

Stagnation Traps

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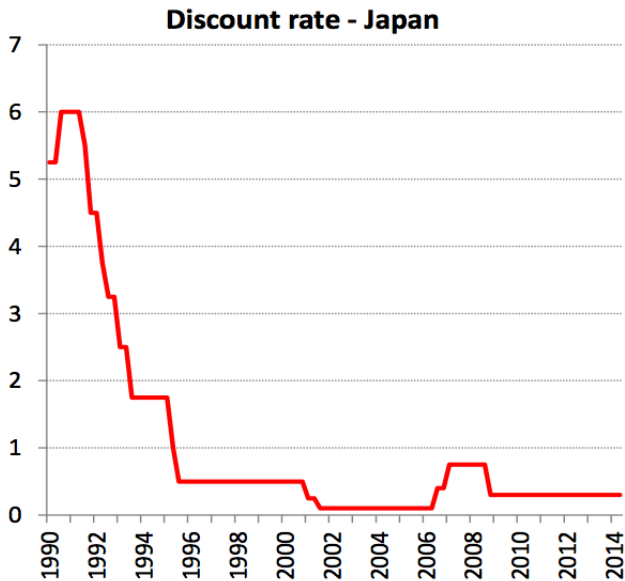
May 2015

RESEARCH QUESTION AND MOTIVATION

Can insufficient aggregate demand lead to economic stagnation?

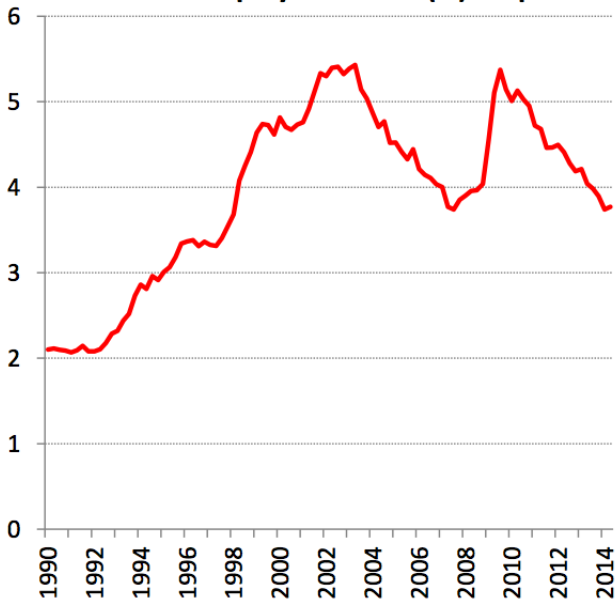
- This question goes back, at least, to the Great Depression
- Recently, renewed interest due to:
 - ▶ Two decades-long stagnation affecting Japan since early 1990s
 - ▶ Slow recoveries from 2008 financial crisis in US and Euro Area
- All these episodes featured policy rates close to the zero lower bound

DISCOUNT RATE - JAPAN (1990-2014)



UNEMPLOYMENT - JAPAN (1990-2014)

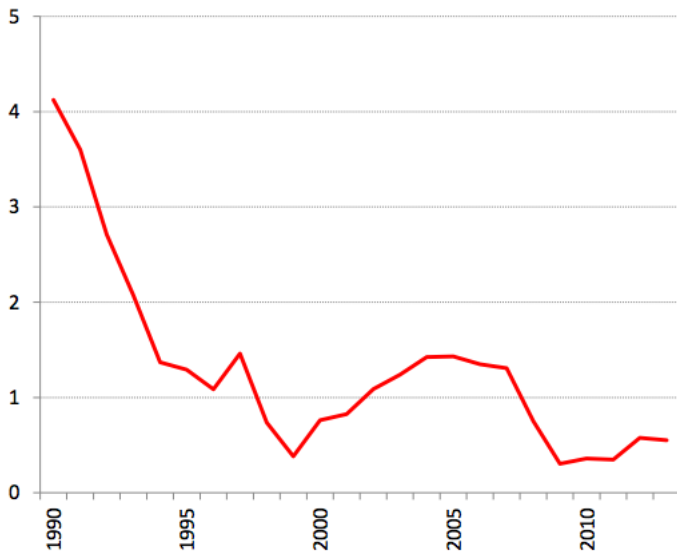
Unemployment rate (%) - Japan



POTENTIAL OUTPUT GROWTH - JAPAN (1990-2014)

Real Potential GDP Growth (%) - Japan

Source: WEO



THIS PAPER

- Keynesian Growth framework
 - ▶ Unemployment due to weak demand when monetary policy is constrained by the zero lower bound
 - ▶ Growth is the result of investment by profit-maximizing firms
- Two-way interaction between aggregate demand, interest rates and growth
 - ▶ Weak aggregate demand has a negative impact on firms' profits and investment, resulting in low growth
 - ▶ Low growth depresses interest rates, thus undermining the central bank's ability to sustain demand by cutting policy rates

OVERVIEW OF RESULTS

- **Key result:** Permanent, or very persistent, slumps characterized by high unemployment and low growth are possible
- Two steady states
 - ▶ Full employment, high growth, positive nominal rate
 - ▶ High unemployment, low growth, zero lower bound binds → **stagnation trap**
- Fluctuations determined by expectations and sunspots
- Policies that foster growth can eliminate the stagnation trap equilibrium if they are sufficiently aggressive

MODEL

Grossman and Helpman (1991) model of vertical innovation, augmented with nominal wage rigidities and zero lower bound on nominal interest rate

- Infinite-horizon closed economy
- Continuum of measure one of differentiated goods produced by monopolistic firms
- Continuum of measure one of households that supply labor and consume
- Central bank that sets monetary policy

HOUSEHOLDS

- Representative household with expected lifetime utility

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) \right]$$

$$C_t \equiv \exp \left(\int_0^1 \ln q_{jt} c_{jt} dj \right) \equiv \underbrace{Q_t}_{\exp(\int_0^1 \ln q_{jt} dj)} \exp \left(\int_0^1 \ln c_{jt} dj \right)$$

- One unit labor endowment, no labor disutility, but unemployment possible due to nominal wage rigidities
- Own firms, have access to nominal bonds paying interest rate i

$$\int_0^1 P_{jt} c_{jt} dj + \frac{b_{t+1}}{1+i_t} = W_t L_t + b_t + d_t$$

HOUSEHOLDS - OPTIMALITY CONDITIONS

- Households' optimization gives the Euler equation

$$c_t^\sigma = \frac{g_{t+1}^{\sigma-1}}{\beta(1+i_t)E_t [c_{t+1}^{-\sigma}/\pi_{t+1}]}$$

$$\text{where } g_{t+1} \equiv \frac{Q_{t+1}}{Q_t} \quad \text{and} \quad \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

- Focus on $\sigma > 1$: increase in growth ($\uparrow g_{t+1}$) generates rise in demand for consumption ($\uparrow c_t$)

FIRMS

- Every sector j is populated by

A leader producing quality q_j

Competitors producing quality $\frac{q_j}{\gamma}$ with $\gamma > 1$

- One unit of labor is needed to manufacture one unit of good \rightarrow marginal cost = W_t
- Optimal price set by leader

$$P_{jt} = \gamma W_t = P_t$$

- In equilibrium every leader faces demand y_t and earns profits

$$y_t (P_{jt} - W_t) = y_t W_t (\gamma - 1)$$

FIRMS AND PRODUCTION

- Output produced using labor $y_t = L_t$
 - ▶ $y_t = 1 \rightarrow$ full employment
 - ▶ $y_t < 1 \rightarrow$ unemployment and negative output gap
- Output can be consumed or invested in research

$$y_t = c_t + \iota_t$$

RESEARCH AND INNOVATION

- Outsiders can innovate on a product and capture monopoly profits by investing in research
- Value of a successful innovation

$$V_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\underbrace{y_{t+1} W_{t+1} (\gamma - 1)}_{\text{profits in } t+1} + \underbrace{(1 - \chi_{t+1}) V_{t+1}}_{\text{value of leadership in } t+1} \right) \right]$$

- Innovation probability

$$\chi^{\iota_t}$$

- Growth rate of the economy (productivity growth)

$$g_t \equiv \frac{Q_t}{Q_{t-1}} = \exp(\chi^{\iota_{t-1}} \ln \gamma)$$

NOMINAL WAGE RIGIDITIES AND MONETARY POLICY

- Nominal wages are rigid and follow

$$W_t = \psi(y_t) W_{t-1} \quad \text{with} \quad \psi' \geq 0, \psi(1) = \bar{\pi}$$

- Focus on special case of constant nominal wage inflation

$$W_t = \bar{\pi} W_{t-1}$$

- Prices are proportional to wages, so CPI inflation is constant and equal to $\bar{\pi}$
- Central bank follows the interest rate rule

$$1 + i_t = \max \left((1 + \bar{i}) y_t^\phi, 1 \right)$$

- Monetary policy constrained by zero lower bound $i \geq 0$

EQUILIBRIUM IN COMPACT FORM

■ Euler equation

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)E_t [c_{t+1}^{-\sigma}]} \quad (EU)$$

■ Growth equation

$$\frac{g_{t+1}^{\sigma-1}}{\beta} = E_t \left[\frac{c_t^\sigma}{c_{t+1}^\sigma} \left(\frac{\chi(\gamma-1)}{\gamma} y_{t+1} + 1 - \frac{\ln g_{t+2}}{\ln \gamma} \right) \right] \quad (GG)$$

■ Market clearing

$$c_t = y_t - \frac{\ln g_{t+1}}{\chi \ln \gamma} \quad (MK)$$

■ Policy rule

$$1 + i_t = \max \left((1 + \bar{i}) y_t^\phi, 1 \right) \quad (PR)$$

- Rational expectation equilibrium is a set of processes $\{y_t, c_t, g_{t+1}, i_t\}_{t=0}^{+\infty}$ satisfying equations (EU) , (GG) , (MK) and (PR)

NON-STOCHASTIC STEADY STATES

■ Aggregate demand

$$\max \left((1 + \bar{i}) y^\phi, 1 \right) = \frac{g^{\sigma-1} \bar{\pi}}{\beta} \quad (\text{AD})$$

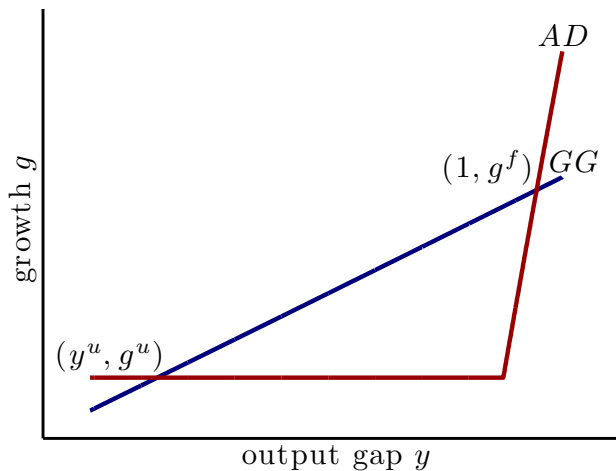
■ Growth equation

$$\frac{g^{\sigma-1}}{\beta} + \frac{\ln g}{\ln \gamma} = \chi \frac{\gamma - 1}{\gamma} y + 1 \quad (\text{GG})$$

■ Market clearing

$$c = y - \frac{\ln g}{\chi \ln \gamma} \quad (\text{MK})$$

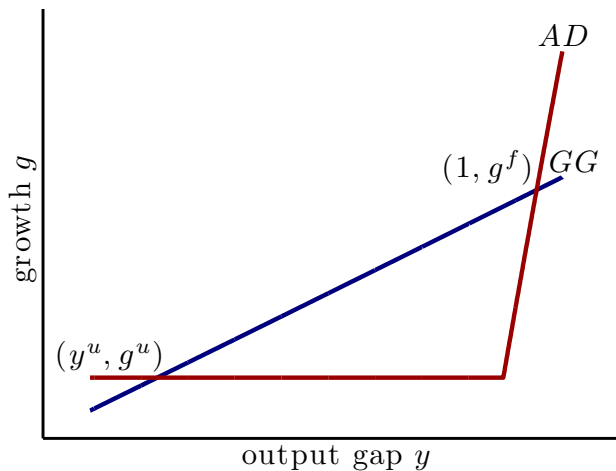
TWO STEADY STATES



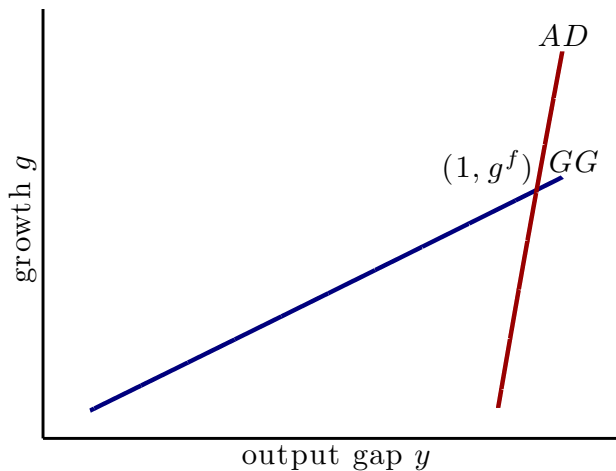
UNDERSTANDING STAGNATION TRAPS

- Aside from the usual full employment steady state, the economy can find itself in a permanent liquidity trap with:
 - ▶ Negative output gap ($y^u < 1$)
 - ▶ Weak growth ($g^u < g^f$)
 - ▶ Monetary policy constrained by zero lower bound ($i^u = 0$)
- The liquidity trap steady state can be seen as a **stagnation trap**, the combination of a liquidity and growth trap
- Pessimistic expectations can generate a stagnation trap
- The zero lower bound constraint and the dependence of growth from the current output gap are both crucial in generating a stagnation trap

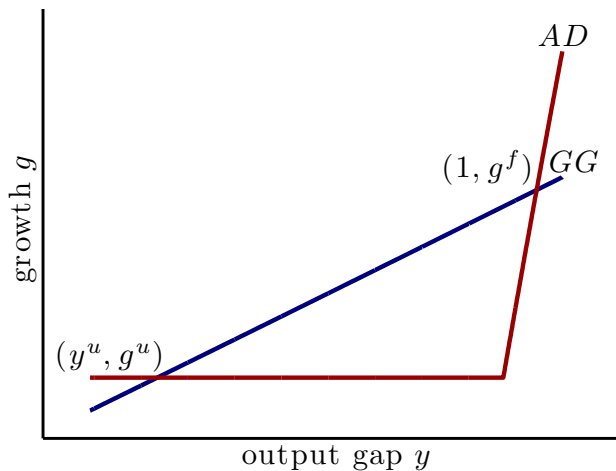
NO ZERO LOWER BOUND



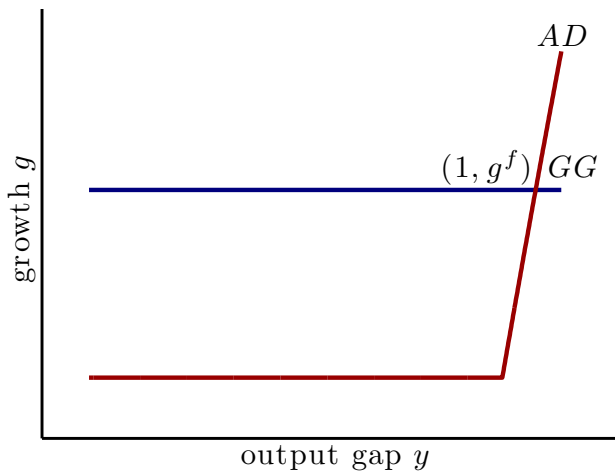
NO ZERO LOWER BOUND



NO DEPENDENCE OF GROWTH FROM OUTPUT GAP



NO DEPENDENCE OF GROWTH FROM OUTPUT GAP



SOME EXTENSIONS

1. Model can generate temporary liquidity traps arising from pessimistic expectations about future growth [link](#)
2. With precautionary savings, it is possible to have a liquidity trap steady state with positive inflation and positive growth [link](#)
3. Results robust to the introduction of a Phillips curve [link](#)

INTRODUCING A PHILLIPS CURVE

- Assume that nominal wages follow

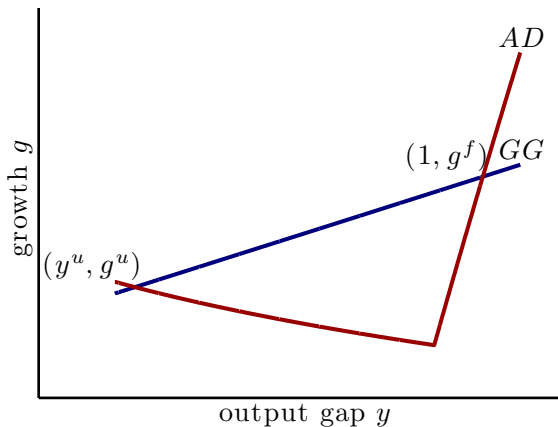
$$W_t = \psi(y_t) W_{t-1} \quad \text{with } \psi' > 0, \psi(1) = \bar{\pi}$$

- Wages more downwardly flexible if unemployment is higher
- Full employment steady state is not affected ($y = 1$, $g = g^f$, $i = i^f$ and $\pi = \bar{\pi} \equiv \pi^f$)
- Growth in the unemployment steady state is now

$$g^u = \left(\frac{\beta}{\psi(y^u)} \right)^{\frac{1}{\sigma-1}}$$

- \uparrow output gap , \uparrow inflation, \downarrow real interest rate, \downarrow growth

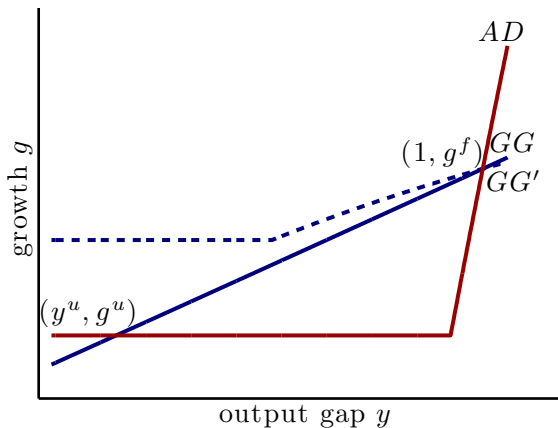
STEADY STATE DETERMINATION WITH VARIABLE INFLATION



GROWTH POLICIES DURING A STAGNATION TRAP

- Recent emphasis on job creating growth
- Indeed, an appropriately designed growth policy can eliminate liquidity traps driven by confidence shocks
- Consider a countercyclical subsidy $s_t = s(1 - y_t)$
- If s is sufficiently large, this policy rules out the liquidity trap steady state, while leaving unchanged the full employment steady state

COUNTERCYCLICAL SUBSIDY (CONT'D)



CONCLUSIONS

- We develop a **Keynesian growth** model in which endogenous growth interacts with the possibility of slumps driven by weak aggregate demand
- The model features two steady states. One is a **stagnation trap**, a permanent liquidity trap characterized by weak growth
- Large policy interventions to support growth can lead the economy out of a stagnation trap

THANK YOU!

SUNSPOTS AND TEMPORARY LIQUIDITY TRAPS

- We can also have liquidity traps of finite expected duration
- Denote a sunspot by ξ_t . Agents form their expectations after observing ξ
- Two-state discrete Markov process, $\xi_t \in (\xi_o, \xi_p)$
- ξ_o is an absorbing optimistic equilibrium, in which agents expect to remain forever around the full employment steady state
- ξ_p is a pessimistic equilibrium with finite expected duration $1/(1 - q_p)$. In this state the economy is in a liquidity trap with unemployment

SUNSPOTS AND TEMPORARY LIQUIDITY TRAPS (CONT'D)

- In the pessimistic sunspot state the equilibrium is described by

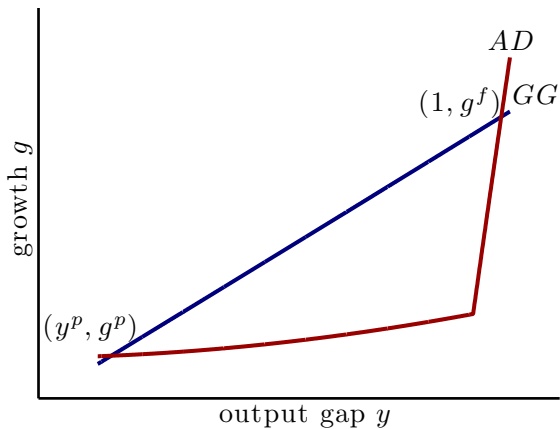
$$(g^p)^{\sigma-1} = \frac{\beta}{\bar{\pi}} \left(q_p + (1 - q_p) \left(\frac{c^p}{c^f} \right)^\sigma \right)$$

$$\frac{(g^p)^{\sigma-1}}{\beta} = q_p \left(\chi \frac{\gamma - 1}{\gamma} y^p + 1 - \frac{\ln g^p}{\ln \gamma} \right) +$$

$$+(1 - q_p) \left(\frac{c^p}{c^f} \right)^\sigma \left(\chi \frac{\gamma - 1}{\gamma} + 1 - \frac{\ln g^f}{\ln \gamma} \right)$$

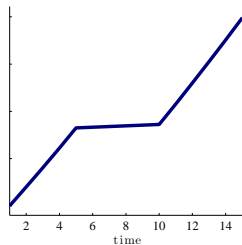
$$\frac{c^p}{c^f} = \frac{y^p - \frac{\ln g^p}{\chi \ln \gamma}}{1 - \frac{\ln g^f}{\chi \ln \gamma}}$$

SUNSPOTS AND TEMPORARY LIQUIDITY TRAPS (CONT'D)

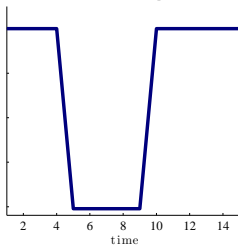


SUNSPOTS AND TEMPORARY LIQUIDITY TRAPS (CONT'D)

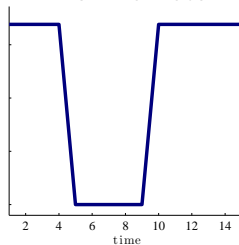
Potential output (log)



Output gap



Nominal rate



[back](#)

PRECAUTIONARY SAVINGS, INFLATION AND GROWTH

- In the benchmark model, positive inflation and positive growth cannot coexist in a permanent liquidity trap

$$g^u = \left(\frac{\beta}{\bar{\pi}} \right)^{\frac{1}{\sigma-1}}$$

- Assume that every period a household becomes unemployed with probability p
- An unemployed household receives a benefit, such that its income is equal to a fraction b of the income of employed households
- Unemployed households cannot borrow

PRECAUTIONARY SAVINGS, INFLATION AND GROWTH (CONT'D)

- Aggregate demand is given by the Euler equation of employed households

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t [c_{t+1}^{-\sigma}]}$$

$$\rho \equiv 1 - p + p/b^\sigma > 1$$

- The unemployment steady state is now characterized by

$$g^u = \left(\frac{\rho\beta}{\bar{\pi}} \right)^{\frac{1}{\sigma-1}}$$

- Since $\rho > 1$, an unemployment steady state in which both inflation and growth are positive is now possible

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- Assume that nominal wages are downwardly rigid

$$W_t \geq \psi(y_t) W_{t-1} \quad \text{with } \psi' > 0, \psi(1) = \bar{\pi}$$

- Wages more downwardly flexible if unemployment is higher
→ non-linear Phillips curve
- Full employment steady state is not affected ($y = 1$,
 $g = g^f$, $i = i^f$ and $\pi = \bar{\pi} \equiv \pi^f$)
- Growth in the unemployment steady state is now

$$g^u = \left(\frac{\beta}{\psi(y^u)} \right)^{\frac{1}{\sigma-1}}$$

- \uparrow output gap , \uparrow inflation, \downarrow real interest rate, \downarrow growth

STEADY STATE DETERMINATION WITH VARIABLE INFLATION

