Monetary Policy and Sovereign Debt Vulnerability*

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Abstract

We investigate the trade-offs between price stability and the sustainability of sovereign debt, using a small open economy model where the government issues nominal defaultable debt and chooses fiscal and monetary policy under discretion. Inflation reduces the real value of outstanding debt, thus making it more sustainable; but it also raises nominal yields and entails direct welfare costs. We compare this scenario with a situation in which the government gives up the ability to deflate debt away, e.g. by issuing foreign currency debt or joining a monetary union with an anti-inflationary stance. We find that the benefits of giving up such adjustment margin dominate its costs, both for our preferred calibration and for a wide range of parameter values.

Keywords: monetary-fiscal interactions, discretion, sovereign default, continuous time, optimal stopping.

JEL codes: E5, E62,F34.

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1 Introduction

One of the main legacies of the 2007-9 financial crisis and the subsequent recession has been the emergence of large fiscal deficits across the industrialized world. The consequence has been a sharp increase in government debt, with debt-to-GDP ratios near or above record levels in countries such as the United States, United Kingdom, Japan or the Euro area periphery (Greece, Ireland, Italy, Portugal, Spain). Before the summer of 2012, Euro area periphery economies experienced dramatic spikes in their sovereign yields, whereas other highly indebted countries did not. Many observers emphasized that a key difference between both groups of countries was that, whereas the US, UK and Japan had the option to deflate away the real burden of nominal debt though inflation, the Euro countries were forced to repay debt solely through fiscal surpluses. At the same time, the experience of a number of developing countries such as Mexico or Brazil, in which sovereign debt is often issued directly in foreign currency, illustrates situations in which governments sometimes renounce the possibility of deflating away their debts.

These developments raise the question as to what role monetary policy should have, if any, in guaranteeing the sustainability of sovereign debt, in view of the existing trade-offs between the latter and price stability. Broadly speaking, on the one hand it can be argued that central banks should provide a 'monetary backstop' that reassures investors in sovereign debt.[1] On the other hand, such a course of action may presumably give rise to inflation, with the resulting costs and distortions. Moreover, while using inflation temporarily for debt-deflation purposes may not largely affect inflation expectations in countries (such as the US or UK) where monetary authorities are perceived to have a clear and credible commitment towards price stability, the same may not be true in countries with a poorer inflation record and/or weaker monetary credibility, thus limiting the effectiveness of debt deflation policies.

In this paper, we try to shed light on the above issues by studying the trade-offs between price stability and sovereign debt sustainability when the government cannot make credible commitments about inflation. With this purpose, we build a general equilibrium, continuous-time model of a small open economy in which a benevolent government issues long-term sovereign nominal bonds to foreign investors. At any time, the government may default on its debt if it finds it optimal to do so. Default produces some costs due to temporary exclusion from capital markets and a drop in the output endowment. We show that the default decision is characterized by an optimal default threshold for the model's single state variable, the debt-to-GDP ratio. In addition, the government chooses fiscal and monetary policy optimally under discretion. That is, the government cannot commit to a future path for primary deficit and inflation. When choosing inflation, the government trades off benefits and costs. On the one hand, inflation reduces the real value of debt; ceteris

[1] This view is shared e.g. by Krugman (2011) or De Grauwe (2011).

paribus, this improves sovereign debt sustainability by making default a less likely outcome. On the other hand, inflation entails a direct welfare cost. Moreover, expectations of future inflation worsen such trade-off by raising nominal yields for new bond issuances, thus making primary deficits more costly to finance.

We calibrate our model to capture some salient features of the EMU periphery economies, including their observed inflation record prior to joining the euro. Under our baseline calibration, the optimal inflation policy function increases roughly linearly with the debt ratio, and then increases steeply as the latter approaches the optimal default threshold. Importantly, the government allows for relatively high inflation rates at debt ratios for which default is still perceived as rather distant by investors. We refer to this baseline scenario as the 'inflationary regime'.

We then compare the baseline inflationary regime with a scenario in which inflation is zero at all times. In other words, the government effectively renounces the possibility of deflating debt away. Given our assumption that the government cannot make credible inflation commitments, this 'no inflation' regime is best interpreted as a situation in which the government directly issues foreign currency debt, or in which it joins a monetary union with a very strong and credible anti-inflationary stance. We find that welfare in the no-inflation regime is *higher* at *any* debt ratio. The reason is that the no-inflation regime avoids the costs of inflation (in terms of higher yields and direct welfare costs) while not compromising too much the sustainability of sovereign debt. Indeed, while renouncing the option to deflate debt away does make debt more vulnerable by making default more likely, such an event is still perceived as rather distant by investors at all debt ratios except for those very close to default. We also find that optimal default thresholds are nearly identical in both regimes, and that welfare under no-inflation is higher too at such threshold.[2]

Having characterized equilibrium in both regimes at each point of the state space, we then compute the stationary distribution of the main variables so as to analyze the average performance of both regimes. We find that the inflationary regime shifts the distribution of the debt ratio to the left vis-à-vis the no-inflation one. This improves sovereign debt sustainability, lowering average sovereign risk premia. However, this effect is dominated by higher average inflation premia, the net effect being higher average bond yields. Moreover, the presence of trend inflation creates direct welfare costs. As a consequence, the inflationary regime produces a loss in average welfare relative to the no-inflation scenario, one that is of first-order magnitude in our baseline calibration. We show that our findings are robust to alternative calibrations of parameters that determine bond maturity, bond recovery rates and output losses upon default.

Finally, as an alternative to giving up the debt deflation margin altogether, we investigate an

[2]Intuitively, the prospect of defaulting is less favorable in the inflationary regime, because following the exclusion period the government returns to capital markets with basically the same debt burden as in the no-inflation regime, only with lower welfare due to the inflationary distortions.

intermediate arrangement in which the government delegates monetary policy to an independent central banker with a greater distaste for inflation than society as a whole. We find that delegating monetary policy to such a 'conservative' central banker allows to achieve superior welfare outcomes vis-à-vis the baseline inflationary regime, in which the benevolent government chooses inflation discretionarily. As it turns out, however, average welfare never reaches that of the 'no inflation' regime: it increases monotonically with the central banker's distaste for inflation, converging asymptotically to its level under the latter regime.

Taken together, our results offer an important qualification of the conventional wisdom that individual countries may benefit from retaining the option to deflate away their sovereign debt. In particular, our analysis suggests that such countries may actually be better off by renouncing such a tool if their governments are unable to make credible commitments about their future inflation policy. This qualification may be relevant for most EMU peripheral economies, in view of their inflation record (relative e.g. to that of Germany) in the decades prior to joining the euro. Our findings may also rationalize why a number of developing countries with limited inflation credibility typically resort to issuing debt in terms of a hard foreign currency.

Literature review. Our paper relates to recent theoretical papers that analyze the link between sovereign debt vulnerability and monetary policy, such as Aguiar et al. (2013), Corsetti and Dedola (2014), and Camous and Cooper (2014). These papers consider self-fulfilling debt crises along the lines of Calvo (1988) or Cole and Kehoe (2000). We complement this literature by considering a framework in which sovereign default is instead an optimal government decision based on fundamentals, in the tradition of Eaton and Gersovitz (1981).[3] Also, the above contributions are qualitative, working in environments with two periods or two-period-lived agents (Corsetti and Dedola, 2014; Camous and Cooper, 2014) or without fundamental uncertainty (Aguiar et al., 2013).[4] By contrast, we adopt a fully dynamic, stochastic approach, which makes our model potentially useful for quantitative analysis. In particular, we show that our model can replicate well average sovereign yields and risk premia in the peripheral EMU economies, while also matching average external sovereign debt stocks. We also show that our model can rationalize the reduction in sovereign bond yields across EMU periphery countries relative to the pre-EMU period, which suggests that investors perceived the reduction in inflation expectations as more important than the presumable increase in default risk.

Our modeling of inflation disutility costs is based on Aguiar et al. (2013), who interpret the

[3] In Corsetti and Dedola (2014) default crisis can also be due to weak fundamentals.

[4] Da Rocha et al. (2013) and Araujo et al. (2013) analyze the connection between inflation and the possibility of self-fulfilling debt crisis in fully dynamic, stochastic frameworks. Da Rocha et al. (2013) analyze optimal debt and exchange rate policy in a model with foreign currency debt where the government is exposed to both self-fulfilling defaults and devaluations. Araujo et al. (2013) consider the welfare gains or losses from issuing debt in local versus foreign currency, in a framework where the costs of local currency debt are due to an exogenous inflation shock.

weight on such disutility as the government's 'inflation credibility'.[5] These authors find that, under certain conditions (such as a moderate inflation credibility and intermediate debt levels), issuing domestic currency debt may achieve superior welfare outcomes relative to issuing foreign currency debt. In our framework with fundamental default à la Eaton-Gersovitz, by contrast, giving up the option to deflate debt away (e.g. by issuing foreign currency debt) consistently outperforms issuing domestic currency debt and deflating it at discretion, regardless of the degree of inflation credibility.

In modeling optimal default à la Eaton-Gersovitz in a quantitative framework, our model is more in line with the literature on quantitative sovereign default models initiated by Aguiar and Gopinath (2006) and Arellano (2008).[6] We build on this literature by introducing nominal bonds and studying the optimal inflation policy when the government cannot commit not to inflate in the future. This allows us to address the trade-offs between price stability and sovereign debt vulnerability in a unified framework. Another difference with respect to the quantitative sovereign default literature is our reliance on continuous time. Continuous-time methods are standard in the corporate default literature initiated by Merton (1974) and Leland (1994) due to their tractability.[7] We show how one can extend this analysis to the pricing of defaultable nominal sovereign debt.

In studying the effects of delegating monetary policy to an independent, conservative central banker, our analysis revisits an old theme initiated by Rogoff (1985) and further discussed e.g. in Clarida, Galí and Gertler (1999), although it does so in a very different context.[8] In particular, we explore the effects of delegation in a framework in which the benefit of allowing for inflation is not to exploit a short-run output/inflation trade-off, as in the mainstream New Keynesian literature, but rather to make sovereign debt more sustainable. Contrary to the linear(ized) frameworks typically used in the New Keynesian literature, our framework takes full account of the strong non-linearities that emerge in the presence of equilibrium sovereign default. As in that literature, we find that there are welfare gains from delegating discretionary monetary policy to an independent authority with a greater distaste for inflation than that of society. What is perhaps more striking is that, whereas the optimal 'delegated' inflation distaste in the above literature is relatively large but finite, in our framework welfare is maximized when such distaste is arbitrarily large, i.e. when the government completely abandons the option of adjusting inflation.

[5] In an environment similar to Aguiar et al. (2013), Aguiar et al. (2015) study the impact of the composition of debt in a monetary union on the occurrence of self-fulfilling debt crises. One important difference between Aguiar et al. (2013, 2015) and our paper is that we consider quadratic (as opposed to linear) inflation costs, which allows us to obtain interior solutions for optimal inflation.

[6] Other notable contributions to this literature include Benjamin and Wright (2009), Chatterjee and Eyigungor (2012), Hatchondo and Martínez (2009) and Yue (2010). Mendoza and Yue (2012) integrate an optimal sovereign default model into a standard real business cycle framework with endogenous production.

[7] See Sundaresan (2013) for a survey of this literature.

[8] Lohmann (1992) generalizes Rogoff's analysis by considering a conservative central banker that can be overriden by the government in the presence of extremely large shocks.

Finally, we make a technical contribution by introducing a new numerical method to find the equilibrium in continuous-time models with several agents. In particular we extend the recent literature about finite difference methods applied to stochastic control in economics, such as Achdou et al. (2014) or Nuño and Moll (2014), to analyze a recursive *optimal stopping* problem in which one of the agents employs both continuously chosen controls and discrete adjustments of the state variables. Optimal stopping problems are typically solved using optimal splitting methods, as in Barles, Daher and Romano (1995). However, the recursivity of our problem and the fact that it only has one state variable makes our method better suited to this particular problem.

The structure of the paper is as follows. In section 2 we introduce the model. Section 3 provides the main results. Section 4 introduces monetary policy delegation. Section 5 concludes.

2 Model

We consider a continuous-time model of a small open economy.

2.1 Output, price level and sovereign debt

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ be a filtered probability space. There is a single, freely-traded consumption good which has an international price normalized to one. The economy is endowed with Y_t units of the good each period (real GDP). The evolution of Y_t is given by

$$dY_t = \mu Y_t dt + \sigma Y_t dW_t, \tag{1}$$

where W_t is a $\mathcal{F}_t$-Brownian motion, $\mu \in \mathbb{R}$ is the drift parameter and $\sigma \in \mathbb{R}_+$ is the volatility. The local currency price relative to the World price at time t is denoted P_t. It evolves according to

$$dP_t = \pi_t P_t dt, \tag{2}$$

where π_t is the instantaneous inflation rate.

The government trades a nominal non-contingent bond with risk-neutral competitive foreign investors. Let B_t denote the outstanding stock of nominal government bonds; assuming that each bond has a nominal value of one unit of domestic currency, B_t also represents the total nominal value of outstanding debt. We assume that outstanding debt is amortized at rate $\lambda > 0$ per unit of time. The nominal value of outstanding debt thus evolves as follows,

$$dB_t = B_t^{new} dt - \lambda dt B_t,$$

where B_t^{new} is the flow of new debt issued at time t. The nominal market price of government bonds at time t is Q_t. Each bond pays a proportional coupon δ per unit of time. Also, the government incurs a nominal primary deficit $P_t(C_t - Y_t)$, where C_t is aggregate consumption.[9] The government's flow of funds constraint is then

$$Q_t B_t^{new} = (\lambda + \delta) B_t + P_t (C_t - Y_t).$$

That is, the proceeds from issuance of new bonds must cover amortization and coupon payments plus the primary deficit. Combining the last two equations, we obtain the following dynamics for nominal debt outstanding,

$$dB_t = \left[\left(\frac{\lambda + \delta}{Q_t} - \lambda \right) B_t + \frac{P_t}{Q_t} (C_t - Y_t) \right] dt. \tag{3}$$

We define the debt-to-GDP ratio as $b_t \equiv B_t / (P_t Y_t)$. Its dynamics are obtained by applying Itô's lemma to equations (1)-(3),

$$db_t = \left[\left(\frac{\lambda + \delta}{Q_t} - \lambda + \sigma^2 - \mu - \pi_t \right) b_t + \frac{c_t}{Q_t} \right] dt - \sigma b_t dW_t, \tag{4}$$

where $c_t \equiv (C_t - Y_t) / Y_t$ is the primary deficit-to-GDP ratio. Equation (4) describes the evolution of the debt-to-GDP ratio as a function of the primary deficit ratio, inflation and the bond price. In particular, *ceteris paribus* inflation π_t allows to reduce the debt ratio by reducing the real value of nominal debt. We also impose a non-negativity constraint on debt: $b_t \geq 0$.

2.2 Preferences

The representative household has preferences over paths for consumption and domestic inflation given by

$$U_0 \equiv \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u(C_t, \pi_t) dt \right]. \tag{5}$$

We assume that instantaneous utility takes the form

$$u(C_t, \pi_t) = \log(C_t) - \frac{\psi}{2} \pi_t^2, \tag{6}$$

where $\psi > 0$. We follow Aguiar et al. (2013) in posing a reduced-form specification for the welfare costs of inflation, $(\psi/2)\pi_t^2$. Unlike in Aguiar et al. (2013), though, we consider a quadratic

[9] As in Arellano (2008), we assume that the government rebates back to households all the net proceedings from its international credit operations (i.e. its primary deficit) in a lump-sum fashion. Denoting by T_t the primary deficit, we thus have $P_t C_t = P_t Y_t + T_t$. This implies $T_t = P_t (C_t - Y_t)$.

functional form, which allows us to obtain interior solutions for optimal inflation.[10] Using $C_t = (1 + c_t) Y_t$, we can express welfare in terms of the primary deficit ratio c_t as follows,

$$\begin{aligned} U_0 &= \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \left(\log(1 + c_t) + \log(Y_t) - \frac{\psi}{2} \pi_t^2 \right) dt \right] \\ &= \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \left(\log(1 + c_t) - \frac{\psi}{2} \pi_t^2 \right) dt \right] + V_0^{aut}, \end{aligned} \tag{7}$$

where

$$V_0^{aut} \equiv \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log(Y_t) dt \right] = \frac{\log(Y_0)}{\rho} + \frac{\mu - \sigma^2/2}{\rho^2} \tag{8}$$

is the (exogenous) value at time $t = 0$ of being in autarky forever.[11] Thus, welfare increases with the primary deficit ratio c_t, as this allows households to consume more for a given exogenous output; and it decreases with squared inflation deviations from zero.

2.3 Fiscal and monetary policy

The government chooses fiscal policy at each point in time along two dimensions: it sets optimally the primary surplus ratio c_t, and it chooses whether to continue honoring debt repayments or else to default. In addition, the government implements monetary policy by choosing the inflation rate π_t at each point in time. We now present the sovereign default scenario, which affects the boundary conditions of the general optimization problem.

2.3.1 The default scenario

In case of default, the government suffers a double punishment. First, it is excluded from international capital markets temporarily. The duration of this exclusion period, τ, is random and follows an exponential distribution with average duration $1/\chi$. Second, during the exclusion period the country's output endowment declines. Suppose the government defaults at an arbitrary debt ratio b. Then during the exclusion period the country's output endowment is given by $Y_t^{def} = Y_t \exp[-\epsilon \max\{0, b - \hat{b}\}]$, with $\epsilon, \hat{b} > 0$, such that the loss in (log)output equals $\epsilon \max\{0, b - \hat{b}\}$. Therefore, the country suffers an output loss only if it defaults at a debt ratio higher than a threshold $\hat{b}$. This specification of output loss is similar to the one in Arellano (2008) and, as in that paper, it helps the model achieve realistic default probabilities and risk premia.[12]

[10] Aguiar et al. (2013) adopt instead a linear inflation disutility, and restrict inflation to be within a closed interval. As explained by the authors, this gives rise to bang-bang inflation equilibria.

[11] Notice that (1) and Itô's Lemma imply $d \log Y_t = \left(\mu - \sigma^2/2\right) dt + \sigma dW_t$. Solving for $\log Y_t$ and taking time-0 conditional expectations yields $\mathbb{E}_0 \left(\log Y_t\right) = \log Y_0 + \left(\mu - \sigma^2/2\right) t$, which combined with the definition of V_0^{aut} gives us the right-hand side of (8).

[12] In Arellano (2008), the output loss following default equals $Y_t - Y_t^{def} = \max\{0, Y_t - \hat{Y}\}$, for some threshold output level $\hat{Y}$. Specifying our output loss function in terms of b_t (as opposed to Y_t) allows us to retain the

During the exclusion phase, households simply consume the output endowment, $C_t = Y_t^{def}$, which implies

$$\log(C_t) = \log(Y_t) - \epsilon \max\{0, b - \hat{b}\}.$$

The main advantage of defaulting is of course the possibility of reducing the debt burden. During the exclusion period, which we may interpret as a renegotiation process between the government and the investors, the latter receive no repayments. We assume that at the end of the exclusion period (with random duration τ) both parties reach an agreement by which investors recover only a fraction $\theta Y_{\hat{t}+\tau} P_{\hat{t}+\tau} / (Y_{\hat{t}} P_{\hat{t}})$ of the nominal value of outstanding bonds at the time of default (where the latter is denoted by $\hat{t}$), for some parameter $\theta > 0$.[13] This specification captures in reduced form the idea that the terms of the debt restructuring agreement are somehow sensitive to the country's macroeconomic performance.[14] Importantly, it allows us to keep the set of state variables restricted to the debt ratio only. To see this, notice that upon regaining access to capital markets, the debt ratio is

$$b_{\hat{t}+\tau} = \left(\theta \frac{Y_{\hat{t}+\tau} P_{\hat{t}+\tau}}{Y_{\hat{t}} P_{\hat{t}}}\right) B_{\hat{t}} \frac{1}{Y_{\hat{t}+\tau} P_{\hat{t}+\tau}} = \theta b_{\hat{t}},$$

where $b_{\hat{t}} = B_{\hat{t}} / Y_{\hat{t}} P_{\hat{t}}$ is the debt ratio at the time of default. Therefore, the government reenters capital markets with a debt ratio that is a fraction θ of the ratio at which it defaulted. It follows that the government has no incentive to create inflation during the exclusion period, as that would generate direct welfare costs while not reducing the debt ratio upon reentry; we thus have $\pi_t = 0$ for $t \in (\hat{t}, \hat{t} + \tau)$.

Taking all these elements together, we can express the value of defaulting at $\hat{t} = 0$ as $U_0^{def} = V_0^{def} + V_0^{aut}$, where V_0^{aut} is the autarky value as defined in (8), and $V_0^{def} \equiv V_{def}(b_0)$ is the value of defaulting net of the autarky value, given by

$$\begin{aligned} V_{def}(b) &= \mathbb{E}_0 \left\{ -\int_0^{\tau} e^{-\rho s} \epsilon \max\{0, b - \hat{b}\} ds + e^{-\rho \tau} V(\theta b) \right\} \\ &= \int_0^{\infty} \chi e^{-\chi \tau} \left(-\int_0^{\tau} e^{-\rho s} \epsilon \max\{0, b - \hat{b}\} ds + e^{-\rho \tau} V(\theta b) \right) d\tau \\ &= -\frac{\epsilon \max\{0, b - \hat{b}\}}{\rho + \chi} + \frac{\chi}{\rho + \chi} V(\theta b), \end{aligned} \tag{9}$$

where in the second equality we use our assumption that τ is exponentially distributed, and where $V(\cdot)$ is the value function of the government, to be defined later. For future reference, the slope

convenient model feature that b_t is the only relevant state variable.

[13] Notice that $Y_{\hat{t}}$ and $Y_{\hat{t}+\tau}$ represent, respectively, the output levels exactly at the time the government decides to default (i.e. right before output drops) and exactly upon regaining access to capital markets (i.e. right after output recovers again). Therefore, they do not incorporate the output loss during exclusion.

[14] See Benjamin and Wright (2009) and Yue (2010) for studies that endogenize the recovery rate upon default, in models with explicit renegotiation betwen the government and its creditors.

of the default value function is

$$V'_{def}(b) = -\frac{\epsilon}{\rho+\chi}\mathbf{1}(b > \hat{b}) + \frac{\chi}{\rho+\chi}V'(\theta b)\,\theta, \tag{10}$$

where $\mathbf{1}(\cdot)$ is the indicator function.

2.3.2 The general problem

As mentioned before, at every point in time the government decides optimally whether to default or not, in addition to choosing the primary deficit ratio and the inflation rate. Following a default, and once the government regains access to capital markets, it starts accumulating debt and is confronted again with the choice of defaulting. This is a sequence of recursive *optimal stopping* problems, as one of the policy instruments is a sequence of stopping times. The solution to this problem will be characterized by an optimal *default threshold* for the debt ratio, which we denote by b^*. This threshold defines an "inaction region" of the state space, $[0, b^*)$, in which the government chooses not to default, and a region $[b^*, \infty)$ in which the government defaults. We denote by $T(b^*)$ the *time to default*. The latter is a stopping time with respect to the filtration $\{\mathcal{F}_t\}$, defined as the smallest time t' such that $b_{t+t'} = b^*$, i.e. $T(b^*) = \min\{t' : b_{t+t'} = b^*\}$.[15]

The government maximizes social welfare (i.e. it behaves benevolently) under discretion. The value function of the government (net of the exogenous autarky value) at time $t = 0$ can then be expressed as

$$V(b) = \max_{b^*, \{c_t, \pi_t\}} \mathbb{E}_0 \left\{ \int_0^{T(b^*)} e^{-\rho t}\left(\log(1+c_t) - \frac{\psi}{2}\pi_t^2\right) dt + e^{-\rho T(b^*)} V_{def}(b^*) \mid b_0 = b \right\}, \tag{11}$$

subject to the law of motion of the debt ratio, equation (4). The optimal default threshold b^* must satisfy the following two conditions,

$$V(b^*) = V_{def}(b^*), \tag{12}$$

$$V'(b^*) = V'_{def}(b^*), \tag{13}$$

where $V_{def}(b^*)$ and $V'_{def}(b^*)$ are given respectively by equations (9) and (10) evaluated at $b = b^*$. Equation (12) is the *value matching* condition and it requires that, at the default threshold, the value of honoring debt repayments equals the value of defaulting. Equation (13) is the *smooth pasting* condition, and it requires that there is no kink at the optimal default threshold.[16] Both

[15] Therefore $\hat{t} = t + T(b^*)$ or $T(b^*) = \hat{t} - t$.

[16] To be precise, the smooth pasting condition holds as long as $b^* \neq \hat{b}$ as the continuation value $V_{def}(b)$ is not differentiable at $\hat{b}$.

are standard conditions in optimal stopping problems; see e.g. Dixit and Pindyck (1994), Oksendal and Sulem (2007) and Stokey (2009). These conditions imply that the value function is continuous and continuously differentiable: $V \in C^1([0,\infty))$.[17]

The solution of this problem must satisfy the Hamilton-Jacobi-Bellman (HJB) equation,

$$\rho V(b) = \max_{c,\pi} \left\{ \log(1+c) - \frac{\psi}{2}\pi^2 + \left[\left(\frac{\lambda+\delta}{Q(b)} - \lambda + \sigma^2 - \mu - \pi \right) b + \frac{c}{Q(b)} \right] V'(b) + \frac{(\sigma b)^2}{2} V''(b) \right\}, \tag{14}$$

$\forall b \in [0, b^*)$, together with the boundary conditions (12) and (13).[18] The term in squared brackets in (14) is the drift of the state variable (see equation 4). The optimal primary deficit ratio and inflation rate are given by the following first order conditions,

$$c = \frac{Q(b)}{-V'(b)} - 1 \equiv c(b), \tag{15}$$

$$\pi = -\frac{b}{\psi} V'(b) \equiv \pi(b). \tag{16}$$

Therefore, the optimal primary deficit ratio increases with bond prices and decreases with the slope of the value function (in absolute value). The intuition is straightforward. Higher bond prices (equivalently, lower bond yields) make it cheaper for the government to finance primary deficits. Likewise, a steeper value function makes it more costly to increase the debt burden by incurring primary deficits. As regards optimal inflation, the latter increases both with the debt ratio and the slope (in absolute value) of the value function. Intuitively, the higher the debt ratio the larger the reduction in the debt burden that can be achieved through a marginal increase in inflation. Similarly, a steeper value function increases the incentive to use inflation so as to reduce the debt burden.

2.3.3 The 'no inflation' regime

So far we have analyzed the decision problem of a benevolent government that cannot make credible commitments about its future fiscal policy (including the possibility of defaulting) and monetary policy. In particular, the inability to commit *not* to use inflation in the future so as to deflate debt away implies that the government is unable to steer investor's inflation expectations in a way that favors welfare outcomes. While lacking commitment, however, we can think of situations in which the government effectively relinquishes the ability to deflate debt away. Formally, we may consider a monetary regime in which inflation is zero in all states: $\pi(b) = 0$, for all b. The government's

[17] In addition to these two boundary conditions, there exists the state constraint $b \geq 0$ introduced above.

[18] Obviously, $\forall b \in [b^*, \infty)$, $V(b) = V_{def}(b)$.

problem is given by (14) with $\pi = 0$ replacing the optimal inflation choice, and with boundary conditions given again by (12) and (13).

We may interpret such a 'no inflation' scenario in alternative ways. One can first think of a situation in which the government appoints an independent central banker with a strong, in fact arbitrarily great, distaste for inflation. Even under discretion, such a central banker would always choose $\pi = 0$. One problem with this interpretation, though, is that it is unlikely that a government that cannot make credible commitments about monetary policy would appoint a central banker with such extreme preferences towards inflation.[19]

A second, perhaps more plausible interpretation is that the government directly issues bonds denominated in *foreign currency*. In that case, the possibility of deflating debt away simply disappears, and with it the only benefit of inflating in this model. As a result, optimal inflation is always zero in such a scenario.

Finally, we may think of a situation in which the government joins a *monetary union* in which the common monetary authority has an extreme distaste for inflation. If the costs of exiting the monetary union are very high, then joining it signals a credible anti-inflationary commitment.

In what follows, we will simply refer to this scenario as the 'no inflation regime', keeping in mind that such scenario admits several interpretations along the lines just discussed.

2.4 Foreign investors

When choosing fiscal and monetary policy, the government takes as given the mapping between the debt ratio and the nominal price of bonds, $Q(b)$. We now characterize such bond price function. The government sells bonds to competitive risk-neutral foreign investors that can invest elsewhere at the risk-free real rate $\bar{r}$. As explained before, bonds pay a coupon rate δ and are amortized at rate λ. Following a default, and during the exclusion period of the government, investors receive no payments. Once the exclusion/renegotiation period ends, investors recover a fraction $\theta \frac{P_{\hat{t}+\tau} Y_{\hat{t}+\tau}}{P_{\hat{t}} Y_{\hat{t}}} = \theta \frac{Y_{\hat{t}+\tau}}{Y_{\hat{t}}}$ of the nominal value of each bond, where we have used the fact that optimal inflation is zero during the exclusion period, such that $P_{\hat{t}+\tau} = P_{\hat{t}}$.[20] They also anticipate that the government's debt ratio at the time of reentering financial markets will be θb^*, such that their outstanding bonds will carry a market price $Q(\theta b^*)$. Therefore, the nominal price of the bond at

[19] In section 4 we will consider a more general scenario in which the government appoints a conservative central banker whose distaste for inflation is greater than that of society, but not so extreme as to imply zero inflation at all times.

[20] The average recovery rate equals $\mathbb{E}\left[\theta Y_{\hat{t}+\tau}/Y_{\hat{t}}\right] = \theta \mathbb{E}_\tau \left\{\mathbb{E}\left[Y_{\hat{t}+\tau}/Y_{\hat{t}} \mid \tau\right]\right\} = \theta \int_0^\infty \chi e^{-\chi\tau} e^{\mu\tau} d\tau = \theta\chi/(\chi-\mu)$, where we have used $\mathbb{E}\left[Y_{\hat{t}+\tau}/Y_{\hat{t}} \mid \tau\right] = \exp(\mu\tau)$ and the fact that τ is exponentially distributed.

time $t = 0$ for a current debt ratio $b \leq b^*$ is given by

$$Q(b) = \mathbb{E}_0 \left[\int_0^{T(b^*)} e^{-(\bar{r}+\lambda)t - \int_0^t \pi_s ds} (\lambda + \delta)\, dt + e^{-\bar{r}(T(b^*)+\tau) - \lambda T(b^*) - \int_0^{T(b^*)} \pi_s ds} \theta \frac{Y_{T(b^*)+\tau}}{Y_{T(b^*)}} Q(\theta b^*) \mid b_0 = b \right], \tag{17}$$

where again $T(b^*)$ denotes the smallest time to default.[21] Applying the Feynman-Kac formula, we obtain the following recursive representation,

$$Q(b)(\bar{r} + \pi(b) + \lambda) = (\lambda + \delta) + \left[\left(\frac{\lambda + \delta}{Q(b)} - \lambda + \sigma^2 - \mu - \pi \right) b + \frac{c(b)}{Q(b)} \right] Q'(b) + \frac{(\sigma b)^2}{2} Q''(b), \tag{18}$$

for all $b \in [0, b^*)$. To determine the boundary condition for $Q(b)$, we calculate the expected value of outstanding bonds at the time of default ($T(b^*) = 0$),

$$\begin{aligned} Q(b^*) &= \mathbb{E}_0 \left[e^{-\bar{r}\tau} \theta \frac{Y_\tau}{Y_0} Q(\theta b^*) \right] = \int_0^\infty \chi e^{-\chi \tau} \mathbb{E}_0 \left[e^{-\bar{r}\tau} \theta \frac{Y_\tau}{Y_0} Q(\theta b^*) | \tau \right] d\tau \\ &= \int_0^\infty \chi e^{-(\bar{r}+\chi-\mu)\tau} \theta Q(\theta b^*)\, d\tau = \frac{\chi}{\bar{r} + \chi - \mu} \theta Q(\theta b^*), \end{aligned} \tag{19}$$

where in the third equality we have used $\mathbb{E}_0 \left[\frac{Y_\tau}{Y_0} | \tau \right] = e^{\mu \tau}$. The partial differential equation (18), together with the boundary condition (19), provide the risk-neutral pricing of the nominal defaultable sovereign bond.[22]

2.5 Some definitions

Given a current bond price $Q(b)$, the implicit *bond yield* $r(b)$ is the discount rate for which the discounted future promised cash flows from the bond equal its price. The discounted future promised payments are $\int_0^\infty e^{-(r(b)+\lambda)t} (\lambda + \delta)\, dt = \frac{\lambda+\delta}{r(b)+\lambda}$. Therefore, the bond yield function is

$$r(b) = \frac{\lambda + \delta}{Q(b)} - \lambda. \tag{20}$$

The gap between the yield $r(b)$ and the riskless real rate $\bar{r}$ reflects both (a) the risk of sovereign default, i.e. a *risk premium*, and (b) the anticipation of inflation during the life of the bond, i.e. an *inflation premium*. In order to disentangle both factors, we define the *riskless yield* as $\tilde{r}(b) = \frac{\lambda+\delta}{\tilde{Q}(b)} - \lambda$, where $\tilde{Q}(b)$ is the price that the investor would pay for a *riskless* nominal bond with the same promised cash flows as the risky nominal bond. Appendix B defines $\tilde{Q}(b)$ and

[21] Notice that the recovery payoff $\theta \frac{Y_{T(b^*)+\tau}}{Y_{T(b^*)}} Q(\theta b^*)$ is discounted by $e^{-\lambda T(b^*)}$, as opposed to $e^{-\lambda(T(b^*)+\tau)}$, because no principal is repaid during the exclusion period (of length τ).

[22] Again, there also exists the state constraint $b \geq 0$.

explains how to solve for it. We then decompose

$$r(b) - \bar{r} = (r(b) - \tilde{r}(b)) + (\tilde{r}(b) - \bar{r}),$$

where $r(b) - \tilde{r}(b)$ is the risk premium, and $\tilde{r}(b) - \bar{r}$ is the inflation premium. In the no inflation regime, the riskless rate is simply $\tilde{r}(b) = \bar{r}$, the inflation premium is zero, and the risk premium is $r(b) - \bar{r}$.

Finally, we define the *expected time to default*, given a current debt ratio b, as

$$T^e(b) \equiv \mathbb{E}_0 \left[T(b^*) | b_0 = b \right] = \mathbb{E}_0 \left[\int_0^{T(b^*)} 1 dt \mid b_0 = b \right]. \tag{21}$$

Appendix C shows how to compute $T^e(b)$ numerically.

2.6 Equilibrium

We define our equilibrium concept:

Definition 1 *A* Markov Perfect Equilibrium *is an interval $\Phi = [0, b^*)$, a value function $V : \Phi \to \mathbb{R}$, a pair of policy functions $c, \pi : \Phi \to \mathbb{R}$ and a bond price function $Q : \Phi \to \mathbb{R}_+$ such that*

1. *Given prices Q, for any initial debt $b_0 \in \Phi$ the value function V solves the government problem (14), with boundary conditions (*12*) and (*13*); the optimal inflation is π, the optimal deficit ratio is c, and the optimal debt threshold is b^*.*

2. *Given the optimal inflation π, deficit ratio c and the interval Φ, bond prices satisfy the pricing equation (18).*

The government takes the bond price as given and chooses inflation and deficit (continuous policies) and default (stopping policy) to maximize its value function. The investors take these policies as given and price government bonds accordingly.

3 Quantitative analysis

Having laid out our theoretical model, we now use it in order to analyze the trade-off between price stability and the sustainability of sovereign debt. We are not able to find an analytical solution to our model. Therefore we resort to numerical techniques. We next describe our solution algorithm.

3.1 Computational algorithm

Here we propose a computational algorithm aimed at finding the equilibrium. The structure of the model complicates its solution as it comprises a pair of coupled ordinary difference equations (ODEs): the HJB equation (14) and the bond pricing equation (18). The policies obtained from the HJB are necessary to compute the bond prices and simultaneously, bond prices are necessary to compute the drift in the HJB equation.

In order to solve the HJB and bond pricing equations, we employ an *upwind finite difference* method.[23] It approximates the value function $V(b)$ and the bond price function $Q(b)$ on a finite grid with steps Δb: $b \in \{b_1, ..., b_I\}$, where $b_i = b_{i-1} + \Delta b = b_1 + (i-1)\Delta b$ for $2 \leq i \leq I$, with bounds $b_1 = 0$ and $b_{I+1} = b^*$.[24] We use the notation $V_i^{(n)} \equiv V^{(n)}(b_i)$, $i = 1, ..., I$, where $n = 0, 1, 2...$ is the iteration counter, and analogously for $Q_i^{(n)}$.

In order to compute the numerical solution to the recursive competitive equilibrium we proceed in three steps. We consider an initial guess of the bond price function, $Q^{(0)} \equiv \{Q_i^{(0)}\}_{i=1}^{I}$, and the default threshold, $b^*_{(0)}$. Set $n = 1$. Then:

Step 1: Government problem. Given $Q^{(n-1)}$ and $b^*_{(n-1)}$, we solve the optimal stopping problem with variable controls. This means solving the HJB equation (14) in the domain $[0, b^*_{(n-1)}]$ imposing the smooth pasting condition (13) (but not the value matching condition) to obtain an estimate of the value function $V^{(n)} \equiv \{V_i^{(n)}\}_{i=1}^{I}$ and of primary deficit and inflation, $\left(c^{(n)}, \pi^{(n)}\right) \equiv \{c_i^{(n)}, \pi_i^{(n)}\}_{i=1}^{I}$.

Step 2: Investors problem. Given $c^{(n)}$, $\pi^{(n)}$ and $b^*_{(n-1)}$, solve the bond pricing equation (18) and obtain $Q^{(n)}$ in the domain $[0, b^*_{(n-1)}]$. Then iterate again on steps 1 and 2 until both the value and bond price functions converge for given $b^*_{(n-1)}$.

Step 3: Optimal boundary. Given $V^{(n)}$ from step 2, we check whether the value matching condition (12) is satisfied. We compute $V^{(n)}(b^*_{(n-1)}) = V_I^{(n)}$ and $V_{def}^{(n)}(b^*_{(n-1)}) = -\frac{\epsilon \max\{0, b^*_{(n-1)} - \hat{b}\}}{\rho+\chi} + \frac{\chi}{\rho+\chi} V_{\theta I}^{(n)}$. If $V^{(n)}(b^*_{(n-1)}) > V_{def}^{(n)}(b^*_{(n-1)})$, then increase the threshold to a new value $b^*_{(n)}$. If $V^{(n)}(b^*_{(n-1)}) < V_{def}^{(n)}(b^*_{(n-1)})$, then decrease the threshold. Set $n := n + 1$. Proceed again to steps 1 and 2 until the value matching condition $V(b^*) = V_{def}(b^*)$ is satisfied.

Appendix A provides further details on these steps. The idea of the algorithm is to find the equilibrium numerically by moving the default threshold b^* and solving the HJB and bond pricing equations. The algorithm stops when the *value matching* condition (12) is satisfied.

[23] Barles and Souganidis (1991) have proved how this method converges to the unique *viscosity solution* of the problem. The latter is the appropriate concept of a general solution for stochastic optimal control problems (Crandall and Lions, 1983; Crandall, Ishii and Lions, 1992).

[24] We thus have $\Delta b = b^*/I$. We use $I = 800$ grid points in all our simulations.

3.2 Calibration

Let the unit of time by 1 year, such that all rates are in annual terms. Most papers in the literature on quantitative optimal sovereign default models set the world riskless real interest rate and the subjective discount rate to 1% and 5% per quarter, respectively.[25] We thus set $\bar{r} = 0.04$ and $\rho = 0.20$ per year.

In order to calibrate the drift and volatility of the exogenous output process, we use annual GDP growth data for the EMU periphery countries (Greece, Italy, Ireland, Portugal, Spain) over the period 1995-2012.[26] Averaging the mean and standard deviation of GDP growth across these countries, we obtain $\mu = 0.022$ and $\sigma = 0.032$.

The bond amortization rate λ is such that the average Macaulay bond duration, $1/(\lambda + \bar{r})$, is 5 years, which is broadly consistent with international evidence on bond duration (see e.g. Cruces et al. 2002). We set the coupon rate δ equal to $\bar{r}$, such that the price of a riskless real bond, $(\delta + \lambda)/(\bar{r} + \lambda)$, is normalized to 1.

We set χ such that the average duration of the exclusion period is $1/\chi = 3$ years, consistently with international evidence on exclusion periods in Dias and Richmond (2007). The bond recovery rate parameter, θ, is set such that the mean recovery rate, $\theta\chi/(\chi - \mu)$, is 60%, consistent with the evidence in Benjamin and Wright (2009) and Cruces and Trebesch (2011).

The parameters determining the output loss during the exclusion period, $\hat{b}$ and ε, are set in order for the model with zero inflation to replicate (i) the average ratio of external public debt over GDP across EMU periphery economies in our sample period (35.6%) and (ii) an output decline of 6% following default.[27] Regarding the latter, the literature offers a broad range of values, from 2% (Aguiar and Gopinath, 2006) to 13-14% (Mendoza and Yue, 2012; Arellano, 2008). The midpoint of this range would be 8%. We target a more conservative output loss of 6%.

Finally, in order to calibrate the government's dislike for inflation, ψ, we we turn to the inflationary model regime and target an average inflation rate of 3.2%. The latter corresponds to the average CPI inflation differential between the EMU periphery economies and the US during the

[25] The world interest rate is set to 1% per quarter in Aguiar & Gopinath (2006), Benjamin and Wright (2009), Hatchondo and Martinez (2009), Yue (2010), Mendoza and Yue (2012) and Chatterjee and Eyigungor (2012). The subjective discount rate is set equal to or close to 5% per quarter in Arellano (2008), Benjamin and Wright (2009), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012), and in Aguiar & Gopinath's (2006) model extension with bailouts.

[26] See Appendix E for data sources and treatment.

[27] We use the no-inflation scenario as the model counterpart of our sample region and period (the average EMU peripheral economy in euro period). First, as discussed in section 2.3.3, the no-inflation regime can be interpreted as an (anti-inflationary) monetary union. Second, as we explain below, we choose the US CPI as the empirical proxy for the 'World price' in the model, which is furthermore normalized to 1. We thus use CPI inflation differentials (rather than levels) relative to the US as the relevant empirical counterpart for inflation in the model. As we show in section 3.4, the average inflation differential across EMU peripheral economies relative to the US was close to zero (0.4% annual) in our sample period, such that the no-inflation regime provides a good approximation for observed inflation differentials in our sample.

period 1987-1997.[28] We thus use observed inflation differentials in the years *before* the creation of EMU in order to back up the preferences for inflation in such countries at a time when they were able to issue debt in their own currency. Table 1 summarizes the calibration.

Table 1. Baseline calibration

Parameter	Value	Description	Source/Target
$\bar{r}$	0.04	world real interest rate	standard
ρ	0.20	subjective discount rate	standard
μ	0.022	drift output growth	average growth EMU periphery
σ	0.032	diffusion output growth	growth volatility EMU periphery
λ	0.16	bond amortization rate	Macaulay duration = 5 years
δ	0.04	bond coupon rate	price of riskless real bond = 1
χ	0.33	reentry rate	mean duration of exclusion = 3 years
θ	0.56	recovery rate parameter	mean recovery rate = 60%
ϵ	1.50	default cost parameter	output loss during exclusion = 6%
$\hat{b}$	0.332	default cost parameter	average external debt/GDP ratio (35.6%)
ψ	9.15	inflation disutility parameter	mean inflation rate (1987-1997) = 3.2%

3.3 Equilibrium

The green dotted lines in Figure 1 show the equilibrium value function and policy functions in the 'inflationary regime'. As shown by the upper left subplot, the value function declines gently with the country's debt burden. The optimal default threshold equals $b^* = 37.0\%$ and is marked by a green circle. At that point, the government defaults and, following the exclusion period, reenters capital markets with a debt ratio $\theta b^* = 20.7\%$.As regards nominal bond prices $Q(b)$, the latter reflect mainly expected inflation during the life of the bond, except for debt ratios close to default. To see this more clearly, Figure 2 displays how the gap between the nominal yield $r(b) = (\delta + \lambda)/Q(b) - \lambda$ and the riskless real rate $\bar{r}$ is decomposed between the risk and inflation premia, as defined in section 2.5. Indeed, except for debt ratios close to b^*, bond yields reflect the risk of default only marginally, with risk premia in the range from 100 to 150 bp. The reason is that default is still perceived as a very distant outcome, as reflected by an expected time to default around 40 years. However, as debt approaches the default threshold, investors start perceiving default as rather imminent, demanding higher and higher risk premia. This in turn increases the

[28] We thus take the US CPI as the proxy for the 'World price' in the model. Notice also that, since the latter is normalized to 1, we target inflation *differentials* as opposed to inflation levels.

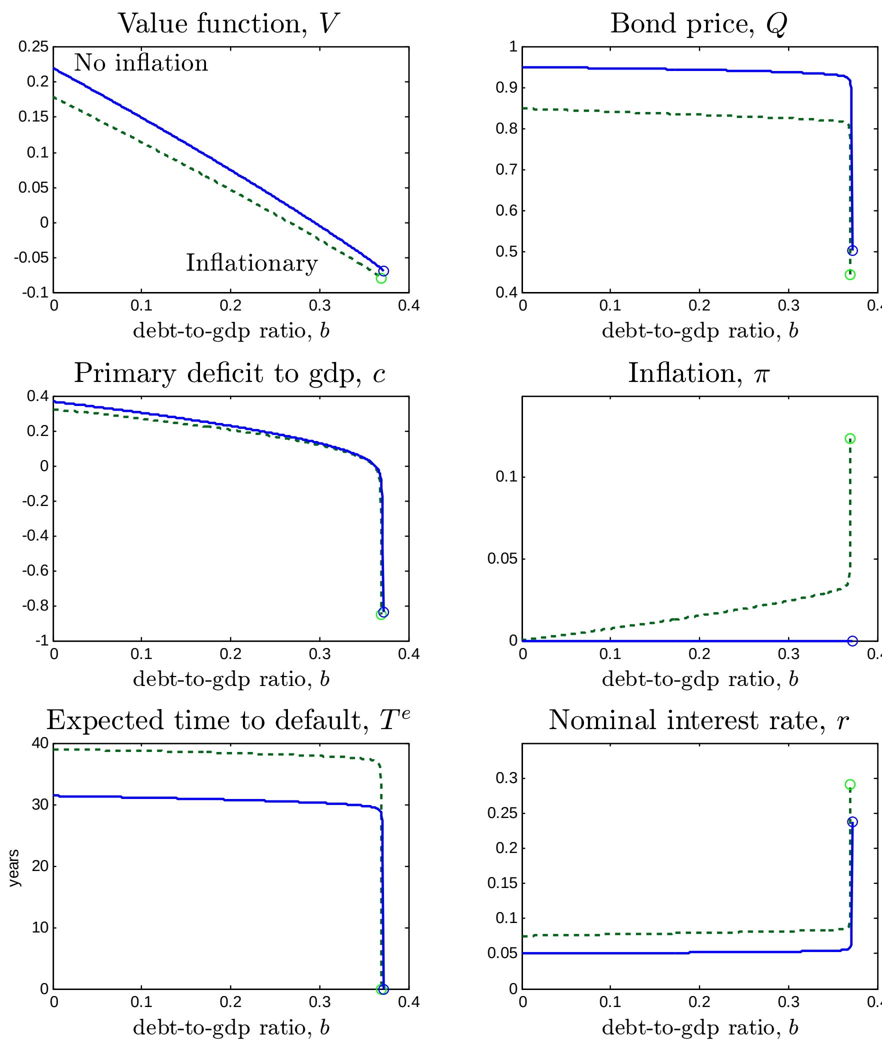

Figure 1: Equilibrium value function and policy functions

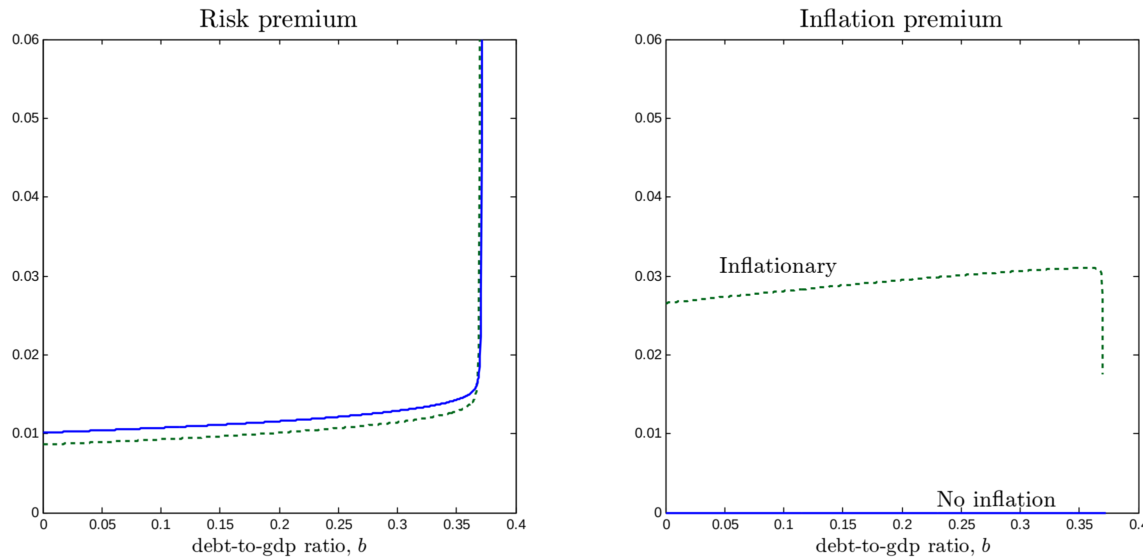

Figure 2: Risk premium and inflation premium

debt burden, making default even more imminent, as shown by the rapid decline in expected time to default in Figure 1.

Regarding inflation, the government's incentive to deflate debt away increase approximately linearly with the debt ratio. This is because the value function is approximately linear, such that the welfare gain per unit of debt reduction is roughly constant. However, in the vicinity of the default threshold, the value function starts declining more and more steeply, such that a marginal reduction in the debt ratio yields a higher and higher marginal gain in welfare. As a result, optimal inflation increases steeply until reaching about 12% at default. Therefore, under discretion, the optimal trade-off between price stability and sovereign debt sustainability prescribes a roughly linear increase in inflation for moderate debt levels, and a strong increase as the economy approaches default.

Consider now the equilibrium in the 'no-inflation regime', depicted by the solid blue lines in Figure 1. As explained in section 2.3.3, this scenario can be interpreted as issuing foreign currency debt or joining a monetary union with a very strong anti-inflationary commitment. The first aspect to notice is that the optimal default threshold ($b^*_{\pi=0} = 37.2\%$; see blue circles) is essentially the same as in the baseline, inflationary regime.[29] This does not mean however that sovereign debt is equally vulnerable. Indeed, for all debt ratios except those very close to default, expected time to default is *lower* (by around 8 years) when the government cannot use inflation to deflate debt away. However, because default is still perceived as rather distant (about 30 years in

[29] Both thresholds are in turn higher than $\hat{b} = 0.332$. Thus, the loss in (log)output during the exclusion period following default is $\epsilon \max\{0, b^* - \hat{b}\} = \epsilon(b^* - \hat{b})$.

expectation), the upward pressure on nominal yields from higher default risk is easily outweighed by the disappearance of the inflation premium (see also Figure 2). As a result, nominal yields are consistently *lower* in the no-inflation regime. This makes it less costly for the government to incur primary deficits, thus raising consumption for given exogenous output. In addition, the direct welfare cost of inflation, $(\psi/2)\,\pi^2$, disappears in the 'no inflation' regime. Thus, both effects (lower nominal yields and no direct welfare costs) imply that the value function is *higher* in the no-inflation regime.

Notice finally that the value function at the default threshold is also higher under no inflation. To understand why, notice that in both regimes the value function at default equals

$$V(b^*) = V_{def}(b^*) = -\frac{\epsilon(b^* - \hat{b})}{\rho + \chi} + \frac{\chi}{\rho + \chi} V\left(\theta b^*\right), \tag{22}$$

where we have used the fact that, under our calibration, $b^* > \hat{b}$ for both regimes. The fact that the default thresholds is very similar in both cases implies that so is the output loss from default, $\epsilon(b^* - \hat{b})$. It also implies that, after the exclusion period, the government reenters capital markets with essentially the same debt ratio ($\theta b^* = 20.7\%$, versus $\theta b^*_{\pi=0} = 20.8\%$). However, at such ratio the value function is higher in the no-inflation regime, for the reasons just discussed.

To summarize the previous discussion, the no-inflation regime achieves superior welfare outcomes at any debt ratio. It does so, first, by avoiding the temptation to inflate at points of the state space where default is still perceived as rather distant, and hence where the stabilizing benefits from deflating debt away are relatively minor. And second, by raising the value of defaulting relative that in the inflationary regime.

3.4 Average performance

So far we have analyzed the equilibrium value function and policy functions, i.e. the optimal choices of fiscal and monetary policy and the associated welfare at each point of the state space. The main result from the previous section is that the no-inflation regime yields higher welfare at any debt ratio, including at the respective default thresholds. This does not guarantee however that unconditional *average* welfare would be higher too. For instance, it could be the case that the inflationary regime delivered lower debt ratios most of the time, which could easily imply higher average welfare.

In order to compute unconditional averages of welfare and other variables, we thus need to solve for the stationary distribution of the state variable, the debt ratio. For this purpose, it is useful to distinguish between (a) 'normal' times in which the country meets its debt obligations and hence can access capital markets and (b) the exclusion periods that follow each default. The stationary

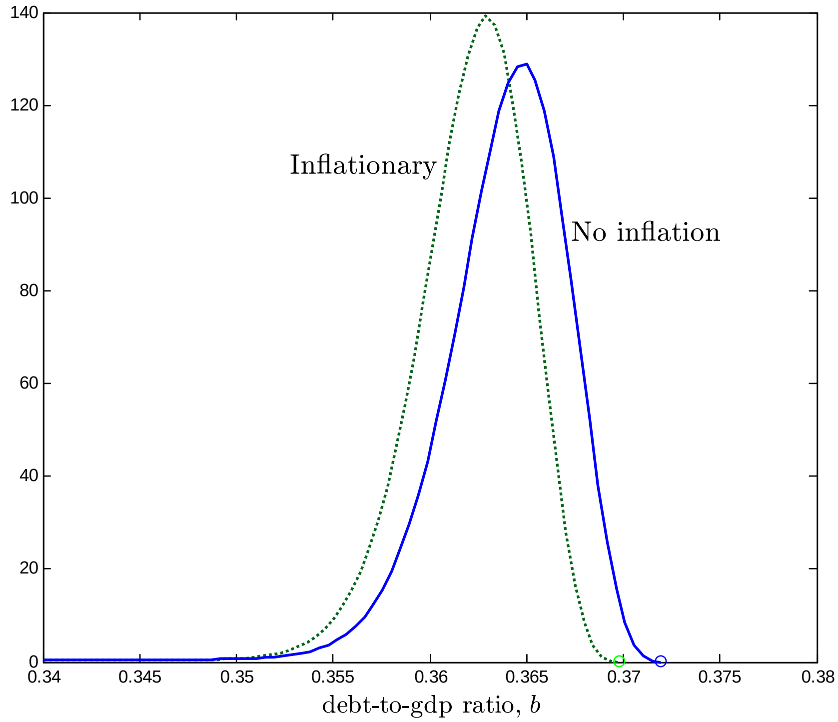

Figure 3: Stationary distribution of debt ratio, $f(b)$

distribution conditional on being in 'normal' times, which we may denote by $f(b)$, satisfies the following *Kolmogorov Forward Equation* (KFE),

$$0 = -\frac{d}{db}\left\{\left[\left(\frac{\lambda+\delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b)\right)b + \frac{c(b)}{Q(b)}\right]f(b)\right\} + \frac{1}{2}\frac{d^2}{db^2}\left[(\sigma b)^2 f(b)\right], \tag{23}$$

with the constraint $1 = \int_0^{b^*} f(b)db$. Appendix D shows how to compute $f(b)$ numerically, using an upwind finite difference scheme similar to the one employed to solve for the value and bond price functions. Figure 3 displays the stationary distributions of the debt ratio for both the baseline and the no-inflation regimes, conditional on being in normal times. In the baseline regime, the possibility of using inflation to deflate debt away allows the government to shift the debt distribution slightly to the left *vis-à-vis* the no-inflation regime.

Conditional on being in an exclusion period, we have already seen that primary deficit and inflation are both zero, $c_t = \pi_t = 0$. Since the rate at which the country reenters capital markets is constant at χ and hence independent of the time elapsed since default, we have that the value function and bond price are equal to their boundary values: $V_t = V(b^*) = V_{def}(b^*)$, $Q_t = Q(b^*)$. Finally, we assume for simplicity that during the exclusion period the debt ratio is equal to b^*, i.e.

the ratio at which the country defaults.[30]

We can now compute the unconditional mean of each variable as the weighted average of the conditional means, using as weights the average time spent in normal and exclusion periods. It is relatively straightforward to show that the stationary probability of being in normal times and in exclusion periods equal $\mathbb{P}[b_t < b^*] = \frac{T^e(\theta b^*)}{1/\chi + T^e(\theta b^*)}$ and $\mathbb{P}[b_t = b^*] = \frac{1/\chi}{1/\chi + T^e(\theta b^*)}$, respectively. Thus, the unconditional mean of a variable x_t equals

$$\begin{aligned} \mathbb{E}[x_t] &= \mathbb{P}[b_t < b^*]\,\mathbb{E}[x_t | b_t < b^*] + \mathbb{P}[b_t = b^*]\,x^* \\ &= \frac{T^e(\theta b^*)}{1/\chi + T^e(\theta b^*)} \int_0^{b^*} x(b)\,f(b)db + \frac{1/\chi}{1/\chi + T^e(\theta b^*)} x^*, \end{aligned}$$

where x^* is the value of x_t during the exclusion period.[31]

Table 2 displays the unconditional averages of a number of key variables in our model for both monetary regimes, as well as their corresponding empirical counterparts across EMU periphery countries[32]. Notice first that, remarkably, the model with no inflation replicates exactly the average bond risk premium (154 bp) conditional on being still on the bond market ($b < b^*$); it also reproduces well the average bond yield. In the inflationary regime, average yields (net of $\bar{r} = 400$ bp) while still in the market (448 bp) reflect mostly the inflation premia (309 bp), rather than risk premia (139 bp). Interestingly, the fact that the no-inflation regime delivers lower average yields than the inflationary regime rationalizes the observed reduction in average sovereign yields across the EMU periphery brought about by the creation of the eurozone, if one interprets both regimes as the model counterparts of the EMU and pre-EMU periods respectively. Indeed, average yields on 10-year peripheral bonds decreased from 12.84% in the period 1987-94 to 5.87% in 1995-2012. Viewed through the lens of our model, this suggests that, when these countries decided to renounce the ability to deflate their debts by joining EMU, the reduction in inflation expectations was a more important factor in investors' pricing of the new euro-denominated bonds than the presumable increase in default risk.

[30] We are thus assuming that during the exclusion/renegotiation period nominal debt outstanding is adjusted at each point in time to changes in the output endowment, such that the debt ratio is kept constant at b^*. We could alternatively assume that, during the exclusion period, nominal debt outstanding is kept constant at its value at the time of default (B_t^*), such that the debt ratio changes with the output endowment. This would complicate the analysis while barely affecting the numerical results, given the relatively short average duration of the exclusion period.

[31] As explained above, $c^* = 0$, $\pi^* = 0$, $V^* = V(b^*)$, and $Q^* = Q(b^*)$.

[32] All data are annual except bond yields and risk premia which are quarterly. We stop the sample for yields and risk premia in 2012:Q2 (included) in order to isolate our analysis from the effects of the annoucement by the European Central Bank of the Outright MonetaryTransactions (OMT) programme in the summer of 2012.

Table 2. Unconditional averages

	units	Data 1995-2012	Model No inflation $b < b^*$	Model No inflation full	Model Inflationary $b < b^*$	Model Inflationary full
debt-to-gdp, b	%	35.6	35.6	35.8	35.6	35.7
primary deficit ratio, c	%	-4.1	-0.01	-0.01	-0.12	-0.11
inflation, π	%	0.4	0	0	3.20	2.97
bond yields (net of $\bar{r}$), $r - \bar{r}$	bp	187	154	317	448	597
risk premium, $r - \tilde{r}$	bp	154	154	317	139	298
inflation premium, , $\tilde{r} - \bar{r}$	bp	33	0	0	309	299
Exp. time to default, T^e	years	-	29.4	29.4	37.1	37.1
Welfare loss, $V - V_{\pi=0}$	% cons.	-	0	0	-0.25	-0.25

Note: Data from IMF, national accounts, and Bloomberg. All data are annual except bond yields and risk premia which are quarterly (annualized) and run through 2012:Q2. See Data Appendix for details. The German 10-year bond yield is used as empirical proxy for the riskless bond yield, $\tilde{r}$. The column labelled '$b < b^*$' displays results conditional on not being in exclusion, the column labelled 'full' displays fully unconditional results. Welfare losses are relative to the no-inflation regime and are expressed in terms of permanent percentage loss in consumption.

From a welfare perspective, we find that average welfare is *higher* in the no-inflation regime, i.e. when the government renounces the possibility of deflating debt away. For instance, under our baseline calibration ($\psi = 9.15$) the welfare gains from not using debt deflation are equivalent to a 0.25% increase in consumption forever. Therefore, the leftward shift in the debt distribution shown in Figure 3 is not sufficient to compensate for the fact that the value function is higher at any debt ratio. In terms of costs and benefits, our results indicate that while reducing average inflation makes sovereign debt more vulnerable (as implied by higher risk premia and lower expected time-to-default on average), this is more than compensated by the reduction in nominal yields in new bond issuances and the elimination of direct welfare costs.

3.5 Robustness

We now evaluate the robustness of our main results to alternative calibrations. We will focus on (i) the amortization rate λ, (ii) the bond recovery parameter θ, and (iii) the default cost parameter $\hat{b}$.

The amortization rate λ determines the average Macaulay bond duration, $1/(\lambda + \bar{r})$, for given riskless real return $\bar{r}$. Table 3 displays averages of a number of key variables for bond durations of

3 and 6 years, both for the no-inflation and the baseline inflationary regimes. For comparison, it also displays the same statistics for the benchmark calibration, with a 5-year bond duration. We find that average welfare continues to be higher in the no-inflation regime. The welfare loss from using discretionary inflation decreases with bond duration. Intuitively, longer bond durations give more stability to the debt ratio, thus reducing the need to use debt deflation. This allows to reduce inflation premia in bond yields and direct utility costs, and hence the welfare loss relative to the no-inflation case.

The bond recovery parameter, θ, controls the average bond recovery rate after default, $\theta\chi/\left(\chi-\mu\right)$, for given reentry and trend growth rates (χ,μ). Table 3 displays results for average recovery rates of 50% and 70% (the benchmark calibration is 60%). Again, average welfare is higher if the government renounces the possibility to deflate debt away. In this case, the welfare gains are fairly similar across different calibrations. As in the baseline calibration, the reduction in average inflation premia from giving up debt deflation clearly dominates the increase in average risk premia.

Finally, $\hat{b}$ controls the loss in (log)output following default, $\epsilon\max\{0,b^{*}-\hat{b}\}$, for given scale parameter ϵ and equilibrium default threshold b^{*}. We consider values of $\hat{b}$ such that, in equilibrium, output declines by 3.5% and 7% upon default (compared to the benchmark 6% loss). In this case, the welfare gains from not deflating debt away, while positive, seem more sensitive to the size of output losses associated to default. The reason is the following. In our model, a positive relationship exists between $\hat{b}$ and average debt ratios. Therefore, lower values of $\hat{b}$ imply lower debt on average and therefore a weaker incentive to deflate the latter away. Lower average inflation in turn reduces inflation premia and direct utility costs, thus reducing the welfare gap with respect to the no-inflation scenario.

Table 3. Robustness analysis

	Welfare	Time to default	Inflation	Risk premium	Inflation premium
	% cons.	years	%	bp	bp
Benchmark					
No inflation	0	29.4	0	317	0
Inflationary	-0.25	37.1	2.97	298	299
Difference	*0.25*	*-7.7*	*-2.97*	*19*	*-299*
Duration = 3					
No inflation	0.1	40.4	0	311	0
Inflationary	-0.36	47.4	3.28	304	331
Difference	*0.35*	*-7.0*	*3.28*	*7*	*-331*
Duration = 7					
No inflation	0.0	26.0	0	309	0
Inflationary	-0.17	34.7	2.75	278	278
Difference	*0.17*	*-8.7*	*-2.75*	*31*	*-278*
Recovery rate = 50%					
No inflation	-0.08	30.7	0	401	0
Inflationary	-0.33	38.5	3.00	373	302
Difference	*0.25*	*-7.8*	*-3.00*	*28*	*-302*
Recovery rate = 70%					
No inflation	0.09	28.3	0	246	0
Inflationary	-0.20	35.8	2.94	236	297
Difference	*0.29*	*-7.5*	*-2.94*	*10*	*-297*
Default costs = 3.5%					
No inflation	0.43	29.7	0	318	0
Inflationary	0.33	34.6	1.87	304	189
Difference	*0.10*	*-4.9*	*-1.87*	*14*	*-189*
Default costs = 7%					
No inflation	-0.22	29.6	0	314	0
Inflationary	-0.59	38.7	3.50	293	353
Difference	*0.37*	*-9.1*	*-3.50*	*21*	*-353*

Note: Welfare is calculated with respect to the no-inflation calibration and it is expressed in % of permanent consumption.

4 Monetary policy delegation

So far we have compared two alternative scenarios. In the baseline inflationary regime, a benevolent government maximizes social welfare taking into account households' preferences towards inflation, where such preferences are calibrated to match the observed inflation performance in the EMU peripheral economies in the pre-EMU period. In the 'no inflation' regime, the government, aware of its inability to make inflation commitments, effectively renounces its ability to deflate debt away. Under our baseline calibration for the relative weight on inflation disutility in households' preferences, ψ, we have found that giving up such discretionary stabilization tool actually increases welfare. As explained in section 2.3.3, such a scenario can be interpreted as the government issuing foreign currency debt, or joining a monetary union with a very strong anti-inflationary stance. We also argued that one could view the 'no inflation' regime as a situation in which the government appoints an independent central banker with an extremely great distaste for inflation.

In this section, we consider an intermediate arrangement by which the government delegates (discretionary) monetary policy to an independent central banker whose distaste for inflation is greater than that of society, but not so extreme as to imply zero inflation at all times. The question here is whether one can find intermediate preferences towards inflation that achieve better welfare outcomes than the two regimes considered thus far.

Formally, our maximization problem is modified as follows. On the one hand, the benevolent government retains the primary deficit and default decisions, taking as given the inflation policy function of the independent monetary authority, which we denote by $\tilde{\pi}(b)$. With a slight abuse of notation, let $V(b)$ denote the value function of the government when the latter no longer chooses inflation. The corresponding HJB equation is

$$\rho V(b) = \max_{c,b^*} \left\{ \log(1+c) - \frac{\psi}{2}\tilde{\pi}(b)^2 + \left[\left(r(b) + \sigma^2 - \mu - \tilde{\pi}(b) \right) b + \frac{c}{Q(b)} \right] V'(b) + \frac{(\sigma b)^2}{2} V''(b) \right\}, \tag{24}$$

where the value matching and smooth pasting conditions are given again by equations (12) and (13), respectively.[33] The optimal primary deficit ratio is given again by equation (15). Investors' bond pricing schedule $Q(b)$ is determined exactly as before.

The monetary authority chooses inflation taking as given the government's primary deficit policy, $c(b)$, and optimal default threshold, b^*. Letting $\tilde{V}(b)$ denote the monetary authority's

[33] In equation (24), we have used the definition of the bond yield $r(b)$ (equation 20) for compactness.

value function, the latter satisfies the following HJB equation,

$$\rho\tilde{V}(b) = \max_{\pi}\left\{\log(1+c(b)) - \frac{\tilde{\psi}}{2}\pi^2 + \left[\left(r(b)+\sigma^2-\mu-\pi\right)b + \frac{c(b)}{Q(b)}\right]\tilde{V}'(b) + \frac{(\sigma b)^2}{2}\tilde{V}''(b)\right\}, \tag{25}$$

where $\tilde{\psi} \geq \psi$ captures the central banker's distaste for inflation. $\tilde{V}$ also satisfies a value matching condition analogous to (12). The optimal inflation decision is given by equation (16) with $\tilde{\psi}$ and $\tilde{V}'$ replacing and ψ and V', which defines the new inflation policy function $\tilde{\pi}(b)$. Notice that $\lim_{\tilde{\psi}\to\infty}\tilde{\pi}(b) = 0$ for all b. Thus, as argued in section 2.3.3, the 'no inflation' regime can be viewed as an extreme case of the independent central banker problem laid out here, in which the latter has an arbitrarily great distaste for inflation.

In order to solve this problem we need to extend the numerical algorithm introduced in section 3.1. In particular, we replace the government problem (step 1) by:

Step 1a: Government problem. Given $Q^{(n-1)}$, $\pi^{(n-1)}$ and $b^*_{(n-1)}$, we solve the HJB equation (24) in the domain $[0, b^*_{(n-1)}]$ imposing the smooth pasting condition (13) to obtain an estimate of the government's value function $V^{(n)}$ and of primary deficit $c^{(n)}$.

Step 1b: Central bank problem. Given $Q^{(n-1)}$, $c^{(n)}$ and $b^*_{(n-1)}$, we solve the HJB equation (25) in the domain $[0, b^*_{(n-1)}]$ imposing the smooth pasting condition (13) to obtain an estimate of the central bank's value function $\tilde{V}^{(n)}$ and of inflation $\pi^{(n)}$.

Figure 4 displays the unconditional means of social welfare and other relevant variables as we vary the conservative central banker's distaste for inflation, $\tilde{\psi}$.[34] The main message is that average social welfare increases monotonically with the inflation conservatism of the delegated monetary authority, but it only reaches average welfare under no-inflation ($\tilde{\psi} = \infty$) asymptotically.

To understand this result, let us focus first on the two arguments of the household utility flow, inflation and the primary deficit ratio. On the one hand, average inflation decreases monotonically with the central banker's distaste for inflation. As explained before, lower inflation favors welfare by reducing direct welfare costs and lowering bond inflation premia. On the other hand, average primary deficit follows a hump-shaped pattern, reaching a maximum level at around $\tilde{\psi}/\psi = 3$ and then decreasing gradually towards its value in the 'no inflation' regime. Intuitively, the intense reduction in average inflation and hence in average yields that takes place as $\tilde{\psi}/\psi$ rises above 1 allows to sustain higher primary deficits by making them cheaper to finance; above $\tilde{\psi}/\psi \approx 3$, the reduction in inflation and yields slows down and average deficit rates start declining. However, the

[34] To facilitate interpretation, the x-axes in Figure 4 display the central banker's distaste for inflation relative to that of society, $\tilde{\psi}/\psi$, where ψ is held constant at its calibrated value (see Table 1). Thus $\tilde{\psi}/\psi = 1$ represents our baseline 'inflationary regime' in which inflation is chosen by the benevolent government.

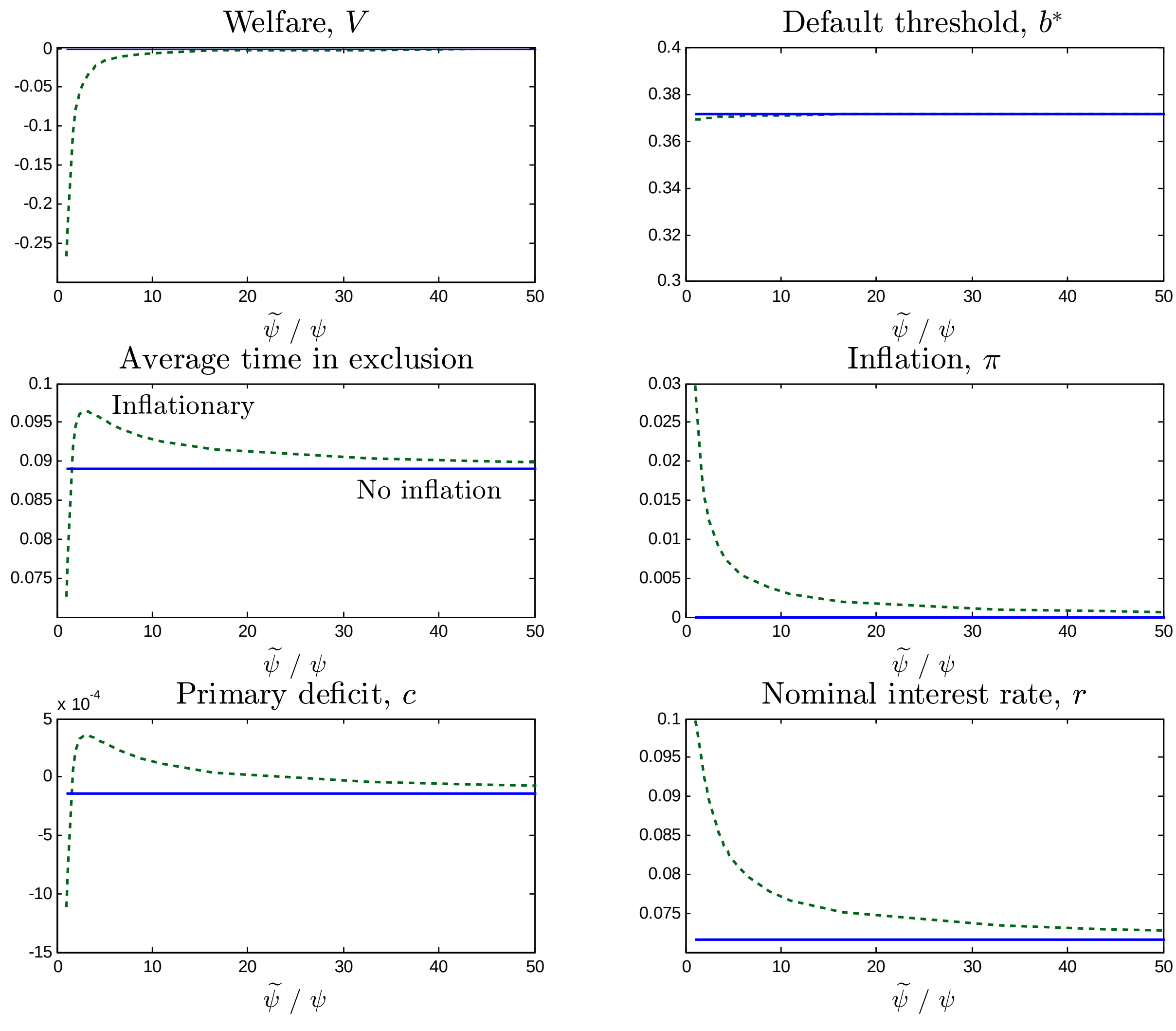

Figure 4: Effect of central bank conservatism under monetary policy delegation

behavior of primary deficits is mirrored by that of average time spent in default/exclusion, which also experiences a hump-shaped pattern with a peak at around $\tilde{\psi}/\psi = 3$. Thus, the downside of increasing average deficits in the range $\tilde{\psi}/\psi \in (1, 3)$ is that the economy also becomes more prone to defaulting. The latter two effects tend to offset each other. This leaves the reduction in the welfare costs of inflation as the dominant force, thus producing a monotonic increase in average welfare.

To summarize our results in this section, we find that if the government is unable to make credible commitments, delegating monetary policy to an independent, relatively conservative central banker achieves better welfare outcomes by reducing expected and current inflation. However, such an institutional solution continues to be dominated by a scenario in which the government fully renounces the ability to deflate debt away, as would be exemplified e.g. by issuing foreign currency debt or joining a monetary union with a very strong and credible anti-inflationary mandate.

5 Conclusions

Motivated by the recent debt crisis in the EMU periphery, in this paper we have analyzed the trade-offs between price stability and the sustainability of sovereign debt. We have done so in the context of a continuous-time, small open economy model where a benevolent government issues nominal defaultable debt to foreign investors. The government is assumed to be unable to make credible commitments regarding fiscal policy (including the possibility of defaulting on sovereign debt) and monetary policy. At each point in time the government optimally chooses primary deficit and inflation, and whether to default or not. A main them of our paper is to compare this situation with an alternative scenario in which the government effectively renounces the option to deflate debt away, e.g. by issuing foreign currency debt or joining an anti-inflationary monetary union. In our quantitative exploration, the government's inflation tolerance is calibrated to replicate observed inflation differentials in the EMU periphery before the start of the euro.

We have found that giving up the option to deflate debt away achieves higher welfare (both at any debt ratio and on average) than retaining such discretionary adjustment margin. The reason lies in the costs and benefits of inflation. On the one hand, inflation allows to reduce the real value of nominal debt and thus make it more sustainable *ceteris paribus*, with the resulting reduction in risk premia. On the other hand, (expected) inflation raises the inflation premium that the government must offer in new bond issuances, and also creates direct welfare costs. In equilibrium, the inflationary costs stemming from higher inflation premia and direct welfare costs outweigh the benefits from reducing risk premia. Our results thus qualify the conventional wisdom that national governments should benefit from retaining the possibility of deflating away their sovereign debt, in the sense that such a benefit may not materialize if such governments are unable to make credible commitments about its future monetary policy. This qualification may be particularly relevant for most EMU peripheral economies, in view of their inflation record (relative e.g. to that of Germany) in the decades prior to joining the euro.

Looking ahead, we note that we have analyzed the problem of a single government in a small open economy setup. Given our interest in recent developments in the euro area, we believe that extending the analysis presented here to the case of a monetary union with a common monetary authority and many national fiscal authorities that differ in their outstanding sovereign debt levels is of great importance. We leave this taks for future research.

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Appendix

A. Numerical algorithm

We describe the numerical algorithm used to jointly solve for the equilibrium value function, $V(b)$, and bond price function, $Q(b)$. The algorithm proceeds in 3 steps. We describe each step in turn.

Step 1: Solution to the Hamilton-Jacobi-Bellman equation

The HJB equation (14) is solved using an *upwind finite difference* scheme following Achdou et al. (2014). It approximates the value function $V(b)$ on a finite grid with step $\Delta b : b \in \{b_1, ..., b_I\}$, where $b_i = b_{i-1} + \Delta b = b_1 + (i-1)\Delta b$ for $2 \leq i \leq I$. The bounds are $b_1 = 0$ and $b_I = b^* - \Delta b$, such that $\Delta b = b^*/I$. We choose θ such that $\theta(I+1) \in \mathbb{N}$. We use the notation $V_i \equiv V(b_i)$, $i = 1, ..., I$, and similarly for the bond price function Q_i and the policy functions (π_i, c_i).

Notice first that the HJB equation involves first and second derivatives of the value function, $V'(b)$ and $V''(b)$. At each point of the grid, the first derivative can be approximated with a forward (F) or a backward (B) approximation,

$$V'(b_i) \approx \partial_F V_i \equiv \frac{V_{i+1} - V_i}{\Delta b}, \tag{26}$$

$$V'(b_i) \approx \partial_B V_i \equiv \frac{V_i - V_{i-1}}{\Delta b}, \tag{27}$$

whereas the second derivative is approximated by

$$V''(b_i) \approx \partial_{bb} V_i \equiv \frac{V_{i+1} + V_{i-1} - 2V_i}{(\Delta b)^2}. \tag{28}$$

In an upwind scheme, the choice of forward or backward derivative depends on the sign of the *drift function* for the state variable, given by

$$s(b) \equiv \left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b) \right) b + \frac{c(b)}{Q(b)}, \tag{29}$$

for $b \leq b^*$, where

$$c(b) = -\frac{Q(b)}{V'(b)} - 1,$$

$$\pi(\pi) = -\frac{b}{\psi} V'(b) = \frac{bQ(b)}{\psi(1+c)}.$$

Let superscript n denote the iteration counter. The HJB equation is approximated by the following upwind scheme,

$$\frac{V_i^{n+1}-V_i^n}{\Delta}+\rho V_i^{n+1}=\log(c_i^n+1)-\frac{\psi}{2}\left(\pi_i^n\right)^2+\partial_F V_i^{n+1}s_{i,F}^n\mathbf{1}_{s_{i,F}^n>0}+\partial_B V_i^{n+1}s_{i,B}^n\mathbf{1}_{s_{i,B}^n<0}+\frac{(\sigma b_i)^2}{2}\partial_{bb}V_i^{n+1},$$

for $i=1,...,I$, where $\mathbf{1}\left(\cdot\right)$ is the indicator function and

$$\begin{aligned} s_{i,F}^n &= \left(\frac{\lambda+\delta}{Q_i}+\sigma^2-\mu-\lambda+\frac{b_i}{\psi}\partial_F V_i^n\right)b_i-\left(\frac{1}{\partial_F V_i^n}+\frac{1}{Q_i}\right), \\ s_{i,B}^n &= \left(\frac{\lambda+\delta}{Q_i}+\sigma^2-\mu-\lambda+\frac{b_i}{\psi}\partial_B V_i^n\right)b_i-\left(\frac{1}{\partial_B V_i^n}+\frac{1}{Q_i}\right). \end{aligned}$$

Therefore, when the drift is positive ($s_{i,F}^n>0$) we employ a forward approximation of the derivative, $\partial_F V_i^{n+1}$; when it is negative ($s_{i,B}^n<0$) we employ a backward approximation, $\partial_B V_i^{n+1}$. The term $\frac{V_i^{n+1}-V_i^n}{\Delta}\to 0$ as $V_i^{n+1}\to V_i^n$. Moving all terms involving V^{n+1} to the left hand side and the rest to the right hand side, we obtain

$$V_{i-1}^{n+1}\alpha_i^n+V_i^{n+1}\beta_i^n+V_{i+1}^{n+1}\xi_i^n=\log(c_i^n+1)-\frac{\psi}{2}\left(\pi_i^n\right)^2+\frac{V_i^n}{\Delta}, \tag{30}$$

where

$$\begin{aligned} \alpha_i^n &\equiv \frac{s_{i,B}^n\mathbf{1}_{s_{i,B}^n<0}}{\Delta b}-\frac{(\sigma b_i)^2}{2\left(\Delta b\right)^2}, \\ \beta_i^n &\equiv \frac{1}{\Delta}+\rho+\frac{s_{i,F}^n\mathbf{1}_{s_{i,F}^n>0}}{\Delta b}-\frac{s_{i,B}^n\mathbf{1}_{s_{i,B}^n<0}}{\Delta b}+\frac{(\sigma b_i)^2}{\left(\Delta b\right)^2}, \\ \xi_i^n &\equiv -\frac{s_{i,F}^n\mathbf{1}_{s_{i,F}^n>0}}{\Delta b}-\frac{(\sigma b_i)^2}{2\left(\Delta b\right)^2}, \end{aligned}$$

for $i=1,...,I$. Notice that the state constraint $b\geq 0$ means that $s_{1,B}^n=0$, which together with $b_1=0$ implies $\alpha_1^n=0$. In equation (30), the optimal primary deficit ratio is set to

$$c_i^n=\left(\frac{-Q_i^n}{\partial V_i^n}-1\right), \tag{31}$$

where

$$\partial V_i=\partial_F V_i\mathbf{1}_{s_{i,F}^n>0}+\partial_B V_i\mathbf{1}_{s_{i,B}^n<0}-\frac{Q(b_i)}{1+\bar{c}_i}\mathbf{1}_{s_{i,F}^n\leq 0}\mathbf{1}_{s_{i,B}^n\geq 0}.$$

In the above expression, $\bar{c}_i$ is the consumption level such that $s\left(b_i\right)\equiv s_i^n=0$, i.e. it solves

$$\left(\frac{\lambda+\delta}{Q_i}+\sigma^2-\mu-\lambda-\frac{b_iQ_i}{\psi\left(1+\bar{c}_i\right)}\right)b_i+\frac{\bar{c}_i}{Q_i}=0.$$

The solution is the higher root of the above equation,

$$\bar{c}_i = \frac{-\left(1+\Gamma_i Q_i\right) + \sqrt{\left(1+\Gamma_i Q_i\right)^2 - 4\left[\Gamma_i Q_i - \frac{b_i^2 Q_i^2}{\psi}\right]}}{2},$$

where $\Gamma_i \equiv \left(\frac{\lambda+\delta}{Q(b_i)} + \sigma^2 - \mu - \lambda\right) b_i$. Given c_i^n, the optimal inflation rate is

$$\pi_i^n = \frac{bQ_i}{\psi\left(1+c_i^n\right)}. \tag{32}$$

The smooth pasting boundary condition (equation 13) can be approximated by[35]

$$\frac{V_{I+1}^{n+1} - V_I^{n+1}}{\Delta b} = -\frac{\epsilon}{\chi+\rho} + \frac{\chi}{\chi+\rho}\theta\partial_F V_{\theta(I+1)}^n \Rightarrow V_{I+1}^{n+1} = V_I^{n+1} + \left(-\frac{\epsilon}{\chi+\rho} + \frac{\chi}{\chi+\rho}\theta\partial_F V_{\theta(I+1)}^n\right)\Delta b. \tag{33}$$

Equation (30) is a system of I linear equations which can be written in matrix notation as:

$$\mathbf{A}^n \mathbf{V}^{n+1} = \mathbf{d}^n, \tag{34}$$

where the matrix $\mathbf{A}^n$ and the vectors $\mathbf{V}^{n+1}$ and $\mathbf{d}^n$ are defined by

$$\mathbf{A}^n = \begin{bmatrix} \beta_1^n & \xi_1^n & 0 & 0 & \cdots & 0 \\ \alpha_2^n & \beta_2^n & \xi_2^n & 0 & \cdots & 0 \\ 0 & \alpha_3^n & \beta_3^n & \xi_3^n & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \cdots & \alpha_{I-1}^n & \beta_{I-1}^n & \xi_{I-1}^n \\ 0 & 0 & \cdots & 0 & \alpha_I^n & \beta_I^n + \xi_I^n \end{bmatrix}, \quad \mathbf{V}^{n+1} = \begin{bmatrix} V_1^{n+1} \\ V_2^{n+1} \\ V_3^{n+1} \\ \vdots \\ V_{I-1}^{n+1} \\ V_I^{n+1} \end{bmatrix}$$

$$\mathbf{d}^n = \begin{bmatrix} \log(c_1^n+1) - \frac{\psi}{2}\left(\pi_1^n\right)^2 + \frac{V_1^n}{\Delta} \\ \log(c_2^n+1) - \frac{\psi}{2}\left(\pi_2^n\right)^2 + \frac{V_2^n}{\Delta} \\ \log(c_3^n+1) - \frac{\psi}{2}\left(\pi_3^n\right)^2 + \frac{V_3^n}{\Delta} \\ \vdots \\ \log(c_{I-1}^n+1) - \frac{\psi}{2}\left(\pi_{I-1}^n\right)^2 + \frac{V_{I-1}^n}{\Delta} \\ \log(c_I^n+1) - \frac{\psi}{2}\left(\pi_I^n\right)^2 + \frac{V_I^n}{\Delta} + \xi_I^n\left(\frac{\epsilon}{\chi+\rho} - \frac{\chi}{\chi+\rho}\theta\partial_F V_{\theta(I+1)}^n\right)\Delta b \end{bmatrix}.$$

Notice that the element (I, I) in A is $\beta_I^n + \xi_I^n$ due to the smooth pasting condition (33).

[35] Notice that we solve for the value function under the guess that the optimal default threshold satisfies $b^* > \hat{b}$, such that $\max\{0, b^* - \hat{b}\} = b^* - \hat{b}$. We verify that our guess is satisfied in equilibrium in all our simulations.

The algorithm to solve the HJB equation runs as follows. Begin with an initial guess $V_i^0 = -b_i$, $i = 1, ..., I$. Set $n = 0$. Then:

1. Compute $\partial_F V_i^n$, $\partial_B V_i^n$ and $\partial_{bb} V_i^n$ using (26)-(28).

2. Compute c_i^n and π_i^n using (31) and (32).

3. Find V_i^{n+1} solving the linear system of equations (34).

4. If V_i^{n+1} is close enough to V_i^n, stop. If not set $n := n + 1$ and go to 1.

Step 2: Solution to the Bond Pricing Equation

The pricing equation (18) is also solved using an upwind finite difference scheme. The equation in this case is

$$Q(b)\left(\bar{r} + \pi(b) + \lambda\right) = (\lambda + \delta) + \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi(b)\right) b + \frac{c(b)}{Q(b)}\right] Q'(b) + \frac{(\sigma b)^2}{2} Q''(b), \tag{35}$$

with a boundary condition

$$Q(b^*) = \frac{\chi}{\bar{r} - \mu + \chi} \theta Q(\theta b^*). \tag{36}$$

This case is similar to the HJB equation. Using the notation $Q_i = Q(b_i)$, the equation can be expressed as

$$\frac{Q_i^{n+1} - Q_i^n}{\Delta} + Q_i^{n+1}\left(\bar{r} + \pi_i + \lambda\right) = \lambda + \delta + \partial_F Q_i^{n+1} s_{i,F}^n \mathbf{1}_{s_{i,F}^n > 0} + \partial_B Q_i^{n+1} s_{i,B}^n \mathbf{1}_{s_{i,B}^n < 0} + \frac{(\sigma b_i)^2}{2} \partial_{bb} Q_i^{n+1}, \tag{37}$$

where:

$$\begin{aligned} Q'(b_i) &\approx \partial_F Q_i \equiv \frac{Q_{i+1} - Q_i}{\Delta b}, && (38) \\ Q'(b_i) &\approx \partial_B Q_i \equiv \frac{Q_i - Q_{i-1}}{\Delta b}, && (39) \\ Q''(b_i) &\approx \partial_{bb} Q_i \equiv \frac{Q_{i+1} + Q_{i-1} - 2Q_i}{(\Delta b)^2} && (40) \end{aligned}$$

and rearranging terms

$$\begin{aligned} Q_{i-1}^{n+1}\alpha_i^n + Q_i^{n+1}\left(\beta_i^n + \bar{r} + \pi_i + \lambda - \rho\right) + Q_{i+1}^{n+1}\xi_i^n &= \lambda + \delta + \frac{Q_i^n}{\Delta}, \ \forall i < I + 1, && (41) \\ Q_{I+1} &= \frac{\chi}{\rho - \mu + \chi} \theta Q(\theta\,(I + 1)). && (42) \end{aligned}$$

Notice the abuse of notation, as

$$s_{i,F}^n = s_{i,B}^n = s_i^n = \left(\frac{\lambda + \delta}{Q^n(b_i)} + \sigma^2 - \mu - \lambda - \pi_i \right) b_i + \frac{c_i}{Q^n(b_i)}.$$

Equation (37) is again a system of I linear equations which can be written in matrix notation as:

$$\mathbf{F}^n \mathbf{Q}^{n+1} = \mathbf{h}^n, \tag{43}$$

where the matrix $\mathbf{F}^n$ and the vectors $\mathbf{Q}^{n+1}$ and $\mathbf{f}^n$ are defined by:

$$\mathbf{F}^n = \begin{bmatrix} \begin{matrix}(\alpha_1^n + \beta_1^n + \pi_1 \\ +\lambda + \bar{r} - \rho)\end{matrix} & \xi_1^n & 0 & \cdots & 0 \\ \alpha_2^n & \begin{matrix}(\beta_2^n + \pi_2 \\ +\lambda + \bar{r} - \rho)\end{matrix} & \xi_2^n & \cdots & 0 \\ 0 & \alpha_3^n & \begin{matrix}(\beta_3^n + \pi_3 \\ +\lambda + \bar{r} - \rho)\end{matrix} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \cdots & \begin{matrix}(\beta_{I-1}^n + \pi_{I-1} \\ +\lambda + \bar{r} - \rho)\end{matrix} & \xi_{I-1}^n \\ 0 & 0 & \cdots & \alpha_I^n & \begin{matrix}(\beta_I^n + \pi_I \\ +\lambda + \bar{r} - \rho)\end{matrix} \end{bmatrix},$$

$$\mathbf{Q}^{n+1} = \begin{bmatrix} Q_1^{n+1} \\ Q_{2,1}^{n+1} \\ Q_{3,1}^{n+1} \\ \vdots \\ Q_{I-1}^{n+1} \\ Q_I^{n+1} \end{bmatrix}, \quad \mathbf{h}^n = \begin{bmatrix} \lambda + \delta + \frac{Q_1^n}{\Delta} \\ \lambda + \delta + \frac{Q_2^n}{\Delta} \\ \lambda + \delta + \frac{Q_3^n}{\Delta} \\ \vdots \\ \lambda + \delta + \frac{Q_{I-1}^n}{\Delta} \\ \lambda + \delta + \frac{Q_I^n}{\Delta} - \xi_I^n \frac{\chi}{\rho - \mu + \chi} \theta Q_{\theta(I+1)}^n \end{bmatrix}.$$

The algorithm to solve the bond pricing equation is similar to the HJB. Begin with an initial guess $Q_i^0 = \frac{\lambda+\delta}{\bar{r}+\lambda}$, set $n = 0$. Then:

1. Find Q_i^{n+1} solving the linear system of equations (43).

2. If Q_i^{n+1} is close enough to Q_i^n, stop. If not set $n := n + 1$ and go to 1.

Step 3: Value Matching

Finally, we iterate until the value matching condition (12) is satisfied:

$$V_{I+1} = -\frac{b_{I+1} - \hat{b}}{\rho + \chi} + \frac{\chi}{\chi + \rho} V_{\theta(I+1)}. \tag{44}$$

Taking into account (33), condition (44) can be rewritten as

$$V_I - \left(-\frac{\epsilon}{\chi + \rho} + \frac{\chi}{\chi + \rho} \theta \partial_F V^n_{\theta(I+1)} \right) \Delta b + \frac{0, b_{I+1} - \hat{b}}{\rho + \chi} - \frac{\chi}{\chi + \rho} V_{\theta(I+1)} = 0.$$

B. The riskless nominal bond

We define a new instrument, a riskless nominal bond. This is a non-defaultable bond issued in the domestic currency. In this case, the nominal price of the bond for a current debt ratio $b \leq b^*$ is given by

$$\tilde{Q}(b) = \mathbb{E} \left[\begin{array}{c} \int_0^{T(b^*)} e^{-(\bar{r}+\lambda)t - \int_0^t \pi_s ds} (\lambda + \delta) \, dt \\ + \int_{T(b^*)}^{T(b^*)+\tau} e^{-(\bar{r}+\lambda)t - \int_0^{T(b^*)} \pi_s ds} (\lambda + \delta) \, dt + e^{-(\bar{r}+\lambda)(T(b^*)+\tau) - \int_0^{T(b^*)} \pi_s ds} \tilde{Q}(\theta b^*) \; |b_0 = b \end{array} \right]. \tag{45}$$

Applying again the Feynman-Kac formula, we obtain

$$\tilde{Q}(b) \left(\bar{r} + \pi(b) + \lambda \right) = (\lambda + \delta) + \left[\left(\frac{\lambda + \delta}{Q(b)} + \sigma^2 - \mu - \lambda - \pi \right) b + \frac{c(b)}{Q(b)} \right] \tilde{Q}'(b) + \frac{(\sigma b)^2}{2} \tilde{Q}''(b), \tag{46}$$

for all $b \in [0, b^*)$. The boundary condition for $\tilde{Q}(b)$ is given by

$$\begin{aligned} \tilde{Q}(b^*) &= \mathbb{E} \left[\int_0^\tau e^{-(\bar{r}+\lambda)t} (\lambda + \delta) \, dt + e^{-(\bar{r}+\lambda)\tau} \tilde{Q}(\theta b^*) \right] \\ &= \int_0^\infty \chi e^{-\chi \tau} \left[\left(1 - e^{-(\bar{r}+\lambda)\tau} \right) \frac{\lambda + \delta}{\bar{r} + \lambda} dt + e^{-(\bar{r}+\lambda)\tau} \tilde{Q}(\theta b^*) \right] d\tau \\ &= \frac{\lambda + \delta}{\bar{r} + \chi + \lambda} + \frac{\chi}{\bar{r} + \chi + \lambda} \tilde{Q}(\theta b^*). \end{aligned} \tag{47}$$

Given the equilibrium default threshold b^*, we solve for the riskless bond price function $\tilde{Q}(b)$ using a finite difference scheme similar to the one used to solve for $Q(b)$ in Step 2 of the general algorithm.

C. Computing the expected time-to-default

Given the definition of the expected time to default (21), applying the Feynman-Kac formula we obtain

$$1+\left[\left(\frac{\lambda+\delta}{Q(b)}+\sigma^2-\mu-\lambda-\pi\right)b+\frac{c}{Q(b)}\right]T^{e\prime}(b)+\frac{(\sigma b)^2}{2}T^{e\prime\prime}(b)=0,$$

with a boundary condition

$$T^e(b^*)=0.$$

This can be solved using a finite difference scheme similar to the one described for the bond price in Appendix A.

D. Solution to the Kolmogorov Forward equation

The stationary distribution of debt-to-GDP ratio, $f(b)$, satisfies the Kolmogorov Forward equation:

$$0 = -\frac{d}{db}\left\{\left[\left(\frac{\lambda+\delta}{Q(b)}+\sigma^2-\mu-\lambda-\pi(b)\right)b+\frac{c(b)}{Q(b)}\right]f(b)\right\}+\frac{1}{2}\frac{d^2}{db^2}\left[(\sigma b)^2 f\right], \tag{48}$$

$$1 = \int_0^{b^*} f(b)db. \tag{49}$$

We solve this equation using an upwind finite difference scheme as in Achdou et al. (2014) or Nuño and Moll (2015). We use the notation $f_i \equiv f(b_i)$. The system can be now expressed as

$$\begin{aligned} 0 = & -\frac{f_i s_{i,F}-f_{i-1}s_{i-1,F}}{\Delta b}\mathbf{1}_{s^n_{i,F}>0}-\frac{f_{i+1}s_{i+1,B}-f_i s_{i,B}}{\Delta b}\mathbf{1}_{s^n_{i,B}<0} \\ & +\frac{f_{i+1}\left(\sigma b_{i+1}\right)^2+f_{i-1}\left(\sigma b_{i-1}\right)^2-2f_i\left(\sigma b_i\right)^2}{2\left(\Delta b\right)^2}, \end{aligned}$$

or equivalently

$$f_{i-1}\xi_i+f_{i+1}\alpha_i+f_i\left(\beta_i-\frac{1}{\Delta}-\rho\right)=0, \tag{50}$$

then (50) is also a system of I linear equations which can be written in matrix notation as:

$$\left(\mathbf{A}-\left(\frac{1}{\Delta}+\rho\right)\mathbf{I}\right)^{\mathbf{T}}\mathbf{f}=\mathbf{0}, \tag{51}$$

where $\left(\mathbf{A}-\left(\frac{1}{\Delta}+\rho\right)\mathbf{I}\right)^{\mathbf{T}}$ is the transpose of $\left(\mathbf{A}-\left(\frac{1}{\Delta}+\rho\right)\mathbf{I}\right)=\lim_{n\to\infty}\left(\mathbf{A^n}-\left(\frac{1}{\Delta}+\rho\right)\mathbf{I}\right)$. In order to impose the normalization constraint (49) we replace the second entry of the zero vector in (51) by 0.1. We solve the system (51) and obtain a solution $\hat{\mathbf{f}}$. Then we renormalize as

$$f_i = \frac{\hat{f}_i}{\sum_{i=1}^{I} \hat{f}_i \Delta b}.$$

E. Data Appendix

Data on GDP, inflation and current account balance for the five EMU periphery countries (Greece, Italy, Ireland, Portugal and Spain), and inflation for the United States, come from the IMF's World Economic Outlook database. The inflation differential is computed as the difference between the average inflation in the EMU periphery and that of the United States for the period 1987-1997.

External public debt is "General Government Gross consolidated Debt held by non-residents of the Member State" and is taken from each country's national accounts. Sovereign risk premia (spreads) are the difference between the average yield on 10-year bonds of EMU periphery countries and that of German bonds, taken from Bloomberg. We use the yield on the German 10-year bond (also from Bloomberg) as the empirical proxy for the model's riskless yield, $\tilde{r}_t$. Bond yields for the pre-EMU period are annual and are taken from the European Commission's macroeconomic database (AMECO).

All data are annual except bond yields and risk premia which are quarterly. We stop the sample for yields and risk premia in 2012:Q2 (included) in order to isolate our analysis from the effects of the annoucement by the European Central Bank of the Outright MonetaryTransactions (OMT) programme in the summer of 2012.