Optimal Monetary and Prudential Policies*

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Abstract

The recent financial crisis has highlighted the interconnectedness between macroeconomic and financial stability, and has raised the question of whether and how to combine monetary and prudential policies. This paper offers a characterization of the jointly optimal monetary and prudential policies, setting the interest rate and bank-capital requirements. The source of financial fragility is the socially excessive risk-taking by banks due to limited liability and deposit insurance. We characterize the conditions under which locally optimal (Ramsey) policy dedicates the prudential instrument to preventing inefficient risk-taking by banks; and the monetary instrument to dealing with the business cycle, with the two instruments co-varying negatively. Our analysis thus identifies circumstances that can validate the prevailing view among central bankers that standard interest-rate policy cannot serve as the first line of defense against financial instability. In addition, we provide conditions under which the two instruments might optimally co-move positively and counter-cyclically.

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1 Introduction

Monetary and prudential policies have traditionally been designed and analyzed in isolation from one another. The 2007-2009 financial crisis, however, has aroused interest in analyzing the interactions between these policies. Policymakers [e.g., Bernanke (2010), Blanchard et al. (2010), Svensson (2010)] have commented on the extent to which monetary policy can or should address concerns about financial stability. And policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Committee on International Economic Policy and Reform (2011)] have summarized alternative views about the potential substitutability or complementarity across policies and the need for policy coordination. There is a general presumption that both policies will be counter-cyclical most of the time, but policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)] have also envisioned scenarios that may put the two policies at odds with each other over the business cycle.

In this paper, we develop a New Keynesian model with banks and use it to study the optimal interactions between monetary and prudential policies. We focus on a prudential policy that sets a state-contingent capital requirement for banks, in the spirit of what the Basel Committee on Banking Supervision (2010) calls the "counter-cyclical capital buffer." We first articulate a benchmark model in which the Tinbergen separation principle applies: it is optimal to relegate the goal of financial stability to prudential policy and assign a mandate of macroeconomic stabilization to interest-rate policy.[1] In this model, the bank capital requirement is optimally used to deter excessive risk taking by banks (countering the risk-taking temptations that arise from limited liability and deposit insurance). Monetary policy cannot deter risk-taking at all and optimally focuses on macroeconomic stabilization, by adjusting the policy rate in response to changes in macroeconomic conditions, including those that reflect optimal changes in prudential policy.[2] In this sense, our benchmark model is a stark rendition of what Smets (2013) calls "the modified Jackson Hole consensus." Although our main goal is to fully articulate a model in which the Tinbergen separation principle applies, we also illustrate how it may fail, by considering a simple extension in which monetary policy does affect risk-taking incentives, and highlighting how this changes the key features of optimal policy interactions.

We depart in two main ways from other recent contributions that study the interactions between monetary and prudential policies from a normative perspective. First, we characterize jointly locally Ramsey-optimal policies, i.e. we determine the state-contingent path for the two policy instruments that locally maximizes the representative household's expected utility.[3] By contrast, the existing literature usually compares simple monetary and prudential policy rules with each other by computing welfare numerically, but does not address the issue of the optimal capital requirement in the steady state.[4] Second, in our model excessive risk taking arises from limited liability and involves the *type* (not

[1] In this context, the Tinbergen separation principle, as articulated by the Committee on International Economic Policy and Reform (2011) among others, refers to the idea that each goal should be pursued with a separate and dedicated instrument.

[2] To be clear, our paper is about optimal assignment of policy instruments, or optimal interactions between instruments, when both policies have the common (Ramsey) objective of maximizing welfare. De Paoli and Paustian (2013) study policy coordination in a different setting that involves separate prudential and monetary authorities with potentially different objectives (and consider the optimal policy interactions under discretion as well as commitment).

[3] We characterize analytically the capital requirement under jointly optimal policies, and this enables us to determine numerically the associated optimal interest rate.

[4] Loisel (2014) summarizes the main features of a number of contributions that consider simple monetary and prudential policy rules [e.g., Angeloni and Faia (2013), Benes and Kumhof (2012), Christensen, Meh, and Moran (2011)]. The notable exception on this front is De Paoli and Paustian (2013), who also study Ramsey-optimal policies but motivate

necessarily the *volume*) of credit extended by banks. Recent work on monetary policy and financial stability emphasizes the credit cycle and the "risk-taking channel" of monetary policy [as discussed, for example, in Borio and Zhu (2008)]. It typically views excessive risk taking in terms of the aggregate volume of credit. Angeloni and Faia (2013), for example, consider a link between the bank leverage ratio and the risk of bank runs; Christensen, Meh and Moran (2011) postulate an externality that links the riskiness of bank projects to the ratio of aggregate credit to GDP. While abstracting from monetary policy, a number of other contributions [e.g., Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010)] similarly view financial instability as the result of excessive borrowing. In these contributions, a pecuniary externality associated with a collateral constraint plays a central role: it makes an economic expansion increase the value of borrowers' collateral and lead to excessive borrowing. A tax on debt can then make borrowers internalize the externality.[5] Benigno et al. (2011) add monetary policy to this setting and examine how it may pursue financial stability in addition to its conventional goals. They also consider the role of a tax on debt, but do not characterize optimal policy. In all these models, economic expansions − following, for example, a favorable productivity shock or a period of low interest rates − lead to excessive risk taking or excessive borrowing and call for a policy response that may be either monetary or prudential.

We find these insights about the recent crisis persuasive.[6] Nonetheless, we can also envision other ways in which monetary and prudential policies may interact with each other, and think that these alternative perspectives can also serve to inform the design of future regulatory frameworks. To make our point, we start with a benchmark model that deliberately abstracts from any connection between risk taking and the volume of credit, and focuses instead on the type of credit, i.e. the composition of banks' loan portfolios. Our model follows a branch of the micro-banking literature [surveyed by Freixas and Rochet (2008)] in which the need for capital requirements arises from limited liability and deposit insurance. These institutional features truncate the distribution of risky returns facing investors, the banks lending to these investors, and the depositors funding the banks; this is the externality that leads to excessive risk taking. In our model, excessive risk taking involves the type of projects that banks may be tempted to finance because limited liability protects them from incurring large losses, and deposit insurance decouples their funding costs from their risk taking.

More specifically, we introduce aggregate risk into a variant of Van den Heuvel's (2008) model of optimal capital requirements, and we embed the resulting model in a DSGE framework with aggregate shocks, sticky prices and monetary policy.[7] Sufficiently high capital requirements can always force banks to internalize the riskiness of their loans and thus tame risk-taking behavior. But monetary policy may not be suited to this task as it works primarily through the volume rather than the composition of credit. In our benchmark model, due to the assumption of perfectly competitive banks operating under constant returns to scale, the interest rate has no effect on risk-taking incentives as it affects the cost of funding *all* (safe or risky) projects equally. From this vantage point, capital

prudential policy differently.

[5] Bianchi (2011) discusses how this tax on debt may be a model proxy for prudential policies (like capital requirements) that work through the banking system.

[6] There is now compelling empirical evidence in support of the risk-taking channel of monetary policy [e.g., Altunbas et al. (2010), Ioannidou et al. (2009), Jimenez et al. (2012)]. Schularick and Taylor (2012, p. 1032) claim that banking crises are "credit booms gone wrong." And Kashyap, Berner and Goodhart (2011) emphasize the relevance of the downside of pecuniary externalities (contractions accompanied by fire sales of assets) for the design of prudential policies.

[7] Martinez-Miera and Suarez (2012) examine capital requirements from a perspective similar to ours, but abstract from aggregate shocks and monetary policy.

requirements and the interest rate are sharply distinct policy tools that do not affect the same margins: monetary policy affects the volume but not the type of credit, while prudential policy affects both the type and the volume of credit. This makes monetary policy ineffective in ensuring financial stability. As such, our framework accords with the standard view among policymakers [expressed, for instance, in Bernanke (2011)] that standard interest-rate policy cannot serve as the first line of defense against financial instability.

Our locally Ramsey-optimal policy sets the capital requirement to the minimum level that prevents inefficient risk taking by banks. Indeed, setting the capital requirement *just below* this threshold level is not optimal because it triggers a discontinuous increase in the amount of inefficient risk taken by banks. This discontinuity is due to our deposit-insurance and limited-liability assumptions, which make banks' expected excess return convex in the amount of risk that they take. And setting the capital requirement *just above* this threshold level is not optimal because it has a negative first-order effect on welfare that cannot be offset by any change in the interest rate around its optimal value (as this change would have a zero first-order effect on welfare). This negative first-order effect on welfare, in turn, is due to the fact that taxes on banks' profits distort banks' funding decisions as they make equity finance more expensive than debt finance for the banks. This tax distortion implies that raising the capital requirement above the threshold level decreases the (bank-loan-financed) capital stock, which is already inefficiently low due to monopolistic competition and the tax distortion itself.[8]

This optimal capital requirement is state dependent: it rises in response to shocks that increase banks' incentives to fund risky projects. In our benchmark model, the interest rate and the capital requirement do not affect the same margins, so there is a clear-cut optimal division of tasks between monetary and prudential policies: in response to shocks that do not affect banks' risk-taking incentives, prudential policy should leave the capital requirement constant, and monetary policy should move the interest rate in a standard way. In response to shocks that increase (decrease) banks' risk-taking incentives, prudential policy should raise (cut) the capital requirement, and monetary policy should cut (raise) the interest rate in order to mitigate the effects of prudential policy on bank lending and output. In the latter case, optimal prudential policy is pro-cyclical (as it is the proximate cause of the contraction of output), while optimal monetary policy is counter-cyclical. So, with this chain of causality, the two policies move in opposite directions over the cycle − a situation envisaged by some policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)].

In this benchmark model, risk taking is exclusively related to the type of credit extended by banks. We can, however, modify our setup to consider situations in which both the type and the volume of credit matter. To illustrate this, we develop an extension that incorporates a risk-taking channel of monetary policy. In this extension, the cost of originating and monitoring safe loans is an increasing function of the aggregate volume of such loans.[9] Consequently, all the shocks that affect the volume of safe loans also affect the cost of such loans and thus banks' risk-taking incentives. Although the

[8] An alternative to our model with the tax distortion would be to follow Van den Heuvel (2008) and model the cost of raising capital requirements as foregone liquidity from holding bank deposits. In his model, liquid deposits and equity are the only sources of funding for bank loans. So, when capital requirements are higher, banks don't issue as much liquid deposits, and households suffer a loss of utility. We don't pursue this track because commercial paper (rather than liquid deposits) is a more likely marginal source of funding for US banks, as Cúrdia and Woodford (2009) point out. For the same reason, following Cúrdia and Woodford (2009) and others, our modelling of optimal monetary policy will abstract from the transactions frictions that motivate the Friedman Rule.

[9] We use this ad-hoc assumption about costs of banking to keep the extension brief. Hachem (2010) develops a full model of this type of externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks.

particular extension that we consider is motivated by tractability, we think it highlights the main features of optimal policy interactions in other environments that link higher output levels and/or lower interest rates to higher risk-taking incentives. Compared to our benchmark model, the main novelty here is that both policies optimally take a countercyclical stance in response to some shocks. A favorable productivity shock, for instance, raises the volume and hence the cost of safe loans, which in turn increases banks' risk-taking incentives. Following this shock, optimal prudential policy raises the capital requirement, and optimal monetary policy raises the interest rate.[10] But the optimal interest-rate hike is smaller than it would be in our benchmark model, because optimal monetary policy mitigates the effects of the rise in the capital requirement on bank lending and output. As we will elaborate below, optimal policy responses to other shocks (shocks that directly increase risk-taking incentives) are also attenuated when we allow risk-taking incentives to rise with the volume of credit. Nonetheless, the qualitative aspects of the optimal policy responses to these shocks do not change: tighter prudential policy tames the risk taking incentives, and easier monetary policy alleviates some of the contractionary consequences.

The rest of the paper is organized as follows. Section 2 presents our benchmark model. Section 3 derives and discusses our analytical results on prudential policy, with proofs relegated to the Appendix. Sections 4 and 5 discuss our calibration and report our numerical results for the optimal monetary and prudential policies in the benchmark model. Section 6 presents two extensions (one with an externality in the cost of banking, the other with correlated shocks) that seem relevant for policy concerns. Section 7 contains concluding remarks.

2 Benchmark Model

To motivate the role of banks in our model, we assume that households must sell their unfurbished capital stock to capital producers –who need to borrow the necessary funds– at the end of each period and buy back the furbished capital at the beginning of the next period. The capital producers have access to two alternative technologies to furbish capital: one is safe and the other risky. The latter technology is less efficient on average, but limited liability tempts the capital producers to use it. Banks are needed to monitor the producers who claim to use the safe technology, to ensure that they do so. Banks themselves, however, may have adverse incentives due to limited liability and deposit insurance, and these adverse incentives give a role to prudential policy.

Each period is divided into two subperiods. At the beginning of the first subperiod, all exogenous shocks are realized, except one, and these realizations are observed by all agents. The only shock that is not realized at the beginning of the first subperiod is the binary shock leading to the success or failure of the risky technology (in the case of failure, forcing any capital producers using this technology to default on their bank loans). This shock is realized at the end of the second subperiod, after households, firms, and banks have made their optimal decisions.

[10] There are also other ways to make both policies optimally counter-cyclical in our setup. As an example, we will present a case with correlated shocks.

2.1 Households

Preferences are defined by the discount factor $\beta \in (0,1)$ and the period utility

$$U(c_t, h_t) = \log(c_t) - \frac{1}{1+\chi} h_t^{1+\chi}$$

over consumption c_t and hours of work h_t, where $\chi > 0$. Households maximize $E_0 \sum_{t=0}^{\infty} \beta^t \; U(c_t, h_t)$. All household decisions are taken in the first subperiod of each period t. We assume that, during this subperiod, households own the furbished capital stock k_t and rent it, at the rental price z_t, to intermediate goods producers. At the end of the subperiod, after production has taken place, households get back $(1-\delta)k_t$ worn-out capital from intermediate goods producers, where $0 < \delta < 1$, and invest i_t in new capital. Unfurbished capital x_t, made of both worn-out capital and new capital, has to be furbished before it can be used for production next period. So, at this stage, households sell their unfurbished capital

$$x_t = (1-\delta)k_t + i_t, \tag{1}$$

at the price q_t^x, to capital goods producers, who can furbish it in the second subperiod of period t. At the beginning of the next period, households buy furbished capital k_{t+1}, at a price q_{t+1}, from capital goods producers.

Households also acquire s_t shares in banks at a price q_t^b. These banks are perfectly competitive and last for only one period. Households face the budget constraint

$$c_t + d_t + q_t^b s_t + q_t k_t + i_t = w_t h_t + \frac{1+R_{t-1}^D}{\Pi_t} d_{t-1} + s_{t-1}\omega_t^b + z_t k_t + q_t^x x_t + (\omega_t^k + \omega_t^f - \tau_t^h), \tag{2}$$

where d_t represents the real value of bank deposits with a gross nominal return R_t^D, $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate in the price index for consumption, w_t is the real wage, ω_t^k and ω_t^f represent the profits of capital producers and firms producing intermediate goods, ω_t^b stands for dividends paid by banks, and τ_t^h is a lump-sum tax paid by households.[11]

Households choose $(c_t, h_t, d_t, s_t, k_t, i_t, x_t)_{t\geq 0}$ to maximize utility subject to (1) and (2). The first-order conditions for optimality are:

$$\begin{aligned}
\frac{1}{c_t} &= \lambda_t, \\
\lambda_t &= \beta\left(1+R_t^D\right) E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}, \\
h_t^{\chi} &= \lambda_t w_t, \\
\lambda_t q_t^x &= \lambda_t^k, \\
\lambda_t &= \lambda_t^k, \\
\lambda_t\left(q_t - z_t\right) &= \lambda_t^k(1-\delta), \\
\lambda_t q_t^b &= \beta E_t\left\{\lambda_{t+1}\omega_{t+1}^b\right\},
\end{aligned} \tag{3}$$

where $E_t\{.\}$ denotes the expectation operator conditional on the information available in the first subperiod of period t, which includes the realization of all the aggregate shocks except the binary

[11] We do not need to model equity stakes in firms as we assume that the representative household owns these firms forever.

shock leading to the success or failure of the risky technology. The optimality conditions imply in particular

$$\begin{aligned} q_t^x &= 1, \\ q_t &= 1 - \delta + z_t. \end{aligned}$$

2.2 Intermediate goods producers

There is a unit mass of monopolistically competitive firms producing intermediate goods. Firm j operates the production function:

$$y_t(j) = h_t(j)^{1-\nu} k_t(j)^{\nu} \exp\left(\eta_t^f\right),$$

where $0 < \nu < 1$, $k_t(j)$ is capital rented by firm j, and η_t^f is an exogenous productivity shock. We assume that firms set their prices facing a Calvo-type price rigidity (with no indexation). Since their optimization problem is standard, we don't present the details. We let α denote the probability that a firm does not get to set a new price at a given date.

The firms' cost minimization problem implies

$$\frac{z_t}{w_t} = \left(\frac{\nu}{1-\nu}\right)\left[\frac{h_t(j)}{k_t(j)}\right].$$

2.3 Final goods producers

Producers of the final good are perfectly competitive and aggregate the intermediate goods $y_t(j)$ to form the final good y_t. The production function is given by

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\sigma-1}{\sigma}} \mathrm{d}j\right)^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\sigma > 1$. Profit maximization leads to the demand for good j

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\sigma} y_t, \tag{5}$$

and free entry lead to the price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\sigma} \mathrm{d}j\right)^{\frac{1}{1-\sigma}}. \tag{6}$$

The final good may be used for consumption, investment, the monitoring of firms, and government purchases.

2.4 Capital goods producers

The capital producing firms are owned by households and are perfectly competitive. They buy unfurbished capital x_t during the second subperiod of period t to produce furbished capital k_{t+1} that they sell to households at the price q_{t+1} in the first subperiod of period $t+1$. Each capital producer chooses to operate either a safe technology (S for "safe" or "storage") or a risky technology (R for

"risky"). Those choosing technology S use x_t^S units of unfurbished capital to produce k_{t+1}^S units of furbished capital with

$$k_{t+1}^S = x_t^S. \tag{7}$$

Producers choosing technology R are subject to a common (systemic) shock θ_t that is independent of all the other shocks. When $\theta_t = 0$, they produce nothing. More specifically, they use x_t^R units of unfurbished capital to produce

$$k_{t+1}^R = \theta_t \exp\left(\eta_t^R\right) x_t^R$$

units of furbished capital, with

$$\begin{aligned} \theta_t &= 0 \text{ with probability } \phi_t, \\ \theta_t &= 1 \text{ with probability } 1 - \phi_t, \end{aligned}$$

where ϕ_t is the exogenous stochastic probability of failure and η_t^R is the exogenous stochastic productivity if the project is successful. We assume that the realization of η_t^R is always positive ($\eta_t^R > 0$), so that in the absence of failure, the risky technology is more productive than the safe one. Producers choose whether to use technology S or technology R after observing the realization of η_t^R and ϕ_t (which occur at the beginning of the first subperiod), but before observing the realization of θ_t (which occurs at the end of the second subperiod).

Our setup with two technologies serves to highlight a familiar connection between limited liability and excessive risk taking: if capital producers are not monitored properly, they may take on more risk than a hypothetical social planner would. We will simplify the exposition − we think, without affecting our main points much − by assuming that using the risky technology to any degree is always inefficient from a planner's perspective, as we elaborate below.[12] Capital producers may have an incentive to use the risky technology, to the extent that they can hide the fact that they do so, only because they have limited liability. There is therefore a need to monitor capital producers who claim to use the safe technology, and we assume that only banks have the appropriate monitoring skills. This motivates a setup with capital producers getting funds from banks to buy unfurbished capital.

More specifically, the risky technology is assumed to be inefficient in the sense that, for all realizations of ϕ_t, η_t^R and Ψ_t,

$$(1 - \phi_t) \exp\left(\eta_t^R\right) \leq 1 - \Psi_t, \tag{8}$$

where $\Psi_t > 0$ is the exogenous marginal resource cost of monitoring a capital producer who claims to use the safe technology.[13] The left-hand side of (8) represents the marginal benefit of allocating one unit of unfurbished capital to the risky technology (the expected output of this technology at the time when decisions are made, i.e. after the realization of all the shocks except the failure shock θ_t). The right-hand side is the opportunity cost, which is the output of the safe technology net of the monitoring cost. This inefficiency condition is stronger than what we actually need for the risky technology to be socially undesirable; but we use it because the necessary and sufficient condition involves the degree of risk aversion and we prefer to define inefficiency only in terms of technology parameters.

[12] One way to extend our model to incorporate efficient risk taking would involve adding a third technology that is risky but can be efficiently combined with the safe technology. This would make the model more realistic by adding some desirable risk, but it would require solving a portfolio problem that does not seem directly relevant for our purposes.

[13] In Section 6, we will consider an extension of the model in which Ψ_t is endogenous.

Our model simplifies (we think in a harmless way) the relationship between capital goods producers, their owners, and the creditor banks. In reality non-bank firms prefer debt finance because they get a tax deduction. They also need some equity, presumably because of the agency problem associated with debt. Their owners absorb losses up to their equity stake. In our model, for simplicity, we abstract from this agency problem and capital goods producers have no equity. So this translates into a framework in which their funding is entirely with loans and they pay no tax; and any profits or losses arising from stochastic disturbances in the absence of failure of the risky technology accrue to households.[14] Thus, a capital producer i choosing technology $j \in \{S, R\}$ borrows

$$q_t^x x_t^j(i) = l_t^j(i) \tag{9}$$

at a nominal interest rate R_t^j.[15] Since capital producers have limited liability, those using the risky technology will default on their loans in the event of failure (when $\theta_t = 0$).

A producer i using technology S chooses $x_t^S(i)$ to maximize

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} k_{t+1}^S(i) - \frac{1+R_t^S}{\Pi_{t+1}} l_t^S(i) \right] \right\}$$

subject to (7) and (9). The optimality condition implies

$$E_t\{\lambda_{t+1} q_{t+1}\} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \left(1 + R_t^S\right) q_t^x. \tag{10}$$

A producer i using technology R chooses $x_t^R(i)$ to maximize

$$(1-\phi_t)\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} \exp\left(\eta_t^R\right) x_t^R(i) - \frac{1+R_t^R}{\Pi_{t+1}} l_t^R(i) \right] \middle| \theta_t = 1 \right\}$$

subject to (9), where $E_t\{.|\theta_t = 1\}$ denotes the expectation operator conditional on the information available in the first subperiod of period t and on the success of the risky technology in the second subperiod of period t. The optimality condition implies

$$E_t\{\lambda_{t+1} q_{t+1} | \theta_t = 1\} \exp\left(\eta_t^R\right) = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} \left(1 + R_t^R\right) q_t^x. \tag{11}$$

Since our model allows for two distinct interest rates, banks need to monitor the capital producers that borrow at the lower rate to ensure that they use the associated technology. Our model has no equilibrium with $R_t^R < R_t^S$.[16] Therefore, there is no need for banks to monitor capital producers that claim to use the risky technology. Accordingly, we will associate a cost with monitoring capital producers that claim to use the safe technology.

As usual, with constant returns to scale, the first-order conditions imply that firms make zero profits. When both (10) and (11) hold, capital producers are indifferent between the two technologies and

$$\frac{1+R_t^R}{1+R_t^S} = \frac{E_t\{\lambda_{t+1} q_{t+1} | \theta_t = 1\}}{E_t\{\lambda_{t+1} q_{t+1}\}} \frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1\right\}} \exp\left(\eta_t^R\right). \tag{12}$$

If the interest-rate ratio on the left-hand side is strictly higher than the critical value on the right-hand side, then capital producers use only technology S.

[14] Our results however would be qualitatively unchanged if capital goods producers were allowed to borrow only a fraction of the funds they need, or if they only needed funding to pay for the investment flow (and financed the rest with equity).

[15] There is no need to work with nominal loan contracts in our model. However, since we will assume that monetary policy sets a nominal interest rate, and for the sake of realism, we make loan contracts nominal.

[16] Indeed, if we had $R_t^R < R_t^S$, then funding the safe projects would strictly dominate funding the risky projects because it would pay more in every state (whatever the realization of θ_t) and incur no monitoring cost.

2.5 Banks

Banks are owned by households. They are perfectly competitive. They incur a cost $\Psi_t l_t^S$ of monitoring safe loans, where Ψ_t satisfies (8). They can fund their loans by raising equity (e_t) or issuing deposits (d_t). They make safe and risky loans (l_t^S and l_t^R). Their balance-sheet identity is

$$l_t^S + l_t^R = e_t + d_t, \tag{13}$$

as e_t is defined net of monitoring costs.

Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), risky projects reduce welfare. So if the regulators could detect any risky project, they would devise a sufficient penalty to prevent it. We need an information friction to rule out a trivial and unrealistic solution in which the regulators directly forbid risk taking. Following Van den Heuvel (2008), we assume that banks can hide some risky loans in their portfolio from regulators. More specifically, we assume that regulators observe the total amount of loans made by each bank but cannot detect its risky loans up to an exogenous fraction γ_t of its safe loans. The prudential authority imposes risk-weighted capital requirements on risky loans above this fraction. We specify the capital requirement as

$$e_t \geq \kappa_t \left(l_t^S + l_t^R\right) + \overline{\kappa} \max\left\{0, l_t^R - \gamma_t l_t^S\right\}. \tag{14}$$

The higher the capital requirement, the more banks internalize the social cost of risk, as they have more "skin in the game." The prudential authority will optimally choose a sufficiently high $\overline{\kappa}$ for $l_t^R \leq \gamma_t l_t^S$ in equilibrium. Therefore, this is equivalent to rewriting the capital requirement as a minimum ratio of equity to loans:

$$e_t \geq \kappa_t \left(l_t^S + l_t^R\right), \tag{15}$$

and imposing the following constraint on banks:

$$l_t^R \leq \gamma_t l_t^S. \tag{16}$$

In the first subperiod of period $t+1$, regulators close the banks that cannot meet their deposit obligations: the banks with

$$\frac{1+R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1+R_t^R}{\Pi_{t+1}} l_t^R - \frac{1+R_t^D}{\Pi_{t+1}} d_t < 0,$$

or equivalently, using (13), those with

$$e_t < -\left(\frac{R_t^S - R_t^D}{1+R_t^D}\right) l_t^S - \left(\theta_t \frac{1+R_t^R}{1+R_t^D} - 1\right) l_t^R.$$

When $l_t^R = 0$ or $\theta_t = 1$, the right-hand side of this inequality is negative as long as lending rates are above the deposit rate, which will be the case in equilibrium because loans either incur a monitoring cost or entail a risk for banks. When $l_t^R > 0$ and $\theta_t = 0$, the right-hand side of this inequality is positive if and only if

$$l_t^R > \left(\frac{R_t^S - R_t^D}{1+R_t^D}\right) l_t^S.$$

We want our model to capture the fact that banks find equity finance more costly than debt finance in reality. We attribute this to a tax distortion (tax deduction for debt finance), although this

interpretation is not essential for our analysis. We take this distortionary tax to be a feature of the environment: the model does not explain why this tax is in place, and the policymakers in our model (the monetary and prudential authorities) cannot set this tax optimally.[17]

The particular way we specify the tax distortion (and the timing of the tax deduction for monitoring costs) ensures that unanticipated changes in the price level cannot cause insolvency.[18] The banks in our model may be insolvent only if they extend too many risky loans, and the risky projects fail. Specifically, we assume that gross revenues from loans are taxed at the constant rate τ after deductions for gross payments on deposits and monitoring costs. The amount of bank equity, net of monitoring costs, is therefore $e_t = q_t^b s_t - (1-\tau)\Psi_t l_t^S$.

The representative bank chooses e_t, d_t, l_t^R and l_t^S to maximize

$$E_t\left\{\beta\frac{\lambda_{t+1}\left(1-\tau\right)\omega_{t+1}^b}{\lambda_t}\right\} - e_t - \left(1-\tau\right)\Psi_t l_t^S,$$

where

$$\omega_{t+1}^b = \max\left\{0, \frac{1+R_t^S}{\Pi_{t+1}}l_t^S + \theta_t\frac{1+R_t^R}{\Pi_{t+1}}l_t^R - \frac{1+R_t^D}{\Pi_{t+1}}d_t\right\}, \tag{17}$$

subject to (13), (15) and (16).

2.6 Government and market-clearing conditions

The government has exogenous purchases G_t and guarantees bank deposits. The lump-sum tax on households balances the budget.[19]

The losses imposed by bank j on the deposit insurance fund amount to

$$\zeta_t(j) = \max\left\{0, \frac{1+R_{t-1}^D}{\Pi_t}d_{t-1}(j) - \frac{1+R_{t-1}^S}{\Pi_t}l_{t-1}^S(j) - \theta_{t-1}\frac{1+R_{t-1}^R}{\Pi_t}l_{t-1}^R(j)\right\},$$

and the lump-sum tax paid by households is

$$\tau_t^h = G_t + \int_0^1 \left\{\zeta_t(j) - \tau[\omega_t^b(j) + \Psi_t l_t^S(j)]\right\} dj.$$

We consider two policy instruments: the deposit rate R_t^D for monetary policy and the capital requirement κ_t for prudential policy. We will discuss our specifications of prudential policy in Sections 3 and 5. For each specification, our monetary policy will be the Ramsey-optimal policy.

Firms producing intermediate goods rent their capital from the representative household; in equilibrium, their choices must satisfy

$$\int_0^1 k_t(j)dj = k_t.$$

Similarly obvious market-clearing conditions must be satisfied in the markets for labor, loans, and unfurbished capital. The market-clearing condition for goods is

$$c_t + i_t + G_t + \Psi_t l_t^S = y_t.$$

[17] This feature of the tax code seems to be one of the primary reasons for banks to lobby against higher capital requirements, at least in the US and the euro area. It is commonly invoked in models with both debt and equity finance [e.g. Jermann and Quadrini (2009, 2012)], to break the Modigliani-Miller theorem about irrelevance of financial structure. We motivate our modeling choice in the conclusion.

[18] In our setting with one-period competitive banks incurring real monitoring costs and extending nominal loans, a change in the price level could lead to insolvency. We don't think this is an interesting feature of the model and have specified our "tax code" to rule it out.

[19] It is harmless to abstract from deposit insurance fees paid by banks and include these in the lump-sum tax paid by households who own the banks.

3 Prudential Policy

This section derives conditions for prudential policy to rule out equilibria with risk taking and ensure the existence of equilibria without risk taking. We first show that our model can only have equilibria at the two corners with $l_t^R = 0$ and $l_t^R = \gamma_t l_t^S$, and that the capital constraint is binding in any equilibrium. Next, we consider a benchmark prudential policy that internalizes the externality (arising from limited liability) by making banks the residual claimants to any losses they may incur. We then characterize the least stringent prudential policy that rules out risk taking, and show that it is locally Ramsey-optimal.

3.1 Ruling out candidate equilibria

We focus on symmetric equilibria in which all banks have the same loan portfolio. We will also assume throughout that the following condition holds:

$$\frac{E_t\left\{\lambda_{t+1} q_{t+1} | \theta_t = 1\right\}}{E_t\left\{\lambda_{t+1} q_{t+1}\right\}} \leq 1. \tag{18}$$

This condition seems plausible because failure of risky projects at date t leads to destruction of the capital stock at date $t+1$, and this by itself should increase both the price of capital (q_{t+1}) and the marginal utility of consumption (λ_{t+1}). However, this condition amounts to an implicit restriction on the set of policies that we consider, as it presumes that policies will not overturn the qualitative effects of the failure of risky projects.

We first show that the banks' optimization problem rules out the existence of equilibria with $0 < l_t^R < \gamma_t l_t^S$. The basic insight follows Van den Heuvel (2008), but since we have added aggregate risk and made other changes to his model, we prove the following proposition in the Appendix.

Proposition 1: *There are no equilibria with $0 < l_t^R < \gamma_t l_t^S$. When $0 < l_t^R < \gamma_t l_t^S$, (a) if banks go bankrupt ($\omega_{t+1}^b = 0$) when risky projects fail ($\theta_t = 0$), then banks can increase their market value by tilting the loan portfolio towards more risky loans; (b) if banks do not go bankrupt ($\omega_{t+1}^b > 0$) when risky projects fail ($\theta_t = 0$), then they can increase their market value by tilting the loan portfolio towards more safe loans.*

The intuition follows. If, given the loan portfolio, bank equity is sufficiently small to be wiped out when risky projects fail, then banks do not internalize the cost of additional risk taking. Additional losses from increasing l_t^R, if risky projects fail, are truncated by deposit insurance and limited liability. Consequently, the only candidate for an equilibrium with the possibility of bank failure involves the corner solution $l_t^R = \gamma_t l_t^S$.

Alternatively, if bank equity is sufficiently large for banks to remain solvent even when risky projects fail, then banks internalize the cost of additional risk taking. In that case, since we assume that the risky technology is inefficient, banks can increase their market value by reducing l_t^R. Accordingly, the only candidate for an equilibrium without the possibility of bank failure involves the corner solution $l_t^R = 0$. In particular, if bank equity is large enough to make banks residual claimants on their risky loans when $l_t^R = \gamma_t l_t^S$, then there does not exist an equilibrium with $l_t^R = \gamma_t l_t^S$.

Next, we show that there are no equilibria in which the capital constraint is lax:

Proposition 2: *In equilibrium, the capital constraint is binding:*

$$e_t = \kappa_t \left(l_t^S + l_t^R \right). \tag{19}$$

This Proposition follows almost directly from our assumption about the tax advantage of debt finance over equity finance, but we provide a proof in the Appendix.

3.2 A benchmark policy

Proposition 1 leads to a sufficient condition for prudential policy to rule out equilibria with $l_t^R > 0$ and ensure the existence of an equilibrium with $l_t^R = 0$: the capital requirement can be sufficiently high to make any bank the residual claimant to the potential losses arising from funding risky projects. This benchmark policy is characterized by the following proposition:

Proposition 3: *(a) A sufficient condition for existence and uniqueness of an equilibrium and for $l_t^R = 0$ in this equilibrium is that*

$$\kappa_t > \widetilde{\kappa}\left(R_t^D, R_t^S\right) \equiv 1 - \frac{1}{1+\gamma_t}\frac{1+R_t^S}{1+R_t^D}; \tag{20}$$

(b) in this equilibrium,

$$\widetilde{\kappa}\left(R_t^D, R_t^S\right) = \widetilde{\kappa}_t \equiv \frac{(1-\tau)(\gamma_t - \Psi_t)}{\tau + (1-\tau)(1+\gamma_t)}; \tag{21}$$

(c) $\widetilde{\kappa}_t$ is increasing in γ_t, and decreasing in Ψ_t.

We prove this proposition in the Appendix, by considering a given bank j that takes the maximum amount of risk ($l_t^R(j) = \gamma_t l_t^S(j)$). We show that this bank will remain solvent when risky projects fail ($\theta_t = 0$) if and only if (20) holds. We then use the banks' optimality conditions at the equilibrium with $l_t^R = 0$ to express $\widetilde{\kappa}\left(R_t^D, R_t^S\right)$ in terms of parameters and exogenous shocks and obtain (21).

We assume $\gamma_t > \Psi_t$, which implies $\widetilde{\kappa}_t > 0$, so that condition (20) may or may not be met depending on the value of κ_t. This restriction states that the temptation to take risk would be present if banks were not subject to any (positive) capital requirements. The threshold $\widetilde{\kappa}_t$ is increasing in γ_t: the higher the fraction of risky loans that a deviating bank can hide, the riskier this bank, and the higher the capital requirement needed to make it remain solvent in case of failure. And $\widetilde{\kappa}_t$ is decreasing in Ψ_t: the higher the cost of monitoring safe loans, the higher the spread between the interest rate on safe loans and that on deposits; thus, the larger the cash flow from safe loans that is available to redeem the deposits, and the lower the capital requirement needed to make a deviating bank remain solvent in case of failure.

Although this benchmark policy suffices to ensure the existence of an equilibrium without risk taking, we show next that it is more stringent than necessary and that the least stringent policy ensuring the existence of an equilibrium without risk taking is locally Ramsey-optimal in our model.

3.3 The locally optimal policy

We now derive a necessary and sufficient condition for prudential policy to ensure the existence of an equilibrium with $l_t^R = 0$, and then show that the least stringent policy satisfying this condition is locally Ramsey-optimal.

Consider a bank j that deviates from a candidate equilibrium with $l_t^R = 0$ to take the maximum amount of risk ($l_t^R(j) = \gamma_t l_t^S(j)$). There exists an equilibrium with $l_t^R = 0$ if and only if this deviating bank has a negative expected excess return. In the Appendix, we derive the threshold value of κ_t that makes its expected excess return negative, and we prove the following proposition:

Proposition 4: *(a) A necessary and sufficient condition for existence of an equilibrium with $l_t^R = 0$ is $\kappa_t \geq \kappa_t^*$, where*

$$\kappa_t^* \equiv (1-\tau)\frac{(1-\phi_t)\gamma_t\left[\exp\left(\eta_t^R\right)-1\right]+\Psi_t\left[(1-\phi_t)\gamma_t\exp\left(\eta_t^R\right)-\phi_t\right]}{\phi_t(1+\gamma_t)-\gamma_t\tau(1-\phi_t)\left[\exp\left(\eta_t^R\right)-1\right]}; \tag{22}$$

(b) $\kappa_t^* < \widetilde{\kappa}_t$; (c) κ_t^* is increasing in the probability of success of the risky technology $1-\phi_t$, the productivity of the risky technology conditionally on its success η_t^R, and the maximum ratio of risky to safe loans γ_t.

The derivations in the Appendix consider a bank j that contemplates a deviation from a candidate equilibrium with $l_t^R = 0$. The same intuition we gave for Proposition 1 (roughly) applies: if there are profitable deviations, the most profitable one is at the corner with maximum risk ($l_t^R(j) = \gamma_t l_t^S(j)$). To derive the value of κ_t^*, we make bank j indifferent between staying at the safe corner and moving to the risky corner. The bank turns indifferent with less equity at stake than what would make it residual claimant (i.e., we have $\kappa_t^* < \widetilde{\kappa}_t$) because the bank has incurred monitoring costs and has a vested interest in remaining solvent to recoup these costs. In a way, monitoring costs in our model work like giving the banks some charter value that they would like to preserve by avoiding bankruptcy.

The preceding intuition also helps us understand the nature of the state dependence, in our model, of the constraint $\kappa_t \geq \kappa_t^*$. Macro-prudential policy must be tight enough to prevent risk taking in equilibrium. The threshold κ_t^* depends negatively on the probability of failure of the risky technology ϕ_t because failure risk, by itself, makes risk-taking less attractive. Similarly, κ_t^* rises with the productivity of the risky technology conditionally on its success η_t^R and with the maximum ratio of risky to safe loans γ_t.[20]

Perhaps a more surprising feature of (22) is that κ_t^* does not depend on the monetary policy instrument R_t^D. This is because, in our model, the deposit rate R_t^D does not affect banks' incentives for risk taking. In particular, it does not affect the spread between the interest rate on risky loans R_t^R and the interest rate on safe loans R_t^S. In a way, this is not a surprising feature for a model with perfect competition and constant returns. Our banks never run out of safe projects to fund and always end up making zero profits. This is the opposite extreme from arguments that (explicitly or implicitly) postulate a fixed number of potential projects and thereby link more lending with more risk taking (as banks run out

[20] Our model could be extended to allow the prudential authority to choose γ_t by incurring some supervision cost, as in Van den Heuvel (2008). This would not change the optimal solution for κ_t as a function of γ_t in (22), but would make γ_t endogenous. In this case, the prudential authority would optimally respond to shocks that increase risk-taking incentives by devoting more resources to supervision (i.e. lowering γ_t) and raising the capital requirement κ_t by less.

of safe lending opportunities). We will revisit this contrast between extreme modelling assumptions in Section 6.

We now turn to normative implications. We define locally Ramsey-optimal policies as follows: a policy $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau\geq 0}$ is locally Ramsey-optimal if there exists a neighborhood of $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau\geq 0}$ such that no other policy in this neighborhood gives a higher value for the representative household's expected utility than $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau\geq 0}$ does. Let $\left(R_\tau^{D*}\right)_{\tau\geq 0}$ denote the monetary policy that is (globally) Ramsey-optimal when the prudential policy is $(\kappa_\tau^*)_{\tau\geq 0}$. The following proposition states that, under a certain condition, setting jointly $\left(R_\tau^D\right)_{\tau\geq 0}$ to $\left(R_\tau^{D*}\right)_{\tau\geq 0}$ and $(\kappa_\tau)_{\tau\geq 0}$ to $(\kappa_\tau^*)_{\tau\geq 0}$ is locally Ramsey-optimal:

Proposition 5: *If the right derivative of welfare with respect to κ_t at $\left(R_\tau^D, \kappa_\tau\right)_{\tau\geq 0} = \left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau\geq 0}$ is strictly negative for all $t \geq 0$, then the policy $\left(R_\tau^D, \kappa_\tau\right)_{\tau\geq 0} = \left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau\geq 0}$ is locally Ramsey-optimal.*

We prove this proposition in the Appendix. The basic idea is the following. First, whatever R_t^D in the neighborhood of R_t^{D*}, setting κ_t just below κ_t^* is not optimal, because it triggers a discontinuous increase in the amount of risk taken by banks. Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that $\left(R_t^D, \kappa_t\right)$ is close enough to $\left(R_t^{D*}, \kappa_t^*\right)$. Second, if the right derivative of welfare with respect to κ_t at $\left(R_t^{D*}, \kappa_t^*\right)$ is strictly negative, then setting κ_t just above κ_t^* is not optimal either, because it has a negative first-order effect on welfare that cannot be offset by any change in R_t^D around its optimal value R_t^{D*} (as this change would have a zero first-order effect on welfare).

The right derivative of welfare with respect to κ_t at $\left(R_t^{D*}, \kappa_t^*\right)$ can be expected to be strictly negative because increasing κ_t from κ_t^* decreases the capital stock, which is already inefficiently low due to the monopoly and tax distortions, without reducing the amount of risk, which is already zero. We check numerically, for the calibration considered in the next section, that this derivative is indeed strictly negative. This derivative is equal to the Lagrange multiplier associated to the constraint $\kappa_t = \kappa_t^*$ in the optimization problem that determines R_τ^{D*}. We first use the program Get Ramsey developed by Levin and López-Salido (2004) and used in Levin, Onatski, Williams and Williams (2005) to get analytically the non-linear first-order conditions of this optimization problem. We then use Dynare to solve numerically, at the first order, the resulting system of constraints and first-order conditions, and thus get the first-order approximation of this Lagrange multiplier (among other variables). We check that this Lagrange multiplier is strictly negative at the steady state, which implies that it is strictly negative for small enough shocks. We also check that it is strictly negative at the first order in the presence of shocks of a standard size.

We suspect (but cannot verify) that the policy $(R_\tau^{D*}, \kappa_\tau^*)_{\tau\geq 0}$ that we have identified is the globally Ramsey-optimal policy when failure of the risky technology is sufficiently costly, given the distortions present in our model. As noted above, values of κ_t higher than κ_t^* bring no additional benefit in terms of reducing risk, and are costly because they reduce the capital stock further. Values of κ_t lower than κ_t^* have the benefit of increasing the capital stock as long as risky projects succeed, and the drawback of recurrent falls in the capital stock. Computing equilibria of our model with $\kappa_t \leq \kappa_t^*$ (e.g. $\kappa_t = 0$) under Ramsey-optimal monetary policy cannot be achieved using log–linearization techniques because our model involves expectations conditional on the realization of a binary shock. This calls

for global approximation techniques which, under Ramsey-optimal policy, would suffer the curse of dimensionality given the number of state variables this equilibrium generates.[21]

3.4 Ruling out equilibria with $l_t^R = \gamma_t l_t^S$

We next formulate a prudential feedback rule that precludes equilibria with $l_t^R = \gamma_t l_t^S$, and coincides with $\kappa_t = \kappa_t^*$ in equilibrium. That is, under this rule, there is a unique equilibrium and, in this equilibrium, $l_t^R = 0$ and κ_t takes the minimum value that is consistent with $l_t^R = 0$.

We will assume throughout that the following condition holds:

$$\frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 1\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}} \leq 1. \tag{23}$$

This condition seems plausible but, as we noted in our discussion of (18), it amounts to an implicit restriction on the set of policies that we consider.[22]

We prove the following proposition in the Appendix:

Proposition 6: *Under the prudential-policy rule*

$$\kappa_t = \frac{1-\phi_t}{\phi_t}\frac{\gamma_t}{1+\gamma_t}\frac{R_t^R - R_t^S}{1+R_t^D} + \frac{1}{\phi_t}\frac{\gamma_t}{1+\gamma_t}\Psi_t - \frac{R_t^S - R_t^D}{1+R_t^D}, \tag{24}$$

there exists a unique equilibrium and, in this equilibrium, $l_t^R = 0$ *and* $\kappa_t = \kappa_t^*$.

Although the formal proof in the Appendix takes a different approach, a heuristic rendition is to start with the equilibrium at the safe corner and define R_t^R as the highest rate that a deviating bank could charge on a loan to a risky firm. In this case, (24) just states κ_t^* as a function of interest-rate spreads. It gives the critical value of κ_t for making the bank indifferent between staying at the safe corner (where all the other banks are) and jumping to the risky corner. The critical value is fairly intuitive. The first two terms represent the temptation to deviate from the safe corner to the risky corner: a deviating bank will pocket $R_t^R - R_t^S$ if risky projects succeed (with probability $1-\phi_t$) and save monitoring costs. The third term represents the opportunity cost $R_t^S - R_t^D$ of this deviation when risky projects fail (with probability ϕ_t).

So, this feedback rule suffices for keeping banks at the safe corner. In the Appendix we show that it also suffices to rule out an equilibrium at the risky corner, because the safe corner becomes even more attractive to an individual bank if there is a mass of banks at the risky corner (in which case the risk is priced).

4 Calibration

The parameters pertaining to households and firms are standard. For the parameters pertaining to the banking sector, we build heavily on Van den Heuvel (2008). The period of time is a quarter.

[21] The set of state variables includes the capital stock, price dispersion, the shocks and the Lagrange multipliers associated to the forward-looking constraints of the Ramsey-optimization problem − a total of 13 state variables.

[22] The condition seems plausible when we consider the pricing of a bond with default risk − a bond that pays \$1 when risky projects succeed and pays nothing when they fail. The inequality (23) says this risky bond has a higher expected real return, compared to a nominal bond with no default risk, in the equilibria we consider.

The discount rate is such that the household discounts the future at the deposit rate, 2.76% per year (see Van den Heuvel, 2008). The labor supply elasticity is set to 1. The value of the elasticity of substitution between intermediate goods σ is related to the degree of monopoly power firms have. Estimates of markups fall in the 10–20 percent range, implying that the elasticity of substitution lies in the 6–11 range. We follow Golosov and Lucas (2007) and set the elasticity of substitution to 7, implying a firms' markup of about 16 percent.

The capital elasticity in the intermediate-good technology is set such that the labor share is 0.66, implying a value for ν of 0.3. The depreciation rate, δ, is set to 0.025, which corresponds to a 10% annual depreciation rate. Firms are assumed to reset their prices every 3 quarters on average, implying the value 2/3 for the Calvo parameter α.[23]

Table 1: Calibration

Parameter	Description	Value
	Preferences	
β	Discount factor	0.993
χ	Inverse of labor supply elasticity	1.000
	Technology	
ν	Capital elasticity	0.300
σ	Elasticity of substitution	7.000
δ	Depreciation rate	0.025
	Nominal rigidities	
α	Price stickiness	0.667
	Banking (steady state)	
τ	Tax rate	0.023
κ^*	Capital requirement	0.100
Ψ	Marginal monitoring cost	0.006
ϕ	Failure probability	0.029
γ	Maximal risky/safe loans ratio	0.427
$exp(\eta^R)$	Productivity of the risky technology	1.005
	Shock processes	
ρ	Persistence	0.950

The parameters pertaining to the banking system are set as follows. Using variables without a time subscript to denote steady-state values, η^R is set such that the annualized lending rate on risky projects is 2 percentage points higher than that on safe projects in the steady state. We assume that the steady-state yield differential $R^S - R^D$ is 3.16% per annum (as in Van den Heuvel, 2008). The tax rate on bank profits is set to 2.29%. This value is chosen to equate the after-tax return on bank equity in our model to the after-tax return in US data.[24] Our calibration of the optimal steady-state capital requirement, κ^*, is 10%. The steady-state monitoring cost, Ψ, is set such that the first-order condition of the representative bank is satisfied:

$$\Psi = \frac{R^S - R^D}{1 + R^D} - \frac{\tau\kappa}{1-\tau}.$$

[23] We use this value as a compromise between higher values considered in the New Keynesian literature and lower values suggested by Bils and Klenow (2004).

[24] In the data, the after-tax return on equity is given by $(1-\tau^c)\pi/e$ where τ^c, π and e respectively denote corporate tax rate, profits and equity. In our model, this quantity is given by $(1-\tau)(\pi+e)/e - 1$ where τ denotes the proper tax rate that applies in our model. By equating these two quantities, and using the fact that the average return on equity is 7% and the tax rate on corporate profits is 35%, we obtain the number reported in Table 1.

This yields the value $\Psi = 0.006$. The steady-state failure probability of risky projects ϕ and the steady-state maximal risky/safe loans ratio γ are set jointly such that the model matches the average failure rate in the US economy (0.86% per quarter) and the optimal steady-state capital requirement is 10%:

$$\frac{\gamma\phi}{1+\gamma} = 0.86 \quad \text{and} \quad (1-\tau)\frac{(1-\phi)\gamma\left[\exp\left(\eta^R\right)-1\right]+\Psi\left[(1-\phi)\gamma\exp\left(\eta^R\right)-\phi\right]}{\phi(1+\gamma)-\gamma\tau(1-\phi)\left[\exp\left(\eta^R\right)-1\right]} = 0.10.$$

This system leads to a quadratic equation in γ, which has a unique positive solution, equal to 0.427, from which we get $\phi = 0.029$. The persistence of all the shocks is set to $\rho = 0.95$.[25] For the impulse-response functions presented in the next section, we set the innovations to the technology shock η_t^f and the fiscal shock G_t equal to 1%, and the innovation to Ψ_t to 10%. We set the innovation to η_t^R such that the annualized risk premium increases from 2% to 3%. And we set the innovation to ϕ_t such that the probability of failure increases by 1/3 of a percentage point.

5 Numerical Results

We consider two alternative prudential policies. Our benchmark prudential policy sets κ_t equal to its locally Ramsey-optimal value κ_t^*, which is 0.10 at the steady state. The other policy keeps κ_t constant at 0.12. This value is high enough, given the size of our shocks, to keep the economy in the safe equilibrium.

For each of these prudential policies, we solve for the Ramsey monetary policy using Dynare and the program Get Ramsey developed by Levin and López-Salido (2004). In both cases, the optimal steady-state inflation rate is zero, given the presence of Calvo-type price rigidity and the absence of monetary distortions in our model.

Figure 1 displays the optimal responses to a favorable productivity shock (positive innovation to η_t^f). The responses, with the exception of those of the interest rates, are expressed as percentage deviations from each steady state. The response of the interest rates is measured in terms of the level of the interest rate as a percentage per annum (rather than a deviation from the steady state). The horizontal dashed line corresponds to the steady-state level of the interest rate, so values below this line represent accommodative monetary policy following the shock, and values above represent restrictive monetary policy.

Since a productivity shock does not create a temptation to take more risk in our model, it does not affect the optimal capital requirement κ_t^*. So the optimal responses of the policy rate, output and inflation are the same, regardless of the prudential policy in place ($\kappa_t = \kappa_t^*$ or $\kappa_t = 0.12$). These optimal responses to a productivity shock are qualitatively similar to optimal responses in the benchmark New-Keynesian (NK) model with capital. Optimal policy essentially keeps inflation at zero. This requires an increase in the deposit rate for a while, because the natural real interest rate rises in the model with capital.[26] Optimal responses to an increase in government purchases (not reported here) are also similar to those from the NK model, and independent of prudential policy.

[25] Note that in order to study the response of the economy to shocks to the failure rate, we assume that $\phi_t = (1+\exp(-(u_t - v)))^{-1}$ where u_t is assumed to follow a zero-mean AR(1) process and $v = 3.393$.

[26] Both the favorable productivity shock and the resulting increase in employment increase the marginal product of capital.

Figure 1: Response to a Favorable Productivity Shock (η_t^f)

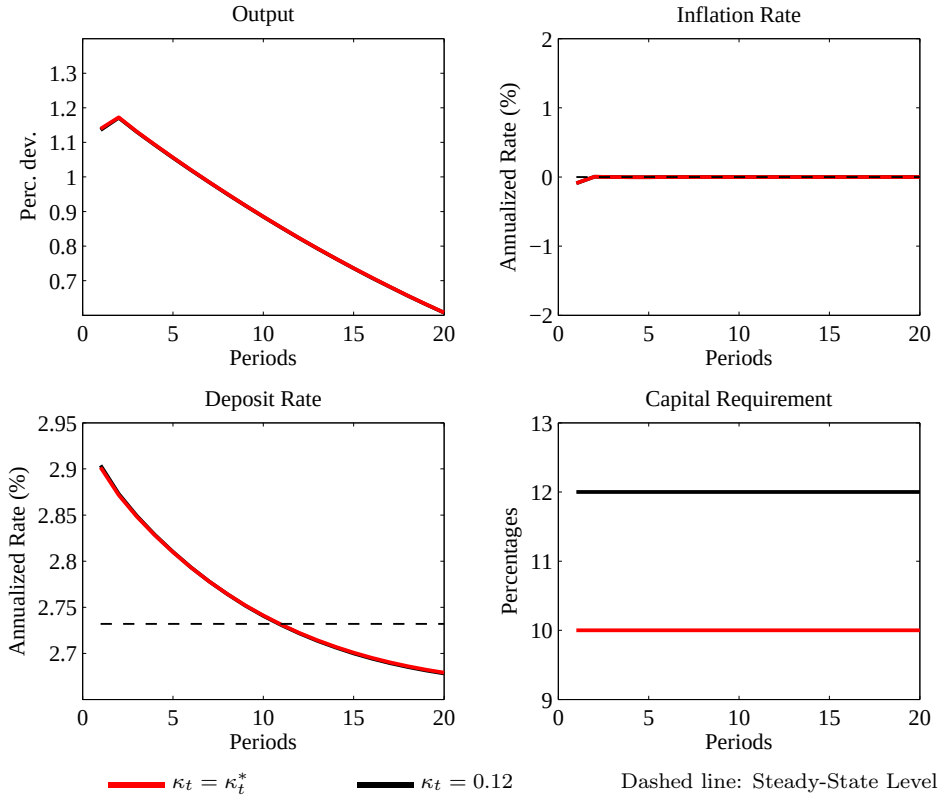
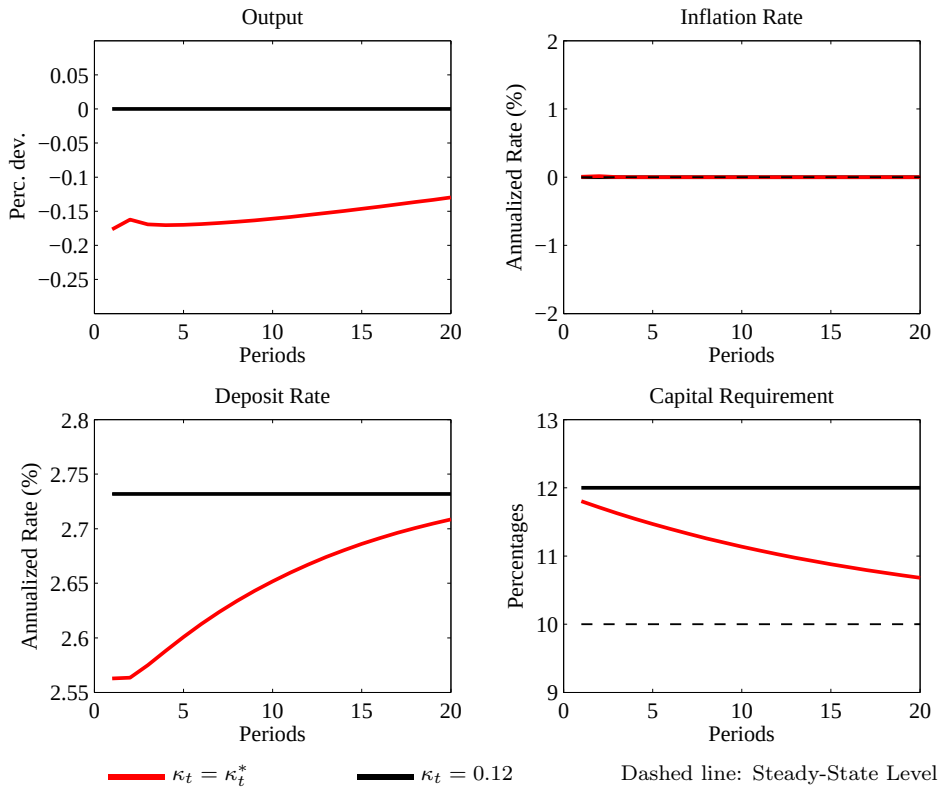


Figure 2: Response to an Increase in the Productivity of the Risky Technology (η_t^R)

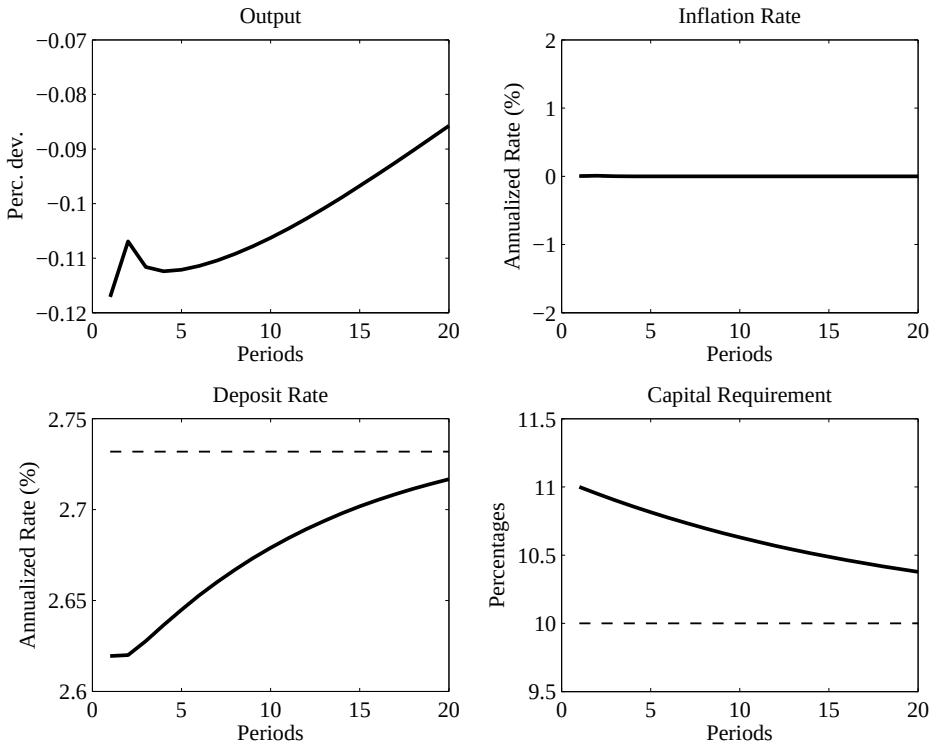


A positive shock to η_t^R is a pure temptation for banks and firms to deviate from the safe equilibrium; it increases the return on risky projects in case they succeed. Figure 2 shows that this shock increases the capital requirement under the optimal prudential policy ($\kappa_t = \kappa_t^*$). By itself, the tightening of capital requirements increases the cost of banking in our model. The optimal monetary-policy response is to cut the deposit rate in order to curb the increase in bank lending rates. The overall effects on output are small, and inflation is essentially zero under optimal policy.

We find this thought experiment quite useful in the context of policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Macklem (2011), Wolf (2012), Yellen (2010)] of how monetary and prudential policies may be substitutes for each other or move to offset each other's effects. In our case, one policy is contractionary and the other expansionary in order to manage risk-taking incentives with the smallest possible adverse effects on investment.

The same observations apply to optimal responses to shocks to the probability of failure of the risky technology (ϕ_t) and the maximal risky/safe loans ratio (γ_t). These shocks affect the economy only though their effect on the optimal capital requirement κ_t^*, which in turn calls for a monetary-policy response to mitigate the macroeconomic effects. Instead of presenting these responses, which are qualitatively the same as those of Figure 2, we present the effects of an exogenous tightening of the capital requirement. Figure 3 shows the responses to an increase in κ_t by one percent (from 0.10 to 0.11). The optimal monetary-policy response is to cut the annualized deposit rate by about 10 basis points. Again, the overall decrease in output is small (about 0.12% on impact) and inflation remains at zero under optimal policy.

Figure 3: Response to an Exogenous Tightening of the Capital Requirement (κ_t)



Dashed line: Steady-State Level

Figure 4: Response to an Increase in the Cost of Making Safe Loans (Ψ_t)

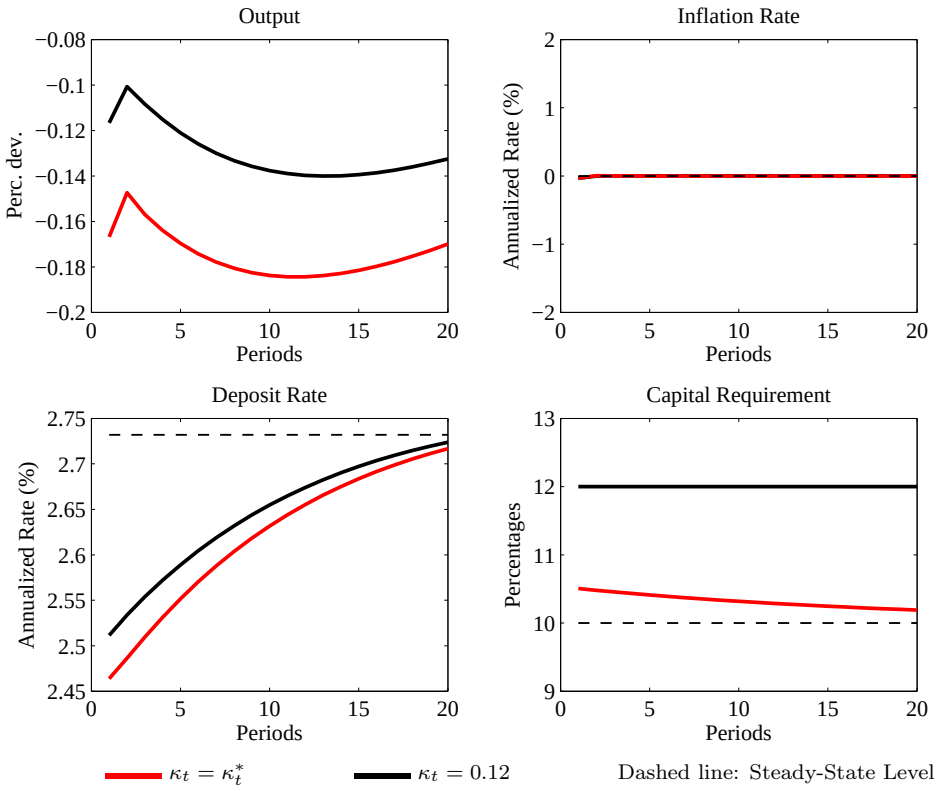


Figure 4 shows responses to a change in the marginal cost of making safe loans Ψ_t. In contrast to the other shocks, this shock has direct macroeconomic effects in addition to its effects on the risk-taking incentives of banks. Under a prudential policy keeping κ_t constant, this shock reduces output in our model, and monetary policy cuts the deposit rate to mitigate this effect. Under the optimal prudential policy ($\kappa_t = \kappa_t^*$), output falls more because, as we explained earlier, the increase in κ_t (needed to prevent risk taking) increases bank lending rates. Monetary policy reacts to the tighter capital requirements by cutting the deposit rate further.

How costly are policies that keep capital requirements constant (at a value high enough to ensure no risk is taken), relatively to the optimal policy? Since our analysis is mainly qualitative, the answer of our calibrated model to this question can, of course, be only suggestive, but we think it is nonetheless informative. To address this question, we compute the welfare cost from unexpectedly switching at date 0 from the optimal prudential policy ($\kappa_t = \kappa_t^*$) to a prudential policy setting κ_t to a constant value $\overline{\kappa}$ equal to either 0.12 or 0.14, assuming that monetary policy is conducted optimally at all dates. We measure this welfare cost by the value of foregone consumption, expressed as the fraction of consumption each period under optimal policy, that would lead to the same welfare loss as the change in policy does – i.e. by the parameter λ implicitly defined by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t U((1-\lambda)c_t^*, h_t^*) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t U(\overline{c}_t, \overline{h}_t)$$

where c_t^* and h_t^* denote consumptions and hours of work at date t under optimal policy, while $\overline{c}_t$ and $\overline{h}_t$ denote consumptions and hours of work at date t in the case where the unexpected policy switch occurs at date 0 ($\kappa_\tau = \kappa_\tau^*$ for $\tau < 0$ and $\kappa_\tau = \overline{\kappa}$ for $\tau \geq 0$). Following Benigno and Woodford (2006,

2012), the welfare computation is performed using a second order perturbation method.

As we are moving from one steady state to another (reached asymptotically), we have to take into account the cost of transition. The first column of Table 2 computes the welfare cost of transiting from one steady state to the other (including the cost of fluctuations due to non-linearities along the way). It is positive because moving from one steady state to the other reduces the capital stock, which is already inefficiently low due to the monopoly and tax distortions. The second column computes the welfare cost due to the difference in fluctuations around each steady state, ignoring the cost of transiting from one steady state to the other. This cost is negative (i.e., corresponds to a welfare gain) because fluctuations are smaller under constant capital requirements, as purely financial shocks (i.e., γ_t, η_t^R, ϕ_t) are not transmitted to the economy. The last column reports the welfare loss associated to both phenomena at the same time. It is positive because the transition cost dominates the fluctuations gain.

Table 2: Welfare costs (percentage points)

	Transition	Fluctuations	Total
$\overline{\kappa}$=0.12	2.2750	-0.9918	1.2606
$\overline{\kappa}$=0.14	3.6704	-0.9465	2.6892

To summarize our main point, our model highlights a distinction across policy instruments that we think deserves more emphasis than it gets in the existing literature: changes in the capital requirement can directly manage risk-taking incentives, while changes in the policy interest rate cannot. When the capital requirement rises to curb risk taking, a contraction ensues, and the policy interest rate is cut. With this chain of causality, optimal prudential policy is pro-cyclical, and optimal monetary policy is counter-cyclical.

Nonetheless, our model also provides a framework for thinking about some scenarios (or extensions) that can make optimal prudential policy counter-cyclical, as we discuss below.

6 Extensions and Policy Concerns

Our benchmark model, while stylized, provide several useful insights. For example, as Angeloni and Faia (2011) elaborate, the leading argument for Basel III-type counter-cyclical capital requirements is the observation that default risk rises during recessions; and risk-weighted (Basel II-type) capital requirements automatically tighten policy in recessions, unless the regulatory rate is lowered.[27] Our model suggests a reason for the latter to happen, that is, for cutting capital requirements when default risk is high: When the banks have enough skin in the game, the additional risk makes banks less inclined to fund risky projects, allowing prudential policy to set lower requirements without undermining the stability of the banking system.

In this subsection, we illustrate how (admittedly ad hoc) extensions can provide additional insights. We consider two extensions: externalities in bank lending, and correlation across shocks affecting the

[27] See Covas and Fujita (2010) for a quantitative assessment of the procyclical effects of bank capital requirements under Basel II.

incentives to take risks and shocks to the business cycle. We show that each of these two extensions can make both policy instruments countercyclical under optimal policy. We also show that, although the first extension gives rise to a risk-taking channel of monetary policy, it does not qualitatively affect the optimal policy responses to shocks that directly affect risk-taking incentives.

6.1 An externality

Our model assumes perfect competition and constant returns in the banking sector. As we noted earlier these assumptions imply that shocks that directly affect the optimal policy interest rate (like standard productivity or fiscal shocks) do not affect the optimal bank-capital requirement. We now consider a simple (ad-hoc) extension that links the cost of banking to the aggregate volume of safe loans and thus allows such shocks to affect both policy margins. Hachem (2010) develops a model with an externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks.[28] Here, we only consider a simple example of such an externality − in order to preserve our earlier derivations that treated Ψ_t as exogenous to the banks' decisions − but we think this example highlights the main features of policy interactions that arise when an economic boom increases risk-taking incentives. Specifically, we assume

$$\log(\Psi_t) = \log(\Psi) + \varrho \left[\log(l_t^S) - \log(l^S)\right] \tag{25}$$

where the term $\log(l_t^S) - \log(l^S)$ is the log-deviation of the aggregate volume of safe loans from its steady-state value, and $\varrho = 0$ corresponds to our benchmark model. We show the impulse responses for $\varrho = 0, 1$, and 5. Figure 5 illustrates the effects of a favorable productivity shock. Following this shock, optimal prudential policy raises the capital requirement in order to discourage risk taking. This makes optimal prudential policy countercyclical, which leads optimal monetary policy to be less restrictive (raises the deposit rate by less, and later on cuts it by more) than in the benchmark model.[29]

Figure 6 shows the optimal responses to an increase in the risk of failure of the risky technology. Absent the externality (looking at the dashed lines in the Figure), optimal prudential policy cuts the capital requirement because banks are naturally less tempted to take risk, while optimal monetary policy raises the deposit rate to curb the expansionary effects of prudential policy. With the externality, the expansion creates a temptation to take more risk (as the cost of making safe loans increases). So, optimal prudential policy cuts the capital requirement by less, and optimal monetary policy raises the deposit rate by less. Figure 7, which is the analogue to Figure 2, makes a similar point about responses to an increase in η_t^R: with the externality, optimal prudential policy increases the capital requirement by less, and optimal monetary policy cuts the deposit rate by less. In terms of optimal output fluctuations in Figures 5−7, the externality always dampens the optimal response (expansion or contraction) of output.

Thus, some key normative implications of the benchmark model are, qualitatively speaking, robust to the introduction of a risk-taking channel of monetary policy (via a lending externality). Optimal policy still uses capital requirements to counter risk-taking incentives, i.e. still raises (respectively

[28] Gete and Tiernan (2011) consider the role of capital requirements in Hachem's (2010) model, but abstract from monetary policy.

[29] Optimal monetary policy actually strikes a balance between this effect and another, smaller effect stemming from the externality (which is that banks have a tendency to lend too much as they ignore the effect of their own lending decision on monitoring costs).

Figure 5: Response to a Favorable Productivity Shock (η_t^f)

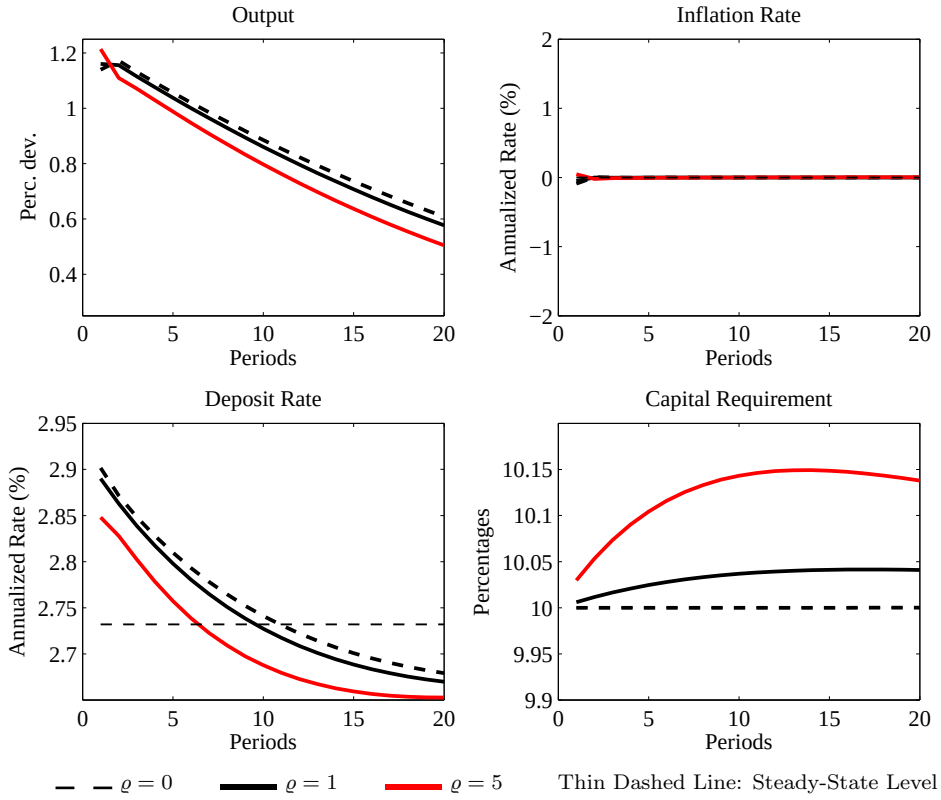


Figure 6: Response to an Increase in the Risk of Failure of the Risky Technology (ϕ_t)

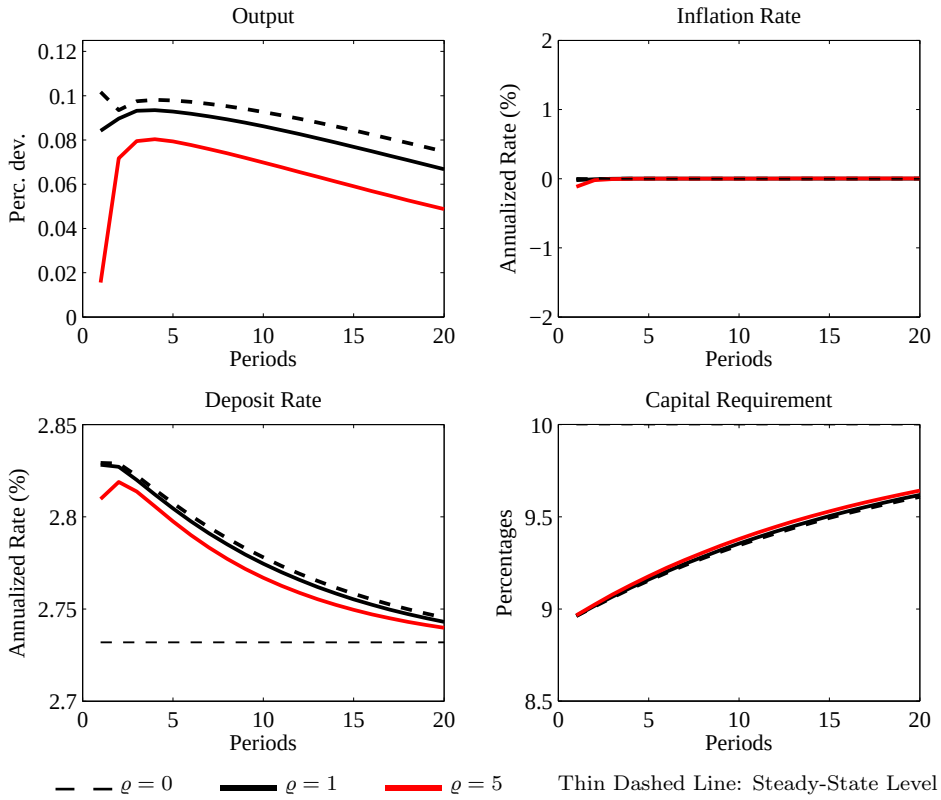
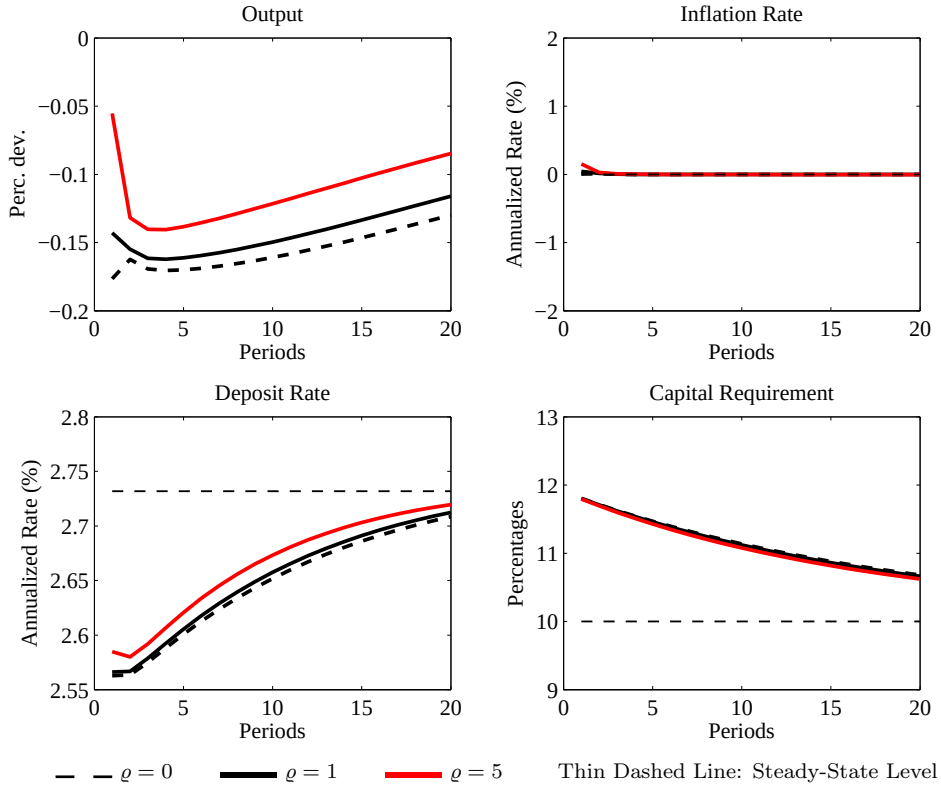


Figure 7: Response to an Increase in the Productivity of the Risky Technology (η_t^R)



cuts) capital requirements in response to shocks that increase (respectively decrease) these incentives. In principle, the deposit rate could have been used for this purpose, since the risk-taking channel of monetary policy implies that it now affects risk-taking incentives. But optimal policy does not use the deposit rate this way in response to shocks that *directly* affect risk-taking incentives (as in Figures 6−7). Instead, in response to these shocks, it still uses the deposit rate to mitigate the macroeconomic effects of capital requirements, i.e. still cuts (respectively raises) the deposit rate when capital requirements are raised (respectively cut). Moreover, as the strength of the risk-taking channel of monetary policy varies, optimal monetary policy becomes more accommodative (or less restrictive) when optimal prudential policy becomes more restrictive (or less accommodative) in response to any given shock.

6.2 Correlated shocks

Correlations across shocks may also link risk-taking incentives to shocks that have direct business-cycle effects and may make both optimal policies countercyclical. As an example, we replace (7) by

$$k_{t+1}^S = \exp\left(\eta_t^S\right) x_t^S,$$

thus adding a shock to the safe technology for producing capital goods, and we allow for the possibility that η_t^S is correlated with η_t^R (the shock to the risky technology). This modification changes our inefficiency condition (8) to

$$(1-\phi_t)\exp\left(\eta_t^R\right) \leq \exp\left(\eta_t^S\right) - \Psi_t,$$

our optimal capital requirement to

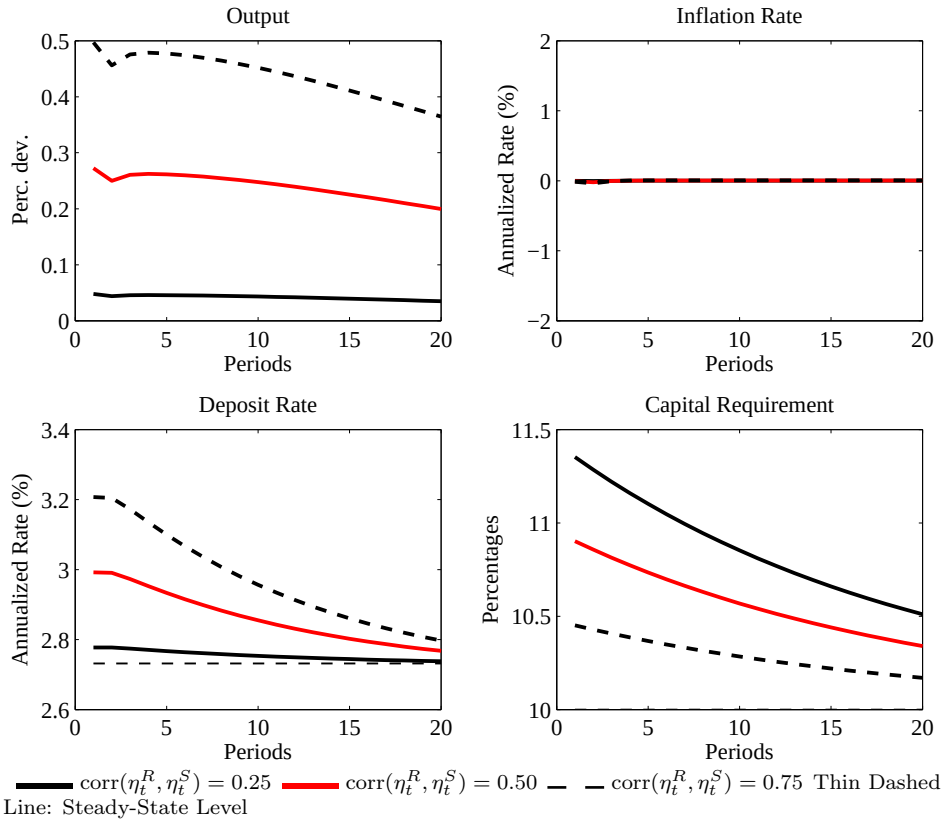
$$\kappa_t^* = (1-\tau)\,\frac{(1-\phi_t)\,\gamma_t\left[\exp\left(\eta_t^R-\eta_t^S\right)-1\right]+\Psi_t\left[(1-\phi_t)\,\gamma_t\exp\left(\eta_t^R-\eta_t^S\right)-\phi_t\right]}{\phi_t\,(1+\gamma_t)-\gamma_t\tau\,(1-\phi_t)\left[\exp\left(\eta_t^R-\eta_t^S\right)-1\right]},$$

and the optimality condition (10) to

$$E_t\left\{\lambda_{t+1}q_{t+1}\right\} = E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}\left(1+R_t^S\right)\exp\left(-\eta_t^S\right)q_t^x.$$

Figure 8 is the analogue of Figure 2; it shows the optimal responses to a positive innovation in η_t^R for three values of its correlation with the innovation to η_t^S: 0.25, 0.50, and 0.75. The correlation makes both optimal policies act in a counter-cyclical way. Optimal prudential policy raises the capital requirement to tame risk taking, and optimal monetary policy raises the deposit rate to tame the effects of the investment boom. The counter-cyclical tendency of the policies is stronger when the correlation across shocks is higher.

Figure 8: Response to an Increase in the Productivity of the Risky Technology (η_t^R)



7 Concluding Remarks

The optimal interaction of monetary with prudential policy is a key issue in policy design that has not been fully addressed in the literature. In this paper we derive jointly optimal policies using a model that views bank capital requirements as a tool for addressing the risk-taking incentives created by limited liability and deposit insurance.

Our benchmark model with perfectly competitive banks and constant marginal costs leads to a simple optimal assignment of tasks to prudential and monetary policies. The locally optimal mandate of prudential policy is to ensure that banks never fund inefficient risky projects, but to accomplish this objective with minimal damage in terms of increased bank lending rates and decreased capital stock. The distortion is minimized if capital requirements are state dependent. The interaction across policies then boils down to cutting (raising) interest rates to moderate the contractions (expansions) caused by changes in the capital requirement. The model also serves to illustrate how time variation in the capital requirement may be in response to shocks that affect the relative attractiveness of risky and safe projects.

The extension with an externality in the cost of banking, however, illustrates that optimal policy interactions may be more complex. In this example, an increase in the aggregate volume of safe loans increases the costs of originating and monitoring safe loans. This feature, which gives rise to a risk-taking channel of monetary policy, matters for the optimal policy interactions. In particular, it makes both policy instruments countercylical under optimal policy in response to certain shocks (like productivity shocks). However, it does not affect the main implications of the benchmark model for the optimal policy responses to shocks that directly affect risk-taking incentives: in responses to these shocks, prudential policy should still be used to tame risk-taking incentives (including those created by monetary policy when it is accommodative) and monetary policy to mitigate the macroeconomic effects of prudential policy.

Our model takes deposit insurance as an institutional feature that does not have to be rationalized within the model.[30] The other institutional feature is our assumption that a tax distortion makes equity finance more expensive than debt finance. We are not aware of any arguments for claiming that this is a feature of optimal policy in some expanded framework. To the contrary, existing discussions of this tax distortion [e.g., Admati et al. (2011), Mooij and Devereux (2011)] note its prevalence in OECD countries and call for removing it. Our motivation for including this policy-induced distortion in our model is this prevalence and the fact that central banks and prudential regulators cannot change the tax code.[31] We think this tax distortion merits more attention in models of how the banking sector matters for monetary-policy analysis.[32]

[30] Presenting an expanded model in which deposit insurance is optimal (rather than taking it as an exogenous feature) seemed too much of a digression to us, but we could motivate deposit insurance as usual [e.g., following Angeloni and Faia (2011)] in terms of ruling out equilibria with bank runs.

[31] Besides, under an arbitrarily small tax distortion, all our analytical results (from Proposition 1 to Proposition 6) still hold, as banks still prefer debt finance to equity finance, though the condition stated in Proposition 5 (the "if" part of this proposition) may not be met. In this case, our model is equivalent, at the first order, to a model in which the preference for debt finance would come from the presence of deposits in the utility function (as in Van den Heuvel, 2008) with an arbitrarily small weight.

[32] For one thing, this may account for the fact that banks extend credit using loan contracts in reality, even though loan contracts are not optimal according to most formal models (with the notable exception of models with costly state verification).

8 Appendix

8.1 Proof of Proposition 1

To show that there is no equilibrium with $0 < l_t^R < \gamma_t l_t^S$, we suppose that there is such an equilibrium and consider a perturbation satisfying $dl_t^S(j) = -dl_t^R(j)$ in the loan portfolio of a given bank j. Note that this perturbation neither tightens nor loosens bank j's balance-sheet identity

$$l_t^S(j) + l_t^R(j) = e_t(j) + d_t(j) \tag{26}$$

and its capital requirement

$$e_t(j) \geq \kappa_t \left[l_t^S(j) + l_t^R(j) \right],$$

given that $l_t^S(j) + l_t^R(j)$ is left unchanged. So this perturbation should not increase bank j's expected excess return. The derivations of the effect of this perturbation on bank j's expected excess return involves two cases, depending on whether firms' default leads to bank j's default.

If firms' default leads to bank j's default, then the change in bank j's expected excess return is

$$(1-\tau)\left[\beta(1-\phi_t) E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 1\right\}\frac{R_t^R - R_t^S}{\lambda_t} + \Psi_t\right] dl_t^R(j),$$

since bank j ignores the effect of its loan portfolio change on aggregate variables like λ_{t+1} or Π_{t+1}. As discussed in the main text, we must have $R_t^R \geq R_t^S$ in equilibrium. Therefore, bank j's expected excess return is increasing in $l_t^R(j)$. This means that bank j would like to take more risk, contradicting our conjecture about the existence of an equilibrium with $l_t^R < \gamma_t l_t^S$. This proves Part (a) of the Proposition.

If firms' default does not lead to bank j's default, then the change in bank j's expected excess return is

$$(1-\tau)\left[\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}}\left[\theta_t\left(1+R_t^R\right) - \left(1+R_t^S\right)\right]\right\} + \Psi_t\right] dl_t^R(j) \equiv M dl_t^R(j).$$

Now,

$$\begin{aligned}
\frac{M}{1-\tau} &= \beta(1-\phi_t)\frac{\left(1+R_t^R\right) - \left(1+R_t^S\right)}{\lambda_t} E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 1\right\} \\
&\quad -\beta\phi_t \frac{1+R_t^S}{\lambda_t} E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 0\right\} + \Psi_t \\
&= \beta(1-\phi_t)\frac{1+R_t^S}{\lambda_t}\left(\frac{1+R_t^R}{1+R_t^S} - 1\right) E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 1\right\} \\
&\quad -\beta\phi_t \frac{1+R_t^S}{\lambda_t} E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 0\right\} + \Psi_t \\
&\leq \beta(1-\phi_t)\frac{1+R_t^S}{\lambda_t}\left[\frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\theta_t = 1\right\}}\exp\left(\eta_t^R\right) - 1\right] E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\theta_t = 1\right\} \\
&\quad -\beta\phi_t \frac{1+R_t^S}{\lambda_t} E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t = 0\right\} + \Psi_l
\end{aligned}$$

where the last inequality comes from (12) and (18).

Therefore,

$$\begin{aligned}\frac{M}{1-\tau} &\leq \beta(1-\phi_t)\frac{1+R_t^S}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}\exp\left(\eta_t^R\right)-\beta(1-\phi_t)\frac{1+R_t^S}{\lambda_t}E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t=1\right\}\\ &\quad -\beta\phi_t\frac{1+R_t^S}{\lambda_t}E_t\left\{\left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right)\middle|\theta_t=0\right\}+\Psi_t,\end{aligned}$$

which implies

$$\frac{M}{1-\tau}\leq\beta(1-\phi_t)\frac{1+R_t^S}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}\exp\left(\eta_t^R\right)-\beta\frac{1+R_t^S}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}+\Psi_l$$

and

$$\frac{M}{1-\tau}\leq\beta\frac{1+R_t^S}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}\left[(1-\phi_t)\exp\left(\eta_t^R\right)-1\right]+\Psi_l.$$

Using (3), we get

$$\frac{M}{1-\tau}\leq\frac{1+R_t^S}{1+R_t^D}\left[(1-\phi_t)\exp\left(\eta_t^R\right)-1\right]+\Psi_l.$$

and, using (8),

$$\frac{M}{1-\tau}\leq\Psi_t\left(1-\frac{1+R_t^S}{1+R_t^D}\right),$$

which implies $M<0$ because monitoring costs make $R_t^S>R_t^D$ in equilibrium. Therefore, bank j's expected excess return is decreasing in $l_t^R(j)$. This means that bank j would like to take less risk, contradicting our conjecture about the existence of an equilibrium with $0<l_t^R$. This proves Part (b) of the Proposition.

8.2 Proof of Proposition 2

This appendix proves Proposition 2 by establishing a more general result that will serve us in subsequent appendices. We show that the capital constraint is always binding for a bank j that may deviate from either the candidate equilibrium with $l_t^R=0$, or the candidate equilibrium with $l_t^R=\gamma_t l_t^S$ (the only two candidate equilibria left, given Proposition 1). As a consequence, for a zero deviation, the capital constraint is binding –i.e. (19) holds– in any of the two candidate equilibria of our model, which leads to Proposition 2.

In general, using (26), bank j's expected excess return can be written

$$(1-\tau)E_t\left\{\beta\frac{\lambda_{t+1}\omega_{t+1}^b(j)}{\lambda_t}\right\}-e_t(j)-(1-\tau)\Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j)=\max\left\{0,\frac{R_t^S-R_t^D}{\Pi_{t+1}}l_t^S(j)+\left[\theta_t\frac{1+R_t^R}{\Pi_{t+1}}-\frac{1+R_t^D}{\Pi_{t+1}}\right]l_t^R(j)+\frac{1+R_t^D}{\Pi_{t+1}}e_t(j)\right\}.$$

In the case where $\omega_{t+1}^b(j)>0$ when $\theta_t=0$, using (3), bank j's expected excess return can be rewritten

$$(1-\tau)\left\{\frac{R_t^S-R_t^D}{1+R_t^D}l_t^S(j)+\left[(1-\phi_t)\left(\frac{1+R_t^R}{1+R_t^D}\right)-1\right]l_t^R(j)+e_t(j)\right\}-e_t(j)-(1-\tau)\Psi_t l_t^S(j).$$

Since this expression is strictly decreasing in $e_t(j)$, it is maximized when $e_t(j)$ is minimal, that is to say when $e_t(j)$ satisfies

$$e_t(j)=\kappa_t\left[l_t^S(j)+l_t^R(j)\right]. \tag{27}$$

In the alternative case where $\omega_{t+1}^{b}(j)=0$ when $\theta_t=0$, consider first the candidate equilibrium with $l_t^R=0$. Bank j's expected excess return can then be written

$$(1-\tau)(1-\phi_t)\left\{\frac{R_t^S-R_t^D}{1+R_t^D}l_t^S(j)+\left[\frac{1+R_t^R}{1+R_t^D}-1\right]l_t^R(j)+e_t(j)\right\}-e_t(j)-(1-\tau)\Psi_t l_t^S(j).$$

Since this expression is strictly decreasing in $e_t(j)$, it is maximized for $e_t(j)$ given by (27). Consider next the candidate equilibrium with $l_t^R=\gamma_t l_t^S$. Bank j's expected excess return can then be written

$$(1-\tau)(1-\phi_t)\frac{\beta}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\theta_t=1\right\}\left[\left(R_t^S-R_t^D\right)l_t^S(j)+\left(R_t^R-R_t^D\right)l_t^R(j)\right.$$
$$\left.+\left(1+R_t^D\right)e_t(j)\right]-e_t(j)-(1-\tau)\Psi_t l_t^S(j).$$

This expression is strictly decreasing in $e_t(j)$, since its derivative with respect to $e_t(j)$ is strictly negative:

$$\begin{aligned}
&(1-\tau)(1-\phi_t)\frac{\beta}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\theta_t=1\right\}\left(1+R_t^D\right)-1\\
=\;&(1-\tau)(1-\phi_t)\frac{E_t\left\{\lambda_{t+1}q_{t+1}|\,\theta_t=1\right\}}{E_t\left\{\lambda_{t+1}q_{t+1}\right\}}\exp\left(\eta_t^R\right)\frac{1+R_t^S}{1+R_t^R}-1\\
<\;&(1-\tau)(1-\phi_t)\exp\left(\eta_t^R\right)\frac{1+R_t^S}{1+R_t^R}-1\\
<\;&(1-\tau)(1-\Psi_t)\frac{1+R_t^S}{1+R_t^R}-1\\
<\;&0,
\end{aligned}$$

where the equality comes from (12) and (3), the first inequality from (18), the second inequality from (8), and the third inequality from the fact that $R_t^R>R_t^S$ in equilibrium. Therefore, bank j will choose the minimal capital requirement, i.e. $e_t(j)$ satisfying (27). To sum up, the capital constraint is always binding for a bank j that may deviate from either the candidate equilibrium with $l_t^R=0$, or the candidate equilibrium with $l_t^R=\gamma_t l_t^S$. In particular, for a zero deviation, the capital constraint is binding –i.e. (19) holds– in any of the two candidate equilibria of our model. This establishes Proposition 2.

8.3 Proof of Proposition 3

Consider a bank j that takes the maximum amount of risk by setting $l_t^R(j)=\gamma_t l_t^S(j)$. Using (26) and (27) to eliminate $d_t(j)$ from

$$\omega_{t+1}^{b}(j)=\max\left\{0,\frac{1+R_t^S}{\Pi_{t+1}}l_t^S(j)+\theta_t\frac{1+R_t^R}{\Pi_{t+1}}l_t^R(j)-\frac{1+R_t^D}{\Pi_{t+1}}d_t(j)\right\},$$

it is straightforward to show that this bank remains solvent ($\omega_{t+1}^{b}(j)>0$) when risky projects fail ($\theta_t=0$) if and only if (21) holds. Part (a) of Proposition 3 follows.

Then, consider a candidate equilibrium with $l_t^R=0$. Using (13) to eliminate d_t and (19) to eliminate e_t, the representative bank's expected excess return can be rewritten

$$(1-\tau)E_t\left\{\beta\frac{\lambda_{t+1}\omega_{t+1}^{b}}{\lambda_t}\right\}-\left[\kappa_t+(1-\tau)\Psi_t\right]l_t^S,$$

where

$$\omega_{t+1}^{b}=\left[\frac{R_t^S-R_t^D}{\Pi_{t+1}}+\frac{1+R_t^D}{\Pi_{t+1}}\kappa_t\right]l_t^S.$$

The representative bank chooses l_t^S so as to maximize its expected excess return. Using (3), the first-order condition of this programme can be written

$$(1-\tau)\frac{R_t^S - R_t^D}{1+R_t^D} - \tau\kappa_t - (1-\tau)\Psi_t = 0. \tag{28}$$

We can then use this first-order condition to rewrite $\widetilde{\kappa}\left(R_t^D, R_t^S\right)$, at the candidate equilibrium with $l_t^R = 0$, as (20). Parts (b) and (c) of Proposition 3 follow.

8.4 Proof of Proposition 4

To prove Part (a) of Proposition 4, we look for a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$. This amounts to looking for a necessary and sufficient condition on policy instruments for the demand and supply curves on the risky-loans market to intersect at one or several points $\left(R_t^R, l_t^R\right)$ with $R_t^R \geq 0$ and $l_t^R = 0$. We proceed in several steps.

Step 1: condition for zero demand for risky loans. Given capital producers' programme, the portion of the demand curve that is consistent with $l_t^R = 0$ is characterized by

$$\frac{1+R_t^R}{1+R_t^S} \geq \frac{E_t\left\{\lambda_{t+1}q_{t+1}|\,\theta_t = 1\right\}}{E_t\left\{\lambda_{t+1}q_{t+1}\right\}} \frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 1\right\}} \exp\left(\eta_t^R\right).$$

Because θ_t is independent of any other shock and because the realization of θ_t does not affect the aggregate outcome when $l_t^R = 0$, the latter inequality can be rewritten

$$\frac{1+R_t^R}{1+R_t^S} \geq \exp\left(\eta_t^R\right). \tag{29}$$

Step 2: condition for zero supply of risky loans. The portion of the supply curve that is consistent with $l_t^R = 0$ can be characterized by a necessary and sufficient condition for an individual bank j not to deviate from the candidate equilibrium with $l_t^R = 0$. We now look for such a condition. Appendix 8.1 implies that, if some deviations are profitable, then the most profitable deviation is $l_t^R(j) = \gamma_t l_t^S(j)$. If bank j makes this deviation, then, using (26) to eliminate $d_t(j)$ and (27) to eliminate $e_t(j)$, its expected excess return can be rewritten

$$(1-\tau)E_t\left\{\beta\frac{\lambda_{t+1}\omega_{t+1}^b(j)}{\lambda_t}\right\} - \left[\kappa_t(1+\gamma_t) + (1-\tau)\Psi_t\right] l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max\left\{0, \left[\frac{R_t^S - R_t^D}{\Pi_{t+1}} + \theta_t\gamma_t\frac{1+R_t^R}{\Pi_{t+1}} - \gamma_t\frac{1+R_t^D}{\Pi_{t+1}} + \frac{1+R_t^D}{\Pi_{t+1}}\kappa_t(1+\gamma_t)\right] l_t^S(j)\right\}.$$

Because θ_t is independent of any other shock and because the realization of θ_t does not affect the aggregate outcome in equilibrium (given that $l_t^R = 0$), bank j's expected excess return can be rewritten, using (3),

$$(1-\tau)E_t\left\{\max\left\{0, \left[\frac{R_t^S - R_t^D}{1+R_t^D} + \theta_t\gamma_t\frac{1+R_t^R}{1+R_t^D} - \gamma_t + \kappa_t(1+\gamma_t)\right] l_t^S(j)\right\}\right\}$$
$$- \left[\kappa_t(1+\gamma_t) + (1-\tau)\Psi_t\right] l_t^S(j).$$

Note that the 'max' that features in this expression is strictly higher than zero when $\theta_t = 1$, because both R_t^R and R_t^S are strictly higher than R_t^D in equilibrium. So we will have to consider two cases, depending on whether this 'max' is strictly higher than zero or equal to zero when $\theta_t = 0$.

In the case where this 'max' is strictly higher than zero when $\theta_t = 0$, that is to say in the case where $\kappa_t > \widetilde{\kappa}_t$, we know from Proposition 1 that bank j's deviation is not profitable.

In the alternative case where the 'max' is equal to zero when $\theta_t = 0$, that is to say in the case where $\kappa_t \leq \widetilde{\kappa}_t$, bank j's expected excess return is

$$\left\{(1-\tau)(1-\phi_t)\left[\frac{R_t^S - R_t^D}{1+R_t^D} + \gamma_t \frac{1+R_t^R}{1+R_t^D} - \gamma_t + \kappa_t(1+\gamma_t)\right] - \kappa_t(1+\gamma_t) - (1-\tau)\Psi_t\right\} l_t^S(j).$$

Using (28) to eliminate R_t^S, we can then rewrite bank j's expected excess return as

$$\left\{(1-\tau)(1-\phi_t)\gamma_t \frac{R_t^R - R_t^D}{1+R_t^D} - [\phi_t(1+\gamma_t) + \gamma_t\tau(1-\phi_t)]\kappa_t - \phi_t(1-\tau)\Psi_t\right\} l_t^S(j).$$

Therefore, a necessary and sufficient condition for the deviation not to be profitable is then

$$[\phi_t(1+\gamma_t) + \gamma_t\tau(1-\phi_t)]\kappa_t + \phi_t(1-\tau)\Psi_t \geq (1-\tau)(1-\phi_t)\gamma_t \frac{R_t^R - R_t^D}{1+R_t^D}. \tag{30}$$

To sum up, the portion of the supply curve that is consistent with $l_t^R = 0$ is characterized by the condition that either $\kappa_t > \widetilde{\kappa}_t$, or $\kappa_t \leq \widetilde{\kappa}_t$ and (30) holds.

Step 3: condition for zero risky loans in equilibrium. The demand and supply curves on the risky-loans market intersect at one or several points $\left(R_t^R, l_t^R\right)$ with $R_t^R \geq 0$ and $l_t^R = 0$ if and only if either (i) $\kappa_t > \widetilde{\kappa}_t$, or (ii) $\kappa_t \leq \widetilde{\kappa}_t$, and (30) holds when (29) holds with equality.

Note that, if (29) holds with equality, then, using (28), we can rewrite (30) as

$$\kappa_t \geq \kappa_t^* \equiv (1-\tau)\frac{(1-\phi_t)\gamma_t\left[\exp\left(\eta_t^R\right)-1\right] + \Psi_t\left[(1-\phi_t)\gamma_t\exp\left(\eta_t^R\right) - \phi_t\right]}{\phi_t(1+\gamma_t) - \gamma_t\tau(1-\phi_t)\left[\exp\left(\eta_t^R\right)-1\right]}, \tag{31}$$

since the denominator on the right-hand side of this inequality is strictly positive:

$$\begin{aligned}
& \phi_t(1+\gamma_t) - \gamma_t\tau(1-\phi_t)\left[\exp\left(\eta_t^R\right)-1\right] \\
= \; & \phi_t[1+\gamma_t(1-\tau)] + \gamma_t\tau - \gamma_t\tau(1-\phi_t)\exp\left(\eta_t^R\right) \\
> \; & \phi_t[1+\gamma_t(1-\tau)] + \gamma_t\tau\Psi_t \\
> \; & 0,
\end{aligned}$$

where the last but one inequality comes from (8). As a consequence, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$ is that either $\kappa_t > \widetilde{\kappa}_t$, or $\kappa_t^* \leq \kappa_t \leq \widetilde{\kappa}_t$. This condition can be equivalently rewritten $\kappa_t \geq \min\{\widetilde{\kappa}_t, \kappa_t^*\}$. Now, using (8) to replace $(1-\phi_t)\exp\left(\eta_t^R\right)$ by $1-\Psi_t$ on the right-hand side of (31), we get

$$\begin{aligned}
\kappa_t^* \; & \leq \; (1-\tau)\frac{-\gamma_t\Psi_t^2 + \phi_t(\gamma_t - \Psi_t)}{\gamma_t\tau\Psi_t + \phi_t(1+\gamma_t-\gamma_t\tau)} \\
& = \; \widetilde{\kappa}_t\left\{1 - \frac{\gamma_t\Psi_t}{\gamma_t - \Psi_t}\frac{\gamma_t\tau + \Psi_t(1+\gamma_t)(1-\tau)}{\gamma_t\Psi_t\tau + \phi_t(1+\gamma_t-\gamma_t\tau)}\right\} \\
& < \; \widetilde{\kappa}_t,
\end{aligned}$$

where the last inequality comes from our assumption that $\gamma_t > \Psi_t$. Therefore, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$ is simply $\kappa_t \geq \kappa_t^*$. Parts (a) and (b) of Proposition 4 follow.

Finally, Part (c) of Proposition 4 follows straightforwardly from the fact that the denominator on the right-hand side of (31) is strictly positive, as shown above.

8.5 Proof of Proposition 5

Define welfare as the representative household's expected utility at date 0, $E_0 \sum_{t=0}^{\infty} \beta^t \, U(c_t, h_t)$. For any policy $\left(R_\tau^D, \kappa_\tau\right)_{\tau \geq 0}$, define the distance from $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ as

$$\varepsilon \equiv \max \left[\max_{\tau \geq 0} \left(\left| R_\tau^D - R_\tau^{D*} \right| \right), \max_{\tau \geq 0} \left(\left| \kappa_\tau - \kappa_\tau^* \right| \right) \right].$$

Let us first compare $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ to policies $\left(R_\tau^D, \kappa_\tau\right)_{\tau \geq 0}$ such that ε is arbitrarily small and $\exists t \geq 0$, $\kappa_t < \kappa_t^*$. Moving from $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ to any such policy triggers a discontinuous increase in the amount of risk, as it makes banks' risky loans l_t^R move from 0 to $\gamma_t l_t^S > 0$ at some date $t \geq 0$. Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that ε is small enough. As a consequence, welfare is strictly higher under $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ than under any such policy provided that ε is small enough.

Let us then compare $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ to policies $\left(R_\tau^D, \kappa_\tau\right)_{\tau \geq 0}$ such that ε is arbitrarily small, $\forall \tau \geq 0$, $\kappa_\tau \geq \kappa_\tau^*$, and $\exists t \geq 0$, $\kappa_t > \kappa_t^*$. Using the equilibrium conditions that are independent of policies, rewrite welfare as

$$W \left[\left(R_\tau^D\right)_{\tau \geq 0}, \left(\kappa_\tau\right)_{\tau \geq 0}, H_0 \right],$$

where H_0 captures initial conditions (endogenous variables until date -1, exogenous shocks until date 0). Since $\left(R_\tau^{D*}\right)_{\tau \geq 0}$ is the monetary policy that is Ramsey-optimal when $\left(\kappa_\tau\right)_{\tau \geq 0} = \left(\kappa_\tau^*\right)_{\tau \geq 0}$, we have

$$\forall t \geq 0, \; \frac{\partial W}{\partial R_t^D} \left[\left(R_\tau^{D*}\right)_{\tau \geq 0}, \left(\kappa_\tau^*\right)_{\tau \geq 0}, H_0 \right] = 0.$$

Therefore, the first-order Taylor approximation of $W \left[\left(R_\tau^D\right)_{\tau \geq 0}, \left(\kappa_\tau\right)_{\tau \geq 0}, H_0 \right]$ in a neighborhood of $\left[\left(R_\tau^{D*}\right)_{\tau \geq 0}, \left(\kappa_\tau^*\right)_{\tau \geq 0}, H_0 \right]$ such that $\forall \tau \geq 0$, $\kappa_\tau \geq \kappa_\tau^*$, is

$$W \left[\left(R_\tau^D\right)_{\tau \geq 0}, \left(\kappa_\tau\right)_{\tau \geq 0}, H_0 \right] = W \left[\left(R_\tau^{D*}\right)_{\tau \geq 0}, \left(\kappa_\tau^*\right)_{\tau \geq 0}, H_0 \right] + \sum_{t=0}^{+\infty} \frac{\partial W}{\partial \kappa_t} \left[\left(R_\tau^{D*}\right)_{\tau \geq 0}, \left(\kappa_\tau^*\right)_{\tau \geq 0}, H_0 \right] \left(\kappa_t - \kappa_t^*\right) + O\left(\varepsilon^2\right),$$

where $\frac{\partial W}{\partial \kappa_t}$ is the right derivative of welfare with respect to κ_t and $O\left(\varepsilon^2\right)$ is a term of second order in ε. As a consequence, if

$$\forall t \geq 0, \; \frac{\partial W}{\partial \kappa_t} \left[\left(R_\tau^{D*}\right)_{\tau \geq 0}, \left(\kappa_\tau^*\right)_{\tau \geq 0}, H_0 \right] < 0,$$

then welfare is strictly higher under $\left(R_\tau^{D*}, \kappa_\tau^*\right)_{\tau \geq 0}$ than under any policy $\left(R_\tau^D, \kappa_\tau\right)_{\tau \geq 0}$ such that $\forall \tau \geq 0$, $\kappa_\tau \geq \kappa_\tau^*$ and $\exists t \geq 0$, $\kappa_t > \kappa_t^*$, provided that ε is small enough. Proposition 5 follows.

8.6 Proof of Proposition 6

Using (28) and (29), it is easy to show that, at any candidate equilibrium with $l_t^R = 0$, the prudential-policy rule (24) implies (i) $\kappa_t \geq \kappa_t^*$ and (ii) $\kappa_t = \kappa_t^*$ if and only if (29) holds with equality. Therefore, given Proposition 4, there exists a unique equilibrium with $l_t^R = 0$ under (24) and, at this equilibrium, $\kappa_t = \kappa_t^*$ and (29) holds with equality.

We now show that there exists no equilibrium with $l_t^R = \gamma_t l_t^S$ under (24). To that aim, consider a candidate equilibrium with $l_t^R = \gamma_t l_t^S$. Proposition 1 implies that, if $\omega_{t+1}^b > 0$ when $\theta_t = 0$, then this candidate equilibrium is not an equilibrium. We focus therefore on the case where $\omega_{t+1}^b = 0$ when $\theta_t = 0$. Consider a given bank j, whose expected excess return is

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1-\tau) \omega_{t+1}^b (j)}{\lambda_t} \right\} - e_t (j) - (1-\tau) \Psi_t l_t^S (j),$$

where

$$\omega_{t+1}^b (j) = \max \left\{ 0, \frac{1+R_t^S}{\Pi_{t+1}} l_t^S (j) + \theta_t \frac{1+R_t^R}{\Pi_{t+1}} l_t^R (j) - \frac{1+R_t^D}{\Pi_{t+1}} d_t (j) \right\}.$$

Using (26) to eliminate $d_t (j)$ and (27) to eliminate $e_t (j)$, its expected excess return can be rewritten

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1-\tau) \omega_{t+1}^b (j)}{\lambda_t} \right\} - \kappa_t \left[l_t^S (j) + l_t^R (j) \right] - (1-\tau) \Psi_t l_t^S (j),$$

where

$$\omega_{t+1}^b (j) = \max \left\{ 0, \frac{1+R_t^S}{\Pi_{t+1}} l_t^S (j) + \theta_t \frac{1+R_t^R}{\Pi_{t+1}} l_t^R (j) - \frac{1+R_t^D}{\Pi_{t+1}} (1-\kappa_t) \left[l_t^S (j) + l_t^R (j) \right] \right\}.$$

If bank j does not deviate from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$, then its expected excess return is equal to

$$\left[(1-\phi_t) \beta \frac{(1-\tau) \left[\left(1+R_t^S\right) + \gamma_t \left(1+R_t^R\right) - (1+\gamma_t)(1-\kappa_t) \left(1+R_t^D\right) \right]}{\lambda_t} \right.$$
$$\left. E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t (1+\gamma_t) - (1-\tau) \Psi_t \right] \frac{1}{1+\gamma_t} l_t (j),$$

where $l_t (j) \equiv l_t^S (j) + l_t^R (j)$, since $\omega_{t+1}^b (j) = 0$ when $\theta_t = 0$. Appendix 8.1 implies that, if some deviations from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$ are profitable, then the most profitable deviation is to provide zero risky loans. If bank j makes this deviation, then its expected excess return becomes

$$\left[\phi_t \beta \frac{(1-\tau) \left[\left(R_t^S - R_t^D\right) + \kappa_t \left(1+R_t^D\right) \right]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right.$$
$$\left. + (1-\phi_t) \beta \frac{(1-\tau) \left[\left(R_t^S - R_t^D\right) + \kappa_t \left(1+R_t^D\right) \right]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t - (1-\tau) \Psi_t \right] l_t (j).$$

The change in bank j's expected excess return, from $l_t^R (j) = \gamma_t l_t^S (j)$ to $l_t^R (j) = 0$, is

$$\left[\phi_t \beta \frac{(1-\tau) \left[\left(R_t^S - R_t^D\right) + \kappa_t \left(1+R_t^D\right) \right]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right.$$
$$\left. - (1-\phi_t) \beta \frac{(1-\tau)}{\lambda_t} \frac{\gamma_t}{1+\gamma_t} \left(R_t^R - R_t^S\right) E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - (1-\tau) \frac{\gamma_t}{1+\gamma_t} \Psi_t \right] l_t (j).$$

It is easy to show that this change is strictly positive, and therefore that bank j gains from deviating from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$, if and only if

$$\begin{aligned}\kappa_t &> -\frac{R_t^S - R_t^D}{1+R_t^D} + \frac{\gamma_t}{1+\gamma_t}\frac{(1-\phi_t)\,\beta\frac{R_t^R - R_t^S}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 1\right\} + \Psi_t}{\phi_t\beta\frac{\left(1+R_t^D\right)}{\lambda_t}E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 0\right\}} \\ &\equiv \widehat{\kappa}\left(R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t\right).\end{aligned}$$

Therefore, there exists no equilibrium with $l_t^R = \gamma_t l_t^S$ under the prudential-policy rule (24) if

$$\kappa^*\left(R_t^D, R_t^S, R_t^R, \phi_t\right) > \widehat{\kappa}\left(R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t\right),$$

where $\kappa^*\left(R_t^D, R_t^S, R_t^R, \phi_t\right)$ is the expression on the right-hand side of (24). Using (3), the latter inequality is easily shown to be equivalent to

$$\frac{1-\phi_t}{\phi_t^2}\frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 0\right\}}\left[1 - \frac{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\middle|\,\theta_t = 1\right\}}{E_t\left\{\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}}\right]\left(\frac{R_t^R - R_t^S}{1+R_t^D} + \Psi_t\right) > 0$$

and is therefore satisfied, given (23) and $R_t^R \geq R_t^S$. This establishes Proposition 6.

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