

Corporate Cash and Employment*

Philippe Bacchetta	Kenza Benhima
University of Lausanne	University of Lausanne
Swiss Finance Institute	CEPR
CEPR	

Céline Poilly
University of Lausanne

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Abstract

In the aftermath of the U.S. financial crisis, both a sharp drop in employment and a surge in corporate cash have been observed. In this paper, based on U.S. data, we document that the negative relationship between the corporate cash ratio and employment is systematic, both over time and across firms. We develop a dynamic general equilibrium model where heterogeneous firms need cash in their production process and where financial shocks are made of both credit and liquidity shocks. We show that external liquidity shocks generate a negative comovement between the cash ratio and employment. We analyze the dynamic impact of aggregate shocks and the cross-firm impact of idiosyncratic shocks. In its calibrated version, the model yields a negative correlation that is close to the data.

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1 Introduction

In the aftermath of the U.S. financial crisis, both a sharp decline in employment and an accumulation of cash held by firms have been observed. While both variables are part of firms' decisions, they are typically not considered jointly in the literature. To what extent are these two features related? Holding liquid assets facilitates the firm's ability to pay for the wage bill. But employment and cash decisions also react to changes in firms environment, e.g., changes in credit conditions. Therefore, examining these two variables jointly sheds light on the role of financial shocks on employment, especially during the crisis. The contribution of this paper is twofold. First, it provides stylized facts on the relationship between the corporate cash position and employment. Second, it delivers an explanation to the empirical evidence by building a tractable dynamic general equilibrium framework, including both cash and employment decisions. This framework sheds a new light on the impact of financial shocks by distinguishing between liquidity and credit shocks. Liquidity shocks appear to be crucial to explain the relationship between cash and employment.

We first document a robust negative comovement between the corporate cash ratio and employment on U.S. data, which is not specific to the recent financial crisis. Using Flow-of-Funds data over the period 1980-2011, the correlation between HP-filtered employment and the share of liquid assets in total assets is -0.41 . Moreover, using firm-level data from Compustat, the annual cross-firm correlation between employment and the cash ratio is on average -0.29 over the same period. Section 2 provides a detailed description of this empirical analysis.

To understand the optimal cash and employment decisions, we consider an infinite-horizon general equilibrium model with heterogeneous firms that need liquid funds in their production process. Liquidity is closely related to labor because firms have liquidity needs in order to finance the wage bill, which is part of working capital. We adopt a structure similar to Christiano and Eichenbaum (1995), who divide periods into two subperiods. In the first subperiod, firms use credit to install capital, while they need liquid funds to pay workers in the second subperiod. In contrast to the literature introducing working capital in macroeconomic models (see Christiano et al. 2011, for a survey), we assume that firms do not have full access to external liquidity and cannot borrow all their short-term needs. This generates a demand for cash. Liquidity that is external to the firm may take

several forms, such as credit lines, trade credits, trade receivables to customers, or late wage payments. Liquidity shocks are changes in the availability of external liquidity. We assume that firms may be hit by technology shocks, by changes in their ability to obtain long-term credit (i.e., standard credit shocks) and by liquidity shocks. These shocks can be at the aggregate or at the idiosyncratic level.

The model is designed to be tractable so that several results can be derived analytically. It suggests that liquidity shocks can explain the negative comovement between employment and the corporate cash ratio. A reduction in external liquidity generates two effects. On the one hand, lower liquidity reduces the financial opportunities of firms and depresses labor demand. On the other hand, the reduction in external liquidity makes the production process more intensive in cash to ensure that wages are fully financed. Firms' assets are then tilted towards cash. Combining these two effects implies that the cash ratio increases while employment declines. This analysis points to the crucial role played by the tightening of liquidity conditions in the aftermath of the Lehman crisis. While no initial sharp reduction in credit supply was observed during the recent financial crisis, firms experienced a significant deterioration in their expected liquidity conditions. For example, Gilchrist and Zakrajsek (2012) argue that banks cut the existing corporate lines of credits during the crisis.¹ Also, short-term loans to business firms decreased by 9% between 2008 and 2009 (using Survey of Terms of Business Lending, maturity of less than 30 days) while the liquidity ratio sharply increased from 3.9% to 5%.

Our model sheds a different light on financial shocks, by disentangling the role of liquidity shocks and credit shocks. From the observations of the cash ratio in the data, we are able to identify liquidity shocks. We find that these shocks are highly correlated with short-term loans. Feeding the liquidity shocks back in our model, we find that they generate a negative comovement between the cash ratio and employment. Idiosyncratic liquidity shocks are also important to explain the negative cross-firm correlation between cash and labor. The model is parametrized using moments distribution from firm-level data. Despite its simplicity, the model

¹Ivashina and Scharfstein (2010) show that firms initially drew heavily on their credit lines, but that subsequently credit conditions tightened. Campello et al. (2011) show that some firms had their credit lines canceled and that other firms had to renegotiate their credit lines with a higher cost. More generally, credit line agreements may contain restrictive covenants that may limit the ability of borrowers to draw on their lines. See also Chari et al. (2008) or Kahle and Stulz (2013).

performs relatively well quantitatively to reproduce the negative cross-firm correlation between the cash ratio and employment. Our benchmark calibration gives a correlation of -0.13 , while it is -0.29 in the data.

The optimal choice of corporate liquidity is rarely introduced in macroeconomic models, even in models with financial frictions. When it is, the focus is on investment, not labor. Liquid assets are usually held by households, typically in the form of money, to finance their consumption.² However, firms also have liquidity needs. Papers incorporating firms' liquidity are typically in the spirit of Holmstrom and Tirole (2011) and Woodford (1990); they include Aghion et al. (2010), Kiyotaki and Moore (2012) or Bacchetta and Benhima (2015). However, these papers do not analyze employment fluctuations.

While the link between liquidity and employment has not received much attention so far, our analysis is related to several strands of the literature. First, there is a growing literature that incorporates firms' financial frictions in a macroeconomic context. For instance, Covas and den Haan (2011) and Jermann and Quadrini (2012) analyze corporate external finance decisions over the business cycle, such as debt and equity. However, these papers do not introduce cash. For example, in their theoretical model, Jermann and Quadrini (2012) have working capital that is fully financed by an intra-period loan. Other papers focus more closely on the relationship between financial factors and the labor market. This literature stresses the role of financial frictions influencing labor demand.³ Most of these papers provide a more detailed analysis of the labor market than we do, but they do not consider cash holdings. Our analysis focuses on the impact of liquidity conditions on labor demand.

Our paper is also related to a vast theoretical literature in corporate finance on firms' cash holdings and corporate saving. Our approach shares features with several recent papers that provide analyses at the firm level or in environments with heterogeneous firms. Some papers are particularly close to our approach as they focus on the role of financing conditions on cash decisions.⁴ Our paper differs

²There are obviously some exceptions. For example, Stockman (1981) considers a cash-in-advance constraint both for consumption and capital.

³See for instance Wasmer and Weil (2004), Benmelech et al. (2011), Monacelli et al. (2011), Boeri et al. (2012), Chodorow-Reich (2014), Karabarbounis and Neiman (2012), Pagano and Pica (2012) or Petrosky-Nadeau and Wasmer (2013).

⁴See for example, Bolton et al. (2013), Eisfeldt and Muir (2013), Falato et al. (2013), Gao (2013) and Hugonnier et al. (2014). Some papers consider other determinants of firms' cash holdings (Armenter and Hnatkovska, 2011; Boileau and Moyen, 2012).

from this literature by focusing on business cycle frequency and on employment, which plays a key role in the working capital management. In addition, we provide a general equilibrium analysis which is important in the context of employment as this is an input that is not generated by the firm (in contrast to capital). As a result, market-clearing wage fluctuations can potentially offset partial equilibrium effects. This is particularly relevant in the context of liquidity management as the wage bill affects firms' liquidity needs. Another difference is that we make a clear distinction between liquid and less liquid assets. The recent dynamic models in the corporate finance literature consider cash a negative debt or as a residual between cash flow and investment.⁵

Finally, our approach is consistent with the findings of the empirical literature on the determinants of corporate cash.⁶ This literature stresses in particular the precautionary motive to save cash and shows that this motive increases with cash flow uncertainty or with more uncertain access to capital markets (see for instance Almeida et al., 2004). Some papers have also analyzed the use of short-term credits, like credit lines, and their interaction with corporate cash holdings. They tend to show that cash is a substitute to credit lines, as suggested by our analysis. For instance, Campello et al. (2011) find a negative correlation between cash and credit lines.⁷

The rest of the paper is organized as follows. Section 2 investigates the negative comovement between corporate cash and employment. Section 3 presents the model and shows the basic mechanism that can lead to this negative relationship. In Section 4, we calibrate the model to analyze the dynamic impact of aggregate shocks. In Section 5, we examine the impact of idiosyncratic shocks on cross-firm correlations. Section 6 discusses various extensions and Section 7 concludes. Several results are derived in the Appendices.

⁵This contrasts with an older corporate finance literature, see Holmstrom and Tirole (2011).

⁶See, for example, Bates et al. (2009) and Almeida et al. (2013) for surveys.

⁷Similarly, Sufi (2009) and Lins et al. (2010) show that internal cash is used more in bad times while firms are more likely to use credit lines in good times. Acharya et al. (2013) build a model to show that firms would rather use credit lines instead of cash reserve when they face a low aggregate risk.

2 Stylized Facts

In this section, we document the negative correlation in the U.S. between the corporate cash ratio and employment, both in aggregate terms and at the firm level, and we investigate the determinants of this correlation. We first illustrate the aggregate correlation between corporate cash and employment over the business cycle. We use quarterly data in the non-farm non-financial corporate sector. The cash ratio, defined as the share of corporate liquidity to total assets, is built from the Flow of Funds of the United States. We define cash as the sum of private foreign deposits, checkable deposits and currency, total time and savings deposits and money market mutual fund shares. Corporate employment in logarithm is drawn from the Bureau of Labor Statistics. Figure 1 displays the HP-filtered component of employment and the cash ratio over the sample 1980q1-2011q4.

[insert Figure 1 here]

We observe a negative comovement between the two variables. This negative relationship is particularly striking from the Great Recession since the corporate liquidity ratio experienced a large boom from 2009 while employment has been strongly depressed. Over the whole sample, the contemporaneous correlation between employment and the cash ratio is negative (-0.41) and significant at 1%.⁸ We show below that this negative comovement can be explained by liquidity shocks in our model.

The aggregate correlations that have been documented are driven by macroeconomic shocks common to all firms. In order to capture the heterogeneity among firms, we assess the correlation between the corporate cash ratio and employment using disaggregated firm-level data from Compustat. The sample contains US non-financial firms from 1980 to 2011. We focus only on firms that are active during the whole period, which allows us to have a homogeneous panel. In addition, we drop the 10% largest firms. This is a standard procedure as the largest firms may have a specific behavior (see Covas and den Haan, 2011). For example, the cash holding of multinational companies might be driven by foreign tax incentives (see

⁸In order to avoid any spurious correlation, we also compute the correlation when cash is divided by the one-quarter lagged value of total assets instead of its current value. The correlation is still negative (-0.42) and significant. The correlation by excluding the Great Recession is lower (-0.18) but significant at 10%. Robustness exercises are provided in Table 1 of the online appendix.

Foley et al., 2007). We also exclude financial and utilities firms, firms which are not incorporated in the US market and those engaged in major mergers.⁹ This is justified by the fact that part of the stock of cash holding is affected by acquisition. We also drop all firms with negative or missing values for: total assets, sales, cash and employees.¹⁰ We use the number of employees per firm (Compustat data item #29) as our measure of employment. The corporate cash ratio is defined as the ratio between cash and short term investment (Compustat data item #1) and the book value of assets (Compustat data item #6). A firm-specific linear trend is removed from both employment and the cash ratio. Figure 2 plots the year-by-year cross-firm correlation coefficients between these two variables with their significance level.

[insert Figure 2 here]

Over the period, the cross-section correlation between detrended employment and cash ratio is -0.29 on average and it is significant at 1%.¹¹ The negative correlation is significant in all periods and is stronger in 2007. While we only present unconditional correlations, the negative relationship between employment and cash holding is robust when we use OLS with firms-fixed effects, years-fixed effects, and standard control variables (see Tables 5-7 of the online appendix). In particular, this relationship is not driven only by macroeconomic shocks or by systematic differences across firms. Our model also accounts for this idiosyncratic correlation.

In this paper, we argue that cash holdings and employment are driven by future prospects about the availability of external liquidity. There are alternative potential explanations for the negative correlation between the cash ratio and employment. First, the demand for cash can be driven by the cyclicalities in the cost of cash (e.g., see Azar et al., 2014). For example, during the crisis, the flight to liquidity can partly be explained by the drop in interest rates which decreased the opportunity cost of cash. However, the negative firm-level correlation is robust to the inclusion of years-fixed effects, which indicates that it is not driven exclusively by business cycle effects like the cost of cash. A second alternative explanation emphasizes the role of unexpected shocks. For example, following a negative unex-

⁹Using Compustat data items, we remove firms when $6000 < \text{SIC} < 6999$, $4900 < \text{SIC} < 4949$, $\text{curcd} \neq \text{USD}$ and $\text{sale_fn} = \text{AB}$.

¹⁰The sample is reduced to 14 563 firm-year observations (456 firms). Data description and descriptive statistics are provided in the online appendix.

¹¹The online appendix shows that the correlation is -0.28% (significant at 1%) when we do not exclude the 10% largest firms (see Table 4 of the online appendix).

pected productivity shock, firms lay off workers, which generates more cash flow. However, using our panel of firms from Compustat, we show that the correlation coefficient remains negative and significant when we control for cash flows (see Table 5 in the online appendix). Moreover, the correlation coefficient is still negative and significant when we use the lagged cash ratio and when we control for the size of the firm (see Table 6 in the online appendix). These two pieces of evidence suggest that the correlation between employment and the cash ratio is not driven solely by unexpected productivity shocks. Finally, controlling for cash flows also helps us to account for the “real option value” of cash. Indeed, cash might be held in bad times to finance future investment in good times. The aggregate correlation is also robust when we control for the Fed’s fund rate, as a measure of the cost of cash, and for corporate output, as a measure of other cyclical drivers of the correlation (see Table 2 of the online appendix).

An important assumption of our model is that cash holding decisions are determined by wage bill financing. We use firm-level data to investigate the plausibility of this assumption. Compustat provides wages only for few firms, however, it is sufficient to provide elements on the relationship between *cash level* and wage. From our database, we observe that cash represents 18% of their staff expenses (median value).¹² In our model, we assume that corporate cash is used to finance end-of-period wages. We assess this link by regressing the current cash level on the future amount of staff expenses, which proxy the expected future staff expenses.¹³ Following Bates et al. (2008), we control for several other determinants of cash holding capturing the firm’s assets, investment, Tobin’s q and the level of indebtedness. It is very likely that the correlation depends on the type of sector we consider, so we include year-sector fixed-effects. We find a positive and significant relationship between the current cash level and future staff expenses.¹⁴ This result suggests

¹²The series staff expense (Compustat data item #42) includes salaries, wages, pension costs, profit sharing and incentive compensation, payroll taxes and other employee benefits. To counteract the scarce availability of this variable, we extend the dataset with the 10% largest firms. The sample now consists in 2 435 firm-year observations (159 firms). The online appendix shows that the distribution of firms’ size and cash ratio is slightly affected in this alternative sample (see Table 8 in online appendix). In addition, the correlation between the cash ratio and employment is -0.17 and still significant at 1%.

¹³Note that this means there is a measurement error in the regressor. However, since measurement errors generate an attenuation bias, the coefficient of future staff expenses is a conservative estimate of the effect of expected future staff expenses.

¹⁴The estimation results are provided in Table 9 of the online appendix and show a positive and significant coefficient for future staff expenses, despite the presence of a measurement error.

that firms hold more cash prior to a rise in staff expenses, which is in line with our model's assumption. We investigate further this relationship by looking at data at the industry level. One might expect that labor-intensive industries experience a stronger correlation between cash level and wage. The NBER-CES Manufacturing Industry Database provides a measure of labor share, defined as the ratio between payroll and production, by year and by industry. We merge this database with the Compustat database in order to get the (median) amount of cash, wages and previous control variable by industry. The sample covers 1980-2009 and consists in 46 industries. As previously, we analyze the relationship between current cash holding and future staff expenses (both expressed in log) by controlling by determinants of cash holding used previously, but we now add the interaction of future staff expenses with the labor share. This interaction is significantly positive, suggesting that the correlation between cash and future wages is stronger for industries that rely more on labor.¹⁵ These two pieces of evidence go in favor of our assumption that cash is a key financing source of wages for firms.

3 A Dynamic Model of Corporate Cash Holdings

The single-good economy is inhabited by infinitely-lived heterogeneous entrepreneurs and identical households. Entrepreneurs produce, hire labor, invest, borrow, and hold cash. Households work, consume, lend to entrepreneurs and also hold cash. We abstract from financial intermediaries. Liquidity is modeled by dividing each period in two subperiods, which we refer to as beginning-of-period and end-of-period. The market for illiquid debt only opens at the beginning-of-period. Firms have a liquidity need at the end-of-period as they have to pay for the wage bill.¹⁶ This liquidity need can be covered either by external liquidity or by cash holdings. Therefore, the need for cash is affected by changes in the availability of external liquidity. We first describe the problem of entrepreneurs and then turn to their optimal behavior, focusing on optimal labor demand and cash. We characterize analytically the properties of the model in this partial equilibrium. Finally, we describe the general equilibrium model by introducing households.

¹⁵Estimation results are provided in Table 11 of the online appendix.

¹⁶For convenience we only consider labor as end-of-period input. In a related context, Gao (2013) considers raw material instead of labor.

3.1 Entrepreneurs

There is a continuum of entrepreneurs of length 1. Entrepreneur $i \in [0, 1]$ maximizes:

$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{it+s}) \quad (1)$$

where c_{it+s} is the consumption of entrepreneur i in period $t + s$. Entrepreneur i produces Y_{it} out of capital K_{it} and labor l_{it} through the production function

$$Y_{it} = F(K_{it}, A_{it}l_{it})$$

where F is a standard constant-return-to-scale production function and A_{it} is total factor productivity (TFP). Capital depreciates at rate δ . TFP is composed of an aggregate component and an idiosyncratic one:

$$A_{it} = A_t + \epsilon_{it}^A \quad (2)$$

where A_t follows an AR(1) process and ϵ_{it}^A follows a Markov process, with $E(A_t) = A$ and $\int_0^1 \epsilon_{it}^A di = 0$.

Entrepreneurs enter beginning-of-period t with initial income Ω_{it} and can borrow in illiquid debt D_{it} to pay for their consumption, their capital, and cash M_{it} . Debt D_{it} is illiquid in the sense that it can only be issued at beginning-of-period. We follow Jermann and Quadrini (2012) by assuming that firms benefit from a subsidy on debt, so the gross interest rate on debt is $r_t = \tau R_t$, with $0 < \tau < 1$, where R_t is the before-tax interest rate.¹⁷ Cash bears no interest. The firms' beginning-of-period budget constraint is:

$$\Omega_{it} + D_{it} = c_{it} + K_{it} + M_{it} \quad (3)$$

The cash ratio m_{it} is defined as the proportion of cash to total assets, i.e., $m_{it} \equiv M_{it}/(K_{it} + M_{it})$. As D_{it} is never negative in equilibrium, it is never part of gross assets.¹⁸ Initial income is made of output, the remaining capital stock, and unused

¹⁷This tax advantage of debt is also found in Hennessy and Whited (2005). It reflects the firms' preference for debt over equity (pecking order). In our model, this pecking order is represented by the fact that the firms will have a tendency to consume (which corresponds to distributing dividends) and as a consequence they will be leveraged up to the maximum level.

¹⁸ D_{it} is non-negative because all firms are always constrained due to the debt subsidy and because we abstract from equity issuance. If some firms were unconstrained, they could choose a

cash minus the gross interest rate payment on debt and the cost associated with external liquidity used in the previous subperiod:

$$\Omega_{it} = Y_{it-1} + (1 - \delta)K_{it-1} + \widetilde{M}_{it-1} - r_{t-1}D_{it-1} - r_{t-1}^L L_{it-1} \quad (4)$$

where \widetilde{M}_{it-1} is unused cash, L_{it-1} is external liquidity obtained in the previous end-of-period and $r_t^L \geq 1$ is the cost associated with it.

Liquidity shocks affect the magnitude of external liquidity L_{it} available to firms. At end-of-period t , firms need to pay for wages out of their cash or any liquid funds they obtain in that subperiod. They face the following liquidity constraint:

$$M_{it} + L_{it} \geq w_t l_{it} \quad (5)$$

where w_t is the wage rate. Unused cash is simply defined as $\widetilde{M}_{it} = M_{it} - L_{it} - w_t l_{it}$. It will be equal to zero in most of our analysis. We assume that liquidity is constrained by lenders. Due to standard moral hazard arguments, a fraction $0 \leq \kappa_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used as collateral for debt repayments, i.e.,

$$r_t^L L_{it} \leq \kappa_{it}(1 - \delta)K_{it}. \quad (6)$$

We will assume that $r_t > r_t^L$, so that $r_t^L L_{it} = \kappa_{it}(1 - \delta)K_{it}$.¹⁹ Shocks to κ_{it} are therefore liquidity shocks, i.e., shocks that affect the amount of external liquidity.²⁰

The liquidity shock κ_{it} is assumed to be composed of an aggregate component and an idiosyncratic one:

$$\kappa_{it} = \kappa_t + \epsilon_{it}^\kappa \quad (7)$$

where κ_t follows an AR(1) process and ϵ_{it}^κ follows a Markov process, with $E(\kappa_t) = \kappa$ and $\int_0^1 \epsilon_{it}^\kappa di = 0$. In our benchmark analysis, we simply assume that κ_{it} is known at beginning-of-period t . Assuming that the liquidity shock is anticipated is a convenient way of capturing the perceived availability of liquidity. More generally, we can think of expected changes in the distribution of κ_{it} . In Section 6, we show that anticipated changes in the variance of κ_{it} can have the same effect.

Finally, we assume that the entrepreneur faces a standard credit constraint

negative D_{it} , and thus hold both bonds and cash.

¹⁹In reality, the interest rate on short-term liquidity is not necessarily lower than longer-term borrowing. But the borrowing period is shorter so that the actual borrowing cost is lower.

²⁰External liquidity could also vary with the proportion of wages that have to be paid at end-of-period.

at beginning-of-period t .²¹ A fraction $0 \leq \phi_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used as collateral for debt repayments:

$$r_t D_{it} \leq \phi_{it}(1 - \delta)K_{it} \quad (8)$$

In principle, the two constraints (6) and (8) could be related (e.g., as in Jermann and Quadrini, 2012). However, we specify them independently as we will estimate κ_{it} and ϕ_{it} from the data.

The parameter ϕ_{it} is composed of an aggregate component and a firm-specific one:

$$\phi_{it} = \phi_t + \epsilon_i^\phi \quad (9)$$

where ϕ_t follows an AR(1) process with $E(\phi_t) = \phi$ and $\int_0^1 \epsilon_i^\phi di = 0$. In this paper, we make the distinction between a standard credit shock, ϕ_{it} , and a liquidity shock, κ_{it} . The former can be viewed as a standard disturbance on the banking sector since it affects the long-term credit. The latter corresponds to an exogenous change in the availability of external liquid funds, which may come from different sources.

3.2 Optimal Cash Holding and Employment

Entrepreneurs maximize (1) subject to (3), (5) and (8). The optimization of the entrepreneur is described in details in Appendix A. We assume that shocks are anticipated so the random variables A_{it} , κ_{it} and ϕ_{it} are known at beginning-of-period t . As cash does not yield any interest, one can also verify that (5) is always binding so that $\widetilde{M}_{it} = 0$.

It is convenient to express production as a function of the capital-labor ratio $k_{it} = K_{it}/l_{it}$. We have $F(K_{it}, A_{it}l_{it}) = A_{it}l_{it}f(k_{it}/A_{it})$ where $f(k) = F(k, 1)$. The optimality conditions with respect to l_{it} and K_{it} imply that the capital-labor ratio is described by (see Appendix A):

$$k_{it} = A_{it}\tilde{k}(\tilde{w}_{it}, \phi_{it}, \kappa_{it}) \quad (10)$$

where $\tilde{w}_{it} = w_t/A_{it}$. As shown in the Appendix, $\tilde{k}(\cdot)$ is increasing in \tilde{w}_{it} , ϕ_{it} and

²¹The presence of credit constraints at the beginning-of-period is not crucial to the main mechanisms we analyze, but it allows to study the impact of credit market shocks. Moreover, it is a convenient assumption with heterogenous firms, as it puts a limit to the size of the most productive firms.

κ_{it} . Indeed, a lower wage makes production less intensive in capital as opposed to labor. Besides, as capital is the collateral, lower ϕ_{it} and κ_{it} reduces the collateral value of capital and thus have a negative effect on the capital-labor ratio. The effect of a reduction in TFP, A , is more ambiguous as it reduces both the marginal productivity of labor and capital. In the Cobb-Douglas case where $F(K, Al) = K^\alpha (Al)^{1-\alpha}$, however, we can show that overall, a lower productivity increases the capital-labor ratio when $\delta > 0$. In that case, an reduction in A affects the marginal productivity of labor relatively more than the return on capital, because it does not affect the remaining stock of capital.

The cash ratio, which is a key variable in our analysis because it reflects the cash-intensity of production, can be derived from the above results. Using (5), (10), and $r_t^L L_{it} = \kappa_{it}(1 - \delta)K_{it}$, we find:

$$\frac{M_{it}}{K_{it}} = \frac{1}{\kappa_{it}} [w_t - \kappa_{it}(1 - \delta)\kappa_{it}/r_t^L] = \frac{w_t}{\kappa_{it}} - \kappa_{it}(1 - \delta)/r_t^L \quad (11)$$

The demand for cash per unit of capital is equal to the demand of cash per unit of labor, divided by the capital-labor ratio. The demand for cash per unit of labor is itself simply equal to the liquidity need per unit of labor (w_t), minus external liquidity per unit of labor ($\kappa_{it}(1 - \delta)\kappa_{it}/r_t^L$). A decrease in κ_{it} has two effects: a *direct* negative effect as it diminishes the access to external finance and an *indirect* negative collateral effect as the capital-labor ratio decreases. These two effects both increase the cash ratio. A decrease in ϕ_{it} also increases the cash ratio, but only through the negative negative collateral effect. In contrast, a decrease in A_{it} increases the capital-labor ratio and as a result it decreases the cash ratio. Equation (11) then implies that the cash ratio, which depends solely on M_{it}/K_{it} , comoves negatively with κ_{it} and positively with A_{it} .

To analyze labor demand, we will focus on cases where entrepreneurs are credit-constrained and have log utility. Appendix A shows that the credit constraint is binding whenever the wage paid by firms, w_t , is lower than the marginal return of labor, denoted w_{it}^* . Moreover, with log utility Appendix A shows that optimal consumption is $c_{it} = (1 - \beta)\Omega_{it}$.

In that case, it is useful to rewrite the constraint (3) using (5), (8), and $L_{it} = \kappa_{it}(1 - \delta)K_{it}$. This gives:

$$\beta\Omega_{it} + \frac{\phi_{it}(1 - \delta)K_{it}}{r_t} + \frac{\kappa_{it}(1 - \delta)K_{it}}{r_t^L} = K_{it} + w_t l_{it} \quad (12)$$

Equation (12) gives the budget constraint aggregated over the two subperiods. Total financing of firms, on the left-hand side, pays for inputs, on the right-hand side. Both the long-term and short-term financing conditions, represented respectively by ϕ_{it} and κ_{it} , affect the capacity of firms to finance labor l_{it} . Using (12), the optimal behavior of entrepreneurs is described in the following proposition.

Proposition 1 (Individual policy functions) *Suppose that $u(c_{it}) = \ln(c_{it})$. If $r_t > r_t^L > 1$ and $w_t < w_{it}^*$, where k_{it} is given by (10), then the liquidity constraint (5) and the credit constraints (6) and (8) are binding and the policy functions for K_{it} , M_{it} , l_{it} , D_{it} , and Ω_{it+1} satisfy:*

$$l_{it} = Z_{it}\Omega_{it} \quad (13)$$

$$K_{it} = k_{it}Z_{it}\Omega_{it} \quad (14)$$

$$M_{it} = [w_t - \kappa_{it}(1 - \delta)k_{it}/r_t^L]Z_{it}\Omega_{it} \quad (15)$$

$$D_{it} = \phi_{it}(1 - \delta)k_{it}Z_{it}\Omega_{it}/r_t \quad (16)$$

$$\Omega_{it+1} = [(1 - \kappa_{it} - \phi_{it})(1 - \delta)k_{it} + A_{it}f(k_{it})]Z_{it}\Omega_{it} \quad (17)$$

where

$$Z_{it} = \frac{\beta}{[k_{it} + w_t] - (\kappa_{it}/r_t^L + \phi_{it}/r_t)(1 - \delta)k_{it}}. \quad (18)$$

Proof. See Appendix A. ■

We call Z_{it} the financial multiplier. It measures the impact of a change in income on labor demand. Notice that a decline in the financing conditions ϕ_{it} or κ_{it} implies a smaller Z_{it} , everything else equal. A worsening of financing conditions has thus a negative effect on inputs, including labor. However, it also decreases the capital-labor ratio as the collateral value of capital declines, which has a positive effect on labor. Under standard assumptions, the direct negative effect dominates, as shown in the following corollary.

Corollary 1 *Under the Cobb-Douglas production function, ceteris paribus, firms with lower financing conditions κ_{it} or ϕ_{it} have lower employment l_{it} and a higher cash ratio m_{it} . Moreover, a lower productivity A_{it} affects negatively employment l_{it} but has a negative effect on the cash ratio m_{it} .*

Proof. See Appendix A. ■

Corollary 1 illustrates the main mechanism in the model. An expected decrease in κ_{it} implies a smaller amount of available liquid funds at end-of-period t . As a response, firms naturally increase the proportion of cash in their portfolio, as seen in (11). At the same time, they reduce their labor demand and their production, as outside funding decreases. The same occurs with a decline in ϕ_{it} , but the increase in cash ratio is milder. This increase takes place as firms reduce their capital stock relative to labor and hence relative to their liquidity needs, because of the indirect collateral effect. On the opposite, with a decline in productivity A_{it} , firms increase their capital-labor ratio, which has a negative effect on their cash ratio, as their liquidity needs decline in proportion to capital. At the same time, labor declines.

The next two sections verify numerically the *ceteris paribus* result from Corollary 1 in a dynamic model where the income level Ω_{it} is endogenous and the wage rate w_t is determined in the labor market. Section 4 focuses on aggregate shocks and the time-series dimension, while Section 5 focuses on the cross-firm dimension.

3.3 Closing the Model

The model is closed by introducing households. Since the emphasis is on firms, households are modeled in a simple way and the full description is left for Appendix B. Identical households provide an infinitely elastic supply of funds D_t to firms at interest rate $R = 1/\beta$, where β is the beginning-of-period to beginning-of-period households' discount factor, which is the same as firms'. This is justified by a utility function linear in consumption, the absence of financial frictions for households and the fact that unlike firms, households do not benefit from a subsidy on debt. Similarly, we assume that households' utility is linear in cash so that their supply of cash is infinitely elastic at rate 1.

At end-of-period t households also supply liquid funds L_t at rate $r_t^L = 1/\psi$, where ψ is the household's discount factor between the end-of-period and the beginning-of-period. They always have sufficient cash since they receive their wages at end-of-period t while they consume at beginning-of-period $t + 1$.

Finally, households have a labor supply $l^s(w_t)$ that depends positively on the wage rate. In our specification, we have $l^s(w_t) = (w_t/\bar{w})^\eta$ where $\eta > 0$ is the Frisch elasticity of labor supply and \bar{w} is a positive constant (see Appendix B). The wage rate is then determined endogenously so that $l^s(w_t) = \int_0^1 l_{it} di$ where l_{it} is the labor demand by firm i in period t . According to Proposition 1, $l_{it} =$

$l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it})$, so the equilibrium wage is defined by

$$l^s(w_t) = \int_0^1 l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it}) di, \quad (19)$$

4 Aggregate Shocks

In this section, we focus on the time-series dimension, as described in Figure 1, of the relationship between the cash ratio and employment. For this purpose, we assume that all entrepreneurs are identical and only face aggregate shocks, so $\epsilon_{it}^A = \epsilon_{it}^\kappa = \epsilon_i^\phi = 0$. We also assume that entrepreneurs are always constrained by setting $\tau < 1$ so that $r_t < 1/\beta$. In this context, we calibrate the model to analyze the dynamic impact and the historical behavior of productivity and financial shocks, in the spirit of Jermann and Quadrini (2012). We derive three relevant series: liquidity shocks κ_t , productivity shocks A_t , and standard credit shocks ϕ_t . We show that our model reveals the presence of negative shocks on liquidity and credit over the recent period, the former contributing the most to the negative comovement between cash and labor.

4.1 Equilibrium

In the absence of idiosyncratic shocks, the only potential source of heterogeneity between firms is their wealth. Since labor demand is linear in wealth, we can then write $\int_0^1 l(w_t, A_t, \kappa_t, \phi_t, \Omega_{it}) di = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ where $\Omega_t = \int_0^1 \Omega_{it} di$. We consider a constrained equilibrium defined as follows:

Definition 1 (Constrained equilibrium under aggregate shocks only) *For a given aggregate wealth Ω_t and a given realization of A_t , κ_t and ϕ_t , a constrained period- t equilibrium is a level of employment l_t , of capital K_t , of cash M_t , of debt D_t , of financial multiplier Z_t and of future wealth Ω_{t+1} satisfying Equations (13) to (18), where $r_t = \tau/\beta$, the wage w_t clears the labor market so that $l^s(w_t) = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ with $l^s(w_t) = (w_t/\bar{w})^\eta$ and k_t is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy $w_t < w_t^*$.*

Since aggregate labor demand depends on A_t , κ_t , ϕ_t and Ω_t , the equilibrium wage also depends on those variables: $w_t = w(A_t, \kappa_t, \phi_t, \Omega_t)$. For an individual firm, we saw that the credit constraint is binding whenever $w_t < w_{it}^*$. At the aggregate

level, we can show that there exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t)$ so that $w_t < w_t^*$ is equivalent to $\Omega_t < \Omega^*$. When the wage is low, firms want to use all their resources to produce. However, because firms' resources are limited by the credit constraints, the aggregate labour demand is low when the aggregate wealth is low, which maintains the equilibrium wage at a low level and firms are constrained in equilibrium. In this section, we focus on cases where this condition is satisfied and we discuss the case where firms are unconstrained in Section 6. The following Proposition shows under which conditions the steady state is constrained:

Proposition 2 (Constrained steady state under aggregate shocks only) *The steady state is constrained if and only if $\tau < 1$.*

Proof. See Appendix C. ■

Individual agents and the aggregate economy will fluctuate around a constrained steady state. Intuitively, on the one hand, a wage that is lower than the marginal productivity of labor makes the credit constraint binding, as stated in Proposition 1. On the other hand, the credit constraint makes the equilibrium wage dependent on aggregate wealth. When $\tau < 1$, the net interest rate $r_t - 1$ is below the propensity to consume out of wealth $1/\beta - 1$, so firms never accumulate sufficient wealth to be able to provide an equilibrium wage equal to marginal productivity.

4.2 Calibration

Table 1 shows the calibration used for the parameters. The first five parameters are calibrated on standard values. Following Jermann and Quadrini (2012), we set the firms' discount factor equal to $\beta = 0.9825$ and $\tau = 0.9939$, implying a subsidy on net interest debt payments of 35% and a steady-state annualized real interest rate of 4%. The Frisch parameter, η , is set to unity. We set the share of capital in production, α , to 0.36. We assume that the cost of using liquidity, r_t^L , is lower than the gross interest rate, such that $r_t^L = 1.01$. The other parameters are calibrated to match two empirical targets, using aggregate data. Precisely, the model has to replicate the mean of the cash ratio and the debt to output ratio over the sample, i.e. 3.3% and 50%, respectively.²² It follows that the liquidity parameter κ is set

²²As in Section 2, the cash ratio is defined as the share of liquidity to total assets from the non-financial corporate business sector. The debt to output ratio is measured by the ratio between credit market instruments (liabilities) from the non-financial corporate business sector and the gross value added in the business sector. Data sources are available in the online appendix.

to 0.04 and the credit parameter ϕ equals 0.06. Finally, we normalize A to unity. Finally, the autoregressive parameters for the technology and the credit shocks are calibrated following Jermann and Quadrini (2012), $\rho_A = 0.94$ and $\rho_\phi = 0.97$, respectively; while we set $\rho_\kappa = 0.41$ as suggested by our liquidity series (see Section 4.4).

[insert Table 1 here]

4.3 Impulse Response Functions

We examine the impact of a 1 percent decrease in aggregate liquidity, technology and credit from their steady-state level. The impulse response functions (IRFs) are computed by determining the equilibrium wage, w_t , that clears the labor market and using in turn the policy functions (13) to (17).²³ Figure 3 displays the IRFs to the three shocks. The solid, dotted and dashed lines correspond to a response to κ_t , A_t and ϕ_t , respectively.

[insert Figure 3 here]

The upper panel in Figure 3 displays the responses to the cash ratio and employment to a 1 percent decrease in the three shocks. The two financial shocks generate a negative comovement between employment and the cash ratio. Considering a negative liquidity shock, i.e., a decline in κ_t , firms have smaller external liquid funds to pay for wage bills at end-of-period. The cash ratio m_t rises through two channels. Through the direct effect firms need to compensate for the reduced access to external liquidity by relying more on internal liquidity. Through the indirect collateral effect the collateral value of capital is reduced relative to labor which reduces the scale of assets. Altogether, these two channels drive the cash ratio in the same upward direction. In the case of a negative credit shock, only the collateral motive plays a role on the cash ratio which slightly increases. The reason of this modest increase is that the credit shock does not *directly* affect the structure of the portfolio between internal and external liquidity. On the other hand, a reduction in financial opportunities (i.e., shortage in external liquidity and credit) lowers labor demand at beginning-of-period through the financial multiplier.

²³We check that we do have $w_t < w_t^*$ every period.

Therefore, employment l_t decline. When it comes to a negative technology shock, the comovement between employment and the cash ratio is different. As explained above, a decline in productivity A_t rises the capital-labor ratio which increases in turn the scale of assets as compared to liquidity needs and generates a slight reduction in the cash ratio. Production is therefore less intensive in cash. The other effect, more standard, is to decrease employment through a tighter financial multiplier.

The lower panel in Figure 3 shows the remaining IRFs. The three recessionary shocks generate a decline in wages and therefore a reduction in liquidity needs. The response of debt is mostly driven by negative credit shocks although it evolves in the same pattern as labor in all experiments, which is in line with Covas and den Haan (2012) and Jermann and Quadrini (2012) who stress that debt is procyclical. Capital also decreases in response to the financial shocks while it increases on impact in response to the TFP shock, due to the rise in the capital-labor ratio mentioned above.

4.4 Inspecting the Liquidity Shock

We argue that external liquidity shocks are good candidates to generate a negative comovement between the cash ratio and employment. A natural test of our approach consists in building up a model-based liquidity measure and to compare it with an empirical one.

The model-based liquidity measure is the liquidity shock κ_t . Let \hat{x}_t denote the log-deviation of the variable x_t from its deterministic trend, corresponding to HP-filtered empirical data (detailed below). The liquidity series is constructed using the liquidity and short-term credit constraints, see Equations (5) and (6),

$$\hat{\kappa}_t = \left[\frac{wl/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} (\hat{w}_t + \hat{l}_t) - \left[\frac{M/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} \hat{M}_t - \hat{K}_t. \quad (20)$$

All the parameters are taken from the calibration and the model's steady-state. The wage bill $(\hat{w}_t + \hat{l}_t)$ is measured as the hourly compensation index \times hours worked in the nonfarm business sector from BLS. Capital (\hat{K}_t) is measured using total capital expenditures and consumption of fixed capital of non-financial corporate business sector from Flow of Funds, as in Jermann and Quadrini (2012). Cash is defined as the sum of private foreign deposits, checkable deposits and currency, total time

and savings deposits and money market mutual fund shares from Flow of Funds. All the nominal series are deflated by the price index for gross value added in the business sector from NIPA.²⁴

The empirical measure of liquidity is short-term loans. The Survey of Terms of Business Lending provides measures of banking loans to business firms for different maturities. We can therefore distinguish short-term and long-term loans in the data. The former is defined as the total amount of loans provided by domestic banks of a maturity of less than one year. The later has a maturity of more than one year. Data are available from 1997q2 to 2011q4 and both are HP-detrended. One might expect the model-based liquidity series to be correlated to the measure of short-term loans. Figure 4 confirms this results.

[insert Figure 4 here]

Despite its simplicity, our model is able to generate a series of liquidity shock which fits well the historical evolution of short-term loans over the sample. Our model predicts a shortage in external liquidity on the last quarter of 2005, which is observed in the data. The strong reduction in external liquidity from 2009 to 2010 is also captured in the theoretical model. This correlation is striking given that short-term loans are not used in the construction of our liquidity shock. One might question whether the model-based liquidity shock captures only short-term loans or all the maturities. Figure 5 plots the dynamic correlation of the constructed $\hat{\kappa}_t$ series with short-term and long-term loans.

[insert Figure 5 here]

Interestingly, our measure of liquidity shock is significantly correlated to loans of a maturity of less than one year while the correlation is insignificant at all horizon for loans of a longer maturity. This result confirms that our external liquidity measure captures only short-term borrowing.

This exercise sheds light on the role a particular type of financial shock on the economy, namely liquidity shocks.

²⁴Details on data sources are provided in the online appendix. All data are at a quarterly frequency.

5 Cross-firms Correlations

We now assess whether the calibrated model is able to explain the cross-firm evidence of a negative correlation between cash and employment. To examine this issue, we reintroduce heterogeneous firms that are hit by idiosyncratic productivity shocks ϵ_{it}^A and liquidity shocks ϵ_{it}^κ . Instead we assume for simplicity that the aggregate economy does not fluctuate by setting $A_t = A$, $\kappa_t = \kappa$. As a benchmark, we assume that credit constraints do not vary across firms and time and set $\phi_{it} = \phi$. We relax this assumption later by assuming that firms can have different levels of credit constraints.

5.1 Equilibrium

As in the case with aggregate shocks only, we consider a constrained equilibrium defined as follows:

Definition 2 (Constrained equilibrium under idiosyncratic shocks only) *For a given period- t distribution of wealth, productivity and liquidity $\{\Omega_{it}, A_{it}, \kappa_{it}\}_{i \in [0,1]}$, a constrained period- t equilibrium is given by the firm-specific levels of employment l_{it} , of capital K_{it} , of cash M_{it} , of debt D_{it} , of Z_{it} and of future wealth Ω_{it+1} satisfying Equations (13) to (18), where $r_t = \tau/\beta$, the wage w_t clears the labor market such that (19) is satisfied with $l^s(w_t) = (w_t/\bar{w})^\eta$ and k_t is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy $w_t < w_{it}^*$ for all $i \in [0, 1]$.*

In our simulation exercise, we check ex post that we do have $w_t < w_{it}^*$ for all i .

5.2 Calibration

Beside the parameter values described in the previous section, we aim at calibrating a range for $\kappa_{it} = \kappa + \epsilon_{it}^\kappa$ and $A_{it} = A + \epsilon_{it}^A$. We assume that these shocks can take 10 equidistant possible realizations. The two shocks are assumed to follow an independent first-order Markov process with transition probability of $\frac{0.25}{9}$. More precisely, each firm has a probability of 75% to stay in the same state for κ (A) and a probability of 25% to switch to one of the 9 other states, with an identical probability for each of these states. We calibrate the range for κ_{it} and A_{it} (namely, we set the minimum and maximum values) to match some distribution moments

observed at the firm level. Table 1 provides the interquartile values to match, computed from the Compustat database described in Section 2. The range of the idiosyncratic liquidity and productivity shocks κ_{it} and A_{it} are set to reproduce the interquartile ratio for our two variables of interest, namely the cash ratio and employment. This implies $\kappa_{it} \in [0.01; 0.091]$ and $A_{it} \in [0.94; 1.07]$. All the other parameters are calibrated as described in Section 4.2. The numerical method to obtain the steady-state wage and distribution of firms is described in Appendix D.

5.3 Results

The upper panel of Table 2 displays firm-level moments computed from the stationary distribution. Interestingly, our stylized model provides a negative cross-firm correlation between the cash ratio and employment, equals to -0.13 under our benchmark calibration. This number is somewhat smaller than the number found in the data (-0.29).

[insert Table 2 here]

To understand this result, Figure 6 shows the impact of an idiosyncratic innovation of κ_{it} and A_{it} on the value of the labor normalized by wealth (ℓ_{it}/Ω_{it}) and the cash ratio (m_{it}), both weighted by the distribution probability.

[insert Figure 6 here]

This figure shows that, as κ_{it} decreases, the cash ratio is higher and labor is lower for a given Ω_{it} . Differently, firms facing a negative productivity shock adjust both labor and the cash ratio downward. Consequently, even though the two shocks predict an opposite correlation between employment and the cash ratio, our calibrated liquidity shock is strong enough to generate a reasonable negative correlation. When the amount of liquid funds is reduced, firms are able to finance less labor with the same amount of cash. To accommodate for this shock, they both accumulate more cash in order to pay for the wage bill and diminish their level of labor to limit the wage bill.

However, while the normalized labor (ℓ_{it}/Ω_{it}) is independent of Ω_{it} according to Proposition 1, the level of labor ℓ_{it} is driven by the size of the firm Ω_{it} , which depends on the history of shocks. As a consequence, the correlation between the cash ratio and labor is driven not only by A_{it} and κ_{it} as suggested by Figure 6, but

also by Ω_{it} . The lower panel of Table 2 complements the previous figure by showing the weighted value of these variables by class of firms. While firms with a level of wealth below median have on average a substantially lower level of employment than firms with a level of wealth above median, their cash ratio is about the same on average. On the one hand, idiosyncratic innovations on liquidity (κ) and technology (A) affect the cash ratio and labor, as shown in Figure 6. On the other hand, they also affect firms' wealth and therefore employment for a given level of cash. This heterogeneity of wealth generates noise that further dampens the correlation.

We can also show that the credit constraint affects the correlation between the cash ratio and employment through a multiplier effect. To do so, we consider two different states for ϕ_i such that $\phi_i = \{\phi_L, \phi_H\}$, where $\phi_L < \phi_H$. In order to be consistent with the calibration strategy described above, we set the value of $\phi_{i,L}$ and $\phi_{i,H}$ in order to match the interquartile ratio for debt to sales from our Compustat database. This strategy implies that most financially-constrained firms are those with $\phi_i = \phi_L$ where $\phi_L = 0.02$, while less constrained firms have $\phi_i = \phi_H$, where $\phi_H = 0.10$. Our model shows that ϕ_L -type firms exhibit a less negative correlation between the cash ratio and labor than ϕ_H -type firms (-0.11 and -0.15 , respectively). Therefore, the simulation results reveal that the correlation between cash and labor is stronger for less financially-constrained firms. Those firms have a larger financial multiplier since they have more resources through their level of borrowing. Consequently, their labor is more sensitive to productivity and liquidity shocks, while their cash ratio is barely affected by the level of ϕ_i . This implies that the correlation between cash and labor is larger for a large ϕ_i . This is consistent with the data. Smaller firms in terms of sales or debt-to-sales ratio, that are more likely to be credit constrained, have a less negative correlation between the cash ratio and employment. For example, the 25 percent smaller firms in terms of sales have a correlation of -0.24 compared to -0.33 for the top 25 percent. The analogous correlations are -0.24 and -0.35 when we rank firms by their debt-to-sales ratio.

6 Extensions

The benchmark model has abstracted from various elements that could be relevant to the analysis. In this section we describe several extensions. First, we analyze

the case where firms are not credit-constrained. Second, we discuss the impact of liquidity uncertainty with unanticipated liquidity shocks. Third, we discuss the impact of unexpected productivity shocks that provide an alternative explanation for the negative comovement between cash and employment.

6.1 Unconstrained Firms

We assumed so far that $r_t < 1/\beta$, so that firms are always credit-constrained. This has two advantages: it enables us to examine the effect of a standard credit shock and it helps sustain an equilibrium with heterogeneous firms. It is however important to examine how this assumption affects the response of the economy to liquidity and productivity shocks. We show that in the absence of credit constraints, a liquidity shock affects essentially the cash ratio while a productivity shock affects essentially labor. Cash and labor are thus more disconnected than in the benchmark constrained case.

In order to simulate the unconstrained case, we set r_t equal to $1/\beta$ and assume that ϕ is sufficiently high so that firms never hit their credit limit. We otherwise use the same calibration as in the benchmark model. Since $r = 1/\beta$, the level of wealth is undetermined in the steady state. For comparison purposes, we set the initial level of Ω to the same level as in the benchmark steady state. Figure 7 shows the simulation results.

[insert Figure 7 here]

Following a negative liquidity shock, the economy experiences a decrease in employment and an increase in the cash ratio as in the benchmark. Indeed, on the one hand, firms need more cash to produce. On the other hand, as cash is costly, labor becomes less productive, so the demand for labor and the equilibrium wage decrease. Notice, however, that the effect on employment and wage is much milder when firms are unconstrained as compared to the benchmark, where firms are constrained. Indeed, as long as the cost of liquidity r_t^L is not too high, the liquidity shock barely affects labor productivity. Therefore, in the absence of constraint, firms do not change their labor demand dramatically. In the presence of credit constraints, the demand for labor and hence the equilibrium wage depend on firms' resources. Since fewer external resources are available, firms have to cut on labor hiring, generating a stronger reaction of labor demand.

Consider now the effect of a negative productivity shock. While employment

decreases as in the benchmark, the cash ratio remains constant. Indeed, the productivity shock has a direct negative effect on the availability of external liquidity, but it has also a negative indirect, general equilibrium effect on the wage and hence on liquidity needs. In the absence of credit constraints, the equilibrium wage is more sensitive to productivity as compared to the case with credit constraints, where labor demand and the wage depend on wealth. Since, in the latter case, the response of aggregate wealth is sluggish, then so are the responses of labor and the wage. Finally, since the wage, and hence liquidity needs, decrease more when firms are unconstrained, the increase in the cash ratio is mitigated as compared to the benchmark. Actually, the decrease in liquidity needs perfectly compensates for the decrease in external liquidity, leaving the cash ratio unchanged.

6.2 Liquidity Uncertainty

In our analysis, firms know perfectly the amount of external liquidity they can get at the end-of-period, i.e., κ_{it} is known at beginning-of-period t . If instead we assume that only the distribution of κ_{it} is known, we can analyze the impact of an increase in uncertainty in κ_{it} . Not surprisingly, an increase in liquidity uncertainty increases the demand for cash and decreases employment on average.²⁵ In particular, if we assume that labor is set at the beginning of period, then an increase in uncertainty has the same effect as an anticipated negative liquidity shock.

To understand this result, consider the simple case where there are two possible states for κ_{it} : $\kappa_t^L = \kappa - \varpi_t$ and $\kappa_t^H = \kappa + \varpi_t$, with $\varpi_t > 0$. The magnitude of ϖ_t , and thus the variance of κ_{it} , is known at the beginning of period but κ_{it} is revealed only at the end of period. When ϖ_t increases, the firm increases its cash holdings. When labor is predetermined at the end-of-period, firms actually hold just enough cash to be able to finance the wage bill in the worst case where $\kappa_{it} = \kappa_t^L$. The reason is that insufficient cash would leave the firm with no revenues ($\Omega_{it+1} = 0$).²⁶ This prospect deters firms from putting themselves in such a situation, as the utility is logarithmic and $\log(0) = -\infty$. In the event where $\kappa_{it} = \kappa_t^H$, firms do not draw down on the whole line of credit as it is costly ($r_t^L > 1$), and they set $L_{it} = \kappa_t^L Y_{it}$.

²⁵This hoarding behavior is reminiscent of the literature on precautionary savings initiated by Bewley (1986) and Aiyagari (1994).

²⁶This implicitly assumes that the punishment the firms face for not honoring the contract entails both that households do not work and that money holdings are seized, leaving the firms. This also supposes that money is a perfectly pledgeable asset and that households are credible enough to implement that punishment.

Thus, cash holdings move proportionately to ϖ_t and firms behave exactly as if their anticipated liquidity shock was κ_t^L .

6.3 Unanticipated Productivity Shocks

In this paper we focus on active liquidity management by firms, i.e., the optimal choice of cash holdings M_{it} . However, a proportion of cash holding may come from unexpected unused cash \widetilde{M}_{it} , which has been equal to zero so far in our analysis. This may give an alternative explanation to the negative comovement between cash and employment. Assume that productivity shocks are not known at beginning-of-period t and that firms can adjust their employment within the end-of-period (i.e., employment is not predetermined as in 6.2). In that case, unused cash \widetilde{M}_{it} is no longer necessarily equal to zero. For example, an unexpected decline in A_t implies a lower need for liquidity and thus higher \widetilde{M}_{it} . Thus, we would have a negative comovement between unexpected cash holding \widetilde{M}_{it} and labor demand. However, if the productivity shock is persistent (e.g., as in (2)) the path of productivity in subsequent periods is anticipated as in our benchmark analysis. Overall, except for the effect on impact, the dynamic effect of an unanticipated productivity shock is similar to an anticipated productivity shock.

The model therefore predicts a temporary increase in relative cash holdings. After an initial negative shock, the cash ratio is reduced to adjust for lower expected productivity. In contrast, it is sometimes argued, especially in the wake of the financial crisis, that firms keep holding cash because of low investment opportunities. For this argument to hold in our model, we should assume repeated unanticipated negative productivity shocks. Alternatively, we would need to add some adjustment costs for reducing money holdings or assume that firms' liquidity management is totally passive, i.e., firms would not choose their optimal level of M_{it} .

7 Conclusion

This paper has documented a negative comovement between the corporate cash ratio and employment. Even though such a relationship may appear surprising at first sight, we show that it can be explained by liquidity shocks. These shocks make production less attractive or more difficult to finance, while they also generate a

need for liquidity necessary to pay wage bills, which can be satisfied by holding more cash. Moreover, we argue that our analysis is useful in understanding the motives for firms' cash holdings and in shedding light on the dominant shocks during the financial crisis.

Besides explaining an interesting stylized fact, the simple model developed in this paper could be extended to analyze the role of corporate liquidity in a macroeconomic environment. Several extensions could be of interest. First, instead of focusing on the business cycle frequency, the model could be used to examine longer term developments. The model would actually be consistent with the documented gradual increase in cash holdings if we assume changes in the production process that imply more end-of-period payments (e.g., with more extensive use of just-in-time technologies as reported in Gao, 2013, or with an increase in production outsourcing). A second extension, that would lead to a richer analysis, is to introduce financial intermediaries. Third, for a better analysis of the financial crisis, it would be of interest to introduce demand shocks. Finally, the role of policy intervention would be a natural extension. The last two extensions would be related to the existing DSGE literature incorporating working capital to study monetary policy.

Appendix

A The Entrepreneur's Problem

Entrepreneurs maximize (1) subject to (3), (4), (5), (6) and (8) and $\tilde{M}_t \geq 0$. They also take into account the production function $Y_{it} = F(K_{it}, A_{it}l_{it})$. The production function has constant returns to scale so we can write $Y_{it} = A_{it}l_{it}f(k_{it}/A_{it})$, with $f(k) = F(k, 1)$ and with k the capital-labor ratio K/l . The Lagrangian problem is

$$\begin{aligned} \mathcal{L}_{it} = & E_t \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) \\ & + \gamma_{is} \left[\tilde{M}_{is-1} + Y_{is-1} + (1 - \delta)K_{is-1} - r_{s-1}D_{is-1} - r_{ts-1}^L L_{is-1} + D_{is} - c_{is} - K_{is} - M_{is} \right] \\ & + \eta_{is} \left[M_{is} + L_{is} - w_t l_{is} - \tilde{M}_{is} \right] \\ & + \lambda_{is} [\phi_{is}(1 - \delta)K_{is} - r_s D_{is}] \\ & + \nu_{is} [\kappa_{is}(1 - \delta)K_{is} - r_s^L L_{is}] \\ & + \mu_{is} \tilde{M}_{is} \} \end{aligned}$$

The entrepreneur's program yields the following first-order conditions with respect to l_{it} , c_{it} , D_{it} , M_{it} , \tilde{M}_{it} and L_{it} :

$$w_t \eta_{it} = A_{it} F_{lit} \beta E_t \gamma_{it+1} \quad (21)$$

$$u'(c_{it}) = \gamma_{it} \quad (22)$$

$$\gamma_{it} = \beta r_t E_t \gamma_{it+1} + r_t \lambda_{it} \quad (23)$$

$$\gamma_{it} = \eta_{it} \quad (24)$$

$$\eta_{it} = \beta E_t \gamma_{it+1} + \mu_{it} \quad (25)$$

$$\eta_{it} = \beta r_t^L E_t \gamma_{it+1} + r_t^L \nu_{it} \quad (26)$$

Studying these FOCs indicates which constraints are binding. Since $\gamma = u'(c) > 0$, then $\eta > 0$ according to (24), which implies that both budget constraints are

binding. Moreover, using (21), (23) and (24), we obtain:

$$\beta E_t \gamma_{it+1} \left(\frac{A_{it} F_{lit}}{w_{it}} - r_t \right) = r_t \lambda_{it}$$

This implies that whenever $wr < AF_l$, the long-term credit constraint is binding ($\lambda > 0$). Besides, using (23), (24) and (26), we find:

$$\beta E_t \gamma_{it+1} (r_t - r_t^L) = r_t^L \nu_{it} - r_t \lambda_{it}$$

Therefore, if the long-term credit constraint is binding ($\lambda > 0$) and $r_t > r_t^L$, then the short-term credit constraint is binding too ($\nu > 0$). Finally, using (25) and (26), we find:

$$\beta E_t \gamma_{it+1} (r_t^L - 1) = \mu_{it} - r_t^L \nu_{it}$$

Therefore, if the short-term credit constraint is binding ($\nu > 0$) and $r_t^L > 1$, then the entrepreneurs hold no excess money ($\mu > 0$).

Assume now that $r_t > r_t^L > 1$ and make the guess that $\lambda > 0$ (we will determine later under which conditions the long-term credit constraint is indeed binding). Then all the constraints are binding and we can write $\tilde{M} = 0$, $D = \phi(1 - \delta)K/r_t$ and $M = wl - \kappa(1 - \delta)K/r_t^L$. We can then rewrite the objective as

$$\begin{aligned} \mathcal{L}_{it} = & E_t \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) \\ & + \gamma_{is} [Y_{is-1} + (1 - \delta)K_{is-1}(1 - \kappa_{is-1} - \phi_{is-1}) \\ & - c_{it} - K_{is}[1 - (1 - \delta)(\phi_{is}/r_{is} + \kappa_{is}/r_{is}^L)] - w_{is}l_{is} \} \end{aligned} \quad (27)$$

The optimality conditions with respect to c_{it} , l_{it} and K_{it} are:

$$\gamma_{it} = u'(c_{it}) \quad (28)$$

$$w_t \gamma_{it} = A_{it} F_{lit} \beta E_t \gamma_{it+1} \quad (29)$$

$$[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] \gamma_{it} = \beta E_t \gamma_{it+1} [F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})] \quad (30)$$

Combining (29) with (30), we obtain:

$$\frac{w_t}{A_{it}} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] F_{lit}}{F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})}$$

F has constant returns to scale so we can write: $F(K, Al) = Alf(K/Al)$. Therefore, $F_K(K, Al) = f'(K/Al)$ and $F_l(K, Al) = f(K/Al) - Kf'(K/Al)/Al$. As a consequence, $w_t/Al = \tilde{w}(\tilde{k}_{it}, \phi_{it}, \kappa_{it})$, with $\tilde{k}_{it} = K_{it}/Al_{it}$ and

$$\tilde{w}(\tilde{k}, \phi, \kappa) = \frac{[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)][f(\tilde{k}) - \tilde{k}f'(\tilde{k})]}{f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)} \quad (31)$$

Since F is concave in both arguments, we have $f'' < 0$, which implies that \tilde{w} is strictly increasing in \tilde{k} . If there exists a solution $\tilde{k}(\tilde{w}_t, \phi_{it}, \kappa_{it})$ to that equation, then this solution is unique. Finally, k_{it} is then given by $k_{it} = Al_{it}\tilde{k}(\tilde{w}_t, \phi_{it}, \kappa_{it})$.

Note that the long-term credit constraint is binding whenever $\tilde{w}r < F_l$. Besides, if it is the case, then all the other constraints are binding, as $r_t^L > 1$ and $r_t > r_t^L$. We thus have simply to determine when $\tilde{w}r < F_l$. Combing this inequality with (31), we find that this is equivalent to:

$$f'(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi) > r_t[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)] \quad (32)$$

$$\Leftrightarrow \tilde{k} < (f'^{-1}\left(r_t[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)] - (1 - \delta)(1 - \kappa - \phi)\right))$$

Finally, according to (31), \tilde{k} is increasing in \tilde{w} , so this inequality is satisfied for \tilde{w} lower than some $\tilde{w}^*(\kappa, \phi)$ and thus for w lower than some $w^*(A, \kappa, \phi)$.

In order to study how k is affected by ϕ , we differentiate Equation (31) with respect to it and find after rearranging

$$\frac{\partial \tilde{k}}{\partial \phi} = \frac{-(1 - \delta)[f(\tilde{k}) - \tilde{k}f'(\tilde{k})] \left[\frac{f'(\tilde{k}) - r_t}{r_t} + (1 - \delta) \left(\frac{1 - \kappa}{r_t} + \frac{\kappa}{r_t^L} \right) \right]}{f''(\tilde{k})[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)][f(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k}]}$$

As $f'' < 0$, both the numerator and denominator are negative so $\partial \tilde{k}/\partial \phi_{it} > 0$. Similarly, we find $\partial \tilde{k}/\partial \kappa_{it} > 0$. Then k is also increasing in ϕ and κ .

Differentiating Equation (31) with respect to \tilde{w} , we find after rearranging

$$\frac{\partial \tilde{k}}{\partial \tilde{w}} = \frac{[f'(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)]^2}{-f''(\tilde{k})[1 - (1 - \delta)(\kappa/r_t^L + \phi/r_t)][f(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k}]}$$

Note that $k = A\tilde{k}$ and $w = A\tilde{w}$ so

$$\begin{aligned}\frac{\partial k}{\partial w} &= A \frac{\partial \tilde{k}}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} = \frac{\partial \tilde{k}}{\partial \tilde{w}} > 0 \\ \frac{\partial k}{\partial A} &= \tilde{k} + A \frac{\partial \tilde{k}}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial A} = \tilde{k} + \frac{\partial \tilde{k}}{\partial \tilde{w}} \tilde{w} \\ &= \tilde{k} - \frac{[f'(\tilde{k}) + (1-\delta)(1-\phi-\kappa)][f(\tilde{k}) - \tilde{k}f'(\tilde{k})]}{-f''(\tilde{k})[f(\tilde{k}) + (1-\delta)(1-\phi-\kappa)\tilde{k}]}\end{aligned}$$

In the Cobb-Douglas case, we have

$$\frac{\partial k}{\partial A} = \frac{-(1-\alpha)(1-\delta)(1-\phi-\kappa)f(\tilde{k})}{-f''(\tilde{k})[f(\tilde{k}) + (1-\delta)(1-\phi-\kappa)\tilde{k}]} < 0$$

Proof of Proposition 1 Assume that the credit constraint is binding and that $r_t > r_t^L > 1$. Then the program of the firm is described by (27) and by the FOCs (28)-(30) and by (31). We make the educated guess that there exists χ such that $c_{it} = (1-\chi)\Omega_{it}$. Combining our guess with (3), (5), (6), (8) and (11), we obtain

$$\chi\Omega_{it} = K_{it} + w_t l_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)K_{it} = A_{it}l_{it}[\tilde{k}_{it} + \tilde{w}_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)\tilde{k}_{it}]$$

Replacing \tilde{w}_{it} using (31) and rearranging, we obtain

$$\chi\Omega_{it} = A_{it}l_{it} \frac{[1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)][f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})}$$

As $\Omega_{it+1} = A_{it}l_{it}[f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]$, we have

$$\chi\Omega_{it} = \frac{[1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]\Omega_{it+1}}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})} \quad (33)$$

Using (28) and (30) under log-utility $u(c) = \log(c)$, we obtain the following Euler equation

$$\frac{1}{c_{it}} [1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] = \beta E_t \left\{ \frac{1}{c_{it+1}} \right\} [f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})]$$

Given that shocks are known at the beginning-of-period, $c_{it+1} = \chi\Omega_{it+1}$ is known at the beginning-of-period, so the Euler equation can be written without the ex-

pectations operator

$$\frac{1}{c_{it}}[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] = \beta \frac{1}{c_{it+1}}[f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})]$$

Using our guess $c_{it} = \chi\Omega_{it}$ and $c_{it+1} = \chi\Omega_{it+1}$ to replace c_{it} and c_{it+1} , we obtain

$$\beta\Omega_{it} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]\Omega_{it+1}}{f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})} \quad (34)$$

Combining (33) and (34) yields $\chi = \beta$.

Combining $c_{it} = (1 - \beta)\Omega_{it}$ with the binding constraints (3), (5) and (8), we can easily derive equations (13)-(17) in Proposition 1.

Proof of Corollary 1 According to Equation (11), a decline in κ_{it} increases the cash ratio through a lower level of external liquid funds and through a lower capital-labor ratio. A decline in ϕ_{it} increases the cash ratio through a lower capital-labor ratio. A decline in A_{it} decreases the cash ratio through a higher capital-labor ratio.

According to Equation (13), the effect on labor depends directly on the effect on the financial multiplier Z_{it} . We can rewrite Z_{it} as follows:

$$Z_{it} = \frac{\beta}{w_{it} + A_{it}\tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]}$$

So the effect on Z_{it} depends on the effect on $X_{it} = \tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]$. In the Cobb-Douglas case, we have

$$\frac{\partial X}{\partial \phi} = (1 - \delta)f(\tilde{k}) \frac{-\alpha(1 - \alpha)(1 - \phi - \kappa)/r - (1 - \alpha) \left[1 - (1 - \delta) \left(\frac{1 - \kappa}{r} + \frac{\kappa}{r^L}\right)\right]}{|f''(\tilde{k})|[f(\tilde{k}) + (1 - \delta)(1 - \phi - \kappa)\tilde{k}]} < 0$$

Similarly, we have $\partial X/\partial \kappa < 0$. Therefore, a decline in ϕ or κ decreases the financial multiplier Z and hence has a negative impact on labor.

Note finally that, in the Cobb-Douglas case, k_{it} is decreasing in A_{it} as shown earlier. As a result, Z and l are increasing in A .

B The Household Problem

Identical households have a linear utility U_t with the discount factor β , and no financial frictions:

$$E_t U_t = E_t \sum_{s=0}^{\infty} \beta^s \left[c_{t+s}^{hb} + \psi^{-1} \beta c_{t+s}^{he} + (1 - \beta) M_{t+s}^h - \beta \bar{w} \frac{l_{t+s}^{1+1/\eta}}{1 + 1/\eta} \right] \quad (35)$$

where c^{hb} is households' consumption in the beginning-of-period, c^{he} is households' consumption in the end-of-period and M^h are the household's beginning-of-period money holdings. $\psi\beta$ is the preference for end-of-period consumption relative of beginning of period consumption. As the household has a preference for the beginning-of-period, then $\psi^{-1}\beta < 1$ so $1/\psi < 1/\beta$. the household has a preference for t end-of-period over time $t+1$ beginning-of-period, $\psi^{-1}\beta > \beta$, so $1/\psi > 1$.

Households maximize this utility subject to their beginning-of-period and end-of-period budget constraints

$$R_{t-1} D_{t-1}^h + M_t^h + c_t^h = D_t^h + r_{t-1}^M \tilde{M}_{t-1}^h + r_{t-1}^L L_{t-1} + T_t$$

$$w_t l_t + M_t^h = L_t + \tilde{M}_t^h$$

where D^h is household debt and \tilde{M}^h are the household's end-of-period money holdings. r^M is the return of 1 unit of cash. At end-of-period, households lend part of their wage $w_t l_t$ to the firms. This lending L_t yields r^L . r^L is the equilibrium return on short-term lending. $T_t = R_{t-1}(\tau - 1)$ are taxes that finance the debt subsidy for firms.

Households' optimization then implies that, in equilibrium, $l_t = (w_t/\bar{w})^\eta$, $R_t = 1/\beta$, $r_t^M = 1$ and $r_t^L = 1/\psi$, with $1 < r^L < R$. Note that, in equilibrium, households are indifferent between consuming in the beginning and the end of period. As output is available only at the beginning-of-period, consumption takes place only in the beginning-of-period.

C Equilibrium with aggregate shocks only (Proof of Proposition 2)

Before proving Proposition 2, we establish the following Lemma:

Lemma 1 *There exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t)$ so that the credit constraint is binding whenever $\Omega_t < \Omega^*$. In that case the dynamics of K_t , M_t , D_t , l_t and Ω_{t+1} follow:*

$$l_t = Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (36)$$

$$K_t = k(w_t, A_t, \kappa_t, \phi_t)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (37)$$

$$M_t = (w_t - \kappa_t(1 - \delta)k(w_t)/\psi)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \quad (38)$$

$$D_t = \phi_t(1 - \delta)k(w_t, A_t, \kappa_t, \phi_t)/r_t]Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t/r_t \quad (39)$$

$$\Omega_{t+1} = [(1 - \delta)(1 - \kappa_t - \phi_t)k(w_t, A_t, \kappa_t, \phi_t) + A_t f[k(w_t, A_t, \kappa_t, \phi_t)/A_t]Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t] \quad (40)$$

where

$$Z(w_t, A_t, \kappa_t, \phi_t) = \frac{\beta}{[k(w_t) + w_t] - (1 - \delta)k(w_t, A_t, \kappa_t, \phi_t)(\kappa_t/\psi + \phi_t/r_t)}$$

is the financial multiplier and

$$w_t = w(A_t, \kappa_t, \phi_t, \Omega_t)$$

is the equilibrium wage so that $w(A_t, \kappa_t, \phi_t, \Omega_t)$ is the solution to $l^s(w_t) = Z(w_t, A_t, \kappa_t, \phi_t)\beta r_t \Omega_t$.

Proof. Note that, as shown earlier, the credit constraint is binding whenever $w < w^*(A, \kappa, \phi)$. Since we also have that the constrained equilibrium wage w is increasing in Ω_t , then there exists an increasing function Ω^* so that $w_t < w^*(A_t, \kappa_t, \phi_t)$ is equivalent to $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t)$. The rest of the Lemma derives from Proposition 1. ■

Using this Lemma, we can study the steady state. From Equation (40), we have that the steady-state wage must satisfy:

$$\tilde{w} + \tilde{k} - (1 - \delta)(\kappa/\psi + \phi/r)\tilde{k} = \beta[f(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi)\tilde{k}]$$

Replacing \tilde{w} using (31) and rearranging:

$$1 - (1 - \delta)(\kappa/\psi + \phi/r) = \beta[f'(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi)]$$

Then inequality (32) is satisfied if and only if $1/\beta > r$, which is the case if and only if $\tau < 1$. Therefore, the credit constraint is binding in the steady state ($\lambda > 0$)

whenever $\tau < 1$. This proves Proposition 2.

D Numerical method

The algorithm to compute the steady-state distribution of firms is as follows:

1. We first choose a grid of wealth Ω_{it} . Our grid is a 1000-value grid over $[5, 65]$. We use the Chebychev nodes to make the grid more concentrated on low values of Ω .
2. We allocate an initial uniform and independent distribution to the values of Ω_{i0} , κ_{i0} and A_{i0} , and make an initial guess on the equilibrium wage w_0 .
3. Given the initial distribution on Ω_{it} , κ_{it} and A_{it} and the initial equilibrium wage w_0 , we use Proposition 1 and the Markov Chain to compute the new distribution of Ω_{it+1} , κ_{it+1} and A_{it+1} . Using Proposition 1, we compute the corresponding distribution of labor demand l_{it+1} . We aggregate this labor demand $l_{t+1} = \sum_i l_{it+1} di$, and if $l_{t+1} > l^s(w_t)$ (if $l_{t+1} < l^s(w_t)$), then we update the equilibrium wage w_{t+1} upward (downward).
4. We repeat step 3 until the equilibrium wage is reached, i.e. when aggregate labor demand is fully satisfied.

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Table 1. Calibration Strategy

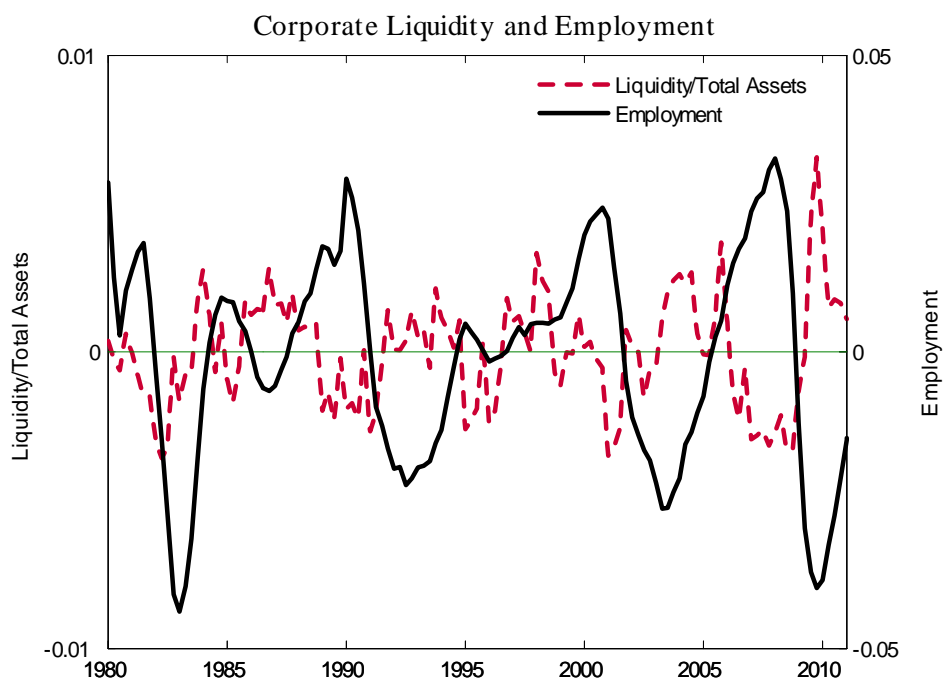
Calibrated Parameters		Value
β	Discount factor	0.9825
τ	Debt subsidy	0.35
r	Gross interest rate on bonds	1.012
r^L	Liquidity cost	1.01
η	Frisch parameter	1
α	Elasticity of output wrt capital	0.36
ϕ	Collateral share for debt	0.0605
κ	s.s collateral share for liquidity	0.0409
κ_i	Firm-specific collateral share for liquidity	[0.01; 0.091]
A	Steady-state productivity shock	1.00
A_i	Firm-specific productivity shock	[0.64; 1.07]
ρ_ϕ	Autoregressive parameter, credit shock	0.9703
ρ_κ	Autoregressive parameter, liquidity shock	0.4133
ρ_A	Autoregressive parameter, productivity shock	0.9457

Table 2. Simulated Moments

Benchmark Calibration		Data	Model
$\frac{m_{75\%}}{m_{25\%}}$	Interquartile ratio of m	7.60	7.68
$\frac{\ell_{75\%}}{\ell_{25\%}}$	Interquartile ratio of ℓ	1.46	1.42
$corr(m, \ell)$	Correlation(cash ratio; labor)	-0.29	-0.13
Average value of labor and cash ratio by class of firms		ℓ	m
Ω_i	bottom 50%	0.65	0.03
	top 50%	1.04	0.03
κ_i	bottom 50%	0.74	0.05
	top 50%	0.77	0.01
A_i	bottom 50%	0.71	0.02
	top 50%	0.80	0.03

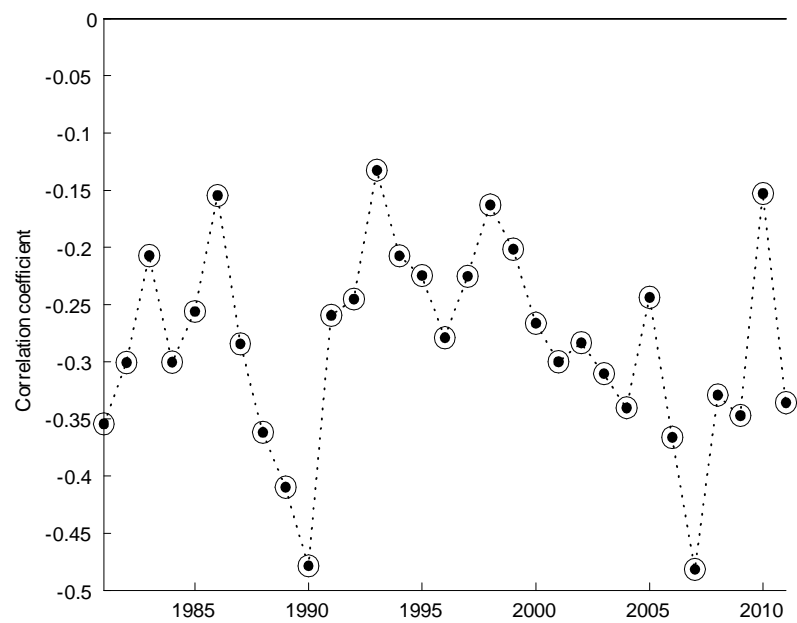
Note: In the upper panel, the empirical correlation between m and ℓ is computed after removing the firm-specific linear trend from data. In the lower panel, all the values of labor and the cash ratio are weighted by the distribution probability.

Figure 1: **Corporate Liquidity and Employment.**



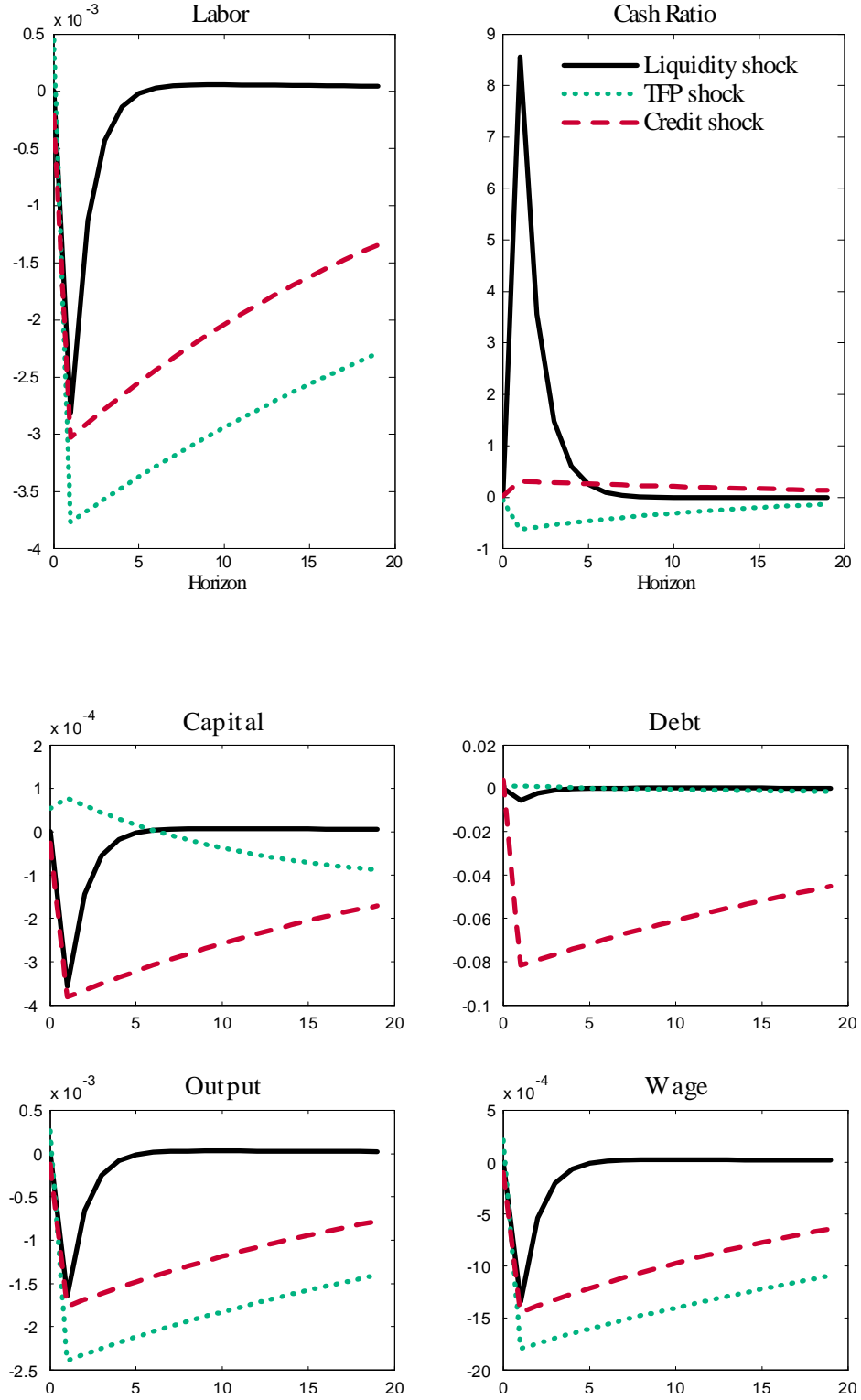
Note: Employment is expressed in logarithm and both variables are HP-filtered.

Figure 2: Cross-section correlation between employment and the cash ratio by year.



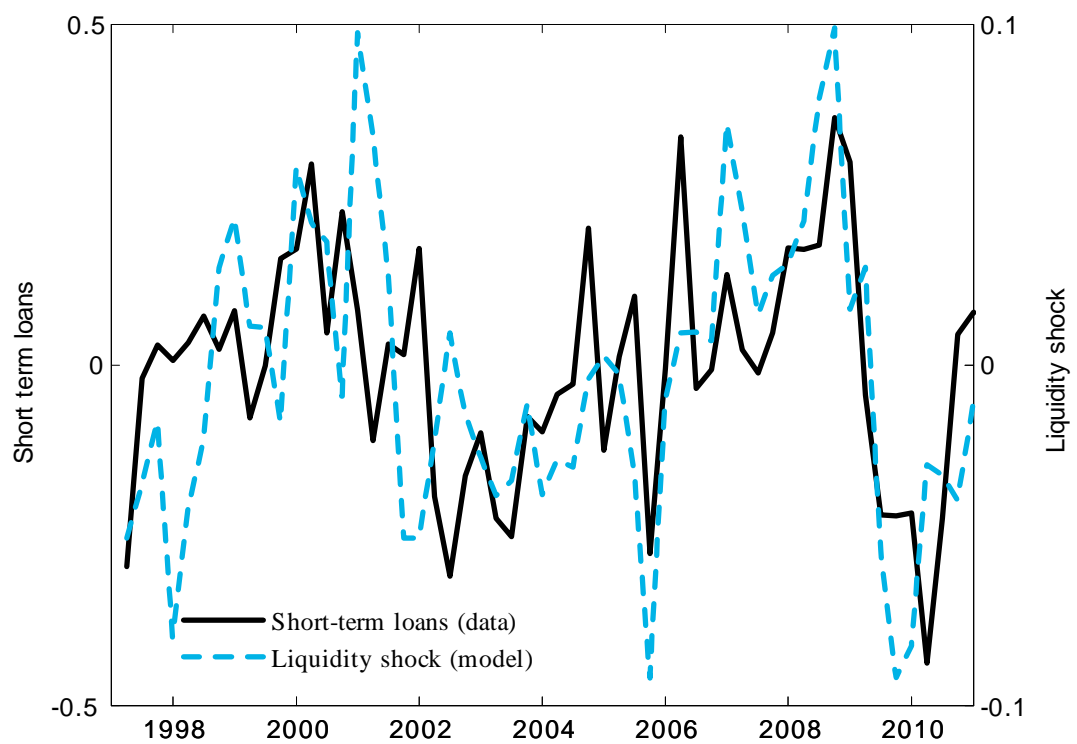
Note: For both variables, we remove the firm-specific linear trend. Markers with circle corresponds to correlation coefficients significant at 1%.

Figure 3: Impulse Response Functions to liquidity, TFP and credit shocks.



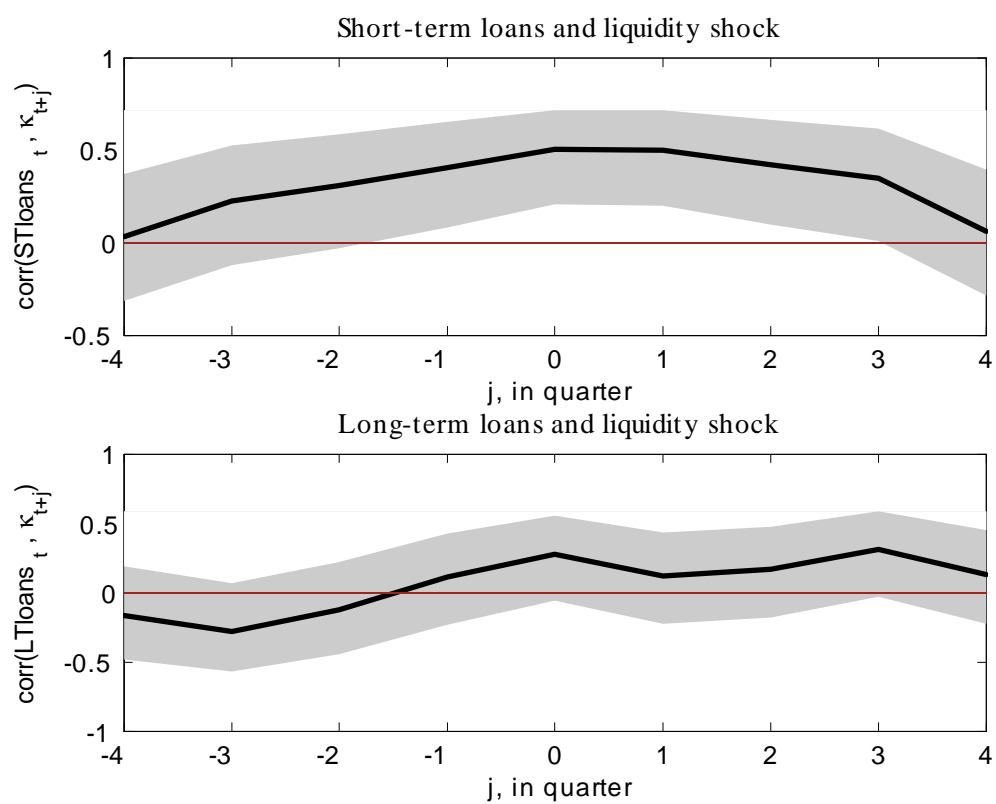
Note: The solid lines correspond to the IRFs to an external liquidity shock (κ). The dotted lines corresponds to the IRFs to a TFP shock (A). The dashed lines correspond to the IRFs to a credit shock (ϕ).

Figure 4: **Model-based liquidity shock and short-term loans.**



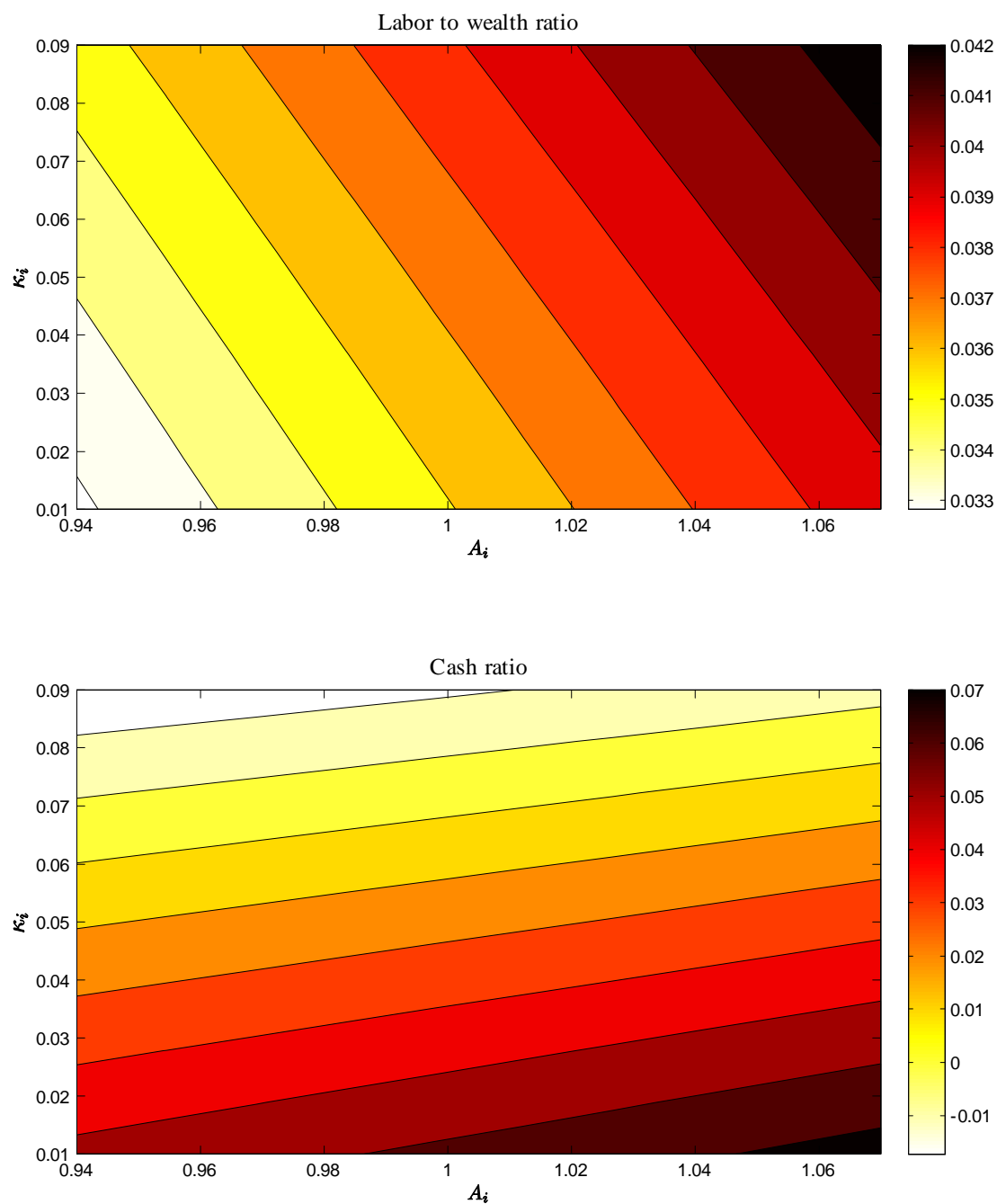
Note: The solid line displays loans to business firms of a maturity of less than one year. The dashed line is the model-based external liquidity series constructed from Equation (20).

Figure 5: **Dynamic correlation between model-based liquidity shock and loans.**



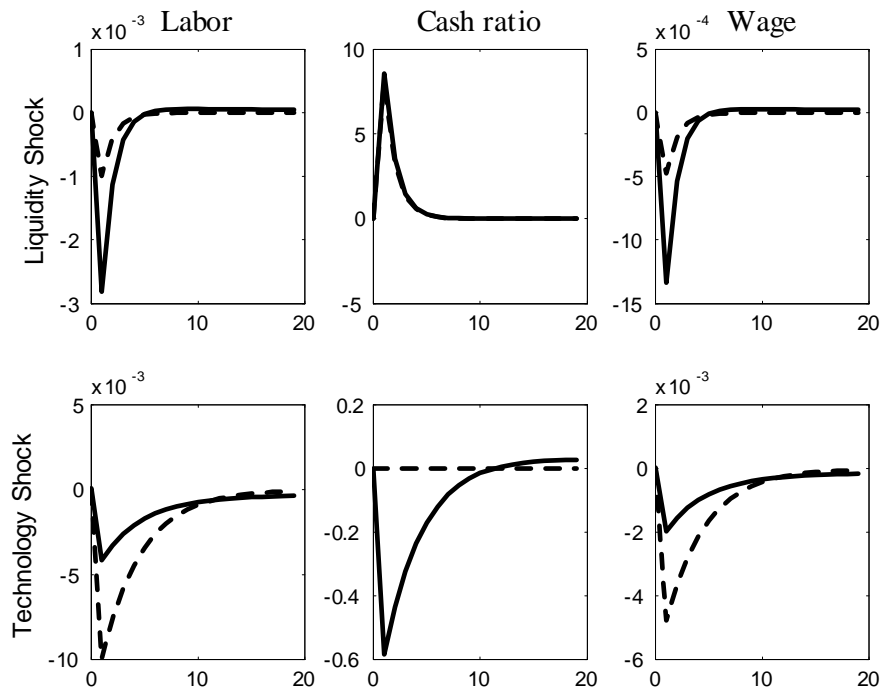
Note: The solid line displays the correlation between the short-term loans series in t and the liquidity series in $t + j$. The grey area corresponds to the 99% confidence interval.

Figure 6: Value of the labor to wealth ratio (l_i/ω_i) and the cash ratio (m_i).



Note: All values of l_i/Ω_i and m_i are weighted by the distribution probability.

Figure 7: Model with credit-unconstrained firms.



Note: The solid lines correspond to the benchmark model. The dashed lines correspond to the model with unconstrained firms. All IRFs are expressed in percentage deviation from the steady-state.