Endogenous Public Information and Welfare in Market Games

Xavier Vives (IESE Business School)

May 30, 2014

Introduction

- Renewed interest in the welfare analysis of economies with private information and in particular on the role of public information (e.g. Morris and Shin (2002), Angeletos and Pavan (2007), Amador and Weill (2010)).
- Themes:
  - Informational efficiency need not be aligned with economic efficiency.
  - Paradox of public information.
  - Agents may not put welfare-optimal weights on public and private information due to payoff and informational externalities.
  - Tax/subsidy schemes to restore efficiency.
- Aim of paper: provide welfare analysis in a market game where the public statistic (price) has both an informational and an allocational role (public information endogenously generated in the RE tradition).

Introduction

- In many markets agents compete in demand and/or supply schedules and therefore condition on prices:
  - financial markets,
  - asset auctions, and
  - some goods markets such as wholesale electricity.
- Prices provide endogenous public information:
  - In financial markets prices are noisy statistics which arise out of the decisions of traders.
  - In goods markets prices aggregate information on the preferences of consumers and the quality of the products.

Introduction

Punchline

- When the public statistics on which agents can condition are prices the presumption that agents put too little weight on private information, and consequently prices contain too little information, need not hold.
- This happens in common scenarios under strategic substitutes competition for reasons other than the well-known Hirshleifer effect of destruction of insurance opportunities.
- Somewhat paradoxically, the fact that agents condition on the price may impair the (social) value of learning from it.
- The results provide a rationalization of pro-cyclical business cycle policy and the Tobin tax on financial transactions.

Three challenges of the welfare analysis of REE

1. Need of a model that can deal in a tractable way with the dual role of prices as conveyors of information and determinants of the budget constraint of traders: Grossman and Stiglitz (1980).
2. Need to have a welfare benchmark to confront market equilibria in an asymmetric information world: team solution (Radner (1979), Vives (1988), Angeletos and Pavan (2007)) in the spirit of Hayek (1945).
3. Need to deal with and disentangle the interaction of payoff and informational externalities: beyond pure information externality models.
    - Informational externality leads to underweighting of private information (Vives (1993, 1997), Amador and Weill (2010, 2012), in the limit to disregard it (Banerjee (1992), Bikhchandani et al. (1992))

Outline and breakdown of usual logic

- Intuition from informational externality models should extend to situations with endogenous public information in economies in which equilibria are restricted efficient when public information is exogenous:
  - Example: In Cournot market with a continuum of firms and private information (Vives (1988)) increasing public information (say with a noisy quantity signal) has to be good marginally, and under regularity conditions the result is global, and more weight to private information is needed (Angeletos and Pavan (2009)).
- This logic breaks down in a market game where agents condition on the price (say firms competing in supply functions):
  - There is a pecuniary externality related to aggregate volatility which makes the market inefficient even if public information is exogenous.
  - The pecuniary externality (with sign depending on whether there is SS or SC) may counteract the learning from the price externality.
- Result: agents may put too much weight on private information in the market game.

The approach

- Tractable linear-quadratic-Gaussian model.
- Uncertainty about a common valuation parameter (e.g. cost) on which agents (e.g. firms) have private information and the price is noisy (e.g. because of demand shock).
- Market game: external effects go through price; pecuniary externalities.
- Rational expectations flavor but in the context of a well-specified game where agents compete in schedules (e.g. supply functions) and allowing for actions to be strategic substitutes or complements.
- Efficiency benchmark with private information: collective welfare or internal welfare of players.
- The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, monopolistic competition, trading in a financial market, or asset auctions.

Summary of results

- Firms correct the slope of their strategy according to what they learn from the price and the character of competition.
    - Under strategic substitutes competition the price's informational and allocational roles conflict: a high price is bad news and the equilibrium schedule is steeper than with full information.
    - Under strategic complements there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information.
- Equilibria will not be team-efficient:
    - Under strategic substitutability, equilibrium prices will tend to convey too little information when the informational role of prices prevails and too much information when their allocational role prevails (normal case).
        - At the boundary of those situations agents use vertical schedules (as in a Cournot game).
    - Under strategic complementarity, prices always convey too little information.

Intuition for the welfare results

- Price-contingent strategies create a double-edged externality in the use of private information:
  - Learning externality, which leads to under-reliance on private information.
  - Pecuniary externality which obtains even if public information is exogenous (i.e. even if firms disregard the information content of the price) because firms are not exposed to aggregate volatility,
    - consumers dislike aggregate output volatility under strategic substitutes but enjoy it under strategic complements;
    - firms respond too much (little) to private information in the strategic substitutes (complements) case.
- With strategic complements we have always underweighting of private information.
- Under strategic substitutes depending on the strength of the learning externality we may overcome or not the overweighting result due to the payoff externality.
- The point where both externalities cancel each other is when firms use vertical supply schedules.

Welfare loss and precision of information

- At the team-efficient solution, more precise public or private information reduces the welfare loss (WL) .
- At the market solution the WL
  - may be increasing with the precision of private information by increasing the response of an agent to his private signal, when the market calls already for a too large response to private information, and this indirect effect may dominate the direct effect;
  - is always decreasing with the precision of public information.

The contribution in the literature (I)

- The paper
  - follows the tradition of the welfare analysis of private information economies (Palfrey 1985, Vives 1988, Angeletos and Pavan 2007) extending it to endogenous public information in market games;
  - subverts the usual intuition of informational externality models (Vives 1997, Angeletos and Pavan 2009, Amador and Weill 2010, 2012) in a market game.
- Competitive equilibria need not be restricted efficient under incomplete markets and private information. Then pecuniary externalities lead to inefficiency since the conditions of the first fundamental welfare theorem are not fulfilled (Greenwald and Stiglitz 1986).
- Information frictions given rise to informational externalities through prices and competitive noisy rational expectations equilibria, in which traders take into account information from prices, are not constrained efficient (as in Laffont 1985).

The contribution in the literature (II)

- In our quasilinear utility model there is no room for the Hirshleifer (1971) effect (fully revealing REE may destroy insurance opportunities by revealing too much information and then REE need not be ex ante efficient).
  - We provide therefore an instance of REE which may reveal too much information on a fundamental on which agents have private information which is independent of the Hirshleifer effect.
- Recent literature about when more public information actually reduces welfare:
  - Burguet-Vives (2000); Morris-Shin (2002); Angeletos-Pavan (2007, 2009); Amador-Weill (2012).
  - Amador-Weill (2010): a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect.
    - Model purely driven by information externalities in the presence of strategic complementarities in terms of responses to private information.

Internal team benchmark

- The results can be extended to the internal team-efficient benchmark (where only the collective welfare of the players is taken into account, e.g. ignoring passive consumers).
- The bias towards too much weight on private information is increased.
- Again endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007):
    - With strategic complementarity in payoffs, agents always rely too little on private information. This is in stark contrast to the case of exogenous information, where agents under strategic complementarity rely too much on private information.

Quadratic payoff market game

- A continuum of players indexed in the interval $[0,1]$.
- Player i has the payoff:

$$\begin{aligned}\pi\left(x_i,\tilde{x}\right) &= \left(p-\theta\right)x_i-\frac{\lambda}{2}x_i^2\\ \text{where } p &= \alpha+u-\beta\tilde{x} \text{ and } \tilde{x}=\int_0^1 x_i di.\end{aligned}$$

- The slope of the best reply of a player (degree of complementarity) is:

$$m\equiv\frac{\frac{\partial^2\pi}{\partial x_i\partial\tilde{x}}}{-\frac{\partial^2\pi}{\left(\partial x_i\right)^2}}=-\frac{\beta}{\lambda}<\frac{1}{2}.$$

- Strategic Substitutes (SS) if $\beta>0$.
- Strategic Complements (SC) if $\beta<0$.

Information structure

- Public statistic (price) $p = \alpha + u - \beta\tilde{x}$ where $u \sim N\left(0, \tau_u^{-1}\right)$ has a dual role:
  - Marginal benefit of taking action level x_i with cost $\theta x_i + \frac{\lambda}{2}x_i^2$.
  - Informational since $\tilde{x}$ will reflect information of agents.
- Player i receives a signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N\left(0, \tau_\varepsilon^{-1}\right)$ and $\theta \sim N\left(0, \tau_\theta^{-1}\right)$.
  - Error terms are uncorrelated across players, with noise u and with θ.
  - $\int_0^1 s_i di = \theta + \int_0^1 \varepsilon_i di = \theta$. (By convention $\int_0^1 \varepsilon_i di = 0$ (a.s.)).
- Both payoff and informational externalities go through the market price p.
  - When $\beta = 0$ there is no informational or payoff externality.

Competition in Supply Functions

A continuum of firms of mass one compete in a homogenous product market facing inverse demand:

$$p = \alpha + u - \beta \tilde{x}$$

where $\tilde{x} = \int_0^1 x_i di$.

The cost function of firm i is given by:

$$C(x_i) = \theta x_i + \frac{\lambda}{2} x_i^2$$

- SS if $\beta > 0$; SC if $\beta < 0$ (network good with upward sloping demand)
- The cost θ could be an ex post pollution damage that is assessed on firm i, say an electricity generator, and for which the firm has an estimate before submitting its supply function.

The market game

- At $t = 0$ the random variables θ and u are drawn but not observed.
- At $t = 1$ each player observes his own private signal and submits a schedule $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$.
- At $t = 2$ the public statistic is formed (the "market clears") by finding a $\hat{p}\left((X_j(s_j, \cdot))_{j \in [0,1]}\right)$ that solves

$$p = \alpha + u - \beta \left(\int_0^1 X_i(s_i, p)\, di \right)$$

  and payoffs are collected.
- Rational expectations flavor in the context of a well specified schedule game with payoffs:

$$\pi_i \left((X_j(s_j, \cdot))_{j \in [0,1]} \right) = (p - \theta)\, x_i - \frac{\lambda}{2} x_i^2$$

where $x_i = X_i(s_i, p)$, $\tilde{x} = \int_0^1 X_j(s_j, p)\, dj$, and $p = \hat{p}\left((X_j(s_j, \cdot))_{j \in [0,1]}\right)$.

Alternative interpretations

- Monopolistic competition
- Demand schedule competition
  - Labor market
  - Financial trading
  - Asset auctions
- We will keep a supply interpretation of the model.

Competition in Supply Functions

A continuum of firms of mass one compete in a homogenous product market facing inverse demand:

$$p = \alpha + u - \beta \tilde{x}$$

where $\tilde{x} = \int_0^1 x_i di$.

The cost function of firm i is given by:

$$C(x_i) = \theta x_i + \frac{\lambda}{2} x_i^2$$

SS and SC:

- SS if $\beta > 0$
- SC if $\beta < 0$ (network good with upward sloping demand)

Equilibrium

- Linear Bayesian Equilibrium of the schedule game for which the public statistic functional is of the type $P(\theta, u)$.
- The solution to

$$\max_{x_i} \mathbb{E}\left[\left(p - \theta - \frac{\lambda}{2} x_i\right) x_i | s_i, p\right]$$

is symmetric across players and the equillibrium will be symmetric:

$$X(s_i, p) = \lambda^{-1}\left[p - \mathbb{E}\left[\theta | s_i, p\right]\right] \text{ with } p = P(\theta, u).$$

Equilibrium

- Suppose players use linear strategies $x_i = \hat{b} + \hat{c}p - as_i$.
- It follows then from $p = \alpha + u - \beta\tilde{x}$ and $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ that, provided $\hat{c} \neq -\beta^{-1}$:

$$p = P(\theta, u) = (1 + \beta\hat{c})^{-1} \left(\alpha - \beta\hat{b} + z\right) \text{ where } z = \beta a\theta + u \ .$$

- Market depth is given by $\left(\frac{\partial P}{\partial u}\right)^{-1} = 1 + \beta\hat{c}$.
- Then if $1 + \beta\hat{c} > 0$, z is informationally equivalent to p (and both move together), and $\mathbb{E}[\theta|s_i, p] = \mathbb{E}[\theta|s_i, z]$.
- Posit strategy

$$x_i = b - as_i + cz$$

and obtain $p = \alpha - \beta b + (1 - \beta c)\, z$.

Equilibrium characterization

- We can solve for the equilibrium in the usual way by identifying coefficients with the candidate linear strategy.
- An alternative way of characterizing the equilibrium (more instructive from the welfare perspective) is to observe that at a equilibrium players make privately efficient use of their signals and efficient use of public information.
- The FOC for player i is $\mathbb{E}\left[p - MC\left(x_i\right) | s_i, z\right] = 0.$
  - Therefore $\mathbb{E}\left[p - MC\left(x_i\right)\right] = 0$ and

$$b = \frac{\alpha}{\beta + \lambda}.$$

Equilibrium characterization

The parameters a and c in $x_i = b - as_i + cz$ are determined by the intersection of

- the (privately) efficient use of private information (which is independent of τ_u):

$$\mathbb{E}\left[\left(p - MC\left(x_i\right)\right) s_i\right] = 0$$

- and of the efficient use of public information (which is independent of τ_ε):

$$\mathbb{E}\left[\left(p - MC\left(x_i\right)\right) z\right] = 0$$

- With SS ($\beta > 0$): Figure 1
- With SC ($\beta < 0$): Figure 2

Equilibrium characterization

Proposition 1. Let $\tau_\varepsilon \geq 0$ and $\tau_u \geq 0$. There is a unique (and symmetric) equilibrium

$$X(s_i, p) = \lambda^{-1}\left[p - \mathbb{E}\left[\theta | s_i, p\right]\right] = \hat{b} - a s_i + \hat{c} p$$

where a is the unique (real) solution of the equation

$$a = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau)} \text{ with } \tau = \tau_\theta + \tau_u \beta^2 a^2,$$

$$\hat{c} = \left((\beta+\lambda)\left(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1}\right)^{-1} - \beta\right)^{-1} \text{and } \hat{b} = \alpha(1 - \lambda\hat{c})(\beta+\lambda)^{-1}.$$

In equilibrium,

- $a \in \left(0, \frac{\tau_\varepsilon}{\lambda(\tau_\theta + \tau_\varepsilon)}\right)$ decreases with λ, $|\beta|$, τ_u and τ_θ, and increases with τ_ε;
- $sign\{\partial\hat{c}/\partial\tau_u\} = sign\{-\partial\hat{c}/\partial\tau_\theta\} = sign\{-\beta\}$ and market depth $1 + \beta\hat{c} > 0$ is decreasing in τ_u and increasing in τ_θ; and
- τ is increasing in $|\beta|$, τ_u and τ_ε; and decreasing in λ.

Interpretation

- Price has an informational role on top of index of scarcity role:
  - SS ($\beta > 0$). Conflicting roles: a high price conveys the news that costs are high and the equilibrium schedule is steeper than with full information. Supply may be downward sloping if information effect is strong enough.
  - SC ($\beta < 0$). The informational and scarcity roles of the price are aligned: a high price conveys the news that costs are low and the equilibrium schedule is flatter than with full information. Supply is upward sloping.
- *Learning (informational) externality* present at the noisy equilibrium: from the viewpoint of an individual firm, public information is perceived as exogenous.
- There is no learning externality (and $\hat{c} = \lambda^{-1}$):
  - When agents have perfect information ($\tau_\varepsilon = \infty$).
    - When $0 < \tau_\varepsilon < \infty$, $\hat{c} < \lambda^{-1}$ (steeper SF) if $\beta > 0$ and $\hat{c} > \lambda^{-1}$ (flatter SF) if $\beta < 0$.
  - When the price contains no information ($\tau_u = 0$); and when signals are uninformative ($\tau_\varepsilon = 0$).

Remarks

- The informational component of the price increases with τ_u and decreases with τ_θ.
- As $\tau_u \to \infty$ the precision of prices also tends to ∞, the weight to private information tends to 0, and the equilibrium becomes fully revealing and collapses.
- The equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy (Vives (2011)).
- "Price impact" is always positive, $\frac{\partial P}{\partial u} = (1 + \beta\hat{c})^{-1} > 0$.
- Slope of excess demand: $\Xi' = -\left(\beta^{-1} + \hat{c}\right)$ and $sgn\left\{\Xi'\right\}$ $= sgn\left\{-\beta\right\}$.

Welfare in the homogenous product market

- Total surplus:

$$\mathrm{TS} = \left(\alpha + u - \beta\frac{\tilde{x}}{2}\right)\tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2}x_i^2\right) di$$

- The TS function is strictly concave for symmetric solutions $(\beta + \lambda > 0)$.
- TS depends on both θ and u.
- The equilibrium is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_\varepsilon < \infty$) and $\mathbb{E}[\mathrm{TS}]$ is strictly greater in the first best (full information) allocation x^o than at the equilibrium:
    - suppliers produce under uncertainty and rely on imperfect estimation of the common cost component and end up producing different amounts despite costs being identical and strictly convex;
    - however, since producers are competitive they produce in expected value the right amount at the equilibrium: $\mathbb{E}[\tilde{x}] = \mathbb{E}[x^o]$.

Team TS solution

- Welfare benchmark: team solution that maximizes expected total surplus subject to the use of linear decentralized strategies (Vives (1988), Angeletos and Pavan (2007)).
- In the economy considered if firms would not condition on prices, i.e. if each firm would set quantities conditioning only on its private information, and possibly an exogenous public signal, then the market solution would be team-efficient (Vives (1988), Angeletos and Pavan (2007)).

Efficiency in the Cournot market

- Firms compete in quantities contingent only on their private information as well as a normally distributed public signal z about θ, with exogenous precision τ: a strategy for firm i is $X_i(s_i, z)$.
- There is a unique Bayesian Cournot equilibrium (it is symmetric and linear):

$$X(s_i, z) = \frac{\alpha}{\beta + \lambda} - \left(as_i + \left((\beta + \lambda)^{-1} - a\right)\mathbb{E}[\theta \mid z]\right)$$

where $a = \frac{\tau_\varepsilon}{\lambda(\tau + \tau_\varepsilon) + \beta\tau_\varepsilon}$. Note that $a \leq (\beta + \lambda)^{-1}$.

- Equilibrium FOC: $\mathbb{E}[p - MC(x_i)| \, s_i, z] = 0$.
- Cournot equilibrium is team efficient since the same FOC hold also for the maximization of $\mathbb{E}[\mathrm{TS}]$ subject to decentralized production strategies:

$$\mathbb{E}\left[\frac{\partial \mathrm{TS}}{\partial x_i} |s_i, z\right] = \mathbb{E}[p - MC(x_i)| \, s_i, z] = 0.$$

Efficiency in the Cournot market

Endogenous quantity signal

- Suppose now that the public signal z comes from an endogenous noisy quantity signal $q = \tilde{x} + \eta$ where $\eta \sim N\left(0, \tau_\eta^{-1}\right)$ is independent of the other random variables in the model.
- Then at a linear equilibrium, $z = a\theta - \eta$ and now $\tau = \tau_\theta + \tau_\eta a^2$ is endogenous.
- Conjecture: the endogenous quantity signal will lead firms to put too little weight on their private information due to the information externality (Vives (1997), Amador and Weill (2012)).
- Argument:
    - The market equilibrium is efficient for given public information.
    - We should improve on the market marginally by increasing the precision of public information, and therefore by having firms put more weight to private information, and under our linear-normal framewok the result is global.
    - This implies that more weight to private information is needed (Angeletos and Pavan (2009)).

Efficiency in the Cournot market

Endogenous quantity signal (proof)

- Candidate team strategy: $X(s_i, z) = \frac{\alpha}{\beta+\lambda} - \left(as_i + \left((\beta+\lambda)^{-1} - a\right)\hat{z}\right)$, $\hat{z} = \mathbb{E}[\theta \mid z]$, with a to be chosen.
- We have that:

$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial a} = \mathbb{E}\left[(p - \mathrm{MC}(x_i))\left(\left.\frac{\partial x_i}{\partial a}\right|_{\hat{z}\text{ ct.}} + \frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right]$$

- At the market solution:
  - $\mathbb{E}\left[(p - \mathrm{MC}(x_i))\left.\frac{\partial x_i}{\partial a}\right|_{\hat{z}\text{ ct.}}\right] = 0$ since firms take z as given.
  - The learning externality term:

$$\mathbb{E}\left[(p - \mathrm{MC}(x_i))\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right] > 0$$

  - Therefore, $\partial \mathbb{E}[\mathrm{TS}]/\partial a > 0$.
- Since $\mathbb{E}[\mathrm{TS}]$ is strictly concave in a we conclude that the information externality leads to a too little response to private information.

Efficiency in the supply function market

- The team efficient solution internalizes the information externalities of the actions of firms and is restricted to use the same type of strategies that the market (decentralized and linear).

$$Max_{a,b,c}\ \mathbb{E}\left[\mathrm{TS}\right]$$

subject to $x_i = b - as_i + cz$, $\tilde{x} = b - a\theta + cz$, and $z = \beta a\theta + u$.

- Equivalently, the team-efficient solution minimizes, over the restricted strategies, the expected welfare loss WL with respect to the full information first best: $\mathrm{WL} = \mathbb{E}\left[\mathrm{TS}^o\right] - \mathbb{E}\left[\mathrm{TS}\right]$.
- We have that:

$$\mathrm{WL} = \left((\beta+\lambda)\,\mathbb{E}\left[(\tilde{x}-x^o)^2\right] + \lambda\mathbb{E}\left[(x_i-\tilde{x})^2\right]\right)\Big/2$$

Team TS solution

Expected deadweight loss: Aggregate and distributive inefficiency

- The candidate team strategy can be written as $x_i = \lambda^{-1}\left(p - \left(\gamma s_i + (1-\gamma)\,\mathbb{E}\left[\theta|z\right]\right)\right)$ (with weight to private information γ $(= a\lambda)$).
- Sources of deadweight loss:
  - $\mathbb{E}\left[(\tilde{x} - \tilde{x}^o)^2\right] = \frac{(1-\gamma)^2}{\tau(\beta+\lambda)^2}$: non-fundamental volatility /aggregate inefficiency (decreases with γ).
  - $\mathbb{E}\left[(x_i - \tilde{x})^2\right] = \frac{\gamma^2}{\tau_\epsilon \lambda^2}$: dispersion/productive inefficiency (increases with γ).
- Team solution minimizes WL among decentralized strategies trading-off both sources:
  - A more informative price (higher τ and γ) reduces allocative inefficiency but increases productive inefficiency.

Team TS solution

Expected deadweight loss

- We have that for a generic strategy with response a to private information:

$$\mathrm{WL}\left(a;\tau\right)=\frac{1}{2}\left(\frac{\left(1-\lambda a\right)^{2}}{\tau\left(\beta+\lambda\right)}+\frac{\lambda a^{2}}{\tau_{\varepsilon}}\right)\text{ where }\tau=\tau_{\theta}+\tau_{u}\beta^{2}a^{2}.$$

- $\mathrm{WL}\left(a;\tau\left(a\right)\right)$ is strictly convex in a.

Exogenous public information: the pecuniary externality

- Suppose that firms receive an exogenous public signal with precision τ.
- The market solution is then a "naive" competitive equilibrium where firms condition on the market price but do not learn from it.
- Firms put Bayesian weights to information to predict θ and solve

$$\mathrm{Min}_a \mathrm{L}(a) = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{\tau\lambda} + \frac{\lambda a^2}{\tau_\varepsilon}\right).$$

- The team solution solves

$$\mathrm{Min}_a \mathrm{WL}(a;\tau) = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{\tau(\beta+\lambda)} + \frac{\lambda a^2}{\tau_\varepsilon}\right).$$

Exogenous public information: the pecuniary externality

- The competitive equilibrium is not team efficient:

$$a_{exo}^{T}\left(\tau\right)=\frac{\tau_{\varepsilon}}{\lambda\left(\tau_{\varepsilon}+\tau\right)+\beta\tau}\gtreqless a_{exo}^{*}\left(\tau\right)=\frac{\tau_{\varepsilon}}{\lambda\left(\tau_{\varepsilon}+\tau\right)}\text{ iff }\beta\lesseqgtr 0.$$

- There is a pecuniary externality in the use of private information since firms do not take into account how their response to private information affects aggregate volatility:
  - With SS, $\beta>0$, (SC, $\beta<0$) outputs move together too much (little) ($\mathbb{E}\left[\mathrm{TS}\right]$ is decreasing (increasing) in $Cov\left[x_i,\tilde{x}\right]=\mathbb{E}\left[(\tilde{x})^2\right]$ with SS (SC)).
  - When $\beta=0$ there is no payoff externality and firms use information efficiently.
  - When there is no private information ($\tau_{\varepsilon}=0$) or information is perfect ($\tau_{\varepsilon}=\infty$) the market is efficient since it is competitive and pecuniary externalities are internalized.

Exogenous public information: the pecuniary externality

- Since the strategy for firm i is of the form $x_i = \lambda^{-1}\left(p - \left(\gamma s_i + (1-\gamma)\ \hat{z}\right)\right)$ we have that

$$\frac{\partial x_i}{\partial a} = \left.\frac{\partial x_i}{\partial a}\right|_{p,\ \hat{z}\ \mathrm{ct.}} + \frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a}$$

and

$$\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a} = \underbrace{\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left.\frac{\partial x_i}{\partial a}\right|_{p,\ \hat{z}\ \mathrm{ct.}}\right]}_{\mathrm{Market}} + \underbrace{\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left(\lambda^{-1}\frac{\partial p}{\partial a}\right)\right]}_{\mathrm{PE}}$$

where the first term is what the market equates to zero and the second is the pecuniary externality in the use of private information.

Exogenous public information: the pecuniary externality

- Using the fact that

$$\partial p/\partial a = \beta\lambda\left(\beta+\lambda\right)^{-1}\left(\theta - \mathbb{E}\left[\theta|z\right]\right),$$

at the market solution for given τ

$$\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left(\lambda^{-1}\frac{\partial p}{\partial a}\right)\right] = -\beta\lambda\left(\beta+\lambda\right)^{-1} a^*_{\mathrm{exo}}\left(\tau\right)\sigma^2_{\varepsilon}.$$

- Therefore:

$$sign\left\{\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a}|_{a=a^*_{\mathrm{exo}}}\right\} = sign\left\{\mathrm{PE}\right\} = sign\left\{-\beta\right\}.$$

Endogenous public information: the learning externality

- When firms take into account the information content of the price, public information is endogenous and a affects the precision of the price $\tau = \tau_\theta + \tau_u \beta^2 a^2$.
- $\mathrm{WL}\,(a; \tau\,(a))$ is a strictly convex function of a and the following FOC characterizes the team solution a^{T}:

$$\frac{d\mathrm{WL}}{da} = \frac{\partial \mathrm{WL}}{\partial a} + \underbrace{\frac{\partial \mathrm{WL}}{\partial \tau}}_{-} \underbrace{\frac{\partial \tau}{\partial a}}_{+} = 0.$$

- Therefore, *at the team solution with exogenous public precision τ by increasing a the welfare loss is reduced; i.e.*

$$\mathrm{dWL}\left(a^{\mathrm{T}}_{\mathrm{exo}}\,(\tau)\,;\tau\right)/da < 0.$$

The combined effect of the externalities

- From the FOC $dWL(a;\tau(a))/da = 0$ we obtain that a^T fulfills

$$a = \frac{\tau_\varepsilon}{\lambda(\tau(a)+\tau_\varepsilon)+\beta\tau(a)-\Delta(a)}$$

  where $\beta\tau(a)$ corresponds to the payoff externality and $\Delta(a)>0$ to the learning externality.
- With strategic substitutes ($\beta>0$), at the market solution $(*)$, the payoff and learning externalities cancel each other when $\beta\tau=\Delta$, in which case and $a^*=a^{\mathrm{T}}$. This happens when $c^*=0$.
  - We have that $\beta\tau-\Delta>0$ when $c^*>0$ and $\beta\tau-\Delta<0$ when $c^*<0$. This suggests that
    - $a^*<a^{\mathrm{T}}$ when $c^*<0$ (for τ_u large the supply function is downward sloping because the informational component of the price prevails and the learning externality is strong and prevails) and
    - $a^*>a^{\mathrm{T}}$ when $c^*>0$ (for τ_u low the supply function is upward sloping because the allocational effect of the price prevails and the learning externality is weak and overpowered by the payoff externality).
- With strategic complements ($\beta<0$) the payoff and learning externalities reinforce each other and $a^*<a^{\mathrm{T}}$.

Team TS solution

Proposition 2. Let $\tau_\varepsilon > 0$. Then the team problem has a unique solution with $\lambda^{-1} > a^T > 0$, and $sgn\left\{a^* - a^T\right\} = sgn\left\{\beta c^*\right\}$.
Proof: From the expression for WL we obtain

$$\left.\frac{d\mathrm{WL}}{da}\right|_{a=a^*} = \lambda a^* \sigma_\varepsilon^2 \beta c^*.$$

Alternatively, the strategy for firm i is of the form $x_i = \lambda^{-1}\left(p - \left(\lambda a s_i + (1-\lambda a)\ \hat{z}\right)\right)$, where $\hat{z} = \mathbb{E}\left[\theta | z\right]$. We have that

$$\frac{\partial x_i}{\partial a} = \left.\frac{\partial x_i}{\partial a}\right|_{p,\hat{z}\ \mathrm{ct.}} + \left.\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a}\right|_{\hat{z}\ \mathrm{ct.}} + \frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}$$

where the last term corresponds to the learning externality. At the market solution

$$\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right] > 0$$

and the result follows adding up the effects of the externalities.

Team vs. Market: Summary

- With strategic substitutability ($\beta > 0$) there is excessive (insufficient) weight to private information whenever τ_u is small (large) and supply functions are increasing, $c^* > 0$ (decreasing, $c^* < 0$).
  - In the first case the market displays too litte aggregate inefficiency (the price contains too much information); in the second too much, the price is too little informative, and there is too little productive inefficiency.

  (See Figure 1))
- With strategic complementarity ($\beta < 0$) we have that $c^* > 0$ and $sign\left\{\partial \mathbb{E}\left[\mathrm{TS}\right]/\partial a\right\} = sign\left\{-\beta c^*\right\} > 0$ always: firms put too little weight on private information and the market displays too much allocative inefficiency.

  (See Figure 2)

Team vs. Market

- When $\beta = 0$ there is neither a payoff nor an informational externality and the team and market solutions coincide.
- For $\beta \neq 0$, $\tau_\varepsilon > 0$ and $\tau_u > 0$, the solutions coincide only if $c^* = 0$ (the intermediate case in Figure 1 when $\beta > 0$).
- When signals are uninformative ($\tau_\varepsilon = 0$) or perfect ($\tau_\varepsilon = \infty$) there is no private information, there is no learning externality and the payoff externality is internalized at the competitive equilibrium.
- When the price contains no information ($\tau_u = 0$), then only the payoff externality remains and $\operatorname{sgn}\left\{a^* - a^{\mathrm{T}}\right\} = \operatorname{sgn}\{\beta\}$.

Corollary 1 (market quality). At the market solution:

- In relation to the team optimum, when $\beta c^* > 0$ price informativeness τ and dispersion $\mathbb{E}\left[(x_i - \tilde{x})^2\right]$ are too high, and aggregate inefficiency $\mathbb{E}\left[\left(\tilde{x} - x^0\right)^2\right]$ too low. The opposite is true when $\beta c^* < 0$.
- In relation to the first best (where $\mathbb{E}\left[\left(\tilde{x} - x^0\right)^2\right] = \mathbb{E}\left[(x_i - \tilde{x})^2\right] = 0$), price informativeness is too low, and aggregate inefficiency and dispersion are too high.

Corollary 2 (implementation of team solution). The team solution a^T can be implemented with a quadratic tax $(\delta/2)\,x_i^2$ on firms, where

$$\delta = \frac{\beta\tau\left(a^T\right) - \triangle\left(a^T\right)}{\tau\left(a^T\right) + \tau_\varepsilon}.$$

Proof: At the team solution

$$a = \frac{\tau_\varepsilon}{\lambda\left(\tau\left(a\right) + \tau_\varepsilon\right) + \beta\tau\left(a\right) - \triangle\left(a\right)} = \frac{\tau_\varepsilon / \left(\tau\left(a\right) - \tau_\varepsilon\right)}{\lambda + \delta},$$

which is the expression for the market a when the quadratic cost parameter is $\lambda + \delta$.

- The implementation of the team solution requires a tax $\delta > 0$ when $a^* > a^T$ and a subsidy $\delta < 0$ when $a^* < a^T$.
- The tax or subsidy may be returned/charged in expectation to the firms and therefore it can satisfy budget balance in expected terms.

Comparative statics of the Welfare Loss

$$\mathrm{WL}(a) = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{\left(\tau_\theta + \tau_u \beta^2 a^2\right)(\beta+\lambda)} + \frac{\lambda a^2}{\tau_\varepsilon}\right)$$

Proposition 3

- $\mathrm{WL}\left(a^{\mathrm{T}}\right)$ at team solution is decreasing in τ_ε, τ_u and τ_θ.
  - Proof: WL is decreasing in τ_ε,τ_u and τ_θ for a given a and $\mathrm{WL}'\left(a^{\mathrm{T}}\right) = 0$.
- $\mathrm{WL}(a^*)$ at market solution is decreasing in τ_θ and τ_u (direct effect always dominates indirect effect) but it may be increasing in τ_ε (it will for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ small enough, in which case necessarily $a^* > a^{\mathrm{T}}$).

$$\frac{d\mathrm{WL}}{d\tau_y}(a^*) = \underset{\binom{+}{-}}{\frac{\partial \mathrm{WL}}{\partial a}}\ \underset{(-)}{\frac{\partial a^*}{\partial \tau_y}} + \underset{(-)}{\frac{\partial \mathrm{WL}}{\partial \tau_y}} < 0\ ;\ y = \theta, u$$

$$\frac{d\mathrm{WL}}{d\tau_\varepsilon}(a^*) = \underset{\binom{+}{-}}{\frac{\partial \mathrm{WL}}{\partial a}}\ \underset{(+)}{\frac{\partial a^*}{\partial \tau_\varepsilon}} + \underset{(-)}{\frac{\partial \mathrm{WL}}{\partial \tau_\varepsilon}}$$

Welfare loss and precision of information: summary and discussion

- More precise public or private information reduces the welfare loss (WL) at the team-efficient solution.
- WL at the market solution may be increasing with the precision of private information when the market calls already for a too large response to private information:
  - increasing the firm's response to his private signal and this indirect effect may dominate.
- WL at the market solution is always decreasing with the precision of public information.
- Result in contrast with recent literature where more public information may be damaging to welfare (because more public information discourages the use and/or acquisition of private information):
  - Burguet and Vives (2000); Morris and Shin (2002); Angeletos and Pavan (2007); Amador and Weill (2011).
  - Amador and Weill (2010): a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect.

What about incentives? (Messner and Vives (2005))

- The qualitative results hold replacing the team benchmark with the incentive-efficient solution
- Furthermore, partially revealing equilibrium prices will have the wrong informational mix when uncertainty is multidimensional.
    - Even if aggregate production is at its first best level at the equilibrium, it may pay to distort allocative efficiency to improve productive efficiency to raise welfare in an incentive compatible way.

Business cycle application (1)

- "Standard" island business cycle model with CRR utilities and CES aggregators augmented with incomplete information (Angeletos and Lao (2013)).
- Reduced form: players are the islands in the economy (with representative household and firm), actions are productions (which can be SS or SC), and types the local information sets consisting of exogenous and endogenous private and public signals (endogenous public signal is a noisy aggregate quantity, say a macro forecast).
- Equilibrium is in log-linear strategies (of the Bayesian-Cournot type) and it is unique under certain parametric conditions.
- The team welfare function is analogous to ours but in constant elasticity form and the optimum is found in the class of log-linear decentralized strategies.

Business cycle application (2)

- If prices are flexible and there are no endogenous signals the equilibrium is team efficient (as in our Cournot economy).
- With endogenous signals there is an informational externality and the equilibrium is not team efficient.
- Optimal policy is countercyclical: it induces agents to put more weight on their private information to internalize the informational externality.
- Where does the aggregate signal come from? If it is an average price across islands then the results of the paper apply:
    - Even if prices are flexible and agents do not learn from prices the equilibrium is not team efficient.
    - In the normal case with SS, agents put too much weight on their private information and optimal policy should be pro-cyclical.

Demand function competition

- Profits of (risk neutral) buyer/trader i of homogenous good with ex post value θ

$$\pi_i = (\theta - p)\, y_i - \frac{\lambda}{2} y_i^2.$$

- Inverse supply:

$$p = \alpha + u + \beta \tilde{y}$$

- Model fits our set up letting $y_i \equiv -x_i$
- Example: Firms demanding labor of uncertain productivity, $\beta > 0$, and facing ajustment costs in labor stock.

Demand function competition

Financial markets

Traders compete in demand schedules for a risky asset.

- λ proxies for risk aversion/transaction cost of informed traders.
- Demand of liquidity suppliers (with mass $\frac{1}{|\beta|}$):

$$\frac{\alpha + u - p}{\beta}$$

- Volume of liquidity supply (decreasing in $|\beta| \tau_u$).
- When $\beta > 0$, there is SS on informed payoffs and with high τ_u the demand of informed traders slopes upwards.
- When $\beta < 0$ (program traders as in Gennote and Leland (1990)), there is SC on informed payoffs and the slope of total excess demand is positive (alternative explanation to risk management VAR constraint, Shin 2010).
- Increasing the mass of liquidity suppliers increases the weight given to the private information by informed traders but decreases the informativeness of prices.

Demand function competition

Financial markets and the Tobin tax

- With $\beta > 0$ and in the normal case with downward-sloping demand schedules for informed traders, prices will contain too much information about θ.
  - This will tend to happen when the volume of liquidity trading is high (i.e. when $\beta^2 \tau_u$ is low).
  - In this case the response of informed traders induces excessive volatility of aggregate quantities from the perspective of liquidity traders and total surplus.
  - Efficiency is restored with a Tobin tax $\delta > 0$ on the transactions of informed.
- With $\beta > 0$ and upward sloping demand schedules for the informed, or with $\beta < 0$, prices will contain too little information about θ.
  - Efficiency is restored with a Tobin subsidy on the transactions of informed.

Demand function competition

Financial markets and a general Tobin tax

- To levy a tax only on privately informed speculators may not be feasible but an appropriate tax on all transactions will also work.
- Consider a tax δ on both informed, $(\delta/2)\, x_i^2$, and liquidity, $(\delta/2)\, \tilde{y}^2$, traders.
- Then the inverse supply of liquidity traders is $p = \alpha + u + (\beta + \delta)\, \tilde{y}$ and at the market solution $a^*(\delta)$,

$$a = \frac{\tau_\varepsilon}{(\lambda + \delta)\left(\tau_\varepsilon + \tau_\theta + (\beta + \delta)^2\, \tau_u a^2\right)},$$

- If $\beta > 0$ and $c^* > 0$, there is a common transaction tax that implements the team solution: $\delta > 0$ for which $a^*(\delta) = a^T$.
  - We have that $a^* > a^T > 0$ when $\delta = 0$; $a^*(\delta)$ decreases with δ, and ranges from a^* to 0 as δ goes from 0 to ∞.
  - δ is strictly lower than the transaction tax targeted only to speculators.

Demand function competition

Asset auctions

- Competitive bidders use demand functions and bid $\tilde{y}$ in the aggregate.
- Noncompetitive bidders bid according to $\frac{u}{\beta}$, $\beta > 0$.
- Inverse supply of asset facing competitive bidders:
  $p = \alpha + \beta\left(\frac{u}{\beta} + \tilde{y}\right) = \alpha + u + \beta\tilde{y}$,
- Value of asset for bidder i is θ.
- Marginal benefit for bidder i is $\theta - \lambda y_i$, where λy_i is a transaction/opportunity cost or risk aversion component
- Example: Banks bidding for liquidity/Treasury auction
  - θ related to price in secondary interbank market; λ reflects the structure of a counterparty's pool of collateral.

Demand function competition

Asset auctions

- Prices (from the viewpoint of general welfare or of competitive bidders) contain too much information in the usual case of downward sloping demand schedules (which obtain when the volume of noncompetitive bids is large, τ_u small).
- When the volume generated by noncompetitive bids is small (high τ_u) then demand schedules are upward sloping and prices will contain too little information (from the viewpoint of general welfare) and may contain too little information (from the viewpoint of competitive bidders) for intermediate values of τ_u within the high value region for τ_u.

Team-profit benchmark

At the (internal) team efficient solution expected profit, $\mathbb{E}[\pi_i]$ ($\pi_i = (\alpha + u - \beta\tilde{x} - \theta)\, x_i - (\lambda/2)\, x_i^2$), is maximized under the constraint that firms use decentralized linear strategies.

- This is the cooperative solution from the point of view of the players (joint profits are maximized):

$$Max_{a,b,c}\ \mathbb{E}[\pi_i]$$

subject to $x_i = b - as_i + cz, \quad \tilde{x} = b - a\theta + cz$ and $\quad z = \beta a\theta + u.$

- The market solution does not internalize
  - the competition payoff externality: if $\beta \neq 0$ it will produce an expected output $\mathbb{E}[\tilde{x}^*]$ which is too high (low) with strategic substitutes (complements) in relation to the full information cooperative solution $\mathbb{E}\left[x^{\mathrm{M}}\right]$;
  - the externalities in the use of information arising from price-contingent strategies.

Team-profit benchmark

Expected loss: Aggregate and distributive inefficiency

- Equivalently, the team-efficient solution minimizes, over the restricted strategies, the expected loss Ω with respect to the full information first best:

$$\Omega = \left((2\beta + \lambda) \mathbb{E}\left[\left(\tilde{x} - x^{\mathrm{M}} \right)^2 \right] + \lambda \mathbb{E}\left[\left(x_i - \tilde{x} \right)^2 \right] \right) / 2$$

- The candidate internal team strategy can be written as $x_i = (\lambda + \beta)^{-1} (p - (\gamma s_i + (1 - \gamma) E[\theta|z]))$ where $\gamma = (\lambda + \beta) a$.
- Sources of deadweight loss:
  - $\mathbb{E}\left[\left(\tilde{x} - x^{\mathrm{M}} \right)^2 \right] = \frac{(1-\gamma)^2}{\tau(2\beta+\lambda)^2}$: non-fundamental volatility /allocative inefficiency (decreases with γ).
  - $\mathbb{E}\left[(x_i - \tilde{x})^2 \right] = \frac{\gamma^2}{\tau_\epsilon (\lambda+\beta)^2}$: dispersion/productive inefficiency (increases with γ).
- Now the internal team solution optimally trades off the sources of the loss with respect to the responsiveness to private information among decentralized strategies which internalize the competition payoff externality.

Team-profit benchmark

Proposition 3. Let $\tau_\varepsilon > 0$. The internal team problem has a unique solution with $(\lambda+\beta)^{-1} > a^{\mathrm{IT}} > 0$. We have that:

$$sgn\left\{a^* - a^{\mathrm{IT}}\right\} = sgn\left\{\beta\left(c^*\lambda\sigma_\varepsilon^2 + \left(c^*\beta - 1\right)^2\sigma_\theta^2\right)\right\}.$$

Proof:

$$\begin{aligned}\frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a} &= \underbrace{\mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.}\right]}_{\text{Market}} + \\ &\quad \underbrace{\mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{PE+LE}} + \\ &\quad \underbrace{\mathbb{E}\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a}\right)\right]}_{\text{CE}}\end{aligned}$$

- Note: $\frac{\partial p}{\partial \tilde{x}} = -\beta x_i$,$\frac{\partial \tilde{x}}{\partial a} = \theta \left(c\beta - 1 \right)$, and $\frac{\partial \tilde{x}}{\partial a} = z$. Then

$$\frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a} = \mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(-s_i + c\beta\theta\right) - \beta x_i \theta \left(c\beta - 1\right)\right] \;=\; 0$$

- At the equilibrium:

$$\frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a}|_{a=a^*} = -\beta a^* \left(c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2 \right)$$

- We obtain:

$$\begin{aligned} sgn\left\{a^{\mathrm{IT}} - a^*\right\} &= sgn\left\{ \frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a}|_{a=a^*} \right\} \\ &= sgn\left\{ -\beta \left(c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2 \right) \right\} \end{aligned}$$

Team-profit benchmark

- As before, $\operatorname{sgn}\{\mathrm{PE}+\mathrm{LE}\} = \operatorname{sgn}\{-\beta c^*\}$.
- Now, $\operatorname{sgn}\{\mathrm{CE}\} = \operatorname{sgn}\{-\beta\}$ since $(c^*\beta - 1)^2 \sigma_\theta^2 > 0$, and therefore the CE term will call for a lower (higher) response to private information with strategic substitutes (complements) than the market solution.
    - If $\beta > 0$ there is adverse selection and a high price indicates high costs. If, say, costs are high $(\theta - \bar{\theta} > 0)$ then an increase in a will increase p ($\frac{\partial p}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial a} = -\beta (c\beta - 1) \left(\theta - \bar{\theta}\right) > 0$ since at the market solution $c\beta - 1 < 1$) while x_i will tend to be low (since at the market solution $\mathbb{E}\left[\left(\theta - \bar{\theta}\right) x_i\right] = a\sigma_\theta^2 (c\beta - 1) < 0$). This means that $\mathrm{CE} < 0$ and that a must be reduced.
    - Similarly, we have that $\mathrm{CE} > 0$ if $\beta < 0$.
- The results on CE are in line with the results obtained by Angeletos and Pavan (2007) with exogenous public signals (but the effect of the learning externality may overturn this result when $c^* < 0$).

Team-profit benchmark (summary)

- Maximize (average) expected profits subject to decentralized strategies.
- The same qualitative results as in team-TS hold but competition externality biases towards too much weight on private information:
  - Firms reliance on private information
    - With SS: too little (but this need not arise when $c^* < 0$)/ too much.
    - With SC: too little.

Monopolistic competition

Quantity setting:

- Substitutes (Complements) if $\beta > 0 \ (<) \, 0$.
- Inverse demand for firm i: $p_i = \alpha + u - \beta \tilde{x} - \frac{\lambda}{2} x_i$.
- Costs for firm i: θx_i
- Supply function: $X(s_i, p_i)$
- Observing the price p_i is informationally equivalent for firm i to observe

$$p = \alpha + u - \beta \tilde{x}.$$

- Welfare analysis of collusion with internal team benchmark.

Comparative statics

Proposition 4. Let $\tau_\varepsilon > 0$ and $\tau_u > 0$. In equilibrium:

i. The responsiveness to private information a decreases with λ, $|\beta|$, τ_u and τ_θ, and increases with τ_ε.

ii. The responsiveness to the public statistic $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞.
   - Market depth $1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

iii. Price informativeness τ is increasing in $|\beta|$, τ_u and τ_ε; decreasing in λ.

iv. Dispersion $\mathbb{E}\left[(x_i - \tilde{x})^2\right]$ decreases with λ, $|\beta|$, τ_u and τ_θ.

Comparative Statics on the Equilibrium Strategy

$$X\left(s_i, p\right) = \hat{b} - as_i + \hat{c}p$$

sgn	$\partial\tau_u$	$\partial\tau_\theta$	$\partial\tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial\hat{c}$	$-\beta$	β	$\beta\left(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta\left(\tau_\varepsilon - \tau_\theta\right)\right)$

- Market depth $\left(\partial P/\partial u\right)^{-1} = 1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

Comparative statics

Informational component of the price

- How the equilibrium weights to private and public information vary with the deep parameters of the model help to explain the results:

$$\mathbb{E}\left[\theta \mid s_i, z\right] = \gamma s_i + hz$$

where $a = \gamma/\lambda$ and $c = (1-h)/(\beta+\lambda)$.

- The informational component of the price is the weight $|h|$ on public information z, with $sgn\{h\} = sgn\{\beta\}$:
  - When $\beta > 0$ there is adverse selection (a high price is bad news about costs) and $h > 0$.
  - When $\beta < 0$, $h < 0$ and there is favorable selection (a high price is good news).
  - Effect of increasing sensitivity of price to output: $\partial a/\partial|\beta| < 0$ and $\partial\tau/\partial|\beta| > 0$ (direct effect prevails).
  - $\partial|h|/\partial\tau_\theta < 0$ and $\partial|h|/\partial\tau_u > 0$. The effect of τ_ε is ambiguous.

Comparative statics

Change in τ_u and τ_θ (Case SS, $\beta > 0$)

- As τ_u increases from 0 then $\hat{c}$ decreases from λ^{-1}(and the slope of supply increases) because of the increased informational component of the price.
  - Agents are more cautious when seeing a high price because it may mean higher costs.
- As τ_u increases more, $\hat{c} = 0$ at some point, turns negative, and as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.
  - In the particular case where the scarcity and informational effects balance, agents set a zero weight ($\hat{c} = 0$) on the public statistic. (See Figure 1)
- If τ_θ increases then the informational component of the price diminishes, since the agents are endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply).

Comparative statics

Change in τ_ε (Case SS, $\beta > 0$)

- An increase in the precision of private information τ_ε increases always the responsiveness to the private signal.
- The parameter $\hat{c}$ is U-shaped in relation to τ_ε: $\hat{c} = \lambda^{-1}$ both when $\tau_\varepsilon = \infty$ and when $\tau_\varepsilon = 0$ and $\hat{c} < \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$.
    - If τ_ε is high a further increase in τ_ε (less noise in the signals) lowers adverse selection and increases $\hat{c}$. If τ_ε is low then the price is not very informative and an increase in τ_ε increases adverse selection and lowers $\hat{c}$.

Comparative statics

Changes in precisions (Case SC, $\beta < 0$)

- A high price conveys good news both from the point of view of the scarcity and informational effects and supply is always upward sloping: $\hat{c} > \lambda^{-1}$ if $\beta < 0$. (See Figure 2)
  - A high price conveys the good news that average quantity tends to be high and therefore costs low.
- Increasing τ_u (which reinforces the informational component of the price) increases $\hat{c}$, the opposite of what happens when τ_θ increases.
- An increase in the precision of private information τ_ε increases the responsiveness to the private signal.
- The parameter $\hat{c}$ is hump-shaped in relation to τ_ε since $\hat{c} > \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$ and $\hat{c} = \lambda^{-1}$ in the extremes of the interval $(0, \infty)$.

Comparative Statics on the Equilibrium Strategy

$$X(s_i, p) = \hat{b} - as_i + \hat{c}p$$

sgn	$\partial\tau_u$	$\partial\tau_\theta$	$\partial\tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial\hat{c}$	$-\beta$	β	$\beta\left(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta(\tau_\varepsilon - \tau_\theta)\right)$

- Market depth $(\partial P/\partial u)^{-1} = 1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

Comparative statics

Change in the Degree of Complementarity

Let $m \equiv -\beta/\lambda$ and fix β (it makes sense to keep β fixed since β also affects the public statistic p).

Effects of a Change in the Degree of Complementarity (β fixed)

sgn	∂a	$\partial \gamma$	$\partial \tau$	$E\left[(x_i - \tilde{x})^2\right]$
∂m	$-\beta$	β	$-\beta$	$-\beta$

Comparative statics

Change in the Degree of Complementarity

- An increase in the degree of strategic complementarity m makes agents rely less on private information and decreases dispersion $\mathbb{E}\left[(x_i - \tilde{x})^2\right]$ in the strategic substitutes case $(\beta > 0)$ and the opposite happens in the strategic complements one $(\beta < 0)$.
    - With endogenous public information the precision of the public signal changes with the degree of complementarity.
    - With more complementarity m (a lower λ) and $\beta < 0$, agents rely less on private information ($\gamma = \lambda a$) decreases) but respond more to private information (a increases), and dispersion increases as well as the public precision increases. The opposite happens when $\beta > 0$.

Summary of results (I)

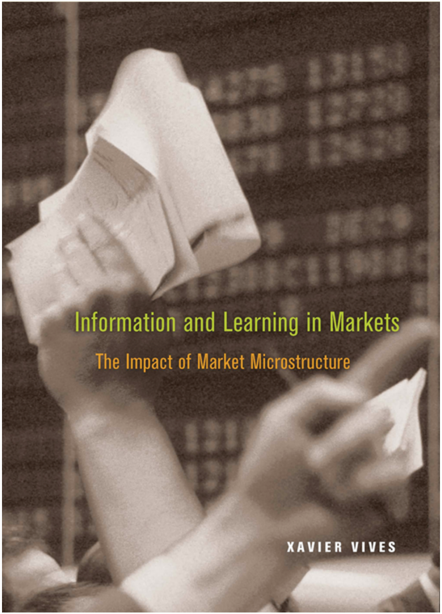
- We show that allowing agents to condition on prices may undermine the social value of learning from them:
  - Price-contingent strategies introduce a pecuniary externality in the use of private information which sign depends on whether competition is of the strategic substitutes or complements variety.
  - When the learning externality from the price is not very strong then agents may put too much weight to private information and the price may be too informative.
- Under SS prices will tend to convey too much information in the normal case where the allocational role of prices prevails over their informational role and too little in the opposite situation.
- Under SC prices convey too little information.
- More precise private information under SS may hurt welfare (but not more precise public information).

Summary of results (II)

- In market games with strategic substitutes the presumption that prices will contain too little information will typically not hold. Therefore:
    - subsidizing information acquisition is not warranted;
    - a Tobin transaction tax on the informed may improve welfare.
- The results can be extended to economies not efficient with full information:
    - The bias towards too much weight on private information is increased.
    - Received results on the relative weights to private and public information (when the latter is exogenous) may be overturned.

Extensions

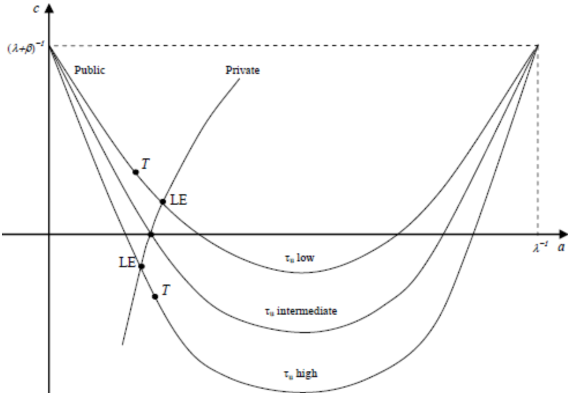
- Explore fully tax-subsidy schemes to implement team-optimal solutions (Angeletos and Pavan 2009, Lorenzoni 2010, Angeletos and La'O 2012).
- Incentives to acquire information (Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009; Myatt and Wallace 2012; Llosa and Venkateswaran 2012; Colombo et al. 2013).



Princeton University Press

Comparative statics

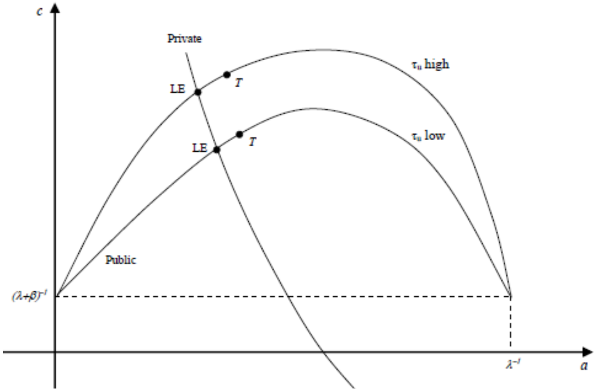
Figure 1. SS case



Return

Comparative statics

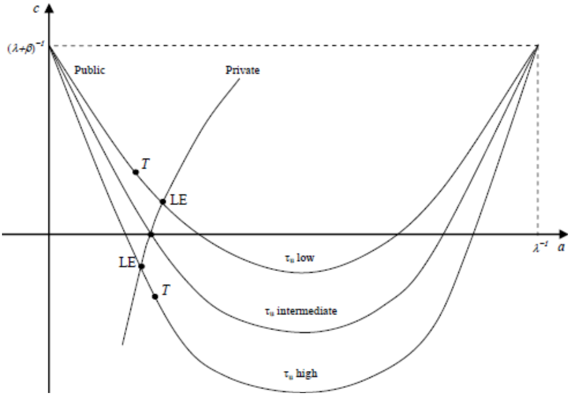
Figure 2. SC case



Return

Comparative statics

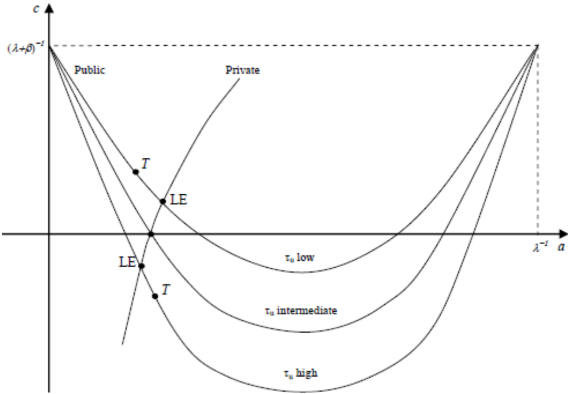
Figure 1. SS case



Return

Comparative statics

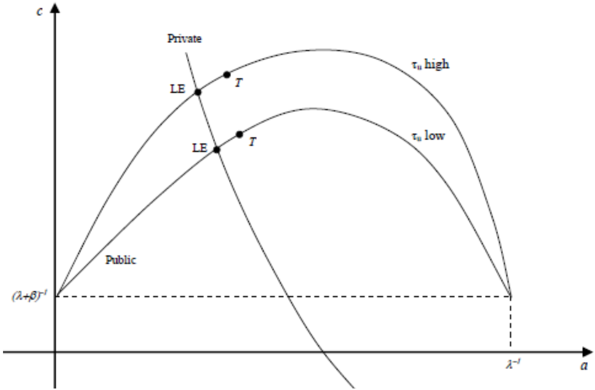
Figure 1. SS case



Return

Comparative statics

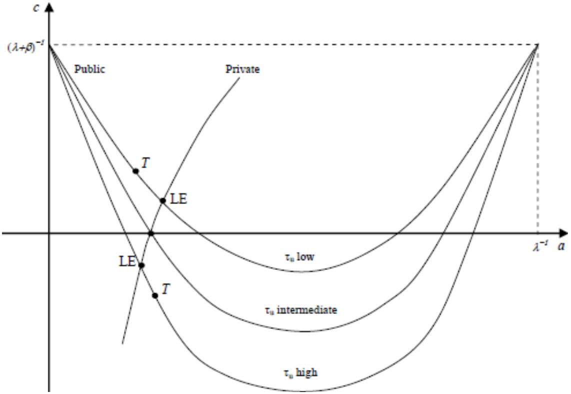
Figure 2. SC case



Return

Comparative statics

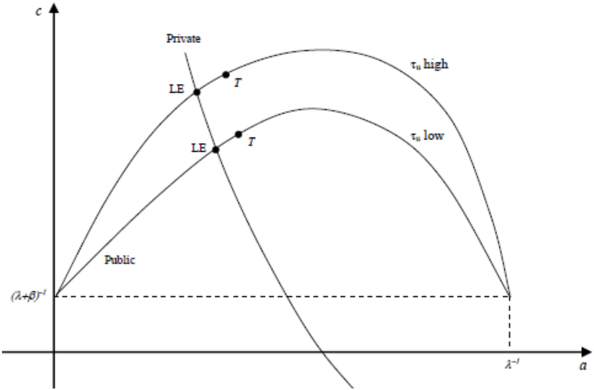
Figure 1. SS case



Return

Comparative statics

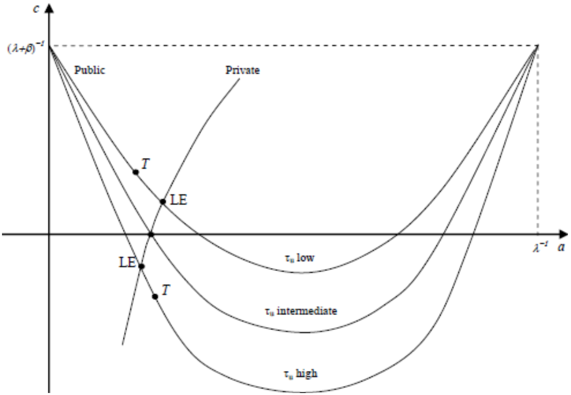
Figure 2. SC case



Return

Comparative statics

Figure 1. SS case



Return

Team TS solution: combined externalities effect

$$\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a} = \underbrace{\mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.}\right]}_{\text{Market}} + \underbrace{\mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{PE+LE}}$$

$$\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a} = \mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\left(-s_i + c\beta\theta\right)\right] = 0 \text{ (with } \frac{\partial x_i}{\partial z} = c \text{ , } \frac{\partial z}{\partial a} = \beta\theta\text{)}$$

Team TS solution

- At the equilibrium:

$$\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a}|_{a=a^*} = \beta c \mathbb{E}\left[\left(p - MC\left(x_i\right)\right)\theta\right] = -\lambda a^* \sigma_\varepsilon^2 \beta c^*$$

- Since $\mathbb{E}\left[\mathrm{TS}\right]$ is single-peaked for $a > 0$ with maximum at $a^T > 0$, conclude

$$sign\left\{\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a}|_{a=a^*}\right\} = sign\left\{\mathrm{PE} + \mathrm{LE}\right\} = sign\left\{-\beta c^*\right\} = sgn\left\{a^T - a^*\right\}.$$

Team TS solution

- The sign of the $\mathrm{PE} + \mathrm{LE}$ term depends on whether we have strategic substitutes or complements competition and on whether supply slopes upwards or downwards.

$$\mathrm{PE} + \mathrm{LE} \;= \mathbb{E}\left[\left(p - MC\left(x_i\right)\right) \left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} \right) \right] \text{ (with } \frac{\partial x_i}{\partial z} = c \text{ , } \frac{\partial z}{\partial a} = \beta \left(\theta - \bar{\theta}\right) \text{)}$$

- If supply is upward sloping $(c^* > 0)$ and good is normal $(\beta > 0)$:
  - If say, costs are high $(\theta - \bar{\theta} > 0)$ then an increase in a will increase x_i $\left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} = c\beta \left(\theta - \bar{\theta}\right) > 0\right)$ while $\left(p - \mathrm{MC}\left(x_i\right)\right)$ will tend to be low (since at the market solution $\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right) \left(\theta - \bar{\theta}\right) \right] < 0$).
  - This means that $\mathrm{PE} + \mathrm{LE} < 0$ and that a must be reduced.

Team TS solution

- If supply is downward sloping ($c^* < 0$) in the same situation an increase in a will decrease x_i, which is welfare enhancing.
- The same will happen if $\beta < 0$ since then $c^* > 0$ and an increase in a will decrease x_i. Then $\mathrm{PE} + \mathrm{LE} > 0$ and a must be increased.
- The team optimal solution uses public information efficiently but is not bound by the privately efficient use of information.

Demand function competition

Double auction with noise traders

- Noise traders demand a random amount u as a limit case of the model as $\beta \to \infty$:

$$\frac{\alpha + \hat{u} - p}{\beta} \text{ with } \hat{u} = \beta u.$$

Then $\beta^{-1}(\alpha + \beta\hat{u} - p) \to u$ as $\beta \to \infty$, and market clearing yields $u + \tilde{y} = 0$.

- With a diffuse prior we obtain in equilibrium: $X(s_i, p) = a(p - s_i)$.

Introduction
Evidence

- Initial evidence of herding of analysts forecasts (Gallo et al. 2002 for GDP forecasts; Trueman 1994, Hong et al. 2000 for securities) has been reversed by strong new evidence of anti-herding behavior in forecasting stock, metal prices or macro variables (e.g. Bernardt et al. 2006 and Pierdzioch et al. 2010)
  - Effinger and Polborn (2001) and Levy (2004) explain anti-herding behavior with reputational concerns.
  - We will offer a novel explanation of why agents may put excessive weight to private information in the context of market games.