How should government debt maturity be structured?
Motivation

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- One motive: Hedge against shocks
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- One motive: Hedge against shocks

- Results in Angeletos (2002) and Buera and Nicolini (2004):
  - Governments purchase short-term assets, and issues long-term debt
  - Positions are very large (several multiples of GDP)
  - Debt positions are constant, not actively managed
Conclusions of previous research not robust to lack of commitment
This paper

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- In practice, government chooses debt and taxes sequentially
  \[\Rightarrow\text{ Commitment problem}\]
  - Government can change market value of outstanding debt ex-post
  - Ex-post policy not optimal ex-ante
Conclusions of previous research not robust to lack of commitment

In practice, government chooses debt and taxes sequentially
⇒ Commitment problem
  - Government can change market value of outstanding debt ex-post
  - Ex-post policy not optimal ex-ante

This paper: Optimal debt maturity under lack of commitment
  - Focus on Markov Perfect Equilibrium
Preview of the Model

- Standard model of optimal fiscal policy [Lucas-Stokey (1983)]
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Add two frictions: non-contingent bonds and no gov’t commitment
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- Debt maturity can solve either problem separately

  Non-contingent Bonds vs. Lack of Commitment
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  Non-contingent Bonds vs. Lack of Commitment
  ↓
  Large and Tilted Positions ↓ Flat Maturity
Main Results

- If debt positions are large and titled $\rightarrow$ **Lack of commitment** is costly
  - Expectation of future deviation raises ex-ante borrowing costs
  - $\rightarrow$ High *average* tax/spending distortions
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- If debt positions are flat $\rightarrow$ **Lack of insurance** is less costly
  - Flat debt position $\rightarrow$ Low fluctuation in market value of debt
  - $\rightarrow$ High *volatility* in tax/spending distortions
Main Results

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- **High volatility** of distortions less costly than high average distortions
Main Results

- If debt positions are large and titled → **Lack of commitment** is costly
  - Expectation of future deviation raises ex-ante borrowing costs
  - → High *average* tax/spending distortions

- If debt positions are flat → **Lack of insurance** is less costly
  - Flat debt position → Low fluctuation in market value of debt
  - → High *volatility* in tax/spending distortions

- **High volatility** of distortions less costly than *high average* distortions

- Optimal maturity is quantitatively nearly flat
  - Reducing borrowing costs more important than insurance
  - Optimal policy approximated by active consol management
Government debt maturity under lack of commitment

- **This paper:** Economy without risk of default or surprise inflation
Related Literature

- Government debt maturity under lack of commitment
  - **This paper:** Economy without risk of default or surprise inflation

- Optimal fiscal policy under non-contingent debt and full commitment
  - **This paper:** Long-term debt. No inefficiencies under full commitment
Related Literature

- Government debt maturity under lack of commitment
  - This paper: Economy without risk of default or surprise inflation

- Optimal fiscal policy under non-contingent debt and full commitment
  - This paper: Long-term debt. No inefficiencies under full commitment

- Optimal fiscal policy under contingent debt and lack of commitment
  - This paper: Long-term debt. No inefficiencies under complete markets
Outline

1. Model

2. Lack of commitment benchmark

3. Lack of insurance benchmark

4. Maturity management under both frictions
$t \in \{0, 1, \ldots \}$. Shock $\theta_t \in \Theta$. 

**The Model**

Lucas and Stokey (1983)
The Model
Lucas and Stokey (1983)

- $t \in \{0, 1, \ldots\}$. Shock $\theta_t \in \Theta$.

- Representative household. Preferences:

$$
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) + \theta_t v(g_t)
$$
The Model
Lucas and Stokey (1983)

- \( t \in \{0, 1, \ldots \} \). Shock \( \theta_t \in \Theta \).

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\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) + \theta_t v(g_t)
\]

- Household budget constraints

\[
c_t = n_t (1 - \tau_t) + \sum_{j=1}^{\infty} q_{t+j}^t \left( B_{t-1}^{t+j} - B_{t+1}^{t+j} \right) + B_{t-1}^t
\]
The Model
Lucas and Stokey (1983)

- $t \in \{0, 1, \ldots\}$. Shock $\theta_t \in \Theta$.

- Representative household. Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) + \theta_t v(g_t)$$

- Household budget constraints

$$c_t = n_t (1 - \tau_t) + \sum_{j=1}^{\infty} q^{t+j}_t \left( B^{t+j}_{t-1} - B^{t+j}_t \right) + B_{t-1}$$

- Government budget constraint

$$\tau_t n_t - g_t = \sum_{j=1}^{\infty} q^{t+j}_t \left( B^{t+j}_{t-1} - B^{t+j}_t \right) + B_{t-1}$$
Government strategy: choose $\tau_t, g_t, \left\{ B_{t+j}^t \right\}_{j=1}^\infty$ given $\theta_t, \left\{ B_{t-1+j}^t \right\}_{j=1}^\infty$. 
Markov Perfect Competitive Equilibrium

- Government strategy: choose $\tau_t, g_t, \left\{B^{t+j}_t\right\}_{j=1}^\infty$ given $\theta_t, \left\{B^{t+j}_{t-1}\right\}_{j=1}^\infty$

- Household allocation: choose $c_t, n_t, \left\{B^{t+j}_t\right\}_{j=1}^\infty$ given $\tau_t, \left\{q^{t+j}_t, B^{t+j}_{t-1}\right\}_{j=1}^\infty$
Markov Perfect Competitive Equilibrium

- Government strategy: choose \( \tau_t, g_t, \left\{ B^{t+j}_t \right\}_{j=1}^{\infty} \) given \( \theta_t, \left\{ B^{t+j}_{t-1} \right\}_{j=1}^{\infty} \)

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- Markov Perfect Competitive Equilibrium:
  1. Government strategy optimal
  2. Household allocation optimal
  3. Bond prices \( q^{t+j}_t \) clears the market
Equilibrium conditions

Primal approach

- Intertemporal condition:

\[ q_{t+j}^t = \beta_j E_t u_{c,t+j} / u_{c,t} \]
Equilibrium conditions

Primal approach

- Intertemporal condition:
  \[ q_{t+j}^t = \beta^j \mathbb{E}_t \frac{u_{c,t+j}}{u_{c,t}} \]

- Intratemporal condition:
  \[ 1 - \tau_t = -\frac{u_{n,t}}{u_{c,t}} \]
Equilibrium conditions

Primal approach

- Intertemporal condition:
  \[ q_t^{t+j} = \beta_j \mathbb{E}_t \frac{u_{c,t+j}}{u_{c,t}} \]

- Intratemporal condition:
  \[ 1 - \tau_t = -\frac{u_{n,t}}{u_{c,t}} \]

- Budget Constraint (implementability condition):
  \[
  \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u_{c,t} \left[ c_t + \frac{u_{n,t}}{u_{c,t}} n_t \right] = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} B_{t-1}^{t+j}
  \]
  Primary Surpluses \( S(\theta_t) \)
  Value of Debt
Perfect insurance: fiscal policies only depend on $\theta_t$, not on the history.
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Optimal spending:

$$u_{c,t} \left[ (1 + \mu) + \mu \frac{u_{cc,t} c_t + u_{cn,t} n_t}{u_{c,t}} \right] = \theta_t v_{g,t}$$

Optimal taxes:

$$u_{c,t} = -u_{n,t} - \frac{\mu}{1 + \mu} \left[ (u_{cc,t} + u_{cn,t}) c_t + (u_{cn,t} + u_{nn,t}) n_t \right]$$
• Perfect insurance: fiscal policies only depend on $\theta_t$, not on the history

• Optimal spending:

$$u_{c,t} \left[ (1 + \mu) + \mu \frac{u_{cc,t} c_t + u_{cn,t} n_t}{u_{c,t}} \right] = \theta_t v_{g,t}$$

• Optimal taxes:

$$u_{c,t} = -u_{n,t} - \frac{\mu}{1 + \mu} \left[ (u_{cc,t} + u_{cn,t}) c_t + (u_{cn,t} + u_{nn,t}) n_t \right]$$

• Government may choose to reduce these distortions ex-post (i.e. change $\mu$)
$t = 0, 1, 2. \ \theta_0 > \theta_1 = \theta_2 = 1$ (high spending at date 0)
Example of Three Period Economy

- $t = 0, 1, 2$. $\theta_0 > \theta_1 = \theta_2 = 1$ (high spending at date 0)

- Suppose that tax revenues are exogenously fixed
  - e.g., applies under GHH preference with commitment to $\tau$

Example of Three Period Economy

- \( t = 0, 1, 2. \, \theta_0 > \theta_1 = \theta_2 = 1 \) (high spending at date 0)

- Suppose that tax revenues are exogenously fixed
  - e.g., applies under GHH preference with commitment to \( \tau \)

- Government welfare:
  \[
  (1 - \psi) \log c + \psi \theta g
  \]

- Consider the limit as \( \psi \to 1 \)
The government solves the following problem

\[
\min \theta_0 c_0 + \beta c_1 + \beta^2 c_2
\]

s.t.
Example of Three Period Economy (cont’d)

The government solves the following problem

\[
\begin{align*}
\text{min} & \quad \theta_0 c_0 + \beta c_1 + \beta^2 c_2 \\
\text{s.t.} & \quad [c_0 - n (1 - \tau)] + 
\end{align*}
\]

At an optimum:

\[c_1 = c_2 = n (1 - \tau) + B\]

where \(B\) is the primary surplus at date 1 and 2.

**REMARK 1:** It implies a bond price at date 1

\[q_2^1 = \beta c_1 c_2 = \beta\]

**REMARK 2:** It can be implemented with any maturity structure, such that

\[
B_1 + \beta B_2 = (1 + \beta)B
\]
Example of Three Period Economy (cont’d)

- The government solves the following problem

\[
\begin{align*}
\min & \quad \theta_0 c_0 + \beta c_1 + \beta^2 c_2 \\
\text{s.t.} & \quad [c_0 - n (1 - \tau)] + \beta \frac{c_0}{c_1} [c_1 - n (1 - \tau)] + \beta^2 \frac{c_0}{c_2} [c_2 - n (1 - \tau)] \geq 0
\end{align*}
\]

REMARK 1: It implies a bond price at date 1

\[q_2^0 = \beta c_1 \]

REMARK 2: it can be implemented with any maturity structure, such that

\[B_1^0 + \beta B_2^0 = (1 + \beta) B_0 \]

The government solves the following problem

\[
\min \theta_0 c_0 + \beta c_1 + \beta^2 c_2 \\
\text{s.t. } [c_0 - n(1 - \tau)] + \beta \frac{c_0}{c_1} [c_1 - n(1 - \tau)] + \beta^2 \frac{c_0}{c_2} [c_2 - n(1 - \tau)] \geq 0
\]

At an optimum:

\[c_1 = c_2 = n(1 - \tau) + \overline{B}\]

where \(\overline{B}\) is the primary surplus at date 1 and 2.
Example of Three Period Economy (cont’d)

- The government solves the following problem

  \[
  \min \theta_0 c_0 + \beta c_1 + \beta^2 c_2 \\
  \text{s.t.} \quad \left[ c_0 - n (1 - \tau) \right] + \beta \frac{c_0}{c_1} \left[ c_1 - n (1 - \tau) \right] + \beta^2 \frac{c_0}{c_2} \left[ c_2 - n (1 - \tau) \right] \geq 0
  \]

- At an optimum:

  \[ \begin{align*}
  c_1 &= c_2 = n (1 - \tau) + \overline{B} \\
  \end{align*} \]

  where \( \overline{B} \) is the primary surplus at date 1 and 2.

- REMARK 1: It implies a bond price at date 1

  \[ q_1^2 = \beta \frac{c_1}{c_2} = \beta \]
The government solves the following problem

\[
\begin{align*}
\min & \quad \theta_0 c_0 + \beta c_1 + \beta^2 c_2 \\
\text{s.t.} & \quad [c_0 - n (1 - \tau)] + \beta \frac{c_0}{c_1} [c_1 - n (1 - \tau)] + \beta^2 \frac{c_0}{c_2} [c_2 - n (1 - \tau)] \geq 0
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q_1^2 = \beta \frac{c_1}{c_2} = \beta
\]

**REMARK 2:** it can be implemented with any maturity structure, such that

\[
B_0^1 + \beta B_0^2 = (1 + \beta) \overline{B}
\]
A Simple Example
No Uncertainty
A Simple Example: No Uncertainty
A Simple Example
No Uncertainty

Short-term or Long-Term?
Only **Short-term** debt
Only **Short-term** debt
Only **Short-term** debt

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</tbody>
</table>
```


Only **Short–term** debt

New debt issued at date 1

- $\text{Taxes}_0$
- $\text{Taxes}_1$
- $\text{Taxes}_2$

$t=0$    $t=1$    $t=2$

$r_0^1 \text{B}_0^1$  $r_1^2 \text{B}_1^2$
Only Long-term debt
Only Long-term debt
Only **Long-term** debt

![Diagram showing financial flows at different time periods](image-url)

- **t=0**: Initial state with debt and taxes.
- **t=1**: Intermediate state showing surplus at date 1.
- **t=2**: Final state with future debt and taxes.

Key elements:
- **$B_0^2$**, **$Taxes_0$**, **$G$** at t=0
- **$B_1^2$**, **$Taxes_1$**, **$G$** at t=1
- **$B_0^2$**, **$B_1^2$**, **$Taxes_2$**, **$G$** at t=2

Arrows indicate financial flows and growth factors $r_1^2B_1^2$ and $r_0^2B_0^2$.
Lack of Commitment
The Commitment Problem

- If government is allowed to deviate in period 1
- ... given a maturity structure $B_0^1$ and $B_0^2$

\[
\min c_1^{\tau} + \beta c_2^{\tau-1} [ c_1^{\tau-1} - n(1-\tau)] + \beta c_1^{\tau} c_2^{\tau-1} [ c_2^{\tau-1} - n(1-\tau)] \\
\geq B_0^1 + \beta c_1^{\tau} c_2^{\tau-1} B_0^2
\]

Is it still optimal to choose $c_1 = c_2$?

The FOC implies:

\[
c_1^{\tau} c_2^{\tau-1} = \left( n(1-\tau) + B_0^1 n(1-\tau) + B_0^2 \right)^{1/2}
\]

FLAT MATURITY solves commitment problem:

$c_1 = c_2$ if and only if $B_0^1 = B_0^2$

Debortoli-Nunes-Yared (2015)
The Commitment Problem

- If government is allowed to deviate in period 1
- ... given a maturity structure $B_0^1$ and $B_0^2$

$$\begin{align*}
\min & \quad c_1 + \beta c_2 \\
\text{s.t.} & \quad [c_1 - n (1 - \tau)] + \beta \frac{c_1}{c_2} [c_2 - n (1 - \tau)] \geq B_0^1 + \beta \frac{c_1}{c_2} B_0^2
\end{align*}$$
The Commitment Problem

- If government is allowed to deviate in period 1
- ... given a maturity structure $B_0^1$ and $B_0^2$

$$\min \quad c_1 + \beta c_2$$

$$s.t. \quad [c_1 - n (1 - \tau)] + \beta \frac{c_1}{c_2} [c_2 - n (1 - \tau)] \geq B_0^1 + \beta \frac{c_1}{c_2} B_0^2$$

- Is it still optimal to choose $c_1 = c_2$?
The Commitment Problem

- If government is allowed to deviate in period 1
- ... given a maturity structure $B_0^1$ and $B_0^2$

$$\min \ c_1 + \beta c_2$$

$$s.t. \quad [c_1 - n (1 - \tau)] + \beta \frac{c_1}{c_2} \ [c_2 - n (1 - \tau)] \geq B_0^1 + \beta \frac{c_1}{c_2} B_0^2$$

- Is it still optimal to choose $c_1 = c_2$?
- The FOC implies

$$\frac{c_1}{c_2} = \left( \frac{n (1 - \tau) + B_0^1}{n (1 - \tau) + B_0^2} \right)^{1/2}$$
The Commitment Problem

- If government is allowed to deviate in period 1
- ... given a maturity structure $B_1^1$ and $B_2^2$

$$\begin{aligned}
&\text{min} \quad c_1 + \beta c_2 \\
&\text{s.t.} \quad [c_1 - n (1 - \tau)] + \beta \frac{c_1}{c_2} [c_2 - n (1 - \tau)] \geq B_1^1 + \beta \frac{c_1}{c_2} B_2^2
\end{aligned}$$

- Is it still optimal to choose $c_1 = c_2$?
- The FOC implies

$$\frac{c_1}{c_2} = \left( \frac{n (1 - \tau) + B_0^1}{n (1 - \tau) + B_0^2} \right)^{1/2}$$

- **FLAT MATURITY** solves commitment problem: $c_1 = c_2 \iff B_0^1 = B_0^2$
A simple example: Commitment Problem
A deviation from original plan at t=1
A simple example: Commitment Problem
A deviation from original plan at $t=1$
A simple example: Commitment Problem
A deviation from original plan at t=1
A simple example: Commitment Problem
A deviation from original plan at t=1

r₁^2 can be changed in t=1

r₀^2 was set in t=0
A simple example: Commitment Problem
A deviation from original plan at $t=1$

INCENTIVE
Spend more in $t=1$ → $B_1^2$ lower → $r_1^2$ higher
A simple example: Commitment Problem
A deviation from original plan at t=1

INCENTIVE
Spend more in t=1
$\Rightarrow B_1^2$ lower
$\Rightarrow r_1^2$ higher
FLAT MATURITY solves COMMITMENT PROBLEM
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FLAT MATURITY solves COMMITMENT PROBLEM

No incentive to change $r_1^2$ ex-post
Government wants to reduce value of what it owes:

\[ B_0^1 + \beta \frac{c_1}{c_2} B_0^2 \]

If \( B_0^1 = 0 \) and \( B_0^2 > 0 \) \( \Rightarrow \) \( \uparrow \) \( c_2 \) and \( \downarrow \) \( c_1 \) (deviation: \( \uparrow \) \( B_1^2 \))
Why Does a Flat Debt Position Fix Commitment?

- Government wants to reduce value of what it owes:

  \[ B_0^1 + \beta \frac{c_1}{c_2} B_0^2 \]

  - If \( B_0^1 = 0 \) and \( B_0^2 > 0 \) ⇒ \( \uparrow c_2 \) and \( \downarrow c_1 \) (deviation: \( \uparrow B_1^2 \))

- Government wants to increase value of what it issues:

  \[ \beta \frac{c_1}{c_2} B_1^2 \]

  - If \( B_0^1 > 0 \) and \( B_0^2 = 0 \) ⇒ \( B_1^2 > 0 \) ⇒ \( \downarrow c_2 \) and \( \uparrow c_1 \) (deviation: \( \downarrow B_1^2 \))
Why Does a Flat Debt Position Fix Commitment?

- Government wants to reduce value of what it owes:

\[ B_0^1 + \beta \frac{c_1}{c_2} B_0^2 \]

If \( B_0^1 = 0 \) and \( B_0^2 > 0 \) \( \Rightarrow \) \( c_2 \) and \( c_1 \) (deviation: \( B_1^2 \))

- Government wants to increase value of what it issues:

\[ \beta \frac{c_1}{c_2} B_1^2 \]

If \( B_0^1 > 0 \) and \( B_0^2 = 0 \) \( \Rightarrow B_1^2 > 0 \) \( \Rightarrow \) \( c_2 \) and \( c_1 \) (deviation: \( B_1^2 \))

If \( B_0^2 = B_0^1 = \bar{B} \) \( \Rightarrow \) No gains from deviation

True since it implies \( B_0^2 = B_1^2 = \bar{B} \)
Proposition. Let $B_0^1 + \beta B_0^2 = \bar{B} (1 + \beta)$. The higher is $|B_0^2 - B_0^1|$, the higher the cost of lack of commitment.
Cost of Lack of Commitment Depends on Maturity

**Proposition.** Let $B_0^1 + \beta B_0^2 = \bar{B} (1 + \beta)$. The higher is $|B_0^2 - B_0^1|$, the higher the cost of lack of commitment.

Date 1 government wants to relax implementability condition:

$$
\frac{c_1 - n (1 - \tau)}{c_1} + \beta \frac{c_2 - n (1 - \tau)}{c_2} \geq \frac{B_0^1}{c_1} + \beta \frac{B_0^2}{c_2}
$$

- e.g. if $B_0^2 > B_0^1$, reducing $c_1 / c_2$ reduces RHS
Cost of Lack of Commitment Depends on Maturity

- **Proposition.** Let $B_0^1 + \beta B_0^2 = B (1 + \beta)$. The higher is $|B_0^2 - B_0^1|$, the higher the cost of lack of commitment.

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  \]
  
  e.g. if $B_0^2 > B_0^1$, reducing $c_1 / c_2$ reduces RHS

  ... BUT this tightens the constraint at date 0
  \[
  \frac{c_0 - n(1 - \tau)}{c_0} + \beta \left( \frac{B_0^1}{c_1} + \beta \frac{B_0^2}{c_2} \right) \geq 0
  \]
Cost of Lack of Commitment Depends on Maturity

**Proposition.** Let $B^1_0 + \beta B^2_0 = \overline{B} (1 + \beta)$. The higher is $|B^2_0 - B^1_0|$, the higher the cost of lack of commitment.

Date 1 government wants to relax implementability condition:

$$\frac{c_1 - n(1-\tau)}{c_1} + \beta \frac{c_2 - n(1-\tau)}{c_2} \geq \frac{B^1_0}{c_1} + \beta \frac{B^2_0}{c_2}$$

- e.g. if $B^2_0 > B^1_0$, reducing $c_1 / c_2$ reduces RHS
- ... BUT this tightens the constraint at date 0

$$\frac{c_0 - n(1-\tau)}{c_0} + \beta \left( \frac{B^1_0}{c_1} + \beta \frac{B^2_0}{c_2} \right) \geq 0$$

- If $|B^2_0 - B^1_0| \uparrow \Rightarrow$ deviation at time 1↑
Cost of Lack of Commitment Depends on Maturity

- **Proposition.** Let $B_0^1 + \beta B_0^2 = \bar{B} (1 + \beta)$. The higher is $|B_0^2 - B_0^1|$, the higher the cost of lack of commitment.

- Date 1 government wants to relax implementability condition:
  
  $$ \frac{c_1 - n(1 - \tau)}{c_1} + \beta \frac{c_2 - n(1 - \tau)}{c_2} \geq \frac{B_0^1}{c_1} + \beta \frac{B_0^2}{c_2} $$

  - e.g. if $B_0^2 > B_0^1$, reducing $c_1 / c_2$ reduces RHS
  - ... BUT this tightens the constraint at date 0

  $$ \frac{c_0 - n(1 - \tau)}{c_0} + \beta \left( \frac{B_0^1}{c_1} + \beta \frac{B_0^2}{c_2} \right) \geq 0 $$

- If $|B_0^2 - B_0^1| \uparrow \iff$ deviation at time 1↑
  \[ \Rightarrow \] the tighter the constraint at date 0 \[ \Rightarrow \] welfare at time 0↓.
Generalizable Insights from Example

- Government can deviate ex post to relax budget constraint
  - Method: Increase consumption in direction of maturity of debt
  - Deviation incentives larger if debt more tilted
  - Relaxing budget allows reducing ex-post tax/spending distortions
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  - Higher borrowing rates + large debt positions tighten ex-ante budget
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- Tighter ex-ante budget $\rightarrow$ Higher initial tax/spending distortions
Quantitative Assessment of Lack of Commitment

- Three period environment $t = 0, 1, 2$

Preferences and parameters [following Chari, Christiano and Kehoe (1995)]

$\log c + \eta \log (1 - n) + \theta t \log g$

$\beta = 0.9644$ (yearly model)

$\eta = 3.33$ (implies $n = 0.23$)

$\theta_1 = \theta_2 = 0.2195$ (implies $g_1/y_1 = g_2/y_2 = 0.18$)

$\theta_0 = 0.2360$, implies $g_0/y_0 = 0.19$ (std(g) = 0.07)

Calculate welfare cost of no commitment given $B_1 + \beta B_2 = B(1 + \beta)$

Consider lack of commitment to spending and to taxes separately

Main result: Welfare cost rises in tilt of maturity structure
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  \[ B_0^1 + \beta B_0^2 = \bar{B} (1 + \beta) \]
  - Consider lack of commitment to spending and to taxes separately
  - Main result: Welfare cost rises in tilt of maturity structure
Cost of Lack of Commitment Rises with Tilt of Debt

Positions exceeding 100% of GDP costs more than 1% of consumption
Lack of Insurance
Proposition (Angeletos, Buera and Nicolini):

If \# available maturities equals \# states of the world

\[\downarrow\]

solution under incomplete markets = solution under complete markets.
Tilted Maturity Fixes Lack of Insurance

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If \(#\) available maturities equals \(#\) states of the world

\[\downarrow\]

solution under incomplete markets = solution under complete markets.

- Implemented with time-invariant non-contingent debt
- Insurance through fluctuations in market value of debt
Example in a three-period model

- Suppose $\theta_1 \in \{\theta_H, \theta_L\}$ is stochastic.
- Let $S^*(\theta_H)$ and $S^*(\theta_L)$ be the value of surpluses under complete markets.
- One can find $B_0^S$ and $B_0^L$ such that,

\[
\begin{bmatrix}
S^*(\theta_H) \\
S^*(\theta_L)
\end{bmatrix} =
\begin{bmatrix}
1 & q_{0,1}^* \\
1 & q_{0,2}^*
\end{bmatrix}
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**Lemma.** If $\theta_1$ stochastic in three-period model, optimal policy:

$$B_0^1 < 0 \text{ and } B_0^2 > 0$$

- Market value of debt declines when $\theta_1$ high, $g_1$ high, and $c_1$ low:

$$B_0^1 + \beta \frac{c_1}{c_2} B_0^2$$
Maturity is Tilted and Large under Commitment

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Welfare Cost of Flat Maturity Rises with Volatility

Cost is below 0.05% under empirical volatility of spending

Flat debt position $\rightarrow$ Low fluctuation in market value of debt
Lack of Insurance Less Costly than Lack of Commitment

- Flat debt position $\rightarrow$ Low fluctuation in market value of debt

- Low insurance $\rightarrow$ High volatility in tax/spending distortions
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- Flat debt position $\rightarrow$ Low fluctuation in market value of debt

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- High volatility of distortions less costly than high average distortions
  - Similar argument to Lucas (1987)
Quantitative Analysis: Infinite Horizon
Let $t = \{0, \infty\}$. $\theta_t = \{\theta^L, \theta^H\}$ with persistence $\rho$. 
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Available maturities: One period bond \( (B^S) \) and a consol \( (B^L) \)

\[
\tau_t n_t - g_t = -q^S_t B^S_t + q^L_t \left( B^L_{t-1} - B^L_t \right) + \left( B^S_{t-1} + B^L_{t-1} \right)
\]

Initial debt consistent with avg. level and maturity of US (1980 - 2008)
- Total Debt 60% of GDP, of which 30% with maturity \( \leq 1 \) year.
Optimal Maturity in Infinite Horizon

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- Initial debt consistent with avg. level and maturity of US (1980 - 2008)
  - Total Debt 60% of GDP, of which 30% with maturity \( \leq 1 \) year.

- No inefficiency under full commitment or full insurance
  - Full commitment: Angeletos and Buera-Nicolini result apply
  - Full insurance: Consol enforces perfect smoothing
    - If there is full commitment to either taxes or spending
Optimal Maturity Is Nearly Flat
Average Debt Positions at Market Value (% of GDP) - Model with Exog. g
Optimal Maturity Is Nearly Flat

Average Debt Positions (% of GDP)

[Graphs showing the relationship between debt positions and various economic parameters such as standard deviation of growth, risk aversion, and curvature of leisure.]
Why Is Optimal Maturity Is Nearly Flat?

- Cost of incompleteness low for empirical volatility of spending
Why Is Optimal Maturity Is Nearly Flat?

- Cost of incompleteness low for empirical volatility of spending
- Cost of lack of commitment high under standard preferences
  - Incentives to deviate strong given large tilted debt needed for hedging
  - Anticipation of future deviation increases cost of financing today
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Debortoli-Nunes-Yared (2015)
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  - Incentives to deviate strong given large tilted debt needed for hedging
  - Anticipation of future deviation increases cost of financing today
  - Significant hedging would require high tax/spending distortions
- Optimal policy goal should be to minimize average distortion
  - Reducing volatility of distortions is second order
Debt Is Actively Managed

Debortoli-Nunes-Yared (2015)
Optimal Government Debt Maturity
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Fiscal Policy is History Dependent (no perfect insurance)

Robustness 2: Different models

Average Debt Positions at Market Value (% of GDP)

Debortoli-Nunes-Yared (2015)
Optimal Government Debt Maturity
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Robustness 3: Different Maturities ($\gamma = 0.5$)

Average Debt Positions (% of GDP)

Debortoli-Nunes-Yared (2015)
Optimal Government Debt Maturity
May 2015
Conclusion

- Results of previous literature not robust to lack of commitment
  - Tradeoff between hedging and commitment
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  - Active management of consol in response to shocks
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- Considerations for future research
  - Monetary policy interactions
  - Debt maturity and financial frictions
  - Redistributive taxation