

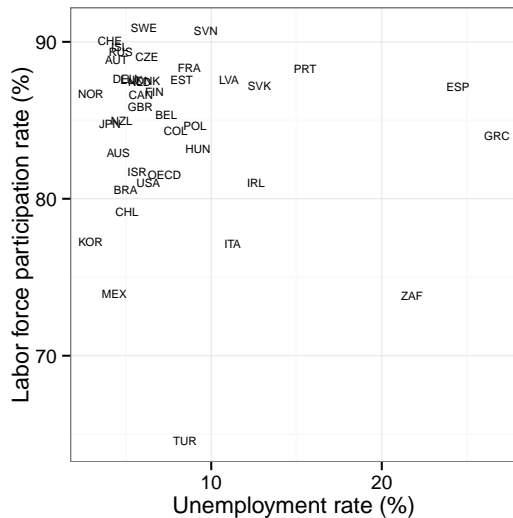
# Reverse-engineering labor market flows

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# Motivation



- models for participation
  - frictionless, labor/leisure margin
  - Hansen (1985), Rogerson (1988), ...
- models for unemployment
  - frictional, driven by job finding and separation processes
  - Diamond-Mortensen-Pissarides model family
- synthesis 1:
  - Merz (1995), Andolfatto (1996), Gomes, Greenwood, and Rebelo (2001)
  - put frictions in the RBC model, no distinction between unemployment and inactivity
- synthesis 2:
  - Ljungqvist and Sargent (1998), Ljungqvist and Sargent (2007), Alvarez and Veracierto (2000), and Veracierto (2008)
  - account for stocks, but not gross flows
- synthesis 3:
  - Garibaldi and Wasmer (2005), Krusell et al. (2011), & their other papers
  - fully frictional, participation margin, target gross flows

Using matched CPS data,

$$P_{1m} = \begin{array}{c} E \\ U \\ I \end{array} \begin{bmatrix} E & U & I \\ 0.980 & 0.009 & 0.010 \\ 0.286 & 0.569 & 0.145 \\ 0.078 & 0.039 & 0.883 \end{bmatrix}$$

- 1 Can our models match these flows?
- 2 What are the necessary ingredients?
- 3 What do the flows identify?

- reasonable benchmark model can only replicate a **limited** subset of the domain for short term flows
- **data is outside this subset**
- reason:  $I \leftrightarrow U$  flows constrain processes

# Results

- reasonable benchmark model can only replicate a **limited** subset of the domain for short term flows
- **data is outside this subset**
- reason:  $I \leftrightarrow U$  flows constrain processes

Extensions (✓: works, ✗: doesn't work)

- state-dependent separation rates ✗✗
- match-specific productivity ✗✗
- generalized productivity process ✗
- fluctuations in search cost ✗
- persistently inactive workers ✓
- skill loss for non-employed ✓
- skill gain for employed ✓✓
- generalized skill process ✓
- misclassification ✓

# The benchmark model

- imposes structure on the labor market state process
- nests Garibaldi and Wasmer (2005)
- continuous time, linear utility, discount rate  $r$
- partial equilibrium: only workers, exogenous
- passive search: costless, rate  $\kappa\lambda$
- active search: costly, more effective, rate  $\lambda$
- exogenous separation: rate  $\sigma$
- state variables: employment status, general state  $x$  that determines productivity on the market  $w(x)$  and at home  $b(x)$
- change events arrive at rate  $\gamma$ , then  $x$  redrawn from a distribution  $x \sim F(x)$  (IID in benchmark model, can be relaxed)

# Value functions and HJB equations

$$rN(x) = b(x) + \max\left\{\overbrace{\lambda(W(x) - N(x))^+}^{\text{active search}} - c, \overbrace{\kappa\lambda(W(x) - N(x))^+}^{\text{passive search}}\right\} + \overbrace{\gamma(E[N(x')] - N(x))}^{\text{change event}}$$
$$rW(x) = w(x) + \underbrace{\sigma(N(x) - W(x))}_{\text{exogenous separation}} + \underbrace{\gamma(E[N(x') \vee W(x')] - W(x))}_{\text{change event}} \quad \text{possible endog. sep.}$$

Introduce the surplus value and the flow surplus

$$S(x) = W(x) - N(x) \quad \text{and} \quad s(x) = w(x) - b(x)$$

and subtract the two HJB equations to obtain

$$(r + \sigma + \gamma)S(x) = s(x) + \gamma E[S(x')^+] - \max\{\lambda S(x)^+ - c, \kappa\lambda S(x)^+\}$$

Solve for  $S$ , which determines the policy function (accept/reject, active/passive search).

# Three regions

- **low** region: worker is not productive and would never accept a job (so won't pay for active search)

$$L = \{x : S(x) < 0\}$$

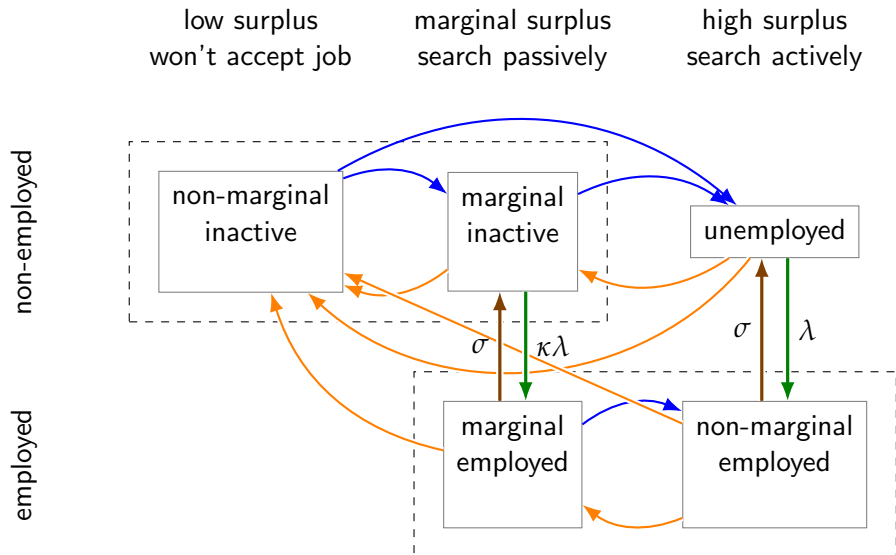
- **middle** or **marginal** region: would accept a job, but search passively

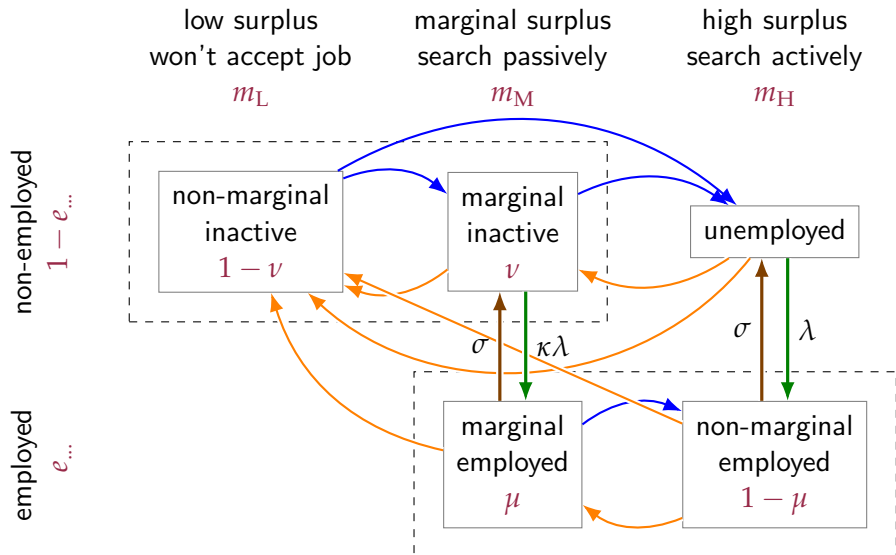
$$M = \left\{ x : 0 \leq S(x) < \frac{c}{(1-\kappa)\lambda} \right\}$$

- **high** region: search actively (unemployed)

$$H = \left\{ x : \frac{c}{(1-\kappa)\lambda} \leq S(x) \right\}$$

- choices for the **non**employed, but reflected in the state of the **employed**.





## Notation: masses and ratios

- $m_L$ ,  $m_M$ , and  $m_H$  for **all** (employed + nonemployed) workers in each region, respectively
- employment **rates**  $e_M$  and  $e_H$
- ratio of **marginal** inactives

$$\nu = \frac{\text{mass of marginal nonemployed}}{\text{mass of inactives}} = \frac{m_M(1 - e_M)}{m_L + m_M(1 - e_M)}$$

- ratio of **marginal** employed

$$\mu = \frac{\text{mass of marginal employed}}{\text{mass of all employed}} = \frac{m_M e_M}{m_M e_M + m_H e_H}$$

- let  $p_L$ ,  $p_M$ ,  $p_H$  be the probabilities of drawing a state  $x$  in each region
- effective transition rates:

$$q_L = \gamma \cdot p_L \quad q_M = \gamma \cdot p_M \quad q_H = \gamma \cdot p_H$$

# Observed flows

$$\begin{array}{lll} \lambda_{EI} = \mu\sigma + q_L & \lambda_{UI} = q_L + q_M & \lambda_{IE} = \nu\kappa\lambda \\ \lambda_{EU} = (1 - \mu)\sigma & \lambda_{IU} = q_H & \lambda_{UE} = \lambda \end{array}$$

Define  $\lambda_{EA} = \lambda_{EI} + \lambda_{EU}$  and  $\alpha = \frac{\lambda_{EI}}{\lambda_{EA}}$

- match  $\lambda_{EA}$ ,  $\lambda_{UI}$ ,  $\lambda_{IU}$ ,  $\lambda_{IE}$ ,  $\lambda_{UE}$ , pinning down **five** parameters
- characterize  $\alpha$  as a function of the **free parameter**  $q_L$

$$\sigma = \lambda_{EA} - q_L \quad \lambda = \lambda_{UE} \quad q_M = \lambda_{UI} - q_L \quad q_H = \lambda_{IU}$$

$$\begin{aligned} \alpha(q_L) &= \frac{\lambda_{EI}}{\lambda_{EA}} = \frac{\mu(q_L)\sigma + q_L}{\lambda_{EA}} = \frac{\mu(q_L)(\lambda_{EA} - q_L) + q_L}{\lambda_{EA}} \\ &= \mu(q_L) + (1 - \mu(q_L)) \frac{q_L}{\lambda_{EA}} \end{aligned}$$

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- direct ( $q_L$ ) and composition ( $\mu(q_L)$ ) effects

# Flow balance equations

$$m_L \cdot (q_M + q_H) = (m_M + m_H) \cdot q_L$$

$$m_M \cdot (q_L + q_H) = (m_L + m_H) \cdot q_M$$

$$m_H \cdot (q_L + q_M) = (m_L + m_M) \cdot q_H$$

$$m_M e_M \cdot (q_L + \sigma + q_H) = m_H e_H \cdot q_M + m_M (1 - e_M) \cdot \kappa \lambda$$

$$m_H e_H \cdot (q_L + \sigma + q_M) = m_M e_M \cdot q_H + m_H (1 - e_H) \cdot \lambda$$

The solution to the first three and  $m_L + m_M + m_H = 1$  is simple,

$$m_L = \frac{q_L}{q_L + q_M + q_H} \quad m_M = \frac{q_M}{q_L + q_M + q_H} \quad m_H = \frac{q_H}{q_L + q_M + q_H}$$

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But solving for  $e_M$  and  $e_H$  is complicated, because of **asymmetry**.

Define

$$\zeta = \frac{q_H}{q_L + q_M + q_H + \lambda + \sigma} = \frac{\lambda_{IU}}{\lambda_{UE} + \lambda_{IU} + \lambda_{UI} + \lambda_{EA} - q_L}$$

If  $\kappa \rightarrow 1$ , then because of symmetry,

$$e_M = e_H = \frac{\lambda}{\lambda + \sigma + q_L} = \frac{\lambda_{UE}}{\lambda_{UE} + \lambda_{EA}}$$

In general,

$$\mu = \frac{q_M}{q_M + q_H + (q_L + \sigma) \frac{(1-\kappa)\zeta}{(1-\kappa)\zeta + \kappa}} = \frac{\lambda_{UI} - q_L}{\lambda_{UI} - q_L + \lambda_{IU} + \lambda_{EA} \underbrace{\frac{(1-\kappa)\zeta}{(1-\kappa)\zeta + \kappa}}_{\in [0,1]}}$$

Even though we can back out  $\kappa \in [0, 1]$  from  $\lambda_{IE}$ , leave it in as a free parameter for now. Then

$$\mu(q_L, \kappa) \geq \underline{\mu}(q_L) = \frac{\lambda_{UI} - q_L}{\lambda_{EA} + \lambda_{IU} + \lambda_{UI} - q_L}$$

# Simple bounds for $\mu$ and $\alpha$

Then, since  $\lambda_{EA} > q_L$ ,

$$\alpha(q_L, \kappa) = (1 - \mu(q_L, \kappa)) \frac{q_L}{\lambda_{EA}} + \mu(q_L, \kappa) \geq (1 - \underline{\mu}(q_L)) \frac{q_L}{\lambda_{EA}} + \underline{\mu}(q_L) \equiv \underline{\alpha}(q_L)$$

We can show that  $\underline{\alpha}'(q_L) > 0$ , and consequently

$$\alpha(q_L, \kappa) \geq \underline{\alpha}(0) = \underline{\mu}(0) = \frac{\lambda_{UI}}{\lambda_{EA} + \lambda_{IU} + \lambda_{UI}}$$

Intuition

- 1 higher  $q_L \Rightarrow$  lower  $\mu(q_L)$ : **composition effect**, rather weak,
- 2 **direct effect of  $q_L$  always dominates**

General case: back out  $\kappa$  from  $\lambda_{IE} = \nu(\kappa, q_L)\kappa\lambda$ , results are similar.

# What we learn from the benchmark model

- 1 matching the other five moments,  $\alpha$  is bounded from below
- 2 bound is obtained when  $q_L = 0$ : all inactives are marginal,  $\alpha = \mu$

Intuition: too many marginal employed.

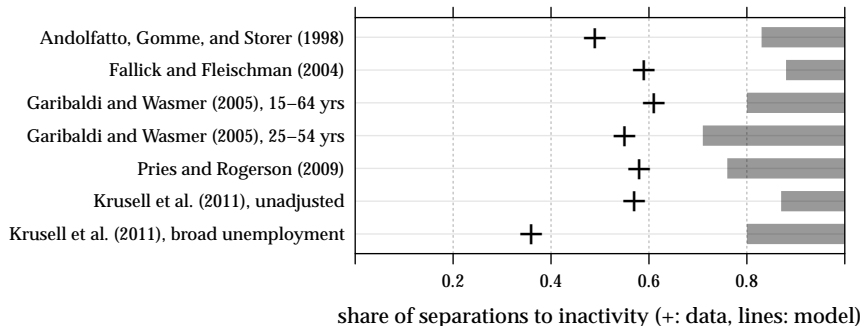
Typical magnitudes: lower bound

$$\alpha(0) = \frac{\lambda_{UI}}{\lambda_{EA} + \lambda_{IU} + \lambda_{UI}} \approx \frac{\mathbf{0.33}}{0.044 + 0.035 + \mathbf{0.33}} \approx 0.81$$

but in the data

$$\alpha_{\text{data}} = \frac{\lambda_{EI}}{\lambda_{EA}} = \frac{0.026}{0.044} \approx 0.60$$

# Exact bounds for $\alpha$ , using data from various papers



- more general (non-IID process): same intuition, algebra hell
- state-dependent separation rates:  $\sigma$  higher for marginal employed
  - 1 composition effect:  $\mu$  decreases
  - 2 direct effect:  $\lambda_{IE}$  increases relative to  $\lambda_{UE}$ , **dominates**,  $\alpha \uparrow$
- match-specific productivity
  - 1 marginal matches dissolve with higher probability
  - 2 special case of state-dependent separation rates
- fluctuations in search cost: can be mapped to a benchmark model, same result



- fraction  $\pi \in [0, 1]$  of **inactive** workers be **permanently inactive**: they don't transition to any other state
- otherwise the model is the same as the benchmark model
- observed flows: mixture of the benchmark model (') and permanently inactive (0)

$$\lambda_{IE} = (1 - \pi)\lambda'_{IE}$$

$$\lambda_{IU} = (1 - \pi)\lambda'_{IU}$$

$$\lambda_{EU} = \lambda'_{EU}$$

$$\lambda_{EI} = \lambda'_{EI}$$

$$\lambda_{UE} = \lambda'_{UE}$$

$$\lambda_{UI} = \lambda'_{UI}$$

Intuition

$$\pi \uparrow \quad \Rightarrow \quad \lambda'_{IU} = \frac{\lambda_{IU}}{1 - \pi} \uparrow \quad \Rightarrow \quad \alpha \downarrow$$

Lowest value of  $\pi$  for matching the flows:

	$\pi$
Andolfatto, Gomme, and Storer (1998)	0.86
Fallick and Fleischman (2004)	0.84
Garibaldi and Wasmer (2005) 15–64 yrs	0.66
Garibaldi and Wasmer (2005) 25–54 yrs	0.55
Pries and Rogerson (2009)	0.63
Krusell et al. (2011) unadjusted	0.84
Krusell et al. (2011) broad unemployment	0.89

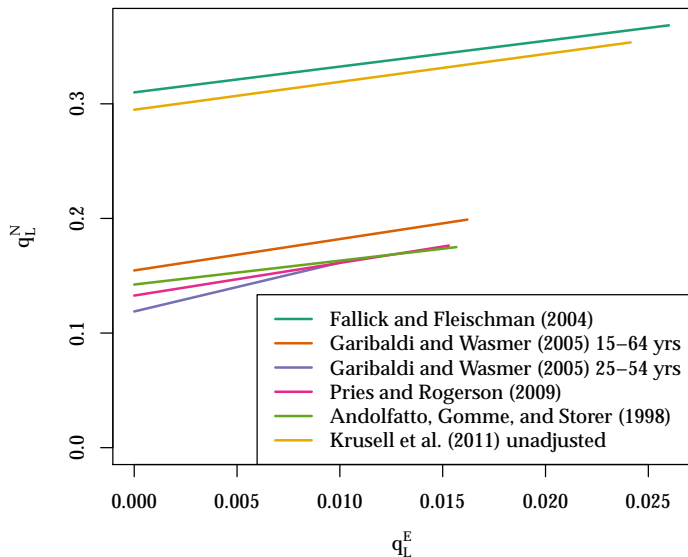
Not very plausible.



Key to breaking the result of the benchmark model: decouple processes for employed and nonemployed. Various possible parameterizations:

- 1 different  $q_L$ : non-employed transition to  $L$  with higher rate (“skill loss”), can obtain very low  $\alpha$
- 2 different  $q_H$ : employed transition to  $H$  with higher rate (“learning on the job”), can go to  $\alpha = 0$  in the limit
- 3 richer processes, for example: non-employed draw from  $N(0, 1)$ , employed draw from  $N(\bar{x}^E, 1)$
- 4 ...

# Skill depreciation — locus for just matching $\alpha$





calibration ( $q_L = 0$ )	$q_H^N$	$q_H^E$
Andolfatto, Gomme, and Storer (1998)	0.035	0.48
Fallick and Fleischman (2004)	0.035	0.40
Garibaldi and Wasmer (2005) 15–64 yrs	0.046	0.42
Garibaldi and Wasmer (2005) 25–54 yrs	0.038	0.38
Pries and Rogerson (2009)	0.022	0.26
Krusell et al. (2011) unadjusted	0.037	0.46
Krusell et al. (2011) broad unemployment	0.064	0.87

- impose consistency with observation error framework
- when questioned, **marginal** inactives respond that they are **unemployed** with probability  $\phi_I$
- similarly, **unemployed** respond that they are **inactive** with probability  $\phi_U$
- calculation:
  - 1 having observed  $U$  or  $I$  at  $t - 1$ ,
  - 2 conditional probability of actually being  $I$  or  $U$ ,
  - 3 conditional on that, use transition matrix to calculate state at  $t$ ,
  - 4 conditional on that, draw observed state at  $t$
- calibration: match monthly flows (✓)
- result:  $\phi_I \approx 0.5$ ,  $\phi_U$  not identified strongly (can be 0)

$$P_{1m} = \begin{matrix} & E & U & I \\ \begin{matrix} E \\ U \\ I \end{matrix} & \begin{bmatrix} 0.980 & 0.009 & 0.010 \\ 0.286 & 0.569 & 0.145 \\ 0.078 & 0.039 & 0.883 \end{bmatrix} \end{matrix}$$

$$P_{12m} = \begin{matrix} & E & U & I \\ \begin{matrix} E \\ U \\ I \end{matrix} & \begin{bmatrix} 0.952 & 0.021 & 0.027 \\ 0.592 & 0.227 & 0.181 \\ 0.174 & 0.037 & 0.789 \end{bmatrix} \end{matrix} \quad P_{1m}^{12} = \begin{matrix} & E & U & I \\ \begin{matrix} E \\ U \\ I \end{matrix} & \begin{bmatrix} 0.893 & 0.025 & 0.082 \\ 0.791 & 0.035 & 0.174 \\ 0.649 & 0.046 & 0.305 \end{bmatrix} \end{matrix}$$

- $P_{1m}^{12} \neq P_{12m}$ , they are very different, in particular,  $P_{1m}^{12}[U, E]$  and  $P_{1m}^{12}[I, E]$  are too high (more on this later)
- implied stocks are close but not perfectly aligned with data, even after accounting for demographics, etc

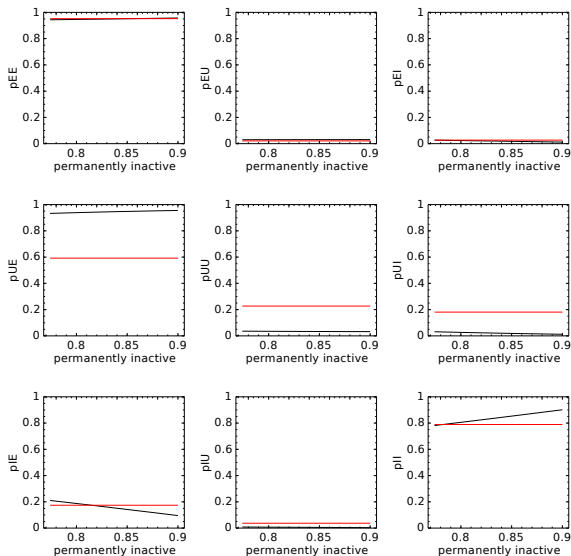
## Exercise

- for any of the ✓ models before,
- match the short-term (1 month) flows,
- then calculate the long-term (12 month) flows.

## Results so far

- UE and IE flows: much lower in the data
- rest: basically OK
- an ingredient is still missing

# Permanently inactive workers — 12m flows (data, model)



- benchmark model cannot even match the short-term flows (impossibility result)
- neither can some plausible extensions
- promising: permanently inactive workers, decoupled processes, misclassification
- none of them come close for 12-month flows, especially difficult: UE and IE too high in the model
- next idea to explore: short term job volatility
- trade-off: makes short-term flows more difficult to match