The Great Moderation and the Great Leverage: Collateralisation bubbles when investors disagree about risk

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Motivating Facts

1. **Increase in Leverage**
   - of US Households since 1990s
   - of US Commercial Banks since 2000
   - of European Banks since late 1990s

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    - Stock prices remain elevated even after .com-boom-bust
    - Boom in (US) house prices through 2005
    - Wide-spread boom in collateralised bonds

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   - Stock prices remain elevated even after .com-boom-bust
   - Boom in (US) house prices through 2005
   - Wide-spread boom in collateralised bonds
4. **Change in (perceptions of) aggregate risk**
   - Great Moderation: Fall in Macro-Volatility since mid-1980s
   - Heterogeneous beliefs about GM: No consensus about origin (and thus persistence) of Great Moderation among academics or investors

Collateralisation and heterogenous risk perception

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- Outright asset purchase: $\pi(s) = s$

  $\Rightarrow$ *Linear payoffs, symmetric weighting of up / downside risk, expectation insensitive to risk perception*

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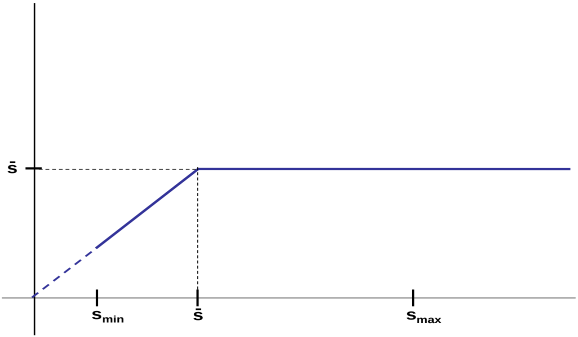
- Claim $x(s)$ collateralised by s pays: $\pi(s) = min\{s, x(s)\}$

  $\Rightarrow$ *Profits non-linear in s, expectation typically sensitive to risk perception*

Payoffs from collateralised loans

Payoffs from collateralised loans



- CL of face value $\bar{s}$ pays: $\pi_{col}(s) = min\{s, \bar{s}\}$
- Expected payoff falls with Var_s

Payoffs from leveraged asset purchase

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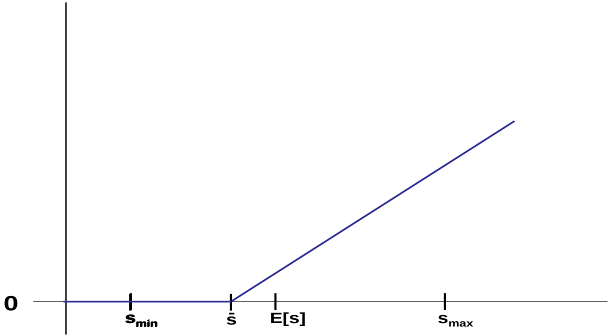

- Asset used as collateral for loan of face value $\bar{s}$:

  $\pi_{lev}(s) = max\{0, s - \bar{s}\}$
- Expected payoff rises with Var_s

This paper

- Analyses equilibrium collateralised asset trade and prices when investors disagree about second, rather than first moments

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- Analyses equilibrium collateralised asset trade and prices when investors disagree about second, rather than first moments
- Non-linearity of profits from collateralisation crucial when investors disagree about risk:
  - J's inequality effects imply perceived gains from trade
  - Can trade downside vs upside risk (Coll Loans, CDOs)
  - More generally allows trading of any contingent asset (CDO^2)

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5. Synthetic CDOs (CDO^2) at least double asset bubble
6. Dynamic model w temporary belief disagreements: price rise of 5 to 30 %

Literature I: Asset Prices w heterogeneous beliefs

- Miller (77): Without short-selling, heterogeneity in expected asset payoffs increases prices above the average valuation
- Harrison and Kreps (78): "Speculation value" of selling to optimists next period increases asset price even when no optimists invest today
- Geanakoplos (01): Possibility to issue loans to buy assets makes marginal buyer more optimistic
- Geanakoplos (12): CDO trade may increase prices further, but CDS reduce prices
- Simsek (13):
  - Leverage disciplines "downside optimism" about low payoffs, amplifies "upside optimism"
  - CDO trade modelled as cash-backed AD securities, increase supply of assets, may increase or decrease asset price

Literature II: Great Moderation and asset prices

- Lettau, Ludvigson and Wachter (2008): GM increases asset prices only when expected to be (essentially) permanent
- Broer and Kero (2013): Learning about persistence of GM can increase asset prices strongly, and above full-information level

This paper:

- Simple ad-hoc learning mechanism with heterogeneous prior tightness leads to temporary disagreement in beliefs, modest rise in asset prices without risk aversion

Comparison to Disagreement about Mean Payoffs (Geanakoplos, Simsek)

Disagreement about mean payoffs

- raises prices w/o collateralisation
- collateralisation may amplify the effect of disagreement by increasing optimists' resources
- increase in disagreement can raise or lower price

Disagreement about risk at equal mean payoffs

- has no effect without collateralisation
- always increases asset price with collateralisation, up to twice fundamental value
- increase in disagreement always raises prices of BOTH collateral asset and debt

Outline

1. Disagreement about GDP growth: Evidence from professional forecasts
2. Paper: Leverage and heterogeneous risk perceptions in a 2 Period Continuuum Economy
3. A 2 Type Environment
4. Collateralised loans with endogenous face value
5. CDO trade
6. Dynamic Example: Heterogeneous 'speed of learning' the GM

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     $\mu_{it} = \widehat{\mu_{it}} + \mu_t,$
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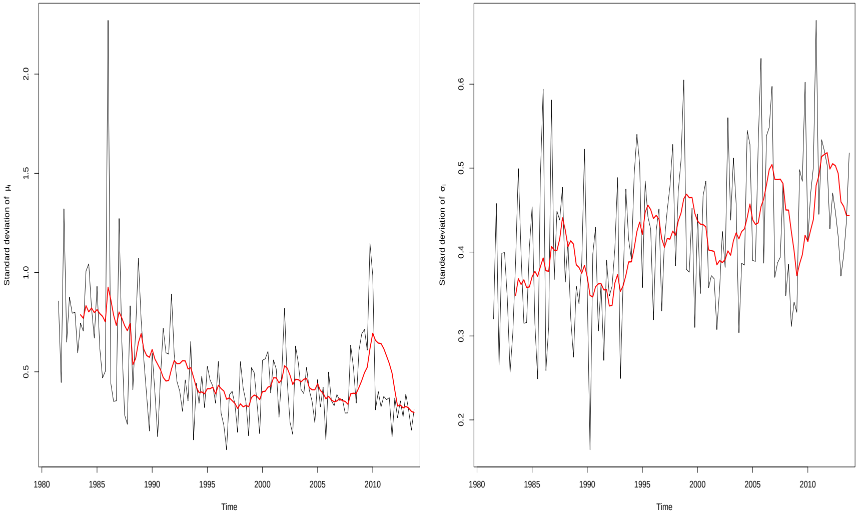
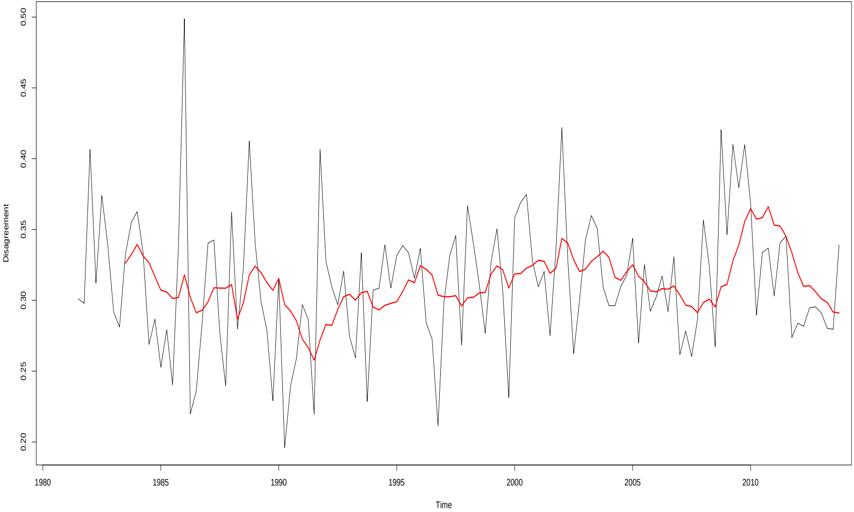
  2. $d = \frac{1}{N^2} \sum_i \sum_j \int |f_i(g_y) - f_j(g_y)| dg_y$
     $\rightarrow$ split into d_μ and d_σ

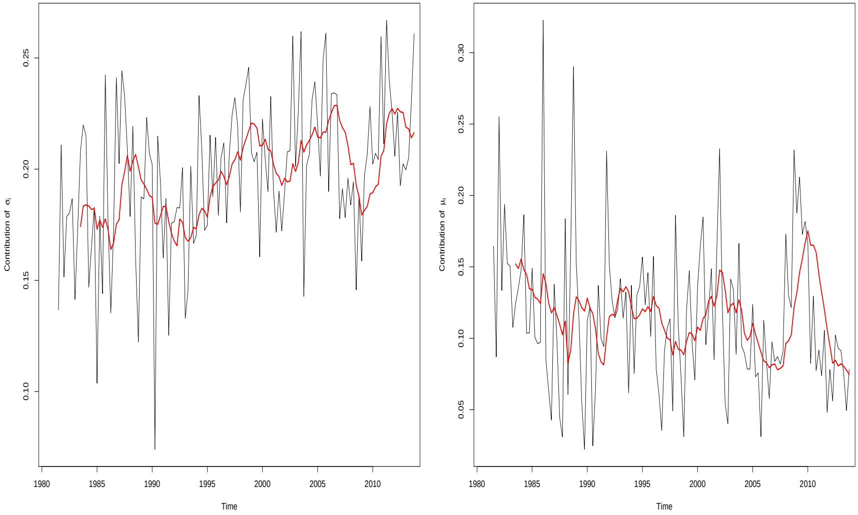
$StDev$ (across forecasters) of $\widehat{\mu_{it}}, \widehat{\sigma_{it}}$



Total disagreement $d = \frac{1}{N^2} \sum_i \sum_j \int |f_i(g_y) - f_j(g_y)| dg_y$



Contributions of μ_{it} and σ_{it} to disagreement



Summary: Evidence on Disagreement about short-term GDP growth

1. Forecasters disagree about both mean and dispersion of 1y ahead growth distribution
2. Disagreement about mean growth has fallen during late 1980s and 1990s
3. Disagreement about growth dispersion has risen

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2 type environment (following Simsek, Etrica 2013)

- 2 dates: 1,2
- 1 consumption good
- 1 asset ("tree"), pays random $s \in S = [s_{min}, s_{max}]$, $s_{min} > 0$
- 2 types of agents: 0 (steady) and 1 (volatile), of mass 1 each, with perceived pdf f_i on S
  - **Endowments**: $\bar{a} = 1$ unit of asset, n_i units of consumption
  - **Preferences**: $U_i = c_i + \frac{1}{R} E(c_i')$,
  - **Beliefs about payoffs**: Summarised by pdf f_i on S
  - **A1**:
    - $E_i(s) = E_s \forall i$
    - f_0 second order stochastically dominates f_1
    - Single-crossing point of CDFs at $s^\star$

Asset Markets at end of $t = 0$

- **Risky assets**: Agents buy $a_i - \bar{a}$ units at price p

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- **Risky assets**: Agents buy $a_i - \bar{a}$ units at price p
- **Debt instruments without commitment**
  - Promises $\bar{s}(s)$ in $t = 1$ have to be collateralised by asset payoffs
  - Payoff: $min\{s, \bar{s}(s)\}$

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   - 4.1 Profits
   - 4.2 Partial Equilibrium
   - 4.3 General Equilibrium
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Collateralised loans

- Promise constant face value $\bar{s}$ $\forall s$ in $t = 1$, collateralised by 1 unit of the asset
- Unit payoff: $min\{s, \bar{s}\}$
- Agents buy b_i units at price $q(\bar{s})$
- For now: exogenous $\bar{s}$

A. Collateralised loans: Profits

Expected Profits

1. **Outright Asset Purchases**:

$$\Pi_i^{ao} = a_i[\frac{E_s}{R} - p]$$

Independent of beliefs.

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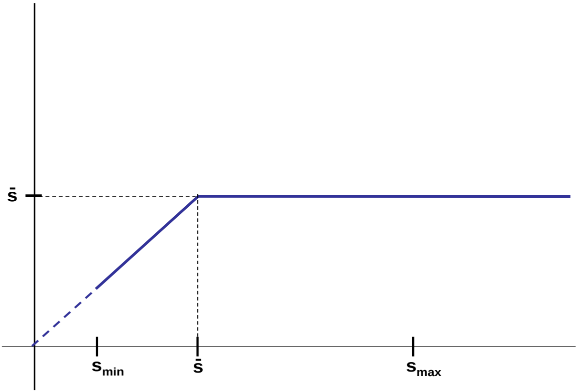
2. **Collateralised loans**

Per-asset profits from collateralised loan purchase



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2. **Collateralized Loan Contracts**:

$$\Pi_i^{l} = b_i[\frac{E_i[min\{s, \bar{s}\}]}{R} - q(\bar{s})]$$

Decreasing in dispersion of beliefs, so $\Pi_0^l > \Pi_1^l$

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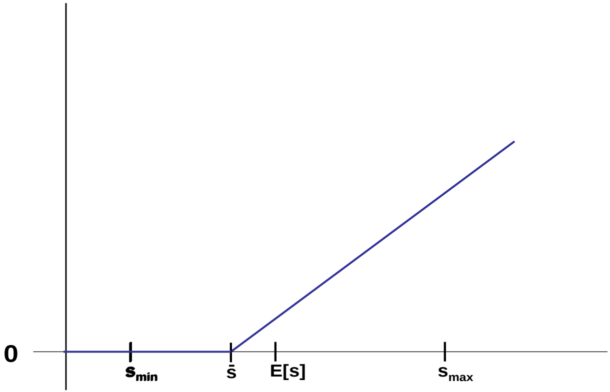
3. **Leveraged Asset Purchase**

Per-asset profits from leveraged asset purchase



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3. **Leveraged Risky Assets**:

$$\Pi_i^a = a_i[\frac{[E_i(s) - E_i(\min(s, \bar{s}))]}{R} - (p - q)] \qquad (1)$$

Increasing in dispersion of beliefs, so $\Pi_0^a < \Pi_1^a$

Assumption: Cash-rich type 0

A3: n_0 large, so $q(\bar{s}) = \frac{E_0[\min\{s, \bar{s}\}]}{R}$

B. Collateralised loans: Partial equilibrium

Partial equilibrium results

Partial equilibrium results

1. Investor specialisation: type 1 agents make leveraged asset purchases, type 0 buys collateralised loans
   $\Rightarrow$ Reduces problem to type 1's choice of c_1, $\bar{s}$

Type 1's problem with $b_1 = -a_1$

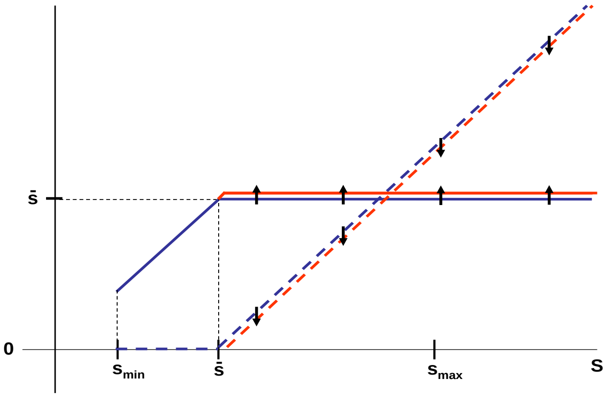
$$\max_{c_1,\bar{s}} \quad U_1 = c_1 + \frac{(n_1 + p - c_1)}{R} R_1^a(\bar{s})$$

$$\text{for} \quad R_1^a(\bar{s}) \doteq \frac{[E_s - E_1(\min\{s,\bar{s}\})]}{p - \frac{E_0\{\min\{s,\bar{s}\}\}}{R}}$$

Partial equilibrium results

1. Investor specialisation: type 1 agents make leveraged asset purchases, type 0 buy collateralised loans
2. Unique interior choice of $\overline{s}^{\star}$

Choice of $\bar{s}$ given $p > \frac{E_s}{R}$



Choice of $\overline{s}$ given $p > \frac{E_s}{R}$

$$R_1^a(\overline{s}) \doteq \frac{[E_s - E_1(\min\{s, \overline{s}\})]}{p - \frac{E_0\{\min\{s, \overline{s}\}\}}{R}} \tag{2}$$

1. $R_1^a(s_{min}) < R$, $R_1^a(s_{max}) = 0$ and $R_1^a(.)$ continuous
2. So if p is such that $R_1^a(\overline{s}) > R$ for some $\overline{s}$, then R_1^a has a unique maximum at $\overline{s}^\star$.

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3. FOC

$$R_1^a(\overline{s})\frac{1 - F_0}{R} - (1 - F_1) = 0 \tag{3}$$

General Equilibrium: Uniqueness

1. Optimal choice of $\bar{s}$

$$PS^1 \quad R_1^a(\bar{s})(1 - F_0) - R(1 - F_1) = 0 \tag{5}$$

2. Asset market clearing

$$\begin{aligned} p &= max\{\bar{p}, p^\star\} \\ \bar{p} &\doteq \frac{E_s + E_0[min\{s, \bar{s}\}] - E_1[min\{s, \bar{s}\}]}{R} \\ PS^2 \; p^\star &: \quad a_1(p^\star) = \frac{n_1 + p^\star}{p^\star - \frac{E_0[\min\{s, \bar{s}\}]}{R}} = 2 \end{aligned}$$

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- PS^2 is upward-sloping. PS^1 is downward-sloping.

  $\Rightarrow$ Uniqueness of Equilibrium

General Equilibrium: Comparative statics

1. Increasing type 1 endowments $\frac{dp}{dn_1} \geq 0$

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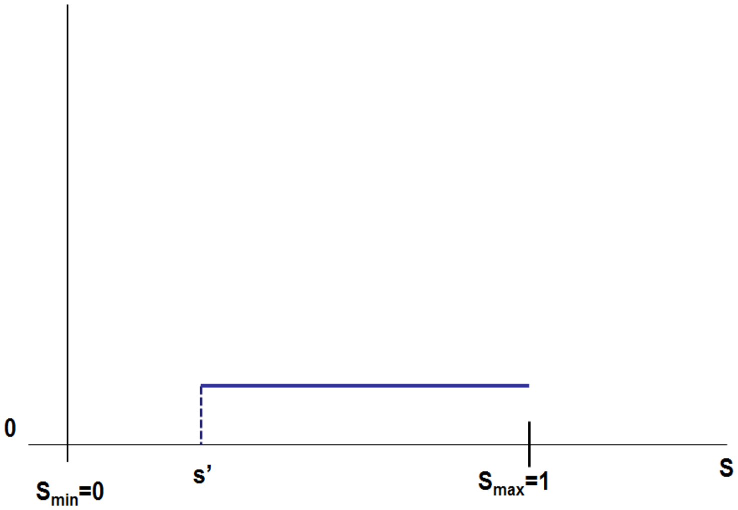
2. Increasing belief-divergence

Dynamic Example: Heterogeneous 'speed of learning' the GM

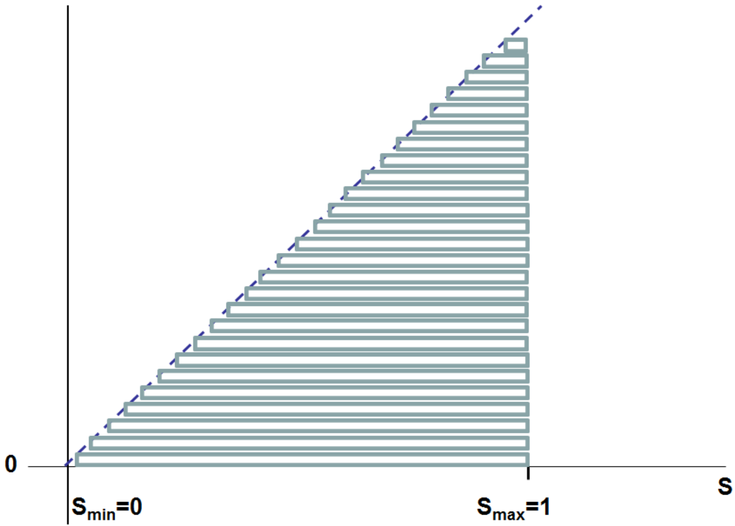
CDO

- Debt obligation backed by xth percentile of a loan pool

Payoffs from CDO



CDO 'tranching'



A. Primary CDOs

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- Agent i values CDO as $\frac{1-F_i(x)}{R}$
- Assume symmetry $n_1 = n_0 \geq \frac{E_s}{R}$
- Implies $Q(x) = max\{1 - F_1(x), 1 - F_0(x)\}$
- So type 0 agents (1) buy CDOs with $x < (\geq) s^\star$

Proposition 6: Equilibrium with CDOs

The equilibrium asset price p and consumption values in the economy with trade in CDOs equal those in an economy with trade in collateralised loans.

Proof:

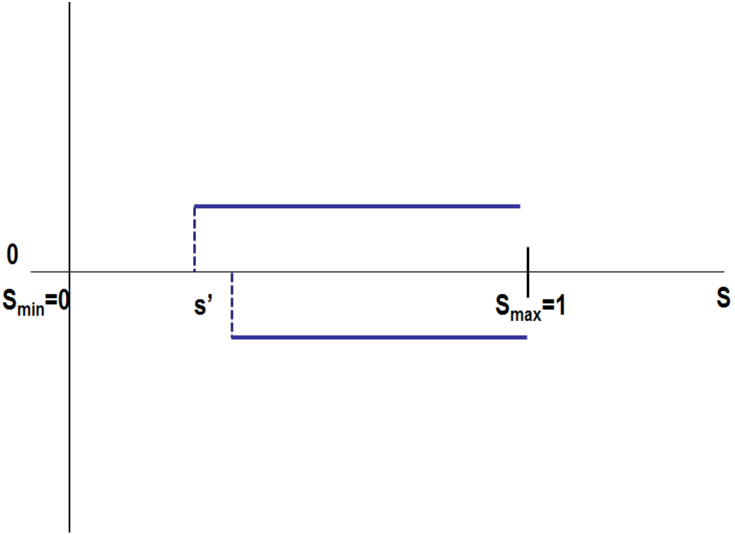
- $max\{1 - F_1(x), 1 - F_0(x)\}$ implies type 0 agents buy all CDO with $x \leq s^\star$ at reservation price.
- So type 0 buys same claims at same price as with leverage.
- Type 1 drives up asset price to her reservation value $\overline{p}$.
- So payments in $t = 0$ are the same, claims to $t = 1$ payments too.

B. CDO^2 *trade*

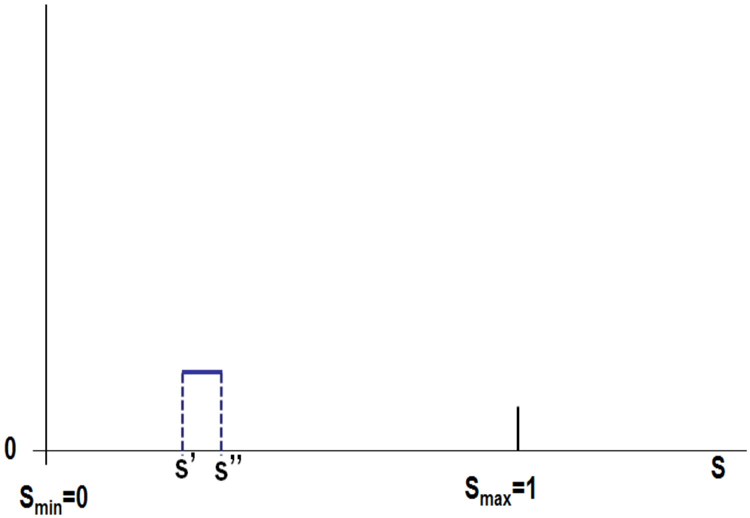
CDO^2 *trade*

- Use CDO(x) as collateral for more junior CDO
- Completes the asset-market, subject to collateralisation

Gross payoffs from CDO purchase and issuance



Net Payoffs from CDO purchase and issuance



Proposition 7: Asset price with CDO^2 trade

Asset prices with trade in synthetic CDOs equal

$$p^{SCDO} = \frac{\int_0^1 s \quad max_i\{f_i(s)\}}{R} ds \tag{6}$$

Proposition 8: Asset bubble doubles

When f_1, f_0 are symmetric, the asset price bubble $p^{SDCO} - \frac{E_s}{R}$ with trade in synthetic CDOs is at least twice as large as that with trade in collateralised loans or primary CDOs.

Proposition 8: Proof

$$
\begin{aligned}
p^{SDCO} * R &= \int_0^1 s\ max\{f_i(s)\}ds = E_s + \int_0^1 s(max\{f_i(s)\} - f_1(s))ds \\
&= E_s + \int_0^{\frac{1}{2}} (s + (1-s))(max\{f_i(s)\} - f_1(s))ds \\
&= E_s + 2\int_0^{\frac{1}{2}} \frac{1}{2}(f_0 - f_1))ds] \\
&\geq E_s + 2[\int_0^{\frac{1}{2}} s(f_0 - f_1))ds + \int_0^{\frac{1}{2}} \frac{1}{2}(f_0 - f_1)ds] \\
&= E_s + 2[E_0[min\{\frac{1}{2}, s\} - E_1[min\{\frac{1}{2}, s\}] \\
&\Rightarrow p^{SDCO} - \frac{Es}{R} \geq 2(p^{LEV} - \frac{Es}{R})
\end{aligned}
$$

Proposition 9: With disjoint f_1, f_0, $p = 2\frac{E_s}{R}$

Whenever f_1 and f_0 are disjoint, the equilibrium price of the asset equals twice its expected discounted payoff $p^{SDCO} = 2\frac{E_s}{R}$.

Proposition 9: Proof

With disjoint f_1, f_0

$$p^{SCDO} = \frac{\int_0^1 s \quad max_i\{f_i(s)\}}{R} ds = \frac{\int_0^1 s \ (f_1 + f_0)}{R} ds = 2\frac{E_s}{R} \qquad (7)$$

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6.1 Stationary dynamic equilibrium

6.2 Simple example with heterogeneous learning

Model dynamics have 3 dimensions

1. Endogenous evolution of relative wealth
2. Endogenous price fluctuations, with potentially different beliefs about the process of future prices p_t by type 1 and 2 agents
3. Learning about the distribution of s_t

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Strategy:

1. Choose S,f_1, f_0, n_i s.t. $p_t = \overline{p}$
2. Look at simple GM scenario, where type 1 updates prior more slowly to observed fall in macro-volatility

Dynamic model: Changes to the two-type environment

- t=0,1,2,...
- s_t is i.i.d. across time
- Agents maximise $U_t = \sum_s \frac{1}{R^{s-t}} c_s$
- Consumption endowment $n_{it} = n_i > 0, \; \forall t$
- Type 0 agents hold all assets in period $t = 0$.

Period t budget constraint w collateralised loans

$$a_{it+1}(p_t - \frac{E_j[min\{p_{t+1} + s_{t+1}, \bar{s}\}]}{R}) + c_{it} \leq n_i + max\{a_{it}(p_t + s_t - \bar{s}), 0\}) \qquad (8)$$

Equilibrium Definition

1. Sequences of prices and quantities as functions of the state of the economy $(s_t, \bar{s}_t, a_{it}, a_{jt})$ s.t.
2. Agents optimise given belief $f_{it} = f_i$
3. Markets for consumption and assets clear

Proposition 10: Stationary Dynamic Model

If $n_1 \geq 2\frac{E_s - E_1\{min[s,s^\star]}{R}$, there is an equilibrium with

- $p_t = \overline{p} = \frac{E_s + E_0(min(s,\overline{s})) - E_1(min\{s,\overline{s}\})}{R-1} \ \forall\ t \geq 0$
- $\overline{s} = s^\star + \overline{p} \ \forall\ t \geq 0$

The Great Moderation: A simple numerical example with learning

The Great Moderation: A simple numerical example

- GM: Mean preserving contraction in distribution of output from f_{pre} to f_{post} on unchanged support
- Dividends equal output
- Agents observe the fall in volatility, update their subjective probability distribution by constant fraction ξ_i every period

$$f_{it} = (1 - \xi_i) f_{it-1} + \xi_i f_{post} \quad (9)$$

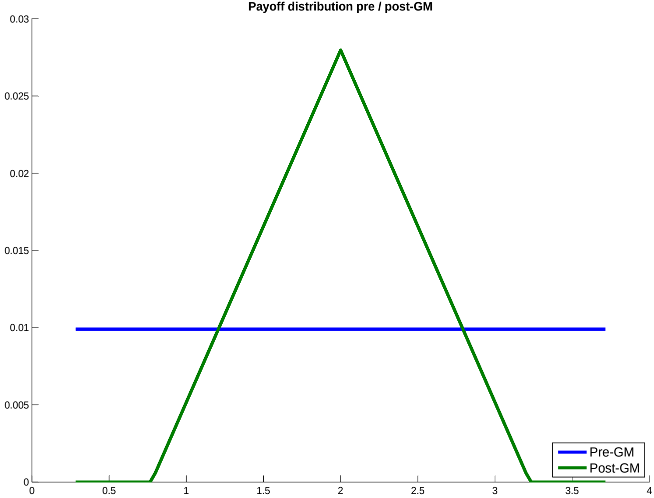
- Type 1 updates slower to low volatility environment $\xi_1 < \xi_0$
- Anticipated utility: Decisions take $f_i = f_{it}$ as given

Calibration

- Choose uniform distribution for f_{pre}, triangular distribution for f_{post}, unchanged S such that:
  - $E_s = 2$
  - $Stdev_{pre} = 1$
  - $Stdev_{post} = \frac{1}{2}$

  $\Rightarrow S = [0.3, 3.7]$

Payoff distributions



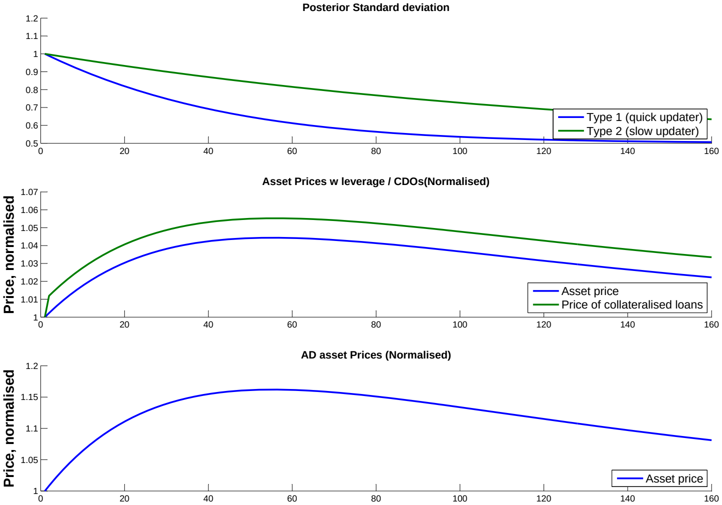
Calibration

- Choose ξ_i s.t.
  - Type 1 has adjusted 50% at end of GM
  - Boom in price has maximum after 55 quarters

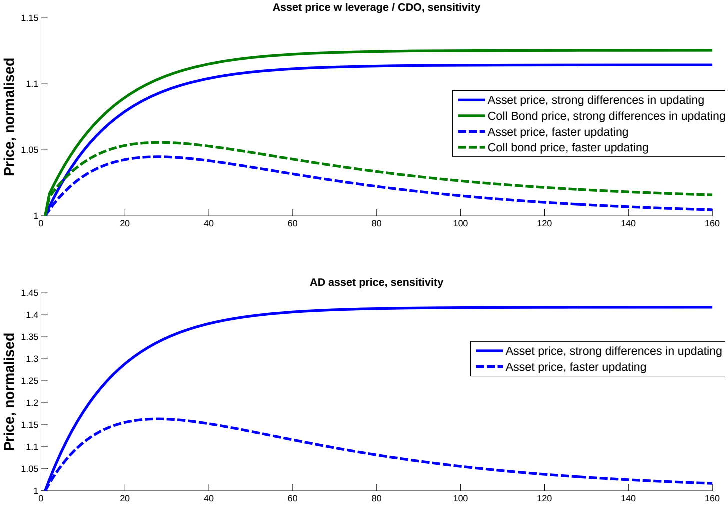
  $\Rightarrow \xi_1 = 1\%$, $\xi_0 = 3\%$
- $R = 1.01$

Results

Prices


Posterior Standard deviation
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100
120
140
160
Type 1 (quick updater)
Type 2 (slow updater)
Asset Prices w leverage / CDOs(Normalised)
Price, normalised
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
Asset price
Price of collateralised loans
AD asset Prices (Normalised)
Price, normalised
1.2
1.15
1.1
1.05
1
Asset price

Prices


Asset price w leverage / CDO, sensitivity
Price, normalised
Asset price, strong differences in updating
Coll Bond price, strong differences in updating
Asset price, faster updating
Coll bond price, faster updating
AD asset price, sensitivity
Price, normalised
Asset price, strong differences in updating
Asset price, faster updating

Conclusion

- Disagreement about macro-growth dispersion has increased in the US
- With leveraged investments, differences in risk perceived by risk-neutral investors imply gains from trade
- Prices increase above common fundamental value, become sensitive to distribution of investor wealth
- More sophisticated collateralisation schemes can raise asset prices substantially
- In a simple example, the effect on asset prices is moderate

The Great Moderation and the Great Leverage: Collateralisation bubbles when investors disagree about risk

Tobias Broer, IIES Stockholm University and CEPR
Afroditi Kero, University of Cyprus

Essim, May 2014

Increasing trade in collateralised debt instruments

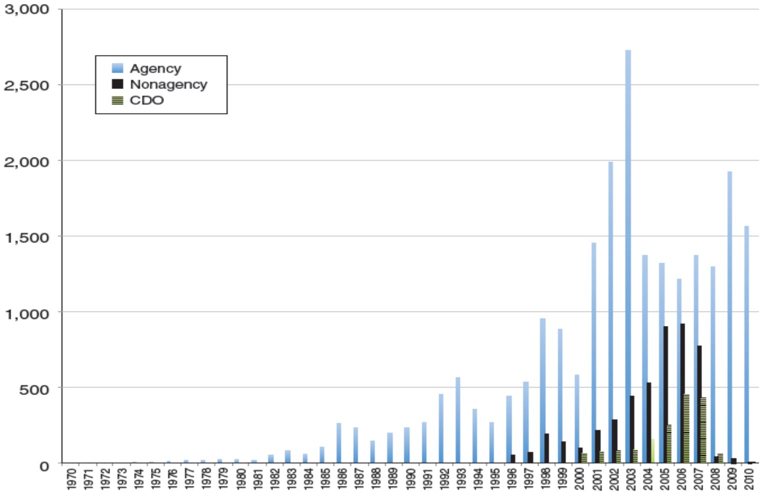


Figure 2. Securitization/Tranching

Source: Fostel and Geanakoplos 2012

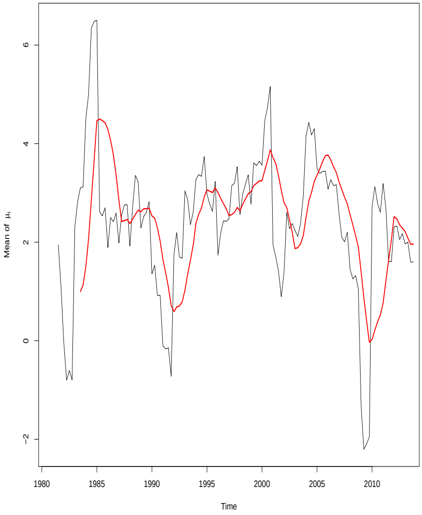
SPF Forecast Ranges 2006 Q3

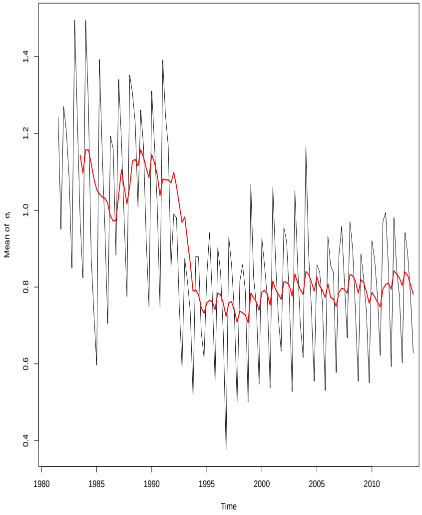
Section 3 Probabilities of Changes in Real GDP and the GDP Price Index

Please indicate what probabilities you would attach to the various possible percentage changes this year and the next in chain-weighted real GDP and the chain-weighted GDP price index (annual averages). The probabilities of these alternative forecasts should, of course, add up to 100, as indicated.

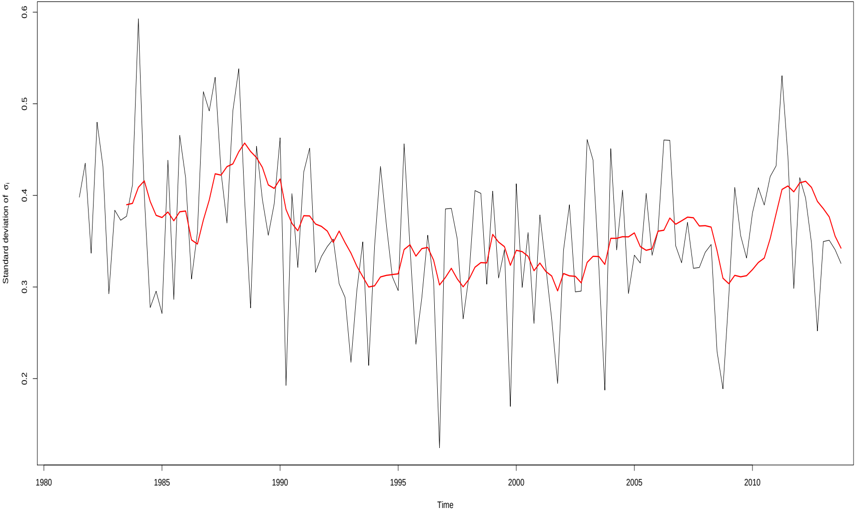
	Probability of indicated percent change in real (chain-weighted) GDP			Probability of indicated percent change in chain-weighted GDP price index	
	2005-2006	2006-2007		2005-2006	2006-2007
+6 percent or more			+8 percent or more		
+5.0 to +5.9 percent			+7.0 to +7.9 percent		
+4.0 to +4.9 percent			+6.0 to +6.9 percent		
+3.0 to +3.9 percent			+5.0 to +5.9 percent		
+2.0 to +2.9 percent			+4.0 to +4.9 percent		
+1.0 to +1.9 percent			+3.0 to +3.9 percent		
+0.0 to +0.9 percent			+2.0 to +2.9 percent		
-1.0 to -0.1 percent			+1.0 to +1.9 percent		
-2.0 to -1.1 percent			+0.0 to +0.9 percent		
Decline more than 2%			Will decline		
TOTAL			TOTAL		

Mean & St Dev of g_y*: Average across forecasters*





StDev (across forecasters) of $t\mu_{it}, \sigma_{it}$



II. 2 Period Continuum Economy: General Environment

- 2 dates: 1,2
- 1 consumption good
- 1 asset ("tree"), pays random $s \in S = [s_{min}, s_{max}]$, $s_{min} > 0$
- Continuum of agents $i \in [0, 1]$, with CDF $G(i) \longrightarrow [0, 1]$
  - **Endowments**: $n_i = n \ \forall i$ units of consumption goods, $\overline{a} = 1$ unit of the asset
  - **Preferences**: $U_i = c_i + \frac{1}{R} E(c_i')$,
  - **Beliefs about payoffs**: Summarised by pdf f_i on S
  - **A1**:
  - $E_i(s) = E_s \forall i$
  - f_i second order stochastically dominates f_j whenever $i < j$

2 Period Continuum Economy: Intuition

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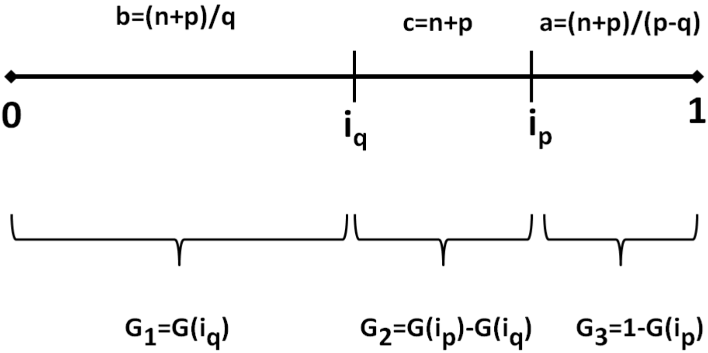
1. Order continuum of agents acc to perceived payoff dispersion on $i \in [0, 1]$. As i rises ...
   - ... expected profits from leveraged assets rise
   - ... expected profits from collateralised loans fall

2 Period Continuum Economy: Intuition

1. Order continuum of agents acc to perceived payoff dispersion on $i \in [0, 1]$. As i rises ...
   - ... expected profits from leveraged assets rise
   - ... expected profits from collateralised loans fall
2. Look for two marginal investors $i_p > i_q$ s.t.
   - $i \geq i_p$ invest whole endowment in leveraged assets
   - $i \leq i_q$ invest whole endowment in collateralised loans
   - $i : i_q \leq i \leq i_p$ consume today

2 Period Continuum Economy: Intuition



A. General Environment Continuum Economy

- 2 dates: 1,2
- 1 consumption good
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- Continuum of agents $i \in [0,1]$, with CDF $G(i) \longrightarrow [0,1]$
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  - $E_i(s) = E_s \forall i$
  - f_i second order stochastically dominates f_j whenever $i < j$

General Equilibrium for a given $\bar{s} \leq E_s$

A set of prices (p, q) and allocations $(c_i, c_i', a_i, b_i)_{i \in [0,1]}$, such that

- agent $i \in [0, 1]$ behaves optimally given f_i, p, q and $\bar{s}$
- the demand for assets equals the fixed supply,

$$\int_{i \in I} a_i = 1 \tag{10}$$

- the collateralized loan maket clears,

$$\int_{i \in I} b_i = 0.$$

Collateralised loans: Results

Collateralised loans: Results

1. Uniqueness equilibrium
2. Comparative Statics: Increased belief dispersion raises asset prices
3. Extension: Endogenous leverage $\bar{s}$

1. Uniqueness of equilibrium

Proposition 1: Uniqueness of equilibrium

There is a unique equilibrium with trade in assets of riskyness $\bar{s}$.

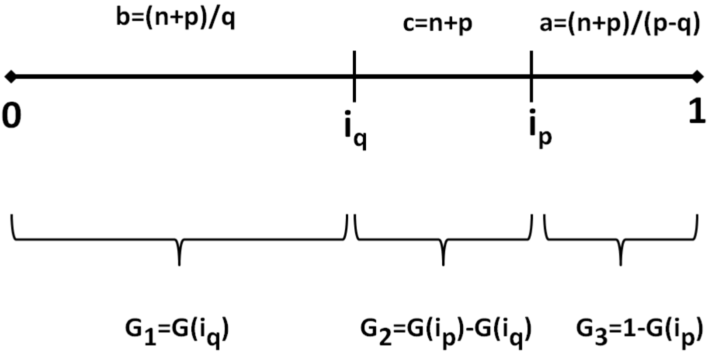
This equilibrium has the following properties:

- $\frac{E_s - E_1[min(s,\bar{s})]}{R} + q \doteq \overline{p} \geq p > \underline{p} \equiv \frac{Es}{R}$
- $q : \underline{q} \equiv \frac{E_1(min\{s,\bar{s}\})}{R} < q < \overline{q} \equiv \frac{E_0(min\{s,\bar{s}\})}{R}$
- $i_q < i_p$ s.t.
  - $\forall i > i_p$ invest $n + p\bar{a}$ in leveraged assets, $c_i = 0$
  - $\forall i < i_q$ invest $n + p\bar{a}$ in collateralised loans, $c_i = 0$
  - $i : i_q \leq i \leq i_p$ consume $c_i = n + p\bar{a}$

Intuition

Investors in assets / collateralised loans are located at extremes of belief distribution and make positive expected profits. So some agents in the middle have to consume.

Illustration



Proof

1. For $q \leq \overline{q} \doteq \frac{E_0(min\{s,\overline{s}\})}{R}$ define $i_q : q = \frac{E_{i=i_q}(min\{s,\overline{s}\})}{R}$

Proof

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4. Market clearing for loans defines $q(p)$:
   $\int_0^{i_q} b_i g(i) = \int_0^{i_q} \frac{n+p}{q} g(i) = 1$
   $\Rightarrow (n+p)G(i_q) = q$

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5. Combine with asset market clearing
   $\int_{i_p}^1 a_i g(i) = \int_{i_p}^1 \frac{n+p}{p-q} g(i) = \bar{a} = 1$
   $\Rightarrow (n+p)(1 - G(i_p)) = (p-q)$

Proof

1. For $q \leq \overline{q} \doteq \frac{E_0(min\{s,\bar{s}\})}{R}$ define $i_q : q = \frac{E_{i=i_q}(min\{s,\bar{s}\})}{R}$
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6. To get $p(1 - \int_0^{i_q} 1g(i) - \int_{i_p}^1 1g(i)) = n(\int_0^{i_q} g(i) + \int_{i_p}^1 g(i))$

$$\Rightarrow p = n(\frac{1}{G(i_p) - G(i_q)} - 1) = n\frac{G1 + G3}{G2} \qquad (11)$$

Proof

1. For $q \leq \overline{q} \doteq \frac{E_0(min\{s,\overline{s}\})}{R}$ define $i_q : q = \frac{E_{i=i_q}(min\{s,\overline{s}\})}{R}$
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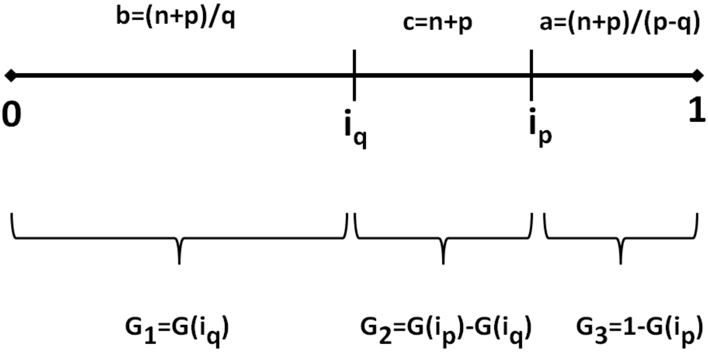
$$\Rightarrow p = n\left(\frac{1}{G(i_p) - G(i_q)} - 1\right) = n\frac{G1 + G3}{G2} \qquad (11)$$

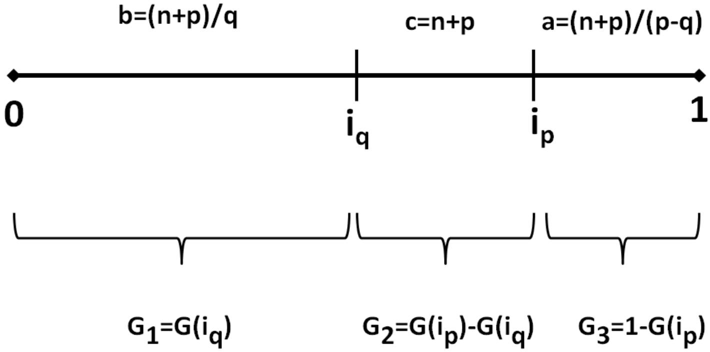
7. Can rule out $p < \underline{p}, q < \underline{q}, p > \overline{p}, q > \overline{q}$

2. Comparative statics: increased belief dispersion raises asset prices

Comparative statics: increased belief dispersion and asset prices

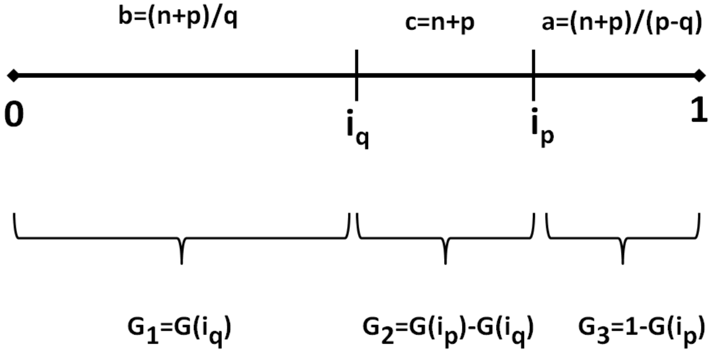


Comparative statics: increased belief dispersion and asset prices



- Increased belief dispersion: $dG_1 \geq 0,\ G_3 \geq 0$. Implies $d\widetilde{a} = \frac{n+p}{q} dG_3$ and $d\widetilde{b} = \frac{n+p}{p-q} dG_1$.

Comparative statics: increased belief dispersion and asset prices



- Increased belief dispersion: $dG_1 \geq 0,\ G_3 \geq 0$. Implies $d\widetilde{a} = \frac{n+p}{q} dG_3$ and $d\widetilde{b} = \frac{n+p}{p-q} dG_1$.
- Derive dp, dq s.t. $db(dp, dq) = -d\widetilde{b},\ da(dp, dq) = -d\widetilde{a}$.

Proposition 2: Increased belief dispersion and asset prices

A small increse in belief dispersion $dG1, dG3 > 0$ raises prices of both collateralised loans and assets.

Comparative statics: increased belief dispersion and asset prices

$$
\begin{aligned}
-d\widetilde{b} &= \frac{n+p}{q}[\frac{\delta G_1}{\delta q} - G_1\frac{1}{q}]dq \ + \ \frac{1}{q}[\frac{\delta G_1}{\delta p}(n+p) + G_1]dp \\
-d\widetilde{a} &= \frac{n+p}{p-q}[\frac{\delta G_3}{\delta q} + G_3\frac{1}{p-q}]dq + [(\frac{\delta G_3}{\delta p}\frac{n+p}{p-q}) + \frac{G_3-1}{p-q})]dp \\
-d\widetilde{x} &= \mathbb{A}d\mathbb{P} \Rightarrow d\mathbb{P} = -\mathbb{A}^{-1}d\widetilde{x}
\end{aligned}
$$

Since $\mathbb{A}_{ij}^{-1} < 0, \ i,j \in \{1,2\}$, $d\widetilde{b} > 0, d\widetilde{b} > 0$ or both implies $dp, dq > 0$.

Intuition

For given p, q, increased dispersion of beliefs reduces the number of agents in the middle. Market clearing of consumer goods requires spread in cutoffs to increase the middle interval $i_p - i_q$. $dp > 0$ raises i_p and loan demand. $dq > 0$ lowers i_q and raises asset demand. So need both $dp, dq > 0$ to offset $d\widetilde{a} > 0$ or $d\widetilde{b} > 0$ or both.

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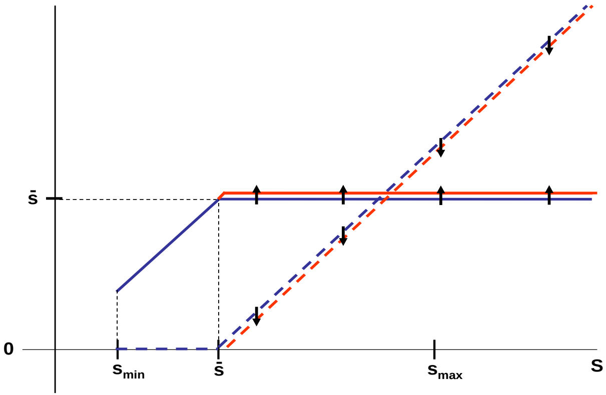
- Note: Effects of change in wealth distribution are similar to change in belief dispersion.

3. Endogenous Choice of $\bar{s}$

Endogenous Choice of $\overline{s}$

- Assume $\overline{s} < \overline{s}^{max}$ (regulatory limit)

Costs and benefits of raising $\bar{s}$



Effect of $d\bar{s}$ on profits

- Leveraged assets

$$\frac{d\Pi_i^a}{d\bar{s}} = \frac{n+p}{p-q}[R_i^a \frac{\delta q(\bar{s})}{\delta \bar{s}} - (1 - F_i(\bar{s}))] \tag{12}$$

Effect of $d\bar{s}$ on profits

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$$\frac{d\Pi_i^a}{d\bar{s}} = \frac{n+p}{p-q}[R_i^a \frac{\delta q(\bar{s})}{\delta \bar{s}} - (1 - F_i(\bar{s}))] \tag{12}$$

- Collateralised Loans

$$\frac{d\Pi_j^l}{d\bar{s}} = \frac{n+p}{q}[(1 - F_j(\bar{s})) - \frac{E_j[min(s,\bar{s})}{q} \frac{\delta q(\bar{s})}{\delta \bar{s}}] \tag{13}$$

Condition for $\overline{s}^{max}$

A2: $\overline{s}^{max} \leq s^\star = min_i min_j(s : F_i(s) = F_j(s) \;\; j, i \in [0,1]) > s_{min}$

Proposition 3: Degenerate choice of $\bar{s}$

Under assumption A_2, only one collateralised loan contract with $\bar{s} = \bar{s}^{max}$ is traded in equilibrium.

Proof

- If i buys $\bar{s}$, must not strictly prefer $\bar{s}^+$

$$\frac{d\Pi^l_{i_q}(d\bar{s})}{d\bar{s}}^+ \leq 0 \Rightarrow \frac{\delta q(\bar{s})}{\delta \bar{s}}^+ \geq \frac{(1 - F_{i_q}(\bar{s}))}{R} \quad (14)$$

- Implies

$$\frac{d\Pi^a_i}{d\bar{s}}^+ \geq \tfrac{n+p}{p-q}[\tfrac{R^a_i}{R}(1 - F_{i_q}(\bar{s})) - (1 - F_i(\bar{s}))] \quad (15)$$

$$\geq \tfrac{n+p}{p-q}[(1 - F_{i_q}(\bar{s})) - (1 - F_{i_p}(\bar{s}))] > 0 \quad (16)$$

Intuition

For agent i_q to prefer loans of face value $\bar{s}$ to $\bar{s}' > \bar{s}$, the price function $q(s)$ must be sufficiently 'steep' at $\bar{s}$. But since $i > i_q$ expects lower loan payments she would always issue $\bar{s}'$.

Summary two-period Model

1. Leverage allows agents to exploit perceived gains from trade: steady agents buy collateralised loans, volatile agents make leveraged asset purchases
2. Asset prices increase above fundamental value ($p > \frac{E_s}{R}$, $q > E_{\bar{i}}[min\{s, \bar{s}\}]$).
3. Further divergence in beliefs increases prices.
4. Equlibrium riskyness is large when endogenous.