Banks, Capital Flows and Financial Crises*

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Abstract

This paper proposes a macroeconomic model with financial intermediaries (banks), in which banks face occasionally binding leverage constraints and may endogenously affect the strength of their balance sheets by issuing new equity. The model can account for infrequent financial crises as a result of the nonlinearity induced by the constraint. It can also capture the interaction between the state of banks' balance sheets and the incidence of financial crises. We show how in an episode of rapid credit expansion triggered by low interest rates, banks diminish the rate of new equity issues, contributing to an increase in the likelihood of a crisis. In the model, policies directed at strengthening banks' balance sheets are useful for enhancing financial stability.

Keywords: Financial Intermediation; Sudden Stops; Leverage Constraints; Occasionally Binding Constraints.

JEL classification: E32; F41; F44; G15

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1 Introduction

The recent wave of financial crises across the globe has put financial stability at the forefront of policy discussions. At the same time, it has led to a renewed interest in macroeconomic frameworks that can capture financial disruptions in a manner consistent with recent experience, and which can be used to analyze financial stability policies. A desirable feature of this class of models is that they be consistent with the main stylized facts associated with financial disruptions. Two salient examples of these stylized facts that been emphasized recently are the following. First, financial crises are often associated with preceding credit booms.[1] Second, they tend to be associated with non-linear dynamics.[2]

This paper makes a contribution to this class of models by developing a small open economy framework with financial intermediaries, which addresses the two stylized facts mentioned above. A novel feature of our framework is that we allow banks to issue new equity as well as short term debt. By modifying the rate at which they issue new equity, banks are thus able to affect the strength of their balance sheets. In turn, the strength of banks' balance sheet is an important determinant of the probability of a subsequent financial crisis. By introducing this new choice margin for banks, we can study how their equity issuance decision changes with economic conditions. As we show, in a credit boom environment, banks will tend to reduce their equity issuance, contributing to an increased likelihood of a subsequent crisis.

As in Gertler and Kiyotaki (2010) and others,[3] in the model an agency problem may limit banks' ability to obtain funds. A novel characteristic of our setup in this respect is that banks' leverage constraints are not permanently binding, but instead may bind only occasionally. We show that the model can generate occasional financial crises nested within regular business cycle fluctuations as a result of the nonlinearity induced by the constraint. When the constraint is not binding, the behavior of the economy is similar to a frictionless neoclassical environment. When banks' are close to the constraint, small adverse realizations of shocks may trigger a full-blown financial crisis. A binding constraint opens the door to a financial accelerator mechanism similar to Bernanke et al. (1999), operating via the adverse loop between net worth, investment and asset prices. As a result, credit and investment collapse, borrowing-lending spreads increase sharply, and output and labor demand fall persistently.[4]

[1] Mendoza and Terrones (2012), for example, show that credit booms are systematically related to a boom-bust cycle in production and absorption, asset prices, capital inflows and external deficits. Moreover, they find that credit boom events often end with financial crises. This holds true for emerging and industrialized countries alike.

[2] Stein (2014), for instance, emphasizes the asymmetries in the relationship between credit spreads and economic activity, with increases in spreads leading to much stronger effects on the economy. See also Kenny and Morgan (2011).

[3] See also Gertler and Karadi (2011) and Gertler et al. (2012).

[4] We find that the financial accelerator is more powerful in the small open economy, relative to what it

We illustrate the workings of our framework via a series of numerical experiments, using a version of the model calibrated to a typical small open economy. We first analyze the economy's response to a decrease in the country interest rate. Low interest rates induce a capital inflow and a credit boom at home, consistent with the evidence that rapid credit expansions are typically financed by borrowing from foreign investors.[5] At the same time, they lead banks to diminish the rate of equity issuance. As we show, this endogenously exposes the economy to a higher risk of a financial crisis in the future.

In a second experiment, we illustrate the nonlinearities induced by the occasionally binding constraint, by comparing the effects of capital quality shocks (which work to reduce banks' net worth) of different sizes. Although negative capital quality shocks are associated with an increase in the probability of a crisis, only a shock that is large enough will bring banks against their leverage constraints, leading to a financial crisis. However, a crisis in the model is not the result of an abnormally adverse shock to capital quality. As we show, the model can deliver occasional financial crises with a frequency comparable to the data. Moreover, the crisis events produced by the model have features that are qualitatively and quantitatively consistent with the data.

Finally, we use the model to assess the desirability of policies directed at enhancing financial stability. In particular, we study a government subsidy which tilts banks' incentives in favor of raising equity, thus strengthening their balance sheet positions. We find that the subsidy is successful at reducing the probability of occurrence of financial crises, thereby increasing welfare.

The model in this paper is related to the literature on financial amplification based on collateral constraints. Mendoza (2010) builds a model focusing on a Fisherian debt-deflation mechanism, where total debt is limited by a collateral constraint that depends on the market price of physical capital. The presence of the collateral constraint amplifies the effects of exogenous shocks, helping the model to explain stylized facts regarding sudden stop episodes in emerging market countries. Benigno et al. (2012) also develop a simple dynamic stochastic general equilibrium model in which crises are endogenous events induced by the presence of an occasionally binding constraint.

The model, as in Mendoza (2010) and Benigno et al. (2012), is able to generate occasional financial crises and sudden stops' nested within normal business cycle fluctuations. The difference with these models, in turn, is that we offer a model which focuses explicitly on the

would be in a closed economy. The reason is that in a closed economy, the drop in investment that results from adverse intermediary balance sheet effects would be in part accommodated by lower domestic interest rates. In an open economy, instead, a capital outflow ensues.

[5]See, among others, Mendoza and Terrones (2012), Gourinchas and Obstfeld (2012) and Bruno and Shin (2012).

role of banks in building up financial risks, and which provides an explicit micro-foundation for the leverage constraint. This paper is also related to recent work by Cespedes et al. (2012), who incorporate financial intermediation subject to an occasionally binding credit constraint into a two-period open economy model. Our focus, instead, is to offer a framework that does not stray too far from the standard quantitative DSGE model used in policy analysis, and that is tractable enough to accommodate the features that are often present in that literature. This focus also differentiates our work from other recent papers introducing financial intermediation within a macroeconomic framework, like Brunnermeier and Sannikov (In Progress) or Boissay et al. (2013).

The remainder of the paper is organized as follows. Section 2 presents the model in detail. Section 3 presents the results and section 4 concludes.

2 The Model

The model is a small open economy in the spirit of Gertler and Kiyotaki (2010). There are banks who make risky loans to non-financial firms and collect deposits from both households and foreigners. Because of an agency problem, banks may be constrained in their access to external funds. In addition, we allow banks to raise new equity from households, so that the evolution of bank net worth reflects banks' endogenously chosen rate of new equity issued, as well as the mechanical accumulation of retained earnings.[6]

Another novel feature of our setup is that banks' constraints are not permanently binding, as in much of the related literature, but instead bind only occasionally. In normal or "tranquil" times, banks' constraints are not binding: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. As we show, when the constraint binds the economy enters into financial crisis mode: credit spreads rise sharply, and investment and credit collapse, consistent with the evidence.

2.1 Households

Each household is composed of a constant fraction $(1 - f)$ of workers and a fraction f of bankers. Workers supply labor to the firms and return their wages to the household. Each banker manages a financial intermediary ("bank") and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

[6] This approach is related to Gertler et al. (2012), in which banks are allowed to issue outside equity as well as debt. We focus on allowing the banks to raise inside equity instead.

Households do not hold capital directly. Rather, they deposit funds in banks. The deposits held by each household are in intermediaries other than the one owned by the household. Bank deposits are riskless one period securities. The consumption, C_t, bond holdings, B_t, and labor decisions, L_t, are given by maximizing the discounted expected future flow of utility

$$Max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \tag{1}$$

where

$$U(C_t, L_t) \quad = \quad \frac{\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1-\gamma} \tag{2}$$

subject to the budget constraint

$$C_t + B_t \quad \leq \quad W_t L_t + R_{t-1} B_{t-1} + \Pi_t \tag{3}$$

$\mathbb{E}_t$ denotes the mathematical expectation operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, given by $1/\epsilon$. Utility is defined as in Greenwood et al. (1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. W_t is the real wage, R_t is the real interest rate received from holding one period bond, Π_t is total profits distributed to households from their ownership of both banks and firms. The first order conditions of the household's problem are presented in Appendix 5.1.

2.2 Banks

Banks are owned by the households and operated by the bankers within them. In addition to its own equity capital, a bank can obtain external funds from both households, b_t, and foreign investors, b_t^*, such that total external financing available to the bank is given by $d_t = b_t + b_t^*$. We assume that both domestic deposits and foreign borrowing are one-period non-contingent debt. Thus, by arbitrage their returns need to be equalized in equilibrium, a condition we impose at the onset.

In addition, banks in period t can raise an amount e_t of new equity. The new equity is available in the following period to make loans, together with the equity accumulated via retained earnings and with any external borrowing d_{t+1}. Accordingly, in each period the bank uses its net worth n_t (which includes equity raised in the previous period) and external funds d_t, to purchase securities issued by nonfinancial firms, s_t, at price Q_t. In turn, nonfinancial firms use the proceeds to finance their purchases of physical capital.

We assume that banks are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and also at enforcing contractual obligations with these borrowers. That is why, as explained below, firms rely exclusively on banks to obtain funds, and the contracting between banks and nonfinancial firms is frictionless. However, as in Gertler and Kiyotaki (2010) and related papers, we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations and instead transfer a fraction of assets to his family, in which case it is shut down and its creditors can recover the remaining funds. In recognition of this possibility, creditors potentially limit the funds they lend to banks. In this setup, banks may be credit constrained, depending on whether their desired asset holdings per unit net worth exceeds the maximum allowed by the incentive constraint.

Figure 1: Period-t Timeline for Bankers

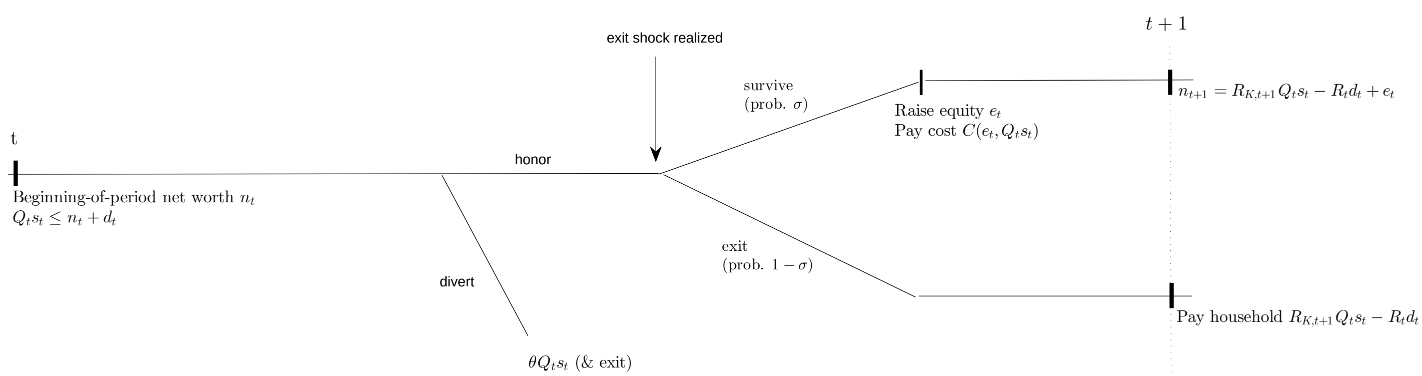


Figure 1 show the timeline for banks. Banks start the period t with net worth n_t.[7] Banks then use their net worth and external funds (issues of short-term bonds) to fund assets $Q_t s_t$, subject to the balance sheet constraint:

$$Q_t s_t \leq n_t + d_t \tag{4}$$

After purchasing the securities, banks can choose to divert fraction θ of assets funded,

[7]The newborn ones that replace the banks that exit receive endowment $\xi Q_{t-1} K_{t-1}$.

in which case they get $\theta Q_t s_t$. The incentive constraint requires that the bank's continuation value be higher than the value of the diverted funds.

At the *end* of the period, surviving banks have the option to raise new equity. In particular, after the bank finds out whether it receives the exit shock, in the case that it continues (which happens with probability σ) it can pay cost $C(e_t, Q_t s_t)$ to raise new equity e_t from the household, which will be available in $t+1$ to fund assets.[8] Therefore, the total net worth available for surviving banks in $t+1$ will be given by

$$n_{t+1} = R_{K,t+1} Q_t s_t - R_t d_t + e_t \tag{5}$$

where $R_{K,t+1}$ denotes the gross rate of return on a unit of the bank's assets from t to $t+1$ and R_t is the rate of return on short-term (risk free) bonds held by the bank's creditors. In the case the bank exits at the end of t (which happens with probability $1-\sigma$), we assume it does not have the option to issue new equity.[9] Accordingly, it simply pays the household the profits from assets funded in t, net of debt repayments: $R_{K,t+1} Q_t s_t - R_t d_t$.

The objective of the bank is to maximize expected terminal payouts to the household, net of the equity transferred by the household and of the cost of the transfer $C(e_t, Q_t s_t)$. The bank values payoffs in each period and state using $\Lambda_{t,t+i}$, the household's stochastic discount factor. Given these considerations, the bank's problem is to choose state-contingent sequences $\{s_t, d_t, e_t\}_{i=0}^{\infty}$ to maximize

$$\mathbb{E}_t \left[\sum_{i=1}^{\infty} \sigma^{i-1} (1-\sigma) \Lambda_{t,t+i} \overline{n}_{t+i} - \sum_{i=1}^{\infty} \sigma^i \left[\Lambda_{t,t+i-1} C(e_{t+i-1}, Q_{t+i-1} s_{t+i-1}) + \Lambda_{t,t+i} e_{t+i-1} \right] \right]$$

subject to (54), (5) and the incentive constraint, where $\overline{n}_t \equiv R_{K,t} Q_{t-1} s_{t-1} - R_{t-1} d_{t-1}$.

Switching to the recursive formulation, we can write a banker's problem as follows:

$$V_t(n_t) = \max_{s_t, d_t, e_t} \; (1-\sigma) \mathbb{E}_t \Lambda_{t,t+1} \left(R_{K,t+1} Q_t s_t - R_t d_t \right) + \sigma \left\{ \mathbb{E}_t \Lambda_{t,t+1} \left[V_{t+1}(n_{t+1}) - e_t \right] - C(e_t, Q_t s_t) \right\} \tag{6}$$

subject to

$$Q_t s_t \leq n_t + d_t \tag{7}$$

[8] The cost is allowed to depend on the overall balance sheet size as well as e_t; later we will specialize to $c(x_t) Q_t s_t$, where $x_t \equiv e_t / Q_t s_t$.

[9] As long as the cost of raising equity is positive, for an exiting bank it would never pay to raise equity, as the new equity would simply be transferred back to the household.

$$n_{t+1} = R_{K,t+1} Q_t s_t - R_t d_t + e_t \tag{8}$$

$$(1-\sigma)\mathbb{E}_t \Lambda_{t,t+1} \left(R_{K,t+1} Q_t s_t - R_t d_t\right) + \sigma \left[\mathbb{E}_t \Lambda_{t,t+1} \left(V_{t+1}(n_{t+1}) - e_t\right) - C(e_t, Q_t s_t)\right] \geq \theta Q_t s_t \tag{9}$$

Equation (54) is the bank's balance sheet constraint. Equation (55) is the law of motion for the banker's net worth, which includes new equity raised e_t. Equation (56) is the incentive constraint: it states that the banker's continuation value must be greater than the value of diverting funds.

To solve the banker's problem, first guess that the value function is $V_t(n_t) = \alpha_t n_t$. Define

$$\begin{aligned}
\Omega_{t+1} &= (1-\sigma) + \sigma \alpha_{t+1} \\
\mu_{K,t} &= \mathbb{E}_t[\Lambda_{t,t+1} \Omega_{t+1} (R_{K,t+1} - R_t)] \\
\nu_t &= \mathbb{E}_t[\Lambda_{t,t+1} \Omega_{t+1}] R_t \\
\nu_{e,t} &= \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\alpha_{t+1} - 1\right)\right]
\end{aligned}$$

Since Ω_{t+1} is the value to the bank of an extra unit of net worth the following period, it acts by "augmenting" banks' stochastic discount factor (SDF), so that their effective SDF is $\Lambda_{t+1}\Omega_{t+1}$. The variable $\nu_{e,t}$ denotes the *net* value today of a transfer by the household that increases bank net worth tomorrow by one unit, conditional on not exiting. In the decision to raise equity, the bank trades off the benefit $\nu_{e,t}$ against the issuing cost.

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t, e_t} \quad \mu_{K,t} Q_t s_t + \nu_t n_t + \sigma \left[\nu_{e,t} e_t - C(e_t, Q_t s_t)\right] \tag{10}$$

subject to

$$\mu_{K,t} Q_t s_t + \nu_t n_t + \sigma \left[\nu_t e_t - C(e_t, Q_t s_t)\right] \geq \theta Q_t s_t \tag{11}$$

Define $x_t \equiv \frac{e_t}{Q_t s_t}$. Specialize to the case where the equity cost takes the following form:

$$C(e_t, Q_t S_t) = c(x_t) Q_t s_t$$

where $c(x_t) = \frac{\kappa}{2} x_t^2$. Then the first order condition for x_t is

$$\begin{aligned}
\nu_{e,t} &= c'(x_t) \\
&= \kappa x_t
\end{aligned}$$

Using this first order condition we can simplify the second part of the value function

$$
\begin{aligned}
\nu_{e,t}e_t - C(e_t, Q_t s_t) &= \nu_{e,t}x_t Q_t s_t - c(x_t)Q_t s_t \\
&= (\nu_{e,t}x_t - \frac{\kappa}{2}x_t^2)Q_t s_t \\
&= \frac{\kappa}{2}x_t^2 Q_t s_t
\end{aligned}
$$

So the value function becomes

$$\mu_{K,t}Q_t s_t + \nu_t n_t + \sigma\frac{\kappa}{2}x_t^2 Q_t s_t$$

Define the "total" excess return on assets μ_t as

$$\mu_t \equiv \mu_{K,t} + \sigma\frac{\kappa}{2}x_t^2$$

Then when the constraint does not bind, $\mu_t = 0$ and the undetermined coefficient is $\alpha_t = \nu_t$. When the constraint binds, bank asset funding is given by the constraint at equality, $Q_t s_t = \phi_t n_t$ where $\phi_t = \frac{\nu_t}{\theta - \mu_t}$, and $\alpha_t = \nu_t + \mu_t\phi_t$.

Since bankers' problem is linear, we can easily aggregate across banks. The law of motion for aggregate net worth is the following:

$$N_t = \sigma\left[(R_{K,t} - R_{t-1} + x_{t-1})\underbrace{Q_{t-1}K_{t-1}}_{Q_{t-1}S_{t-1}} + R_{t-1}N_{t-1}\right] + (1-\sigma)\xi Q_{t-1}K_{t-1} \qquad (12)$$

2.3 Nonfinancial Firms

There are two categories of nonfinancial firms: final goods firms and capital producers. In turn, within final goods firms we also distinguish among "capital storage" firms and final goods producers, in order to clarify the role of bank credit used to purchase capital goods.

2.3.1 Final Goods Firms

We assume that there are two types of firms: "capital storage" firms and final goods producers. The first type of firm purchases capital goods from capital good producers, stores them for one period, and then rents them to final goods firms. The latter type of firm combines physical capital rented from capital goods firms with labor to produce final output.[10] Importantly, capital storage firms have to rely on banks to obtain funding, as explained below.

[10] Sargent and Ljungqvist (2004, Chapter 12) present a similar structure with two types of firms. Firms of type I and II in their notation correspond to our final goods producers and capital storage firms, respectively.

In period $t-1$, a representative capital storage firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to banks which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the realization of the capital quality shock ψ_t determines the effective amount of physical capital in possession of the firm, given by $e^{\psi_t}K_{t-1}$. The firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)e^{\psi_t}K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $e^{\psi_t}[Z_t + (1-\delta)Q_t]$. Since contracting between firms and banks is frictionless, it follows that the return on the securities issued by the firm is given by:

$$R_{K,t} = e^{\psi_t}\frac{Z_t + (1-\delta)Q_t}{Q_{t-1}} \tag{13}$$

Note that this equation ensures that capital storage firms make zero profits state-by-state.

The capital quality shock $\psi_t \sim N(0, \sigma_\psi)$ is a simple way to introduce an exogenous source of variation in the value of capital, which may be thought of as capturing some form of economic obsolescence.[11] As equation (13) makes clear, the random variable ψ_t provides a source of fluctuations in returns to banks' assets. These fluctuations are enhanced by movements in the endogenous asset price Q_t triggered by fluctuations in ψ_t.

In the aggregate, the law of motion for capital is given by

$$K_t = I_t + (1-\delta)e^{\psi_t}K_{t-1} \tag{14}$$

Final goods firms produce final output Y_t using capital, K_{t-1}, and labor, L_t via the following Cobb-Douglas production function:

$$Y_t = (e^{\psi_t}K_{t-1})^\alpha L_t^{1-\alpha} \tag{15}$$

The first order conditions for labor and for physical capital determines are as follows:

$$(1-\alpha)\frac{Y_t}{L_t} = W_t \tag{16}$$

$$\alpha\frac{Y_t}{e^{\psi_t}K_{t-1}} = Z_t \tag{17}$$

2.3.2 Capital Producers

Capital producers make new capital using input of final output and are subject to adjustment costs. They sell new capital to firms at the price Q_t. The price of capital goods is equal

[11] Gertler et. al. (2012) provide an explicit microfoundation.

to the marginal cost of investment goods production:

$$Q_t = 1 + \psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta \right) \tag{18}$$

2.4 International Capital Markets

We follow Schmitt-Grohe and Uribe (2003) and assume that small open economy is subject to debt elastic interest rate premium in the international markets.

$$R_t = \frac{1}{\beta} + \varphi(e^{\frac{B_t^*}{Y} - \bar{b}} - 1) + + e^{R_t^* - 1} - 1 \tag{19}$$

where $\bar{b}$ governs the steady state foreign debt to GDP ratio and R_t^* is the risk-free world interest rate, which is assumed to follow an AR(1) process in logs:

$$\log(R_t^*) = \rho_R \log(R_{t-1}^*) + \epsilon_{R,t} \tag{20}$$

where $\epsilon_{R,t} \sim N(0, \sigma_R)$.

2.5 Resource Constraint and Market Clearing

The resource constraint of this economy is determined by the budget constraint of the representative household:

$$Y_t = C_t + \left[1 + \frac{1}{2} \psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta \right)^2 \right] I_t + \sigma \frac{\kappa}{2} x_t^2 Q_t K_t + NX_t \tag{21}$$

The balance of payments equation is:

$$R_{t-1} B_{t-1}^* - B_t^* = NX_t \tag{22}$$

where NX stands for the net exports.

(Proved in Appendix 5.2)

3 Model Analysis

We now proceed to illustrate the features of the model via a series of numerical experiments. We solve the model using the Parameterized Expectations Algorithm (PEA), described in Appendix 5.4. Since this method allows us to solve the model fully nonlinearly, we can capture banks' occasionally binding incentive constraint. Moreover, the risk-taking behavior of banks is appropriately captured, as the method fully accounts for shock uncertainty.

3.1 Calibration and Stochastic Steady State

Table 1 reports parameter values. Our model includes ten conventional preference and technology parameters, for which we choose standard values. We then need to assign values to the four parameters relating to financial intermediaries: the survival rate of bankers σ, the fraction of assets that bankers can divert θ, the parameter determining the cost of raising equity κ, and the transfer to entering bankers ξ. We calibrate σ to 0.95 as in Gertler and Kiyotaki (2013), implying that bankers survive for about 5 years on average. We set θ to 0.3, to generate a frequency of financial crises of about 2% annually, in line with the data. We set κ to target a leverage ratio (assets to equity) of about 4 in the stochastic steady state. Finally, we set the transfer rate ξ to a very small number (0.00025) to ensure that this parameter does not significantly affect any of our results while still allowing the entering bankers to start operations.[12]

We calibrate the exogenous shocks process using data from several emerging/advanced economies. The foreign interest rate process is estimated using the sovereign borrowing rate faced by small open economies in international financial markets.[13] Finally, we calibrate the volatility of the capital quality shock (which is *iid*) so that the model delivers a realistic standard deviation of output growth.

Table 2 contains some moments describing the stochastic steady state of the economy, defined as the point at which the economy settles in the absence of exogenous shocks (but in which agents still expect that shocks might occur in the future).[14] Note that the constraint does not bind in the stochastic steady state. In fact, it is considerably far from binding: the probabilities of a financial crisis are 1.4% and 5%, 2-quarters-ahead and 1-year-ahead, respectively. But because banks anticipate that they may be constrained in the future, they raise equity at a positive rate in the stochastic steady state (absent the prospect of a binding constraint in the future, banks would prefer to set $x = 0$).

[12] We have verified that setting ξ to smaller values has virtually no effect on any of the results reported below.

[13] See, among others, Uribe and Yue (2006) and Akinci (2013).

[14] To calculate the stochastic steady state, we simply simulate the economy for many periods without any exogenous shocks, until the system converges to a point at which all endogenous variables are constant. This point is what we call the stochastic steady state.

3.2 Credit Booms and Bank Risk-Taking

In the first experiment, documented in Figures 2 and 3, we illustrate the consequences in the model of a lower country interest rate, R^*. We consider a two standard deviation decrease in $\epsilon_{R,t}$, the innovation to the interest rate. Figure 2 shows the responses of the real economy to the shock: as in standard small open economy models, lower country interest rates lead to a boom at home, with rising output, consumption, investment and asset prices. The boom is accompanied by an increase in credit (bottom panel on the right column), and by a compression in credit spreads $\mathbb{E}_t(R_{K,t+1}) - R_t$, consistent with the data. The novelty in our framework is that we can assess the implications of the shock for the probability of a financial crises, as discussed next.

The blue solid line in Figure 3 shows the probability of a financial crisis occurring within the next two quarters (left panel) and within the next year (right panel), in response to the decline in interest rates. Both probabilities increase as the credit boom progresses. The reason is twofold: along the boom, the net foreign indebtedness of the economy increases, which puts it closer to the region where the constraint binds. In addition, in response to the decrease in interest rates, banks *endogenously* decrease the pace at which they raise equity: note the substantial fall in x in Figure 2.

To illustrate the latter point clearly, we analyze the following counterfactual scenario: suppose that instead of allowing the banks to endogenously adjust x as they desire in response to the shock, we force x to stay constant at its stochastic steady state level. The green dashed lines in Figures 2 and 3 show the path in the counterfactual experiment with fixed rate of equity raising. Note that banks' net worth now is substantially larger along the boom, as a result of the faster pace of new equity issued. Interestingly, the crisis probabilities now are essentially unchanged: with stronger equity positions, banks are now further away from the constrained region along the credit boom, mitigating the risk of a financial crisis.

3.3 Nonlinearity and Amplification

We now perform a simple experiment to illustrate the nonlinearity induced by the borrowing constraint, and the amplification via the financial accelerator mechanism that occurs when the banks' incentive constraint binds.

We begin by plotting the responses to a 2.5% capital quality shock (green dashed line in Figure 4 and Figure 5). The shock leads net worth to drop about 10% on impact (roughly the size of the shock times banks' leverage). The decline in net worth, however, is not large enough to trigger the constraint implied by banks' agency problem. As a consequence, the shock has only modest effect on investment, asset prices, foreign debt and credit spreads (note that the shock does induce a sizable decline in output, purely due to the physical destruction of capital).

From the green dashed line in Figure 5, note that even if the shock does not trigger the constraint, it does increase the probability of a subsequent crisis: by weakening banks' balance sheets, the shock makes it more likely that in the future a crisis event occurs. Note also that the rate of equity issuance x increases substantially as a consequence of the shock: in response to their deteriorated balance sheets, which brings banks closer to their constraints, they work to rebuild their net worth.

We next perform the same experiment, now with a capital quality shock twice as large (blue solid line). The resulting decline in bank net worth is now large enough to bring banks up against their constraints. As a consequence, the spread $\mathbb{E}_t(R_{K,t+1} - R_t)$ jumps to almost 1500 basis points annually. The decline in net worth is about 25% on impact, two and a half times the decline that occurs with a capital quality shock of half the size. This more-than-proportional decline is explained by the financial accelerator mechanism that operates when the constraint binds: low net worth leads investment to drop, which drives asset prices down, leading net worth to drop further. As a consequence, there is a severe drop in investment, of about 30 percent – much more than the decline of less than 5 percent that occurs with the smaller capital quality shock.

An interesting point is that the financial crisis that arises from the adverse effect of the shock on banks' balance sheets induces a sudden stop in capital inflows: the stock of foreign debt, shown in the second panel in the second column, drops by more than 8 percent in the case of the 5% capital quality shock, while it barely moves in response to the 2.5% capital quality shock.

3.4 Average Financial Crisis

We now turn to describing what an average financial crisis looks like in our framework. In particular, we simulate the economy for 100,000 periods, select financial crisis events, and compute averages across all the events that we find. We define a financial crisis event simply as a period in which banks' incentive constraints bind.

Figure 6 displays the results. Financial crises have the usual features of sharp reductions in bank capital, which lead the constraint to bind and credit spreads to rise by about 500 basis points – with the consequent adverse effects for the real economy, including depressed output, investment and asset prices. Note also the sudden stop in capital inflows that accompanies the financial crises, exemplified by the sharp decline in the economy's foreign borrowing. Total domestic credit declines sharply as well.

From the last two panels, note that the crisis is ultimately the result of adverse realizations of both exogenous shocks: crisis events are triggered by a negative sequence of capital quality shocks, together with an increase in foreign interest rates.

Notice that x increases as banks see the crisis coming and their net worth deteriorates. However, the crisis ultimately still comes as a surprise: note that the spread jumps sharply in one quarter. Note also that credit spreads are especially compressed until right before the crisis, consistent with the argument in Stein (2014) that credit spreads are often particulary low before sharply increasing.

3.5 Policy Experiment

We now turn to an analysis of the effectiveness of a macroprudential policy. In particular, we consider a regulatory subsidy on equity issuance, which the government finances by levying a tax on bank assets. The goal of the regulatory tax/subsidy scheme is to induce banks to raise more equity, thereby strengthening their balance sheet position.

In particular, the government is assumed to set a subsidy on equity issuance equal to τ^s, and a tax proportional to the value of banks' assets equal to τ_t. The subsidy works to reduce the net cost of issuing equity for banks. With the subsidy, the banks' first order condition for equity issues x_t is now

$$\nu_{e,t} + \tau^s = c'(x_t) \tag{23}$$

Everything else equal, the subsidy induces banks to choose a higher x. Thus, the policy has the flavor of a capital requirement, as it distorts banks' decision in favor of raising more equity.

With the tax τ_t, the banks' balance sheet constraint now becomes

$$(1 + \tau_t)Q_t s_t \leq n_t + d_t \tag{24}$$

Thus, the tax τ_t increases the cost of financing a balance sheet of a given size. We assume that the government sets τ_t to balance the budget period-by-period, which amounts to setting $\tau_t = \sigma\tau^s x_t$.[15]

We restrict attention to a constant subsidy τ^s. In particular, we study its impact on the stochastic steady state of the model economy, including crisis probabilities, and also on the average frequency that the economy spends at the constraint. We also compute the welfare effect of the policy. Our criterion is welfare at the stochastic steady state; since in the

[15] See Appendix 6 for details on how the tax/subsidy scheme affects the banker's problem.

stochastic steady state agents still expect shocks to hit in the future, the criterion incorporates the welfare impact of reducing the probability of future crises.

Table 2 reports the policy results. We describe the effects of the policy, for different values of τ^s. Note that the policy is successful in raising x. As a consequence, banks' net worth is higher. In turn, this helps in reducing the likelihood of future crises, as well as the average time that the economy is constrained. Note also that the policy increases welfare (although for the subsidy equal to 0.04 welfare is lower than with $\tau^s = 0.02$). Interestingly, note that current period utility tends to be lower with the tax/subsidy scheme in place (consumption is higher, but hours are too). However, total welfare is still larger, reflecting the reduced probability of crises occurring in the future.

4 Conclusion

We have developed a small open economy framework with banks that face occasionally binding leverage constraints. The latter feature implies that the model can generate the type of nonlinear dynamics usually associated with financial crises and sudden stops. Our model can produce episodes of financial crises nested within normal business cycle fluctuations, and does not need to rely on unusually large shocks to produce a crisis. A virtue of our approach is that by analyzing a fully nonlinear solution, we can adequately capture the risk-taking behavior of banks. Moreover, by allowing banks to issue equity, we can capture how banks endogenously adjust the strength of their balance sheet in response to economic conditions.

Our focus has been to produce a framework that is tractable enough to accommodate easily the features used in the DSGE literature to enhance the quantitative performance and to facilitate policy analysis. In ongoing work, we use the model to analyze time-varying policies that are explicitly contingent on a credit boom episode, such as the countercyclical capital buffers that have been proposed by several policymakers.[16] The model can also provide useful insight into the relative benefits of capital controls vis-á-vis bank capital requirements. Finally, another interesting avenue of future research would be to augment the model with nominal rigidities, and use it to analyze the implications of financial stability considerations for the conduct of monetary policy.

[16] See Norges Bank (2013) or Basel Committee on Banking Supervision (2010).

References

Akinci, O., 2013. Global financial conditions, country spreads and macroeconomic fluctuations in emerging countries. Journal of International Economics. 13

Basel Committee on Banking Supervision, 2010. Guidance for national authorities operating the countercyclical capital buffer. Tech. rep., Bank for International Settlements. 16

Benigno, G. D., Chen, H., Otrok, C., Rebucci, A., Young, E. R., 2012. Optimal policy for macro-financial stability. Cep discussion papers, Centre for Economic Performance, LSE.
URL http://EconPapers.repec.org/RePEc:cep:cepdps:dp1172 1

Bernanke, B. S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J. B., Woodford, M. (Eds.), Handbook of Macroeconomics. Vol. 1 of Handbook of Macroeconomics. Elsevier, Ch. 21, pp. 1341–1393.
URL http://ideas.repec.org/h/eee/macchp/1-21.html 1

Boissay, F., Collard, F., Smets, F., Feb. 2013. Booms and systemic banking crises. Working Paper Series 1514, European Central Bank.
URL http://ideas.repec.org/p/ecb/ecbwps/20131514.html 1

Brunnermeier, M. K., Sannikov, Y., In Progress. International credit flows, pecuniary externalities and capital controls. 1

Bruno, V., Shin, H., 2012. Capital flows and the risk-taking channel of monetary policy. In: Paper presented at the 11th BIS Annual Conference. Vol. 22. p. 23. 5

Cespedes, L. F., Chang, R., Velasco, A., October 2012. Financial intermediation, exchange rates, and unconventional policy in an open economy. Working Paper 18431, National Bureau of Economic Research.
URL http://www.nber.org/papers/w18431 1

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58, 17–34. 3

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. In: Friedman, B. M., Woodford, M. (Eds.), Handbook of Monetary Economics. Vol. 3 of Handbook of Monetary Economics. Elsevier, Ch. 11, pp. 547–599.
URL http://ideas.repec.org/h/eee/monchp/3-11.html 1, 2, 2.2

Gertler, M., Kiyotaki, N., 2013. Banking, liquidity and bank runs in an infinite-horizon economy. Tech. rep., National Bureau of Economic Research. 3.1

Gertler, M., Kiyotaki, N., Queralto, A., 2012. Financial crises, bank risk exposure and government financial policy. Journal of Monetary Economics 59, 17–34. 3, 6

Gourinchas, P.-O., Obstfeld, M., January 2012. Stories of the twentieth century for the twenty-first. American Economic Journal: Macroeconomics 4 (1), 226–65.
URL http://ideas.repec.org/a/aea/aejmac/v4y2012i1p226-65.html 5

Greenwood, J., Hercowitz, Z., Huffman, G. W., June 1988. Investment, capacity utilization, and the real business cycle. American Economic Review 78 (3), 402–17.
URL http://ideas.repec.org/a/aea/aecrev/v78y1988i3p402-17.html 2.1

Guerrieri, L., Iacoviello, M., 2012. A toolkit to solve models with occasionally binding constraints easily. Working paper, Federal Reserve Board. 0

Kenny, G., Morgan, J., 2011. Some lessons from the financial crisis for the economic analysis. Tech. rep., European Central Bank. 2

Mendoza, E., 2010. Sudden Stops, Financial Crises, and Leverage. American Economic Review 100 (5), 1941–66. 1

Mendoza, E. G., Terrones, M. E., Sep. 2012. An anatomy of credit booms and their demise. NBER Working Papers 18379, National Bureau of Economic Research, Inc.
URL http://ideas.repec.org/p/nbr/nberwo/18379.html 1, 5

Norges Bank, 2013. Criteria for an appropriate countercyclical capital buffer. Tech. rep., Norges Bank. 16

Schmitt-Grohe, S., Uribe, M., October 2003. Closing small open economy models. Journal of International Economics 61 (1), 163–185.
URL http://ideas.repec.org/a/eee/inecon/v61y2003i1p163-185.html 2.4

Stein, J., 2014. Incorporating financial stability considerations into a monetary policy framework. 2, 3.4

Uribe, M., Yue, V. Z., 2006. Country spreads and emerging countries: Who drives whom? Journal of international Economics 69 (1), 6–36. 13

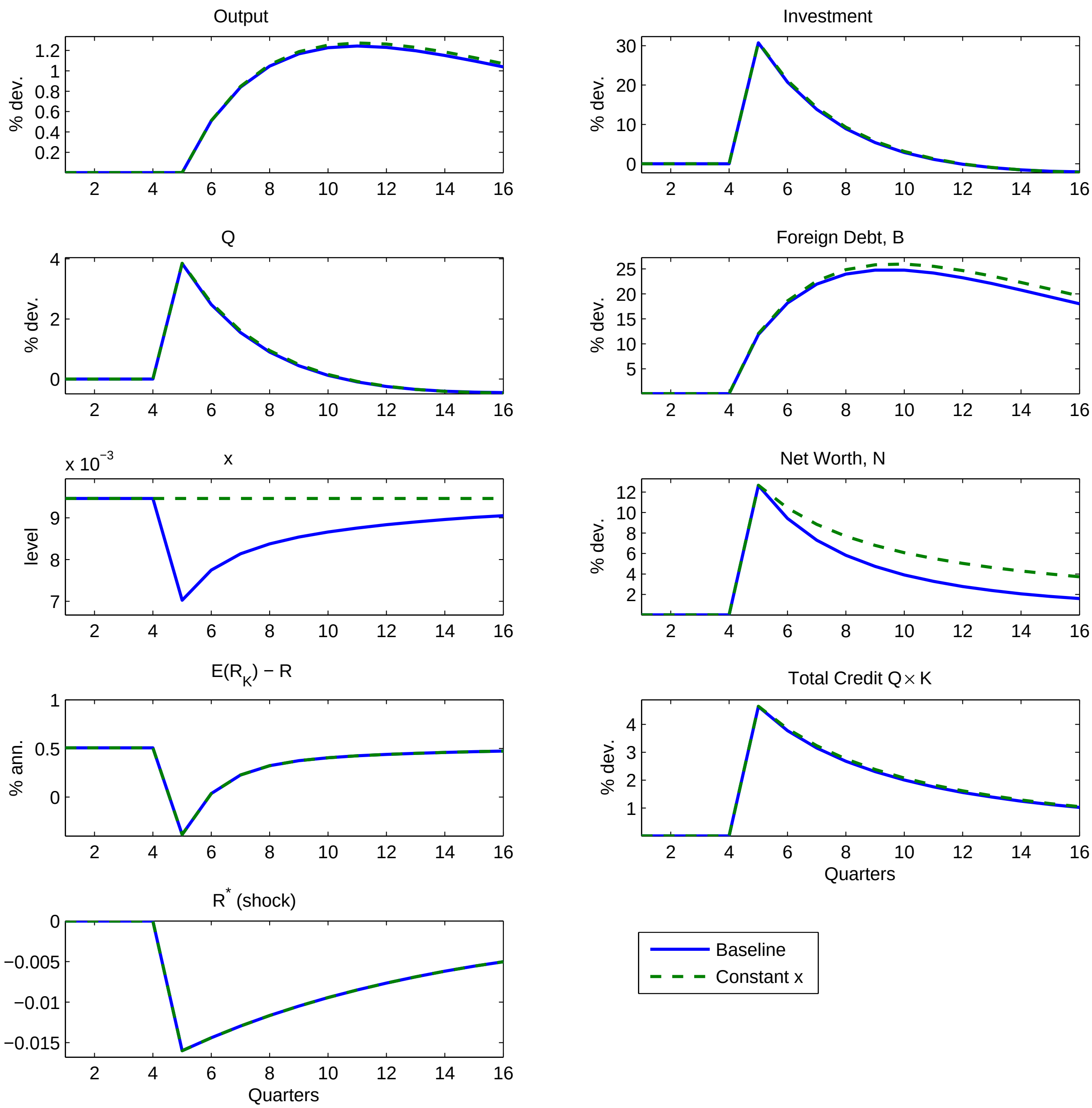
Table 1: Calibration

Parameter	Symbol	Value	Source/Target
Conventional			
Discount factor	β	0.985	Interest rate (6%, ann.)
Risk aversion	γ	2	
Inverse Frisch elast.	ϵ	1/3	Frisch lab. sup. elast. (inv)
Labor disutility	χ	2.8125	Steady state labor (20%)
Capital share	α	0.33	Standard RBC value
Capital depreciation	δ	0.025	Mendoza (2010)
Investment adj. cost	Ψ_I	5	BGG (2000)
Debt elast. of interest rate	φ	0.05	
Reference debt/output ratio	$\bar{b}$	0.6	Steady state B/Y of 60&
Financial Intermediaries			
Survival rate	σ	0.95	Expected horizon of 5 yrs, as in GK (2013)
fraction divertable	θ	0.26	Frequency of crises (2% ann.)
Transfer rate	ξ	0.0001	
Cost of raising equity	κ	5	Steady State Leverage of 4
Shock Processes			
Persistence of interest rate	ρ_R	0.9	
SD of interest rate innov.	σ_R	0.008	
SD of capital quality	σ_ψ	0.01	

Table 2: Stochastic Steady State

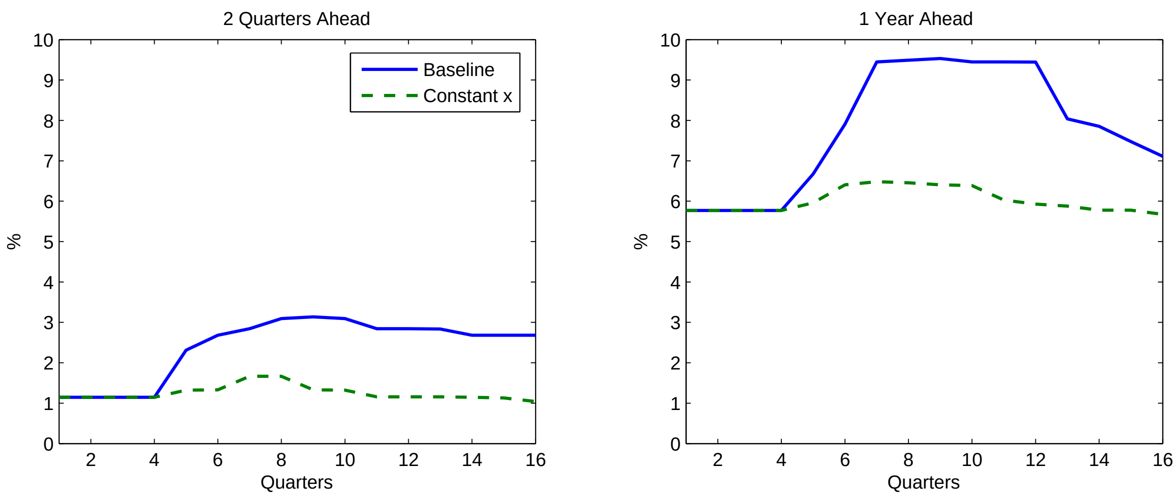
Variables	**No Policy**	$\tau^s = 0.02$	$\tau^s = 0.04$
Y	0.8379	0.8387	0.8399
C	0.6594	0.6597	0.6602
L	0.2986	0.2989	0.2992
K	6.8065	6.8163	6.8309
N	1.9162	1.9809	2.0777
x	0.0095	0.01	0.0108
QK/N	3.55	3.44	3.29
B/Y	0.58	0.58	0.59
$U(C, L)$	-3.1966	-3.1973	-3.1987
Moments			
Time at the constr. (%)	1.98	1.31	0.70
2-qtr-ahead crisis prob. (%)	1.15	0.45	0.08
1-yr-ahead crisis prob. (%)	5.77	3.76	1.84
SD(annual g_Y)	0.0181	0.0180	0.0178
SD(Y)/$\mathbb{E}$(Y)	0.0595	0.0606	0.0587
SD(C)/$\mathbb{E}$(C)	0.0577	0.0589	0.0571
SD(I)/$\mathbb{E}$(I)	0.2314	0.2289	0.2246
SD(NX/Y)	0.0485	0.0481	0.0477
Welfare	-213.705	-213.4275	-213.4302

Figure 2: Responses to Decline in Country Interest Rate



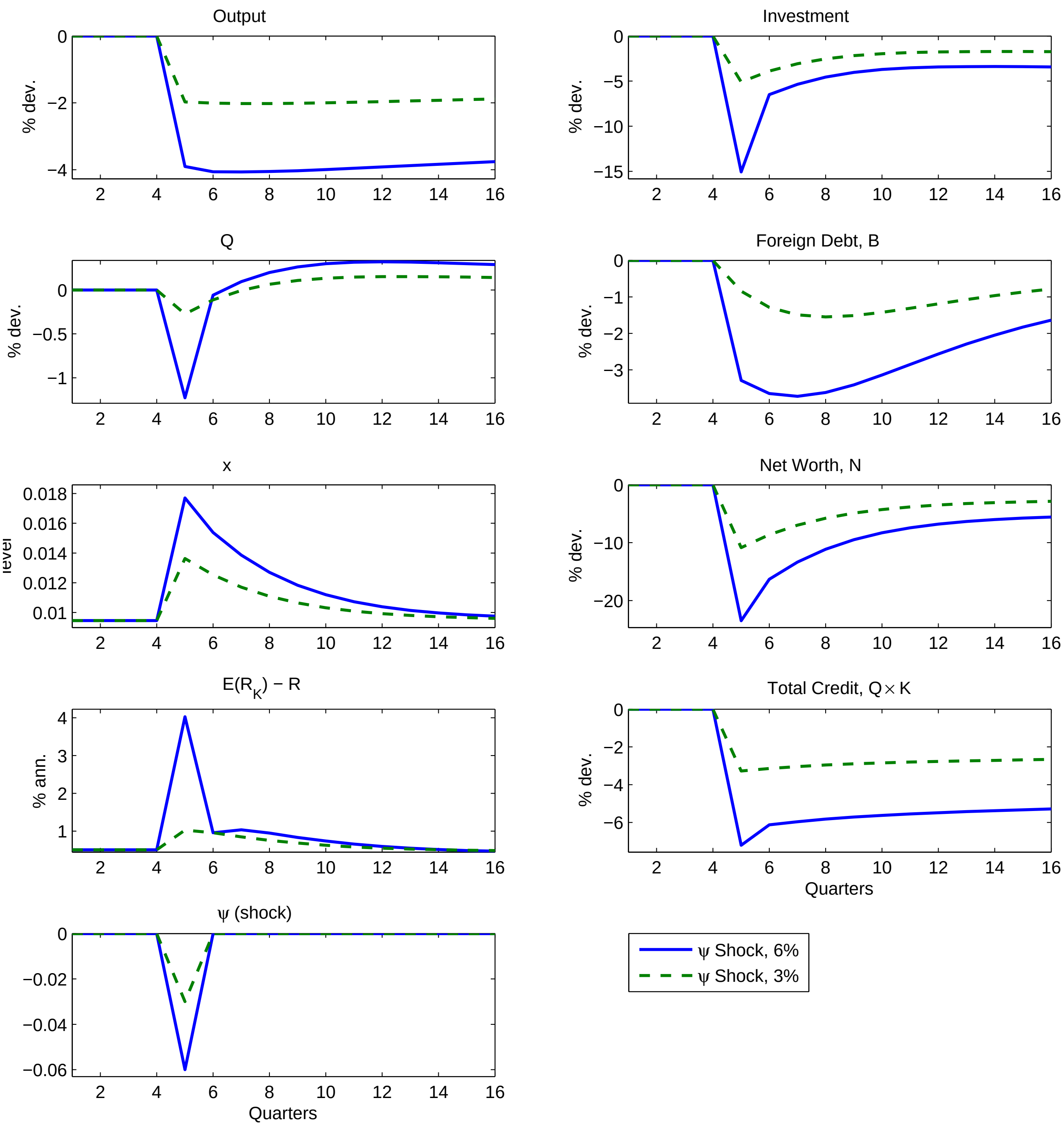
Note: The figure shows the responses to a negative innovation to the foreign interest rate of 2 standard deviations. The blue solid line shows the responses in the baseline model, and the green dashed line shows the counterfactual where the rate of equity issuance, x, remains constant. Variables indicated "% dev." computed as percent deviations from their stochastic steady state value.

Figure 3: Responses to Decline in Country Interest Rate, Crisis Probabilities



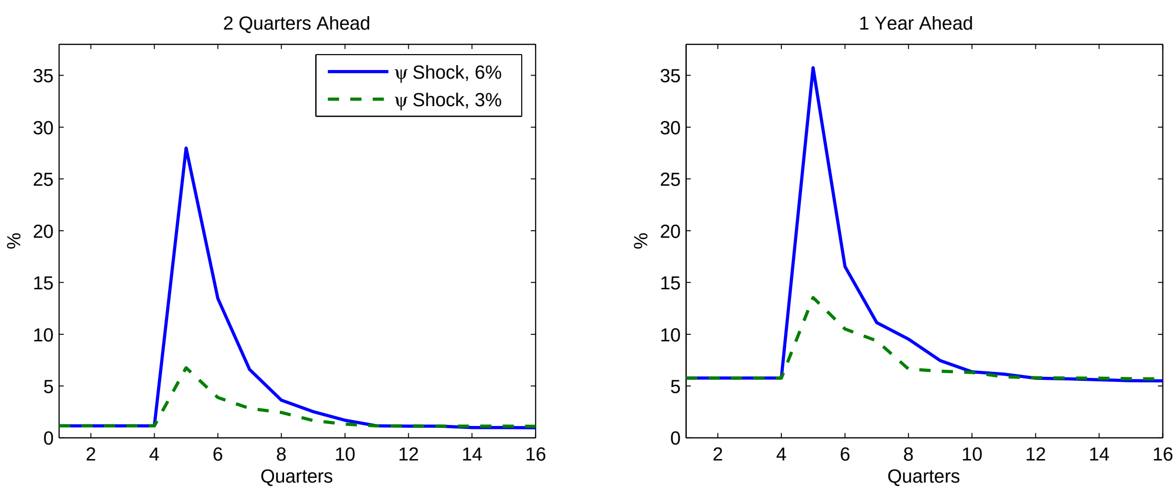
Note: The figure shows crisis probabilities at different horizons in response o a negative innovation to the foreign interest rate of 2 standard deviations. The blue solid line shows the responses in the baseline model, and the green dashed line shows the counterfactual where the rate of equity issuance, x, remains constant.

Figure 4: Responses to Capital Quality Shocks



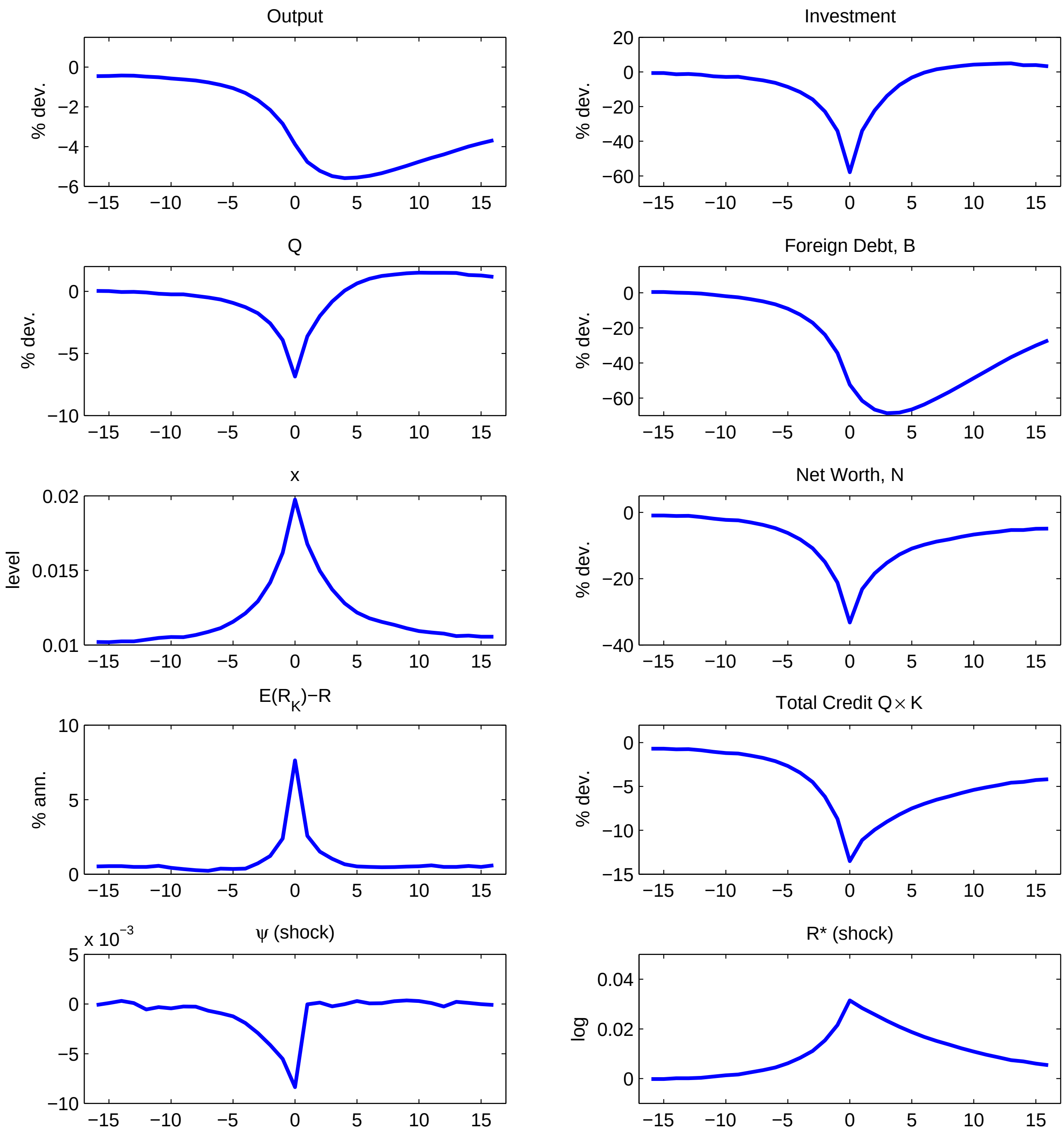
Note: The figure shows the responses to a 5% capital quality shock (blue solid line) and to a 2.5% capital quality shock (green dashed line). The constraint binds as a consequence of the 5% shock, but it does not bind in response to the smaller shock. Variables indicated "% dev." computed as percent deviations from their stochastic steady state value.

Figure 5: Responses to Quality Shocks, Crisis Probabilities



Note: The figure shows the responses of crisis probabilities to a 5% capital quality shock (blue solid line) and to a 2.5% capital quality shock (green dashed line).

Figure 6: Average Financial Crisis



Note: We simulate the economy for 100,000 periods and compute average across financial crisis events. A crisis event is defined as a period in which banks' incentive constraint binds. Variables indicated "% dev." computed as percent deviations from their average value in the simulation.

5 Appendix

5.1 Appendix 1: Household's Optimality Conditions

$$\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{-\gamma} = \lambda_t \tag{25}$$

$$\chi L_t^{\epsilon} = W_t \tag{26}$$

$$\mathbb{E}_t(\Lambda_{t,t+1} R_t) = 1 \tag{27}$$

Household's stochastic discount factor is defined as

$$\Lambda_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \tag{28}$$

where $\lambda_t = U_{c,t}$ is the marginal utility of consumption.

5.2 Appendix 2: Resource Constraint and Balance of Payments

Aggregate the bank's budget constraint across banks and combine with the household's budget constraint and with the market clearing condition for claims on capital ($S_t = K_t$) to obtain

$$Q_t K_t + R_{t-1} B_{t-1}^* + C_t + \sigma \frac{\kappa}{2} x_t^2 Q_t K_t \leq W_t L_t + R_{K,t} Q_{t-1} K_{t-1} + B_t^* + \Pi_t^F + \Pi_t^C \tag{29}$$

The last two terms, Π_t^F and Π_t^C, are the profits of final goods firms and capital producers, respectively. They are given by their respective budget constraints:

$$Y_t + Q_t (1-\delta) e^{\psi_t} K_{t-1} = \Pi_t^F + W_t L_t + R_{K,t} Q_{t-1} K_{t-1} \tag{30}$$

$$\Pi_t^C = Q_t I_t - \left[1 + \frac{1}{2}\psi_I (\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta)^2\right] I_t \tag{31}$$

Using these expressions, we can derive the resource constraint and the balance of payments equation for the economy as the following:

$$Y_t = C_t + \left[1 + \frac{1}{2}\psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta\right)^2\right] I_t + \sigma \frac{\kappa}{2} x_t^2 Q_t K_t + NX_t \tag{32}$$

$$R_{t-1}B^*_{t-1} - B^*_t = NX_t \tag{33}$$

5.3 Appendix 3: Model Policy Functions

To further illustrate the nonlinear features of the model, arising due to banks' occasionally binding constraints, Figure 7 report policy functions for the state variables. The effective capital stock $\overline{K}_t$ refers to capital stock at the beginning of the period, after the capital quality shock realizes: $\overline{K}_t \equiv e^{\psi_t}K_{t-1}$. The predetermined amount of net worth refers to the part of time-t net worth that is predetermined in period $t-1$, i.e. that does not depend on the realization of capital quality and asset prices in period t:

$$\overline{N}_{t-1} \equiv \sigma \left[x_{t-1}Q_{t-1}K_{t-1} - R_{t-1}\left(\underbrace{Q_{t-1}K_{t-1} - N_{t-1}}_{\equiv D_{t-1}} \right) \right] + (1-\sigma)\xi Q_{t-1}K_{t-1}$$

$\overline{N}_{t-1}$ can be interpreted as the total equity raised in period $t-1$ (including the equity brought in by the newborn bankers who replace the exiting ones, captured by the last term above) net of the total financial sector debt incurred in period $t-1$ (including interest). $\overline{N}_{t-1}$ is one of the key state variables of the aggregate system.

Total aggregate net worth is then:

$$N_t = \sigma R_{K,t}Q_{t-1}K_{t-1} + \overline{N}_{t-1} = \left[Z_t + (1-\delta)Q_t\right] e^{\psi_t}K_{t-1} + \overline{N}_{t-1}$$

The remaining state variables are the stock of external debt (including interest) $\overline{B}_{t-1} \equiv R_{t-1}B_{t-1}$, and the exogenous shock to the country interest rate, R^*_t. Note that for all state variables, there exists a threshold level after which the constraint starts to bind.

5.4 Appendix 4: Complete Model and Solution Method

5.4.1 The Complete Model

$$Y_t + B_t = C_t + \left[1 + \frac{1}{2}\psi_I\left(\frac{I_t}{e^{\psi_t}K_{t-1}} - \delta\right)^2\right] I_t + \sigma\frac{\kappa}{2}x_t^2 Q_t K_t + R_{t-1}B_{t-1} \tag{34}$$

$$K_t = I_t + (1-\delta)e^{\psi_t}K_{t-1} \tag{35}$$

$$Q_t = 1 + \psi_I\left(\frac{I_t}{e^{\psi_t}K_{t-1}} - \delta\right) \tag{36}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}\right)R_t = 1 \tag{37}$$

$$\Lambda_{t-1,t} = \beta \frac{U_{C,t}}{U_{C,t-1}} \tag{38}$$

$$U_{C,t} = \left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon} \right)^{-\gamma} \tag{39}$$

$$R_{K,t} = e^{\psi_t} \frac{\alpha \frac{Y_t}{e^{\psi_t} K_{t-1}} + (1-\delta) Q_t}{Q_{t-1}} \tag{40}$$

$$Y_t = (e^{\psi_t} K_{t-1})^{\alpha} L_t^{1-\alpha} \tag{41}$$

$$\mu_{K,t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{K,t+1} - R_t \right) \right] \tag{42}$$

$$\mu_t = \mu_{K,t} + \sigma \frac{\kappa}{2} x_t^2 \tag{43}$$

$$\nu_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \Omega_{t+1} \right) R_t \tag{44}$$

$$\Omega_t = 1 - \sigma + \sigma(\nu_t + \phi_t \mu_t) \tag{45}$$

$$\nu_t^e = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\nu_{t+1} + \mu_{t+1} \phi_{t+1} \right) \right] \tag{46}$$

$$N_t = \sigma R_{K,t} Q_{t-1} K_{t-1} + \overline{N}_{t-1} \tag{47}$$

$$\overline{N}_t = \sigma \left(x_t Q_t K_t - R_t Q_t K_t + R_t N_t \right) + (1-\sigma) \xi_t Q_t K_t \tag{48}$$

$$(1-\alpha) \frac{Y_t}{L_t} = \chi L_t^{1+\epsilon} \tag{49}$$

$$R_t = \frac{1}{\beta} + \varphi \left(e^{\frac{B_t}{Y_t} - \bar{b}} - 1 \right) + e^{R_t^* - 1} - 1 \tag{50}$$

$$\phi_t = \frac{\nu_t}{\theta - \mu_t} \tag{51}$$

$$x_t = \frac{\nu_t^e}{\kappa} \tag{52}$$

If the constraint does not bind, we must have $\mu_t = 0$. If it binds, then we have $Q_t K_t = \phi_t N_t$ (and $\mu_t > 0$). Define the *effective* amount of capital at the beginning of t as $\overline{K}_t \equiv e^{\psi_t} K_{t-1}$, and the stock of external debt plus interest as $\overline{B}_{t-1} \equiv R_{t-1} B_{t-1}$. The state variables in this economy are $\overline{K}_t$, $\overline{N}_{t-1}$, R_t^* and $\overline{B}_{t-1}$.

5.4.2 Solution Method

The four state variables at the beginning of period t are the following: the effective amount of capital $\overline{K}_t$, the predetermined part of aggregate net worth $\overline{N}_{t-1}$, the stock of external debt (including interest) $R_{t-1} B_{t-1}$, and the foreign interest rate shock R_t^*. Define the state vector at the beginning of period t: $\mathbf{s}_t \equiv \left\{ \overline{K}_t, \overline{N}_{t-1}, R_t^*, \overline{B}_{t-1} \right\}$.

We solve the nonlinear model by Parameterized Expectations. To so, we need to approximate five expectations as a function of the state vector. Let $\varepsilon(\mathbf{s}_t)$ denote the set of the corresponding five functions of the state vector. Knowing the functions $\varepsilon(\mathbf{s}_t)$ and the state $\mathbf{s}_t$, one can solve nonlinearly for all of the model's endogenous variables. We use the following algorithm:

0. Simulate a long time series using OccBin (Guerrieri and Iacoviello (2012)). Use the simulated data to obtain $\varepsilon^0(\mathbf{s}_t)$, by regressing the expectations produced by OccBin on the state variables. At this step we can check on the accuracy of the function used to approximate the expectations, by comparing the expectations produced by OccBin (which captures the kink accurately) with those produced by the parameterized function.
1. Given $\varepsilon^0(\mathbf{s}_t)$, solve the system of equations that characterizes the equilibrium. To do so, at each period t we first solve the system assuming that the constraint does not bind, which implies $\mu_t = 0$. We then check if bank leverage is above the maximum allowed by the constraint. If it is not we proceed; if it is, we again solve the system, this time imposing that the constraint binds. At each period, we obtain one-period-ahead expectations by quadrature.
2. Obtain a new set of expectation functions $\varepsilon^1(\mathbf{s}_t)$ by running regressions using the data simulated in step 1.
3. Compare $\varepsilon^1(\mathbf{s}_t)$ with $\varepsilon^0(\mathbf{s}_t)$. If they are close, stop. If not, update $\varepsilon^0(\mathbf{s}_t)$ and go back to step 1.

5.5 Appendix 5: Computation of Crisis Probabilities

At period t_0, we compute the crisis probability at horizon t_0+j, defined as the probability of at least one crisis in periods t_0+1 until t_0+j, as follows. First, obtain draws for the exogenous innovations $\{\epsilon_{R,t}, \psi_t\}_{t_0+1}^{t_0+j}$, together with their associated probabilities. For each history of realizations of shocks h, defined as each possible sequence of realizations of $\{\epsilon_{R,t}, \psi_t\}_{t_0+1}^{t_0+j}$, let the set of histories in which there is at least one crisis be H. Then the probability of a crisis (for a given horizon j) is the sum of the probabilities of each of the histories in H, i.e. $\sum_{h \in H} p(h)$. In the body of the paper we report the results for $j = 2, 4$.

5.6 Appendix 6: Banker's Problem with Policy

In this section we lay out the banking problem in the presence of government subsidy which tilts banks' incentive in favor of raising more equity. In particular, we suppose that the government offers banks a fixed subsidy of τ^s per unit of new equity issued and finances the subsidy with a tax τ_t on total assets such that $\tau_t = \sigma\tau^s x_t$ to achieve balanced budget for the government.

Bankers' problem is now given by

$$V_t(n_t) = \max_{s_t,d_t,e_t} \; (1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_ts_t - R_td_t\right)+\sigma\left\{\mathbb{E}_t\Lambda_{t,t+1}\left[V_{t+1}(n_{t+1}) - e_t\right] - C(e_t,Q_ts_t) + \tau^s e_t\right\} \tag{53}$$

subject to

$$(1+\tau_t)Q_ts_t \leq n_t + d_t \tag{54}$$

$$n_{t+1} = R_{K,t+1}Q_ts_t - R_td_t + e_t \tag{55}$$

$$(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_ts_t - R_td_t\right)+\sigma\left[\mathbb{E}_t\Lambda_{t,t+1}\left(V_{t+1}(n_{t+1}) - e_t\right) - C(e_t,Q_ts_t) + \tau^s e_t\right] \geq \theta Q_ts_t \tag{56}$$

As before, to solve the banker's problem, first guess that the value function is $V_t(n_t) = \alpha_t n_t$. Define

$$\begin{aligned}
\Omega_{t+1} &= (1-\sigma) + \sigma\alpha_{t+1} \\
\mu^\tau_{K,t} &= \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{K,t+1} - (1+\tau_t)R_t)] \\
\nu_t &= \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}]R_t \\
\nu_{e,t} &= \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\alpha_{t+1} - 1\right)\right]
\end{aligned}$$

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t,e_t} \; \mu^\tau_{K,t}Q_ts_t + \nu_t n_t + \sigma\left[\nu_{e,t}e_t - C(e_t,Q_ts_t) + \tau^s e_t\right] \tag{57}$$

subject to

$$\mu^\tau_{K,t}Q_ts_t + \nu_t n_t + \sigma\left[\nu_t e_t - C(e_t,Q_ts_t) + \tau^s e_t\right] \geq \theta Q_ts_t \tag{58}$$

The first order condition for x_t is[17]

$$\begin{aligned}
\nu_{e,t} + \tau^s &= c'(x_t) \\
&= \kappa x_t
\end{aligned}$$

[17] The equity cost takes the following form, as before: $C(e_t, Q_tS_t) = \frac{\kappa}{2}x_t^2 Q_ts_t$.

Using this first order condition we can simplify the second part of the value function

$$\begin{aligned}
\nu_{e,t}e_t - C(e_t, Q_t s_t) + \tau^s e_t &= \nu_{e,t}x_t Q_t s_t - c(x_t)Q_t s_t + \tau^s x_t Q_t s_t \\
&= \left[(\nu_{e,t} + \tau^s)x_t - \frac{\kappa}{2}x_t^2\right] Q_t s_t \\
&= \frac{\kappa}{2}x_t^2 Q_t s_t
\end{aligned}$$

So the value function becomes

$$\mu^{\tau}_{K,t} Q_t s_t + \nu_t n_t + \sigma \frac{\kappa}{2} x_t^2 Q_t s_t$$

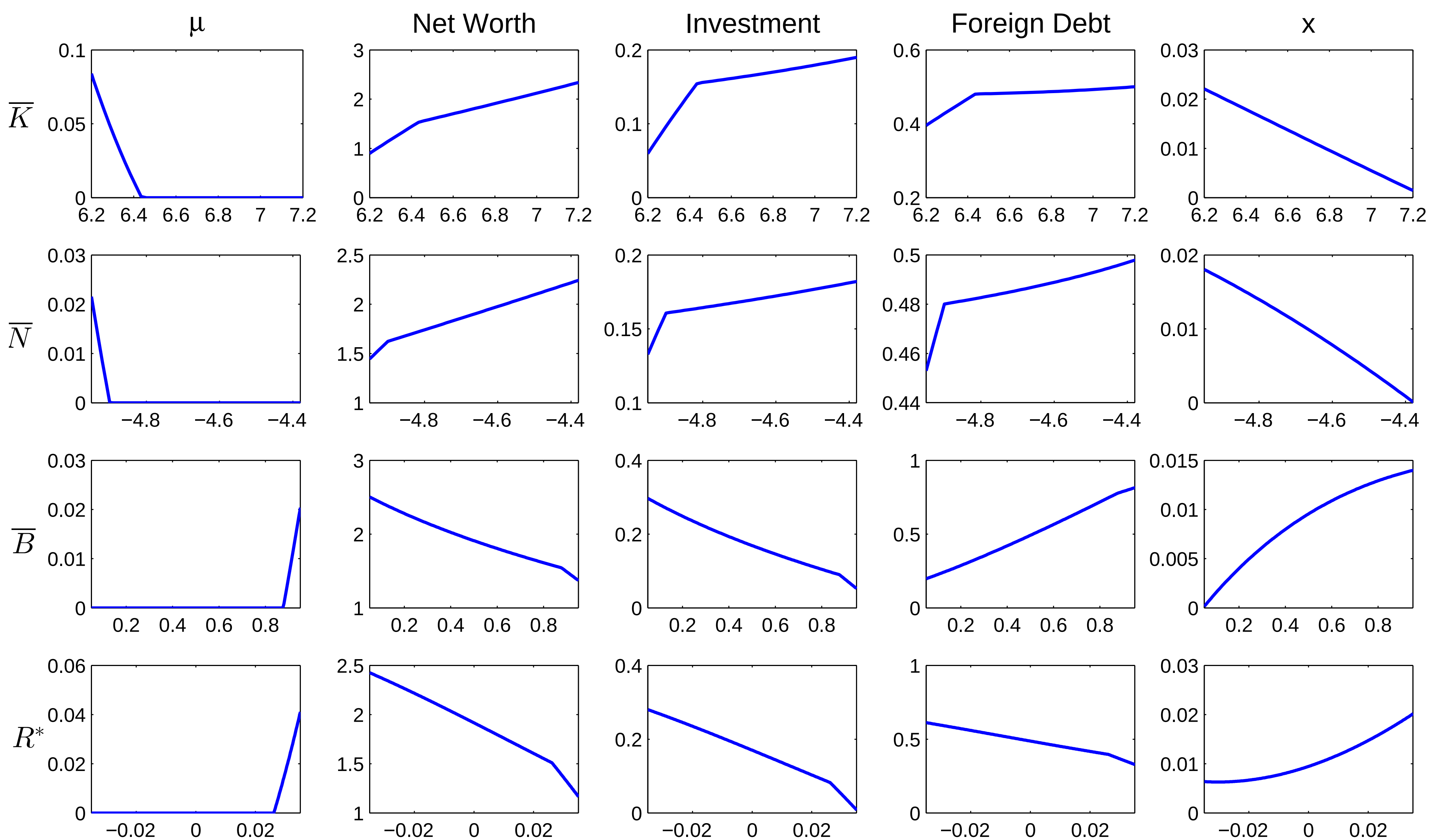
Define the "total" excess return on assets μ_t as

$$\mu_t \equiv \mu^{\tau}_{K,t} + \sigma \frac{\kappa}{2} x_t^2$$

The law of motion for aggregate net worth is now

$$N_t = \sigma \left[(R_{K,t} - (1 + \tau_{t-1})R_{t-1} + x_{t-1}) \underbrace{Q_{t-1}K_{t-1}}_{Q_{t-1}S_{t-1}} + R_{t-1}N_{t-1} \right] + (1-\sigma)\xi Q_{t-1}K_{t-1} \tag{59}$$

Figure 7: Model Policy Functions



Note: Model policy functions. Each row of the figure moves along the corresponding state variable, while keeping the others at their stochastic steady state value.