Is Government Spending at the Zero Lower Bound Desirable?*

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Abstract

Government spending at the zero lower bound (ZLB) is not necessarily welfare enhancing, even when its output multiplier is large. We illustrate this point in the context of a standard New Keynesian model. In that model, when government spending provides direct utility to the household, its optimal level is at most 0.5-1 percent of GDP for recessions of -4 percent; the numbers are higher for deeper recessions. When spending does not provide direct utility, it is generically welfare-detrimental: it should be kept unchanged at a long run-optimal value.

Keywords: Government spending multiplier, zero lower bound, welfare

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1 Introduction

A series of recent papers have argued that, once an economy faces a binding zero lower bound (ZLB) constraint on the nominal interest rate, government spending as a stabilization tool is particularly effective. This is the message of Christiano, Eichenbaum and Rebelo (2011) (CER 2011 henceforth), Eggertsson (2010), and Woodford (2011), among others. The key reason it that, at the ZLB, the output multiplier of government spending can be much larger than in normal times. In a model with sticky prices, if the nominal interest rate is constrained by the ZLB, a persistent increase in government spending raises labor demand and therefore the real marginal cost; this translates into higher expected inflation, hence into a negative real interest rate (given a zero nominal interest rate), inducing a substitution from future into current consumption, which raises output.

The academic literature on the ZLB has focused on the case of government spending that provides direct utility to the representative agent. A possibly misleading interpretation of this literature (although one that has not been formalized yet) is that, precisely because government spending has a very large multiplier, even *wasteful* government spending might have a positive welfare effect at the ZLB by reducing the output gap.[1]

In this paper, we ask three questions. First, does a large output multiplier translate into a large positive welfare effect? Second, is the optimal government spending increase at the ZLB large? Third, can even wasteful government spending be beneficial at the ZLB? We address these questions across several possible specifications and solution methods of the standard New Keynesian model, and the answer we reach is consistently "no".

A standard approach to answering these questions is to vary the parameter configuration - in particular, the size of the discount rate shock that takes the economy into a recession and to the ZLB, its persistence, and the degree of price stickiness - and show that, for some configurations, the optimal government spending increase can be very large. However, in general such configurations also deliver declines in GDP that can be several times the decline of a typical recession. Thus, our strategy is to fix the decline in GDP at the ZLB at 4 percent - a sizeable recession - and to study the optimal increase in government spending across different specifications and solution methods.

We start with the same specification and the same calibration as CER (2011), which features a stochastic duration of the discount rate shock and assumes that government

[1] In a different setup, not specific to the ZLB, Galí (2014) shows that an increase in wasteful government spending, financed with money creation, can increase welfare.

spending provides direct utility to the representative agent. In the loglinearized version of this model ("LS model" henceforth), we find, like many others, that the consumption multiplier of government spending is quite large, at about 2; still, the optimal increase in government spending in a 4 percent recession is just .5 percent of steady state GDP. Also, it is enough to reduce slightly the persistence of the ZLB shock or the slope of the Phillips curve (the latter to values more consistent with most of the existing empirical literature) for the optimal government spending increase to be arbitrarily close to zero.

Larger values of optimal spending at the ZLB obtain only for parameter values that imply otherwise unreasonably large recessions. The key intuition is that, as the recession gets larger and the economy approaches the starvation point (the point where private consumption is zero), there are two important consequences. First, the marginal utility of consumption is very high. Second, the multiplier of government spending on private consumption is also very high, and can indeed become unboundedly large, as already emphasized in other contributions (see e.g. CER, 2011; Woodford, 2012 or Eggertsson, 2009). The welfare effect in this extreme parameter region is driven entirely by this explosive behavior: government spending is very effective in boosting private consumption, and at the same time consumption is highly valued because the recession is deep.

The explosive behavior of multipliers observed in the LS model does not arise in a model with deterministic duration of the ZLB (Carlstrom, Fuerst and Paustian, 2013), nor in a stochastic model when solved nonlinearly (Braun, Körber, and Waki, 2013; Christiano and Eichenbaum, 2013). In addition, loglinearization of the model, whether with stochastic or deterministic duration of the ZLB, is likely to provide a poor approximation to the true solution because the underlying shock is quite large. Thus, we compare the LS model both with the nonlinear solution of the same model ("NLS model" henceforth) and with the loglinearized solution of the deterministic duration model ("LD model" henceforth). Essentially the same conclusions apply: at the ZLB associated with a 4 percent recession, the optimal increase in government spending at the ZLB is modest, between 1.1 percent in the LD model and .8 percent in the NLS model.

We next address the third question, namely whether even *wasteful* government spending can be beneficial at the ZLB, simply because it reduces a large (negative) output gap that cannot by definition be reduced using monetary policy at the ZLB. To formalize this notion, we assume that the increase in government spending that occurs at the ZLB is pure waste, while the steady state amount of government spending still delivers utility

as before. We now find that, at the baseline parameter values, the optimal increase in government spending at the ZLB is zero in all models and solutions, despite the fact that the multiplier can still be very large. The optimal increase in wasteful government spending is positive on an extreme, and very small, range of parameter values in the stochastic duration models, where once again the result is driven by the explosive behavior near the starvation point discussed above.

In a model with useful government spending, Woodford (2011) has argued that the case for welfare enhancing government spending at the ZLB can be made only for large shocks that induce a recession of the magnitude observed under the Great Depression. Thus, we next use Woodford's approach to replicate stylized facts of the Great Depression, i.e., a 28.8 per cent fall in GDP and a 10 per cent annual deflation. In the LS version of this model with useful spending, like Woodford (2011), we find that there is a large welfare scope for increasing government spending at the ZLB: the optimal value is about 14.5 per cent of GDP. When we assume that spending is wasteful (unlike Woodford 2011), we still find an optimal increase in spending of 13.5 percent of GDP.

We show, however, that these findings hinge upon two features of the calibration: first, the economy is close to the starvation point, where it exhibits the explosive behavior emphasized earlier; second, and in order to replicate the deflation evidence, the Phillips curve must be extremely flat, implying a very large degree of price stickiness (translated in Calvo terms, a price duration of 20 quarters). The latter feature implies that the welfare cost of the ZLB, stemming from the negative output gap, is very high.[2] In fact, in both the LD and the NLS models, we find, conditional on the same Great Depression calibration, that once again the optimal level of wasteful government spending is zero.

Our analysis is related to Werning (2011). He studies the general determinants of a liquidity trap in a New Keynesian model, as well as the optimal decomposition of government spending into "stimulus" and "opportunistic".[3] Werning's analysis differs from ours in four main respects. First, it focuses on the joint determinants of optimal

[2] We define GDP as output net of the price adjustment cost; this distinction is relevant only in the nonlinear model, insofar as the price adjustment cost is quadratic in inflation (and hence drops out when taking a linear approximation). The large difference between output and GDP here occurs precisely because the price adjustment cost needed to fit deflation numbers is so high–an issue discussed also by Braun, Körber, and Waki (2013) when analyzing the Great Depression in a nonlinear model.

[3] A subtle but important difference between our analysis and Werning (2011) is that he defines opportunistic spending as the time-varying level of spending consistent with the "Samuelson" condition for the optimal provision of public goods; whereas we define "Samuelson spending" as the (constant) value of efficient spending in the steady state, and measure cyclical spending in deviation from that.

monetary and fiscal policy in a liquidity trap, depending on whether the fiscal and/or the monetary authority can commit (in a way more similar to Nakata 2012). Second, it focuses exclusively on a linearized and perfect-foresight, deterministic environment, whereas we compare a stochastic and a deterministic environment, and analyze both the linearly approximated and the full nonlinear model solution. Third, it conducts the analysis in a continuous time environment, which (relative to ours) is less prone to a quantitative evaluation. Fourth, it does not study the case of wasteful spending. In this vein, we view our paper as a complement to his.

The outline of the paper is as follows. Section 2 presents the model. In section 3 we solve the loglinearized version of the stochastic duration model. Section 4 discusses the welfare effects of government spending at the ZLB, optimal government spending, and robustness to variations in the three key parameters described above. Section 5 presents the nonlinear solution of the stochastic duration model, and the loglinearized solution of the deterministic duration model. Section 6 discusses the key features of the model in a calibration that delivers a decline of GDP as in the Great Depression. Section 7 concludes.

2 The model

To facilitate comparison with what is now a standard model in the literature on the ZLB, we start from exactly the same specification as CER (2011).[4] We present its main features here, leaving the full solution to Appendix A.

A representative household maximizes the expected discounted value of momentary utility, $\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j \right) U(C_t, N_t, G_t)$, where C_t is consumption, N_t is hours worked and G_t is government spending on goods produced by the private sector. The discount factor is $\beta_j = 1$ for $j = 0$ and $\beta_j = (1 + \rho_j)^{-1}$ for $j \geq 1$; the discount rate ρ_j varies exogenously, in a way specified below (if ρ_j were a constant, the cumulative discount factor would simply be $\prod_{j=0}^{t} \beta_j = \beta^t$). Preferences are non-separable in consumption and hours:

$$U(C_t, N_t, G_t) = \frac{\left[C_t^{\zeta} (1 - N_t)^{1-\zeta} \right]^{1-\sigma} - 1}{1 - \sigma} + \chi_G \frac{G_t^{1-\sigma} - 1}{1 - \sigma}, \tag{1}$$

[4] The only difference is that, as in Christiano and Eichenbaum (2012), we use Rotemberg pricing rather than Calvo pricing. That is because we also solve the nonlinear model and, as it is by now well understood, the Rotemberg model is much easier to solve nonlinearly–because it has an explicit nonlinear Phillips curve, and it does not introduce any extra state variable. This difference is immaterial insofar as the solution of the linearized model is concerned.

where $\sigma > 0$, $0 < \zeta < 1$, and $\chi_G \geq 0$ parameterizes the utility benefit of public spending.[5]

In (1), C_t is a basket of a continuum of individual varieties indexed by z, with constant elasticity of substitution ε:

$$C_t = \left(\int_0^1 C_t(z)^{(\varepsilon-1)/\varepsilon} dz \right)^{\varepsilon/(\varepsilon-1)} \quad \varepsilon > 1.$$

Each differentiated good is produced by a different monopolistically competitive firm, with a linear production function: $Y_t(z) = N_t(z)$. Each firm chooses its price subject to a convex adjustment cost (as in Rotemberg 1982) in order to maximize the present discounted value of its profits. The government purchases a basket of the consumption goods G_t with the same composition as the private consumption basket and levies lump-sum taxes to finance this spending.

There is a constant sales subsidy that corrects the markup distortion in steady-state and makes steady-state profits equal to zero by inducing marginal cost pricing.

Useful vs. wasteful spending. We study two cases: *useful* and *wasteful* government spending. This distinction pertains only to the spending occurring at the ZLB. In both cases, and in the steady state away from the ZLB, government spending is determined optimally by the typical Samuelson condition for efficient public good provision:

$$U_C(Y - G) = U_G(G), \tag{2}$$

where a variable without time subscript denotes a steady-state value. Condition (2) states that the marginal utilities of private and public expenditure should be equalized.

This condition implies the following expression for the utility weight of government spending (see Appendix A):

$$\chi_G = \zeta \left(\frac{G}{Y}\right)^{\sigma} \left(1 - \frac{G}{Y}\right)^{\zeta(1-\sigma)-1} \left(\frac{1-N}{N}\right)^{(1-\zeta)(1-\sigma)}. \tag{3}$$

Assuming $G/Y = .2$ (in line with the average US postwar experience), together with the other parameters in the baseline calibration described below, gives us a value for χ_G.

In the *useful* government spending case, the extra government spending at the ZLB yields utility in precisely the same way as in the steady state away from the ZLB. In other words, the utility weight in (1) is given by (3). In the *wasteful* government spending

[5] Notice that in the case $\sigma = 1$, the utility in (1) reduces to a separable log-log specification.

case, the extra spending at the ZLB yields no direct utility: hence, the last term in (1) becomes $\chi_G \left(G^{1-\sigma} - 1\right) / \left(1 - \sigma\right)$. Note that if we assumed that in the wasteful government spending case χ_G is zero even outside the ZLB, optimal government spending in steady state would be zero.

We call (steady-state) G "structural" government spending, and the extra government spending that might occur at the ZLB "cyclical" government spending.[6] Thus, our distinction between useful and wasteful spending allows for the possibility, often discussed both in theory and in the policy debate, that government spending in the recession occurring at the ZLB might be of a different nature than in "normal" times.[7]

3 The loglinearized stochastic model

In order to obtain analytical results, we start from a loglinear approximation of the equilibrium conditions around the steady state. With a slight abuse of terminology, we label this the "LS model", to distinguish it from other solutions of the same model and from other models, that we introduce below. Let a lower case letter indicate a log deviation from the steady state. The exceptions are π_t and i_t, which are already in percentage points and are expressed here in levels (steady-state inflation is zero). We obtain the following expressions for the consumption Euler equation and for the Phillips-curve:

$$\begin{aligned} c_t = \mathbb{E}_t c_{t+1} &- \frac{(1-\zeta)(1-\sigma)}{1-\zeta(1-\sigma)} \frac{N}{1-N} \left(n_t - \mathbb{E}_t n_{t+1}\right) \\ &- \left[1 - \zeta(1-\sigma)\right]^{-1} \left(i_t - \mathbb{E}_t \pi_{t+1} - \rho_t\right) \end{aligned} \tag{4}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(1 + \frac{N}{1-N} \frac{Y-G}{Y}\right) c_t + \kappa \frac{N}{1-N} \left(\frac{G}{Y}\right) g_t \tag{5}$$

[6] See Werning (2012) for a related decomposition.

[7] In the wasteful spending case, what the government does at the ZLB is similar to "fill(ing) old bottles with bank-notes (and) bury(ing) them at suitable depths in disused coal-mines" (J.M. Keynes, *The General Theory of Employment, Interest and Money* (London: Macmillan, 1936), p. 129). The metaphor is not exactly right because in our model the government taxes people in order to buy a good produced by the private sector that has a positive marginal cost, but provides no utility once purchased by the government. A better analogy is with cars bought by the government for the police. In our model, these cars are useful up to the point where the Samuelson condition holds. Extra cars beyond that point have zero utility. However, the metaphor is still useful in that buying these extra cars in our model does reduce the output gap.

where N are steady state hours, $\kappa \equiv (\varepsilon - 1)/\nu$, and ν is the convex price adjustment cost parameter (the higher ν, the higher the degree of price stickiness).[8] The two equations above describe the dynamics of the economy for arbitrary exogenous (stochastic or deterministic, see below for more details) processes ρ_t and g_t.

The discount rate To model the ZLB in a tractable form, we make the same Markovian assumption as CER (2011), Woodford (2011), and several others: if the discount rate ρ_t takes the negative value $\rho_L < 0$, with probability p it will be ρ_L in period $t+1$ as well; with probability $1-p$ it will revert to the steady state value ρ; once it returns to the steady state, it remains there. We assume that the steady state value of the discount rate is $\rho = \beta^{-1} - 1 = .01$ in the benchmark case. Formally:

$$\begin{aligned} &\Pr\{\rho_{t+1} = \rho_L | \rho_t = \rho_L\} = p; \\ &\Pr\{\rho_{t+1} = \rho | \rho_t = \rho_L\} = 1 - p; \\ &\Pr\{\rho_{t+1} = \rho_L | \rho_t = \rho\} = 0. \end{aligned} \tag{6}$$

Similarly, the process for g_t can either take the values $g_L > 0$ (in the liquidity trap state) or 0 (in the steady state); since the process is perfectly correlated with the discount rate shock generating the ZLB, it inherits the same transition matrix (transition probabilities p and $1-p$, with the steady state as an absorbing state).

At this stage, it is useful to review the intuition of why a negative shock to the discount rate can take the economy to the ZLB, and why government spending can have a large multiplier at the ZLB. When the discount rate falls (the discount factor increases), the agent would like to save more, hence to reduce current consumption. In equilibrium, savings must be zero. With flexible prices, the real interest rate would become negative, so as to convince the agent to make zero savings; as the real interest rate tracks the natural interest rate, the output gap would also be zero.

When prices are sticky, however, the slack in the economy generates expected deflation, and this induces an *increase* in the real interest rate. Hence, it is the nominal interest rate that bears all the downward adjustment on the real interest rate, so as to reduce savings. Thus, the nominal interest rate falls as much as it can, to zero. If the fall in the discount rate is sufficiently large, this is not enough to reduce savings to zero; the rest of

[8] The log-linear Phillips curve of the convex adjustment cost model is isomorphic to that obtained using a Calvo-Yun setup; in the latter case, the slope of the Phillips curve would read $\kappa = \alpha^{-1}(1-\alpha)(1-\alpha\beta)$, where α is the probability that the price remains fixed in any given quarter.

the adjustment is borne by income, which falls until net savings is zero. Thus, a discount rate shock causes the economy to enter a recession and the nominal interest rate to reach the ZLB.

In this situation, a persistent increase in government spending raises labor demand and therefore the real marginal cost; this translates into higher expected inflation, hence into a negative real interest rate (given a zero nominal interest rate). Thus, government spending has a particularly large multiplier because, by reducing the real interest rate, it tilts the Euler equation towards today's private consumption; this raises private consumption and output today.

Monetary authority The monetary authority sets the short-term nominal interest rate according to the feedback rule:

$$i_t = \max\left(\iota_t + \phi_\pi \pi_t, 0\right) \tag{7}$$

We assume that the intercept is $\iota_t = \rho$.

Solution The solution of the model consists of time-invariant equilibrium responses of consumption and inflation that apply as long as the ZLB is binding. Their expressions are

$$\begin{aligned} c_L &= \frac{1-\beta p}{\Omega}\rho_L + M_c \frac{G}{Y-G} g_L \\ \pi_L &= \frac{\kappa\left(1+\frac{N}{1-N}\frac{Y-G}{Y}\right)}{\Omega}\rho_L + M_\pi \frac{G}{Y}\, g_L, \end{aligned} \tag{8}$$

where $\Omega \equiv (1-\beta p)(1-p) - \kappa p\left(1+\frac{N}{1-N}\frac{Y-G}{Y}\right)$ and the consumption and inflation multipliers[9] are, respectively:

$$\begin{aligned} M_c &\equiv \frac{\left[(1-\beta p)(1-p)\zeta(\sigma-1) + \kappa p \frac{N}{1-N}\frac{Y-G}{Y}\right]}{\Omega} \\ M_\pi &\equiv \frac{(1-p)\kappa\left[\left(\frac{Y}{Y-G}+\frac{N}{1-N}\right)\zeta(\sigma-1) + \frac{N}{1-N}\right]}{\Omega} \end{aligned}$$

Bifurcation point The economy has two steady states: one is the zero inflation steady state, and the other the ZLB. We assume the economy starts from the former. When $\Omega > 0$

[9] Notice that the in this linearized environment without investment the output multiplier is $\mathcal{M}_y = 1 + \mathcal{M}_c$.

the economy only visits the ZLB for a while because the zero inflation steady state is the absorbing state of the Markov process. When $\Omega < 0$ the economy is subject to sunspot-driven fluctuations, i.e., it can be driven into the liquidity trap state by pure sunspot shocks with persistence p, even when $\rho_L = \rho > 0$.[10] Hence, $\Omega = 0$ is a bifurcation point and, in the loglinearized model, an asymptote: the elasticities of endogenous variables to shocks tend to infinity.[11] We will focus on the more standard case $\Omega > 0$, where liquidity traps occur because of fundamental, rather than sunspot changes. *Ceteris paribus*, this restriction is satisfied, under both utility specifications, when shocks have small persistence (p low), and prices are sticky (κ low). We show below that the value of Ω, and therefore of the multipliers, is highly sensitive to the values of several parameters of the model.

Starvation point. Another restriction on parameters obtains by imposing non-negativity of private consumption at the ZLB ($C_L > 0$, or $1 + c_L > 0$), with no fiscal policy intervention ($G_L = G$). From the expression for c_L in (8), this condition boils down to $1 + (1 - \beta p)\,\Omega^{-1}\rho_L > 0$. Thus, since $\rho_L < 0$, the economy reaches the starvation point as it approaches (and before it reaches) the bifurcation point, since $\lim_{\Omega \to 0} c_L = -\infty$.

Calibration. We start with the same parameter values as CER (2011). In particular, we assume $\rho_L = -0.0025$, implying a natural interest rate at the ZLB of $-$ 1 percent per annum. In turn, this implies that output falls by 4 percent per annum, regardless of the value of σ. Table 1 describes the main parameter values in this baseline case.

Table 1. Baseline calibration

Parameter	Description	Value
p	transition probability	0.8
ρ_L	quarterly discount rate	$-.0025$
β	discount factor in steady state	0.99
σ	relative risk aversion	2
φ	inverse labor elasticity	$N/(1-N)$
κ	slope of the Phillips curve	0.028
ϕ_π	Taylor rule coefficient	1.5

[10] See Benhabib, Schmitt-Grohe, and Uribe (2002) for an analysis, and Mertens and Ravn (2012) for the implications in terms of consumption multipliers.

[11] Formally, the limits of the elasticities x are $\lim_{\Omega \searrow 0} x(\Omega) = +\infty$ and $\lim_{\Omega \nearrow 0} x(\Omega) = -\infty$ for $x(\Omega) = \{\mathcal{M}_c,\ \mathcal{M}_\pi, \partial c_L/\partial \rho_L, \partial \pi_L/\partial \rho_L\}$.

To put things in a "Calvo probability" perspective, $\kappa = 0.028$ corresponds, in a linearized equilibrium and conditional on a price elasticity of demand of 6, to a probability of not being able to reset the price of 0.85, or an average price duration of 6.7 quarters. Finally, given $N = 1/3$, $G/Y = 0.2$ and the optimal steady state subsidy, we have that $\zeta = 0.2857$ (see Appendix A for details). Notice that, under the baseline calibration described above, the starvation point is reached at $p = 0.82319$ while the bifurcation point is $p = 0.82435$.[12]

4 Welfare and optimal spending in the loglinearized stochastic model

We now turn to the central theme of our analysis, the welfare implication of government spending at the ZLB. We define the *welfare gap* $\tilde{U}_L$ as:

$$\tilde{U}_L(g_L) = 100 \cdot \frac{U_L(g_L) - U_L(0)}{|U_L(0)|}, \tag{9}$$

where U_L is the present discounted value of the household's utility conditional on the economy being at the the ZLB. Hence the welfare gap is the percentage variation in utility between a scenario where spending increases at the ZLB, $U_L(g_L)$, and a scenario where spending is kept constant at its steady-state value G, $U_L(0)$. See Appendix B for a formal derivation of U_L.

4.1 Approximation method

As a large literature dealing with optimal monetary policy has recognized in the context of welfare analyses using a second order approximation to the utility function (see Woodford, 2003 Ch. 6 and Woodford, 2012 in a ZLB context), second order terms in the equilibrium conditions are important in sticky price models for capturing the welfare costs of inflation. The inflation distortion (be it through a real resource cost, as in the Rotemberg model, or through relative price dispersion, as in the Calvo model) has second order effects through the resource constraint, and hence it matters for welfare, although it is negligible to first order when approximated about a zero inflation steady-state. In particular, in our model

[12] The same thresholds for the Phillips curve slope are (given $p = 0.8$) $\kappa = 0.03669$ and $\kappa = 0.03716$ for the starvation and bifurcation points, respectively.

the resource constraint of the economy reads:

$$C_t + G_t = \frac{N_t}{\Delta_t}\left(= \frac{Y_t}{\Delta_t}\right) \tag{10}$$

where $\Delta_t \equiv \left(1 - \frac{\nu}{2}\pi_t^2\right)^{-1} \geq 1$ represents the distortion coming from inflation costs, and the second equality uses the production function. A second order approximation of the resource constraint about zero inflation gives:

$$y_L = n_L = \frac{Y - G}{Y} c_L + \frac{G}{Y} g_L + \frac{1}{2}\nu\pi_L^2 \tag{11}$$

To capture the distortion associated with imperfect price adjustment, and the way in which government spending can alleviate it, we use (11) in the nonlinear utility function in order to evaluate welfare.[13] We call this the "second order" approximation of the LS model.

In contrast, CER (2011) evaluate welfare by replacing the first order approximation of the resource constraint (10),

$$y_L = \frac{C}{Y} c_L + \frac{G}{Y} g_L, \tag{12}$$

into the nonlinear utility function. To keep the comparison with CER clear, we also study this approximation method, which we label "first order" approximation of the LS model.

4.2 Welfare analytics

In this section we clarify the channels through which government spending affects welfare at the ZLB. Therefore, we study the effect on welfare of an increase in G_L, conditional on being at the ZLB.

Welfare at the ZLB is:

$$\Psi_L \times \left[U\left(C_L, N_L\right) + v\left(G_L\right)\right],$$

where $\Psi_L \equiv \frac{1+\rho_L}{1+\rho_L - p}$. The derivative of welfare with respect to G_L (ignore Ψ_L as it is invariant to G_L):

$$U_C\left(C_L, N_L\right)\frac{dC_L}{dG_L} + U_N\left(C_L, N_L\right)\frac{dN_L}{dG_L} + v'\left(G_L\right) \tag{13}$$

[13] Differently from Woodford (2012), we use the nonlinear utility function rather than taking a second-order approximation of the utility function; in other words, our approach captures terms of order three or higher in utility, although these are likely to be negligible. Below, we also solve the full nonlinear model.

The intratemporal optimality condition $U_N(C_L, N_L) = -W_L U_C(C_L, N_L)$ implies that W_L is the *marginal rate of substitution (MRS)* of leisure for consumption, i.e., the number of consumption units the household is willing to give up in order to have one extra hour of leisure. From the resource constraint, $C_L = N_L/\Delta_L$, where $1/\Delta_L$ is the *marginal rate of transformation* (*MRT*) of leisure into consumption, i.e., the number of units of the good the economy must give up in order to have one more hour of leisure and stay on the production possibility frontier. Replacing these equilibrium conditions into the derivative of utility we obtain:

$$\frac{dU_L}{dG_L} = W_L \Delta_L U_C(C_L, N_L) \left[\underbrace{\left(\frac{1}{W_L \Delta_L} - 1 \right) \frac{dC_L}{dG_L}}_{\text{multiplier channel}} \underbrace{-1}_{\text{income effect}} \underbrace{-\frac{C_L}{\Delta_L} \frac{d\Delta_L}{dG_L}}_{\text{inflation distortion } = -\nu \pi_L \Delta_L^2 \frac{d\pi_L}{dG_L}} \right] + v'(G_L) \tag{14}$$

For simplicity, we just consider the "wasteful" case, corresponding to the terms in square brackets (i.e., we ignore the last additive term $v'(G_L)$ which is positive anyway).

The term labeled "multiplier channel" implies a positive effect on welfare if the term $(W_L \Delta_L)^{-1} - 1$ and the nonlinear multiplier dC_L/dG_L have the same sign. If the multiplier is positive, the effect is positive if and only if $(W_L \Delta_L)^{-1} > 1$. The left-hand side of this inequality, $(W_L \Delta_L)^{-1}$, is the ratio of the MRT to the MRS as defined above. In a steady state without an optimal subsidy, and due to the monopolistic competition distortion, this term exceeds one.[14] At the ZLB, with a large negative output gap, the same term widens, due to the countercyclicality of markups.[15] More precisely, $(W_L \Delta_L)^{-1}$ is higher when W_L is low – because that is when the relative price of leisure in consumption units is low –, and when the distortion Δ_L is low – because this is when the household gets more consumption out of one unit of extra labor (the marginal rate of transformation is high). The right-hand side of the inequality $(W_L \Delta_L)^{-1} > 1$ captures the idea that producing consumption requires extra work, which is costly to the household – thus representing a negative effect on welfare associated with an increase in consumption.

To summarize, the multiplier channel implies a positive effect on welfare when the MRT exceeds the MRS, and is increasing in the multiplier. As emphasized above, a

[14] Note that, in the steady state and under an optimal subsidy, we have $W = \Delta = 1$, and this channel is shut off.

[15] In other words, and for a given non-linear consumption multiplier, the term $(W_L \Delta_L)^{-1}$ captures movements in the so called "labor wedge".

positive consumption multiplier is a defining feature of the ZLB. Hence, in the presence of a negative output gap, the multiplier channel has a generally positive contribution to welfare at the ZLB.

In the ZLB equilibrium, whether the MRT exceeds the MRS will depend on the equilibrium value of inflation. Indeed, replacing the ZLB equilibrium value of W_L as a function of inflation (from the ZLB Phillips curve), one can derive a threshold value for inflation (deflation) such that this condition holds and the effect of the multiplier channel on welfare is positive (as long as the multiplier itself is positive).[16] The intuition is as follows. When deflation is larger than this threshold (in absolute value), the MRT (marginal product of labor, i.e., Δ_L^{-1}) still increases; but the real wage (marginal cost) increases too, and does so by more than the marginal product of labor (MRT). That is because in the NKPC there is a term that is linear in inflation, and one that is quadratic. At "low" values of deflation the first, linear term dominates and the marginal cost goes down when inflation goes down. But for large enough deflations, the second quadratic term dominates, and the marginal cost increases when inflation falls. Of course this logic is not captured in the linearized model because in a small neighborhood of the steady state only the linear term matters.

The term labeled "income effect" is simply the negative income effect of taxation, or the crowding out effect of government spending. This term is independent of being at the ZLB or not, and is indeed independent of whether prices are sticky or not.

The term labeled "inflation distortion" captures the inefficiency stemming from movements in inflation in a sticky price environment. That term is positive as long as the derivative of the distortion Δ_L with respect to G_L is negative (i.e., an increase in spending reduces the distortion). Note that $d\Delta_L/dG_L = \nu\pi_L\Delta_L^2\frac{d\pi_L}{dG_L}$ and, since $\pi_L < 0$, government spending at the ZLB reduces the distortion and increases welfare if it is inflationary. Intuitively, creating inflation alleviates the deflation occurring at the ZLB and allows more resources to be allocated to consumption rather than paying the adjustment cost. The stickier prices, the larger ν, and the stronger this channel. This alleviation of the inflation distortion through the inflationary effect of a government spending increase constitutes a de facto efficiency gain because it expands the production possibility frontier. Notice

[16] Specifically, $(W_L\Delta_L)^{-1} = \frac{\left(1-\frac{\nu}{2}\pi_L^2\right)}{\frac{\nu}{\varepsilon}\left(1-\frac{p}{1+\rho_L}\right)\pi_L(1+\pi_L)+\frac{\varepsilon-1}{\varepsilon}(1+s)}$. Under an optimal subsidy, the threshold is $\pi_L > -\left(1-\frac{p}{1+\rho_L}\right)/\left(1+\frac{\varepsilon}{2}-\frac{p}{1+\rho_L}\right)$ which in our baseline calibration implies $\pi_L > -0.062$ per quarter, while in the GD calibration $\pi_L > -0.028$ per quarter.

that the "inflation distortion" term is not captured in a simple first-order approximation of the model, because it is of order two. Thus, a first-order approximation of Δ_L around a zero-inflation steady state always equals zero.

4.3 Welfare and optimal spending in the baseline scenario

Figure 1 plots the welfare gap as a function of the increase in government spending, for a domain such that the ZLB keeps binding.[17] Notice that government spending at the ZLB is measured in *units of steady state output*. The left-hand and right-hand panels display the cases of useful and wasteful government spending, respectively, for the two approximations. In each panel, the difference between the two curves is hence a measure of the welfare effect of ZLB government spending due exclusively to the second order inflation distortion term.

Two results are worth emphasizing. First, in the useful spending case, the optimal increase in government spending at the ZLB is just 0.5 percent of steady state output in both approximations. This value is much lower than the one in CER (2011), who report a value of optimal government spending of 30 percent of its own steady state, which in turn corresponds to 6 percent of steady state GDP. The main reason for this difference lies in the calculation of the optimal utility weight of government spending χ_G from (3). The *erratum* by CER (2013) provides simulation results in line with those in the left panel of Figure 1.[18]

Second, when government spending is wasteful, utility is monotonically decreasing at a very fast rate in the first order approximation; as a result, the level of g_L that maximizes welfare under useful spending - 0.5 percent of GDP - would cause a decline in welfare by 300 percent under wasteful spending.[19] Utility changes much less in the second order approximation: the optimal level of wasteful ZLB spending is 0.12 percent of GDP. The

[17] The domain over which the ZLB keeps binding is determined by the condition $\rho + \phi_\pi \pi_L < 0$, which (upon replacing the equilibrium value of π_L from (8)) implies a threshold for g_L.

[18] There is still a small residual difference, in that the value of optimal spending obtained by CER (2013) is slightly higher, 0.8 percent of steady-state output. The reason is the same slight difference in the value of the Phillips curve slope κ (0028 versus 0.03) that we highlighted in the Introduction. This tiny difference also will play a role in sections 4.3 and 4.5. CER (2013) is available at http://faculty.wcas.northwestern.edu/~lchrist/research/cer_gov/erratum.pdf

[19] Notice that, because of our distinction between structural and cyclical spending, an argument for cutting government spending cannot be made in the wasteful case. Our finding merely implies that, if cyclical spending is wasteful, its optimal value is zero, while structural, steady-state spending is kept at its optimal value dictated by the Samuelson principle.

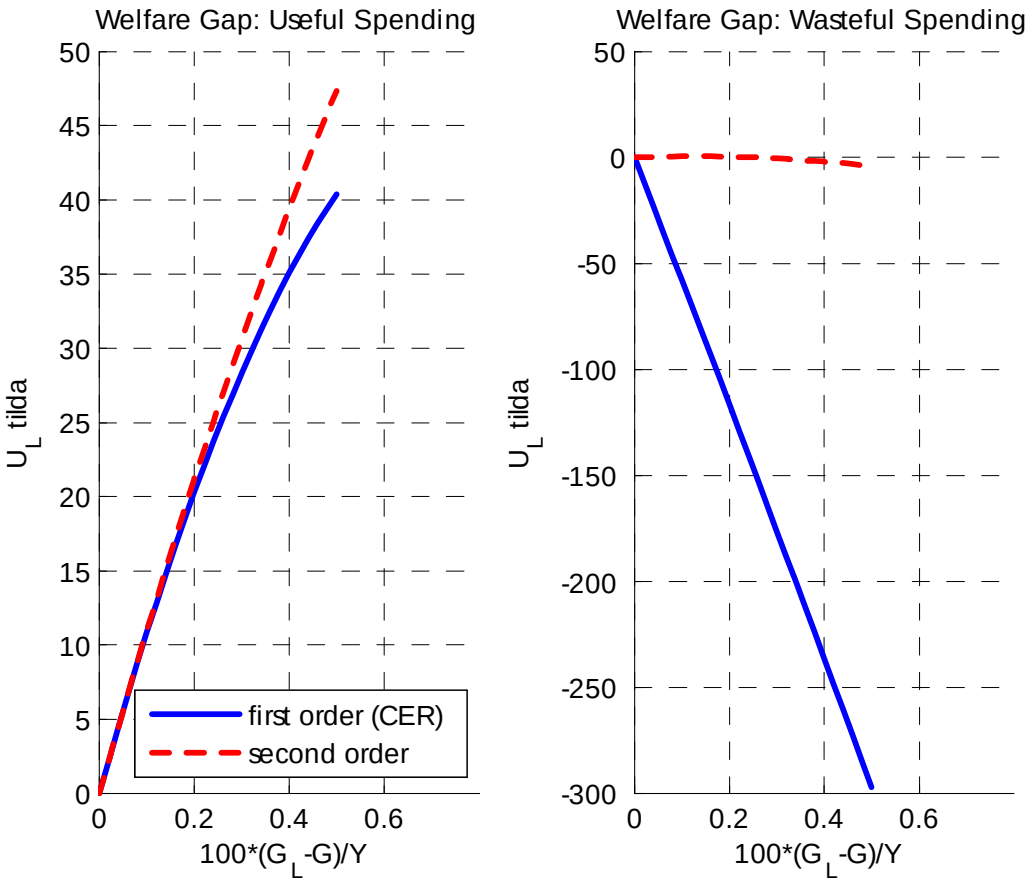

Figure 1: Welfare gaps and government spending at the ZLB. LS model.

intuition for the difference between the two approximation methods is that deflation at the ZLB causes a positive loss due to the quadratic term; increasing government spending at the ZLB reduces deflation and therefore the utility loss. This difference is particularly noticeable in the wasteful spending case, while in the useful spending case the first order direct increase in utility brought about by government spending dominates the second order effect on utility through the inflation distortion.

4.4 The role of the ZLB persistence and of price rigidity

We now study how the optimal government spending depends on three key parameters: the ZLB persistence p, the slope of the Phillips curve κ, and the ZLB discount rate ρ_L. Recall that p measures the probability that, conditional on the economy being in the liquidity trap in a given period, it will remain in that state in the following period. Hence $1/(1-p)$ measures the *expected* duration of the trap and also, given the perfect correlation between the discount rate shock and the government spending shock, the expected duration of the increase in government spending. A higher p therefore means a higher expected present value of government spending at the ZLB, hence higher expected inflation (or lower expected deflation) and a larger decline in the real interest rate. Thus, the multiplier is increasing in p.

It is by now well known that in the LS model the relation between the consumption multiplier and p is also highly non-linear: as the value of p approaches the bifurcation point, the multiplier increases sharply.[20] This is illustrated in Figure 2 (obviously this figure applies to both approximation methods). From this figure, it is also clear that the value of $p = 0.8$ of the baseline calibration (highlighted by a vertical dotted line) is a point at which a marginal increase in p generates a very large increase in the multiplier, and a huge decline in private consumption (the latter becomes exactly zero at the starvation point $p = 0.82319$); both the multiplier and consumption at the ZLB are very steep functions of p.

We now show that not only the multiplier, but also the optimal increase in government spending is highly nonlinear in p. The first panel of Figure 3 plots $g_L^* = \arg\max \widetilde{U}_L$, i.e., the optimal increase in government spending at the ZLB (expressed in percentage points of steady-state GDP) as a function of p. The domain for p is limited to the left by the requirement that the ZLB be binding,[21] and to the right by the starvation point ($p = 0.82319$ in the baseline scenario).[22] Optimal government spending at the ZLB increases with p, to reach a maximum of 1.9 percent of GDP in the useful spending case. The picture for the wasteful spending case is similar, except that the optimal increase

[20] See, e.g., CER (2011) and Woodford (2011). A similar discussion applies to the nonlinearity in κ, the slope of the Phillips curve.

[21] Formally, the lower limit is obtained by replacing the equilibrium value of inflation at the ZLB into the Taylor rule: $\rho + \phi_\pi \kappa \left(1 + \frac{N}{1-N}\frac{Y-G}{Y}\right)\Omega^{-1}\rho_L < 0$. For the baseline calibration, this requires $p > 0.79$.

[22] Since the size of the discount rate shock is given and the fall in GDP/consumption is not otherwise limited here, the non-starvation condition binds.

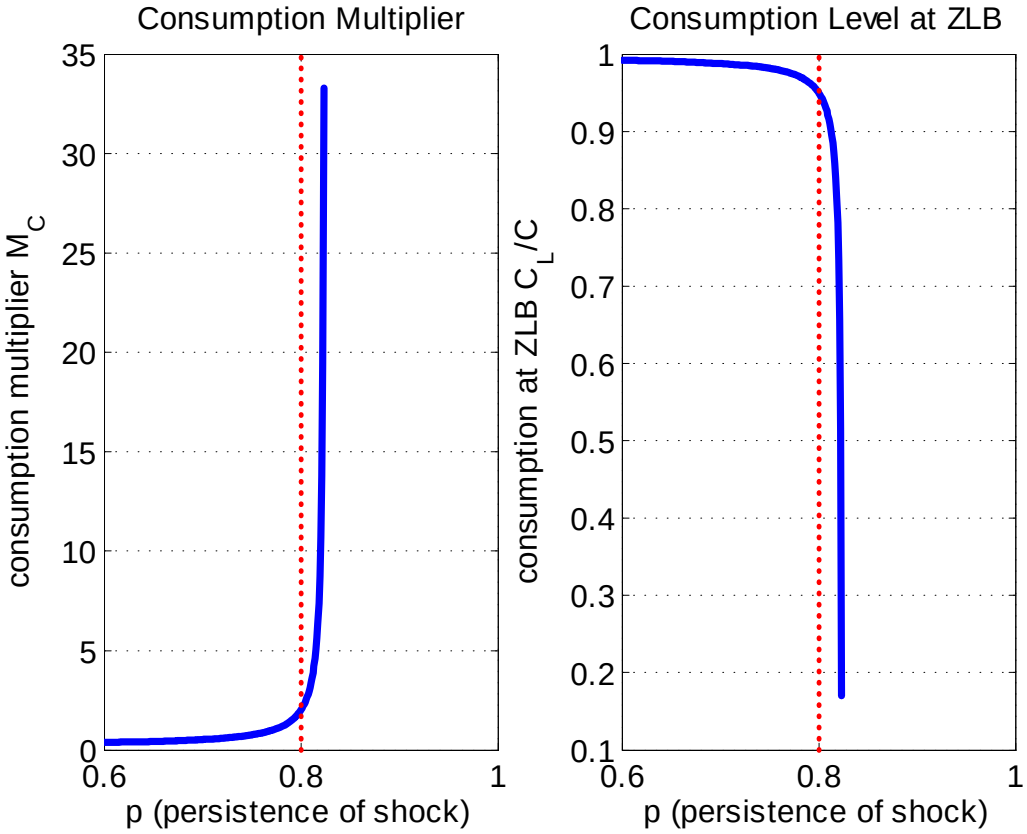

Figure 2: Consumption multiplier (left panel) and consumption level (right panel) as a function of p. LS model.

in government spending starts being positive for a slightly higher value of p. In the first order approximation (second panel) the optimal increase in wasteful government spending is 0 except when approaching the starvation point, i.e. at $p = 0.816$ in our grid. Thus, using the original linearization method of CER (2011) would reinforce our conclusion, that the optimal increase in wasteful government spending is zero except on a very small range, and at very high declines of GDP.

The third panel of Figure 3 displays the decline in GDP when the economy enters the ZLB, also as a function of p. This decline too is highly nonlinear in p: as the economy approaches the starvation point, GDP falls by a dramatic 70 percent.

Thus, the larger values of optimal government spending occur when the decline in GDP and consumption is particularly high, and much higher than in any "normal" recession. The intuition is that government spending has a very large multiplier exactly when consumption is low as a consequence of the discount rate shock, and the marginal utility of consumption is very large. In the limit, as consumption at the ZLB is particularly low and close to starvation, it becomes irrelevant what type of government spending is pursued (whether it provides direct utility or not), only its multiplier matters.

A nearly identical pattern is displayed in Figure 4, which plots the optimal increase in government spending and the decline in GDP as a function of κ, the slope of the Phillips curve when expressed in terms of real marginal cost; thus, κ is inversely related to the degree of price rigidity. As κ increases the economy gets closer to the starvation point, implying a larger multiplier, a higher optimal increase in government spending, and a larger decline in GDP when the economy enters the ZLB.

Like in the case of p, the domain of κ in Figure 4 is dictated, respectively, by the condition that the ZLB constraint is binding and that the economy remains below the starvation point. In our baseline calibration, the admissible range of κ is between about 0.0253 and 0.0367. The range of empirical estimates of κ is typically between 0.002 and 0.03; thus, our baseline value of 0.028 would be at the upper end of this range.[23]

[23] For instance, in the classic study of Galí and Gertler (1999) for the US, the estimate of the Phillips curve slope coefficient *on the real marginal cost*, in a specification with no lagged term on inflation, as ours, is 0.023. In our setup, that would imply a value of optimal government spending of nearly zero (provided that the ZLB is binding). In general, values of κ for which, in our simulations, optimal government spending exceeds 1 percent of steady state GDP are well above available empirical estimates. See also Erceg and Linde (2013) on this point. Braun, Körber, and Waki (2013) find a posterior mode estimate of the Rotemberg adjustment cost parameter of 458.4. Given their value for $\varepsilon = 7.67$, this implies a value of $\kappa = 0.0214$ in their case (and a value of $\kappa = 0.0109$ in our case, given $\varepsilon = 6$).

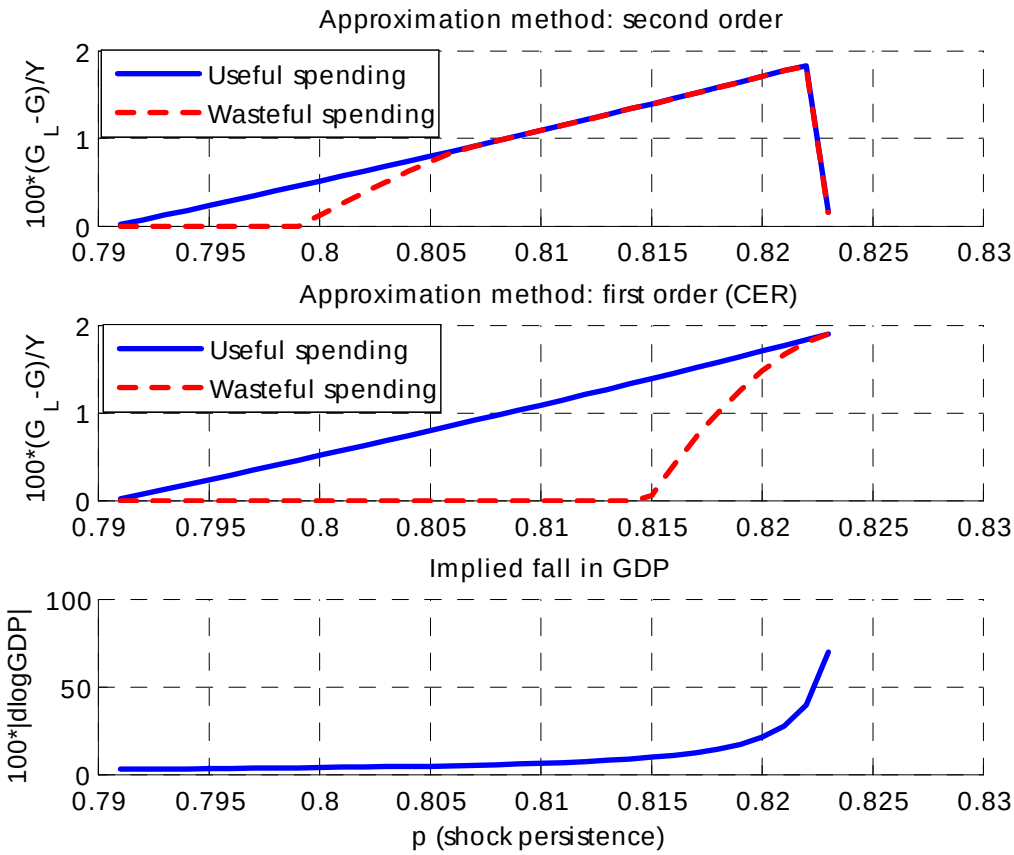

Figure 3: Optimal increase in government spending at the ZLB and decline in GDP as a function of p. LS model.

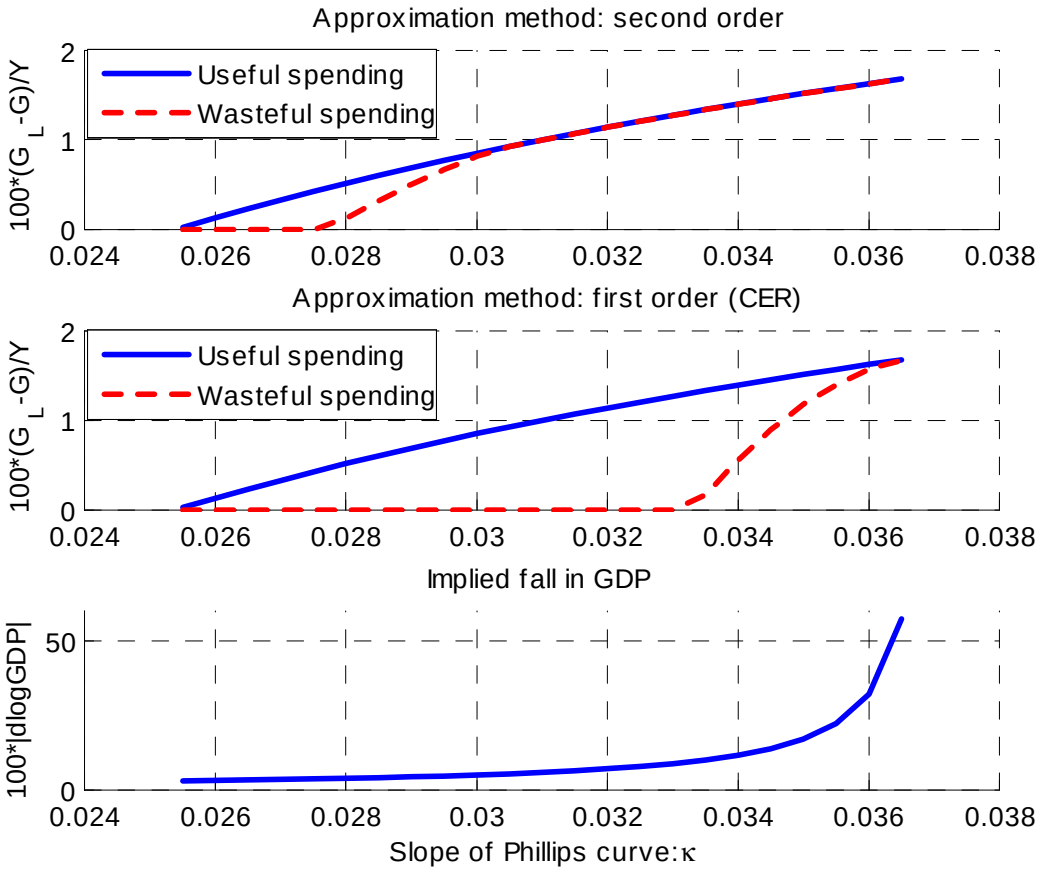

Figure 4: Optimal increase in government spending at the ZLB and decline in GDP as a function of κ. LS model.

4.5 Holding constant the decline in GDP

Because the fall in GDP depends strongly on p and κ, the exercise we have performed so far - studying the optimal increase in government spending as a function of p and κ, holding constant $\rho_L = -.0025$ - can lead to misleading conclusions. For instance, when p is at its maximum admissible level, we find that the optimal increase in government spending is about 1.9 percent of steady state GDP, or 6.3 percent of actual GDP. However, GDP has declined by 70 percent from its steady state; this makes this case not particularly interesting, and difficult to evaluate.

To address this problem, we calculate the optimal increase in spending as a function of p and κ, respectively, but at the same time varying ρ_L so that the annual decline in GDP remains constant at its baseline value, -4 percent. We do not have a feel for the appropriate value of the discount rate shock (which we interpret as a shortcut for whatever causes the economy to hit the ZLB), while a 4 percent GDP decline is a reasonable definition of a sizeable recession.

Figure 5 displays the optimal increase in government spending as a function of p (as usual, the increase in spending is expressed in units of steady state GDP); the implied absolute values of the discount rate are plotted in the bottom panel, in annualized terms.[24] Two findings emerge. In the useful spending case, the optimal increase in government spending is low for most of the domain, reaching a maximum value of about 0.6 percent of steady-state GDP at $p = 0.78$. However, as p increases further and approaches the bifurcation point $p = 0.824$, the optimal ZLB spending falls sharply back to zero.[25]

In the wasteful spending case, on the other hand, optimal ZLB spending is identically zero except for values of p between about 0.80 and 0.82. Once again, in the first order approximation the range over which the optimal increase in wasteful government spending is positive is even smaller: in our grid search, from 0.821 to 0.823. Near the bifurcation, optimal government spending is small because, as the multiplier is so large, the discount rate shock required to achieve a 4 percent decline in GDP is minuscule; as a consequence, the ZLB stops binding at low values of government spending. In the same region, optimal

[24] Note that very low values of p require somewhat implausibly large values of the discount factor shock in order to deliver a 4 percent fall in GDP: e.g. -9 percent per annum when $p = 0.5$.

[25] Since the fall in GDP (and implicitly, consumption) is limited to 4 percent here, starvation never occurs. Therefore, all figures which are plotted for a given fall in GDP have a domain for p or κ that goes arbitrarily close to the bifurcation point. Since multipliers become arbitrarily large when approaching bifurcation, the discount rate shock becomes arbitrarily small.

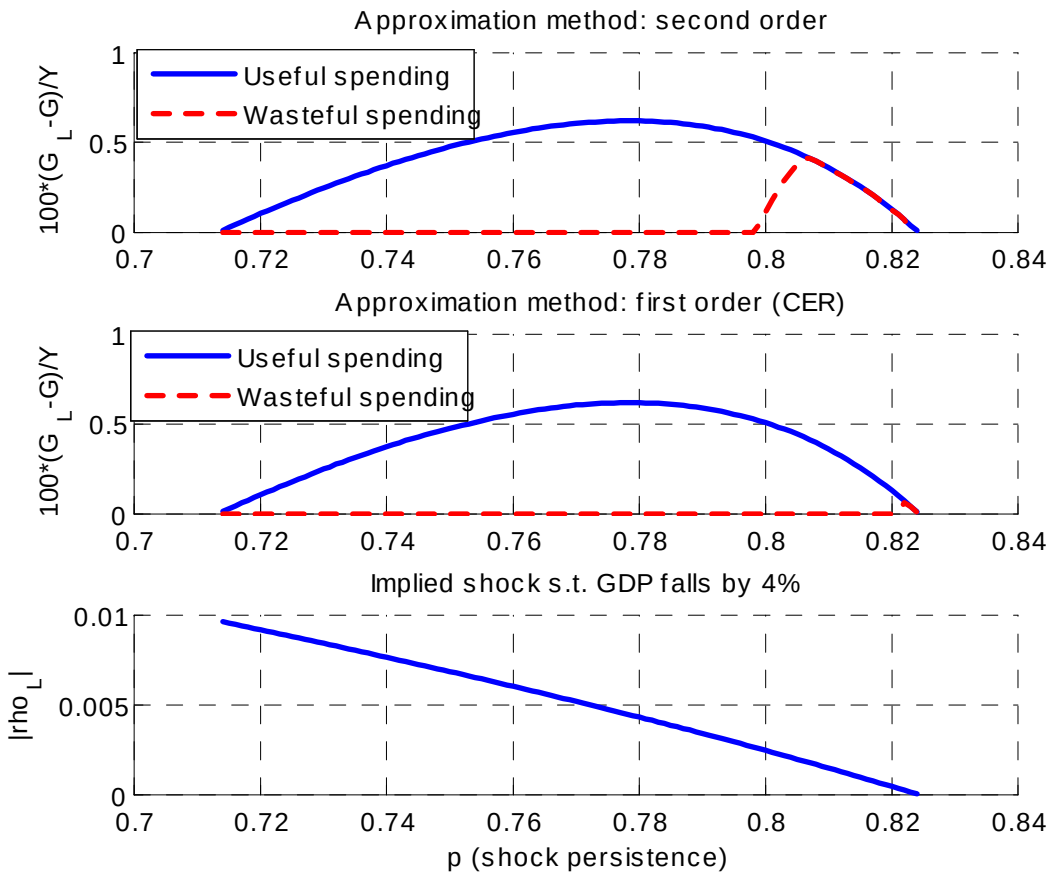

Figure 5: Optimal increase in government spending at the ZLB and implied value of ρ_L as a function of p. Decline in GDP is constant at 4 percent. LS model.

government spending is identical in the useful and wasteful spending cases, because the extremely large multipliers - of both spending itself and of the discount rate shock - make the welfare effect of spending through boosting private consumption dominate the direct utility effect, which becomes irrelevant.

Figure 6 plots the optimal increase in government spending at the ZLB as a function of κ. Again, as κ varies we vary also ρ_L so that the decline in GDP is constant at 4 percent per annum. The pattern is the same as in Figure 5. The largest value of the optimal spending in the useful spending case is just 0.5 percent of steady state GDP, and it is achieved around the point corresponding to the baseline calibration $\kappa = 0.028$; as we

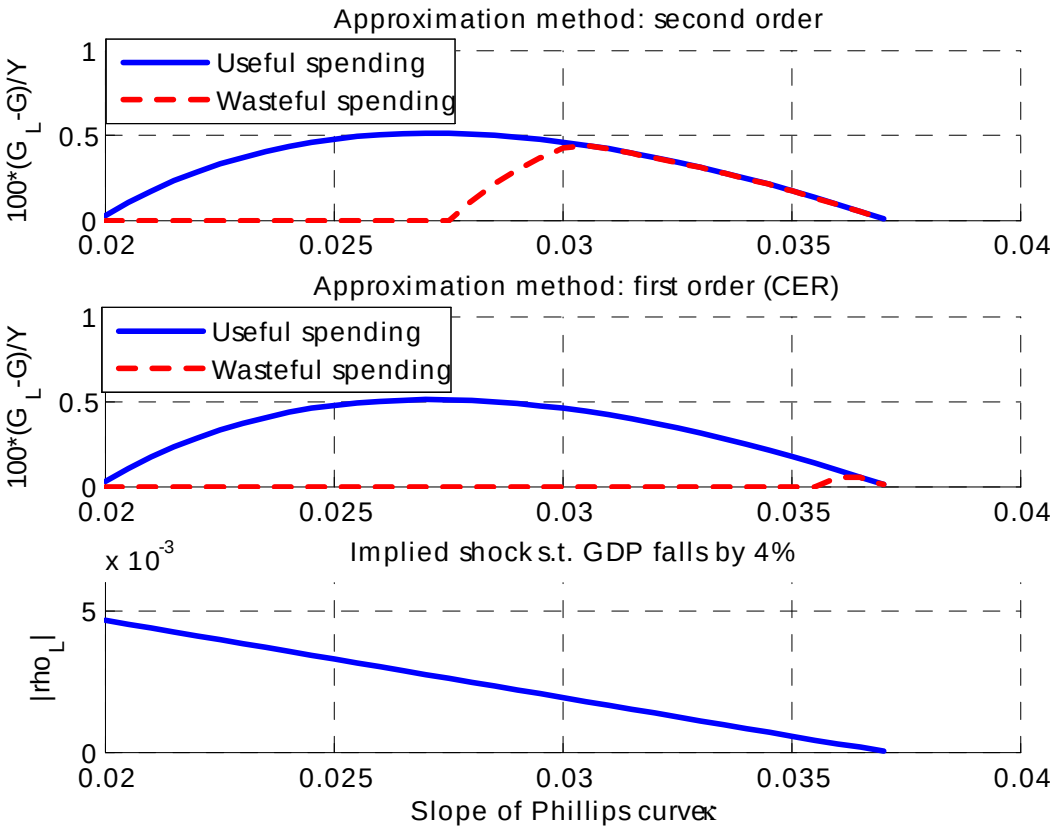

Figure 6: Optimal increase in government spending at the ZLB and implied value of ρ_L as a function of κ. Decline in GDP is constant at 4 percent. LS model.

approach the bifurcation region, optimal spending at the ZLB decreases abruptly. Like before, in the wasteful spending case the optimal increase in spending is zero on a larger range in the second order approximation than in the first order approximation.

This last result for wasteful spending is in apparent contradiction with that of CER (2013), who argue that utility is increasing in government spending even when the latter is wasteful. However, CER (2013) depart from the baseline calibration previously used in CER (2011) (the first order approximation in our paper) in two respects. First, they assume $\kappa = 0.03$ instead of $\kappa = 0.028$. Second, they assume a larger value for the discount rate shock, $\rho_L = -0.01$ rather than $\rho_L = -0.0025$. These two seemingly small differences generate radically different welfare conclusions.

The left panel of Figure 7 illustrates this point by plotting optimal wasteful spending at the ZLB as a function of κ, for the two values of the shock mentioned above and for the first order approximation only (in the case of $\rho_L = -0.0025$, this replicates the middle panel of Figure 4). The right panel plots the implied fall in GDP, in percentage points. When $\rho_L = -0.0025$, optimal spending at the ZLB is zero for $\kappa = 0.03$, and indeed for any $\kappa < 0.034$. When $\rho_L = -0.01$, optimal spending at the ZLB is 1 percent of steady-state GDP precisely at $\kappa = 0.03$. However, this latter calibration also implies a very large fall in output of 20 percent - larger than any peacetime recession experienced in the developed world in modern history, except for the Great Depression. In fact, recall from Figure 6 that when the size of the fall in GDP is fixed at 4 percent, optimal spending in the wasteful case is generally zero under this approximation method.

5 Alternative models and solution methods

There are two reasons why the conclusions from the LS model might not hold with generality. First, the nonlinearity of the model: the shock that makes the ZLB bind might be too large for the loglinear approximation to be sufficiently accurate. Indeed, Braun, Körber, and Waki (2012) and Christiano and Eichenbaum (2012) have shown that, when solving the full nonlinear stochastic model, the multiplier does not explode when reaching the bifurcation point. To address this concern, we next derive the full nonlinear solution of the stochastic model. We label this case *NLS model.*[26]

Second, the bifurcation issue arises only in models with stochastic duration of the ZLB. Carlstrom, Fuerst and Paustian (2013) have shown that, when both the shock generating the liquidity trap and government spending follow *deterministic* processes with a given duration, the multiplier is much smaller than in the stochastic case; in addition, no bifurcation occurs, and the multiplier is monotonically increasing in the duration. We label the loglinearized version of such a model the "linear deterministic" model, or *LD model.* In the baseline case, we assume that the duration of the liquidity trap is $T_L = 5$, to make it comparable with the expected duration of the ZLB in the stochastic duration

[26] The results reported below for the NLS model are derived under the assumption that there is no steady state optimal subsidy (the steady state is inefficient). As emphasized by Benigno and Woodford (2003), when the steady state is distorted government spending acts like a cost-push shock, so it has an additional welfare-damaging effect not captured by the model when linearized around an efficient steady state. Results for the NLS model under an optimal subsidy are, however, qualitatively similar.

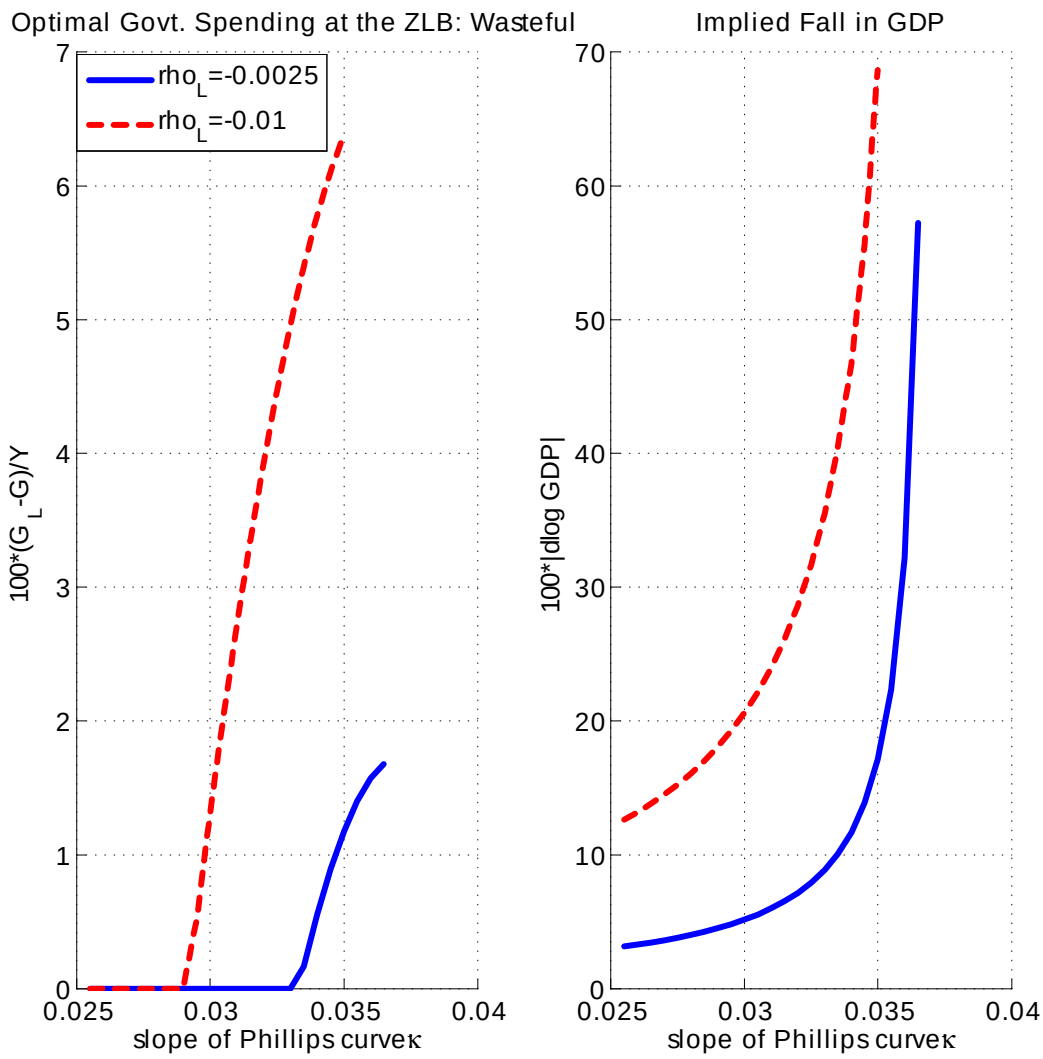

Figure 7: Optimal increase in government spending at the ZLB (left panel) and implied decline in GDP (right panel) as a function of κ,alternative value of ρ_L.LS model.

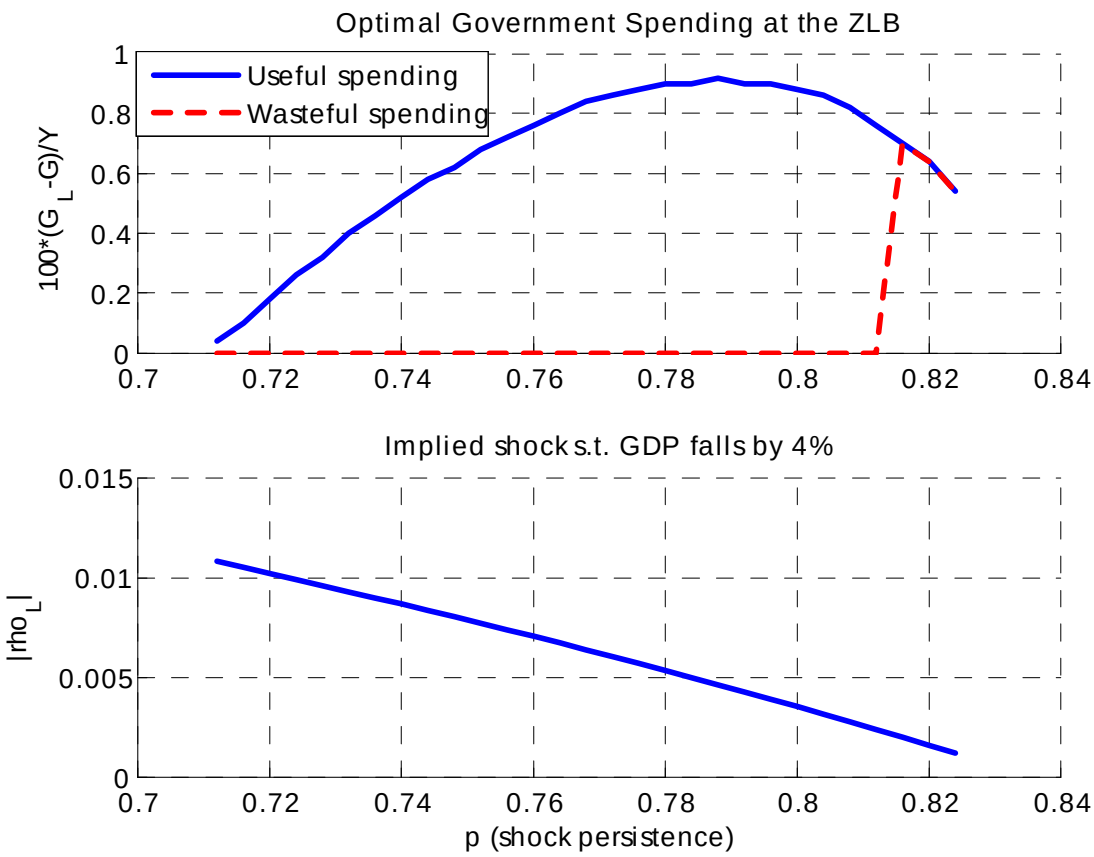

Figure 8: Optimal increase in government spending at the ZLB and implied value of ρ_L as a function of p. Decline in GDP is constant at 4 percent. NLS model.

model and baseline calibration, which is $1/(1-0.8) = 5$. Appendix C describes the solution method in more detail for each case.

5.1 The nonlinear stochastic model

In the NLS model, the multiplier (not shown) is effectively almost a linear function of p; in particular, now it does not explode close to the bifurcation point. Despite this difference, the optimal increase in government spending at the ZLB does not differ substantially from what we have seen in the LS case. Figures 8 and 9 are the analogues of Figures 5 and 6 respectively, but for the NLS model instead of the LS model.

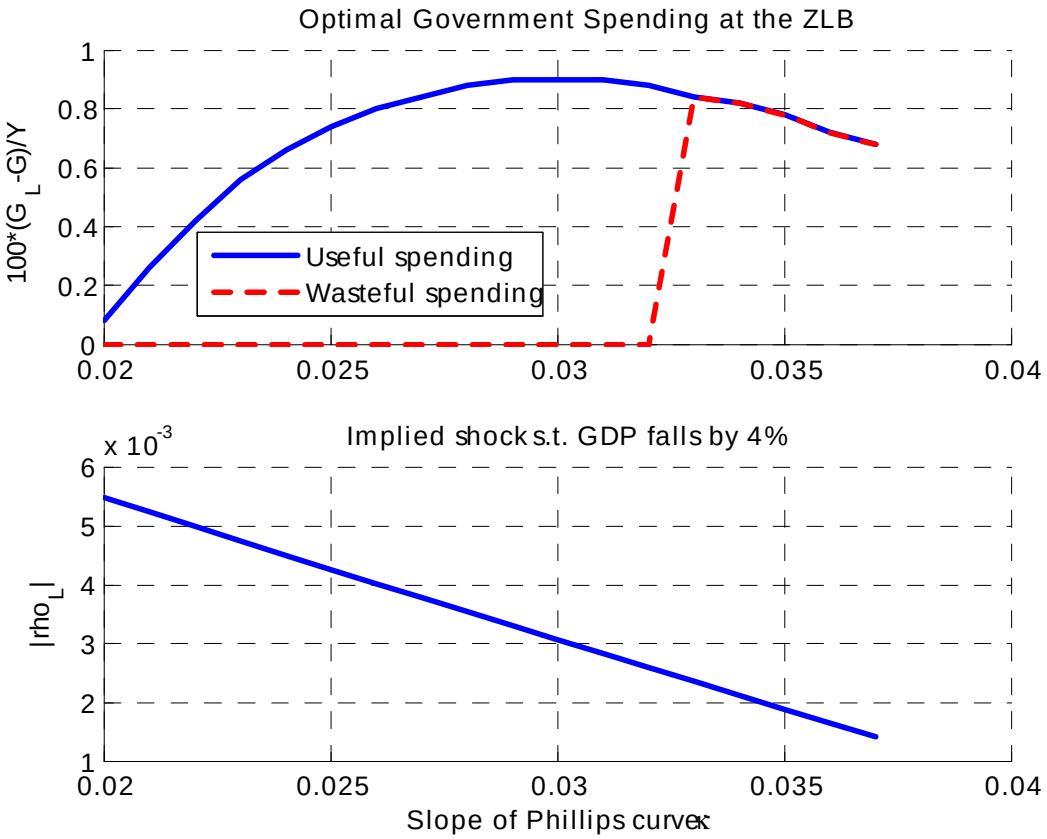

Figure 9: Optimal increase in government spending at the ZLB and implied value of ρ_L as a function of κ. Decline in GDP is constant at 4 percent. NLS model.

Now the maximum optimal increase in government spending for any given p or κ is slightly higher than in the LS model, due to the approximation error in the latter. On the other hand, in the wasteful spending case, the increase in government spending is positive only on a much smaller region than in the second-order approximation of the LS model.

5.2 The linear deterministic model

The results from the LD model are displayed in Figures 10 and 11.[27] In the useful spending case solved with the first order approximation method (middle panels), the optimal increase in government spending is around 0.4 percent of steady state output for values of $T_L \leq 5$ (Figure 10) and for values of κ in the range considered previously (Figure 11).[28] For higher values of T_L, which in the LS model would take the economy to the sunspot region, the optimal spending in the useful case increases to values as high as 0.7 percent (Figure 10); the same result applies for higher values of κ (Figure 11). Lastly, when the LD model is solved with the second order approximation method (top panels), optimal ZLB spending in the useful case is higher, for reasons that are by now clear.

Nevertheless, in the wasteful case, optimal spending at the ZLB is uniformly zero, for both the first- and second-order approximations, on the whole admissible ranges of p and κ. This reinforces the point that the positive values of optimal wasteful ZLB spending found with the previous solution methods (and in a very small and extreme range of parameter values) are specific to the stochastic setting. Note also that, in the LD model, the shock needed to achieve a 4 percent decline in GDP is much larger than in the stochastic model. The main reason is that, in the LD model, the effects of a decline in ρ_L are significantly smaller.

5.3 Summary of results

Table 2 provides a summary of the results. We consider two experiments for each solution method. In the first (column 1), the discount rate at the ZLB is kept constant at $\rho_L = -.0025$ (1 percent *per annum*); in the second (column 2), ρ_L is such that the resulting

[27] In this deterministic environment, we consider a slightly different monetary policy rule. The intercept is now given by the time varying discount rate rather than the discount rate in the steady state : $\iota_t = \rho_t$ in (7). This ensures that the duration of the ZLB, which is now endogenous, coincides with the exogenous duration of the shock. Carlstrom, Fuerst and Paustian (2013) use the same specification.

[28] Nakata (2013) also argues that, at least under commitment, optimal government spending at the ZLB is higher in a stochastic environment than in a deterministic one.

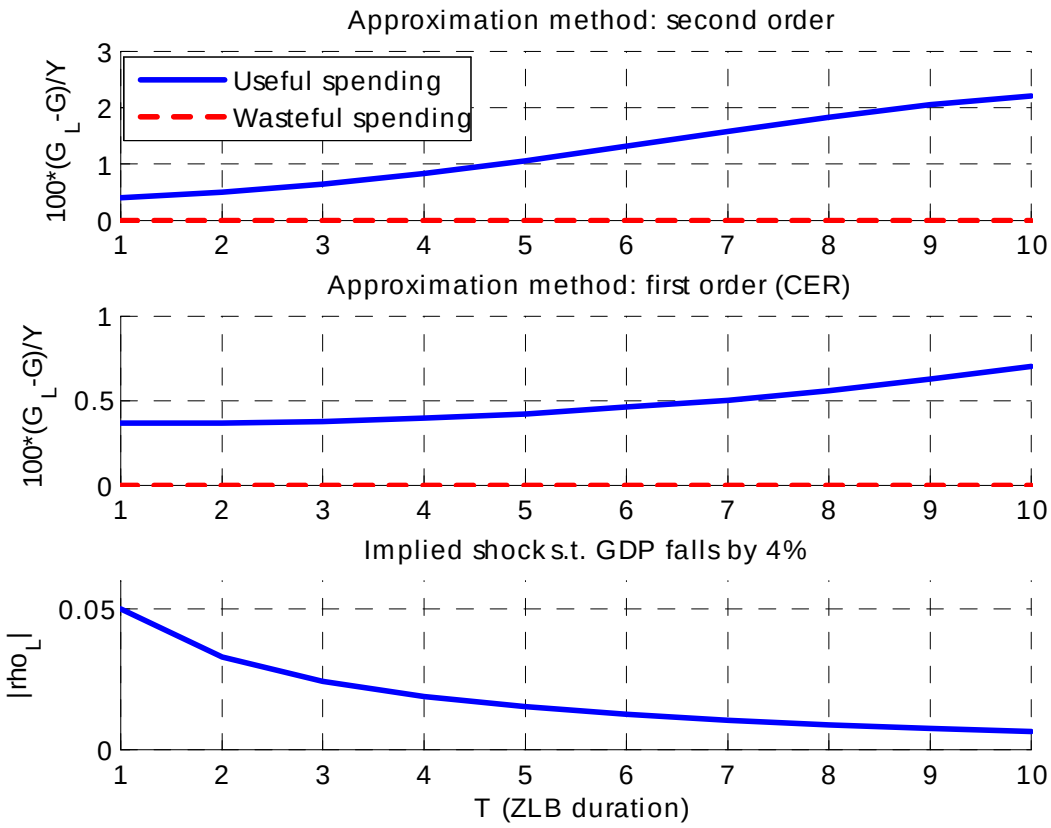

Figure 10: Optimal increase in government spending and implied value of ρ_L as a function of the duration T of the ZLB. Decline in GDP is constant at 4 percent. LD model.

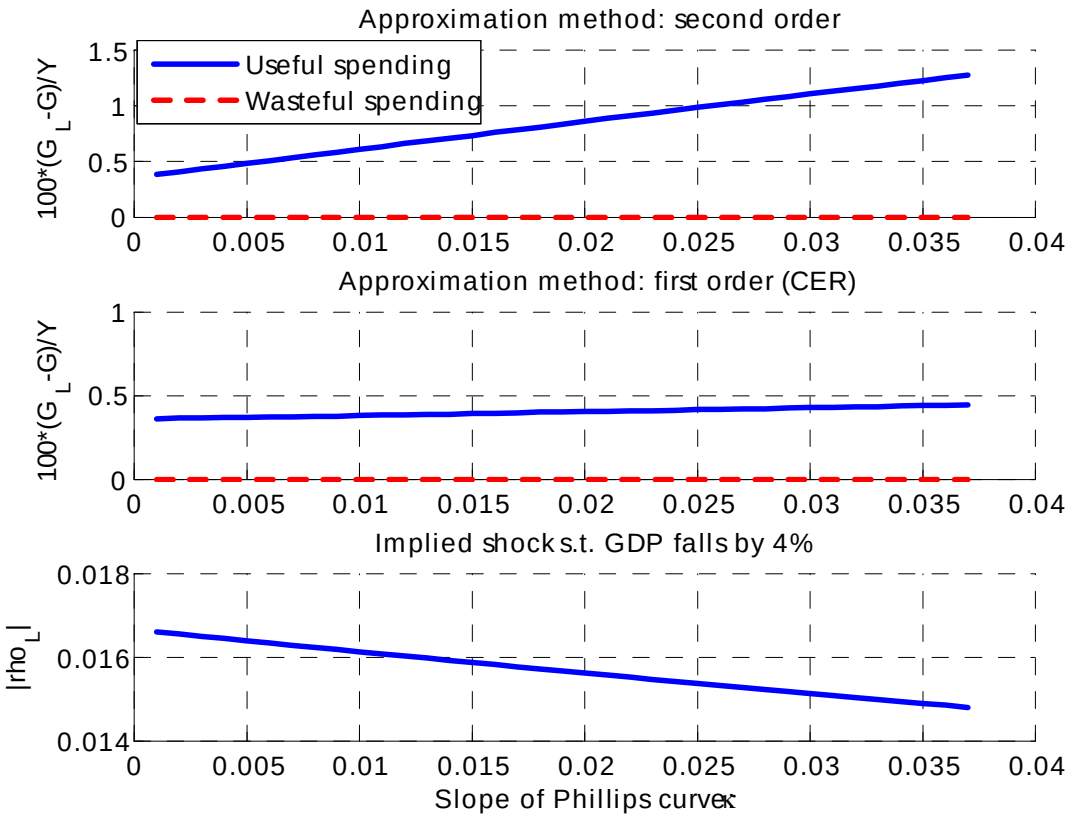

Figure 11: Optimal increase in government spending at the ZLB and implied value of ρ_L as a function of κ. Decline in GDP is constant at 4 percent. LD model.

decline in GDP is -4 percent *per annum*. We denote these two values of ρ_L as ρ_L^1 and ρ_L^2 respectively. Columns (3) and (4) display the annualized decline in GDP at the zero lower bound in these two cases. Columns (5) and (6) display the consumption multipliers at the zero lower bound: for the LD model, we report the impact multiplier, and for the NLS model we report the midpoint of the range of multipliers corresponding to the range of cyclical spending.[29] Columns (7) and (8) display the optimal cyclical spending under useful spending, in percentage points of steady-state GDP, for the two values of ρ_L; columns (9) and (10) display the optimal cyclical spending under wasteful government spending.

Table 2: Alternative solution methods, baseline calibration

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	ρ_L^1	ρ_L^2	ΔY^1	ΔY^2	M_C^1	M_C^2	$g_{L,u}^{*,1}$	$g_{L,u}^{*,2}$	$g_{L,w}^{*,1}$	$g_{L,w}^{*,2}$
LS, 1st order	-0.0025	-0.0025	-4.0	-4.0	2.02	2.02	0.5	0.5	0.0	0.0
LS, 2nd order	-0.0025	-0.0025	-4.0	-4.0	2.02	2.02	0.5	0.5	0.1	0.1
LD, 1st order	-0.0025	-0.0150	-0.7	-4.0	0.60	0.60	0.0	0.4	0.0	0.0
LD, 2nd order	-0.0025	-0.0150	-0.7	-4.0	0.60	0.60	0.0	1.1	0.0	0.0
NLS	-0.0025	-0.0035	-3.0	-4.0	1.10	1.10	0.1	0.8	0.0	0.0

Columns (1), (2): ρ_L: value of ρ at ZLB.
Columns (3), (4): ΔY: percentage change in GDP at ZLB.
Columns (5), (6): M_C: consumption multiplier at ZLB(change in private consumption as a share of steady-state GDP divided by change in government spending as a share of steady-state GDP).
Columns (7), (8): $g_{L,u}^*$: optimal increase in useful government spending at ZLB, as share of steady-state GDP.
Columns (9), (10): $g_{L,w}^*$: optimal increase in wasteful government spending at ZLB, as a share of steady-state GDP.
Columns (1), (3), (5), (7) , (9) (indexed by superscript "1"): rho_L is -.0025.
Columns (2), (4), (6), (8) , (10) (indexed by superscript "2"): rho_L is such that decline in GDP is fixed at 4 percent.
"LS, 1st order": Stochastic model, solved by inserting the loglinearized first order conditions and the first order approximaztion of the resource constraint into the utility function, as in CER (2011).
"LS, 2nd order": Stochastic model, solved by inserting the loglinearized first order conditions and the second order approximaztion of the resource constraint into the utility function.
"LD, 1st order": Deterministic model, solved by inserting the loglinearized first order conditions and the first order approximaztion of the resource constraint into the utility function.
"LD, 2nd order": Deterministic model, solved by inserting the loglinearized first order conditions and the second order approximaztion of the resource constraint into the utility function.
"NLS": Full nonlinear solution of the stochastic model.

[29] In the NLS model, consumption is a non-linear function of government spending, so the multiplier is not constant with respect to the level of spending. The range of cyclical spending is dictated by the requirement that the ZLB binds.

Two conclusions stand out. First, in the useful spending case the optimal increase in government spending varies between 0.0 and 0.5 percentage points of GDP when $\rho_L = -0.0025$ (column 7), whereas it varies between 0.4 and 1.1 when the decline in GDP is kept constant at 4 percent (column 8). In the relevant case of the NLS model, optimal useful spending varies between 0.1 and 0.8 percentage points. Second, in the wasteful case, the optimal government spending increase is always zero in all models and in all scenarios except in the second order approximation of the LS model, where it is again a very small 0.1 percent.

6 Optimal spending in a Great Depression

Our analysis thus far has focused mostly on recessions that, while substantial, are still moderate in size when compared to the Great Depression. An argument could be made that government spending at the ZLB is particularly desirable when the ensuing recession is exceptionally deep. In our analysis, several alternative calibrations could deliver a GDP collapse of 28.8%, which is in line with the Great Depression data; with such calibrations, optimal ZLB spending would be higher, of the order of 2 to 4-5percent of steady-state output.[30] However, the issue with replicating a Great Depression with this calibration is that it implies movements in deflation that are unrealistically large, i.e. annualized deflations of about 32 to 40 percent, while annualized deflation during that period has been 10 percent.

6.1 The Great Depression in the LS model

To address these concerns, in Appendix D we study a slightly different setup, similar to Woodford (2011), with a calibration that can deliver a Great Depression along both dimensions: GDP collapse of 28.8 percent and deflation of 10 percent, both in annualized terms. This is due chiefly to the value of two key parameters: the Phillips curve is much flatter, $\kappa = 0.003147$ (i.e., consistent with a sufficiently high degree of price stickiness in order to avoid too large a collapse in inflation), and the persistence of the shock is higher,

[30] For instance, start by looking at Figures 3, 4, and 7. A calibration such that GDP falls by 28.8 percent implies $p = 0.821$ in Figure 3 or $\kappa = 0.0359$ in Figure 4 . Optimal ZLB spending in the useful case is around 1.7 percent of GDP while in the wasteful case it is about 1.5 percent. When $\rho_L = -0.01$ as in Figure 7, $\kappa = 0.032$ delivers a Great Depression and optimal ZLB spending in the wasteful case of almost 4 percent (see Figure 7) while in the useful case it is 5.5 percent (not shown).

$p = 0.903$. This calibration has important implications for the welfare results that we describe next.

Table 3 mirrors Table 2 in presenting the main results for the Great Depression environment. Note that the second row, headed "LS, second order", replicates the results of Woodford (2011).[31] The optimal increase in useful government spending at the ZLB is large, 11.5 percent of steady state GDP in the first order approximation and 14.5 in the second order approximation. When we assume that spending is wasteful, we still find a sizeable optimal increase of 5.5 percent for the first order and 13.5 percent for the second order approximation, respectively. The reason why these numbers are so high is twofold.

Table 3: Great Depression calibration

	(1)	(2)	(3)	(4)	(5)
	ρ_L	ΔY	M_C	$g^*_{L,u}$	$g^*_{L,w}$
LS, 1st order	-0.010	-28.8	1.29	11.5	5.5
LS, 2nd order	-0.010	-28.8	1.29	14.5	13.5
LD, 1st order	-0.055	-28.8	0.25	9.5	0.0
LD, 2nd order	-0.055	-28.8	0.25	10.0	0.0
NLS	-0.017	-28.8	0.55	25.5	0.0

Column (1): ρ_L: value of ρ at ZLB.
Column (2): ΔY: percentage change in GDP at ZLB.
Column (3): M_C: consumption multiplier at ZLB(change in private consumption as a share of steady-state GDP divided by change in government spending as a share of steady-state GDP).
Column (4): $g^*_{L,u}$: optimal increase in useful government spending at ZLB, as share of steady-state GDP.
Column (5): $g^*_{L,w}$: optimal increase in wasteful government spending at ZLB, as a share of steady-state GDP.
"LS, 1st order": Stochastic model, solved by inserting the loglinearized first order conditions and the first order approximaztion of the resource constraint into the utility function, as in CER (2011).
"LS, 2nd order": Stochastic model, solved by inserting the loglinearized first order conditions and the second order approximaztion of the resource constraint into the utility function.
"LD, 1st order": Deterministic model, solved by inserting the loglinearized first order conditions and the first order approximaztion of the resource constraint into the utility function.
"LD, 2nd order": Deterministic model, solved by inserting the loglinearized first order conditions and the second order approximaztion of the resource constraint into the utility function.
"NLS": Full nonlinear solution of the stochastic model.

[31] Woodford (2011), however, considers only the case of useful government spending.

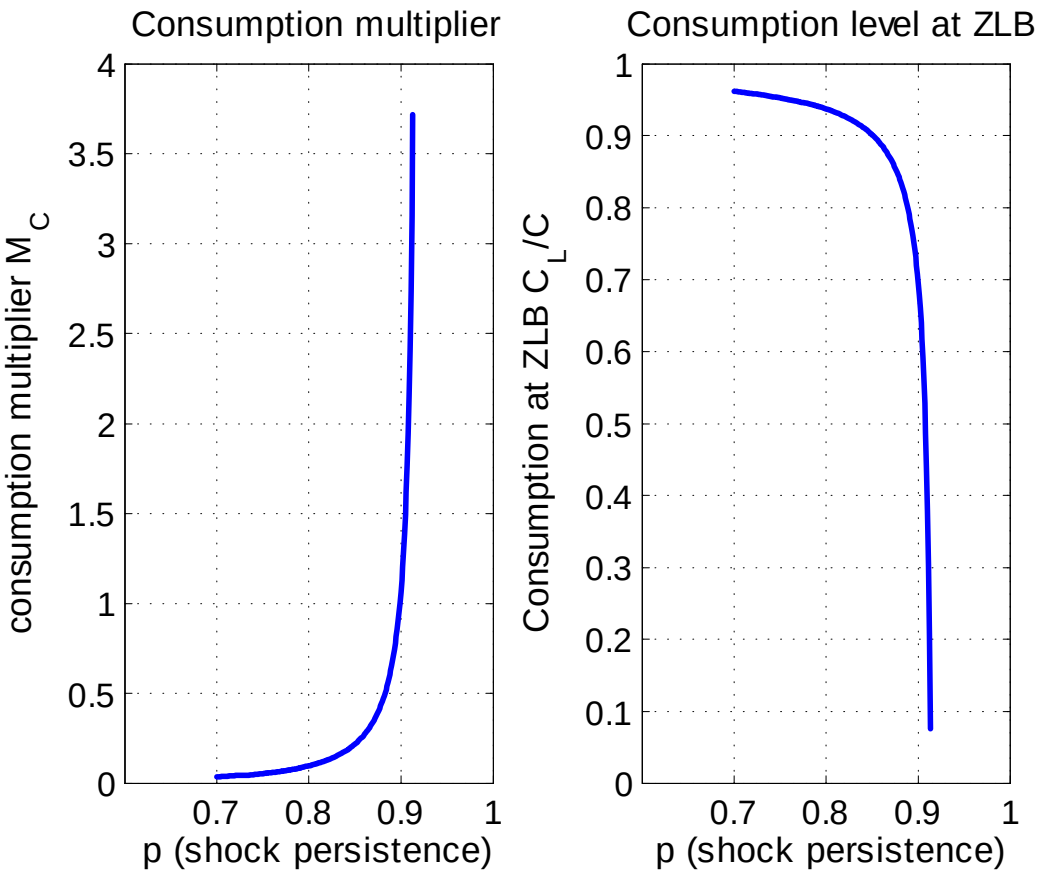

Figure 12: Consumption multiplier (left panel) and consumption level (right panel) as a function of p under the Great Depression calibration. LS model.

First, the starvation/bifurcation issue described before is particularly acute here, because the Great Depression calibration is very close to the bifurcation point. To illustrate this, Figure 12 plots (like Figure 2) the multiplier and the level of consumption at the ZLB as a function of persistence p. It is clear from the picture that the value $p = 0.903$ is a point at which an arbitrarily small increase in p leads to an explosive multiplier and brings the economy arbitrarily close to the starvation point (consumption becomes exactly zero at $p = 0.91346$).[32] In other words, both the multiplier and ZLB consumption are almost vertical at $p = 0.903$.

[32] A similar picture holds with the Phillips curve slope on the x-axis: starvation occurs at $\kappa = 0.004$, so given an increase in κ of merely 0.0008 from the calibrated value.

Second, in order to replicate the deflation data associated with the Great Depression, we had to assume an extremely large degree of price rigidity, implying a value of $\kappa = 0.003147$. The Rotemberg price adjustment cost parameter consistent with this value of κ is $\nu = 1588$. Translated in the Calvo framework, this implies a 0.95 probability of not adjusting the price in any given quarter, or an expected price duration of 20 quarters (five years).[33]

This extreme price stickiness translates into an enormous welfare cost of the output gap – the more so, when the shock is large (as it is here, $\rho_L = -.01$) and hence the output gap itself is large. This large welfare cost of the ZLB explains why, in this framework, there is scope for increasing government spending at the ZLB even when it is wasteful.

6.2 The Great Depression in the NLS and LD models

The LS model of the Great Depression displays a more extreme version of the usual problems: since the calibrations are so close to the bifurcation point, an arbitrarily small change in one of the parameters can generate very large changes in the conclusions, making any welfare inference unreliable. In addition, the large size of the shock necessary to obtain a large fall in GDP renders the loglinear approximation potentially inaccurate. Thus, like before, we now turn to the NLS and the LD models.

In the NLS model we still find a very large optimal increase in government spending in the useful case, by 25.5 percent of GDP. Strikingly, however, as Table 3 shows the optimal increase in wasteful spending is still zero. The reason is that now the output gap falls by much less than GDP. To see this, note that $GDP = C + G$, while output is GDP net of the price adjustment cost, $Y_t = (C_t + G_t) / \left(1 - \frac{\nu}{2}\pi_t^2\right)$. In the LS case, the price adjustment cost is loglinearized around the zero inflation steady state, hence it is always zero: output falls by as much as GDP, 28.8 percent. In the NLS case, the price adjustment cost is positive because of the large deflation; hence, after the discount rate shock output must be larger then GDP. In fact, output now falls by only 8.5 percent, implying a smaller output gap than in the LS case. This explains why at the ZLB spending in the wasteful

[33] It is important to note that these calculations are based on the standard New Keynesian model with *homogeneous* labor types, as used in our case and, e.g., in Woodford (2011). Eggertsson (2009), from which the calibration in Woodford (2011) is taken, uses a model of the labor market with *differentiated* labor types, hence obtaining (by standard arguments pertaining to real rigidities) the same value of the Phillips curve parameter with a lower degree of price stickiness. However, Eggertsson (2009) does not study welfare. We use the same model and calibration as Woodford (2011) in order to facilitate the comparison.

case is still zero.

However, in order to generate the Great Depression, the LD model requires a phenomenally large discount rate shock of 22 percent per annum, more than five times the shock required in the LS model to generate the same decline in GDP. Still, the optimal increase in useful spending is less than in the LS model, 9.5 percent of steady state GDP in the linear approximation and 10 percent in the second order approximation, against 11.5 percent and 14.5 percent in the LS model, respectively. This illustrates once more how the LS model amplifies the value of optimal spending, simply because of the stochastic nature of the model.

Like in the NLS model, the optimal increase in wasteful government spending in the LD case is still zero, in both approximations. Put differently, the LS model delivers positive ZLB spending in the wasteful case both because it is linear and because it is stochastic. Dropping either of those two features eliminates the scope for wasteful spending–even when the welfare distortion associated with the ZLB is large.

On the other hand, it is true that in the Great Depression case we find very large optimal increases in government spending in the useful case, even in a deterministic setup and even more so in a nonlinear setup (which accounts for the distortions fully). But these large values stem from extremely high welfare distortions associated with the zero lower bound, coming from what one might view as an implausibly high degree of price rigidity–a feature which is necessary, to start with, for the model's ability to replicate the deflation observed during the Great Depression.

7 Conclusion

For sizeable recessions, and in the context of a standard New Keynesian model, the optimal increase in government spending at the ZLB is small, or zero. At the benchmark values of the parameters of the model that have typically been used in the literature, the optimal increase in government spending in a 4 percent recession is just 0.8 percent. Larger optimal increases in (useful) government spending obtain only at parameter values that imply extremely large output declines, in the range observed during the Great Depression - and even in these cases, the model requires a somewhat extreme degree of price rigidity to generate the combination of output decline and (relatively small) deflation observed in that historical episode.

Perhaps more importantly, we have shown that when cyclical government spending

does *not* provide direct utility, the optimal increase in government spending at the ZLB is zero in all versions of our model - stochastic or deterministic, even in a scenario like the Great Depression where output falls by almost 30 percent. Thus, this simple model does not provide support for the often heard notion that anything that reduces the output gap and prevents deflation - including wasteful government spending - should be used at the ZLB. This is a particularly surprising result, for it suggests that within the class of models typically used to analyze the aggregate implications of liquidity traps (mostly New Keynesian models) the welfare cost of being at the ZLB is of second-order relative to the (first-order) welfare cost of increasing government spending and, therefore, taxation.

Our results suggest that for higher government spending to be welfare enhancing at the ZLB the underlying model economy must be able to generate significant welfare costs of being at the ZLB. Promising examples in this vein are models with equilibrium unemployment and imperfect consumption insurance, such as Christiano, Walentin and Trabandt (2012), Rendahl (2013) and Michaillat (2012); and/or models where the main nominal stickiness is some form of downward rigidity in wages, such as Schmitt-Grohe and Uribe (2013).

Finally, despite government spending not being welfare-improving in a certain economic environment, other fiscal instruments may well be. In a companion paper (Bilbiie, Monacelli and Perotti, 2014) we study the effects of *tax cuts* financed by public debt at the ZLB, as a form of implicit transfer from unconstrained savers to constrained borrowers. In that framework, a uniform tax cut financed by public debt is Pareto improving because it alleviates the constraint on private debt for borrowers and allows savers to frontload their savings, something that is prevented by the presence of the ZLB. That paper substantiates a claim often made in policy circles that if a liquidity trap is due to excess savings, there are potential benefits for the government to step in and borrow via a tax cut.

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A Appendix A: The model

The setup is a standard New Keynesian model along the lines of Woodford (2003) and CER (2011), except for the assumption of convex costs of price adjustment (as opposed to Calvo pricing). There is a representative household,with period utility given in the text, who solves the intertemporal problem:

$$\max_{\{C_t,N_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t U\left(C_t, N_t, G_t\right)$$

subject to the period-by-period budget constraint:

$$C_t + B_{t+1} \leq \frac{1+I_{t-1}}{1+\pi_t} B_t + W_t N_t - \tau_t, \tag{A.1}$$

where W_t is the real wage, $1+\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, B_t is a portfolio of one-period bonds issued in $t-1$ on which the household receives nominal interest I_{t-1} (in equilibrium the net supply of these bonds is nil), and τ_t are lump-sum taxes.

Under the assumption that $U\left(C_t, N_t, G_t\right)$ is as in (1), the intratemporal optimality condition and Euler equation for bond holdings are respectively:[34]

$$\frac{(1-\zeta)}{\zeta} \frac{C_t}{1-N_t} = W_t \tag{A.2}$$

$$\frac{\left[C_t^{\zeta}\left(1-N_t\right)^{1-\zeta}\right]^{1-\sigma}}{C_t} = \beta \mathbb{E}_t \left(\frac{1+I_t}{1+\pi_{t+1}} \frac{\left[C_{t+1}^{\zeta}\left(1-N_{t+1}\right)^{1-\zeta}\right]^{1-\sigma}}{C_{t+1}} \right). \tag{A.3}$$

Each individual good is produced by a monopolistic competitive firm, indexed by z, using a technology given by: $Y_t(z) = N_t(z)$. Cost minimization taking the wage as given, implies that real marginal cost is W_t/P_t. The problem of producer z is to maximize the present value of future profits, discounted using the stochastic discount factor of their shareholders, the households:

$$\max_{P_t(z)} \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \left[(1+s) P_t(z) Y_t(z) - W_t N_t(z) - \frac{\nu}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 P_t Y_t \right],$$

where $Q_{0,t} \equiv \beta^t P_0 C_0 \left[C_0^{\zeta}\left(1-N_0\right)^{1-\zeta}\right]^{\sigma-1} / P_t C_t \left[C_t^{\zeta}\left(1-N_t\right)^{1-\zeta}\right]^{\sigma-1}$ is the marginal rate of intertemporal substitution between times 0 and t, and s is a sales subsidy. Firms

[34] These conditions must hold along with the usual transversality conditions.

face demand for their products from three sources: consumers, government and firms themselves (in order to pay for the adjustment cost); the demand function for the output of firms z is $Y_t(z) = (P_t(z)/P_t)^{-\varepsilon} Y_t$. Substituting this into the profit function, the first order condition is, after simplifying:

$$\begin{aligned} 0 = {} & Q_{0,t} \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \left[(1+s)(1-\varepsilon) + \varepsilon \frac{W_t}{P_t} \left(\frac{P_t(z)}{P_t} \right)^{-1} \right] \\ & - Q_{0,t} \nu P_t Y_t \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right) \frac{1}{P_{t-1}(z)} + \\ & + \mathbb{E}_t \left\{ Q_{0,t+1} \left[\nu P_{t+1} Y_{t+1} \left(\frac{P_{t+1}(z)}{P_t(z)} - 1 \right) \frac{P_{t+1}(z)}{P_t(z)^2} \right] \right\}. \end{aligned} \tag{A.4}$$

In a symmetric equilibrium all producers make identical choices (including $P_t(z) = P_t$). Defining net inflation $\pi_t \equiv (P_t/P_{t-1}) - 1$, and noticing that

$$Q_{0,t+1} = Q_{0,t} \frac{\beta}{1+\pi_{t+1}} \left(\frac{C_t}{C_{t+1}} \right) \left[\frac{C_t^{\zeta} (1-N_t)^{1-\zeta}}{C_{t+1}^{\zeta} (1-N_{t+1})^{1-\zeta}} \right]^{\sigma-1},$$

equation (A.4) becomes:

$$\begin{aligned} \pi_t (1+\pi_t) = {} & \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \left[\frac{C_{t+1}^{\zeta} (1-N_{t+1})^{1-\zeta}}{C_t^{\zeta} (1-N_t)^{1-\zeta}} \right]^{1-\sigma} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1+\pi_{t+1}) \right] + \\ & + \frac{\varepsilon - 1}{\nu} \left[\frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{P_t} - (1+s) \right]. \end{aligned} \tag{A.5}$$

Since Ricardian equivalence holds, we assume without loss of generality that the budget is balanced every period

$$G_t = \tau_t$$

A monetary authority sets the nominal interest rate subject to a zero lower bound, as described in (7) and/or (A.7). In an equilibrium of this economy, all agents take as given prices (with the exception of monopolists who reset their good's price in a given period), as well as the evolution of exogenous processes. A rational expectations equilibrium is then as usual a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time t. Specifically, labor market clearing requires that labor demand equal total labor supply,

and private debt is in zero net supply $B_{t+1} = 0$. Finally, by Walras' Law, the goods market also clears. The resource constraint specifies that all produced output will be used, either for private or government consumption or by firms to pay the adjustment cost:

$$C_t + G_t = \left(1 - \frac{\nu}{2}\pi_t^2\right) Y_t. \tag{A.6}$$

The monetary authority sets the interest rate according to

$$1 + i_t = \max\left\{1,\ \beta^{-1}\left(1+\pi_t\right)^{\phi_\pi}\right\}, \tag{A.7}$$

unless specified otherwise.

Steady state and loglinearized equilibrium As described in text, we solve the model by loglinearizing the equilibrium conditions around the steady state, with $\pi = 0$. Letting a capital letter without a time subscript indicate a steady state value, we have

$$\begin{aligned} W &= (1+s)\frac{\varepsilon-1}{\varepsilon} \\ \frac{(1-\zeta)}{\zeta}\frac{C}{1-N} &= (1+s)\frac{\varepsilon-1}{\varepsilon}. \end{aligned} \tag{A.8}$$

We calibrate $G = 0.2Y$ and $Y = N = 1/3$, implying that the second equation pins down the value of ζ:

$$\zeta = \frac{1}{1 + \frac{1-N}{N}\frac{G}{Y-G}(1+s)\frac{\varepsilon-1}{\varepsilon}}.$$

Note that s is the subsidy that takes values between 0 and $(\varepsilon-1)^{-1}$. We solve the loglinearized model assuming that there is a constant subsidy that induces marginal cost pricing in steady state and makes profits equal to zero, $s = (\varepsilon-1)^{-1}$ and $W = 1$.

A loglinear approximation of the Phillips curve (A.5) around a zero-inflation steady state delivers:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa w_t.$$

where w_t denotes deviations of the real wage (or real marginal cost) from the deterministic steady state. Loglinearizing labor supply (A.2) we have: $\frac{N}{1-N}n_t = w_t - c_t$, where which combined with the production function $y_t = n_t$ and a first-order approximation of the economy resource constraint (A.6), $y_t = (1-G_Y)\,c_t + G_Y g_t$ gives

$$w_t = \left(1 + \frac{N}{1-N}\frac{Y-G}{G}\right)c_t + \frac{N}{1-N}\frac{G}{Y}g_t. \tag{A.9}$$

Replacing this in the loglinearized Phillips curve, we obtain the Phillips curve used in text. Finally, we calibrate $\kappa \equiv (\varepsilon - 1)/\nu$ and we obtain ν by using the calibrated value for κ and $\varepsilon - 1 = 5$.

B Appendix B: Utility at the zero lower bound

In this Appendix we derive the analytical expression for the present discounted value of utility conditional upon being at the ZLB used for the welfare calculations in text. Utility at the ZLB is, in the "useful G" case (recall that in the "wasteful" case we simply replace G_L by G) reads:

$$U_L = \Psi_L \left\{ \frac{\left[C_L^{\zeta} (1 - N_L)^{1-\zeta}\right]^{1-\sigma} - 1}{1 - \sigma} + \chi_G \frac{G_L^{1-\sigma} - 1}{1 - \sigma} \right\},$$

where $\Psi_L \equiv \frac{1+\rho_L}{1+\rho_L - p}$. In the text we presented log-linearized model solutions, where lower-case variables are the percentage deviations defined as $c_L = \frac{C_L - C}{C} \simeq (1 + c_L) C$, and so on. Note:

$$y_L = n_L = \frac{N_L - N}{N} = -\frac{1 - N}{N} \frac{1 - N_L - (1 - N)}{1 - N} \rightarrow 1 - N_L = \left(1 - \frac{N}{1 - N} n_L\right)(1 - N)$$

Next, we rewrite utility in order to have the percentage deviations as arguments. This yields:

$$U_L = \Psi_L \left\{ \frac{\left[C^{\zeta} (1 - N)^{1-\zeta} (1 + c_L)^{\zeta} \left(1 - \frac{N}{1-N} y_L\right)^{1-\zeta}\right]^{1-\sigma} - 1}{1 - \sigma} + \chi_G \frac{G^{1-\sigma} (1 + g_L)^{1-\sigma} - 1}{1 - \sigma} \right\}.$$

We can simplify this further by replacing the steady-state optimality conditions. First, under the optimal subsidy that makes real wage (markup) equal to one in steady state, we have that:

$$\zeta = \left[1 + \frac{1 - N}{N} \frac{Y}{Y - G}\right]^{-1} < 1.$$

Second, the Samuelson condition for public goods provision requires that the marginal utilities of private and public expenditure be equal. This implies that, in steady state:

$$\zeta \frac{\left[C^{\zeta} (1 - N)^{1-\zeta}\right]^{1-\sigma}}{C} = \chi_G G^{-\sigma}.$$

Solving for χ_G yields:

$$\chi_G = \zeta \left(\frac{G}{Y}\right)^{\sigma} \left(1 - \frac{G}{Y}\right)^{\zeta(1-\sigma)-1} \left(\frac{1-N}{N}\right)^{(1-\zeta)(1-\sigma)}.$$

We use the above expression to simplify the welfare function, abstracting from a constant additive term (which anyway disappears when we look at percentage deviations of welfare, as we do):

$$U_L = \hat{\Psi}_L \left\{ \frac{\left[(1+c_L)^{\zeta}\left(1 - \frac{N}{1-N} y_L\right)^{1-\zeta}\right]^{1-\sigma} - 1}{1-\sigma} + \zeta \frac{G}{Y-G} \frac{(1+g_L)^{1-\sigma} - 1}{1-\sigma} \right\} + \text{constant}, \tag{B.1}$$

where $\hat{\Psi}_L \equiv \Psi_L \left[C^{\zeta}(1-N)^{1-\zeta}\right]^{1-\sigma}$.

We define the welfare gap $\tilde{U}_L$ as:

$$\tilde{U}_L(g_L) = 100 \cdot \frac{U_L(g_L) - U_L(0)}{|U_L(0)|}, \tag{B.2}$$

namely, the percentage variation in utility between the scenario whereby spending increases at the ZLB $U_L(g_L)$ and a scenario where spending is kept constant at its steady-state value G, $U_L(0)$.

When using the "linearized" approximation method, we replace y_L in the nonlinear utility (B.1) by using the first-order approximation of the resource constraint (12). When using the "second-order inflation cost" approximation method, we replace y_L in the nonlinear utility (B.1) by using the second-order approximation of the resource constraint (11).

C Appendix C: Different solution methods

In this appendix we outline the model solution in the two cases considered in text: nonlinear stochastic (NLS) and linearized deterministic (LD).

C.1 Nonlinear stochastic model

In this case, we solve directly for the nonlinear equations from Appendix A assuming the same Markov structure for shocks as in the linearized model[35]. The steady state is solved as previously, with $\pi = 0$, namely (A.8) and $C = Y - G$. In all simulations for the nonlinear model, we assume that there is no subsidy, i.e., $s = 0$.

The liquidity trap state is, like in the linearized model, a time-invariant solution, but in this case to the nonlinear system, holding as long as ZLB binds. Denoting with L subscript the value of a variable in the liquidity trap state, the equilibrium is determined by the system (note that π in the regular steady state is zero, which simplifies considerably the Phillips curve):

$$W_L = \frac{1-\zeta}{\zeta}\frac{C_L}{1-N_L}, \tag{C.1}$$

$$\frac{\left[C_L^{\zeta}\left(1-N_L\right)^{1-\zeta}\right]^{1-\sigma}}{C_L} = \frac{1}{1+\rho_L}\left(p\frac{1}{1+\pi_L}\frac{\left[C_L^{\zeta}\left(1-N_L\right)^{1-\zeta}\right]^{1-\sigma}}{C_L} + (1-p)\frac{\left[C^{\zeta}\left(1-N\right)^{1-\zeta}\right]^{1-\sigma}}{C}\right), \tag{C.2}$$

$$\left(1-\frac{p}{1+\rho_L}\right)\pi_L\left(1+\pi_L\right) = \frac{\varepsilon-1}{\nu}\left[\frac{\varepsilon}{\varepsilon-1}W_L - (1+s)\right], \tag{C.3}$$

$$C_L + G_L = \left(1-\frac{\nu}{2}\pi_L^2\right)Y_L. \tag{C.4}$$

Reducing further, we obtain the two core equations to be solved for:

$$\frac{C_L^{1-\zeta(1-\sigma)}}{\left(1-\frac{C_L+G_L}{1-\frac{\nu}{2}\pi_L^2}\right)^{(1-\zeta)(1-\sigma)}} = \frac{\left(1+\rho_L\right)\left(1+\pi_L\right)-p}{\left(1+\pi_L\right)\left(1-p\right)}\frac{C^{1-\zeta(1-\sigma)}}{\left(1-N\right)^{(1-\zeta)(1-\sigma)}} \tag{C.5}$$

$$\left(1-\frac{p}{1+\rho_L}\right)\pi_L\left(1+\pi_L\right) = \frac{\varepsilon-1}{\nu}\left[\frac{\varepsilon}{\varepsilon-1}\frac{1-\zeta}{\zeta}\frac{C_L}{1-\frac{C_L+G_L}{1-\frac{\nu}{2}\pi_L^2}} - (1+s)\right], \tag{C.6}$$

which delivers the equilibrium values of consumption and inflation at the zero lower bound. Output, hours and real wage are determined residually using $N_L = Y_L = \left(C_L + G_L\right)/\left(1-\frac{\nu}{2}\pi_L^2\right)$ and $W_L = \frac{1-\zeta}{\zeta}\frac{C_L}{1-N_L}$. Once solutions C_L and N_L are found, utility

[35] Note that discount factor shocks affect the Phillips curve too (see also Braun et al. 2013 and Christiano and Eichenbaum 2013). This was not the case in the linearized model because loglinearization around a zero-inflation steady state implies that discount factor shocks have no first-order effects on the Phillips curve.

is calculated as

$$U_L = \frac{1+\rho_L}{1+\rho_L-p}\left[\frac{\left[C_L^{\zeta}\left(1-N_L\right)^{1-\zeta}\right]^{1-\sigma}-1}{1-\sigma}+\chi_G\frac{G_L^{1-\sigma}-1}{1-\sigma}\right], \tag{C.7}$$

where χ_G is defined above. In the "wasteful" case, G replaces G_L in the last term above.

We solve the model while

$$\pi_L \leq \beta^{\frac{1}{\phi_\pi}}-1,$$

which, given the Taylor rule, ensures that the zero lower bound is binding ($i \leq 0$).

C.2 Linearized deterministic model

Under the assumption of deterministic shocks of given duration, we assume that the shocks take the values $\rho_t = \rho_L$ and $g_t = g_L$ from 1 to an arbitrary time T, and zero thereafter. Note that T is a parameter. For t from 1 to T, we need to solve the system $x_t = Ax_{t+1} + Bu_t$ where

$$x = (c,\pi)'\,;\; u = \left(\rho_t,\; \frac{G}{Y}g_t\right)',$$
$$A \equiv \begin{pmatrix} 1 & \omega \\ \delta & \beta+\delta\omega \end{pmatrix}; B \equiv \begin{pmatrix} \omega & 0 \\ \delta\omega\psi & \kappa\varphi \end{pmatrix},$$

where we used the extra notation

$$\varphi \equiv \frac{N}{1-N};\; \delta \equiv \kappa\left(1+\varphi\left(1-\frac{G}{Y}\right)\right);$$
$$\omega \equiv \left[1-\zeta\left(1-\sigma\right)+\left(1-\zeta\right)\left(1-\sigma\right)\varphi\left(1-\frac{G}{Y}\right)\right]^{-1}.$$

It can be shown by standard difference equations methods (details available upon request) that the solution is, for any t :

$$x_t = \left(A^{T-t}D+\left(I-A\right)^{-1}\left(I-A^{T-t}\right)B\right)u_L,$$

where

$$D \equiv \begin{pmatrix} \omega & -\left(1-\zeta\right)\left(1-\sigma\right)\varphi\omega \\ \delta\omega & \left[\kappa\varphi-\delta\left(1-\zeta\right)\left(1-\sigma\right)\varphi\omega\right] \end{pmatrix}.$$

The solution holds as long as the zero lower bound binds–which is dictated by the Taylor rule. As already mentioned in text, the policy rule we use in this setup is (as in Carlstrom, Fuerst and Paustian, 2013) with $\iota_t = \rho_t$ in (7). This rule ensures that the now endogenous duration of the ZLB coincides with the duration of the exogenous shock T (in other words, when using ρ instead of ρ_t the ZLB stops binding earlier than T).

Welfare is computed as:

$$U_L = \sum_{t=1}^{T} (1+\rho_L)^{-t} \left\{ \frac{\left[(1+c_t)^{\zeta}\left(1-\frac{N}{1-N}y_t\right)^{1-\zeta}\right]^{1-\sigma} - 1}{1-\sigma} + \zeta \frac{G}{Y-G} \frac{(1+g_t)^{1-\sigma}-1}{1-\sigma} \right\} + \text{const.}, \tag{C.8}$$

where in the "wasteful" case $g_t = 0$ in the last term in curly brackets.

The other objects in the Table are computed as follows. Let the solution matrix be $Q_t \equiv A^{T-t}D + (I-A)^{-1}\left(I - A^{T-t}\right)B = \begin{pmatrix} q_t^{c\rho} & q_t^{cg} \\ q_t^{\pi\rho} & q_t^{\pi c} \end{pmatrix}$ where the $q's$ are essentially the (time-varying) multipliers. More specifically, the consumption multiplier dC/dG at any time t is $\left(1-\frac{G}{Y}\right) q_t^{cg}$, and the output collapse at the ZLB is $\left(1-\frac{G}{Y}\right) q_t^{c\rho}$.To get the average object for each of this, we simply take the time average e.g. $\left(1-\frac{G}{Y}\right)\left(\sum_{t=1}^{T} q_t^{c\rho}\right)/T$.

D Appendix D: Government spending at the ZLB and welfare in a Great Depression calibration

In this Appendix, we study optimal government spending in a slightly different setup that has been analyzed in Woodford (2011); this setup and the calibration studied below deliver a ZLB-driven recession that is of the size of the Great Depression. The utility specification — indicated with a superscript S — is separable in consumption and hours:

$$U^S(C_t, N_t, G_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} + \chi_G \frac{G_t^{1-\gamma}-1}{1-\gamma}, \tag{D.1}$$

where $\gamma > 0$ and $\varphi > 0$.

The loglinearized equilibrium conditions are, for arbitrary exogenous processes:

$$c_t = \mathbb{E}_t c_{t+1} - \gamma^{-1}\left(i_t - \mathbb{E}_t \pi_{t+1} - \rho_t\right) \tag{D.2}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(\gamma + \varphi\left(1 - \frac{G}{Y}\right)\right) c_t + \kappa\varphi \frac{G}{Y} g_t. \tag{D.3}$$

The solution at the ZLB is:

$$c_L = \frac{1-\beta p}{\Omega^S}\rho_L + M_c^S \frac{G}{Y-G} g_L$$
$$\pi_L = \frac{\kappa\left(\gamma + \varphi\left(1-\frac{G}{Y}\right)\right)}{\Omega^S}\rho_L + M_\pi^S \frac{G}{Y}\, g_L,$$

where $\Omega^S \equiv \gamma(1-p)(1-\beta p) - \kappa p\left[\gamma + \varphi\left(1-\frac{G}{Y}\right)\right]$.

To facilitate comparison with Woodford (2011) who assumes optimal monetary policy, we assume that the monetary policy rule takes the form: $i_t = \max(r_t^*, 0)$, where r_t^* is the flexible-price natural interest rate[36]. In particular, r_t^* is calculated as follows. From the Phillips curve (D.3), the natural level of consumption under separable utility reads: $c_t^* = -\varphi\left[\varphi\left(1-\frac{G}{Y}\right)+\gamma\right]^{-1}\frac{G}{Y}g_t$. Replacing this in the Euler equation (D.2), and simplifying, yields the following expression for the natural interest rate :

$$r_t^* = \rho_t + \frac{\varphi\gamma}{\varphi\left(1-\frac{G}{Y}\right)+\gamma}\frac{G}{Y}\left(g_t - \mathbb{E}_t g_{t+1}\right)$$

The consumption and inflation multipliers at the ZLB are:

$$M_c^S \equiv \frac{p\kappa\varphi\left(1-\frac{G}{Y}\right)}{\Omega^S},$$
$$M_\pi^S \equiv \frac{\gamma\varphi(1-p)\kappa}{\Omega^S},$$

with the same requirements as before ruling out sunspot fluctuations $\Omega^S > 0$ and starvation $C_L/C = 1 + (1-\beta p)\left(\Omega^S\right)^{-1}\rho_L > 0$. The conditions are satisfied when labor supply is elastic (φ low), and the intertemporal elasticity of substitution is low (γ high)—in addition to the conditions on p and κ already operating in the setup studied in text.

The Great Depression calibration

We first describe how the calibration of Woodford (2011), which delivers a Great Depression, is obtained in our model. The utility function used by Woodford is: $u(C) + g(G) - v(N)$,with elasticities

$$\eta_u = -\frac{u''Y}{u'};\ \eta_g = -\frac{g''Y}{g'};\ \eta_v = \frac{v''Y}{v'}$$

[36] We abstract from well-understood local determinacy issues associated with the equilibrium outside the ZLB, since our focus is on the ZLB equilibrium.

For our utility function in (D.1) the mapping is hence:

$$\eta_u = \gamma \frac{Y}{Y-G};\ \eta_g = \gamma \frac{Y}{G};\ \eta_v = \varphi$$

Note that since $G/Y = 0.2$ and we have the same curvature for utility in G and C, we can only consider one of the cases considered by Woodford (2011) in his Figure 4, namely case B in which $\eta_g = 4\eta_u$.The parameters calibrated by Woodford are:

$$\begin{aligned}
\Gamma &\equiv \frac{\eta_u}{\eta_u + \eta_v} = 0.425 \\
\psi &\equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}(\eta_u + \eta_v) = 0.00859 \\
\sigma &\equiv (\eta_u)^{-1} = 0.862
\end{aligned}$$

In terms of our parameters, this can be expressed (replacing the mapping found above) as:

$$\begin{aligned}
\frac{\gamma}{\gamma + \varphi\left(1 - \frac{G}{Y}\right)} &= 0.425 \\
\kappa\left(\frac{\gamma}{1-\frac{G}{Y}} + \varphi\right) &= 0.00859 \\
\frac{1-\frac{G}{Y}}{\gamma} &= 0.862,
\end{aligned}$$

which given $G/Y(= 0.2)$ can be solved easily to deliver:

$$\gamma = 0.928\,07;\ \kappa = 0.0031469;\ \varphi = 1.5695$$

Lastly, given $\beta = 0.997$, the value of κ implies, in the Calvo model with homogenous labor as in Woodford (2011), a probability of not adjusting the price of $\alpha = 0.946\,82$, or an average price duration of 20 quarters.[37]

[37] It is important to note that these calculations are based on the standard New Keynesian model with *homogeneous* labor types, as used in our case and, e.g., in Woodford (2012). Eggertsson (2009), from which the calibration in Woodford (2012) is taken, uses a model of the labor market with *differentiated* labor types, hence obtaining (by standard arguments pertaining to real rigidities) the same value of the Phillips curve parameter with a lower degree of price stickiness. However, Eggertsson (2009) does not study welfare. We use the same model and calibration as Woodford (2012) in order to facilitate the comparison.

Table 4. Great Depression calibration			
Parameter	Description		
p	transition probability		0.903
ρ_L	quarterly discount rate		$-.01$
β	discount factor in steady state		0.997
γ	relative risk aversion		0.862
φ	inverse labor elasticity		1.5695
κ	slope of the Phillips curve		0.003147
Note. values based on Woodford (2011).			

Welfare at the ZLB Lifetime welfare conditional upon being at the ZLB is, in the separable-utility case:

$$U_L^S = \frac{1+\rho_L}{1+\rho_L-p}\left[\frac{C^{1-\gamma}(1+c_L)^{1-\gamma}-1}{1-\gamma} - \chi\frac{N^{1+\varphi}(1+n_L)^{1+\varphi}}{1+\varphi} + \chi_G\frac{G^{1-\gamma}(1+g_L)^{1-\gamma}-1}{1-\gamma}\right], \tag{D.4}$$

With the optimal steady-state subsidy in place, $C^{-\gamma} = \chi N^{\varphi} = \chi Y^{\varphi}$. The steady state "Samuelson condition" equating the marginal utility of private and public spending reads in this case:

$$C^{-\gamma} = \chi_G G^{-\gamma} = \chi Y^{\varphi}.$$

Replacing the above expression in the welfare function and dividing by $\chi N^{1+\varphi}$ yields, abstracting from a constant additive term (which anyway disappears when we look at percentage deviations of welfare, as we do):

$$U_L^S = \frac{1+\rho_L}{1+\rho_L-p}C^{-\gamma}Y\left[(1-G_Y)\frac{(1+c_L)^{1-\gamma}-1}{1-\gamma} - \frac{(1+n_L)^{1+\varphi}}{1+\varphi} + G_Y\frac{(1+g_L)^{1-\gamma}-1}{1-\gamma}\right], \tag{D.5}$$

Using the Samuelson condition combined with the resource constraint, we therefore obtain:

$$\chi_G = \left(\frac{G}{Y-G}\right)^{\gamma},$$

which, for $G_Y = 0.2$, delivers a unique χ_G. Finally, the weight of labor in utility $\chi = N^{-(\varphi+\gamma)}(1-(G/Y))^{-\gamma}$ can be chosen to match steady-state hours worked.

Under the Great Depression calibration, including a discount rate at the ZLB of $\rho_L = -.01$, such that the natural real interest rate falls to -4 percent per annum,[38] the model

[38] Notice that we are constrained in our choice of the size of the shock. Considering even larger shocks, under the Great Depression, leads easily to a violation of the non-negativity constraint on consumption.

delivers a contraction in output of -28.8 percent (in annualized terms), in line with the Great Depression data—se the row labelled "LS" in Table 5. In the case of useful spending, the optimal increase in government spending is 11.5 percent of GDP. In the wasteful government spending case, the optimal increase in government spending is 5.5 percent of GDP. Importantly, this is the *only case* across all our experiments (including different solution methods for this same calibration—see below) in which we find that government spending can increase welfare at the ZLB under the assumption of wasteful spending.

The optimal increase in government spending under the Great Depression calibration is large because prices are now extremely sticky (an expected duration of 20 quarters), implying that the welfare cost of deflation is also extremely large; hence the rationale for using government spending in order to close the gap with respect to the flexible-price equilibrium is at its strongest. In addition, the shock is very persistent ($p = .903$), implying a very large output collapse, but also a large government spending multiplier, hence a large incentive to use government spending as a stabilization tool. In other words, the economy is very close to the bifurcation point that we discussed above; in fact, the parameter Ω, is now 0.0028.

However, the results are once again extremely sensitive to the expected duration of the liquidity trap. Table 5 below summarizes the effect of varying, ceteris paribus, the conditional probability $p$on the optimal size of government spending at the ZLB when all remaining parameters are kept equal to their values under the Great Depression calibration. If p is reduced from $p = 0.903$ to $p = 0.8$ (i.e., the value under the benchmark calibration), or to $p = 0.7$, the optimal increase in government spending plunges, in the case of useful spending, from 11.5 percent to 1.5 percent and 0.8percent, respectively; while for the rest of the domain for which the ZLB keeps binding (up to around 30 respectively 40 percent of GDP under these calibrations), utility decreases abruptly. Moreover, in the case of wasteful spending, utility is sharply decreasing in government spending. Already for $p = 0.8$ the optimal increase in government spending is zero.

Table 5. Optimal Increase in Government Spending		
Great Depression calibration		
p	useful G	wasteful G
0.903	11.5	5.5
0.8	1.5	0.0
0.7	0.8	0.0
Note. Great Depression calibration for all other parameters. Entries are in % of steady state output.		

It is worth noticing that the size of the shock under the Great Depression calibration is rather extreme. This raises some concern about the accuracy of the first order approximation, especially in an equilibrium where the policy function, in principle, should exhibit a kink due to the presence of a ZLB constraint. We turn to these considerations next. As for the baseline calibration, we study the implications of solving the model for the Great Depression calibration using the other two approaches: nonlinear stochastic (NLS), and linearized deterministic (LD). Detail of the solution methods parallel those outlines above for the benchmark model. The results are presented in Table 3 in text, following the same structure as in Table 2. For the linear deterministic model, we assume that the duration of the liquidity trap is $T_L = 10$, to make it comparable with the expected duration in the stochastic trap case, which is $1/(1 - 0.903) = 10.3$.

As with the baseline calibration, the multiplier is smaller in the NLS model and one order of magnitude smaller in the deterministic model. Likewise, the output collapse is smaller in the NLS model and several times smaller in the deterministic models. When cyclical spending provides a direct utility benefit, optimal spending is one order of magnitude smaller in the deterministic model than in the stochastic models, which (as usual) are plagued by the bifurcation issue discussed above.[39] Finally, even under this extreme calibration optimal cyclical spending in the wasteful case is zero in all cases, except for the linearized stochastic case.

[39] Note that in the NLS model, as in the loglin stochastic case, when we decrease the probability p to 0.7 optimal spending in the useful case is much smaller, about 1 percent of steady state output.