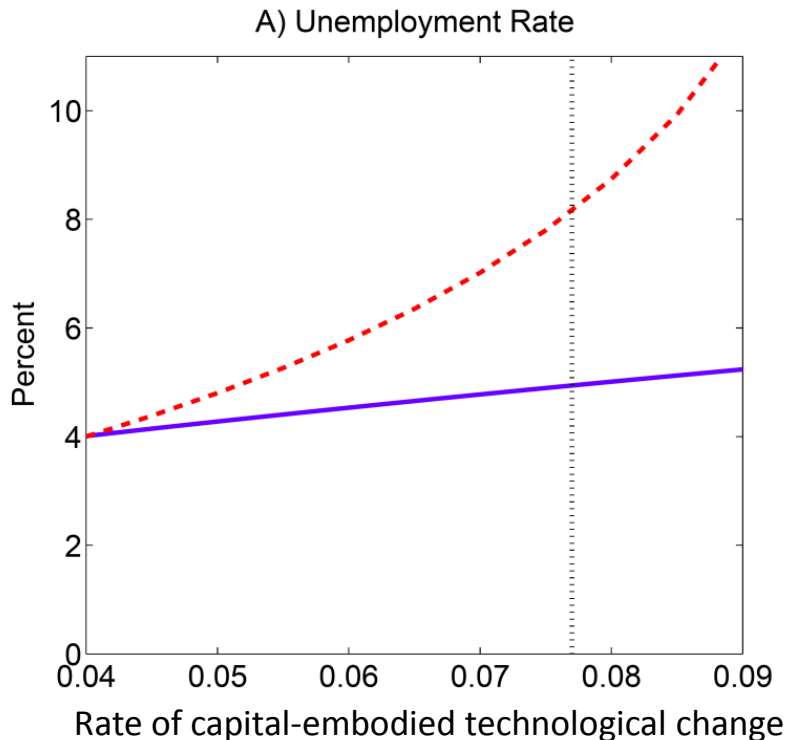
- A job is created by investing in a one-worker machine (no vacancy costs).
- Capital-embodied productivity is determined by the exogenously moving technology frontier at the time of investment.
- Two inputs into a single matching function; homogenous unemployed workers and vacant machines of different vintages.
- Nash bargaining results in vintage-specific wage rates.
- All worker-machine pairs are subject to exogenous separation shocks.
- Endogenous job destruction when a vintage productivity has fallen too far behind the current technology frontier.

Absolute quantities (investment cost,  $b$  and  $\tau$ ) grow at the economy's growth rate.

# Hornstein, Krusell and Violante (2007): Vintage capital model of European unemployment

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Rate of capital-embodied technological change

pre-1970 0.04

post-1990 0.077

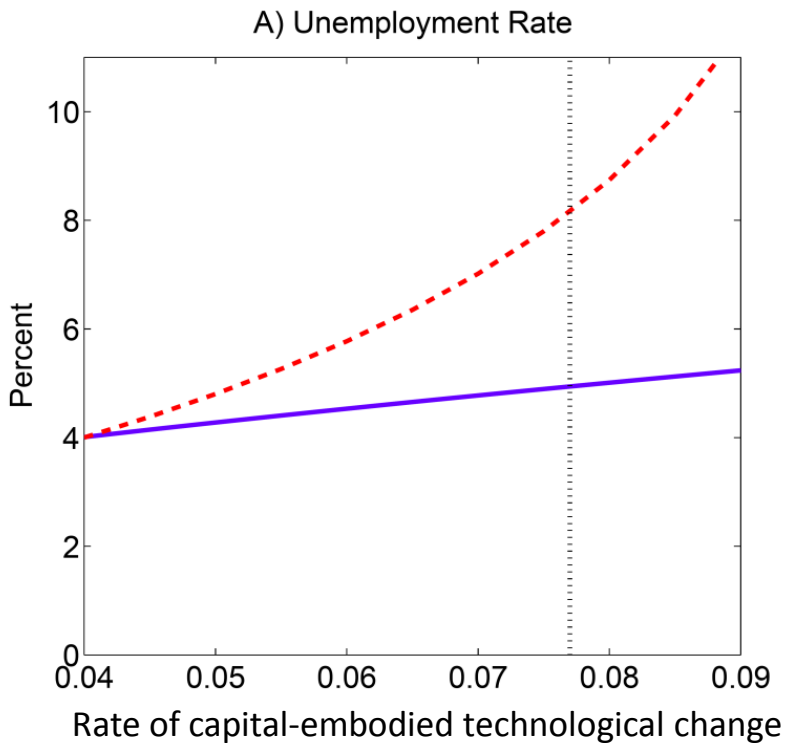
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Only unemployment benefits ( $b$ )

<u>Europe</u>	<u>U.S.</u>
85%	13%

in 1970

Postulating different exogenous separation rates, unemployment rates are 4% in both Europe and the U.S. in 1970.



Rate of capital-embodied technological change

- pre-1970      0.04
- post-1990    0.077

Let's apply the perspective of the **fundamental surplus fraction**.

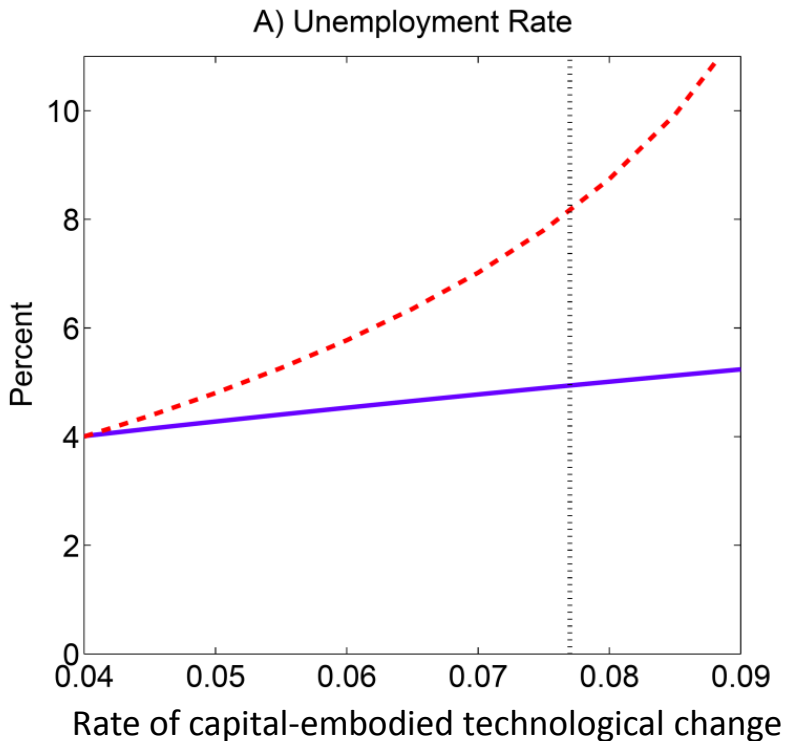
In 1990, machines become obsolete faster – tantamount to 'higher vacancy costs'.

The fundamental surplus fraction shrinks because

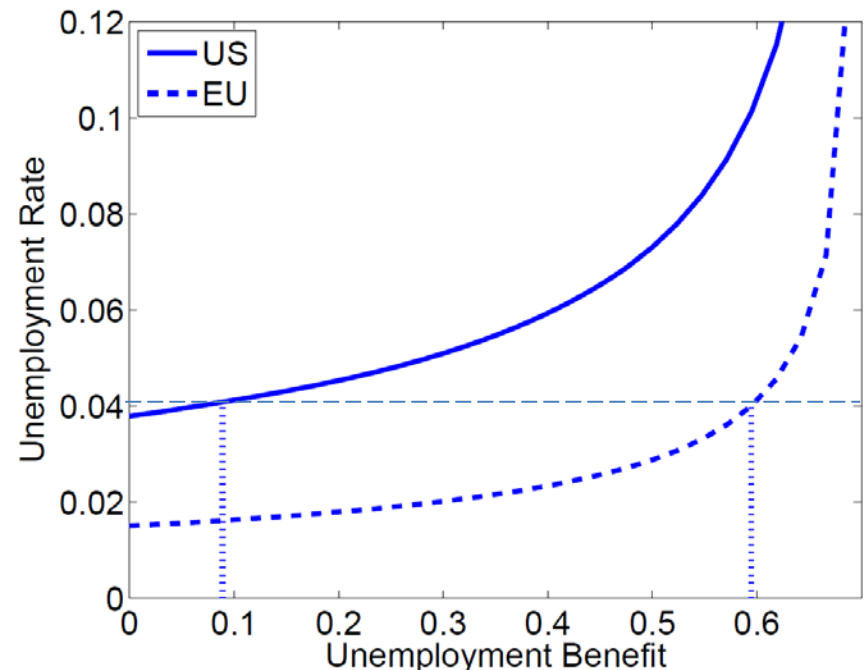
- ❑ the annuitized value of layoff costs increases (with shorter machine lifespans)
- ❑ both  $b$  and  $\tau$  increase faster with a higher overall growth rate (while a machine's productivity evolves in the same way as in pre-1970)
- ❑ the replacement rate increases (because growth-adjusted wages fall relative to pre-1970)

Only unemployment benefits ( $b$ )

Europe	U.S.	
85%	13%	in 1970



Unemployment in 1970 for different  $b$



Government policies

Europe

unemployment benefit ( $b$ )	75% (83%)
layoff tax ( $\tau$ )	1 (1.14)
Income / payroll taxes	.24 / .21

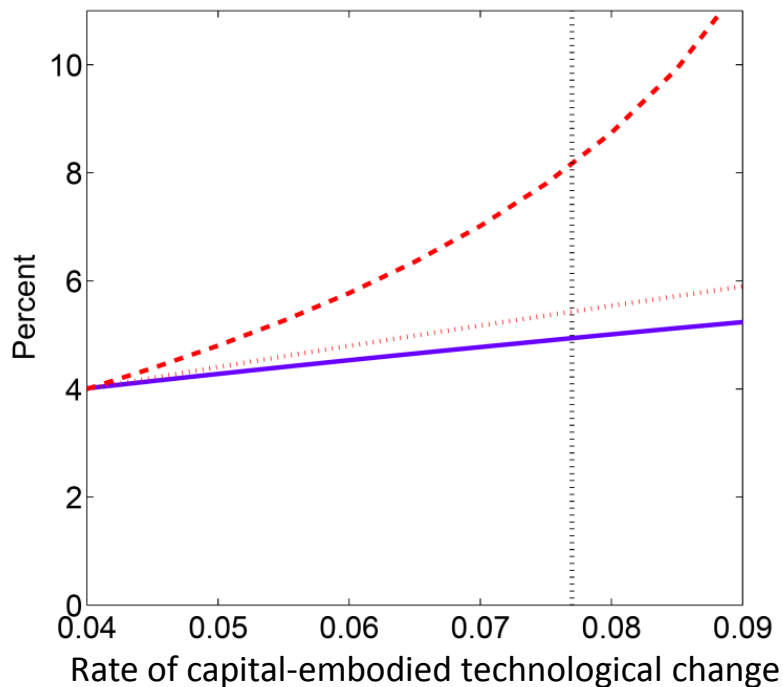
Only unemployment benefits ( $b$ )

Europe

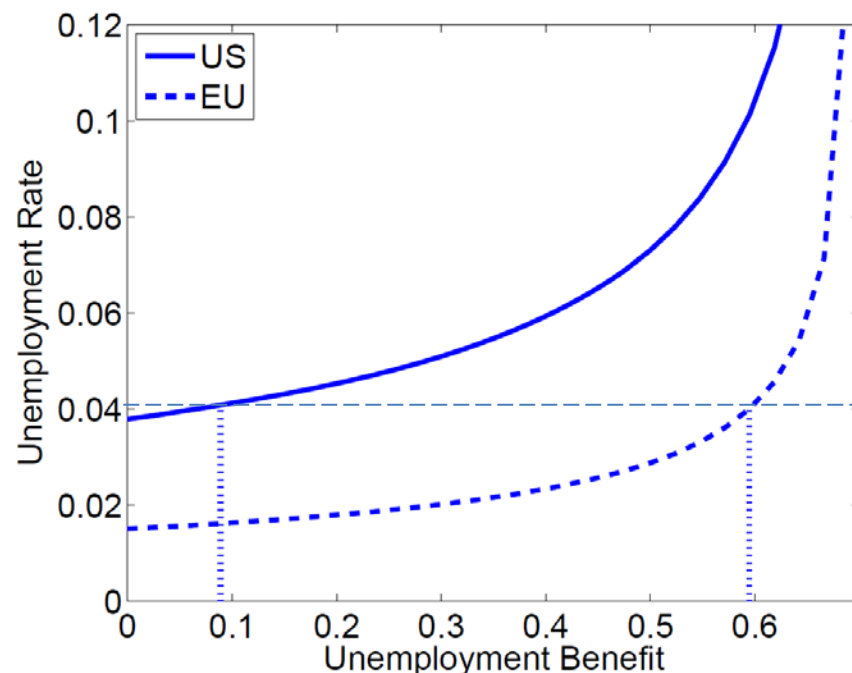
85% (93%) in 1970 (1990)

If instead the replacement rate and the layoff cost in terms of the average wage remain constant at pre-1970 values, the dotted curve depicts European outcomes.

A) Unemployment Rate



Unemployment in 1970 for different  $b$



Hornstein, Krusell and Violante (2007):

the technology–policy interaction is much starker when the three policies are considered together: as  $\gamma$  increases, if one estimated the total role of policy by merely summing the effects of the individual policies, one would only account for less than one-third of the total technology–policy interaction predicted by the model with all policies jointly considered.<sup>24</sup> We conclude that, first, it would be inaccurate to point to one particular institution as the culprit and second, looking ahead, reforming any one institution could reduce dramatically the elasticity of the unemployment rate to obsolescence shocks.

*by increasing the fundamental surplus fraction.*

Finally, there are some important connections between our results and an emerging set of studies pointing to two shortcomings of the standard matching model.

One lesson from these studies (Hagedorn and Manovskii, 2006, in particular) is that significant volatility requires large values of a worker’s utility when unemployed—the value of leisure and unemployment benefits ( $\ell + b$  in our model). ~~This leads to a low-wage elasticity to variations in productivity; thus, productivity changes have a strong effect on profits and hence on firm inflow as well as unemployment.~~ Although we focus on steady states rather than short-run dynamics, we also emphasize that in the presence of strict labour-market policy (e.g. high unemployment benefits  $b$ ), productivity changes (here, growth in capital-embodied technology) have a significant impact on quantities: market tightness falls sharply as a result of significant withdrawal of firm entry. Thus, our interpretation of the recent European experience builds on similar mechanisms as those discussed in the recent literature on short-run dynamics. *namely, the fundamental surplus fraction must be small.*

## Wasmer and Weil (2004): A financial accelerator

- Matching in a credit market precedes matching in the labor market.
- Free entry of entrepreneurs and financiers in the credit market.
- Entrepreneur-financier pairs match with workers in the labor market (sticky wage  $\hat{w}$ ).

Petrosky-Nadeau and Wasmer (2013) characterize the financial accelerator as contributing multiplicatively to the elasticity of market tightness,

$$\eta_{\theta,y} = \Upsilon^{\text{sticky}} \frac{J}{J - K} \frac{y}{y - \hat{w}}$$

where  $K$  is the average search cost for the formation of an entrepreneur-financier pair in the credit market, who goes on to post a vacancy in the labor market where the value of a filled job is

$$J = \frac{y - \hat{w}}{\beta(r + s)}$$


$$\eta_{\theta,y} = \Upsilon^{\text{sticky}} \frac{y}{y - \hat{w} - a}$$

where  $a = (r + s)\beta K$  is the annuitized value of credit search costs.

# Fundamental surplus versus match surplus

Eran Yashiv's steady-state formula

$$\frac{y - z}{y} = \frac{r + s + \phi \theta q(\theta)}{1 + r} \frac{S}{y}$$

Hagedorn-Manovskii (HM) calibration versus a standard calibration (no superscript)

$$\frac{y - z^{\text{HM}}}{y - z} = \frac{r^{\text{HM}} + s^{\text{HM}} + \phi^{\text{HM}} \theta^{\text{HM}} q^{\text{HM}}(\theta^{\text{HM}})}{r + s + \phi \theta q(\theta)} \frac{1 + r}{1 + r^{\text{HM}}} \frac{S^{\text{HM}}}{S}$$

Except for the value of leisure and a worker's bargaining weight, HM adopt common parameter values,  $r^{\text{HM}} = r$  and  $s^{\text{HM}} = s$ , as well as calibration targets for the job finding rate and total vacancy costs,

$$\theta^{\text{HM}} q^{\text{HM}}(\theta^{\text{HM}}) \approx \theta q(\theta),$$

$$(1 - \phi^{\text{HM}}) S^{\text{HM}} \approx (1 - \phi) S.$$

# Fundamental surplus versus match surplus

Eran Yashiv's steady-state formula

$$\frac{y - z}{y} = \frac{r + s + \phi \theta q(\theta)}{1 + r} \frac{S}{y}$$

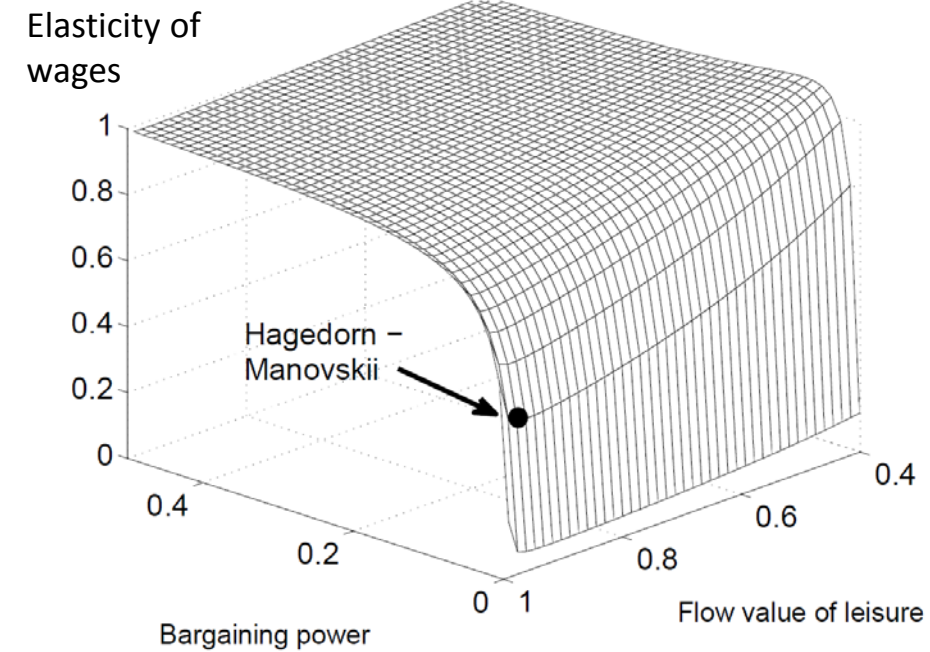
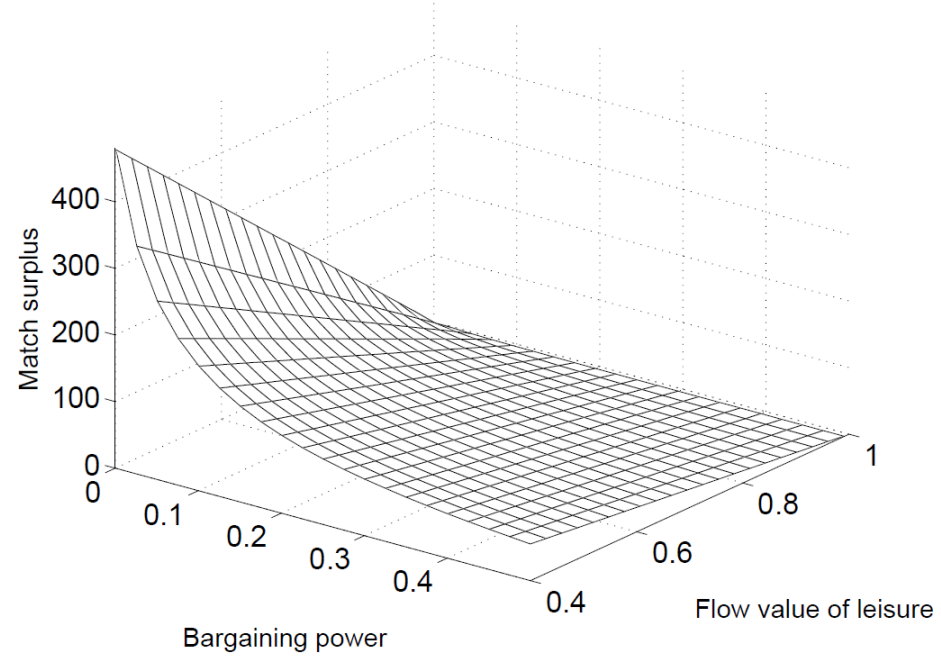
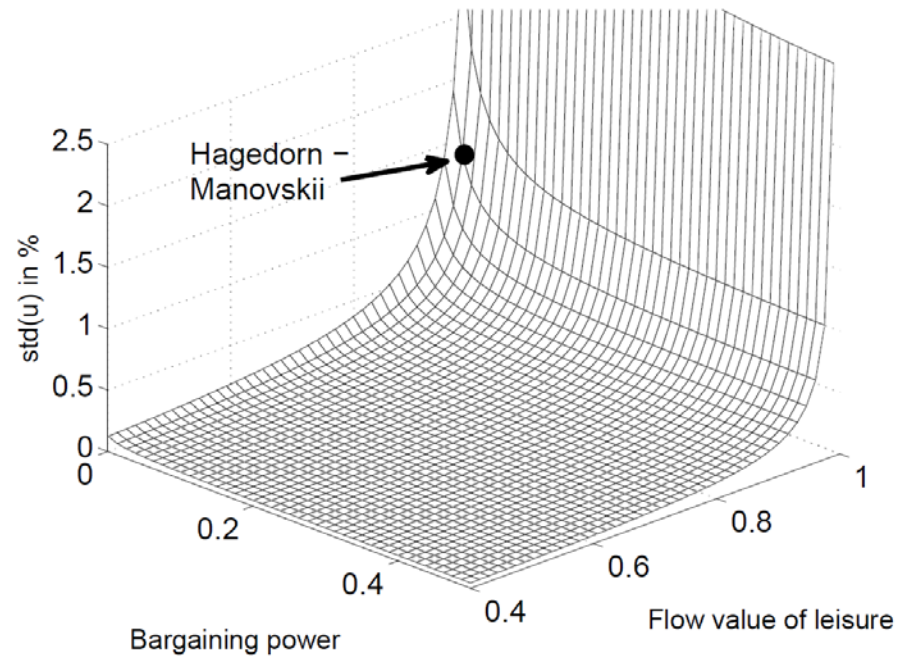
Hagedorn-Manovskii (HM) calibration versus a standard calibration (no superscript)

$$\frac{1 - z^{\text{HM}}}{1 - z} = \frac{r + s + \phi^{\text{HM}} \theta q(\theta)}{r + s + \phi \theta q(\theta)} \frac{1 - \phi}{1 - \phi^{\text{HM}}}$$

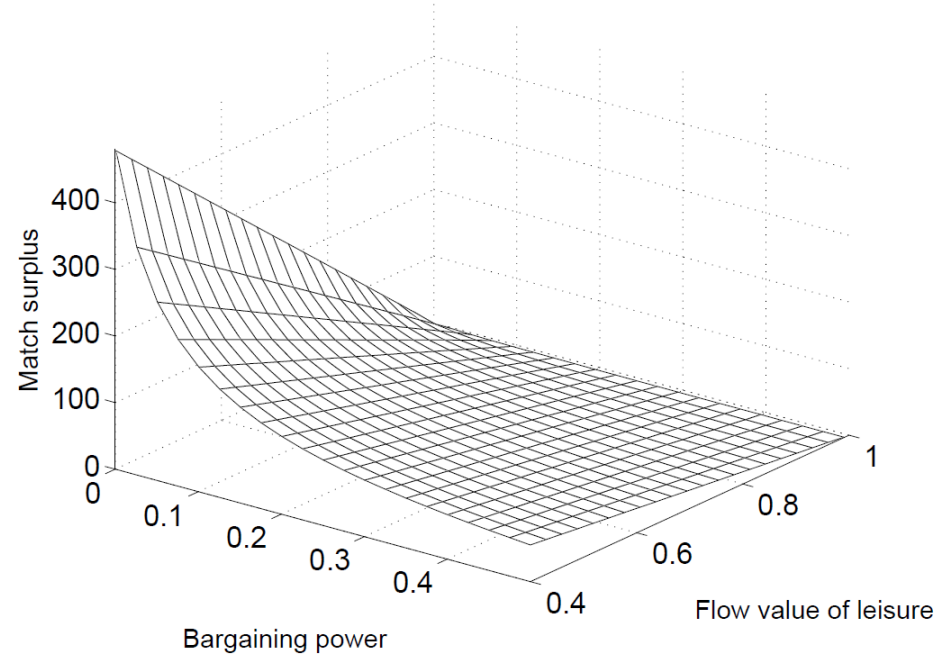
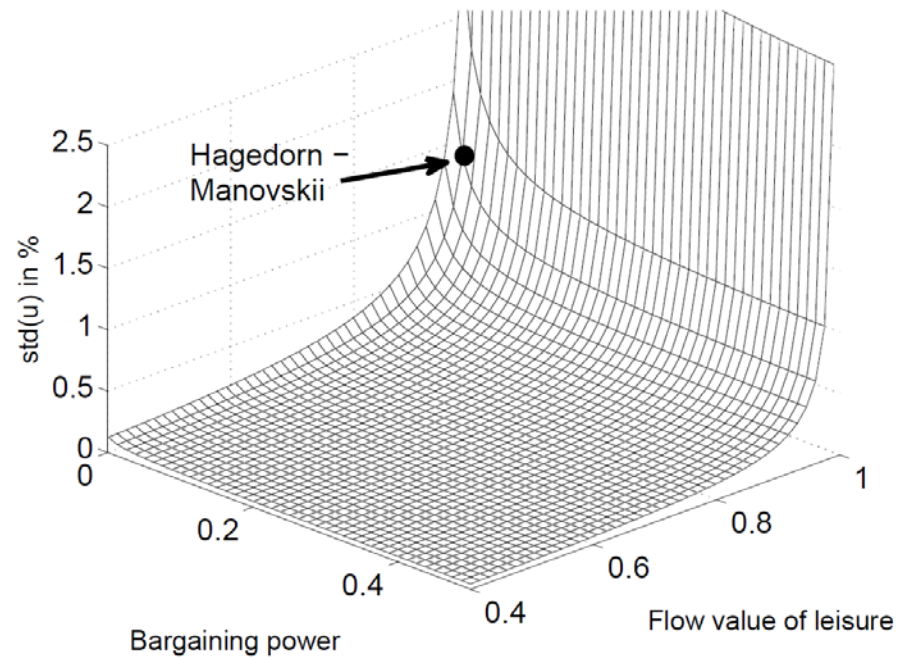
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$$\begin{aligned} \theta^{\text{HM}} q^{\text{HM}}(\theta^{\text{HM}}) &\approx \theta q(\theta), \\ (1 - \phi^{\text{HM}}) S^{\text{HM}} &\approx (1 - \phi) S. \end{aligned}$$

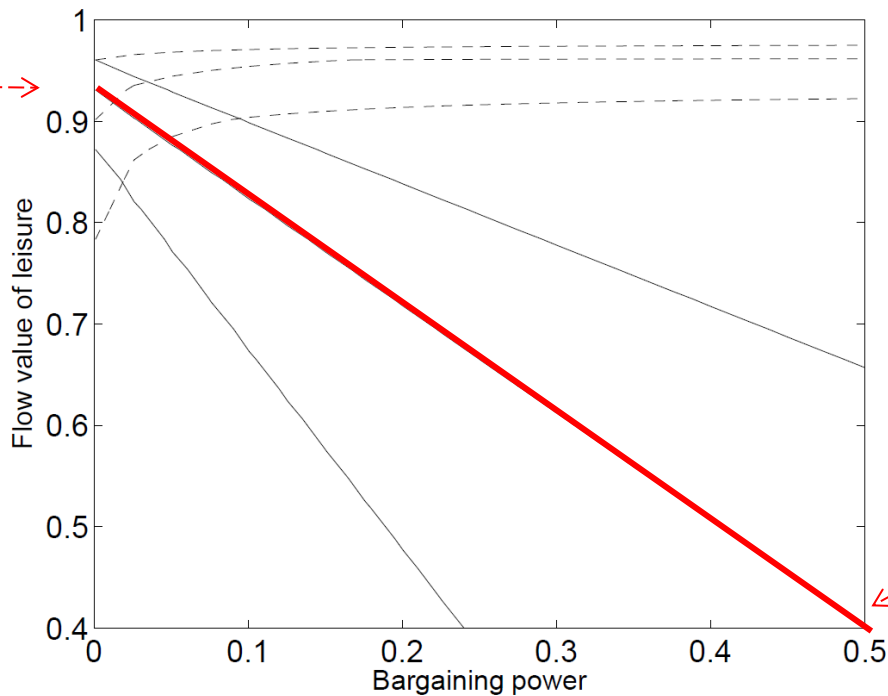
Suppose calibration targets are met



Hall's (2005) stochastic environment with mean productivity equal to one.



Hagedorn-Manovskii type of calibration



Isoquants for  $\text{std}(u)$  (dashed lines)

Isoquants for match surplus (solid lines)

Hall's (2005) calibration of a standard matching model



## Fundamental surplus fraction:

key determinant of the elasticity of market tightness  
with respect to productivity

$$\frac{y - x^j}{y}$$

$x^j$  is the part of a job's output  $y$  that the "invisible hand"  
cannot divert to vacancy creation