Reverse engineering labor market flows*
(work in progress)

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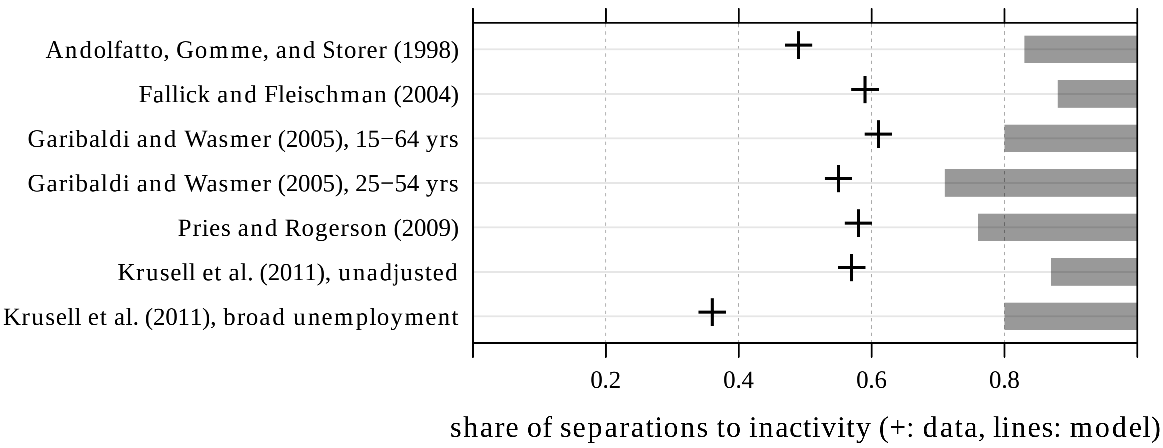
Abstract

We show that the labor market flows that can be generated by a general class of frictional labor market models with a participation margin and an individual-specific state can always match five out of six moments of the labor marker transition rates, but conditional on that can only match the sixth moment if it is within a certain range, which we characterize analytically for a benchmark model. Based on this result, we introduce a scalar, dimensionless discrepancy index between models and the data that can be calculated from observed flows only. We discuss related papers and show that the flows they use for calibration are outside the range of the model class, thus explaining why they have been unable match the gross labor market flows. We consider extensions to the benchmark model to see how this puzzle can be resolved: more general processes for the individual-specific state do not change the basic result, and nor does state-specific exogenous separation probability. However, allowing the process for the worker's surplus to depend on the worker's employment status allows us to match the data.

Graphical abstract



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1 Introduction

Labor market participation and unemployment rates display significant cross-country variation, as illustrated by Figure 1. While the stocks themselves have important policy consequences — for example, changes in the participation rate have implications for productivity, growth, and the sustainability of pay-as-you-go pension systems — the fact that gross flows between employment, unemployment and nonparticipation are large also suggests that transitions between various labor market states have a large potential welfare effect. This is especially true if typical shocks that lead to changes in labor market status are persistent and only partially insurable at best. Consequently, in order to arrive at satisfactory models of the labor market for policy purposes, explaining the gross flows is particularly important.

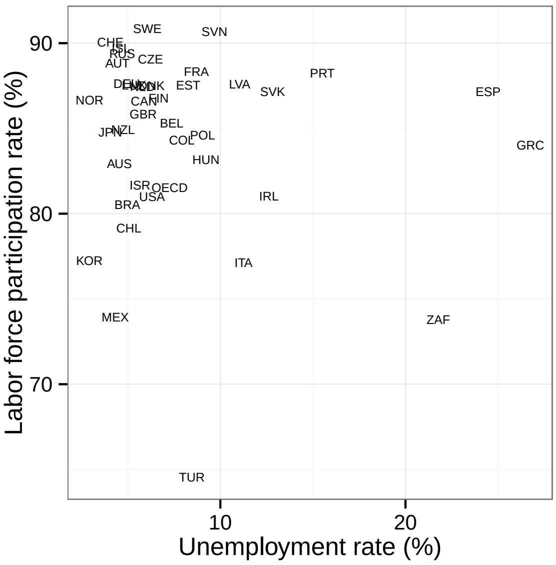

Figure 1: Labor force participation and unemployment rates in OECD countries, 2013.

This paper takes the first step in examining the ability of a family of frictional labor market models with idiosyncratic shocks and an endogeneous participation margin to reproduce the labor market flows found in the data. First, we build a *benchmark model* from standard elements: a persistent process that determines the difference between the flow utility of working or not working, meant to capture idiosyncratic shocks such as shocks to market opportunity, health shocks, family shocks, and changes to preferences, a frictional labor market with exogeneously given job offer rates, exogeneous and endogeneous separation, and a simple binary choice for search activity that allows modeling active and passive search. We examine the implications of calibrating this model to transition rates between employment (E), unemployment (U), and non-participation or inactivity (I).

The most important finding of the paper is that even with seemingly enough degrees of freedom, the benchmark model can only reproduce labor market transition rates if they are within a certain subset. On an abstract level, if we consider the model as a mapping from its fundamental parameters to labor market flows, we show that the *range* of this mapping is limited to a subset of all possible values. Moreover, we also provide a complete and tractable characterization of all labor market transition rates that the model can generate: we show that if the model is calibrated to *all* separations from employment, transition rates between unemployment and inactivity, and job finding rates, then it can match

$$\alpha = \frac{\text{EI flows}}{\text{EI and EU flows}} = \frac{\text{rate of separations to } \textit{inactivity}}{\text{rate of } \textit{all} \text{ separations}} \tag{1}$$

if and only if it is within a certain range $[\alpha^*, 1]$. The lower limit

$$\alpha^* = \inf\{\alpha(\text{free parameters}) \mid \text{matching the other flows}\}$$

is shown to have a simple form that can be calculated from the other five moments that are matched. Figure 2 summarizes the empirical relevance of this finding: for all estimates that we have examined in the literature, $\alpha_{\text{data}} \gg \alpha^*$, which demonstrates an important discrepancy between the models and the data.

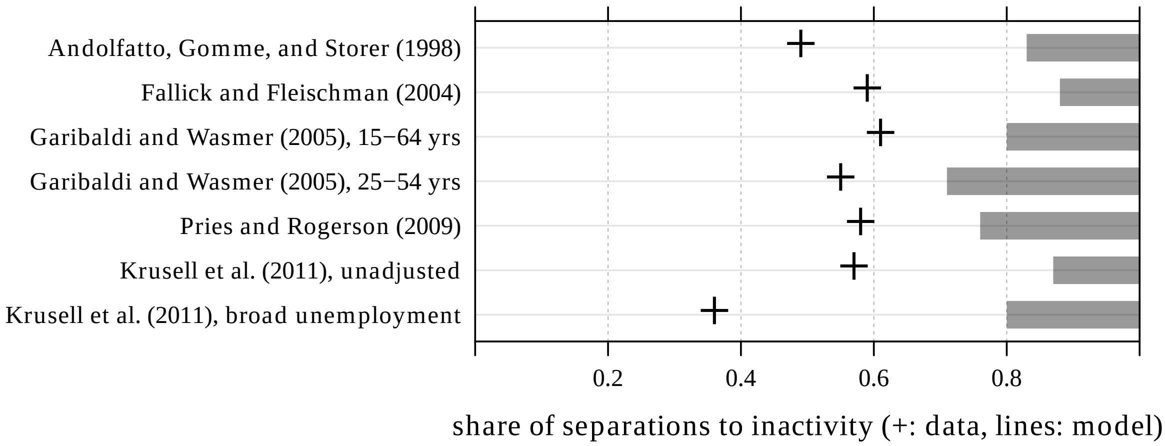

Figure 2: $\alpha = \lambda_{\text{EI}}/(\lambda_{\text{EU}} + \lambda_{\text{EI}})$ (see (1)) in the data (+'s) and the range that can be generated by the benchmark model when calibrating to the other five moments from flows for various estimates in the literature.

The intuition behind this result is the following. In the model, the workers have two state variables: employment/nonemployment, and an individual-specific state variable that captures the effect labor market and home productivity. Nonemployed workers who are highly productive in the market will search actively, and thus report themselves as *unemployed*, while less productive workers will only search passively, or not accept jobs at all, and consequenly are counted as *inactive*. Even though transition rates between unemployment and inactivity do not completely determine the individual-specific process, they constrain its calibration.

This is relevant in the benchmark model since the same process is imposed on the employed, where the individual-specific state plays a role in determining whether workers end up unemployed or inactive in case of an exogeneous separation. We call the latter group *marginal* workers, since they would choose passive search because their flow surplus is low. In the calibration of the model, α is closely related to the endogeneous fraction of marginal employed among all employed workers: exogeneous separations generate EI flows for marginal employed, and EU flows the rest of the workers.

We then consider extensions to the benchmark model to see if it is possible to escape this constraint. First, allowing marginal workers to have higher separation rates simply exacerbates the original problem, as it leads to even higher EI flows. In contrast, allowing employed and non-employed workers to have different productivity processes allows matching the labor marker transition rates observed in the data, since churning between unemployment and inactivity does not constrain the calibration for transitions among employed workers. We examine two parametrizations: one in which marginal employed workers can transition into non-marginal states with a higher probability, modeling learning on the job in a stylized manner, and another in which unemployed workers experience transitions into states of lower surplus, which loosely corresponds to skill loss among the unemployed. Finally, we show that allowing some inactive workers to be permanently inactive and thus not participate in the flows can also be used to match labor market transition rates: effectively, permanently inactive workers "dilute" observed flows between unemployment and inactivity, allowing fundamental flows to be different, with higher UI transitions than observed.

Our analysis is related to the various approaches in the literature that aim to explain participation and/or unemployment. As noted by Krusell et al. (2011), historically, frictionless versions of the standard growth model were mainly used to explain participation, mapping it to a choice on the labor/leisure margin: for example Hansen (1985) and Rogerson (1988), while models in the Diamond-Mortensen-Pissarides model family[1] have been used to explain unemployment with labor market frictions. However, recognizing that satisfactory models should account for both unemployment and the participation margin, many papers incorporated the latter into frictional models of the labor market. Ljungqvist and Sargent (1998), Ljungqvist and Sargent (2007), Alvarez and Veracierto (2000), and Veracierto (2008) are models that are similar to the one discussed in this paper along many dimensions, but they do not attempt to account for labor market flows across the three states. Coming from the other direction, Merz (1995), Andolfatto (1996), Gomes, Greenwood, and Rebelo (2001) include labor market frictions in the standard growth model, but do not distinguish unemployment and inactivity.

The two papers most closely related to this one are Garibaldi and Wasmer (2005) and Krusell et al. (2011). In particular, our model nests the structure of Garibaldi and Wasmer (2005), formalizing the reason for the discrepancy between observed and generated flows in their paper in a more general setting. What we call marginal workers they term "employment hoarding", since it results from the irreversibility of separations. Krusell et al. (2011) also focus on the flows, in a model that is essentially similar to ours except for the fact that they also allow

[1]See Pissarides (2000) for an introductory overview.

risk aversion and saving. However, as noted in the paper, this does not have a significant effect on the flows, and thus would only complicate our analysis. Krusell et al. (2011) also argue that marginally inactive in the data should be counted as unemployed when accounting for the flows. However, this turns out to increase UI transition rates, increasing the distance between the model and the data even further. Both papers note the discrepancy between calibrated transition rates and the data, but focus on its consequences on the UI and IU flows.

The structure of the paper is as follows: in Section 2, we introduce the notation and the model framework. Section 3 discusses an important special case of the benchmark model, which provides most of the intuition, while Section 4 derives the results for the benchmark model. Section 5 discusses related literature, both for related models and they data they have estimated and calibrated in the context of our results. Section 6 examines the potential of various extensions for bridging the gap between the model and the data.

Most the results in the paper are analytical, but illustrated with calculations using empirical data. However, in order to avoid making the paper unreadable, we relegated most steps in the analytical proofs to the appendix, and only included important equations in the main text. All analytical proofs and calculations have been checked using the symbolic algebra software Maxima (2014) and are available as an online appendix.

2 The model class, notation and observed flows

In this section we introduce the model class used in the paper, aiming to find a reasonable compromise between tractability and generality: because characterizing the range of a nonlinear mapping is a difficult problem, the model needs to be as simple as possible, while at the same time the model needs to be general enough to nest or at least closely approximate models used in the literature, and also possible extensions. Below we briefly discuss the modeling choices and trade-offs motivated by these possibly conflicting requirements.

First, we use a *partial equilibrium model* in which job offer rates are exogenous (conditional on search intensity). The advantage of this is that a general equilibrium formulation with job creation would just provide additional restrictions, and thus the range of labor market flows generated by a partial equilibrium model is necessarily larger than it would be for general equilibrium one. Using the latter would just complicate the formulation and would add little to our result. A partial equilibrium formulation also precludes discussion of aggregate fluctuations and policy experiments, but both of those directions are outside the scope of this paper.

Second, the model needs to be able to generate apparent flows from seemingly inactive workers (ie those workers who are neither employed nor unemployed, in the sense that they do not search actively) into employment: as we discuss in Section 5, even though a fraction of these transition can be explained by time aggregation (inactive workers becoming unemployed and then employed between two observations), this flow is too large in the data to be assumed away. We generate these flows by assuming that workers who choose not to search actively also receive job offers, albeit at a lower rate.

Third, we include an individual-specific state that we think of as a proxy for market opportunities, family, health, and preference shocks. Krusell et al. 2008 show that it is difficult to generate labor market flows without these shocks even in a very rich model with precautionary savings. However, we aim to keep state spaces as general as possible at this stage, potentially allowing for very elaborate processes. We make wages and benefits a function of this state, without restricting functional forms, for the sake of generality.

Finally, it is important to note that we will mostly use the flows generated by this model in the paper and only discuss other features tangentially. Nevertheless, we believe that specifying the model in detail is important because it allows us to relate to the models in the literature, and also puts discipline on the exercise of accounting for observable flows, as it rules out arbitrary statistical processes.

With these considerations in mind, consider a partial equilibrium frictional labor market model in which the only state of workers is their individual-specific state $s \in \mathcal{S}$ and their employment status, employed (E) or nonemployed (N). We allow $\mathcal{S}$ to be completely general: for example it could be a set of discrete states, a subset of $\mathbb{R}^n_+$ for some n, a Cartesian product of these, or any other arbitrary set. We assume that when the worker is employed, wages $w(s)$ are a function of this state only. When the worker is unemployed, his utility is $b(s)$, which, as usual, incorporates both the unemployment benefit and home production. Time is continuous, and the discount rate is r.

Individual productivity follows a Markov process, potentially changing at random points (called *change events*) in time. The arrival of change events follows a Poisson process with rate γ. Conditional on a change event, a new value for s is drawn, with the cumulative distribution $F(s' \mid s)$. Employed workers have the option to quit (endogenous separations), but are also separated exogenously at rate σ. Non-employed workers can pay a flow cost c and search *actively*, in which case offers arrive at rate λ, or search for free *passively*, getting offers at rate $\kappa\lambda$, where $\kappa \in [0, 1)$ is the relative search efficiency of passive compared to active search.[2] Consequently, the HJB equations are

$$
\begin{aligned}
rN(s) = b(s) + \max\Big\{&\underbrace{\kappa\lambda \max\{W(s) - N(s), 0\}}_{\text{passive search}}, \underbrace{-c + \lambda\big(W(s) - N(s)\big)}_{\text{active search}}\Big\} \\
&+ \underbrace{\gamma\, \mathbb{E}_{F(s'|s)}\big[N(s') - N(s)\big]}_{\text{state change}}
\end{aligned} \tag{2}
$$

$$
rW(s) = w(s) + \underbrace{\sigma\big(N(s) - W(s)\big)}_{\text{exogenous separation}} + \underbrace{\gamma\, \mathbb{E}_{F(s'|s)}\Big[\max\{W(s'), N(s')\} - W(s)\Big]}_{\text{state change, maybe endogenous separation}} \tag{3}
$$

Equation (2) states that a nonemployed worker receives benefits (which are function of s), and can choose between passive and active search. For the former, offers are only accepted when working is preferable to non-employment, for the latter, the formulation above anticipates that when agents choose to search actively, they accept the job they find. The exogenous state

[2] Table 1 summarizes the gist of the notation for this section and the whole paper.

Model setup	
s	individual-specific state
$\mathcal{S}$	space of individual-specific states, $s \in \mathcal{S}$
γ	arrival rate of change events for s
$F(s' \mid s)$	transition kernel for s, conditional on a change event
$w(s)$	wage when employed
$b(s)$	flow value of nonemployment (unemployment benefit and leisure)
λ, $\kappa\lambda$	arrival of job offers for passive and active search, $\kappa \in [0, 1)$ relative search efficiency
c	flow cost of active search
σ	rate of exogenous separation
$W(s)$, $N(s)$, $S(s)$	present discounted value of employment, non-employment, and their difference (the surplus)
Regions and flows	
$L \subset \mathcal{S}$	low surplus: no active search, non-employment preferred
$M \subset \mathcal{S}$	marginal surplus: passive search, employment preferred
$H \subset \mathcal{S}$	high surplus: active search
λ_{IU}, λ_{IE}, ...	observed transition rates between Inactivity, Unemployment, and Employment
μ_{E}, μ_{I}	share of marginal workers among the Employed and Inactive
$\lambda_{\text{E}*}$	transition rate out of employment, $\lambda_{\text{EI}} + \lambda_{\text{EU}}$
α, α_D	share of λ_{EI} in $\lambda_{\text{E}*}$ in general, and in the Data
The corner case (C)	
p_{M}, p_{H}	endogenous probability of transitioning to region M and H from the other
α_C	α for the corner case
n_{M}, n_{H}, e_{M}, e_{H}	mass of non-employed and employed workers in M and H
The benchmark model (B)	
q_{L}, q_{M}, q_{H}	transition rates to the corresponding regions
n_{L}	mass of non-employed workers in L
State-dependent separation rates (S)	
σ_M, σ_H	separation rates for Marginal and High surplus employed
δ_σ	the parameter which governs their difference
Three-point distribution (T)	
q_{LM}, q_{LH}, ...	non-iid transition rates between L, M, H
Human capital model (N, E)	
q^{E}_{L}, q^{N}_{L}	transition rate to L for Employed and Non-employed (skill depreciation)
q^{E}_{H}, q^{N}_{H}	transition rate to H for Employed and Non-employed (learning on the job)

Table 1: Notation

change always leaves a nonemployed worker nonemployed, and thus it generates flows between inactivity and unemployment. For an employed worker, (3) shows that the flow payoff is the wage, and the two possible transitions are exogenous separations and state changes, which can result in continued employment, or a transition to non-employment.

Lemma 1. *Equations* (2)–(3) *uniquely determine* $N(s)$ *and* $W(s)$ *as a function of the parameters* $w(s)$, $b(s)$, r, γ, σ, c, λ, *and* κ.

Proof. Can be shown with a standard contraction argument. □

Below we discuss the partitioning of the parameter space into regions what will be relevant when constructing observable worker flows from the latent Markov chain. The actions of unemployed agents depend only on the surplus

$$S(s) = W(s) - N(s)$$

and $c/(\lambda(1-\kappa))$. This partitions $\mathcal{S}$ into three regions which characterize the policy function:

$$\begin{aligned} L &= \{s : S(s) < 0\} \\ M &= \{s : 0 \le S(s) < c/(\lambda(1-\kappa))\} \\ H &= \{s : c/(\lambda(1-\kappa)) \le S(s)\} \end{aligned}$$

In region $L \subset \mathcal{S}$ (low surplus), nonemployed workers do not search actively, and if they encounter a job, they choose to remain nonemployed because their surplus from the job would not be positive. In region $M \subset \mathcal{S}$ (middle or marginal surplus), nonemployed workers still do not search actively because their surplus does not justify the cost, but would accept a job if they were offered one. Finally, in region $H \subset \mathcal{S}$ (high surplus), nonemployed workers search actively, and thus from now on we will call them *unemployed*. In contrast, workers in regions $L \cup M$ are *inactive* according to standard definitions of unemployment.

Even though only nonemployed workers have a nontrivial choice in this model, when we account for the distributions it is important to also distinguish employed workers based on the partition above. Naturally, there are no employed workers in L, since employed workers ending up in this region after a change event quit their job (endogenous separation). In contrast, employed workers ending up in region M after a change event do not quit, but they would not search actively if they got separated. This justifies calling them *marginal* workers.

Since L, M and H characterize the policy function, combined with γ, F, σ, λ and κ they characterize the stochastic process for the state space $\mathcal{S} \times \{N, E\}$, which then determines the steady state distributions of employed and nonemployed workers.

We now consider how the flows above map to observed flows (see Figure 3 for an illustration). Nonemployed in $L \cup M$ are counted as *inactive*, but from M, they find jobs at rate $\kappa\lambda$ so the model has *inactive to employed* (IE) transitions. The observed transition rate is

$$\lambda_{\mathrm{IE}} = \mu_{\mathrm{I}} \kappa \lambda$$

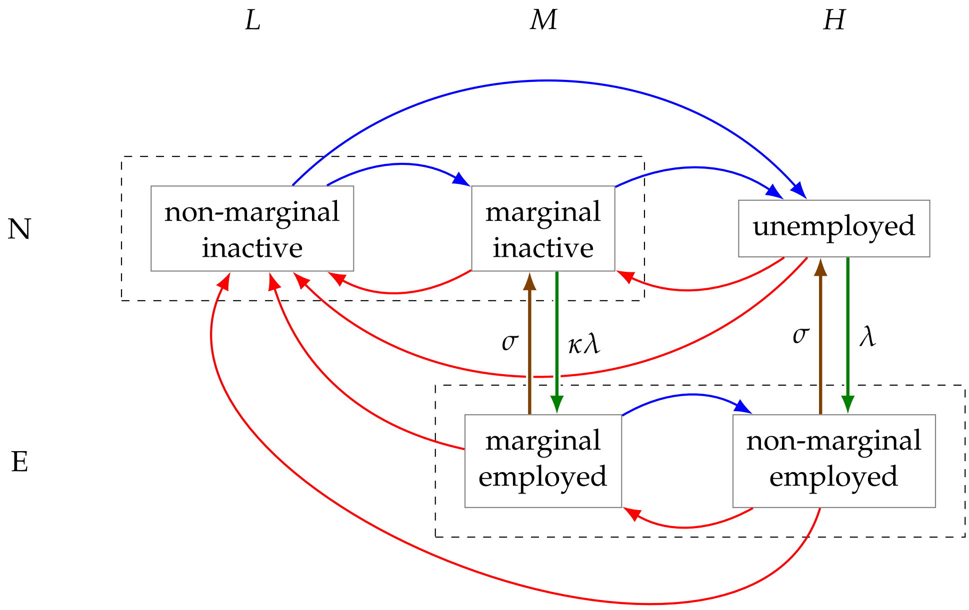

Figure 3: Latent and observed flows. Blue arrows correspond to transitions to a region with a higher surplus for the worker, while red arrows denote transitions to a lower surplus. When these happen between L and M for nonemployed workers (both of which are counted as inactive) or M and H for employed workers, they are not observed as transitions in a dataset with three states, otherwise they show up as UI, IU, or EI transitions. Brown arrows are exogenous separations (EI or EU, depending on whether s is in M or H), while green arrows correspond to job finding (IE or UE), similarly depending on s.

where

$$\mu_{\text{I}} = \frac{\text{mass of non-employed workers in } M}{\text{mass of non-employed workers in } L \cup M}$$

denotes the share of marginal workers among the inactive, and is endogenous, while for unemployed (in H),

$$\lambda_{\text{UE}} = \lambda$$

can be matched directly. Similarly,

$$\lambda_{\text{EI}} = \mu_{\text{E}}\sigma + \gamma\,\mathbb{P}(M \cup H \to L \mid \text{employed})$$
$$\lambda_{\text{EU}} = (1 - \mu_{\text{E}})\sigma$$

where

$$\mu_{\text{E}} = \frac{\text{mass of employed workers in } M}{\text{mass of employed workers in } M \cup H}$$

denotes the share of marginal workers among employed, and is also endogenous. Finally, λ_{IU} and λ_{UI} just depend on γ and the steady state distribution of workers and F.

Despite the generality of this formulation, it is important to note two limitations of the model as formulated above: first, the process for s is independent of the employment state by construction, and thus precludes the possibility of incorporating processes which depend on employment status or search intensity—for example, skill loss during unemployment or human capital accumulation for the employed. Second, there is no firm- or match-specific[3] state in the model, which eliminates an important source of cross-sectional heterogeneity. We consider both of these extensions in Section 6.

3 The corner parameterization

In this section we construct a parameterization that matches five out of six moments of the flows. This is important for two reasons: first, this demonstrates that the discrepancy between models and the data can be summarized by a scalar index (for the sixth, unmatched moment); second, this parameterization is an important baseline case for the special case to which we can compare other models—specifically, when we discuss variations and extensions, we will see that in some cases the corner parametrization is the closest the model can get to the data.

We arrive at the corner parameterization by ensuring that there are no workers in the low region. This can be achieved as a limiting case $b(s) \to -\infty$ if the steady state distribution has infinite support, otherwise by making b smaller than the smallest possible $w(s)$, both of which ensure $L = \emptyset$. Alternatively, we can choose F such that workers would never transition to L in the steady state. Then, without loss of generality, $\mathcal{S} = M \uplus H$, and we only need to consider two relevant transition rates in the steady state: the transition probabilities between marginal

[3]The distinction between the two is not relevant in partial equilibrium.

and high surplus:[4]

$$p_{\mathrm{M}} = \mathbb{P}(H \to M)$$
$$p_{\mathrm{H}} = \mathbb{P}(M \to H)$$

both of which depend on the steady state distribution but are well-defined. In this parameterization of the model, *all inactive workers would accept a job, and there are no endogenous separations.* Consequently, the observed transition rates are

$$\begin{aligned}
\lambda_{\mathrm{EI}} &= \mu_{\mathrm{E}}\sigma \\
\lambda_{\mathrm{EU}} &= (1-\mu_{\mathrm{E}})\sigma \\
\lambda_{\mathrm{IE}} &= \kappa\lambda \\
\lambda_{\mathrm{IU}} &= \gamma p_{\mathrm{H}} \equiv q_{\mathrm{H}} \\
\lambda_{\mathrm{UE}} &= \lambda \\
\lambda_{\mathrm{UI}} &= \gamma p_{\mathrm{M}} \equiv q_{\mathrm{M}}
\end{aligned}$$

where we have introduced rates q_{M} and q_{H}. Let e_{M} and e_{H} denote the mass of employed workers in the marginal and non-marginal regions, and similarly n_{M} and n_{H} for non-employed workers. In the steady state, inflows match outflows, so

$$\begin{aligned}
n_{\mathrm{M}}q_{\mathrm{H}} + \kappa\lambda n_{\mathrm{M}} &= (n_{\mathrm{L}} + n_{\mathrm{H}})\, q_{\mathrm{M}} + \sigma e_{\mathrm{M}} \\
n_{\mathrm{H}}q_{\mathrm{M}} + \lambda n_{\mathrm{H}} &= (n_{\mathrm{M}} + n_{\mathrm{L}})\, q_{\mathrm{H}} + \sigma e_{\mathrm{H}} \\
e_{\mathrm{M}}q_{\mathrm{H}} + \sigma e_{\mathrm{M}} &= e_{\mathrm{H}}q_{\mathrm{M}} + \kappa\lambda n_{\mathrm{M}} \\
e_{\mathrm{H}}q_{\mathrm{M}} + \sigma e_{\mathrm{H}} &= e_{\mathrm{M}}q_{\mathrm{H}} + \lambda n_{\mathrm{H}}
\end{aligned} \tag{4}$$

with

$$n_{\mathrm{M}} + n_{\mathrm{H}} + e_{\mathrm{M}} + e_{\mathrm{H}} = 1 \tag{5}$$

The share of marginal employed μ_{E} is a function of the steady state masses

$$\mu_{\mathrm{E}} \equiv \frac{e_{\mathrm{M}}}{e_{\mathrm{M}} + e_{\mathrm{H}}} \tag{6}$$

and, of course, $\mu_{\mathrm{I}} = 1$ by construction.

This model has five parameters (γ, p_{M}, and p_{M} are counted as only two, since they do not occur independently), so it is unlikely that it can match all six of the labor market transition rates above. Consequently we select *five* moments to match, and leave the sixth one

[4] Transition rates from L do not affect flows and in this sense they are irrelevant.

unmatched. We map λ_{EI} and λ_{EU} to

$$\begin{aligned}\lambda_{\mathrm{E}*} &= \lambda_{\mathrm{EU}} + \lambda_{\mathrm{EI}} \\ \alpha_D &= \frac{\lambda_{\mathrm{EI}}}{\lambda_{\mathrm{EI}} + \lambda_{\mathrm{EU}}} = \frac{\lambda_{\mathrm{EI}}}{\lambda_{\mathrm{E}*}}\end{aligned} \tag{7}$$

Here, α_D is the *share of separations to inactivity in the data*. This $\mathbb{R}_+^2 \setminus 0 \mapsto \mathbb{R}_+ \times [0,1]$ mapping is a bijection.[5]

Notation 2. *Consider a model family M, parameterizing by some parameters $p \in \mathcal{P}_M$. Fix $\Lambda^* = [\lambda_{\mathrm{UI}}, \lambda_{\mathrm{IU}}, \lambda_{\mathrm{UE}}, \lambda_{\mathrm{IE}}, \lambda_{\mathrm{E}*}]$, and let $\Lambda_M(p)$ denote the corresponding transition rates generated by parameterization p in the steady state, similarly, let $\alpha_M(p)$ denote $\lambda_{\mathrm{IE}}/(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{UE}})$ for p.*

1. *Let $\mathcal{M}_M(\Lambda^*) = \{p : \Lambda(p) = \Lambda^*\}$. We say that the model* matches Λ^* *with p whenever $p \in \mathcal{M}_M(\Lambda^*)$.*
2. *Let $\mathcal{A}_M(\Lambda^*) = \alpha(\mathcal{M}_M(\Lambda^*))$ denote all the α's that the model can generate when it matches Λ^*.*

The following constructive lemma shows that it is easy to match five out of the six moments with a corner parameterization of the model, and calculates α that is a function of the other five transition rates.

Lemma 3. *Consider labor market flows (λ_{EU},λ_{EI},λ_{IE},λ_{IU},λ_{UE},λ_{UI}) and a given distribution $F(s' \mid s)$. Let $\lambda = \lambda_{\mathrm{UE}}$, $\kappa = \lambda_{\mathrm{IE}}/\lambda_{\mathrm{UE}}$, and $\sigma = \lambda_{\mathrm{EI}} + \lambda_{\mathrm{EU}}$. If, for this parameterization, there exists a cost c such that*

$$\frac{\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{IU}}} = \frac{p_{\mathrm{M}}}{p_{\mathrm{H}}}$$

then the flows generated by the model match $\lambda_{\mathrm{EU}} + \lambda_{\mathrm{EI}}$, λ_{IE}, λ_{IU}, λ_{UE}, λ_{UI}, and the model yields

$$\alpha_C = \mu_{\mathrm{E}} = \left(1 + \frac{\lambda_{\mathrm{IU}}}{\lambda_{\mathrm{UI}}} \cdot \frac{\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}} + \lambda_{\mathrm{IE}}\lambda_{\mathrm{UE}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{UE}}}{\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}} + \lambda_{\mathrm{IE}}\lambda_{\mathrm{UE}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}}\right)^{-1} \tag{8}$$

and $\mathcal{A}_C = \{\alpha_C\}$.

Proof. Follows from setting

$$\gamma = \frac{\lambda_{\mathrm{UI}} + \lambda_{\mathrm{IU}}}{p_{\mathrm{M}} + p_{\mathrm{H}}}$$

and then some algebra.[6] □

The formula in (8) looks a bit complicated, but it is easy to get the intuition. If $\lambda_{\mathrm{IE}} = \lambda_{\mathrm{UE}}$ then workers in the M and H regions find jobs and separate with equal probability, and since all separations are exogenous, the share of workers in M is the same among the employed and the non-employed; consequently

$$\alpha = \mu_{\mathrm{E}} = \frac{\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{IU}} + \lambda_{\mathrm{UI}}}$$

[5] Except, of course, around $\lambda_{\mathrm{E}*} = 0$, which is why we exclude that; also, it is not relevant empirically.
[6] Also, the equation is a special case of Lemma 6.

is determined solely by the exogenous state process. However, if $\lambda_{\text{IE}} < \lambda_{\text{UE}}$ (which is usually what happens in the data), then $\alpha = \mu_{\text{E}}$ is lower, since we have a higher share of workers in H in employment, compared to non-employment.

The example below shows how to match the five moments we have selected with an embedded AR(1) process.

Example 4. *Consider the AR(1) process embedded in continuous time, ie*

$$s' = \rho s + \varepsilon$$

with $\varepsilon \sim \text{N}(0, 1)$*, IID.*[7] *The ergodic distribution of* s *is* $s \sim \text{N}(0, 1/(1-\rho^2))$*. Let*

$$M = (\infty, x] \qquad \textit{and} \qquad H = (x, \infty) \qquad \textit{for some } x \in \mathbb{R}.$$

Since $p_{\text{M}}/p_{\text{H}}$ *is strictly increasing in* x*, and goes from* 0 *to* ∞ *on* $\mathbb{R}$*, we can use Lemma 3 to match the five moments.*

4 The benchmark case: IID transitions

Another important case is IID transitions, ie $F(s' \mid s) = F(s')$, we denote this family with the subscript B and will refer to as the *benchmark model*. Note that the terminology is somewhat misleading: the transition to a new s is IID *conditional on a change event*, but the process for s is of course not IID, as γ still affects its persistence.[8]

In this case, we can analytically characterize the bounds on α globally. In order to do this, let's introduce the following notation for the transition rates into the three surplus regions:

$$\begin{aligned} q_{\text{L}} &= \gamma\, \mathbb{P}(\mathcal{S} \to L) \\ q_{\text{M}} &= \gamma\, \mathbb{P}(\mathcal{S} \to M) \\ q_{\text{H}} &= \gamma\, \mathbb{P}(\mathcal{S} \to H) \end{aligned}$$

Then the observed transition rates are

$$\lambda_{\text{EI}} = q_{\text{L}} + \mu_{\text{E}}\sigma \tag{9}$$

$$\lambda_{\text{EU}} = (1-\mu_{\text{E}})\sigma \tag{10}$$

$$\lambda_{\text{IE}} = \mu_{\text{I}}\kappa\lambda$$

$$\lambda_{\text{IU}} = q_{\text{H}} \tag{11}$$

$$\lambda_{\text{UE}} = \lambda$$

$$\lambda_{\text{UI}} = q_{\text{L}} + q_{\text{M}} \tag{12}$$

[7] Normalizing the variance to unity is without loss of generality.

[8] Versions of this simple persistent process are used frequently in the literature for tractable solutions, eg Hornstein, Krusell, and Violante (2011, Section III).

where μ_{E} and μ_{I} are functions of $(q_{\mathrm{L}}, q_{\mathrm{M}}, q_{\mathrm{H}}, \sigma, \lambda, \kappa)$ that can be derived using (6) and the flow balance equations

$$\begin{aligned} n_{\mathrm{L}}\left(q_{\mathrm{M}}+q_{\mathrm{H}}\right) &= \left(n_{\mathrm{M}}+n_{\mathrm{H}}+e_{\mathrm{M}}+e_{\mathrm{H}}\right) q_{\mathrm{L}} \\ n_{\mathrm{M}}\left(q_{\mathrm{L}}+q_{\mathrm{H}}+\kappa\lambda\right) &= \left(n_{\mathrm{L}}+n_{\mathrm{H}}\right) q_{\mathrm{M}}+\sigma e_{\mathrm{M}} \\ n_{\mathrm{H}}\left(q_{\mathrm{M}}+q_{\mathrm{L}}+\lambda\right) &= \left(n_{\mathrm{M}}+n_{\mathrm{L}}\right) q_{\mathrm{H}}+\sigma e_{\mathrm{H}} \\ e_{\mathrm{M}}\left(q_{\mathrm{L}}+q_{\mathrm{H}}+\sigma\right) &= e_{\mathrm{H}} q_{\mathrm{M}}+\kappa\lambda n_{\mathrm{M}} \\ e_{\mathrm{H}}\left(q_{\mathrm{M}}+q_{\mathrm{L}}+\sigma\right) &= e_{\mathrm{M}} q_{\mathrm{H}}+\lambda n_{\mathrm{H}} \end{aligned} \tag{13}$$

with

$$n_{\mathrm{L}}+n_{\mathrm{M}}+n_{\mathrm{H}}+e_{\mathrm{M}}+e_{\mathrm{H}}=1 \tag{14}$$

From (9), (10) and (12), the admissible range for q_{L} is

$$0 \leq q_{\mathrm{L}} \leq \min(\lambda_{\mathrm{E}*}, \lambda_{\mathrm{UI}})$$

Making the following assumption allows us to simplify the algebra.

Assumption 5. *The rate of all separations from employment is (weakly) smaller than the rate of transitions from unemployment to inactivity, ie*

$$\lambda_{\mathrm{E}*} \leq \lambda_{\mathrm{UI}}$$

As we shall see in Section 5, this assumption is innocuous, as in the data $\lambda_{\mathrm{E}*} \ll \lambda_{\mathrm{UI}}$. Making this assumption allows us to consider the range

$$0 \leq q_{\mathrm{L}} \leq \lambda_{\mathrm{E}*}$$

Lemma 6 (Lower bound on α, IID model)**.** *Consider a setup with IID transitions, characterized by* (9)–(12) *and* (13)–(14)*. For all parameters* $(q_{\mathrm{L}}, q_{\mathrm{M}}, q_{\mathrm{H}}, \sigma, \lambda, \kappa) \in \mathbb{R}_{+}^{5} \times [0,1)$ *such that the resulting flows match* $\lambda_{\mathrm{E}*} = \lambda_{\mathrm{EU}} + \lambda_{\mathrm{EI}}$*,* λ_{IE}*,* λ_{IU}*,* λ_{UE}*,* λ_{UI}*, the following are true:*

1. *The* μ_{E} *generated by the model is strictly decreasing in* q_{L}*, and has the* upper bound $\alpha_C = \mu_{\mathrm{E}}(q_{\mathrm{L}})$ *at* $q_{\mathrm{L}} = 0$*,*
2. *The* $\alpha_B(q_{\mathrm{L}})$ *generated by the model is strictly increasing in* q_{L}*, and has the* lower bound $\alpha_C = \alpha_B(q_{\mathrm{L}})$ *at* $q_{\mathrm{L}} = 0$*,*
3. *At* $q_{\mathrm{L}} = \lambda_{\mathrm{E}*}$*,* $\alpha_B = 1$*. Consequently, the range of* α_B *that the model can match is* $\mathcal{A}_B = [\alpha_C, 1]$*.*

Proof. 1. First, we solve (9)–(12) to obtain μ_E as a function of q_L:

$$\begin{aligned} q_M &= \lambda_{UI} - q_L \\ q_H &= \lambda_{IU} \\ \lambda &= \lambda_{UE} \\ \sigma &= \lambda_{EU} + \lambda_{EI} - q_L \end{aligned}$$

Then we find the steady state distributions using (13)–(14) and, with a bit of algebra, obtain

$$\mu_E(q_L) = \left(1 + \frac{\lambda_{IU}\left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE} + \lambda_{IE}\lambda_{UE} + \lambda_{E*}\lambda_{UE}\right)}{\lambda_{UI}\left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE} + \lambda_{IE}\lambda_{UE} + \lambda_{E*}\lambda_{IE}\right) - \left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE}\right) q_L}\right)^{-1} \tag{15}$$

Note that $0 \leq \mu_E \leq 1$, since

$$0 \leq q_L \leq \min(\lambda_{UI}, \lambda_{EI})$$

Also, (15) shows that μ_E is strictly decreasing in q_L.

2. With some algebra,

$$\alpha(q_L) = \frac{\left(\lambda_{IE}\lambda_{IU} - \lambda_{E*}\lambda_{IE}\right)\lambda_{UI} + \left(\lambda_{IU}^2 + \lambda_{IE}\lambda_{IU}\right)\lambda_{UE}}{\lambda_{E*}\lambda_{IE}\lambda_{UI} + \lambda_{E*}\lambda_{IU}\lambda_{UE}} + \frac{C_0}{C_1 - C_2 q_L}$$

where

$$\begin{aligned} C_0 &= \lambda_{IU}\left(\lambda_{UI} + \lambda_{IU}\right)\left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE} + \lambda_{IE}\lambda_{UE}\right)\left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE} + \lambda_{IE}\lambda_{UE} + \lambda_{E*}\lambda_{UE}\right) \\ C_1 &= \lambda_{E*}(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE})(\lambda_{IE}\lambda_{UI}^2 + \lambda_{IU}\lambda_{UE}\lambda_{UI} + \lambda_{IE}\lambda_{UE}\lambda_{UI} + \lambda_{IE}\lambda_{IU}\lambda_{UI} \\ &\qquad + \lambda_{E*}\lambda_{IE}\lambda_{UI} + \lambda_{IU}^2\lambda_{UE} + \lambda_{IE}\lambda_{IU}\lambda_{UE} + \lambda_{E*}\lambda_{IU}\lambda_{UE}) \\ C_2 &= \lambda_{E*}\left(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE}\right)^2 \end{aligned}$$

which is decreasing in q_L and has its minimum at $q_L = 0$.

3. Follows from setting $q_L = \lambda_{E*}$, also cf (16) below.

□

Even though the algebra behind the result is a bit complicated the intuition relatively simple. First, holding λ_{UI} constant means that the higher q_L, the lower q_M is (cf (12)), and thus fewer employed workers will be in the marginal region, making the share of marginal workers μ_E decrease.

In order to understand why α is increasing it is useful to rearrange the definition as

$$\alpha(q_L) = \frac{q_L + \mu_E\sigma}{\lambda_{E*}} = \frac{(1-\mu_E)q_L + \mu_E\lambda_{E*}}{\lambda_{E*}} \tag{16}$$

Since λ_{E*} is matched, q_L changes α via two channels: *directly* and via μ_E. The direct effect makes α increasing in q_{L}, while the effect of μ_{E}, which we will call the *composition effect*, works in the other direction. However, as we have shown above, the direct effect always overwhelms the composition effect, making α increasing in q_{L}. This is because the effect of q_{L} on μ_{E} is very small and indirect.

The algebra of Lemma 6 is complicated because of the need to match λ_{IE}, which involves the endogenous share μ_{I}, which is itself a function of the parameters — since the latter is an eigenvalue of the unit eigenvector, there is no simple expression for it in the general case. Ignoring this condition and leaving κ in as a free parameter would allow one to match a larger range of α compared to the benchmark model. Exploring this allows us to investigate what role does the need to match flows from inactivity to employment play in the results, both for the lower bound on α and the sign of its derivative with respect to q_{L}. The following lemma characteries μ_{E} and κ as a function of q_{L} and κ.

Lemma 7. *When we match λ_{UE}, $\lambda_{\mathrm{E*}}$, λ_{IU} and λ_{UI}, but not λ_{IE}, with q_{L} and κ as free parameters, the following are true:*

1. *$\mu_{\mathrm{E},K}(q_{\mathrm{L}},\kappa)$ is strictly increasing in q_{L} and κ over its whole domain.*
2. *$\alpha_K(q_{\mathrm{L}},\kappa)$ is strictly increasing in q_{L} and κ over its whole domain, and thus it assumes its minimum at $q_{\mathrm{L}}=\kappa=0$.*

Proof. We solve the system (9), (10), (11)–(14) to obtain

$$\mu_{\mathrm{E},K}=1+\frac{\lambda_{\mathrm{IU}}\left(\kappa q_{\mathrm{L}}-\kappa\lambda_{\mathrm{UI}}-\kappa\lambda_{\mathrm{UE}}-\lambda_{\mathrm{IU}}-\lambda_{\mathrm{E*}}\right)}{\kappa\left(\frac{\kappa\lambda_{\mathrm{UE}}+(\kappa+1)\lambda_{\mathrm{IU}}+\kappa\lambda_{\mathrm{E*}}}{2\kappa}+\lambda_{\mathrm{UI}}-q_{\mathrm{L}}\right)^2-C_0}$$

where $C_0>0$ is a constant.[9] Clearly, μ_{E} is increasing in q_{L}. Differentiating by κ yields

$$\frac{\partial\mu_{\mathrm{E},K}}{\partial\kappa}\propto\lambda_{\mathrm{E*}}\lambda_{\mathrm{IU}}\left(\lambda_{\mathrm{UI}}-q_{\mathrm{L}}\right)\left(\lambda_{\mathrm{UI}}+\lambda_{\mathrm{UE}}+\lambda_{\mathrm{IU}}+\lambda_{\mathrm{E*}}-q_{\mathrm{L}}\right)>0$$

Then

$$\alpha_K=\frac{1}{\lambda_{\mathrm{E*}}}\frac{C_1-\kappa\lambda_{\mathrm{IU}}\left(\frac{\kappa\lambda_{\mathrm{UI}}+\kappa\lambda_{\mathrm{UE}}+\lambda_{\mathrm{IU}}+(\kappa+1)\lambda_{\mathrm{E*}}}{2\kappa}-q_{\mathrm{L}}\right)^2}{\kappa\left(\frac{\kappa\lambda_{\mathrm{UE}}+(\kappa+1)\lambda_{\mathrm{IU}}+\kappa\lambda_{\mathrm{E*}}}{2\kappa}+\lambda_{\mathrm{UI}}-q_{\mathrm{L}}\right)^2-C_0} \tag{17}$$

where

$$C_1=\frac{\lambda_{\mathrm{IU}}\left(\kappa\lambda_{\mathrm{UI}}+\kappa\lambda_{\mathrm{UE}}+\lambda_{\mathrm{IU}}-\kappa\lambda_{\mathrm{E*}}+\lambda_{\mathrm{E*}}\right)^2}{4\kappa}$$

In the quadratic expression in the numerator, since $\lambda_{\mathrm{E*}}+\lambda_{\mathrm{UI}}>2q_{\mathrm{L}}$, the numerator is increasing in q_{L}, while the denominator is decreasing in q_{L}, and thus consequently α is increasing in q_{L}. Similarly to μ_{E}, differentiating (17) can used to show that α_K is increasing in κ. □

[9]

$$C_0=\frac{\kappa^2\lambda_{\mathrm{UE}}^2-2\kappa^2\lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}+2\kappa\lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}+2\kappa^2\lambda_{\mathrm{E*}}\lambda_{\mathrm{UE}}+\kappa^2\lambda_{\mathrm{IU}}^2-2\kappa\lambda_{\mathrm{IU}}^2+\lambda_{\mathrm{IU}}^2+2\kappa^2\lambda_{\mathrm{E*}}\lambda_{\mathrm{IU}}-2\kappa\lambda_{\mathrm{E*}}\lambda_{\mathrm{IU}}+\kappa^2\lambda_{\mathrm{E*}}^2}{4\kappa}$$

When comparing to the benchmark case in Lemma 6, it is important to note that if we matched λ_{IE}, too, then the result above would still hold, except that κ would no longer be a free parameter, but a function of q_{L}. Comparing the two lemmas, it is apparent that this effect is so strong that it makes μ_{E} decreasing in q_{L} when it also changes κ. However, since the composition effect is not very strong anyway, the effect on α is the same. The intuition for κ is straightforward: when it is increased, marginal non-employed find jobs are a higher rate, and thus their share amond the employed μ_{E} increases. Since κ itself has no direct effect on α, only the composition effect via μ_{E} increases α. The following corollary is useful for understanding the most important determinants of the numerical value of α_C, discussed in Section 5.

Corollary 1 (Bounds for α_C)**.** *Since the corner case is a special case of this model for some $\kappa \in [0, 1)$,*

$$\underbrace{\frac{\lambda_{\text{UI}}}{\lambda_{\text{UI}} + \lambda_{\text{IU}} + \lambda_{\text{E}*}}}_{\alpha_K(0,0)} \leq \alpha_C < \underbrace{\frac{\lambda_{\text{UI}}}{\lambda_{\text{UI}} + \lambda_{\text{IU}}}}_{\alpha_K(0,1)}$$

Proof. Follows from (17), after algebraic simplification. □

The upper bound above has very simple intuition. When $\kappa = 1$, not only separation but also job finding occur at the same rate for workers, regardless of whether they have a marginal or a high surplus. Since the process for s is independent of job flows, the share of workers in region M and H, conditional on employment status, has to be the same, and it depends only on the probabilities that govern transitions between regions of $\mathcal{S}$. When $\kappa = 0$, we are at the opposite extreme: the only inflow to marginal workers is from high-surplus workers, hence their share is lower. As we shall see below, in the data $\lambda_{\text{UI}} \gg \lambda_{\text{IU}} + \lambda_{\text{E}*}$, so the lower bound is already high, and in fact will not be very far away from α_C.

5 Data and related literature

Lemma 6 shows that we can always match five out of six moments with the model introduced in Section 2 but when using an IID F we can only match α_D in the data when it is above α_C implied by the other five moments. This motivates the introduction of a *discrepancy index*

$$\Delta_\alpha = \alpha_D - \alpha_C$$

where α_D is calculated according to (7) and α_C is calculated using (8). Consequently Δ_α is a scalar that can be calculated from the flows only and it can be used to measure the discrepancy between models and the data: when $\Delta_\alpha > 0$ the model cannot match the labor market flows. In this section we calculate α_D, α_C, and their discrepancy for various papers that either provide estimates of labor market transition rates or calibrate a model with a participation margin to labor market flows. In the latter case we also provide a brief summary of the differences between the model that is discussed in each paper and our benchmark model in Section 2. Table 2 provides a compact summary of the flows the αs and Δ_α.

	λ_{EU}	λ_{EI}	λ_{UE}	λ_{UI}	λ_{IU}	λ_{IE}	α_D	α_C	Δ_α
Andolfatto, Gomme, and Storer (1998)	.016	.016	.310	.155	.026	.022	.49	.83	.34
Fallick and Fleischman (2004)	.018	.026	.404	.330	.045	.035	.59	.88	.29
Garibaldi and Wasmer (2005) 15–64 yrs	.010	.016	.259	.166	.035	.044	.61	.80	.19
Garibaldi and Wasmer (2005) 25–54 yrs	.008	.010	.256	.133	.046	.034	.55	.71	.16
Pries and Rogerson (2009)	.011	.015	.234	.144	.038	.043	.58	.76	.19
Krusell et al. (2011) unadjusted	.018	.024	.385	.318	.041	.038	.57	.87	.30
Krusell et al. (2011) broad unemployment	.029	.016	.343	.336	.029	.064	.36	.80	.43

Table 2: Summary of various calibrations. Observed transition rates are monthly, corrected for time aggregation when necessary, displayed with 3 significant digits (calculations of course use the unrounded values). The last three columns contain the corresponding α_D, α_C, and Δ_α displayed with 2 significant digits. 0s before the decimal dot are omitted in order to obtain a compact table. Note that $\Delta_\alpha > 0$ for all papers indicating that the models cannot fit the data.

Andolfatto, Gomme, and Storer (1998) were among the first to emphasize the importance of the participation margin for modeling labor markets. Similarly to this paper they use a frictional labor market model that allows job offers for inactive workers with a probability that is lower compared to unemployed workers who search actively. They use $(w, v) \in \mathcal{S} = \mathbb{R}^2_+$ as a state for the workers where potential w is the wage and v is the potential value of home production. This formulation has the consequence that the unemployed in their model are those who have a have drawn a low wage and home production because if either one is larger than the other the worker will search actively or remain inactive. Also in their model the rate at which changes arrive to w is endogenous, because search will increase the probability of new offers and unemployment benefits are history-dependent. Despite these differences their model is very similar to our benchmark model, so it is not surprising that they cannot match labor market flows: the Δ_α calculated for their data is 0.34. They argue that the model has difficulties matching flows into and out of the labor force, but we have seen in Section 3 that this is not the case; however this view has influenced the subsequent literature.

Fallick and Fleischman (2004) provide a detailed and methodologically thorough descriptive summary of gross labor market flows using CPS data between 1994:1–2003:12. After adjusting for time aggregation[10] we find that the discrepancy between the corner parameterization and the data is $\Delta_\alpha = 0.29$.

Garibaldi and Wasmer (2005) present a model that is very close to the one in this paper—in fact the worker side of their baseline model is nested by our benchmark model in this paper, but their model is general equilibrium one, and is thus closed by modeling job creation. They use CPS data between 1995:10–2001:12 and calculate transition rates using the Abowd and Zellner (1985) correction. They argue that EI and IE flows are the result of time aggregation and misclassification and calibrate their baseline model accordingly. They also allow for a positive job finding rate for the inactive ("jobs bump in to people") similarly to our model in their extended model. Even with these adjustments they can't match labor market flows which is in line with the results of Section 4, as Δ_α is 0.16 (ages 25–54) and 0.19 (ages 16-64) for their

[10] Using the method in Shimer (2012).

dataset. Following Andolfatto, Gomme, and Storer (1998) they see the main discrepancy in the UI and IU flows and try to extend the model accordingly, but as we have seen in Section 4 this in itself cannot fix the problem.

Pries and Rogerson (2009) use model similar to the one in Section 2 to motivate an explanation for cross-country differences in participation patterns. The most important difference between their model and the one in this paper is that theirs has a job-specific state and thus it can potentially provide richer flow patterns. Our model nests all other components of theirs as both feature linear utility and binary search decisions, and their only individual-specific state is a scalar that represents the cost associated with labor force participation and can take two values in their parameterization, and thus $\mathcal{S} = \{x_b, x_g\}$. They use March CPS data between 1990–2000, restricting ages between 16–64 years which yields $\Delta_\alpha = 0.19$. Consequently their model cannot match labor market flows, but following Andolfatto, Gomme, and Storer (1998) they also emphasize the model's inability to explain the magnitude of IU and UI flows.

Krusell et al. (2011) construct a three-state model with asset accumulation and nonlinear utility arguing that linear utility imposes implicit assumptions on income and substitution effects. In their model workers have a scalar productivity state s_t which evolves stochastically following an AR(1) process which is later extended with temporary shocks. Saving and consumption decisions also play a role in labor market transitions but these differences turn out to play a minor role in practice—Section 6 of their paper discusses a setup with complete markets which is effectively similar to linear utility. The most important difference is that they allow only a single search intensity, arguing based on time-use surveys that search costs are small. In order to account for IE transitions they adjust flow rates by extending the notion of unemployment to include marginally attached workers. Calculation of Δ_α for both the unadjusted CPS data ($\alpha_D = 0.30$) and the flows with the extended unemployment state ($\alpha_D = 0.49$) suggest that this data adjustment makes it even more difficult to bring the model close to the data, which is apparent in their Table 6, which shows that the model missed α_D by a large margin. This demonstrates that a seemingly innocuous and potentially useful adjustment to the data can have unexpected consequences.

In summary even though the papers discussed above use various modeling approaches and datasets (though mostly variants of the CPS), they cannot match labor market transition rates in the data. While formally the benchmark model presented in Section 2 (and with the IID special case in Section 4) only nests special parameterizations of these models, the corresponding α_Ds combined with Lemma 6 suggests an explanation for this discrepancy. Also even though many of these papers talk about the difficulty of matching IU and UI flows, Lemma 3 suggests that the difficulty lies in matching the share of EI flows of all separations.

6 Extensions

Considering that $\Delta_\alpha > 0$ for all the calibrations reviewed in Section 5, we conclude that not only does the benchmark model discussed in Section 4 have a limited range of labor market flows it can generate, but the data appears to lie outside this range and thus the problem is empirically relevant.

In this section we discuss various extensions and check if they alleviate this problem. First, in Section 6.1 we explore how state-dependent separation rates affect labor market flows and whether this is relevant to our problem. Then in Section 6.2 we relax the assumption of IID distributions in a simplified and analytically tractable setting. In Section 6.3 we generalize the IID model in another direction, allowing transition rates to differ based on employment status. We aim for global analytical results in all of these discussions.

6.1 State-dependent separation rates

We extend the model by allowing the rates for exogenous separation to depend on the state. For analytical simplicity we only distinguish separation rates for marginal (σ_M) and non-marginal workers (σ_H). Also since α is the closest to the data in the corner case we don't derive the full model but only consider the modification of the model in Section 3.

The observed transition rates are consequently

$$\begin{aligned} \lambda_{\text{EI}} &= \mu_{\text{E}}\sigma_M \\ \lambda_{\text{EU}} &= (1-\mu_{\text{E}})\sigma_H \\ \lambda_{\text{IE}} &= \kappa\lambda \qquad (18) \\ \lambda_{\text{IU}} &= q_{\text{H}} \\ \lambda_{\text{UE}} &= \lambda \\ \lambda_{\text{UI}} &= q_{\text{M}} \qquad (19) \end{aligned}$$

where μ_{E} is endogenous, analogously to (8).[11] For analytical convenience we parameterize the deviation from the corner case with the parameter δ_σ as

$$\sigma_M = \sigma \cdot (1 + (1-\mu_{\text{E}})\delta_\sigma) \qquad \sigma_H = \sigma \cdot (1 - \mu_{\text{E}}\delta_\sigma) \tag{20}$$

where $\sigma_M, \sigma_H \geq 0$ implies that

$$-\frac{1}{1-\mu_{\text{E}}} \leq \delta_\sigma \leq \frac{1}{\mu_{\text{E}}} \tag{21}$$

When $\Delta_\sigma > 0$ marginal workers experience exogenous separations at a higher rate. The formulation in (20) is convenient because when we add up all the separations,

$$\lambda_{\text{E}*} = \lambda_{\text{EI}} + \lambda_{\text{EU}} = \mu_{\text{E}}\sigma_M + (1-\mu_{\text{E}})\sigma_H = \sigma \tag{22}$$

and thus the calculation of all five parameters κ, λ, q_{M}, q_{H}, and σ is straightforward.

Lemma 8 (State-dependent separation rates)**.** *When the five other moments are matched to the observed flows, $\mu_{\text{E},S}(\delta_\sigma)$ is decreasing while $\alpha_S(\delta_\sigma)$ is increasing in δ_σ.*

[11]Flow balance equations are analogous to (4)–(5), *mutatis mutandis*, and are not repeated here.

Proof. Since the calculation of the model parameters is straightforward (see (18)–(19) and (22)) it is the determination of μ_{E} that is difficult, especially because the rates σ_M and σ_H depend on it themselves. When we calculate the shares of workers in marginal and non-marginal employment in the steady state and then equate the ratio of marginal workers to μ_{E}, we obtain the following implicit characterization in terms of δ_σ:

$$0 = \delta_\sigma \mu_{\mathrm{E}}^2 - \delta_\sigma \mu_{\mathrm{E}} - a\mu_{\mathrm{E}} + b$$

where

$$a = \frac{\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}^2 + \left(\left(\lambda_{\mathrm{IU}} + \lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}} + \lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}}}{\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}}$$

$$b = \frac{\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}^2 + \left(\left(\lambda_{\mathrm{IU}} + \lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UI}}}{\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}}\lambda_{\mathrm{UE}}}$$

It is easy to check that

$$a > 1 \tag{23}$$

Implicit differentiation yields

$$\frac{d\mu_{\mathrm{E}}}{d\delta_\sigma} = \frac{\mu_{\mathrm{E}}(1-\mu_{\mathrm{E}})}{2\delta_\sigma\mu_{\mathrm{E}} - \delta_\sigma - a} \tag{24}$$

Because of (21) and (23), the denominator on the right hand side of (24) is negative, and thus $d\mu_{\mathrm{E}}/d\delta_\sigma < 0$. Now we express

$$\alpha_S(\delta_\sigma) = \frac{\mu_{\mathrm{E}}\sigma_M}{\lambda_{\mathrm{E}*}} = \mu_{\mathrm{E}} \cdot (1 + (1-\mu_{\mathrm{E}})\delta_\sigma) = (1-a)\mu_{\mathrm{E}} + b$$

Because of (23), α_S is decreasing in μ_{E} and consequently it is increasing in δ_σ. □

The intuition again comes from direct and composition effects. The direct effect of increasing δ_σ increases σ_M and thus α, since it makes marginal workers separate with a higher rate. However, because of this, their share among all employed workers diminishes, and thus μ_{E} decreases. The net effect is that the direct effect overwhelms the composition effect.

Empirically, we consider $\delta_\sigma > 0$ the plausible parameterization: which drives α further away from the data, and thus we conclude that introducing different separation rates it not a solution to the puzzle.

6.2 Three-point discrete distributions

The IID case discussed in Section 4 turned out to be analytically tractable because conditional on a state change the distribution of the new state was IID and consequently the cross-sectional distributions in the regions $\{LMH\} \times \{\mathrm{NE}\}$ were just truncated versions of $F(s')$.[12] However when we allow s' to depend on s this no longer holds because of the asymmetric inflows and outflows: for example in the region (M, N) workers enter from (L, N) and (H, N) but for the

[12] Naturally (LE) has no workers in equilibrium.

employed in (M, E) they only enter from (H, E). Consequently flows can only be calculated precisely if we keep track of the whole cross-sectional distribution, which makes the problem difficult to handle analytically.

In order to simplify this setup consider a parameterization such that $\mathcal{S}$ consists of three elements l, m, h which are in L, M, and H respectively, and that the transition probabilities according to F are $p_{\mathrm{LM}}, p_{\mathrm{LH}}, \ldots$. Let $q_{\mathrm{LM}} = \gamma p_{\mathrm{LM}}, \ldots$ denote the corresponding transition rates. This model has nine parameters: the rates σ, κ, and λ and the six transition rates. In order to relate to the IID model we parameterize the latter as

$$
\begin{aligned}
q_{\mathrm{ML}} &= q_{\mathrm{L}} + \delta_L & \qquad q_{\mathrm{HL}} &= q_{\mathrm{L}} - \delta_L \\
q_{\mathrm{LM}} &= q_{\mathrm{M}} + \delta_M & \qquad q_{\mathrm{HM}} &= q_{\mathrm{M}} - \delta_M \\
q_{\mathrm{MH}} &= q_{\mathrm{H}} + \delta_H & \qquad q_{\mathrm{LH}} &= q_{\mathrm{H}} - \delta_H
\end{aligned}
$$

Clearly $\delta_L = \delta_M = \delta_H = 0$ yields the IID model which only had q_{L} as a free parameter. The transformation into δs is advantageous because it also allows us to think about the process in terms of economics: for example $\delta_L > 0$ and $\delta_H > 0$ would imply that conditional on a change event s is more likely to move into a closer state (on the other hand the sign of δ_M has no similar interpretation). We use the subscript T("three point") to denote this model family.

In the steady state,

$$
\begin{aligned}
n_{\mathrm{L}} \left(q_{\mathrm{LM}} + q_{\mathrm{LH}}\right) &= (n_{\mathrm{M}} + e_{\mathrm{M}})q_{\mathrm{ML}} + (n_{\mathrm{H}} + e_{\mathrm{M}})q_{\mathrm{HL}} \\
n_{\mathrm{M}} \left(q_{\mathrm{ML}} + q_{\mathrm{MH}} + \kappa\lambda\right) &= n_{\mathrm{L}} q_{\mathrm{LM}} + n_{\mathrm{H}} q_{\mathrm{HM}} + \sigma e_{\mathrm{M}} \\
n_{\mathrm{H}} \left(q_{\mathrm{HM}} + q_{\mathrm{HL}} + \lambda\right) &= n_{\mathrm{M}} q_{\mathrm{MH}} + n_{\mathrm{L}} q_{\mathrm{LH}} + \sigma e_{\mathrm{H}} \\
e_{\mathrm{M}} \left(q_{\mathrm{ML}} + q_{\mathrm{MH}} + \sigma\right) &= e_{\mathrm{H}} q_{\mathrm{HM}} + \kappa\lambda n_{\mathrm{M}} \\
e_{\mathrm{H}} \left(q_{\mathrm{HM}} + q_{\mathrm{HL}} + \sigma\right) &= e_{\mathrm{M}} q_{\mathrm{MH}} + \lambda n_{\mathrm{H}}
\end{aligned}
$$

Lemma 9.

$$
\alpha_T(q_{\mathrm{L}}, \delta_L, \delta_M, \delta_H) \geq \alpha_C \qquad \textit{for all feasible } \delta_L, \delta_M, \delta_H
$$

The proof is rather complicated and thus it is included in the appendix. However, intuitively the result merely refines Lemma 6 and the intuition is the same: δ_L has a direct and a composition effect, but when considering q_{ML} and q_{HL}, α is increasing in both. δ_M and δ_H only affect μ_{E}, and thus have no direct effect.

6.3 Human capital model

Empirical evidence suggests the possibility that human capital deteriorates during non-employment, and also that workers may accumulate human capital while working (learning by doing), or lose human capital during unemployment. In order to incorporate features like this into the model we allow the transition kernels to be different for employed and non-employed workers. However for tractability we return to an IID distributions for the transition kernel F

— as we have seen in Section 6.2, non-IID transition processes do not lead to a lower α, so that direction of generality does not extend the model in a way that would bring it closer to the data.

We consider two different extensions. In the first one, non-employed workers have a different (possibly higher) probability of transitioning to the L region, compared to employed workers. We call this the *skill depreciation* model, as it allows a stylized representation for human capital loss or skill depreciation during unemployment. In the second extension, we allow employed workers to have a different (again, possibly higher) probability of transitioning to the H region, compared to non-employed workers. We call this the *skill accumulation* model, as it allows a stylized representation of skill accumulation for employed workers. We consider the implications of these two extensions below.

6.3.1 Skill depreciation for nonemployed

For the skill depreciation model, we denote the transition rates to region L by q_L^E and q_L^N for employed and non-employed workers, respectively. The observed labor market flows are

$$\lambda_{EI} = \mu_E\sigma + q_L^E \tag{25}$$
$$\lambda_{EU} = (1-\mu_E)\sigma$$
$$\lambda_{UI} = q_L^N + q_M \tag{26}$$
$$\lambda_{IU} = q_H$$
$$\lambda_{UE} = \lambda$$
$$\lambda_{IE} = \mu_I\kappa\lambda \tag{27}$$

Only (25) and (26) are different from the IID setup in Section 4. Again, we will leave in q_L^N and q_L^E as free parameters, and solve for the other parameters and endogenous terms μ_I, μ_E, and α in terms of these. In the steady state,

$$\begin{aligned}
n_L(q_M+q_H) &= (n_M+n_H)q_L^N + (e_M+e_H)q_L^E \\
n_M(q_L^N+q_H+\kappa\lambda) &= (n_L+n_H)q_M + \sigma e_M \\
n_H(q_L^N+q_M+\lambda) &= (n_M+n_L)q_H + \sigma e_H \\
e_M(q_H+q_L^E+\sigma) &= e_H q_M + \kappa\lambda n_M \\
e_H(q_M+q_L^E+\sigma) &= e_M q_H + \lambda n_H
\end{aligned}$$

Furthermore, $q_M \geq 0$ and $\mu_E\sigma \geq 0$ implies that

$$0 \leq q_L^E \leq \lambda_{E*} \qquad 0 \leq q_L^N \leq \lambda_{UI} \tag{28}$$

Lemma 10 (Skill depreciation for the unemployed)**.** *Consider a setup characterized by* (25)–(27)*, and let the subscript N denote this model family. For all parameters* $(q_L^N, q_L^E, q_M, q_H, \sigma, \lambda, \kappa) \in \mathbb{R}_+^6 \times [0,1)$ *such that the resulting flows match* $\lambda_{E*} = \lambda_{EU} + \lambda_{EI}$*,* λ_{IE}*,* λ_{IU}*,* λ_{UE}*,* λ_{UI}*, the following are true:*

1. *μ_{E} is increasing in $q_{\mathrm{L}}^{\mathrm{E}}$, and decreasing in $q_{\mathrm{L}}^{\mathrm{N}}$.*

2. *α is increasing in $q_{\mathrm{L}}^{\mathrm{E}}$, and decreasing in $q_{\mathrm{L}}^{\mathrm{N}}$.*

3. *The minimal value of α_N^* is at the maximum feasible value of $q_{\mathrm{L}}^{\mathrm{N}} = \lambda_{\mathrm{UI}}$ and the minimum feasible value of $q_{\mathrm{L}}^{\mathrm{E}} = 0$, and is equal to*

$$\alpha_N^* \equiv \min \alpha = \frac{\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}}{\left(\lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}}} < \alpha_C$$

Proof. We solve the steady state flow balance equations and (25)–(27) to obtain

$$\mu_{\mathrm{E}} = 1 + \frac{\lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}}\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}}}{\left(\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}} + \lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}}\right)\cdot q_{\mathrm{L}}^{\mathrm{N}} - \lambda_{\mathrm{IE}}\lambda_{\mathrm{UE}}\cdot q_{\mathrm{L}}^{\mathrm{E}} - C_1}$$

where

$$C_1 = \lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}^2 + \left(\left(\lambda_{\mathrm{IU}} + \lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}} + \lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}}$$

This demonstrates that μ_{E} is increasing in $q_{\mathrm{L}}^{\mathrm{E}}$ and decreasing in $q_{\mathrm{L}}^{\mathrm{N}}$. Similarly we find that

$$\alpha_N = 1 + \frac{C_2 - \left(\lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}}\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}}\right)\cdot q_{\mathrm{L}}^{\mathrm{E}}}{\left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}}\right)\cdot q_{\mathrm{L}}^{\mathrm{N}} - \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UE}}\cdot q_{\mathrm{L}}^{\mathrm{E}} - C_3}$$

where

$$\begin{aligned} C_2 &= \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}}\lambda_{\mathrm{UI}} + \left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}^2\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}} \\ C_3 &= \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{UI}}^2 + \left(\left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UE}} + \lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}}\lambda_{\mathrm{IU}} + \lambda_{\mathrm{E}*}^2\lambda_{\mathrm{IE}}\right)\lambda_{\mathrm{UI}} \\ &\quad + \left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IU}}^2 + \left(\lambda_{\mathrm{E}*}\lambda_{\mathrm{IE}} + \lambda_{\mathrm{E}*}^2\right)\lambda_{\mathrm{IU}}\right)\lambda_{\mathrm{UE}} \end{aligned}$$

which implies that α is decreasing in $q_{\mathrm{L}}^{\mathrm{N}}$. Differentiating by $q_{\mathrm{L}}^{\mathrm{E}}$, it can be shown that

$$\frac{\partial \alpha_N}{\partial q_{\mathrm{L}}^{\mathrm{E}}} = -\frac{C_4\left(q_{\mathrm{L}}^{\mathrm{N}} - \lambda_{\mathrm{UI}} - \lambda_{\mathrm{IU}} - \lambda_{\mathrm{E}*}\right)}{C_5} \qquad \text{where } C_4, C_5 > 0$$

so (28) implies that α is increasing in $q_{\mathrm{L}}^{\mathrm{E}}$. □

The key to understanding this result is, again, the decomposition of α,

$$\alpha_N = \frac{q_{\mathrm{L}}^{\mathrm{E}} + (\lambda_{\mathrm{E}*} - q_{\mathrm{L}}^{\mathrm{E}})\mu_{\mathrm{E}}}{\lambda_{\mathrm{E}*}}$$

which is very similar to (16) except that it has $q_{\mathrm{L}}^{\mathrm{E}}$ instead of q_{L}. Recall that in the benchmark model, the direct effect of q_{L} would increase α, while the composition effect via μ_{E} would decrease it, with the direct effect dominating. However, in this version of the model, we can decouple the direct and the composition effect via $q_{\mathrm{L}}^{\mathrm{E}}$ and $q_{\mathrm{L}}^{\mathrm{N}}$: we can keep $q_{\mathrm{L}}^{\mathrm{E}}$ low (or zero) so that it does not increase α, while at the same time increase $q_{\mathrm{L}}^{\mathrm{N}}$ which will decrease μ_{E} and consequently α. The mechanism for the latter is the following: keeping λ_{IU} constant in (26), increasing $q_{\mathrm{L}}^{\mathrm{N}}$ decreases q_{M}, the transition rate to the marginal region M, which means that the share of marginal employed workers μ_{E} will decrease.

	α_D	α_N^*
Andolfatto, Gomme, and Storer (1998)	0.49	0.17
Fallick and Fleischman (2004)	0.59	0.22
Garibaldi and Wasmer (2005) 15–64 yrs	0.61	0.14
Garibaldi and Wasmer (2005) 25–54 yrs	0.55	0.06
Pries and Rogerson (2009)	0.58	0.12
Krusell et al. (2011) unadjusted	0.57	0.20
Krusell et al. (2011) broad unemployment	0.36	0.11

Table 3: Comparing the data and the model with skill depreciation during unemployment. Second column: α in the data, third column: possible minimum. See Table 2 for discussion of the data.

Table 3 shows that this can lower α significantly and in principle is more than enough to match the labor market flows. Figure 4 shows that locus that matches α (and of course all the other moments). Note that $q_{\mathrm{L}}^{\mathrm{N}} \gg q_{\mathrm{L}}^{\mathrm{E}}$, so this effect needs to be very strong to match labor market flow data.

6.3.2 Learning on the job

We extend the benchmark model of Section 4 in another direction, by allowing *employed* workers to transition into region H with a *higher* rate $q_{\mathrm{H}}^{\mathrm{E}}$, compared to non-employed workers who can expect this transition at rate $q_{\mathrm{H}}^{\mathrm{N}}$. This is motivated by the empirical finding that workers gain experience and possibly human capital while being employed . While it can be argued that the latter would be better represented by a continuous state, our crude approximation provides a tractable illustration with some advantageous properties. The observed labor market flows are

$$\lambda_{\mathrm{EI}} = q_{\mathrm{L}} + \sigma\mu_{\mathrm{E}} \tag{29}$$

$$\lambda_{\mathrm{EU}} = \sigma(1-\mu_{\mathrm{E}})$$

$$\lambda_{\mathrm{UI}} = q_{\mathrm{M}} + q_{\mathrm{L}}$$

$$\lambda_{\mathrm{IU}} = q_{\mathrm{H}}^{\mathrm{N}} \tag{30}$$

$$\lambda_{\mathrm{UE}} = \lambda$$

$$\lambda_{\mathrm{IE}} = \kappa\lambda\mu_{\mathrm{I}}$$

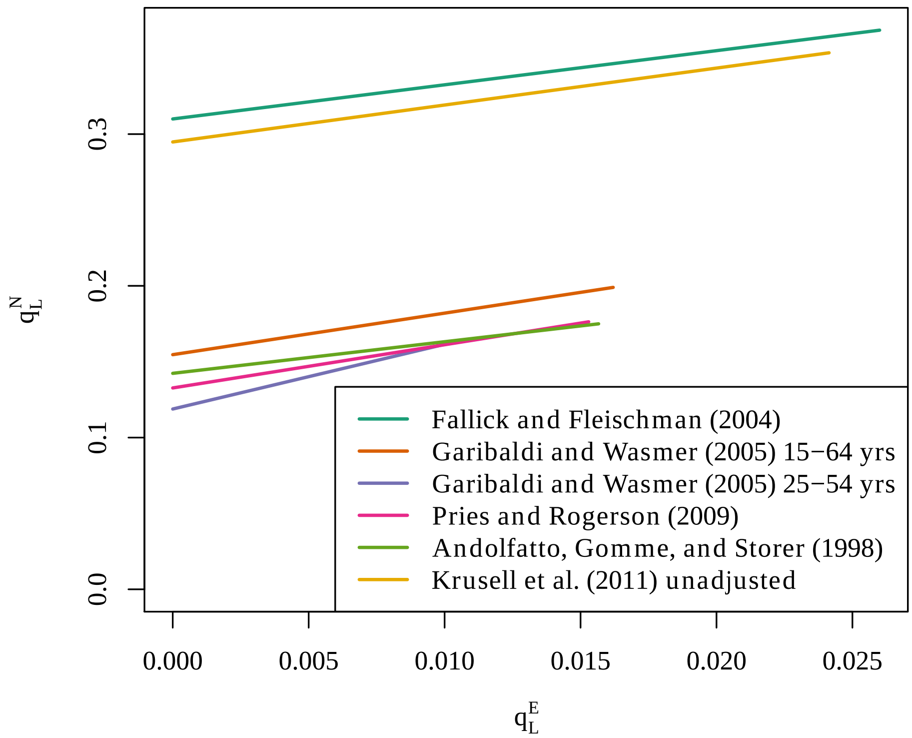

Figure 4: Locus for matching α and all the other five moments, using the model with skill depreciation for the unemployed (equations (25)–(27)). Note that the probability of transitioning to region L is needs to be about 10 times larger for the non-employed (q_L^N) compared to the employed (q_L^E).

and equations (31)–(32) characterize the steady state distribution of workers.

$$
\begin{aligned}
n_{\mathrm{L}}\left(q_{\mathrm{H}}^{\mathrm{N}}+q_{\mathrm{M}}\right) &= (n_{\mathrm{M}}+n_{\mathrm{H}}+e_{\mathrm{M}}+e_{\mathrm{H}})\,q_{\mathrm{L}} && (31)\\
n_{\mathrm{M}}\left(q_{\mathrm{H}}^{\mathrm{N}}+q_{\mathrm{L}}+\kappa\lambda\right) &= (n_{\mathrm{L}}+n_{\mathrm{H}})\,q_{\mathrm{M}}+\sigma e_{\mathrm{M}}\\
n_{\mathrm{H}}\left(q_{\mathrm{M}}+q_{\mathrm{L}}+\lambda\right) &= (n_{\mathrm{M}}+n_{\mathrm{L}})\,q_{\mathrm{H}}^{\mathrm{N}}+\sigma e_{\mathrm{H}}\\
e_{\mathrm{M}}\left(q_{\mathrm{L}}+q_{\mathrm{H}}^{\mathrm{E}}+\sigma\right) &= e_{\mathrm{H}}q_{\mathrm{M}}+\kappa\lambda n_{\mathrm{M}}\\
e_{\mathrm{H}}\left(q_{\mathrm{M}}+q_{\mathrm{L}}+\sigma\right) &= e_{\mathrm{M}}q_{\mathrm{H}}^{\mathrm{E}}+\lambda n_{\mathrm{H}} && (32)
\end{aligned}
$$

We leave q_{L} and $q_{\mathrm{H}}^{\mathrm{E}}$ in as free parameters, and solve for α, μ_{E}, and μ_{I} as functions of these, while matching the usual five moments of the flows. This formulation is convenient because the feasible domain for the free parameters is

$$
0 \le q_{\mathrm{L}} \le \min(\lambda_{\mathrm{E}*}, \lambda_{\mathrm{UI}}), \qquad 0 \le q_{\mathrm{H}}^{\mathrm{E}} \tag{33}
$$

In particular, $q_{\mathrm{H}}^{\mathrm{E}}$ can be as large as desired. The following lemma characterizes the minimum value for α.

Lemma 11 (Learning on the job)**.** *Consider a setup characterized by* (29)–(32)*. Let the subscript E denote this model family. For all parameters* $(q_{\mathrm{L}}, q_{\mathrm{M}}, q_{\mathrm{LH}}, q_{\mathrm{H}}^{\mathrm{E}}, \sigma, \lambda, \kappa) \in \mathbb{R}_{+}^{6} \times [0,1)$ *such that the resulting flows match* $\lambda_{\mathrm{E}*} = \lambda_{\mathrm{EU}} + \lambda_{\mathrm{EI}}$*,* λ_{IE}*,* λ_{IU}*,* λ_{UE}*,* λ_{UI}*, the following are true:*

1. $\mu_{\mathrm{E},E}$ *is decreasing in* q_{L} *and* $q_{\mathrm{H}}^{\mathrm{E}}$*,*
2. α_E *is increasing in* q_{L} *and* decreasing *in* $q_{\mathrm{H}}^{\mathrm{E}}$*,*
3. *With a sufficiently large* $q_{\mathrm{H}}^{\mathrm{E}}$*, arbitrarily low values of* α_E *can be matched, ie*

$$
\lim_{q_{\mathrm{H}}^{\mathrm{E}} \to \infty} \alpha_E(q_{\mathrm{L}}, q_{\mathrm{H}}^{\mathrm{E}}) = 0
$$

Similarly,

$$
\lim_{q_{\mathrm{H}}^{\mathrm{E}} \to \infty} \mu_{\mathrm{E},E}(q_{\mathrm{L}}, q_{\mathrm{H}}^{\mathrm{E}}) = 0
$$

Proof. The proof is conceptually very similar to previous proofs, so I provide the gist here, algebra steps are in the appendix. We solve (29)–(32) for the first two results, we solve for $\mu_{\mathrm{E},E}$ and α_E as a function of q_{L} and $q_{\mathrm{H}}^{\mathrm{E}}$, obtaining first-order rational functions. We examine the derivatives of these and establish the signs using (33). Then at $q_{\mathrm{L}} = 0$, we obtain the limit $q_{\mathrm{H}}^{\mathrm{E}} \to \infty$. □

The intuition behind the result is as follows. Similarly to (16), we decompose the effect of the free parameters on α_E to a direct and a composition effect. Both effects for q_{L} are similar to the one discussed for the benchmark model in Section 4, so the lowest α_E can only be obtained at $q_{\mathrm{L}} = 0$. However, $q_{\mathrm{H}}^{\mathrm{E}}$ has *no* direct effect on α_E, which it only affects via the composition

$\mu_{E,E}$. At $q_L = 0$,

$$\mu_{E,E} = \frac{C}{(\lambda_{IE}\lambda_{UI} + (\lambda_{IU} + \lambda_{IE})\lambda_{UE}) q_H^E + \lambda_{E*}\lambda_{IU}\lambda_{UE} + C}$$

where

$$C = \lambda_{UI}(\lambda_{IE}\lambda_{UI} + \lambda_{IU}\lambda_{UE} + \lambda_{IE}\lambda_{UE} + \lambda_{E*}\lambda_{IE})$$

which shows that we can drive $\mu_{E,E}$ arbitrarily low with a large q_H^E. The intuition is simple: with an increasing probability of transitioning to a state with a higher surplus, the number of workers in the marginal region diminishes. Given that $q_L \leq \lambda_{E*}$, a $\mu_{E,E} \to 0$ will drive $\alpha_E \to q_L/\lambda_{E*}$, which is lowest when $q_L = 0$.

Similarly to Section 6.3.1, we can calculate the locus of q_L, q_H^E that is required to match α_E in addition to the five other moments. However, it is difficult to plot, since when $q_L \to \lambda_{EI}$ (which is the effective constraint in (33) since $\lambda_{EI} \ll \lambda_{UI}$), q_H^E converges to infinity. Instead, we just calculate the values for $q_L = 0$, which is then comparable to the corner case in Section 3. With this restriction, $q_M = \lambda_{UI}$, $q_H^N = \lambda_{IU}$, $\sigma = \lambda_{E*}$, $\lambda = \lambda_{UE}$, $\kappa = \lambda_{IE}/\lambda_{UE}$, and $\mu_I = 1$ since there are no low-surplus nonemployed. q_H^E itself is a linear expression in $1/\alpha_E$:

$$q_H^E = \frac{\frac{1}{\alpha_E} \cdot \left(\lambda_{IE}\lambda_{UI}^2 + ((\lambda_{IU} + \lambda_{IE})\lambda_{UE} + \lambda_{E*}\lambda_{IE})\lambda_{UI}\right) - C}{\lambda_{IE}\lambda_{UI} + (\lambda_{IU} + \lambda_{IE})\lambda_{UE}}$$

where

$$C = \lambda_{IE}\lambda_{UI}^2 + ((\lambda_{IU} + \lambda_{IE})\lambda_{UE} + \lambda_{E*}\lambda_{IE})\lambda_{UI} + \lambda_{E*}\lambda_{IU}\lambda_{UE}$$

Table 4 shows the transition rate into the high surplus region for various calibrations. The required transition rate for the employed q_H^E is approximately 10 to 15 times larger the corresponding rate q_H^N for the nonemployed. Allowing q_L to be larger than zero would only make this disrepancy worse, as it would drive q_H^E higher, in a nonlinear fashion, converging to infinity, while q_H^N does not depend on q_L (cf (30)).

calibration ($q_L = 0$)	q_H^N	q_H^E
Andolfatto, Gomme, and Storer (1998)	0.035	0.48
Fallick and Fleischman (2004)	0.035	0.40
Garibaldi and Wasmer (2005) 15–64 yrs	0.046	0.42
Garibaldi and Wasmer (2005) 25–54 yrs	0.038	0.38
Pries and Rogerson (2009)	0.022	0.26
Krusell et al. (2011) unadjusted	0.037	0.46
Krusell et al. (2011) broad unemployment	0.064	0.87

Table 4: Rates of transitioning to the high-surplus region for nonemployed (q_H^N) and employed (q_H^E), various calibrations, $q_L = 0$.

6.4 Measurement error

6.4.1 Permanently inactive population

In Section 4 we have seen that most important constraint on the model that keeps α higher than it is in the data is that by construction we match flows between unemployment and inactivity. However, neither of these groups need to be homogeneous, in which case the flows that we observe may be lower or higher than the actual ones. In particular, even though in the models we have discussed so far all inactive workers have a chance of finding employment, it would be plausible to assume that a fraction of the population is, for practical purposes, more or less permanently excluded from the labor market, for reasons of age or health status.

Let's assume that a fraction $\zeta \in [0,1)$ of the workers who are observed as inactive are *permanently inactive*, and are separate from the actual labor market, in the sense that they never transition into any other state, and also that workers who are outside this group never end up as permanently inactive. Effectively, this divides the population into two disjoint groups that never interact: those who are not permanently inactive generate labor market flows as in Section 2, while the permanently inactive do not generate any flows.

The rate at which we observe IU transitions is

$$\lambda^*_{\mathrm{IU}} = \zeta \cdot 0 + (1-\zeta) \cdot \lambda_{\mathrm{IU}}$$

where λ_{IU} is the transition rate for the workers who are not permanently inactive. Similarly,

$$\lambda^*_{\mathrm{IE}} = \zeta \cdot 0 + (1-\zeta) \cdot \lambda_{\mathrm{IE}}$$

is the observed IE transition rate. All the other transition rates are unaffected.

Effectively, this extension of the model can be seen as nothing more than a transformation of the data, which imposes

$$\lambda_{\mathrm{IE}} = \frac{\lambda^*_{\mathrm{IE}}}{1-\zeta} \qquad \lambda_{\mathrm{IU}} = \frac{\lambda^*_{\mathrm{IU}}}{1-\zeta} \tag{34}$$

Substituting into (8), it is easy to show the following result.

Lemma 12 (Permanently inactive population.)**.** *Let $\alpha_{C,\text{permanently inactive}}(\zeta)$ denote the lowest α that the model can generate while matching the flows λ^*_{IE}, λ^*_{IU}, λ_{UI}, λ_{UE}, and $\lambda_{\mathrm{E}*}$. Then*

$$\frac{\partial}{\partial \zeta} \alpha_{C,\text{permanently inactive}}(\zeta) < 0 \qquad \text{and} \qquad \lim_{\zeta \to 1} \alpha_{C,\text{permanently inactive}}(\zeta) = 0$$

The results follow from (8) and (34). The intuition is the following: in Section 4 we have seen that the most important constraint that keeps α high is that the IU transition rate is small. A higher ζ concentrates the same number of transitions among fewer workers, effectively increasing λ_{IU}, and thus lowering α_C because fewer of the employed workers will be marginal, since most of them come from unemployment.

In the limiting case where ζ approaches 1, α can be driven arbitrarily low, so in a purely mechanical sense this solves the problem faced by the benchmark model. However, for practical purposes it is interesting to see what ζ is required to align the data with the model, ie

$$\zeta_{\min} \equiv \min\{z : \alpha_{C,\text{permanently inactive}}(\zeta) = \alpha_D\}$$

Table 5 shows $\zeta_{\min}$ for various datasets. Clearly, a large fraction of the inactive workers need to be outside the labor market for this extension of the model to match the data: the lowest value of $\zeta_{\min}$ is 55%, but values between 80%–90% are more common.

	$\zeta_{\min}$
Andolfatto, Gomme, and Storer (1998)	0.86
Fallick and Fleischman (2004)	0.84
Garibaldi and Wasmer (2005) 15–64 yrs	0.66
Garibaldi and Wasmer (2005) 25–54 yrs	0.55
Pries and Rogerson (2009)	0.63
Krusell et al. (2011) unadjusted	0.84
Krusell et al. (2011) broad unemployment	0.89

Table 5: Minimum share of permanently inactive workers that matches the data to the model.

7 Conclusion

(in progress)

We have provided a tractable analytical characterization of the range of labor market transition rates that can be generated by a benchmark model that nests or is similar to commonly used models in the literature. We have introduced a scalar discrepancy index between the models and the data, and demonstrated that the benchmark model cannot match the data.

Having considered various extensions, it seems that only the model which allows the process for the individual-specific surplus to depend on the employment status is able to match the data. However, this is only indirect evidence in favor of this extension, and we need direct, independent empirical findings that confirm this result. We leave this for future work.

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