Countercyclical Job Values

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Abstract

The paper studies the cyclical behavior of the job value, the expected present value of the marginal worker to the firm, estimated to be counter-cyclical in U.S. data. Job values are shown to be the dominant force behind hiring from non-employment. This finding implies that in recessions firms *increase* their hiring rates, from the pools of the unemployed and out of the labor force. The underlying reason is the dynamic behavior of the labor share of GDP, which engenders counter-cyclicality of the expected future profits of the firm.

This counter-cyclicality, which may appear counter-intuitive, is shown to be consistent with well-known business cycle facts, such as pro-cyclical employment, vacancy and job-finding rates and job to job flows. The analysis emphasizes the difference between current labor productivity and forward-looking job values. Changes in job values are also helpful in explaining the reduction in labor market fluidity in the U.S.

Key words: vacancy rate, hiring rate, job value, labor market frictions, cyclical behavior.
JEL codes: E24, E32

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1 Introduction

The paper asks what governs the representative firm recruiting behavior along the business cycle. This behavior is important for our understanding of business cycles and employment dynamics. In particular, I look at the optimality equation of the firm, which equates the marginal cost of worker recruitment and the expected present value of the worker for the firm, i.e., the job value. I empirically examine the issue by estimating alternative specifications of the equation. In doing so, I follow key formulations in the literature, particularly the ones related to search and matching models. Following estimation, I analyze the cyclical behavior of job values. I examine the components of job values, using an approximation and a variance decomposition of realized future values in a truncated sample.

The paper finds the following:

(i) Job values are counter-cyclical in U.S. data. This means that in recessions the value of jobs for firms goes *up*. Note that this value is a forward-looking expected present value of future labor profitability.

(ii) Correspondingly, hiring rates from non-employment (unemployment + out of the labor force) are counter-cyclical: it is worthwhile for firms to increase hiring rates as job values rise in recessions.

(iii) While the afore-mentioned hiring rates are counter-cyclical, vacancy rates and hiring rates from employment (i.e., job to job flows) are pro-cyclical. The differences between points (ii) and (iii) are explained.

(iv) Point (i) is consistent with the findings of recent studies looking at the cyclical behavior of the labor share in GDP. It is the dynamic behavior of the labor share that engenders the counter-cyclicality of the forward-looking job values.

(v) Points (i) and (ii), though counter-intuitive, do not contradict what we know about the cyclical features of the labor market, including pro-cyclical employment and job finding rates. Both points are shown to be consistent with the theoretical dynamics of q models of investment, which are analogous to the current framework.

(vi) The high volatility of vacancy and hiring rates is explained within the same framework. Part of the explanation has to do with job values and another part with the interaction with investment (in capital) behavior.

The paper is related to, but different from, previous work I had done on related issues – Merz and Yashiv (2007) and Yashiv (2014). The former paper employs a framework akin to one out of several specifications examined here and shows that it can account well for the market value of

[1] I am grateful to Avihai Lifschitz, Andrey Perlin and Ziv Usha for excellent research assistance. Any errors are my own.

U.S. firms. The idea is that the value of investment and the value of hiring make up the value of the firm. The latter paper uses the same framework to analyze the joint, forward-looking behavior of hiring and investment, examining their inter-relationships and the determinants of their present values. The current paper focuses on job values and their implications for recruiting behavior over the business cycle. It relates those to the behavior of the labor share over the cycle. It does so using an updated data set, examining alternative specifications, and undertaking decompositions of the determinants of job values and recruitment rates.

The paper proceeds as follows: Section 2 presents the vacancy creation or hiring equation as posited in 14 key papers in the relevant literature and discusses its main elements and the emerging alternative specifications. Section 3 presents the model and the key relations to be empirically examined. The data and methodology are elaborated in Section 4, followed by the presentation of the cyclical behavior of the key data series in Section 5. The results of the empirical work are presented in Section 6 and their implications are elaborated in Section 7, including the phase dynamics implied by the model. Section 8 studies the volatility and cyclicality of the series related to recruitment (vacancy and hiring rates) and relates them to job value behavior. Section 9 elaborates on the connections of the results to the dynamics of the labor share in GDP, recently discussed in other Macro contexts. In Section 10 I use this framework to explain the decline in U.S. labor market fluidity. Section 11 concludes. Derivations and other technical matters are relegated to appendices.

2 Models of Recruiting Behavior in the Literature

The vacancy creation or hiring equation expresses firms' optimal recruiting behavior in the presence of frictions. It relates the marginal costs of vacancies or of hiring which the firm faces with the marginal benefit, which is the expected present value of what the firm will get from the employment relationship. The following expresses this equation in very general terms:

$$MC_t(\cdot) = E_t PV_t(\cdot) \tag{1}$$

In what follows I show how different papers formulate costs (the LHS) and how they treat the present value and its components (the RHS). There are different specifications for the economic mechanisms underlying these components, for the functional form of costs, and for the relevant arguments to be used in the functions. I will derive job values from the estimation of equation (1).

Table 1 lists 14 key studies and reports what these studies have posited with respect to the LHS and the RHS of equation (1). Appendix A presents the equation as formulated by each study. The studies are classified into two groups – those positing linear costs and those positing convex costs.

Table 1

Beyond the differences between linear and convex costs, the table shows that the different permutations of formulating the equation include:

(i) Single job vs large firms.

(ii) Using vacancies or actual hires as arguments of the cost function.

(iii) Formulating labor only or capital and labor as determining productivity.

(iv) Wages determined by the Nash solution, intrafirm bargaining, credible bargaining or sticky wage mechanisms.

(v) Worker separations modeled as exogenous or endogenous, constant or stochastic.

(vi) Discounting the future with a constant or time-varying rate; for the latter, there are different formulations (IMRS, WACC or derived from the stock market).

As an additional point, there is the issue of taking into account general equilibrium effects. If taken into account, as in Gertler and Trigari (2009), Gali (2010), and Christiano et al (2013), these effects take place via discounting and the wage solution.

To fix ideas, note that, using the afore-cited studies, the following expression captures a standard formulation of the RHS of the above equation, the present value of the job to the firm:

$$PV_t = \rho_{t,t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})MC_{t+1} \end{array}\right] \tag{2}$$

where $f_{n_{t+1}}$ is the marginal product netted out of $g_{n_{t+1}}$ (change in costs due to a different level of employment), w_{t+1} is the wage, $\rho_{t,t+1}$ is the discount factor, ψ_{t+1} is the separation rate, and τ_{t+1} is the corporate tax rate. MC_{t+1} is whatever the model used places on the LHS of (1) at time $t+1$. Basically PV_t is the present value of the profit flows from the marginal worker $f_{n_{t+j}} - g_{n_{t+j}} - w_{t+j}$ for $j = 1...\infty$.

3 The Model

Following the discussion above, I formulate alternative versions of the firms' optimality equation. In order to do so, I present a model of firm optimization, which includes capital as well as labor, and formulate the costs function underlying the problem so that the cases shown in Table 1 will be special cases. The presentation follows Yashiv (2014).

3.1 The General Model

Set-Up. There are identical workers and identical firms, who live forever and have rational expectations. All variables are expressed in terms of output.

Worker Flows. Consider worker flows. The flow from non-employment – unemployment (U) and out of the labor force (O) – to employment is to be denoted $OE + UE$ and the separation flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE.

I shall denote:

$$\begin{aligned} \frac{h}{n} &= \left(\frac{h^1}{n}\right) + \left(\frac{h^2}{n}\right) \\ \frac{h^1}{n} &= \frac{OE+UE}{E} \\ \frac{h^2}{n} &= \frac{EE}{E} \end{aligned} \tag{3}$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively.

Separation rates are given in an analogous way by:

$$\begin{aligned} \psi &= \psi^1 + \psi^2 \\ \psi^1 &= \frac{EO+EU}{E} \\ \psi^2 &= \frac{EE}{E} = \frac{h^2}{n} \end{aligned} \tag{4}$$

Employment dynamics are thus given by:

$$\begin{aligned} n_{t+1} &= (1-\psi_t^1-\psi_t^2)n_t + h_t^1 + h_t^2 \\ &= (1-\psi_t)n_t + h_t, \quad 0 \leq \psi_t \leq 1 \\ h_t^2 &= \psi_t^2 n_t \end{aligned} \tag{5}$$

Matching and Separations.[2] Firms hire from non-employment (h_t^1) and from other firms (h_t^2). Each period, the worker's effective units of labor (normally 1 per person) depreciate to 0, in the current firm, with some exogenous probability ψ_t. Thus, the match suffers an irreversible idiosyncratic shock that makes it no longer viable. The worker may be reallocated to a new firm where his/her productivity is (temporarily) restored to 1.

[2] I am indebted to Giuseppe Moscarini for very useful suggestions to this section.

This happens with a probability of ψ_t^2. Those who are not reallocated join unemployment, with probability $\psi_t^1 = \psi_t - \psi_t^2$. So the fraction ψ_t^2 that enters job to job flows depends on the endogenous hiring flow h_t^2. The firm decides how many vacancies v_t to open and, given job filling rates (q_t^1, q_t^2), will get to hire from the pre-existing non-employed and from the pool of matches just gone sour. The matching rates satisfy:

$$\begin{aligned} q_t^1 &= \frac{h_t^1}{v_t} \\ q_t^2 &= \frac{h_t^2}{v_t} \\ q_t &= q_t^1 + q_t^2 \end{aligned} \tag{6}$$

Firms Optimization. Firms make gross investment (i_t) and vacancy (v_t) decisions. Once a new worker is hired, the firm pays him or her a per-period wage w_t. Firms use physical capital (k_t) and labor (n_t) as inputs in order to produce output goods y_t according to a constant-returns-to-scale production function f with productivity shock z_t:

$$y_t = f(z_t, n_t, k_t), \tag{7}$$

Gross hiring and gross investment are subject to frictions, spelled out below, and hence are costly activities. I represent these costs by a function $g[i_t, k_t, v_t, h_t, n_t]$ which is convex in the firm's decision variables and exhibits constant returns-to-scale, allowing hiring costs and investment costs to interact.

In every period t, the capital stock depreciates at the rate δ_t and is augmented by new investment i_t. Similarly, workers separate at the rate ψ_t^1 and the employment stock is augmented by new hires $q_t^1 v_t = h_t^1$. The laws of motion are:

$$k_{t+1} = (1-\delta_t)k_t + i_t, \quad 0 \le \delta_t \le 1. \tag{8}$$

$$n_{t+1} = (1-\psi_t)n_t + q_t v_t, \quad 0 \le \psi_t \le 1 \tag{9}$$

The representative firm chooses sequences of i_t and v_t in order to maximize its profits as follows:

$$\max_{\{i_{t+j}, v_{t+j}\}} E_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \rho_{t+i} \right) (1-\tau_{t+j}) \begin{pmatrix} f(z_{t+j}, n_{t+j}, k_{t+j}) - g\left(i_{t+j}, k_{t+j}, v_{t+j}, h_{t+j}, n_{t+j}\right) \\ -w_{t+j} n_{t+j} - \left(1 - \chi_{t+j} - \tau_{t+j} D_{t+j}\right) \widetilde{p}_{t+j}^I \, i_{t+j} \end{pmatrix} \tag{10}$$

subject to the constraints (8) and (9), and where τ_t is the corporate income tax rate, w_t is the wage, χ_t the investment tax credit, D_t the present discounted value of capital depreciation allowances, $\tilde{p}_t^I$ the real pre-tax price of investment goods, and ρ_{t+j} is a time-varying discount factor. The firm takes the paths of the variables $q_t^1, w_t, p_t^I, \delta_t, \psi_t^1$, τ_t and ρ_t as given. This is consistent with the standard models in the search and matching and Tobin's q literatures. The Lagrange multipliers associated with these two constraints are Q_{t+j}^K and Q_{t+j}^N, respectively. These Lagrange multipliers can be interpreted as marginal Q for physical capital, and marginal Q for employment, respectively. I shall use the term capital value or present value of investment for the former and job value or present value of hiring for the latter.

The first-order conditions for dynamic optimality are:[3]

$$Q_t^K = E_t \left[\rho_{t+1} \left[(1-\tau_{t+1}) \left(f_{k_{t+1}} - g_{k_{t+1}} \right) + (1-\delta_{t+1}) Q_{t+1}^K \right] \right] \tag{11}$$

$$Q_t^K = (1-\tau_t) \left(g_{i_t} + p_t^I \right) \tag{12}$$

$$Q_t^N = E_t \left[\rho_{t+1} \left[(1-\tau_{t+1}) \left(f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \right) + \left(1-\psi_{t+1} \right) Q_{t+1}^N \right] \right] \tag{13}$$

$$Q_t^N = (1-\tau_t) \frac{g_{v_t}}{q_t} \tag{14}$$

I can summarize the firm's first-order necessary conditions from equations (11)-(14) by the following two expressions:

$$(1-\tau_t) \left(g_{i_t} + p_t^I \right) = E_t \left[\rho_{t,t+1} (1-\tau_{t+1}) \left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array} \right] \right] \tag{15}$$

$$(1-\tau_t) \frac{g_{v_t}}{q_t} = E_t \left[\rho_{t,t+1} (1-\tau_{t+1}) \left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1}) \frac{g_{v_{t+1}}}{q_{t+1}} \end{array} \right] \right] \tag{16}$$

Equation (16) is at the focal point of the analysis and gives structure to equation (1) . Following the explicit formulation of the costs function g I shall consider alternative, specific cases.

The costs function g, capturing the different frictions in the hiring and investment processes, is at the focus of the estimation work. Specifically, hiring costs include costs of advertising, screening and testing, matching frictions, training costs and more. Investment involves implementation costs, financial premia on certain projects, capital installation costs, learning the use of new equipment, etc. Both activities may involve, in addition to production disruption, the implementation of new organizational structures within the firm and new production techniques.[4]In sum g is meant to

[3]where I use the real after-tax price of investment goods, given by:

$$p_{t+j}^I = \frac{1 - \chi_{t+j} - \tau_{t+j} D_{t+j}}{1 - \tau_{t+j}} \tilde{p}_{t+j}^I.$$

[4]See Alexopoulos (2011) and Alexopoulos and Tombe (2012).

capture all the frictions involved in getting workers to work and capital to operate in production, and not, say, just capital adjustment costs or vacancy costs. One should keep in mind that this is formulated as the costs function of the representative firm within a macroeconomic model, and not one of a single firm in a heterogenous firms micro set-up.

Functional Form. The parametric form I use is the following, generalized convex function.

$$g(\cdot) = \left[\begin{array}{c} \frac{e_1}{\eta_1}\left(\frac{i_t}{k_t}\right)^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}}+\frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{array} \right] f(z_t, n_t, k_t). \tag{17}$$

This function is linearly homogenous in its arguments $i, k, v, h, n.$ The parameters $e_l, l = 1, 2, 31, 32$ express scale, and the parameters $\eta_1, \eta_2, \eta_{31}, \eta_{32}$ express the convexity of the costs function with respect to its different arguments. λ_1 is the weight in the cost function assigned to hiring from non-employment $(\frac{h_t^1}{n_t})$, λ_2 is the weight assigned to hiring from other firms $(\frac{h_t^2}{n_t})$, and $(1-\lambda_1-\lambda_2)$ is the weight assigned to vacancy $(\frac{v_t}{n_t})$ costs. The weights λ_1 and λ_2 are thus related to the training and production disruption aspects, while the complementary weight is related to the vacancy creation and recruiting aspects. The last two terms in square brackets capture interactions between investment and hiring. I rationalize the use of this form in what follows.

Arguments of the function. This specification captures the idea that frictions or costs increase with the extent of the activity in question – vacancy creation, hiring and investment. This needs to be modelled relative to the size of the firm. The intuition is that hiring 10 workers, for example, means different levels of hiring activity for firms with 100 workers or for firms with 10,000 workers. Hence firm size, as measured by its physical capital stock or its level of employment, is taken into account and the costs function is increasing in the vacancy, hiring and investment rates, $\frac{v}{n}, \frac{h}{n}$ and $\frac{i}{k}$. The function used postulates that costs are proportional to output, i.e., the results can be stated in terms of lost output.

More specifically, the terms in the function presented above may be justified as follows (drawing on Garibaldi and Moen (2009)): suppose each worker i makes a recruiting and training effort h_i; as this is to be modelled as a convex function, it is optimal to spread out the efforts equally across workers so $h_i = \frac{h}{n}$; formulating the costs as a function of these efforts and putting them in terms of output per worker one gets $c\left(\frac{h}{n}\right)\frac{f}{n}$; as n workers do it then the aggregate cost function is given by $c\left(\frac{h}{n}\right)f$.

Convexity. I use a convex function. The use of such a function may be questioned at the micro-level, as non-convexities were found to be significant at that level (plant, establishment, or firm). But a number of recent papers have given empirical support to the use of a convex function in the aggregate, showing that such a formulation is appropriate at the macroeconomic level.[5]

*Interaction.*The terms $\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}}$and $\frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}}$express the interaction of investment and hiring costs. They allow for a different interaction for hires from non-employment (h_t^1) and from other firms (h_t^2). These terms, absent in many studies, have important implications for the complementarity of investment and hiring.

3.2 Alternative Specifications

Beyond looking at the general model spelled out above, I examine two, alternative, special cases.

3.2.1 The Standard Search and Matching Model

The standard search and matching model does not consider investment when formulating costs and refers to linear vacancy costs. In terms of the model above it has $e_1 = e_{31} = e_{32} = 0, \lambda_1 = \lambda_2 = 0$ and $\eta_2 = 1$. It thus formulates the optimality equation for vacancy creation (v) as follows, i.e., this is equation (16) for this particular model.

$$(1-\tau_t)\frac{e_2}{q_t}\frac{f_t}{n_t} = E_t\left[\rho_{t+1}(1-\tau_{t+1})\left[f_{n_{t+1}} - w_{t+1} + (1-\psi_{t+1})\frac{e_2}{q_{t+1}}\frac{f_{t+1}}{n_{t+1}}\right]\right] \tag{18}$$

As shown in the first group of studies in Table 1 above, and further discussed in Appendix A, this is a prevalent formulation, that has total costs be a linear function of vacancies, i.e., $\frac{e_2}{q_t}\frac{f_t}{n_t}v_t$ whereby the cost is proportional to labor productivity $\frac{f_t}{n_t}$ and depends on the average duration of the vacancy $\frac{1}{q_t}$ and q_t is the job filling rate, $q_t = \frac{h_t}{v_t}$.

3.2.2 The Optimality Equation: Hiring and Investment Costs Approach

As shown in the second group of studies in Table 1 above, there is a formulation of the hiring problem following the literature on investment models, mostly the seminal contributions of Lucas and Prescott (1971) and of Tobin (1969) and Hayashi (1982). Appendix B shows the derivation of the following optimality equations, elaborated in Merz and Yashiv (2007), which

[5]Thus, Thomas (2002) and Kahn and Thomas (2008, see in particular their discussion on pages 417-421) study a dynamic, stochastic, general equilibrium model with nonconvex capital adjustment costs. One key idea which emerges from their analysis is that there are smoothing effects that result from equilibrium price changes.

equate the marginal costs of investment g_{i_t} and of hiring g_{h_t} with the expected present value of the benefit. In this case $\lambda_1 = 1, \lambda_2 = 0$.

$$(1-\tau_t)\left(g_{i_t} + p^I_t\right) = E_t\left[\rho_{t,t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p^I_{t+1}) \end{array}\right]\right] \tag{19}$$

$$(1-\tau_t)\, g_{h_t} = E_t\left[\rho_{t,t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})g_{h_{t+1}} \end{array}\right]\right]. \tag{20}$$

The main difference with respect to the more general model is that this specification does not consider vacancy costs, in addition to hiring costs.

4 Methodology and Data

To be empirically evaluated, the afore-going optimality equations will be estimated. I discuss the data, the estimation methodology and a post-estimation approximation and a variance decomposition to be computed.

4.1 Data

The data are quarterly and pertain to the private sector of the U.S. economy. For a large part of the empirical work reported below the sample period is 1994-2013. The start date of 1994 is due to the lack of availability of job to job worker flows (h^2_t) data prior to that. For another part of the empirical work, the sample covers 1976-2013 and the 1976 start is due to the availability of credible monthly CPS data from which the gross hiring flows (h^1_t) series is derived. This longer sample period covers five NBER-dated recessions, including the Great Recession (2007-2009) and its aftermath (2009-2013). The data include NIPA data on GDP and its deflator, capital, investment, the price of investment goods and depreciation, BLS CPS data on employment and on worker flows, and Fed data computations on tax and depreciation allowances. Appendix C elaborates on the sources and on data construction. These data have the following distinctive features: (i) they pertain to the U.S. private sector; (ii) both hiring h and investment i refer to gross flows; likewise, separation of workers ψ and depreciation of capital δ are gross flows; (iii) the estimating equations take into account taxes and depreciation allowances. Table 2 presents key sample statistics.

Table 2

4.2 Estimation

I use the different specifications discussed above. For the production function I use a standard Cobb-Douglas formulation, with productivity shock

z_t:

$$f(z_t, n_t, k_t) = e^{z_t} n_t{}^{\alpha} k_t^{1-\alpha},\ 0 < \alpha < 1. \tag{21}$$

Estimation pertains to the parameters $\alpha; e_1, e_2, e_{31}, e_{32}; \eta_1, \eta_2, \eta_{31}, \eta_{32}, \lambda_1, \lambda_2$, or a sub-set of these parameters.

Estimation of the parameters in the production and costs functions allows for the quantification of the derivatives g_{i_t} and g_{v_t} that appear in the firms' optimality equations. I structurally estimate the firms' first-order conditions – equation (16) and the associated equation (15) – using Hansen's (1982) generalized method of moments (GMM). The moment conditions estimated are those obtained under rational expectations. I formulate the equations in stationary terms by dividing the investment equation by $\frac{f_t}{k_t}$ and the vacancy/hiring equation by $\frac{f_t}{n_t}$. Appendix B spells out the first derivatives included in these equations. Importantly, I check whether the estimated g function fulfills the convexity requirement.

4.3 Post Estimation Approximation and Variance Decomposition

Post estimation I compute an approximated present value, Q_t^N, on which a variance decomposition is computed. Iterating forward the RHS of (16) one gets:

$$PV_{t,T} = \sum_{j=1}^{T} \left[\begin{array}{c} \left(\prod_{l=1}^{j} \rho_{t+l-1,t+l} \frac{\frac{f_{t+l}}{n_{t+l}}}{\frac{f_{t+l-1}}{n_{t+l-1}}} \right) \left(\prod_{l=2}^{j} (1 - \psi_{t+l-1}) \right) (1 - \tau_{t+j}) \\ \left[\alpha - \frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}} - \frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}} \right] \end{array} \right] \tag{22}$$

Following Cochrane (1992), I use the following first-order Taylor expansion to get (see Appendices D and E for details[6]):

$$P_t \equiv (1 - \tau_t) \frac{g_{v_t}}{q_t} = E_t \left[\sum_{j=1}^{\infty} \exp\left[\sum_{l=1}^{j} g^r_{t+l} \right] \exp\left[\sum_{l=1}^{j} g^f_{t+l} \right] \exp\left[\sum_{m=l}^{j} g^s_{t+m-1} \right] MP_{t+j} \right] \tag{23}$$

where

$$MP_{t+j} \equiv (1 - \tau_{t+j}) \left(\alpha - \frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}} - \frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}}) \right) \tag{24}$$

[6]Note, though, that Cochrane (1992) does a second-order rather than a first-order Taylor expansion.

$$g_t^f = \ln\left(\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}\right) \tag{25}$$

$$g_t^s \equiv \ln(1-\psi_t) \tag{26}$$

$$g_t^r \equiv \ln\rho_{t,t+1} \equiv \ln\left(\frac{1}{1+r_t}\right) \tag{27}$$

$$w_t \equiv \left(g_t^f + g_t^s + g_t^r\right) \tag{28}$$

and

$$\Omega^f = e^{E(g_t^f)} \tag{29}$$

$$\Omega^s = e^{E(g_t^s)} \tag{30}$$

$$\Omega^r = e^{E(g_t^r)} \tag{31}$$

$$\Omega = e^{E(w)} = \Omega^f\Omega^s\Omega^r \tag{32}$$

This yields the variance decomposition:

$$\begin{aligned} var(P) \cong\ & \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, g_{t+j}^r) + \\ & \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, g_{t+j}^f) + \\ & \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=2}^{\infty}(\Omega)^{j-1}\,cov(P_t, g_{t+j}^s) + \\ & \Omega^r\Omega^f\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, MP_{t+j}) \end{aligned} \tag{33}$$

The first term relates to future discount rates (g_{t+j}^r), productivity growth (g_{t+j}^f), separation rates (g_{t+j}^s) and marginal profits (MP_{t+j}). I look at the relative size of the different terms on the RHS of equation (33) in order to gauge their relative importance.

5 The Cyclical Behavior of Vacancy and Hiring Rates

Before turning to the results of estimation, it is worthwhile to briefly examine the cyclical behavior of each of the data series themselves: hiring rates ($\frac{h_t^1}{n_t}$) from non-employment (unemployment + OLF); hiring rates ($\frac{h_t^2}{n_t}$) from employment (i.e., job to job flows); and vacancy rates ($\frac{v_t}{n_t}$). I consider each in turn.

5.1 Hiring from Non Employment

I compute $\rho(\frac{h_t^1}{n_t}, f_{t+i})$ where h_t^1 is the CPS gross hiring flow from the pool of unemployment plus out of the labor force and f_{t+i} is NFCB GDP (f), in logged, HP filtered terms (see Appendix C for data definitions and sources).

Table 3 and Figure 1

Hiring rates from non-employment are counter-cyclical.

5.2 Job to Job Flows

I repeat the same computation for job to job flows i.e., $\rho(\frac{h_t^2}{n_t}, f_{t+i})$ where h_t^2 is the CPS gross job to job flows, based on the work of Fallick and Fleischmann (2004), which was updated till 2013 (see Appendix C). The sample here starts in 1994.

Table 4 and Figure 2

Job to job flows, i.e., hiring rates from employment, are pro-cyclical.

5.3 Vacancy Rates

I repeat the same computation for vacancy rates i.e., $\rho(\frac{v_t}{n_t}, f_{t+i})$ where v_t is the adjusted HWI rate taken from Barnichon (2014), as delineated in the Appendix C.

Table 5 and Figure 3

Vacancy rates are pro-cyclical.

5.4 CPS vs JOLTS Hires Data

When using worker flows, a natural question that arises concerns the use of JOLTS data. These data are not used here, as they do not allow for the breakdown of hiring into h_t^1 and h_t^2 and are available only from December 2000. Moreover, there are big differences between CPS and JOLTS data as shown in the following table that pertains to total hires $h_t = h_t^1 + h_t^2$ in the overlapping sample period.

Table 6

The following conclusions emerge from the table: the CPS mean is 1.83 times higher that the JOLTS mean, the CPS median is 1.81 times higher; the c.o.v of CPS is 0.0587, about half of c.o.v for JOLTS at 0.10; the third moment is very different; only the fourth moment is close across the data samples.

5.5 Consistency with Well-Known Facts

The emerging picture from Figures 1-3 and Tables 3-5 is consistent with some well-known facts. The following explains.

In the steady state, hiring to employment h^1 equals separations from employment s:

$$h^1 = s \tag{34}$$

Non-employment in the steady state, i.e., unemployment u plus the pool out of the labor force o, satisfies:

$$\frac{u+o}{pop} = \frac{\psi}{\frac{h^1}{u+o} + \psi} \tag{35}$$

where pop is the working age population and ψ is the separation rate from employment n (i.e., $s = \psi\, n$).

In steady state the hiring rate is the product of the job finding rate, steady state non-employment and the inverse of the employment rate:

$$\frac{h^1}{n} = \frac{h^1}{u+o} \times \frac{u+o}{pop} \times \frac{pop}{n} \tag{36}$$

Using the above formulation of steady-state non-employment:

$$\underbrace{\frac{h^1}{n}}_{\text{hiring rate}} = \underbrace{\frac{h^1}{u+o}}_{\text{job finding}} \times \underbrace{\frac{\psi}{\frac{h}{u+o} + \psi}}_{\text{ss non-emp}} \times \underbrace{\frac{1}{\frac{n}{pop}}}_{\text{inv emp ratio}} \tag{37}$$

To give a sense of how these variables behave outside the steady state, given that it has often been found that actual dynamics are close to the latter, the following table shows the co-movement statistics for these variables.

Table 7

The employment stock n_t and the job finding rate $\frac{h_t^1}{u_t+o_t}$ are pro-cyclical, as is well known. The latter feature has been emphasized by Shimer (2012). Steady state non-employment $\frac{\psi}{\frac{h^1}{u+o}+\psi}$ and the inverse of the employment ratio $\frac{1}{\frac{n}{pop}}$ are counter-cyclical, as widely known too. At the same time the gross hiring rate $\frac{h_t^1}{n_t}$ is counter-cyclical, as shown above. Hence the hiring rate is counter-cyclical as the counter-cyclicality of the last two variables dominates the pro-cyclicality of the job-finding rate.[7]

[7] In this context the following quote from Shimer (2012, page 145) is pertinent: "Still, it is most important point to recognize the differential behavior of the job finding probability and the number of workers finding jobs;..."

Also note the following. Employment dynamics are given by:

$$\frac{n_{t+1} - n_t}{n_t} = \frac{h_t^1}{n_t} - \psi_t^1 \tag{38}$$

Along the cycle the variables in (38) can be shown as follows:

Figure 4

Evidently, in the shaded NBER-dated recessions net employment growth is negative with separations being higher than hires. At the same time in cyclical terms the following applies:

Figure 5

Both rates increase – relative to the HP trend – during recessions, i.e., both are counter-cyclical.

6 Results

I present GMM estimates of equation (16) under alternative specifications and subsequently the results of the variance decomposition defined in (33).

6.1 FOC Estimation

Table 8 reports the results of estimation. The table reports the estimates and their standard errors, Hansen's (1982) J-statistic and its p-value.

Table 8

Table 9 shows the size of the implied costs.

Table 9

I examine alternative specifications of the model discussed above. Following the results of Yashiv (2014), row a examines a quadratic function ($\eta_1 = \eta_2 = 2$) with linear interactions ($\eta_{31} = \eta_{32} = 1$). The weights on the different elements of the hiring process – vacancies, hiring from non-employment, and hiring from other employment – are expressed by the fixed parameters $\lambda_1 = 0.6, \lambda_2 = 0.2$, obtained after some experimentation. The parameters estimated are the scale parameters of the frictions function (e_1,e_2, e_{31} and e_{32}) and the labor share (α) of the production function (21). The J-statistic has a high p-value, the parameters are precisely estimated, and the resulting g function fulfills all convexity requirements; the estimate of α is around the conventional estimate of 0.66. Table 9 indicates very moderate costs estimates.

Row b looks at a quadratic specification, ignoring the other factor of production (here ignoring investment in capital), as often done in Tobin's q type of models. It thus sets $\eta_2 = 2, e_1 = e_{31} = e_{32} = 0$, i.e., has quadratic vacancy and hiring costs, with no role for capital. The results appear reasonable and there is no rejection of the model, but, this specification implies very high, unreasonable costs, as seen in Table 9.

Row c looks at the standard (Pissarides-type) search and matching model formulation with linear vacancy costs and no other arguments, as formulated in (18), such that $\eta_2 = 1, e_1 = e_{31} = e_{32} = \lambda_1 = \lambda_2 = 0$. The emerging estimates imply even higher costs (shown in Table 9) and the parameter α is estimated at a high value (0.77).

6.2 Post Estimation: Approximation and Decomposition

Table 10 reports the results of the variance decomposition defined by (33) following the approximation (23).

Table 10

Table 10 shows that the key determinant of job value volatility ($var(P)$) is the last term, i.e., the sum of the co-variances of job values with future marginal profits $\sum_{j=1}^{\infty} (\Omega)^{j-1} cov(P_t, MP_{t+j})$. Marginal profits MP_{t+j} are net marginal productivity less the wage, all in terms of the average product ($(1-\tau_{t+j})\left(\alpha - \frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}} - \frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}})\right)$). Given the small variability of τ_{t+j} and $\frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}}$, the main driver of volatility are the future labor shares $\frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}}$. All other terms play a very small role.

7 The Counter-Cyclicality of Job Values

Section 5 above presented the cyclical properties of the key data series. This section examines the cyclical properties of estimated job values in the different models in light of these data series.

7.1 Results and Implications for the Basic Equation

Table 11 reports the cyclical behavior of estimated job values, using the point estimates of the LHS of equation (16), i.e. of marginal hiring costs, as reported in Table 8.

Table 11 and Figure 6

The preferred specification indicates counter-cyclicality, the Tobin's q model is weakly pro-cyclical or a-cyclical, while the standard model is strongly pro-cyclical.

Getting back to equation (1) the implications of these results are that they imply two contradictory views of job values.

The standard search and matching (Pissarides-type) model in its simple form is as follows, re-writing equation (18):

$$(1-\tau_t)\frac{e_2}{q_t} = E_t\left[\rho_{t+1}(1-\tau_{t+1})\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}\left[\alpha - \frac{w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}} + (1-\psi_{t+1})\frac{e_2}{q_{t+1}}\right]\right] \tag{39}$$

This equation has a pro-cyclical MC_t on the LHS, as shown in Table 11 and Figure 6. This is to be expected as it depends inversely on the matching rate $q_t = \frac{h_t}{v_t}$, which itself is highly counter-cyclical. This means that job values, the RHS, are pro-cyclical too. The Lucas-Prescott/Tobin approach has a similar result, with marginal costs of hiring being weakly pro-cyclical, as seen in Table 11 and Figure 6.

The preferred specification implies the opposite. The results of Table 11 and Figure 6 indicate counter-cyclicality. Note that this is a broader model. It follows the Pissarides approach of using a vacancy creation equation but MC_t depends on all the relevant rates – $\frac{h_t^1}{n_t}, \frac{h_t^2}{n_t}$ and $\frac{v_t}{n_t}$. This model delivers counter-cyclicality on both sides of the equation, when the weight attached to the counter-cyclical $\frac{h_t^1}{n_t}$ is sufficiently high. In terms of equation (1) this is given by:

$$\underbrace{MC_t(\frac{h_t^1}{n_t}, \frac{h_t^2}{n_t}, \frac{v_t}{n_t})}_{\text{counter-cyclical}} = \underbrace{E_t PV_t(\cdot)}_{\text{counter-cyclical}}$$

and the counter-cyclical $\frac{h_t^1}{n_t}$(see Table 3 and Figure 1) dominates the pro-cyclical $\frac{h_t^2}{n_t}$ and $\frac{v_t}{n_t}$(see Tables 4 and 5 and Figures 2-3).

Which of the approaches is the right one? The analysis, which follows in this section and in the sections below, reinforces the support for counter-cyclical job values.

7.2 Consistency with the Logic of Q Model Dynamics

Consider the Q-theory model dynamics as depicted by the phase diagram of Figure 7. These dynamics are well-known, as can be seen, for example, in Figure 7.1 in Aceomoglu (2008, p. 273). The figure here shows Q_t^N on the vertical axis and the employment stock n on the horizontal axis.

Figure 7

This is derived as follows. Recall the F.O.C (ignoring taxes):

$$Q_t^N = E_t\left[\rho_{t+1}\left[f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} + \left(1-\psi_{t+1}\right)Q_{t+1}^N\right]\right] \tag{40}$$

$$Q_t^N = \frac{g_{v_t}}{q_t} \tag{41}$$

There are two dynamic equations underlying Figure 7.
One is the equation for the evolution of employment, given by:

$$\frac{n_{t+1}-n_t}{n_t} = -\psi_t + q_t\frac{v_t}{n_t}, \quad 0 \le \psi_t \le 1 \tag{42}$$

Using (41) and the preferred specification I get:

$$\frac{v_t}{n_t} = \frac{\frac{Q_t^N}{\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[\left(1-\lambda_1-\lambda_2\right)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} - \frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[\left(1-\lambda_1-\lambda_2\right)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} \tag{43}$$

Hence:

$$\begin{aligned}
\frac{n_{t+1}-n_t}{n_t} &= \frac{\frac{Q_t^N}{\frac{f_t}{n_t}}}{e_2\left[\left(1-\lambda_1-\lambda_2\right)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} - \frac{q_t\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[\left(1-\lambda_1-\lambda_2\right)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} - \psi_t \\
\frac{Q_t^N}{\frac{f_t}{n_t}} &= q_t\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t} + \psi_t e_2\left[\left(1-\lambda_1-\lambda_2\right)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2
\end{aligned} \tag{44}$$

This is the horizontal curve in Figure 7.
The other is the evolution of job values. Consider a re-write of equation (40):

$$Q_t^N = E_t\left[\rho_{t+1}\left[\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}}+w_{t+1}\right)\right) + \left(1-\psi_{t+1}\right)Q_{t+1}^N\right]\right]$$

Hence, under perfect foresight:

$$\begin{aligned}
Q_{t+1}^N - Q_t^N &= Q_{t+1}^N - \left[\rho_{t+1}\left[\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}}+w_{t+1}\right)\right) + \left(1-\psi_{t+1}\right)Q_{t+1}^N\right]\right] \\
&= Q_{t+1}^N\left(1-\rho_{t+1}\left(1-\psi_{t+1}\right)\right) - \left[\rho_{t+1}\left[\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}}+w_{t+1}\right)\right)\right]\right]
\end{aligned}$$

The stationary schedule is given by:

$$
\begin{aligned}
Q^N_{t+1} - Q^N_t &= 0 = Q^N_{t+1}\left(1-\rho_{t+1}\left(1-\psi_{t+1}\right)\right) - \left[\rho_{t+1}\left[\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}} + w_{t+1}\right)\right)\right]\right] \\
Q^N_{t+1} &= \frac{\rho_{t+1}}{\left(1-\rho_{t+1}\left(1-\psi_{t+1}\right)\right)}\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}} + w_{t+1}\right)\right)
\end{aligned}
$$

As it is reasonable to assume $\frac{\partial\left(\alpha\frac{f_{t+1}}{n_{t+1}} - \left(g_{n_{t+1}} + w_{t+1}\right)\right)}{\partial n} < 0$ this is a downward upward sloping curve in Figure 7.

This set-up engenders the saddle path dynamics depicted in Figure 7. Note that when the employment stock (n) is relatively low (lower than n^*), and thus output f is relatively low, the job value (Q^N) is relatively high. As the economy moves along the saddle path, n (and f) rise while Q^N falls. Hence there is a negative correlation between n and f, on the one hand, and Q^N on the other hand. The intuition is clear: when the employment stock is low there is a need for high hiring to bring it to steady state. The same works in the opposite direction for high values of n. Hence these well known theoretical dynamics imply counter-cyclical hiring rates and job values, as found above.

8 Cyclicality and Volatility of Recruitment Rates

In what follows I discuss the cyclicality and volatility of the key variables relating to recruitment, using the results above and relating to the concept of job values. In particular, I seek to explain the high volatility of these variables, widely discussed in the literature.

8.1 Data Moments

I first look at the data moments of the key variables: the hiring rate – both the total one $\frac{h_t}{n_t}$ and the rate from non-employment $\frac{h_t^1}{n_t}$, the vacancy rate $\frac{v_t}{n_t}$ and the job filling rates q_t^1 and q_t^2. I also look at the investment rate $\frac{i_t}{k_t}$. All variables are logged and HP-filtered.

The standard deviations are reported in Table 12a.

Table 12a

Two points can be noted:

(i) The vacancy rate and the job filling rates are much more volatile than the hiring rates. As $\frac{h_t}{n_t} = (q_t^1 + q_t^2)\frac{v_t}{n_t}$ this is the result of the negative comovement of $\frac{v_t}{n_t}$ and $(q_t^1 + q_t^2)$,reported below.

(ii) The vacancy rate is more volatile than the investment rate, which in turn is more volatile than the hiring rate.

Table 12b reports the co-movement of these variables with respect to NFCB GDP f and with respect to the job value $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$, as estimated in Table 8 row 1. First consider the second moments of these reference variables themselves.

Table 12b

Job values are much more volatile than NFCB GDP and are negatively correlated with it, i.e., are countercyclical.

The correlations of the key variables with these reference variables are given as follows.

Table 12c

These series are shown in Figure 8 with the lines indicating the start and end of NBER-dated recessions.

Figure 8

In terms of the business cycle, the well-known moments are the procyclicality of the investment rate and of the vacancy rate and the countercyclicality of job filling rates. Much less known is the weak cyclicality of hiring rates, with the rate of hiring from non-employment, even being counter-cyclical.

As to job values, they have positive co-movement with the flow from non-employment, as expressed by the hiring rate $\frac{h_t^1}{n_t}$ and the job filling rate q_t^1. They negatively co-move with the decision variables of the firm – vacancy and investment rates.

8.2 The Vacancy Rate

To explain the volatility and co-movement of the vacancy rate, I start off from the F.O.C:

$$(1-\tau_t)\frac{g_{v_t}}{q_t\frac{f_t}{n_t}} = \frac{Q_t^N}{\frac{f_t}{n_t}}$$

Using the preferred estimates of Table 8 row 1 I get:

$$\frac{1}{q_t^1+q_t^2}\left[\begin{array}{c} e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2\frac{v_t}{n_t} \\ +\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}\end{array}\right] = \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$$

The vacancy rate can then be expressed as follows:

$$\frac{v_t}{n_t} = \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} - \frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} \tag{45}$$

The vacancy rate is composed of two terms:

(i) The job value $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$, multiplied by a factor $\frac{\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}$, which is a non-linear function of the job filling rates q_t^1 and q_t^2 and model parameters $(e_2, \lambda_1, \lambda_2)$.

(ii) The investment rate $\frac{i_t}{k_t}$, multiplied by another factor $\frac{-\left(e_{31}q_t^1+e_{32}q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}$, which is a non-linear function of the job filling rates q_t^1 and q_t^2 and model parameters $(e_2, e_{31}, e_{32}, \lambda_1, \lambda_2)$.

Table 13 reports the variance decomposition which ensues:

$$\begin{aligned} var(\frac{v_t}{n_t}) = {} & var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right) \\ & + var\left(\frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right) \\ & - 2cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right) \end{aligned} \tag{46}$$

Table 13

The table implies that by far the biggest part of the variance of vacancy rates can be attributed to its second term, i.e., investment rates $\frac{i_t}{k_t}$ multiplied by the factor delineated above. This term becomes zero in the case of no interaction of hiring costs and investment costs ($e_{31} = e_{32} = 0$).

Table 14 shows correlations of the two terms making up $\frac{v_t}{n_t}$ with GDP (f_t) and with job values ($\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$), where all variables have been logged and HP-filtered:

Table 14

Vacancy rates are pro-cyclical (0.91) and negatively correlated (-0.56) with job values. The pro-cyclicality, as well as the negative correlation with job values, is very much engendered by the correlations of the second term with GDP and with job values. In contrast, the first term making up the vacancy rate, is weakly pro-cyclical and has a positive, rather than negative, correlation with job values.

Hence the second moments of the vacancy rate are dominated by the interaction of hiring and investment costs. The latter are a function of investment rates, which are volatile and pro-cyclical, as reported in Table 12 above.

8.3 The Hiring Rate

I now seek to explain the volatility and co-movement of the hiring rate. Note that the total hiring rate $\frac{h_t}{n_t}$ is given by:

$$\begin{aligned} \frac{h_t}{n_t} &= \left(q_t^1 + q_t^2\right) \frac{v_t}{n_t} = \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} \left(q_t^1 + q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2} \\ &\quad - \frac{\left(q_t^1 + q_t^2\right)\left(e_{31} q_t^1 + e_{32} q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2} \end{aligned} \tag{47}$$

I repeat the same computations for hiring rates. The variance of hiring rate is given by:

$$\begin{aligned} var(\frac{h_t}{n_t}) &= var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} \left(q_t^1 + q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2}\right) \\ &\quad + var\left(\frac{\left(q_t^1 + q_t^2\right)\left(e_{31} q_t^1 + e_{32} q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2}\right) \\ &\quad - 2cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} \left(q_t^1 + q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2}, \frac{\left(q_t^1 + q_t^2\right)\left(e_{31} q_t^1 + e_{32} q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2}\right) \end{aligned} \tag{48}$$

This yields the following decomposition in Table 15:

Table 15

The table again implies that by far the biggest part of the variance of hiring rates can be attributed to its second term, i.e., investment rates $\frac{i_t}{k_t}$

multiplied by a factor $\frac{\left(q_t^1+q_t^2\right)\left(e_{31}q_t^1+e_{32}q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}$ which is a non-linear function of the job filling rates q_t^1 and q_t^2 and model parameters $(e_2, e_{31}, e_{32}, \lambda_1, \lambda_2)$. This term becomes zero in the case of no interaction of hiring costs and investment costs ($e_{31} = e_{32} = 0$). Note, too, that the variance of the first term, job values $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$ multiplied by the factor $\frac{\left(q_t^1+q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}$, is only slightly higher than the variance of the hiring rate itself.

Table 16 shows the correlations of the two components of $\frac{h_t}{n_t}$ with GDP (f_t) and with job values $(\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}})$, where all variables have been logged and HP-filtered:

Table 16

Hiring is weakly related to GDP and to job values. Its two constituent terms offset each other, hence the weak correlations.

8.4 The Rate of Hiring from Non-Employment

Turning now to the hiring rate from non-employment, it is given by:

$$
\begin{aligned}
\frac{h_t^1}{n_t} &= q_t^1 \frac{v_t}{n_t} = \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2} \\
&\quad - \frac{q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}
\end{aligned} \tag{49}
$$

Thus:

$$
\begin{aligned}
var(\frac{h_t^1}{n_t}) &= var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right) \\
&\quad + var\left(\frac{q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right) \\
&\quad - 2cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)
\end{aligned} \tag{50}
$$

This yields the following decomposition in Table 17:

Table 17

Here the first term, which depends on the job value, plays the bigger role.

Table 18 shows the correlations of the two components of $\frac{h_t^1}{n_t}$ with GDP (f_t) and with job values ($\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$), where all variables have been logged and HP-filtered:

Table 18

The dominant role of the job value term is seen here by the correlation of the hiring rate ($\frac{h_t^1}{n_t}$) with it (0.78) and by the fact that the hiring rate is counter-cyclical, following the counter-cyclicality of job values.

8.5 The Standard Search and Matching Model.

Now consider the same analysis in terms of the standard search and matching model. The relevant equation is:

$$(1-\tau_t)\,\frac{g_{v_t}}{q_t\frac{f_t}{n_t}} = \frac{Q_t^N}{\frac{f_t}{n_t}}$$

which in this case is given by:

$$(1-\tau_t)\,\frac{e_2}{q_t} = \frac{Q_t^N}{\frac{f_t}{n_t}} \tag{51}$$

With a Cobb Douglas matching function I get:

$$\begin{aligned}
\frac{m_t}{n_t} &= \mu\left(\frac{v_t}{n_t}\right)^{\sigma}\left(\frac{u_t}{n_t}\right)^{1-\sigma} \\
q_t &= \frac{m_t}{v_t} = \frac{m_t}{n_t}\frac{n_t}{v_t} \\
&= \mu\left(\frac{v_t}{n_t}\right)^{\sigma-1}\left(\frac{u_t}{n_t}\right)^{1-\sigma}
\end{aligned}$$

So:

$$(1-\tau_t)\,\frac{e_2}{\mu\left(\frac{v_t}{n_t}\right)^{\sigma-1}\left(\frac{u_t}{n_t}\right)^{1-\sigma}} = \frac{Q_t^N}{\frac{f_t}{n_t}} \tag{52}$$

Solving out for the vacancy rate:

$$
\begin{aligned}
\mu\left(\frac{v_t}{n_t}\right)^{\sigma-1} &= (1-\tau_t)\frac{e_2}{\left(\frac{u_t}{n_t}\right)^{1-\sigma}\frac{Q_t^N}{\frac{f_t}{n_t}}} \\
\frac{v_t}{n_t} &= \left[\frac{e_2}{\mu}\frac{(1-\tau_t)}{\left(\frac{u_t}{n_t}\right)^{1-\sigma}\frac{Q_t^N}{\frac{f_t}{n_t}}}\right]^{\frac{1}{\sigma-1}}
\end{aligned}
$$

In log terms this is given by:

$$
\ln\frac{v_t}{n_t} = \frac{1}{\sigma-1}\left[\ln\frac{e_2}{\mu_t} - \ln\left(\frac{u_t}{n_t}\right)^{1-\sigma} - \ln\left(\frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right)\right]
$$

The variance decomposition is thus given by:

$$
\begin{aligned}
var\left(\ln\frac{v_t}{n_t}\right) &= \left(\frac{1}{\sigma-1}\right)^2 var\left[\ln\frac{e_2}{\mu_t}\right] \\
&+\left(\frac{1}{\sigma-1}\right)^2 var\left[\ln\left(\frac{u_t}{n_t}\right)^{1-\sigma}\right] \\
&+\left(\frac{1}{\sigma-1}\right)^2 var\left[\ln\frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right] \\
&-2\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln\frac{e_2}{\mu_t},\ln\left(\frac{u_t}{n_t}\right)\right] \\
&-2\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln\frac{e_2}{\mu_t},\ln\frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right] \\
&+2\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln\left(\frac{u_t}{n_t}\right)^{1-\sigma},\ln\frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right]
\end{aligned} \tag{53}
$$

Table 19 reports.

Table 19

The dominant term is the job value term, but all terms are sizeable except for the co-variance between job values and the matching technology.

Table 20 shows the correlations of the components of $\frac{v_t}{n_t}$ with GDP (f_t) and with job values ($\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$), where all variables have been logged and HP-filtered.

Table 20

Vacancy rates are pro-cyclical and also positively correlated with job values, which are pro-cyclical in this model.

8.6 Summing Up

The key series pertaining to recruiting display differential behavior.

(i) For both the vacancy rate $\frac{v_t}{n_t}$ and the hiring rate $\frac{h_t}{n_t}$ the following is found:

a. There are two determinants: the job value $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$ multiplied by a factor and the investment rate $\frac{i_t}{k_t}$ multiplied by a different factor, with each factor being a function of the job filling rates and model parameters. These factors are functions of market conditions.

b. Much of the variance comes from the term which depends on the investment rate, hence the interaction of hiring costs and investment costs is key.

c. Both are pro-cyclical due to the fact that the high pro-cyclicality of the investment rate term dominates the counter-cyclicality or the weak pro-cyclicality of the job value term.

(ii) The hiring rate from non-employment $\frac{h_t^1}{n_t}$ behaves differently. It follows the behavior of job values, as the $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$ term dominates.

(iii) In the standard search model the job value term dominates as well, and the vacancy rate is tightly linked to it. In this model the job value is pro-cyclical.

9 The Role of the Labor Share

The labor share in GDP plays a key role in the results here. It has also been the focus of some attention in recent macroeconomic models of the business cycle.

The main reason for the key result of this paper is the cyclical behavior of the labor share. The variance decomposition of the approximated PV_t reported in Table 9 reveals that the main determinant of the present value is the labor share; this is so because I am looking at the following expression (noting that all variables are computed relative to $\frac{f_{t+1}}{n_{t+1}}$) :

$$\frac{f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}} = \alpha - \left(\frac{g_{n_{t+1}} + w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}} \right)$$

As α is constant and the term $\frac{-g_{n_{t+1}}}{\frac{f_{t+1}}{n_{t+1}}}$ is small, the main driver is the labor share, $\frac{w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}}$.

Consider the cyclicality of the labor share:

Table 21

Noting the bolded numbers in the table, *dynamically,* the labor share is pro-cyclical**.** Hence job values are counter-cyclical.

This cyclical behavior has recently been noted by a number of authors. The observation, whereby the labor share first falls in a boom and subsequently rises for a substantial period of time, i.e., is dynamically pro-cyclical, was discussed by Rios-Rull and Santaeulalia-Llopis (2010). Hall (2014b) finds that the labor share is a-cyclical contemporaneously and pro-cyclical subsequently.

Ramey and Nekarda (2013) examine the cyclicality of mark-ups. Essentially they treat the mark-up as inversely related to the labor share (see their equation 5), allowing various modifications to the relationship, such as overhead hours, CES production functions, and differentials between marginal and average wages. Studying both aggregate and four-digit manufacturing data of the U.S. economy, they find that mark-ups are *contemporaneously* pro-cyclical and that *dynamically* they are counter-cyclical. The latter finding means that if GDP is low now (recession), mark-ups will rise henceforth (see their Figure 2). This is similar to the finding here that job values are counter-cyclical, i.e., that the *present value* of profits rises in recessions. It is so for the same reason, namely that the future labor share declines (dynamically the labor share is pro-cyclical).

Ramey and Nekarda (2013) also make the point that their findings contradict the conventional wisdom of New Keynesian models. The latter posit that demand shocks (monetary policy or government spending shocks) increase marginal costs while prices are sticky. Thereby positive shocks in these models lower the mark-ups,i.e., engender counter-cyclical mark-ups, contemporaneously, in contradiction to the Ramey and Nekarda (2013) empirical findings.

10 Explaining the Decline in Labor Market Fluidity

Recently, Davis and Haltiwanger (2014) have documented a decline over time in U.S. labor market fluidity. They provide detailed evidence, in terms of both worker flows and job flows (see, in particular, their Figures 1-10). In terms of the variables in the current analysis this is manifested in the decline of $\frac{h^1}{n}$, $\frac{h^2}{n}$, ψ^1, ψ^2 and $\frac{v}{n}$, which can be seen in Figure9 for the full sample period (noting that job to job flows are measured from 1994 only).

Figure 9

The afore-going analysis can account for these facts. Consider the following equation derived from (14):

$$\frac{Q_t^N}{\frac{f_t}{n_t}} = (1-\tau_t)\,\frac{g_{v_t}}{q_t\frac{f_t}{n_t}} \tag{54}$$

Table 22 and Figure 10 show the LHS and the RHS of equation (54) separately for two sub-periods. It is hard to pinpoint one particular year as the dividing line, so somewhat arbitrarily the following sub-periods were examined: 1976-1995 and 1996-2013. In order to cater for the longest sample period possible, the figure uses the preferred estimates of row 2 in Table 8 for the parameter values and for the average value of the LHS in each sub-sample. It also uses the data averages in the sub-samples for the variables τ_t, q_t^1 and $\frac{i_t}{k_t}$. Note that under this specification, which omits job to job flows, the RHS of (54) is given by, omitting time sub-scripts to denote averages:

$$(1-\tau)\,\frac{g_v}{q\frac{f}{n}} = (1-\tau)\left(e_2\frac{qv}{n} + e_{31}\frac{i}{k}\right) \tag{55}$$

Table 22 and Figure 10

The figure shows that job values ($\frac{Q^N}{\frac{f}{n}}$) – depicted as the horizontal lines – went up, going from the pre-1995 period to the post-1995 period. This rise is consistent with the decline in separation rates (ψ) and in tax rates (τ), but could also have been engendered by other changes. In itself it should have led to *higher* vacancy rates. But the upward sloping curve, expressing marginal costs ($(1-\tau)\,\frac{g_v}{q\frac{f}{n}}$), also went up. The final outcome (shown in the intersection of the dashed lines) was that vacancy rates declined, as the rise in marginal costs dominated the rise in job values. The reason for the rise in marginal costs is the rise in the job filling rate (q) . It should be noted, too, that marginal costs went up also because taxes (τ) declined, but the latter change also contributed to the rise in $\frac{Q^N}{\frac{f}{n}}$. Thus, the term $(1-\tau)$ on both sides of (54) cancels out. The change in q dominated the offsetting effects of a rise in the investment rate ($\frac{i}{k}$). Hence the decline in recruiting activity took place as the result of the rise in the firm job filling rate, which led to higher costs. The rise in costs was bigger than the rise in the expected present value of profitability from workers, both at the margin.

These developments are echoed by Davis and Haltiwanger (2014); on page 12 of their paper they discuss higher training costs as an explanatory variable. Here a higher value of q, which the firm takes as given, leads to a higher rate of hiring and, thus, to higher costs. One needs to note, though, the process depicted here: firms see, and take as exogenous, a rise in job filling rates q as well as in job values $\frac{Q^N}{\frac{f}{n}}$; the former leads them to open vacancies at a lower rate (for a given job value), while the latter leads them to increase the vacancy rate. With the former dominating the latter, across the two sub-periods, vacancy rates declined. As q rose less than vacancy rates $\frac{v}{n}$ went down, the resulting hiring rates $\frac{h}{n}$ were lower, as seen in Figure 9 above.

11 Conclusions

The paper depicted the following picture: using a model of recruiting, whereby costs are a function of vacancy rates and hiring rates, one can fit the data with a moderate costs formulation. The standard matching model or a standard Q type approach are unable to match the data, and produce high costs estimates. The implied job values of the preferred specification are counter-cyclical. This is so because of the dynamic behavior of the labor share, which is the dominant element in driving job value. Hence, somewhat counter-intuitively, hiring from non-employment is counter-cyclical. All the while hiring from employment, job finding rates and employment are all pro-cyclical. These patterns are consistent with phase dynamics of a Q model.

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12 Tables and Figures

Table 1
Alternative Formulations of The Recruiting Equation (1)

Linear Costs Models

	paper	firm size	LHS, costs, arguments
1	Pissarides (2000, chapter 1)	single job	v, p
2	Pissarides (2000, chapter 2)	single job	v
3	Shimer (2005)	single job	v
4	Hall (2005)	single job?	v
5	Mortensen and Nagypal (2007)	single job	v
6	Hall and Milgrom (2008)	large	v
7	Hagedorn and Manovskii (2008)	single job	$v : c^K p + c^w p^\xi$
8	Elsby and Michaels (2013)	large	v
9	Christiano et al (2013)	large	$\frac{h}{n}$
10	Hall (2014a)	single job	v

	RHS, job value			
	f (production)	**w** (wages)	**s** (separation)	ρ (discounting)
1	exo, stoch	Nash	exo, constant	constant
2	exo, stoch	Nash	endo, stoch	constant
3	exo, stoch	Nash	exo, stoch	constant
4	exo, stoch	sticky	exo, constant	constant
5	exo, stoch	Nash /rigid/Calvo	exo, constant	constant
6	exo, stoch	alternating offers	exo, constant	constant
7	exo, stoch	Nash	exo, constant	constant
8	$pxF(n)$	Intrafirm bargaining	endo, stochc	constant
9	$h_t = l_t$;	alternating offers	exo, constant	GE, IMRS
10	exo, stoch	Nash/alternating offers	exo, constant	from stock market

exo=exogenous
endo=endogenous
stoch= stochastic

Convex Costs Models

	paper	size	LHS, costs, arguments	
			arguments	**function**
1	Merz and Yashiv (2007)	large	$\frac{h}{n}, \frac{i}{k}, f$	linear-convex
2	Gertler and Trigari (2009)	large	$\frac{h}{n}$	quadratic
3	Gali (2010)	large	$\frac{h}{u}$	power
4	Acemoglu and Hawkins (2014)	large	v	convex

	RHS, job value			
	f (production)	**w** (wages)	**s** (separation)	ρ (discounting)
1	$e^{z_t} n_t^{\alpha} k_t^{1-\alpha}$	exo, stoch	exog, stoch	WACC
2	$z_t n_t^{\alpha} k_t^{1-\alpha}$	Nash, Calvo	exo, constant	GE, IMRS
3	$A_t N_t^{1-\alpha}$	Nash, Calvo	exo, constant	GE, IMRS
4	$y(n; z)$	Intrafirm bargaining	exo, constant	constant

exo=exogenous
stoch= stochastic

Table 2

Descriptive Sample Statistics
Quarterly, U.S. data

a. 1976:1-2013:4 ($n = 152$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h^1}{n}$	$\frac{v}{n}$	ψ^1	β
Mean	0.14	0.38	0.024	0.02	0.62	0.126	0.031	0.125	0.99
Standard Deviation	0.01	0.05	0.003	0.003	0.02	0.010	0.007	0.010	0.005

b. 1994:1-2013:4 ($n = 80$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h}{n} = \frac{h^1+h^2}{n}$	$\frac{v}{n}$	$\psi = \psi^1 + \psi^2$
Mean	0.15	0.34	0.026	0.02	0.61	0.178	0.028	0.178
Standard Deviation	0.01	0.005	0.002	0.002	0.03	0.012	0.005	0.012

	β
Mean	0.99
Standard Deviation	0.005

Table 3
Hiring Rates from Non Employment and GDP

				$\rho(\frac{h_t^1}{n_t}, f_{t+i})$			
i	-8	-4	-1	0	1	4	8
	-0.15	-0.35	-0.39	-0.30	-0.15	0.11	0.23

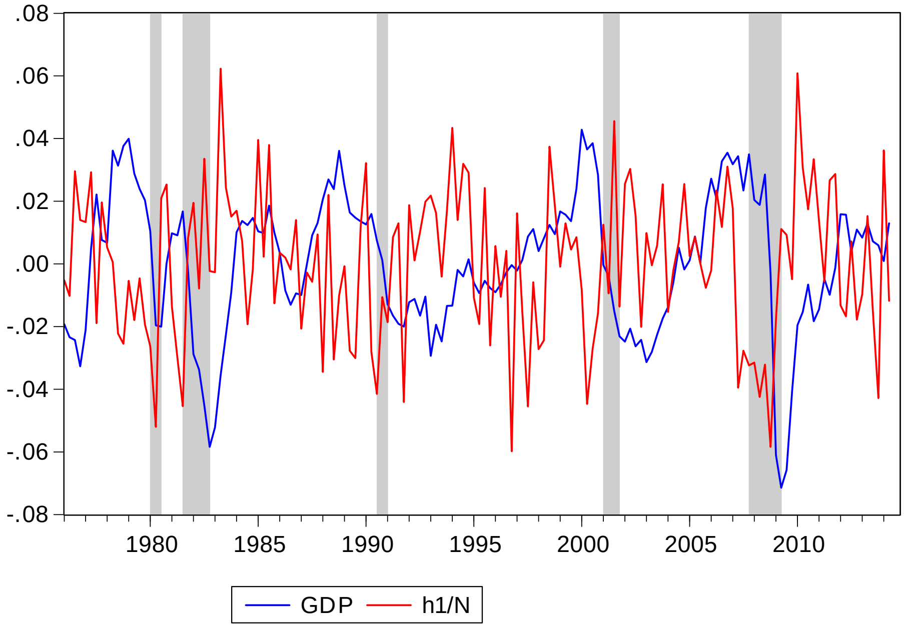

Figure 1: Cyclicality of $\frac{h_t^1}{n_t}$

Table 4
Hiring Rates from Employment (Job to Job flows) and GDP

	$\rho(\frac{h_t^2}{n_t}, f_{t+i})$						
i	-8	-4	-1	0	1	4	8
	-0.13	0.26	0.64	0.68	0.66	0.28	-0.05

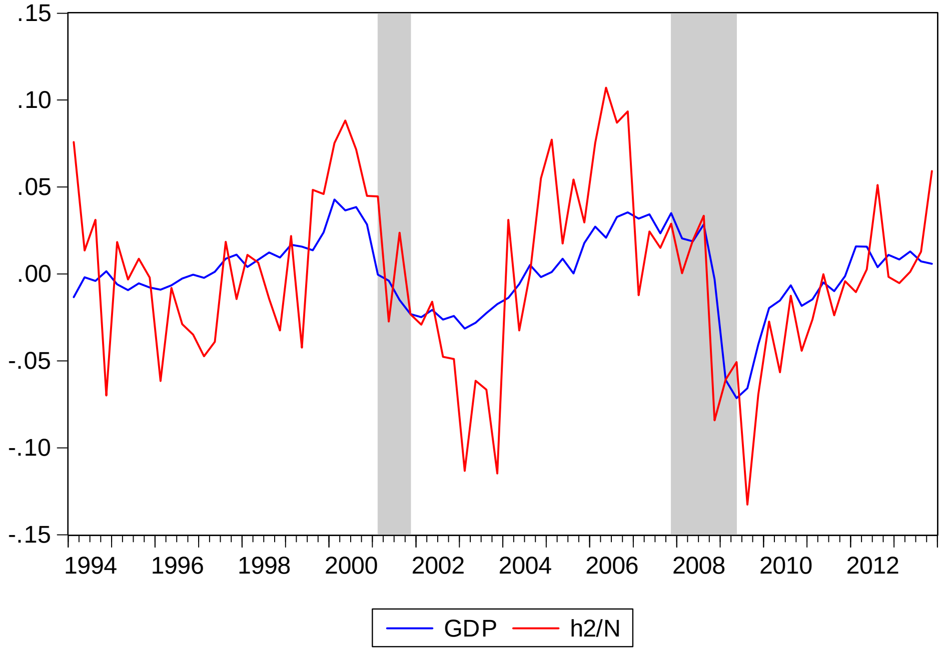

Figure 2: Cyclicality of $\frac{h_t^2}{n_t}$

Table 5
Vacancy Rates and GDP

				$\rho(\frac{v_t}{n_t}, f_{t+i})$			
i	-8	-4	-1	0	1	4	8
	-0.26	0.29	0.82	0.89	0.83	0.39	-0.13

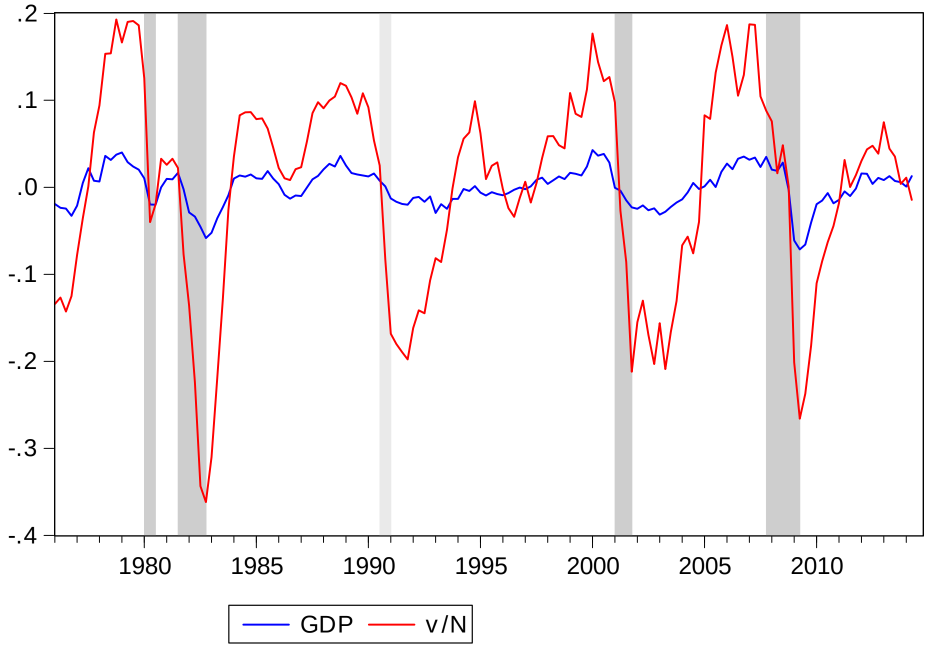

Figure 3: Cyclicality of $\frac{v_t}{n_t}$

Table 6
Total Hiring Flows (NSA, 000s)
Sample: $2001:1-2014:06$

	CPS	**JOLTS**
Mean	8595	4698
Median	8609	4765
Std. Dev.	496	484
C.O.V	0.06	0.10
Skewness	0.25	−0.25
Kurtosis	2.42	2.14

Table 7
Stochastic Behavior of the Gross Hiring Rate
and Other Labor Market Variables

Co-Movement (contemporaneous correlation) with GDP

logged, HP filtered

n_t	$\frac{h_t^1}{n_t}$	$\frac{h_t^1}{u_t+o_t}$	$\frac{\psi}{\frac{h_t^1}{u_t+o_t}+\psi}$	$\frac{1}{\frac{n_t}{POP_t}}$
0.81	−0.25	0.53	−0.39	−0.82

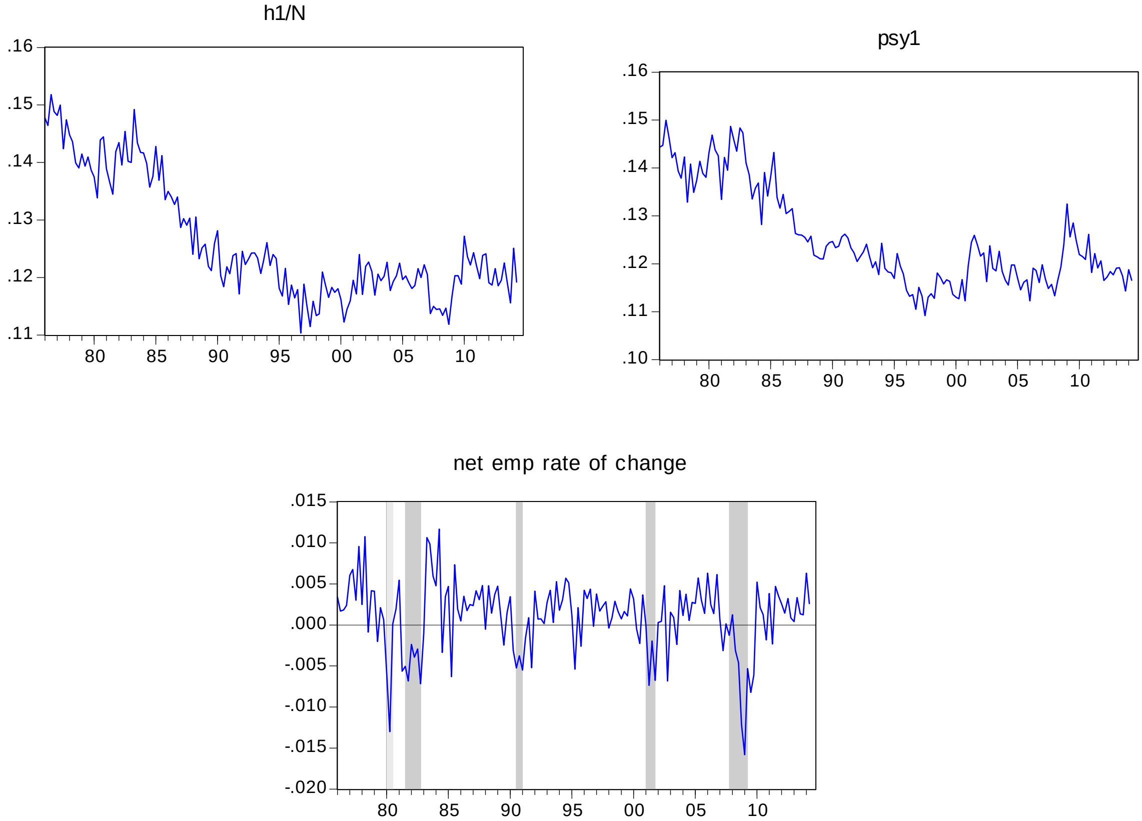

Figure 4: $\frac{h_t^1}{n_t}, \psi_t^1, \frac{n_{t+1}-n_t}{n_t}$

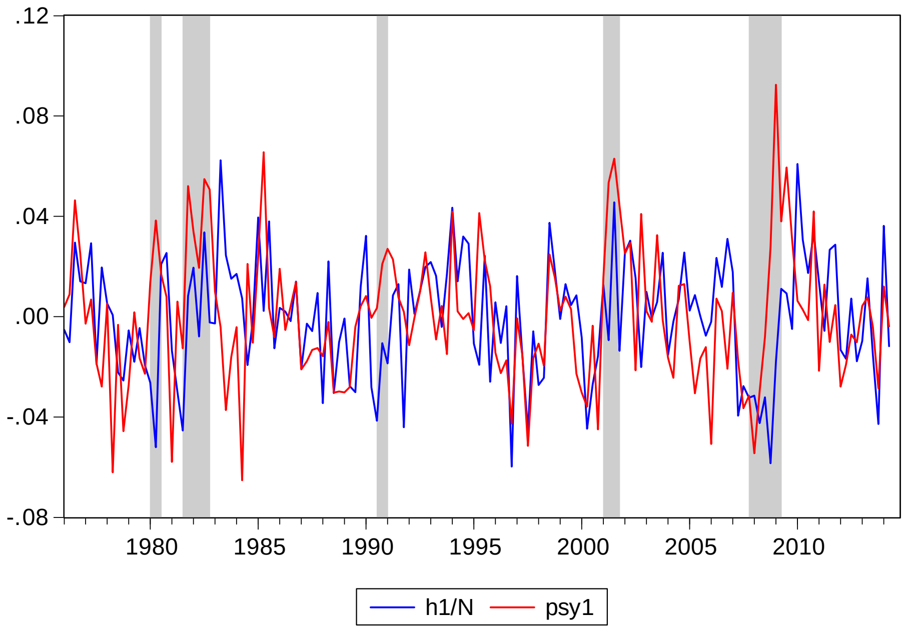

Figure 5: $\frac{h_t^1}{n_t}, \psi_t$ **logged, HP filtered**

Table 8
GMM Estimates

	specification	e_1	e_2	e_{31}	e_{32}	α	J-Statistic
a	benchmark	77.3	9.1	-2.8	-19.6	0.66	51.6
		(6.29)	(0.98)	(1.2)	(0.9)	(0.003)	(0.74)
b	Tobin's q for N	0	30.8	0	0	0.70	61.9
		–	(0.9)	–	–	(0.003)	(0.48)
c	Standard matching model	0	9.3	0	0	0.77	62.5
	$\eta_2 = 1, \lambda_1 = \lambda_2 = 0$	–	(0.1)	–	–	(0.002)	(0.46)

Notes:

1. The table reports point estimates with standard errors in parentheses. The J-statistic is reported with p value in parentheses.

2. The following parameter values are set unless indicated otherwise: $\lambda_1 = 0.6; \lambda_2 = 0.2; \eta_1 = \eta_2 = 2, \eta_{31} = \eta_{32} = 1.$

3. The instrument set for both equations is $\left\{ \frac{w_t}{\frac{f_t}{n_t}}, \frac{i_t}{k_t}, p_t^I \right\}$; for the investment equation also $\frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}$ is used;and for the vacancies equation also $\frac{h_t}{n_t}$ is used; all with lags 1 to 6, 8 and 10.

4. The sample period is 1994:1 – 2013:4.

Table 9
Estimated Marginal Costs –Moments
$1994:1-2013:4$

	benchmark	**Tobin's Q for N**	**Standard matching model**
mean	0.12	0.90	0.97
median	0.12	0.89	1.00
std.	0.03	0.03	0.13
auto-correlation	0.91	0.55	0.92
skewness	-0.07	0.0007	-0.37
kurtosis	2.37	2.49	1.97

Notes:
1. The series in the table are the LHS of the estimated equation namely $(1-\tau_t)\frac{g_{v_t}}{q_t}$.

Table 10
Variance Decomposition ($T = 30$)

	1	2	3
	benchmark	Tobin's q for N	Standard matching model
$var(PV_{t,T})$	0.01	0.008	0.01
$\frac{\frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{T}(\Omega)^{j-1}cov(P_t,g^r_{t+j})}{var(PV_{t,T})}$	0.04	0.09	0.07
$\frac{\frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{T}(\Omega)^{j-1}cov(P_t,g^f_{t+j})}{var(PV_{t,T})}$	-0.02	0.04	0.03
$\frac{\frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=2}^{T}(\Omega)^{j-1}cov(P_t,g^s_{t+j})}{var(PV_{t,T})}$	-0.007	0.06	0.06
$\frac{\Omega^r\Omega^f\sum_{j=1}^{T}(\Omega)^{j-1}cov(P_t,MP_{t+j})}{var(PV_{t,T})}$	0.74	0.78	0.62
residual	0.24	0.02	0.21

Notes:
1. See Appendix E. The basic equation is:

$$
\begin{aligned}
var(P) \quad \cong \quad & \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, g^r_{t+j}) + \\
& \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, g^f_{t+j}) + \\
& \frac{\Omega^r\Omega^f E(MP)}{1-\Omega}\sum_{j=2}^{\infty}(\Omega)^{j-1}\,cov(P_t, g^s_{t+j}) + \\
& \Omega^r\Omega^f\sum_{j=1}^{\infty}(\Omega)^{j-1}\,cov(P_t, MP_{t+j})
\end{aligned}
$$

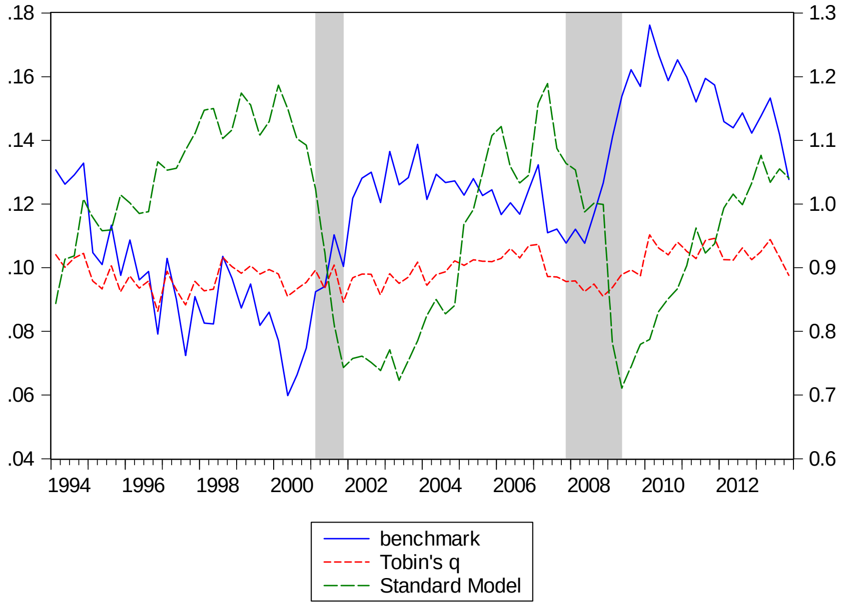

Figure 6: Cyclicality of Job Values

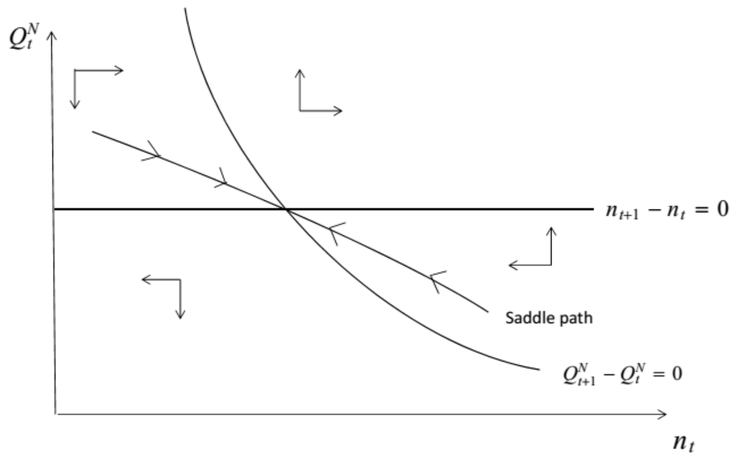

Figure 7: Saddle Path Dynamics

Table 11
Job Value Cyclicality

Benchmark model
$\rho(LHS_t, y_{t+i})$
HP filtered ($\lambda = 1600$)

i	-8	-4	-1	0	1	4	8
y	-0.04	-0.46	-0.67	-0.63	-0.49	0.04	0.33

Tobin's q
$\rho(\ LHS_t, y_{t+i})$
HP filtered ($\lambda = 1600$)

i	-8	-4	-1	0	1	4	8
y	-0.28	-0.24	0.03	0.13	0.27	0.41	0.19

Standard model
HP filtered ($\lambda = 1600$)

i	-8	-4	-1	0	1	4	8
y	-0.26	0.38	0.85	0.90	0.79	0.39	-0.18

Table 12a

Volatility of Recruiting Variables

	std
$\frac{i_t}{k_t}$	0.07
q_t^1	0.12
q_t^2	0.08
$\frac{v_t}{n_t}$	0.11
$\frac{h_t}{n_t}$	0.02
$\frac{h_t^1}{n_t}$	0.02

Table 12b

Moments of the Reference Variables

	f	$\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$
std	0.02	0.12
ρ		-0.63

Table 12c

Co-Movement of Recruiting Variables

ρ(row,column)	f	$\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$
$\frac{i_t}{k_t}$	0.87	-0.83
q_t^1	-0.89	0.67
q_t^2	-0.79	0.39
$\frac{v_t}{n_t}$	0.91	-0.56
$\frac{h_t}{n_t}$	0.27	0.15
$\frac{h_t^1}{n_t}$	-0.28	0.78

Table 13
Variance Decomposition: Vacancy Rate

	variance value	relative to $var(\frac{v_t}{n_t})$
$var(\frac{v_t}{n_t})$	$2.3 * 10^{-5}$	1
$var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$2.7 * 10^{-6}$	0.12
$var\left(\frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$2.5 * 10^{-5}$	1.08
$cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$2.4 * 10^{-6}$	0.11

Table 14
Co-Movement: Vacancy Rate

$\rho\left(\frac{v_t}{n_t}, f_t\right)$	0.91
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	0.26
$\rho\left(\frac{-\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	0.92

$\rho\left(\frac{v_t}{n_t}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	-0.56
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	0.49
$\rho\left(\frac{-\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	-0.77

Table 15
Variance Decomposition: Hiring Rate

	variance value	**relative to** $var(\frac{h_t}{n_t})$
$var(\frac{h_t}{n_t})$	$1.5 * 10^{-4}$	1
$var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$1.8 * 10^{-4}$	1.19
$var\left(\frac{\left(q_t^1+q_t^2\right)\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$4.4 * 10^{-4}$	2.90
$cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{\left(q_t^1+q_t^2\right)\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$2.3 * 10^{-4}$	1.55

Table 16
Co-Movement: Hiring Rate

$\rho\left(\frac{h_t}{n_t}, f_t\right)$	0.27
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	-0.65
$\rho\left(\frac{-\left(q_t^1+q_t^2\right)\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	0.82

$\rho\left(\frac{h_t}{n_t}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	0.15
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)^2}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	0.99
$\rho\left(\frac{-\left(q_t^1+q_t^2\right)\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	-0.86

Table 17
Variance Decomposition: Hiring Rate (from non-employment)

	variance value	relative to $var(\frac{h_t^1}{n_t})$
$var(\frac{h_t^1}{n_t})$	$1.2 * 10^{-5}$	1
$var\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$1.4 * 10^{-4}$	11.4
$var\left(\frac{q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$9.3 * 10^{-5}$	7.6
$cov\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\right)$	$1.1 * 10^{-4}$	9.0

Table 18
Co-Movement: Hiring Rate (from non-employment)

$\rho\left(\frac{h_t^1}{n_t}, f_t\right)$	-0.28
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	-0.68
$\rho\left(\frac{-q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, f_t\right)$	0.84

$\rho\left(\frac{h_t^1}{n_t}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	0.78
$\rho\left(\frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\left(q_t^1+q_t^2\right)q_t^1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	0.997
$\rho\left(\frac{-q_t^1\left(e_{31}q_t^1+e_{32}q_t^2\right)\frac{i_t}{k_t}}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}, \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right)$	-0.84

Table 19
Variance Decomposition: Vacancy Rate, Standard Model

	variance value	**relative to** $var\left(\ln \frac{v_t}{n_t}\right)$
$var\left(\ln \frac{v_t}{n_t}\right)$	0.03	1
$\left(\frac{1}{\sigma-1}\right)^2 var\left[\ln \frac{e_2}{\mu_t}\right]$	0.05	1.8
$v\left(\frac{1}{\sigma-1}\right)^2 var\left[\ln \left(\frac{u_t}{n_t}\right)^{1-\sigma}\right]$	0.05	1.6
$\left(\frac{1}{\sigma-1}\right)^2 var\left[\ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right]$	0.09	2.9
$\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln \frac{e_2}{\mu_t}, \ln\left(\frac{u_t}{n_t}\right)\right]$	0.04	1.3
$\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln \frac{e_2}{\mu_t}, \ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right]$	0.01	0.3
$\left(\frac{1}{\sigma-1}\right)^2 cov\left[\ln \left(\frac{u_t}{n_t}\right)^{1-\sigma}, \ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right]$	-0.03	-1.0

Table 20
Co-Movement: Vacancy Rate, Standard Model

$\rho\left(\ln \frac{v_t}{n_t}, \ln f_t\right)$	0.89
$\rho\left(\ln \frac{e_2}{\mu_t}, \ln f_t\right)$	0.01
$\rho\left(\ln \left(\frac{u_t}{n_t}\right)^{1-\sigma}, \ln f_t\right)$	-0.87
$\rho\left(\ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}, \ln f_t\right)$	0.90

$\rho\left(\ln \frac{v_t}{n_t}, \ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right)$	0.96
$\rho\left(\ln \frac{e_2}{\mu_t}, \ln \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right)$	0.27
$\rho\left(\ln \left(\frac{u_t}{n_t}\right)^{1-\sigma}, \frac{Q_t^N}{\frac{f_t}{n_t}(1-\tau_t)}\right)$	-0.85

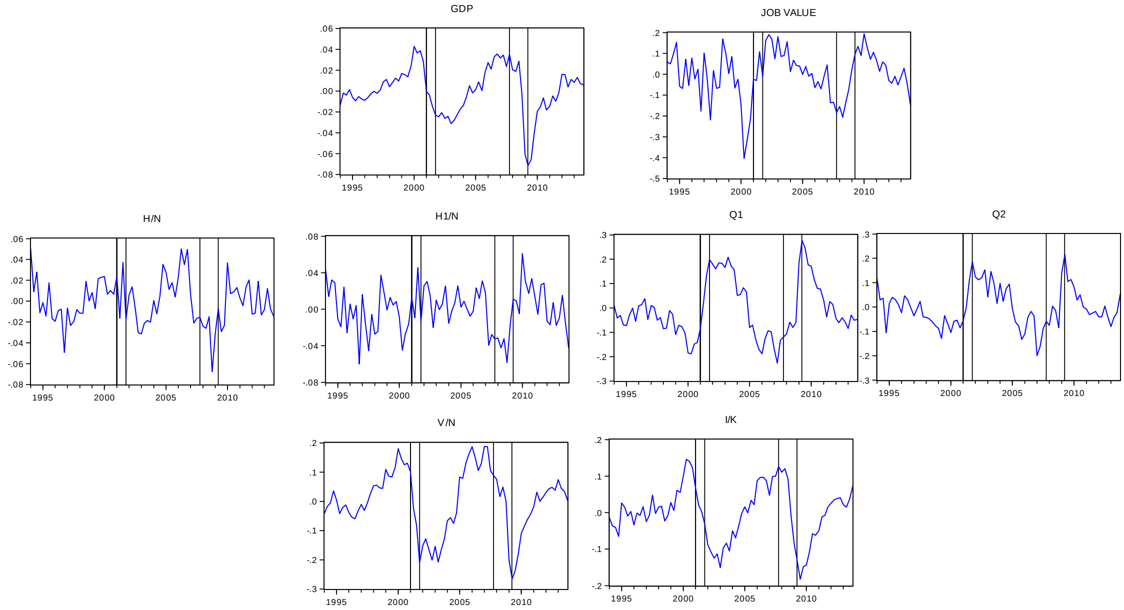

Figure 8: Logged, HP filtered Series

Table 21
The Labor Share and GDP

$$\rho(y_t, \frac{w_{t+i}}{\frac{f_{t+i}}{n_{t+i}}})$$

logged, HP filtered ($\lambda = 1600$)

i	-8	-4	-1	0	1	4	8
	-0.36	-0.38	-0.29	-0.23	-0.02	**0.53**	**0.46**

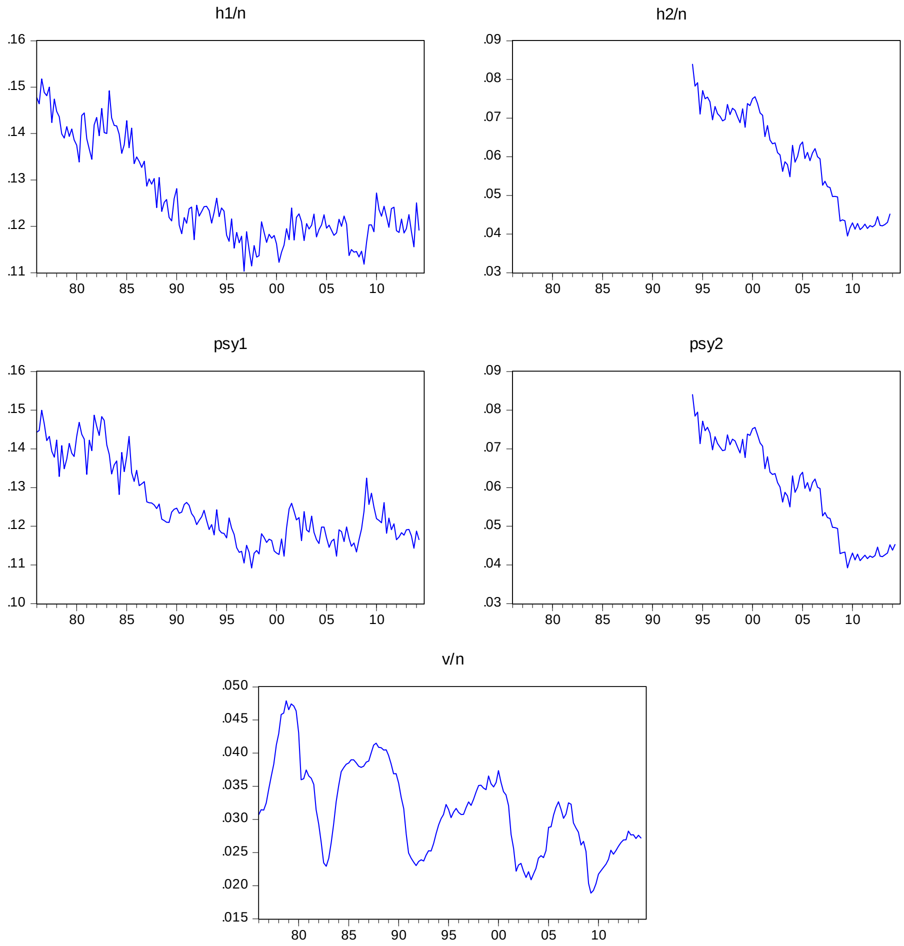

Figure 9: Decline in Worker Flows and Vacancy Rates

Table 22

	1976-1995	**1996-2013**
$\frac{Q_t^N}{\frac{f_t}{n_t}}$	0.7	1.03
τ_t	0.41	0.34
q_t	4.01	6.44
$\frac{i_t}{k_t}$	0.022	0.026

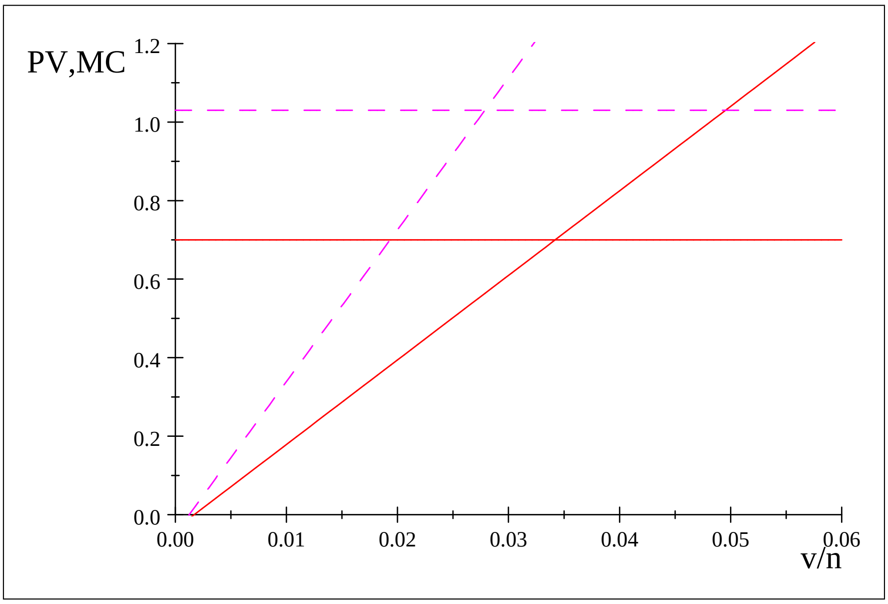

Figure 10: Job Values and Marginal Costs Across Sub-Periods

13 Appendix A: Alternative Formulations of The Recruiting Equation (1)

Linear Costs Models

	paper	the hiring equation
1	Pissarides (2000, ch.1)	$\frac{(r+\lambda+\beta\theta q(\theta))pc}{q(\theta)} = (1-\beta)(p-z)$
2	Pissarides (2000, ch. 2)	$\frac{c}{q(\theta)} = (1-\beta)\frac{1-R}{r+\lambda}$
3	Shimer (2005)	$c[(r+s+\lambda)\frac{1}{q(\theta_{p,s})}+\beta\theta_{p,s}] = \begin{bmatrix} (1-\beta)(p-z) \\ +\lambda c E_{p,s}\frac{1}{q(\theta_{p',s'})} \end{bmatrix}$
4	Hall (2005)	$k = \beta\rho(x_s)\sum_{s'}\pi_{ss'}(J_{s'}-w_{s'});\ J_s = z_s + \begin{bmatrix} \beta(1-\delta) \\ \sum_{s'}\pi_{ss'}(J_{s'}-w_{s'}) \end{bmatrix}$
5	Hall & Milgrom (2008)	$c = q(\theta_i)(P_i - W_i)$
6	Mortensen & Nagypal (2007)	$\frac{c\theta_p}{f(\theta_p)} = J_p = \frac{p-w_p-sJ_p+\lambda(E_pJ_{p'}-J_p)}{r}$
7	Hagedorn & Manovskii (2008)	$c_p = \delta q(\theta_p)E_pJ_{p'}$
8	Elsby & Michaels (2013)	$\begin{bmatrix} pxF'(n) - w(n,x) - w_n(n,x)n \\ -\frac{c}{q}1^+ + \beta\int \Pi_n(n,x')dG(x' \mid x) \end{bmatrix} = 0; \Delta n 1^+ = h$
9	Christiano et al (2013)	$\kappa\frac{1}{Q_t} = J_t = v_t^p - w_t^p = v_t - w_t + \rho\beta E_t\frac{C_t}{C_{t+1}}(v_{t+1}^p - w_{t+1}^p)$
10	Hall (2014a)	$\frac{cx}{H/V} = \frac{1}{1+r}\left(\frac{x-w}{r+s}\right)$

Convex Costs Models

	paper	the hiring equation
1	Merz & Yashiv (2007)	$(1-\tau_t)g_{h_t} = Et\left[m_{t+1}(1-\tau_{t+1})\begin{bmatrix} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})g_{h_{t+1}} \end{bmatrix}\right]$
2	Gertler & Trigari (2009)	$\kappa x = \beta E\begin{bmatrix} \frac{\bar{c}}{c'}((1-\alpha)\frac{y'}{n'} - w' \\ +\frac{\kappa}{2}x'^2 + \rho k x' \mid w, \mathbf{s} \end{bmatrix}; x = \frac{h}{n}$
3	Gali (2010)	$\left(\frac{P_t^I}{P_t}\right)(1-\alpha)A_tN_t(j)^{-\alpha} = \begin{bmatrix} \frac{W_t(j)}{P_t} \\ +\Gamma\left(\frac{H_t}{U_t^0}\right)^{\frac{1-\xi}{\xi}} \\ -(1-\delta)E_t\{\beta\frac{C_t}{C_{t+1}}G_{t+1}\} \end{bmatrix}$
4	Acemoglu & Hawkins (2014)	$c'(v) = qJ_n(n.t;z);\ J_n(n.t;z) = \frac{\phi}{1-\phi}[V(n,t;z) - V^u(t)]$

14 Appendix B: The Estimating Equations

14.1 The Cost Function and its Derivatives

$$g(\cdot) = \begin{bmatrix} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{bmatrix} f(z_t, n_t, k_t). \tag{56}$$

$$\frac{g_{i_t}}{\frac{f_t}{k_t}} = \begin{bmatrix} e_1(\frac{i_t}{k_t})^{\eta_1 - 1} \\ +e_{31}\left(\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}}(\frac{i_t}{k_t})^{\eta_{31}-1} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}}(\frac{i_t}{k_t})^{\eta_{32}-1} \end{bmatrix} \tag{57}$$

$$\frac{g_{v_t}}{\frac{f_t}{n_t}} = \begin{bmatrix} e_2\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2 - 1}\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right] \\ +e_{31}q_t^1\left(\frac{i_t}{k_t}\right)^{\eta_{31}}\left(\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}-1} \\ +e_{32}q_t^2\left(\frac{i_t}{k_t}\right)^{\eta_{32}}\left(\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}-1} \end{bmatrix} \tag{58}$$

$$\begin{aligned} \frac{g_{k_t}}{\frac{f_t}{k_t}} &= -\left[e_1(\frac{i_t}{k_t})^{\eta_1} + e_{31}\left(\frac{q_t^1 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{31}} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{32}}\right] \\ &\quad +(1-\alpha)\begin{bmatrix} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q2 v_t}{n_t}\right)^{\eta_{32}} \end{bmatrix} \end{aligned} \tag{59}$$

$$\begin{aligned} \frac{g_{n_t}}{\frac{f_t}{n_t}} &= -\begin{bmatrix} e_2\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +e_{31}\left(\frac{q_t^1 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{31}} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{32}} \end{bmatrix} \\ &\quad +\alpha\begin{bmatrix} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{bmatrix} \end{aligned} \tag{60}$$

14.2 The Estimating Equation

Replacing expected variables by actual ones and a rational expectations forecast error, the estimating equation is :

$$(1-\tau_t)\,\frac{g_{v_t}}{q_t} = \left[\rho_{t,t+1}\,(1-\tau_{t+1})\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}} \end{array}\right]\right] + j_t \qquad (61)$$

As explained in the text, estimation pertains to $\alpha, e_1, e_2, e_{31}, e_{32}, \eta_1, \eta_2, \eta_{31}, \eta_{32}, \lambda_1, \lambda_2$ or a sub-set of these parameters.

I estimate this equation after dividing throughout by $\frac{f_t}{n_t}$.

14.2.1 The Standard Search and Matching Model

In this case $e_1 = e_{31} = e_{32} = 0, \eta_2 = 1, \lambda_1 = \lambda_2 = 0$ and the equation becomes:

$$(1-\tau_t)\,\frac{e_2}{q_t} = \left[\rho_{t,t+1}\,(1-\tau_{t+1})\,\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}\left[\alpha - \frac{w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}} + (1-\psi_{t+1})\frac{e_2}{q_{t+1}}\right]\right] + j_t \qquad (62)$$

This is estimated for e_2 and α.

14.2.2 Costs Model

In this case $\lambda_1 = 1$.

15 Appendix C: the Data

variable	**symbol**	**definition**
GDP	f	gross value added of NFCB
GDP deflator	p^f	price per unit of gross value added of NFCB
wage share	$\frac{wn}{f}$	numerator: compensation of employees in NFCB
discount rate	r	the rate of non-durable consumption growth minus 1
employment	n	employment in nonfinancial corporate business sector
hiring	h	gross hires
separation rate	ψ	gross separations divided by employment
vacancies	v	adjusted Help Wanted Index
investment	i	gross investment in NFCB sector
capital stock	k	stock of private nonresidential fixed assets in NFCB sector
depreciation	δ	depreciation of the capital stock
price of capital goods	p^I	real price of new capital goods

variable	**symbol**	**source**
GDP	f	NIPA accounts, table 1.14, line 41
GDP deflator	p^f	NIPA table 1.15, line 1
wage share	$\frac{wn}{f}$	NIPA; see note 7
discount rate	r	NIPA Table 2.3.3, lines 3, 8, and 13; see note 1
employment	n	CPS; see note 2
hiring	h	CPS; see note 3
separation rate	ψ	CPS; see note 3
vacancies	v	Conference Board; see note 4
investment	i	BEA and Fed Flow of Funds; see note 5
capital stock	k	BEA and Fed Flow of Funds; see note 5
depreciation	δ	BEA and Fed Flow of Funds; see note 5
price of capital goods	p^I	NIPA and U.S. tax foundation; see note 6

The sample period is 1976:2-2013:4 unless noted otherwise; all data are quarterly.

Notes:

1. *The discount rate and the discount factor*

The discount rate is based on a DSGE-type model with logarithmic utility $U(c_t) = \ln c_t$. Define the discount factor as $\rho_t \equiv \frac{1}{1+r_t}$

In this model:

$$U'(c_t) = U'(c_{t+1}) \cdot (1 + r_t) \tag{63}$$

Hence:

$$\rho_t = \frac{c_t}{c_{t+1}} \tag{64}$$

where c is non-durable consumption (goods and services) and 5% of durable consumption.

2. *Employment*

As a measure of employment in the nonfinancial corporate business sector (n) I take wage and salary workers in non-agricultural industries (series ID LNS12032187) less government workers (series ID LNS12032188), less self-employed workers (series ID LNS12032192). All series originate from CPS databases. I do not subtract workers in private households (the unadjusted series ID LNU02032190) from the above due to lack of sufficient data on this variable.

3. *Hiring and Separation Rates*

The aggregate flow from non-employment – unemployment (U) and out of the labor force (O) – to employment is to be denoted $OE + UE$ and the separation rate ψ_t is rate of the flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE.

I denote:

$$
\begin{aligned}
\frac{h}{n} &= \left(\frac{h^1}{n}\right) + \left(\frac{h^2}{n}\right) \\
\frac{h^1}{n} &= \frac{OE + UE}{E} \\
\frac{h^2}{n} &= \frac{EE}{E}
\end{aligned}
\tag{65}
$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively.

Separation rates are given by:

$$
\begin{aligned}
\psi &= \psi^1 + \psi^2 \\
\psi^1 &= \frac{EO + EU}{E} \\
\psi^2 &= \frac{EE}{E} = \frac{h^2}{n}
\end{aligned}
\tag{66}
$$

Employment dynamics now satisfies:

$$
\begin{aligned}
n_{t+1} &= (1 - \psi_t^1 - \psi_t^2)n_t + h_t^1 + h_t^2 \\
&= (1 - \psi_t)n_t + h_t, \quad 0 \leq \psi_t \leq 1 \\
h_t^2 &= \psi_t^2
\end{aligned}
\tag{67}
$$

To calculate hiring and separation rates for the whole economy I use the following:

a. *The h_t^1 and ψ_t^1 flows.* I compute the flows between E (employment), U (unemployment) and O (not-in-the-labor-force) that correspond to the E,U,O stocks published by the CPS. The methodology of adjusting flows to stocks is taken from BLS, and is presented in Frazis et al (2005).[8]The data till 1990:Q1 were kindly provided by Ofer Cornfeld. The data from 1990:Q2 onwards were taken from the CPS (http://www.bls.gov/cps/cps_flows.htm). Employment is the quarterly average of the original seasonally adjusted total employment series from BLS (LNS12000000).

b. *The h_t^2 and ψ_t^2 flows.* The data on EE, available only from 1994:Q1 onward, were computed by multiplying the percentage of people moving from one employer to another using Fallick and Fleischman (2004)'s[9]data by the NSA population series LNU00000000, taken from the CPS, completing several missing observations and performing seasonal adjustment.

4. *Vacancies*

I use the vacancies series based on the Conference Board Composite Help-Wanted Index that takes into account both printed and web job advertisements, as computed by Barnichon. The updated series is available at

https://sites.google.com/site/regisbarnichon/research/publications.

This index was multiplied by a constant to adjust its mean to the mean of the JOLTS vacancies series over the overlapping sample period (2001:Q1–2013:Q4)

5. *Investment, capital and depreciation*

The goal here is to construct the quarterly series for real investment flow i_t, real capital stock k_t , and depreciation rates δ_t. I proceed as follows:

- Construct end-of-year fixed-cost net stock of private nonresidential fixed assets in NFCB sector, K_t . In order to do this I use the quantity index for net stock of fixed assets in NFCB (FAA table 4.2, line 37, BEA) as well as the 2009 current-cost net stock of fixed assets (FAA table 4.1, line 37, BEA).

- Construct annual fixed-cost depreciation of private nonresidential fixed assets in NFCB sector, D_t . The chain-type quantity index for depreciation originates from FAA table 4.5, line 37. The current-cost depreciation estimates (and specifically the 2009 estimate) are given in FAA table 4.4, line 37.

- Calculate the annual fixed-cost investment flow, I_t:

[8]Frazis, Harley J., Edwin L. Robison, Thomas D. Evans and Martha A. Duff, 2005. Estimating Gross Flows Consistent with Stocks in the CPS, **Monthly Labor Review,** September, 3-9.

[9]Fallick and Fleischman, 2004. "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows," FEDS #2004-34.

$$I_t = K_t - K_{t-1} + D_t$$

- Calculate implied annual depreciation rate, δ_a:

$$\delta_a = \frac{I_t - (K_t - K_{t-1})}{K_{t-1} + I_t/2}$$

- Calculate implied quarterly depreciation rate for each year, δ_{qt}:

$$\delta_q + (1-\delta_q)\delta_q + (1-\delta_q)^2\delta_q + (1-\delta_q)^3\delta_q = \delta_a$$

- Take historic-cost quarterly investment in private non-residential fixed assets by NFCB sector from the Flow of Funds accounts, atabs files, series FA105013005).

- Deflate it using the investment price index (the latter is calculated as consumption of fixed capital in domestic NFCB in current dollars (NIPA table 1.14, line 18) divided by consumption of fixed capital in domestic NFCB in chained 2009 dollars (NIPA table 1.14, line 42). This procedure yields the implicit price deflator for depreciation in NFCB. The resulting quarterly series, i_t_unadj, is thus in real terms.

- Perform Denton's procedure to adjust the quarterly series i_t_unadj from the Federal Flow of Funds accounts to the implied annual series from BEA I_t, using the depreciation rate δ_{qt} from above. I use the simplest version of the adjustment procedure, when the discrepancies between the two series are equally spread over the quarters of each year. As a result of adjustment I get the fixed–cost quarterly series i_t.

- Simulate the quarterly real capital stock series k_t starting from k_0 (k_0 is actually the fixed-cost net stock of fixed assets in the end of 1975, this value is taken from the series K_t) , using the quarterly depreciation series δ_{qt} and investment series i_t from above:

$$k_{t+1} = k_t \cdot (1-\delta_{qt}) + i_t$$

6. *Real price of new capital goods*

In order to compute the real price of new capital goods, p^I, I use the price indices for output and for investment goods.

Investment in NFCB Inv consists of equipment Eq and structures St as well as intellectual property, which I do not include. I define the time-t price-indices for good $j = Eq, St$ as $\tilde{p}_t^j$. The data are taken from NIPA table 1.1.4, lines 10, 11.

I take from http://www.federalreserve.gov/econresdata/frbus/us-models-package.htm the following tax -related rates:

a. The parameter τ – the statutory corporate income tax rate as reported by the U.S. Tax Foundation.

b. The investment tax credit on equipment and public utility structures, to be denoted ITC.

c. The percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit, denoted χ.

d. The present discounted value of capital depreciation allowances, denoted $ZPDE^{St}$ and $ZPDE^{Eq}$.

I then apply the following equations:

$$\begin{aligned} p^{Eq} &= \widetilde{p}^{Eq}\left(1-\tau_{Eq}\right) \\ p^{St} &= \widetilde{p}^{St}\left(1-\tau_{St}\right), \end{aligned}$$

$$\begin{aligned} 1-\tau^{S_t} &= \frac{\left(1-\tau\, ZPDE^{St}\right)}{1-\tau} \\ 1-\tau^{Eq} &= \frac{1-ITC-\tau ZPDE^{Eq}\left(1-\chi ITC\right)}{1-\tau} \end{aligned}$$

Subsequently I compute their change between $t-1$ and t (denoted by Δp_t^j) :

$$\frac{\Delta p_t^{Inv}}{p_{t-1}^{Inv}} = \omega_t \frac{\Delta p_t^{Eq}}{p_{t-1}^{Eq}} + \left(1-\omega_t\right)\frac{\Delta p_t^{St}}{p_{t-1}^{St}}$$

where

$$\omega_t = \frac{\begin{array}{c}(\text{nominal expenditure share of } Eq \text{ in } Inv)_{t-1} \\ +\,(\text{nominal expenditure share of } Eq \text{ in } Inv)_t\end{array}}{2}.$$

The weights ω_t are calculated from the NIPA table 1.1.5, lines 9,11.

I divide the series by the price index for output, p_t^f, to obtain the real price of new capital goods, p^I.

As all of these prices are indices, in estimation I estimate a scaling parameter e^a.

7. *Labor share*

NIPA table 1.14, line 20 (compensation of employees in NFCB) divided by line 17 in the same table (gross value added in NFCB).

16 Appendix D: Approximation

Equation (16) with the RHS iterated forward can be expressed as:

$$(1-\tau_t)\frac{g_{v_t}}{q_t} = E_t\sum_{j=1}^{\infty}\left[\left(\prod_{l=1}^{j}\rho_{t+l-1,t+l}\right)\left(\prod_{l=2}^{j}(1-\psi_{t+l-1})\right)(1-\tau_{t+j})\left[f_{n_{t+j}}-g_{n_{t+j}}-w_{t+j}\right]\right] \tag{68}$$

So the RHS of the F.O.C., to be denoted PV_t, can be written as:

$$PV_t = E_t\sum_{j=1}^{\infty}\left[\begin{array}{c}\left(\prod_{l=1}^{j}\rho_{t+l-1,t+l}\frac{\frac{f_{t+l}}{n_{t+l}}}{\frac{f_{t+l-1}}{n_{t+l-1}}}\right)\left(\prod_{l=2}^{j}(1-\psi_{t+l-1})\right)(1-\tau_{t+j})\\ \left[\alpha-\frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}}-\frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}}\right]\end{array}\right] \tag{69}$$

Using a truncated value going to T rather than ∞ and dropping the expectations operator one gets:

$$PV_{t,T} = \sum_{j=1}^{T}\left[\begin{array}{c}\left(\prod_{l=1}^{j}\rho_{t+l-1,t+l}\frac{\frac{f_{t+l}}{n_{t+l}}}{\frac{f_{t+l-1}}{n_{t+l-1}}}\right)\left(\prod_{l=2}^{j}(1-\psi_{t+l-1})\right)(1-\tau_{t+j})\\ \left[\alpha-\frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}}-\frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}}\right]\end{array}\right] \tag{70}$$

17 Appendix E: Variance Decomposition

The following derivation follows Cochrane (1992), noting that the latter does a second order Taylor expansion while here a first -order one is undertaken. Define:

$$MP_{t+j} \equiv \left(1-\tau_{t+j}\right)\left(\alpha - \frac{g_{n_{t+j}}}{\frac{f_{t+j}}{n_{t+j}}} - \frac{w_{t+j}}{\frac{f_{t+j}}{n_{t+j}}})\right) \tag{71}$$

$$g_t^f = \ln\left(\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}\right) \tag{72}$$

$$g_t^s \equiv \ln(1-\psi_t) \tag{73}$$

$$g_t^r \equiv \ln\rho_{t,t+1} \equiv \ln\left(\frac{1}{1+r_t}\right) \tag{74}$$

$$w_t \equiv \left(g_t^f + g_t^s + g_t^r\right) \tag{75}$$

and

$$\Omega^f = e^{E(g_t^f)} \tag{76}$$

$$\Omega^s = e^{E(g_t^s)} \tag{77}$$

$$\Omega^r = e^{E(g_t^r)} \tag{78}$$

$$\Omega = e^{E(w)} = \Omega^f\Omega^s\Omega^r \tag{79}$$

Then (68) implies the present value relationship:

$$P_t \equiv (1-\tau_t)\frac{g_{v_t}}{q_t} = E_t\left[\sum_{j=1}^{\infty}\exp\left[\sum_{l=1}^{j}g_{t+l}^r\right]\exp\left[\sum_{l=1}^{j}g_{t+l}^f\right]\exp\left[\sum_{m=l}^{j}g_{t+m-1}^s\right]MP_{t+j}\right] \tag{80}$$

Multiply both sides by any variable Z_t observed at time t and take expectations:

$$E\left(Z_tP_t\right) = E\left[Z_t\sum_{j=1}^{\infty}\exp\left[\sum_{l=1}^{j}g_{t+l}^r\right]\exp\left[\sum_{l=1}^{j}g_{t+l}^f\right]\exp\left[\sum_{l=2}^{j}g_{t+l-1}^s\right]MP_{t+j}\right] \tag{81}$$

The first order Taylor expansion of the term in square brackets is:

$$
\begin{aligned}
& Z_t \sum_{j=1}^{\infty} \exp\left[\sum_{l=1}^{j} g^r_{t+l}\right] \exp\left[\sum_{l=1}^{j} g^f_{t+l}\right] \exp\left[\sum_{l=2}^{j} g^s_{t+m-1}\right] MP_{t+j} \qquad (82) \\
\cong\ & Z_t \frac{\Omega^r \Omega^f}{1-\Omega} E(MP) + E(Z) \frac{\Omega^r \Omega^f}{1-\Omega} E(MP) \sum_{j=1}^{\infty} (\Omega)^{j-1} \widetilde{g}^r_{t+j} + E(Z) \frac{\Omega^r \Omega^f}{1-\Omega} E(MP) \sum_{j=1}^{\infty} (\Omega)^{j-1} \widetilde{g}^f_{t+j} \\
& + E(Z) \frac{\Omega^r \Omega^f}{1-\Omega} E(MP) \sum_{j=2}^{\infty} (\Omega)^{j-1} \widetilde{g}^s_{t+j} + E(Z) \Omega^r \Omega^f \sum_{j=1}^{\infty} (\Omega)^{j-1} \widetilde{MP}_{t+j}
\end{aligned}
$$

Multiplying by $P_t - E(P)$ and taking expectations yields the variance decomposition:

$$
\begin{aligned}
var(P) \quad \cong \quad & \frac{\Omega^r \Omega^f E(MP)}{1-\Omega} \sum_{j=1}^{\infty} (\Omega)^{j-1} cov(P_t, g^r_{t+j}) + \qquad (83) \\
& \frac{\Omega^r \Omega^f E(MP)}{1-\Omega} \sum_{j=1}^{\infty} (\Omega)^{j-1} cov(P_t, g^f_{t+j}) + \\
& \frac{\Omega^r \Omega^f E(MP)}{1-\Omega} \sum_{j=2}^{\infty} (\Omega)^{j-1} cov(P_t, g^s_{t+j}) + \\
& \Omega^r \Omega^f \sum_{j=1}^{\infty} (\Omega)^{j-1} cov(P_t, MP_{t+j})
\end{aligned}
$$