

# Search-Based Endogenous Illiquidity and the Macroeconomy

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# Outline

- 1 Intro
  - Motivation
- 2 The Model
  - Set-up
  - Households
  - Bargaining and Final Goods Producers
- 3 Results
  - Some Theoretical Results
  - Impulse Responses
- 4 Final Remark
  - Investment Shocks?
  - Conclusion

# Motivation

- Asset liquidity
  - procyclical variation (turnover, bid-ask spreads....) [▶ cycle](#)
  - holding of liquid assets against possible funding constraints, at least since Keynes (1936) and Tobin (1969)

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- Shocks to asset liquidity as a source of business cycle
  - recently Kiyotaki and Moore (2012), Shi (2012), and Del Negro et al. (2011)...
- But, abnormal asset price dynamics and we need endogenous asset liquidity
  - financial intermediation and markets as a matching function
  - what induces flight to liquidity and push down asset price in recessions?
  - the macro impacts and feedback?

# Endogenous Search-Based Illiquidity

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- Two contributions: a tractable variant of RBC model...
  - liquidity-based asset pricing
  - amplification through imperfect substitutes between assets
- Adverse shocks to productivity or cost of matching
  - Return from assets ↓
  - Search for investment opportunities = demand ↓
  - Resaleability ↓, Price ↓
  - **But different “flight to liquidity” dynamics**

# Literature

- Search-framework captures features of...
  - OTC markets for corporate bonds, Duffie, Garleanu, and Pedersen (2007)
  - investment banking fees for IPOs
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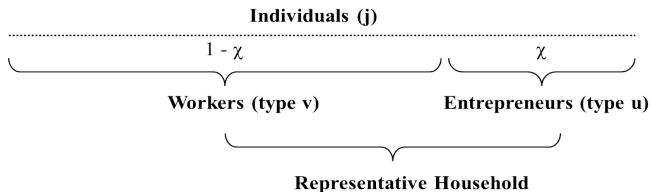
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  - Pure liquidity shocks = supply shocks, asset prices  $\uparrow$
- Interaction between asset liquidity and macro
  - Bacchetta, Benhima, and Poilly (2015)
  - Rocheteau and Wright (2013), Guerrieri and Shimer (2012), Lagos and Rocheteau (2009)

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# Environment

- Infinite time horizon.
- A continuum of households, final goods producers, and financial intermediaries.
- Household structure



- Only entrepreneurs have investment opportunities

# A Model Comparison



# Timing

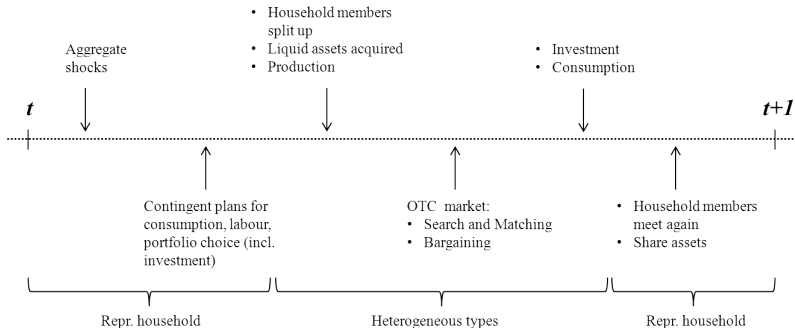


Figure: Timing

# A Representative Household

- Preferences:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[ \chi u\left(\frac{C_{u,t+s}}{\chi}\right) + (1 - \chi) u\left(\frac{C_{v,t+s}}{1 - \chi}\right) - \mu N_{t+s} \right]$$

- Portfolio

Table: Household's Balance Sheet

|                |                 |               |                           |
|----------------|-----------------|---------------|---------------------------|
| liquid bonds   | $R_t B_t / P_t$ | equity issued | $q_t S_t^I$               |
| other's equity | $q_t S_t^O$     |               |                           |
| capital stock  | $q_t K_t$       | net worth     | $q_t S_t + R_t B_t / P_t$ |

$$S_t = \underbrace{S_t^O}_{\text{other's equity}} + \underbrace{K_t - S_t^I}_{\text{unmortgaged capital}}$$

# Some Notation

- $V$ : units quoted from buyers.  $\kappa_V$ : unit cost.  $\phi_V$  fraction of successful match
- $U$ : units quoted from sellers.  $\kappa_U$ : unit cost.  $\phi_U$  fraction of successful match
- $\kappa_V$  and  $\kappa_U$  measures the efficiency of the financial sectors
- effective prices and spreads

$$q_u \equiv q - \frac{\kappa_u}{\phi_u}, \quad q_v \equiv q + \frac{\kappa_v}{\phi_v},$$

$$q_v - q_u = \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u}$$

# Workers' Budget Constraint

- Workers

$$C_{v,t} + \kappa_v V_t + \frac{B_{v,t+1}}{P_t} = w_t N_{v,t} + r_t S_{v,t} - q_t \phi_{v,t} V_t + \frac{R_t B_{v,t}}{P_t},$$

$$S_{v,t+1} = (1 - \delta) S_{v,t} + \phi_{v,t} V_t$$

Rearrange

$$C_v + q_v S'_v + \frac{B'_v}{P} = wL + rS_v + (1 - \delta) q_v S_v + \frac{RB_v}{P} \quad (1)$$

# Entrepreneurs' Budget Constraint

- Entrepreneurs,  $U = e[(1 - \delta)S_u + I]$

$$C_{u,t} + I_{u,t} + \kappa_u U_t + \frac{B_{u,t+1}}{P_t} = r_t S_{u,t} + q_t \phi_{u,t} U_t + \frac{R_t B_{u,t}}{P_t}$$

$$S_{u,t+1} = (1 - \delta)S_{u,t} + I_{u,t} - \phi_{u,t} U_t$$

Rearrange and substitute out investment

$$C_u + q_r S'_u + \frac{B'_u}{P} = r S_u + [\phi_u q_u + (1 - \phi_u) q_r] (1 - \delta) S_u + \frac{R B_u}{P} \quad (2)$$

where

$$q_r \equiv \frac{1 - e \phi_u q_u}{1 - e \phi_u}$$

$$I = \frac{\chi \left[ (r + e \phi_u q_u (1 - \delta)) K + \frac{B}{P} \right] - C_u}{1 - e \phi_u q_u}. \quad (3)$$



# HH Problem

## Problem

$$J(S, B; \Gamma) = \max_{\{N, C_u, C_v, S'_u, S'_v, B'_u, B'_v\}} \chi u \left( \frac{C_u}{\chi} \right) + (1 - \chi) \left[ u \left( \frac{C_v}{1 - \chi} \right) - \mu \frac{N}{1 - \chi} \right] \\ + \beta \mathbb{E}_\Gamma [J(S', B'; \Gamma')]$$

$$\text{s.t. (1), (2)}$$

$$S_u = \chi S, \quad S_v = (1 - \chi)S$$

$$B_u = \chi B, \quad B_v = (1 - \chi)B$$

$$S' = S'_u + S'_v, \quad B' = B'_u + B'_v$$

Entrepreneurs can leverage up and  $B'_u = 0$  because

$q_r < 1 < q < q_v$ . [▶ Leverage](#)

Choose  $e = 1$ , i.e.,  $U = (1 - \delta)S_u + I$  because  $q_r$  decreases with  $e$ .

# Liquidity Service

- Let  $\rho \equiv \frac{q_v}{q_r}$  measures risk-sharing. Optimality conditions:

$$u'(c_v) w = \mu$$

$$u'(c_u) = \rho u'(c_v)$$

$$\mathbb{E}_\Gamma \left[ \frac{\beta u'(c'_v)}{u'(c_v)} [\chi \rho' r'_u + (1 - \chi) r'_v] \right] = 1,$$

$$\mathbb{E}_\Gamma \left[ \frac{\beta u'(c'_v)}{u'(c_v)} (\chi \rho' + 1 - \chi) \frac{R'}{\pi'} \right] = 1$$

where  $r'_u = [r' + (1 - \delta)(\phi' q'_u + (1 - \phi') q'_r)]/q_v$ ,  
 $r'_v = [r' + (1 - \delta)q'_v]/q_v$ , and  $\pi = P/P_{-1}$ .

- Liquidity premium in steady state:
  - $\rho > 1$  and the real interest rate  $R/\pi < 1/\beta$ .

# Final Goods Markets and Financial Markets

- Final goods producers

$$Y = e^{z_a} F(K, L) = e^{z_a} K^\alpha N^{1-\alpha}$$

Then,  $w = e^{z_a} F_N(K, N)$  and  $r = e^{z_a} F_K(K, N)$

- The total matches  $M$  and endogenous buying and selling probabilities:

$$M \equiv \xi U^\eta V^{1-\eta}$$

$$\phi_v \equiv \frac{M}{V} = \xi \left( \frac{V}{U} \right)^{-\eta}, \quad \phi_u \equiv \frac{M}{U} = \xi \left( \frac{V}{U} \right)^{1-\eta}$$

- Cost of matching  $\kappa_u = \bar{\kappa}_u e^{z_\kappa}$  and  $\kappa_v = \bar{\kappa}_v e^{z_\kappa}$
- Note:  $z_a$  and  $z_\kappa$  are exogenous

# Asset Price

Value of buyers and sellers

$$-J_m^v = -q + \beta \mathbb{E}_\Gamma \left[ \frac{J_S(S', B'; \Gamma')}{u'(c_v)} \right]$$

$$J_m^u = q - \phi_u^{-1} + \beta (\phi_u^{-1} - 1) \mathbb{E}_\Gamma \left[ \frac{J_S(S', B'; \Gamma')}{u'(c_u)} \right]$$

Intermediaries choose  $q$  to maximize

$$\omega \ln(J_m^u) + (1 - \omega) \ln(-J_m^v)$$

## Lemma

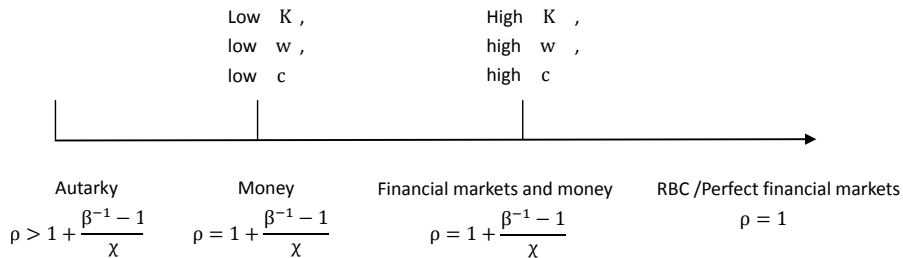
Let  $\gamma \equiv \frac{\omega}{1-\omega} \frac{\kappa_v}{\kappa_u}$ , the bargaining solution is  $\rho = \gamma\theta$ . Equivalently,

$$q = \frac{\gamma\phi_u \left(1 + \frac{\kappa_u}{\omega}\right) - \kappa_v}{\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{1-\eta}} \left[1 + \left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}} - 1\right) \phi_u\right]}. \quad (4)$$

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# Asset Price and Costs of Intermediation

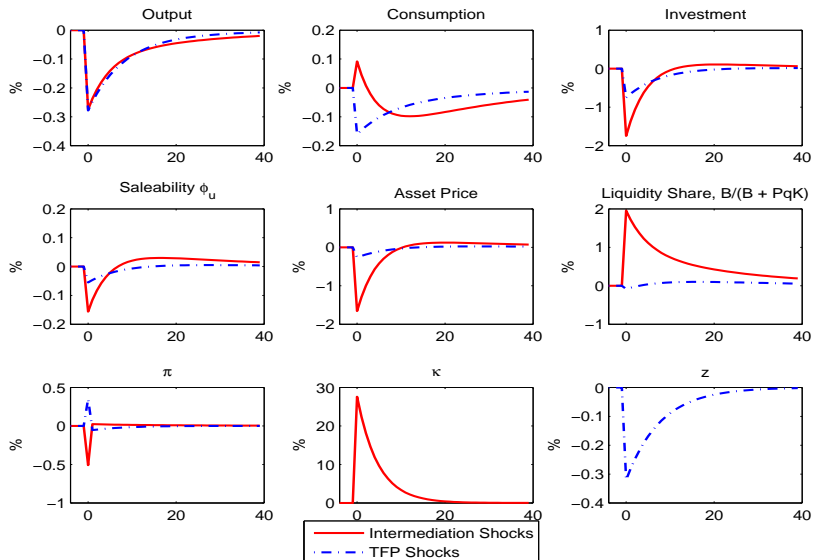
## Proposition

*Suppose the ratio of  $\bar{\kappa}_v/\bar{\kappa}_u = g$  is fixed. Under some regularity conditions, then in steady state asset price  $q$  drops when  $\bar{\kappa}_v$  and  $\bar{\kappa}_u$  increase.*

## Proposition

*Under some regularity conditions,  $q$  correlates positively with asset saleability  $\phi_u$  (i.e.  $\frac{\partial q}{\partial \phi_u} > 0$ ) and negatively with the purchase rate  $\phi_v$  (i.e.  $\frac{\partial q}{\partial \phi_v} < 0$ ).*

# Equilibrium Responses to Shocks



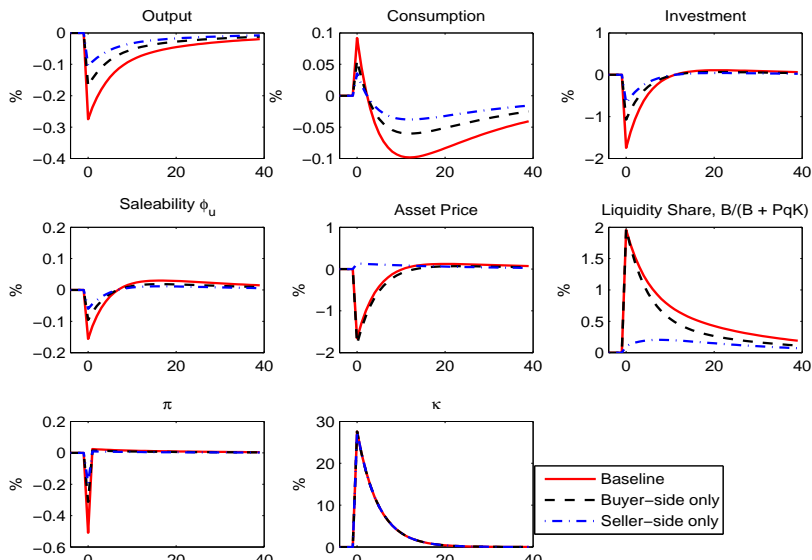
# A Negative Productivity Shock (cont.)

- What do we learn?
- When productivity drops
  - Investors search less; demand for assets drops
  - Harder to sell and price drops
- However: no robust flight to liquidity
  - persistent TFP shock; future investment unattractive
  - no hedging against future illiquidity
  - liquidity share drops

# An Intermediation Cost Shock (cont.)

- What do we learn?
- Cost of matching  $\uparrow$ : liquidity and price drop
  - less investment; decrease supply of financial assets
  - households substitute away from costly search
  - large demand drops endogenously
- Hedging: liquidity share increases
  - capital still highly productive; future investment attractive
  - flight to liquidity to hedge
- Output decreases because of drop of employment

# Shocks to One Side of the Market



# Cycle Statistics

**Table:** Cycle statistics with only aggregate productivity shocks

| Variable $x$    | Relative volatility $\frac{\sigma_x}{\sigma_y}$ |       | Correlation $\rho(x, y)$ |       | 1st auto-co |
|-----------------|---|-------|--------------------------|-------|-------------|
|                 | Data  | Model | Data                     | Model | Data        |
| Output          | 0.02  | 0.02  | 1.00                     | 1.00  | 0.89        |
| Consumption     | 0.44  | 0.67  | 0.88                     | 0.98  | 0.80        |
| Investment      | 3.45  | 2.43  | 0.96                     | 0.98  | 0.85        |
| Liquidity Share | 3.44  | 0.87  | -0.58                    | -0.27 | 0.89        |
| Asset Price     | 5.23  | 0.79  | 0.50                     | 0.88  | 0.81        |

Note: The volatility of output ( $y$ ) is reported as is. The relative volatilities and correlations of other variables are measured against  $y$ .

# Cycle Statistics

Table: Cycle statistics with only intermediation shocks

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| Investment      | 3.45  | 4.61  | 0.96                     | 0.78  | 0.85        |
| Liquidity Share | 3.44  | 7.85  | -0.58                    | -0.96 | 0.89        |
| Asset Price     | 5.23  | 4.36  | 0.50                     | 0.75  | 0.81        |

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# Business Cycle Accounting?

- Shi(2012): aggregate productivity shocks are always needed
- Matching-cost shocks: not productivity shocks

$$C + I + \kappa_v V + \kappa_u U = Y$$

- Investment-specific technology shocks that will change equilibrium employment.
  - related to Chari et al (2007), financial shocks that can be both investment and labor wedges.
- Equity premium puzzle?

# Takeaways

- Endogenous spectrum of liquidity premia
  - procyclical asset liquidity and prices
  
- Matching cost push shocks
  - robust flight to liquidity
  - many stylized business cycle facts
  
- Further work
  - financial intermediaries' balance sheets?
  - implementation of constrained efficient allocation?
  - financial development and growth?

# Appendix

Appendix

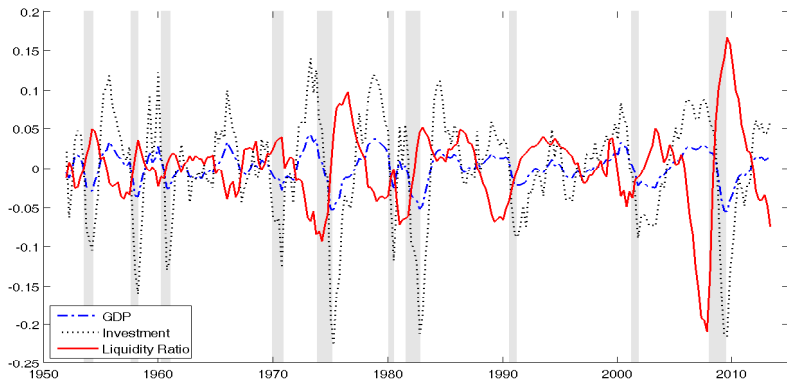
# Asset Liquidity?

- During recessions...
  - ... assets are harder to sell and their prices drop
  - ... firms and banks re-balance towards highly liquid government bonds/money (flight to liquidity)
- Why are fluctuations in asset liquidity important?
  - Adverse effects on financing, investment, employment...
- However, is asset illiquidity an endogenous or exogenous phenomenon?
- What drives pro-cyclical liquidity, flight to liquidity in recessions, and the impacts on investment and employment?

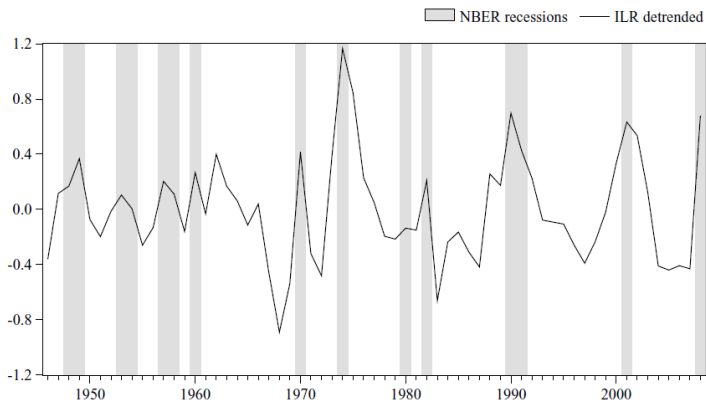
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- 5 Data
  
- 6 Model
  - Leverage
  - Bargaining

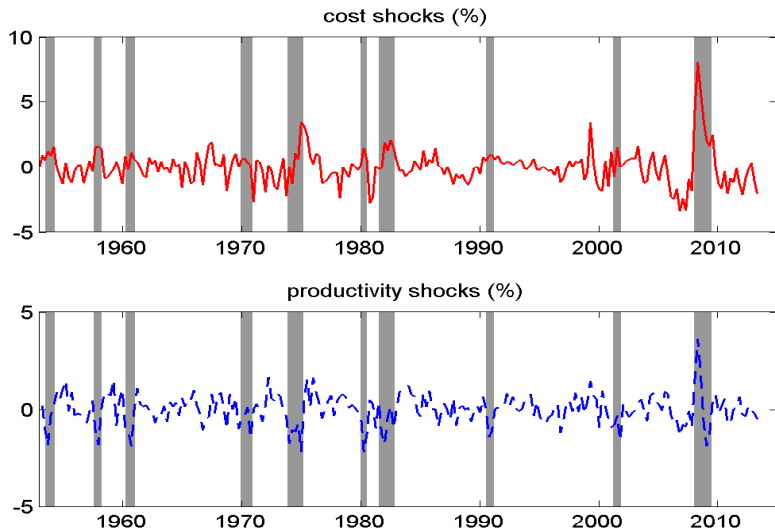
# The Liquidity Share

[▶ back](#)

# (Market) Liquidity Cycles: Naes et al. 2011 JF

[▶ Back](#)

# Back Out Shocks



# Outline

5 Data

6 Model

- Leverage
- Bargaining

# Entrepreneurs' leverage

Rearrange the budget constraint of entrepreneur, one can express

$$S'_u = \chi \frac{(r + (1 - \delta)q_\chi) S + R \frac{B}{P}}{q_r} \quad (5)$$

▶ back

## Bargaining - value

## Problem

Buyers' Value  $J^v(s_v, b_v, b'_v, v, m_v)$

$$J = \int_{j \in u} u(c_j) dj + \int_{j \in v} \left[ u(c_j) - \frac{\mu}{1+\nu} n_j^{1+\nu} \right] dj + \mathbb{E}_\Gamma [J(S', B'; \Gamma')] \quad \text{s.t.}$$

$$c_v + \kappa_v v + \frac{b'_v}{P} = wn_v + rs_v + qm_v + \frac{Rb_v}{P}$$

$$s'_v = (1 - \delta)s_v - m_v$$

$$S' = \int_{j \in v} s'_j dj + \int_{j \in u} s'_j dj, \quad B' = \int_{j \in v} b'_j dj + \int_{j \in u} b'_j dj$$

## Bargaining - Value (cont.)

## Problem

Sellers' Value  $J^u(s_u, b_u, b'_u, u, m_u)$

$$J^u = \int_{j \in u} u(c_j) dj + \int_{j \in v} \left[ u(c_j) - \frac{\mu}{1+\nu} n_j^{1+\nu} \right] dj + \mathbb{E}_\Gamma [J(S', B'; \Gamma')] \quad s.t.$$

$$c_u + \kappa_u u + \frac{b'_u}{p} = r s_u + (1 - \delta) s_u + \left( q - \frac{1}{\phi_u} \right) m_u + \frac{R b_u}{p}$$

$$s'_u = \left( \frac{1}{\phi_u} - 1 \right) m_u$$

$$S' = \int_{j \in v} s'_j dj + \int_{j \in u} s'_j dj, \quad B' = \int_{j \in v} b'_j dj + \int_{j \in u} b'_j dj$$

# Nash Bargaining

Choose  $q$  to maximize  $\omega \ln(J_m^u) + (1 - \omega) \ln(-J_m^v)$  The marginal value of additional match for a buyer (worker) is thus given by

$$\begin{aligned} -J_m^v &= -u'(c_v)q + \beta \mathbb{E}_\Gamma \left[ J_S(S', B'; \Gamma') \frac{\partial S'}{\partial s'_v} \frac{\partial s'_v}{\partial (-m_v)} \right] \\ &= -u'(c_v)q + \beta \mathbb{E}_\Gamma [J_S(S', B'; \Gamma')], \end{aligned}$$

and for a seller (entrepreneur) by

$$\begin{aligned} J_m^u &= u'(c_u) \left( q - \frac{1}{\phi_u} \right) + \beta \mathbb{E}_\Gamma \left[ J_S(S', B'; \Gamma') \frac{\partial S'}{\partial s'_u} \frac{\partial s'_u}{\partial m_u} \right] \\ &= u'(c_u) \left( q - \frac{1}{\phi_u} \right) + \beta \mathbb{E}_\Gamma \left[ J_S(S', B'; \Gamma') \left( \frac{1}{\phi_u} - 1 \right) \right]. \end{aligned}$$