Non-Fundamental Dynamics and Financial Markets Integration [*]

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Abstract

This paper studies the fluctuations in asset prices, capital investment and output in a stylized global equilibrium model of economies with diverse financial development. In autarky asset prices are always at their fundamental level since any non-fundamental dynamics (such as bubble-like fluctuations) are ruled out by the financial depth in economies with high financial development and by the low leveraging potential in economies with low financial development. When financial markets are integrated non-fundamental dynamics can take place in the global economy as a consequence of a global shortage of asset supply for intermediated saving. To the extent that sudden changes in agents beliefs about asset market valuations create the conditions for a financial crisis to precipitate, the model predicts that an increase in markets integration exposes the global economy to more frequent financial crises at a global scale.

Keywords: financial frictions, global imbalances, stochastic bubbles, investors sentiments, liquidity, asset shortage

1 INTRODUCTION

Stylized empirical evidence indicates that episodes of financial crises tend to happen when financial market integration is high in international capital markets. Figure 3, reproduced from Reinhart and Rogoff (2009), shows the strong correlation between banking crises – a tell-tale sign of a major financial crisis – and international capital mobility over a period of 200 years for a set of 66 economies. Banking crises, and financial crises more generally, are usually associated with sudden changes in economic agents beliefs which result in unexpected and large changes in market valuations of financial assets. In many instances it is difficult to relate the sudden change in beliefs to a change in the underlying fundamentals of the assets, which suggests that the value of the assets prior to the crisis might harbor a non-fundamental component, often referred to as a "bubble".

A large literature studies episodes of financial crises in emerging economies with limited financial development as they open up to capital inflows from the rest of the world, providing a host of theoretical explanations for the evidence of Figure 3 based on a small open economy framework (see Caballero and Krishnamurthy (2006)). However, the recent global financial crisis and the ongoing global financial distress indicate that episodes of non-fundamental valuations can arise in both financially undeveloped economies as well as financially mature economies. In particular, the financial crisis of 2008 happened after a period of intense capital market integration across economies with very diversely developed financial systems. Figure 4 reports the Chinn-Ito index of "de-jure" capital account openness for most world economies since the 1970's. As the plot clearly indicates, the world underwent a major structural change in terms of capital markets integration beginning from the early 1990's and accelerating over the last 15 years. The years following the structural change have seen several episodes of booms in asset prices followed by severe financial crises in emerging economies as well as industrialized economies. Figure 5 shows the familiar dynamics of the US stock market, real estate and gold over the last 40 years. Each one of the three assets experienced a sustained persistent boom and a subsequent sharp fall since 1990, a behavior that is hard to reconcile with changing fundamentals. Similar behaviors can be found in asset prices exchanged in other major industrialized economies over the same period of time.

The picture that emerges if one considers such global stylized facts hints at a connection between the integration of financial markets that are diversely developed and the emergence of episodes of non-fundamental assets valuations and dynamics. This paper aims at providing a theoretical framework

where such connection can be formally studied. In particular, the central question that we ask is whether financial markets integration can increase the proneness of the entire global economy to episodes of non-fundamental dynamics.

We address such question by developing a global equilibrium neo-classical growth model in which a financially mature economy and a financially undeveloped economy integrate their financial markets. While under financial autarky both economies do not allow for episodes of non-fundamental valuations, as a result of market integration equilibria with episodes of non-fundamental dynamics become possible in the global economy. Interestingly, the non-fundamental asset can be held by depositors of both economies, thereby exposing both economies, each one in measure of the non-fundamental asset held, to the risk of a financial crisis due to the sudden reversal of expectations on the market valuation of the non-fundamental asset.

Non-fundamental valuations in our framework refer to situations in which an asset is valued not because it is expected to provide a stream dividends or interest payments, but because it is expected to be sold at a competitive value in the future when more attractive investment or consumption opportunities arise. In presence of heterogeneous investment opportunities and financial frictions, the non-fundamental asset operates a transfer of resources from less productive to more productive users that would not be possible otherwise, thereby increasing the efficiency of production in the economy. If the increase in production efficiency raises the income of future savers enough to keep the non-fundamental asset affordable, the non-fundamental valuation can be rationally sustained in equilibrium.

The crucial insight gained from our model is that the mechanism just described is affected by the degree of financial development in a non-monotonic fashion. On the one hand, a financially mature economy harbors a financial system that is capable of producing a supply of assets that satisfies the saving demand and allows the funds to flow to the most efficient users. In such a context a non-fundamental valuation cannot arise since the resources that would be liberated would not generate an excess saving demand that could permanently absorb the non-fundamental asset. On the other hand, a financially undeveloped economy might be so financially constrained that the value of the non-fundamental asset would have to grow at a rate that could not possibly be matched by the growth of the income of future savers. A non-fundamental asset would still provide savers with additional internal funds once the investment opportunity arises, but if the ability to leverage those internal funds is low only a limited amount of resources would be channeled to the most productive users and the increase in

efficiency in production would be severely limited. A higher value of the non-fundamental asset would correspond to more resources transferred, but the value needed to transfer enough resources would always be too big to remain affordable for future savers.

As the two economies integrate their financial markets, the internal funds of the investors in the undeveloped economy inherit some of the leverage potential of the financially mature economy. This raises the efficiency gain in production for any given value of the non-fundamental asset and can thus make the asset affordable. This is not enough for a non-fundamental value to be sustained in a global equilibrium. The saving demand of the financially undeveloped economy is now satisfied by the asset supply of the financially mature economy. If such asset supply is large enough, there would be no excess saving demand to absorb the non-fundamental asset. However, if the asset supply of the financially mature region is not enough to satisfy the excess saving demand coming from the undeveloped region, i.e. if financial integration creates asset supply shortage, a non-fundamental asset becomes sustainable in the equilibrium of the global economy.

We model non-fundamental valuations as arising from aggregate shocks to investors' sentiments, as in Martin and Ventura (2012). Investors' sentiments are always present in the economy, but their implications for asset prices and investment/consumption decisions might be inconsistent with optimal strategies, rational beliefs and market clearing. Under some conditions, however, the same sentiments can affect asset prices and investment in a way that still satisfies all the requirements of a rational expectations equilibrium. In this sense our model provides discipline as to when investors' sentiments have the potential to drive aggregate dynamics. We characterize the conditions under which sentiments cannot affect the dynamics of economies in autarky, but they can become a source of aggregate fluctuations in the integrated global economy.

1.1 Related Literature   Our paper is closely related to Caballero, Farhi, and Gourinchas (2008). CFG are primarily interested on the consequences of asset supply shortage in a global equilibrium model in terms of current account balance, gross cross-country assets holdings and long run interest rates. Unlike our paper, the focus of their paper is not in isolating the effects of non-fundamental valuations, and in their analysis all the equilibria are fundamental valuations equilibria. However, CFG main exercise consists in studying a drop in the supply of assets in emerging economies, which the authors interpret as possibly the bursting of a financial bubble. Therefore, the idea of non-fundamental

valuations, while not formalized, is central to the interpretation of their analysis.[1] In this paper we formally study non-fundamental valuations and the main focus is on the question of *when* an asset supply shortage in the integrated global economy can create the conditions for non-fundamental valuations episodes to appear and *how* such episodes affect fluctuations in macroeconomic aggregates both as they appear and as they collapse.[2]

The way we model the non-fundamental valuations is closely related to Martin and Ventura (2012) which builds upon the work on rational bubbles in general equilibrium of Tirole (1985). As MV we introduce a non-fundamental asset as separated from other assets, and we allow a positive new supply of non-fundamental asset to appear in each period and benefit productive agents. Differently from Martin and Ventura (2012) we allow a market for intermediated savings to coexist with a market for the non-fundamental asset. This allows us to study how the existence conditions for non-fundamental valuations are affected by the change in the pledgeability of future income, a property that is central in understanding the equilibrium of the integrated global economy.[3]

The rest of the paper is organized as follows. Section 2 introduces the model and defines an equilibrium for the closed economy. Section 3 studies the conditions under which non-fundamental dynamics can emerge in the closed economy, in particular with respect to the level of financial development. Section 4 characterizes the equilibrium for the global economy. Section 5 studies the conditions under which a non-fundamental equilibrium, while not possible in autarky, can emerge in the integrated economy. Section 6 shows numerical simulations of the non-fundamental dynamics in the global economy. Section 7 concludes. The proofs of the main results are reported in Appendix A.

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[1] The asset supply side of the CRG model is built so that in the extreme case in which the pledgeability of income is zero, an equilibrium with non-fundamental valuations is the only possible equilibrium (see also Caballero (2006)). The reason for this is that there is no direct investment option for agents in the model and so the demand for savings can be only satisfied by intermediated savings. In our model agents always have the option to use their savings to generate capital goods - a form of storage technology - which implies that there always exists a fundamental equilibrium where market clearing is reached without the necessity of non-fundamental valuations.

[2] See Caballero and Krishnamurthy (2001), Caballero and Krishnamurthy (2006), Caballero and Krishnamurthy (2009), Maggiori (2011), Gourinchas and Rey (2007) for additional works on global asset supply shortage and implications for macroeconomic fluctuations.

[3] The role of the non-fundamental asset in our economy is also very similar to the bubbly liquidity modeled in Farhi and Tirole (2012).

2 THE MODEL

PREFERENCES AND TECHNOLOGY. The individual economy consists of an infinite sequence of over-lapping generations each of measure 1. An individual agent $i \in [0, 1]$ born at time $t - 1$ lives for three periods: young (period $t - 1$), adult (period t) and old (period $t + 1$).[4] When young, each agent i is endowed with one unit of time that she supplies inelastically to the labor market at the unitary wage w_{t-1}. The objective of agent i born at time $t - 1$ is to maximize her expected consumption when old, $\mathbb{E}_{i,t-1}(c_{it+1})$, where c_{it+1} denotes the amount of output good consumed at time $t + 1$. Agents in the economy are risk neutral and their savings demand when young and adult is inelastic and equal to their total wealth.[5]

The output good is produced by a perfectly competitive final good sector where each firm employs labor from the young and capital via a constant return to scale technology

$$y_t = k_t^\alpha \ell_t^{1-\alpha}, \quad \alpha \in (0, 1), \tag{2.1}$$

where k_t denotes capital and ℓ_t labor. The labor market and the rental market for capital are both perfectly competitive, so that each factor is always paid its marginal return. Because agents supply labor inelastically, under any equilibrium $\ell_t = 1$ and the factor prices are given by

$$w_t = (1 - \alpha)k_t^\alpha \quad \text{and} \quad R_t = \alpha k_t^{\alpha-1} \tag{2.2}$$

Capital depreciates completely after use. New capital for production at $t + 1$ is obtained by investing output good at time t. Let x_{it} denote the output good invested at time t by agent i, the investment technology is

$$k_{it+1} = A_{it+1}x_{it}.$$

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[4]The overlapping generations structure is chosen for analytical convenience. A model with infinitely lived agents with stochastic investment opportunities would also allow the derivation of our results, at a cost of a more burdensome notation and less analytical transparency. For alternative frameworks for the analysis of non-fundamental dynamics see Rondina (2012)

[5]The two assumptions are admittedly a simplification of reality, but both risk non-neutrality and intertemporal consumption decision are not essential for the basic mechanism that the model aims at capturing. They are nonetheless relevant, the former in particular, to understand the change in the composition of the external balance sheet of emerging and industrialized economy following financial globalization. See Gourinchas (2012).

Both young and adult agents can operate the direct investment technology, but they differ in terms of their investment productivity. For the individual young agent i at time $t-1$ the investment productivity A_{it} is constant and equal to $\underline{a} > 0$. When adult, investment productivity A_{it+1} is drawn from the continuous distribution with cumulated density G over the support $[\underline{a}, \bar{a}] \subset \mathbb{R}$, independently across time and agents. Both young and adult agents at time t know their own investment productivity for the current period. Young agents, however, do not know their future productivity at adult age. Output produced at period t, y_t, is either consumed or invested, so output market clearing is

$$y_t = c_t + x_t^A + x_t,$$

where c_t stands for aggregate consumption and x_t^A and x_t stand for the aggregate investment of adults at time t and young at time t, respectively.

In addition to directly investing in the capital investment technology, agents have access to intermediated saving and to borrowing. In particular, they can deposit or borrow funds through a representative intermediary which operates in a competitive market with free entry, and offers the same gross financial interest rate R_{t+1}^a on both loans and deposits from period t to $t+1$.[6] The asset position of the agent i with the intermediary at the end of period t is denoted by a_{it+1}, with $a_{it+1} > 0$ if the agent is depositing and $a_{it+1} < 0$ if the agent is borrowing. The optimization problem for the young agent at time $t-1$ can then be written as

$$\max_{c_{it+1}, k_{it}, k_{it+1}, a_{it}, a_{it+1}} \mathbb{E}_{it-1}\big(c_{it+1}\big) \tag{2.3}$$

subject to

$$w_{t-1} \geq a_{it} + \frac{k_{it}}{\underline{a}}, \tag{2.4}$$

$$R_t^a a_{it} + R_t k_{it} \geq \frac{k_{it+1}}{A_{it+1}} + a_{it+1}, \tag{2.5}$$

$$c_{it+1} \leq R_{t+1} k_{it+1} + R_{t+1}^a a_{it+1}. \tag{2.6}$$

Constraint (2.4) requires the total wealth of the young at time $t-1$, equal to the wage earned in that

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[6]For simplicity, we restrict our attention to an environment with one-period debt contracts only.

period, to be either invested directly with productivity $\underline{a}$ or to be deposited with the intermediary. In period t, the total beginning of period wealth available to the adult agent is equal to the return on direct investment in capital, as capital is rented out to the final output sector, plus the return on intermediated savings (or minus the re-payment of any borrowing). The wealth can be allocated to direct investment into capital with a productivity A_{it+1}, or deposited with the intermediary once again. In the final period of her life the agent collects the return from her portfolio and uses it to consume.

FINANCIAL INTERMEDIATION.   The representative intermediary collects deposits and extends loans to agents that find it optimal to directly invest in excess of their internal funds. We refer to the assets representing loans as "fundamental assets". The intermediary can also invest the funds deposited in an asset that, contrary to loans, does not promise any stream of payments but it is held only for the purpose of reselling it at some point in the future, were the need for funds to arise. We refer to the value of such asset as a "non-fundamental valuation". Let the total value of the asset held by the intermediary at the beginning of period t in terms of output good at time t be denoted by b_t. The value b_t will be assumed to have a stochastic structure related to investors' sentiments, or the coordinated willingness of depositors to buy the asset through the intermediary. The specifics of this structure will be given below. The asset can be freely disposed with, which bounds its value to be weakly above zero. The value of the asset is always taken as given by market participants, which implies that the supply of the asset is out of the control of agents and intermediaries. We follow Martin and Ventura (2012) and we assume that a new supply of the non-fundamental asset can be randomly obtained by any individual adult agent that directly invests into capital. The individual investors cannot anticipate the creation of the non-fundamental assets. Under certain conditions, the newly created supply is purchased by the intermediary, and the relative funds accrue to the investing agent in addition to her internal funds and the amount borrowed from the intermediary. The total value of the new asset created at time t and purchased by the intermediary is denoted by b_t^N. The balance sheet of the intermediary at the end of period t can be then written as

$$b_t + b_t^N + l_t = d_t,$$

where l_t denotes the total amount of loans outstanding and d_t the total value of the liabilities of the intermediary to the depositors. Notice that, indirectly, the non-fundamental asset $b_t + b_t^N$ is always held

by depositors in the economy.[7] The expected return on the non-fundamental asset, denoted by R_{t+1}^b, consists of the capital gain from the asset between period t and $t+1$,

$$R_{t+1}^b = \frac{\mathbb{E}_t(b_{t+1})}{b_t + b_t^N}, \tag{2.7}$$

where the expectation is conditional on an information set that is common knowledge across agents at time t. Risk neutrality of agents together with the zero-profit condition for the intermediaries implies that the expected return on the non-fundamental is equal to the return on deposits and loans, i.e. $R_{t+1}^b = R_{t+1}^a$. The risk-neutrality assumption masks an important difference between assets representing loans and b_t. The fundamental assets l_t are "safe" in the sense that their return is exactly known at the time of issuance. The non-fundamental assets are instead "risky": if the value of the asset is higher (lower) than expected the unexpected capital gain (loss) is immediately distributed to (taken out from) depositors.[8]

FINANCIAL FRICTIONS.   When financial markets function smoothly, funds are channeled towards their more productive use. In our economy this means that young agents and adult agents with a low investment productivity transfer their wealth at time t to the adults agents with high productivity. Productive adult agents invest in the capital technology, rent out the capital in the next period and repay the loans with interest.[9]

We assume that in the economy there is a financial friction. In particular, borrowing agents have limited commitment and can only pledge a fraction θ of their future income which imposes an upper

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[7]An alternative modeling choice would be to allow agents to participate directly in the market for loans, deposits and non-fundamental asset, under price taking conditions. While equivalent in terms of results, using a representative intermediary allows a more compact representation of the role of financial markets.

[8]The modeling choice of separating the fundamental and the non-fundamental asset is made for convenience and is meant to capture the idea that some assets contain a component that is not related to the expected value of their stream of payments. For instance, Martin and Ventura (2012) interpret b_t as the value of a firm after production has taken place and capital has depreciated. The fundamental value of the firm is clearly zero, but if the intermediary can "sell" the firm to depositors willing to believe that the firm will be sold in the future at b_{t+1}, the firm assumes a non-fundamental value and becomes a tradable asset. The intermediary is therefore purchasing loans in the credit market, l_t, and old firms in the stock market b_t. Occasionally, new firms are created by investors and sold to the intermediary in the stock market with value b_t^N.

[9]Alternatively, we could have assumed that the loan is issued and repaid all within the same period, so that the lending agents receive capital to carry into period $t+1$ and rent it out in the capital market. The two assumptions are equivalent in our setting, but the former streamlines the presentation of the equilibrium conditions in the asset market in presence of an intermediary.

bound on their borrowing of the form

$$R^a_{t+1} a_{it+1} \geq -\theta R_{t+1} k_{it+1}, \tag{2.8}$$

where $\theta \in [0,1]$. Constraint (2.8) is to be included in the description of the maximization problem of the young agent (2.3)-(2.6). As pointed out in Caballero, Farhi, and Gourinchas (2008), the parameter θ is interpretable as an index of financial development, in the sense that it measures the extent over which property rights over earnings are well defined in the economy and can be exchanged on financial markets.

OPTIMAL PORTFOLIO STRATEGIES.   The solution to the problem of the young agent at time $t-1$ boils down to choosing a portfolio allocation strategy that maximizes the expected wealth once old at $t+1$. At the beginning of each period, given the total wealth available, the agent will decide whether to directly invest into the capital technology by borrowing up to the limit, or whether to deposit the funds with the financial intermediary for intermediated saving. The choice between the two portfolio strategies depends on their relative returns given the productivity of the individual agent. It is useful to define the variable ρ as

$$\rho_{t+1} \equiv \frac{R^a_{t+1}}{R_{t+1}}.$$

An agent at time t with an investment productivity draw A_{it+1} can directly invest in capital and receive $A_{it+1} R_{t+1}$ in return, or she can invest in intermediated savings and receive R^a_{t+1}. Because of the linearity of the objective function, the optimal portfolio strategy of the agent will always be a corner solution: if $A_{it+1} > \rho_{t+1}$ it is optimal to directly invest in capital all the internal funds plus the maximum amount that can be borrowed from the intermediary, which is $a_{it+1} = -\theta \frac{k_{it+1}}{\rho_{t+1}}$; if $A_{it+1} < \rho_{t+1}$ it is optimal to deposit all the wealth with the intermediary. At equality the agent will be indifferent between direct investment or intermediated saving. The value of ρ in equilibrium provides an indication of the severity of financial frictions in the economy. With no financial frictions ($\theta = 1$) ρ would be equal to the highest productivity level $\bar{a}$, while under the most severe of financial frictions ρ would be stuck at $\underline{a}$.

Let us consider first the portfolio choice of the young agent at time $t-1$. The young agent will be indifferent between directly investing or depositing with the intermediary when $\rho_t = \underline{a}$. In this situation,

the amount of deposits will be determined by the value of assets that the intermediary holds. If the deposits issued by the intermediary are not enough to satisfy the demand of the young agents, they will be forced to directly invest the remaining funds. Denote by the direct investment of the young agent in such case as $\delta_t \in [0,1]$. Then the total capital produced by young agents is

$$k_t^Y = \underline{a}\left(\frac{\delta_{t-1}}{1-\theta}\right)w_{t-1}. \tag{2.9}$$

It follows that the total borrowing from the intermediary and the total savings demand are

$$a_t = -\theta\left(\frac{\delta_{t-1}}{1-\theta}\right)w_{t-1} \quad \text{and} \quad (1-\delta_{t-1})w_{t-1}. \tag{2.10}$$

The net wealth that the young generation at $t-1$ expects to carry into t is[10]

$$w_{t|t-1}^A = R_t^a w_{t-1}. \tag{2.11}$$

If the intermediary holds a non-fundamental asset, the return R_t^a cannot be exactly guaranteed, and any discrepancy between b_t and $\mathbb{E}_{t-1}(b_t)$ might affect the actual available wealth at time t. The crucial question is who, between the young turning adult and the adult turning old, is holding the risk of changes in the value of b_t. While not changing the strategies of risk neutral agents, the holding allocation does matter for the aggregate dynamics. We let φ measure the exposure of the young generation turning adult to non-fundamental shocks, and $1 - \varphi$ the exposure of the adult generation when turning old.[11] The realized wealth for the young turning adult is then given by

$$w_t^A = w_{t|t-1}^A + \varphi\big(b_t - \mathbb{E}_{t-1}(b_t)\big).$$

Consider next the adult agent at time t. At the beginning of time t the agent receives the draw of investment productivity A_{it+1} and faces a portfolio choice with internal funds given by w_t^A and with relative return ρ_{t+1}. If $A_{it+1} < \rho_{t+1}$ the agent deposits all her funds with the intermediary. If

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[10]To see this note that the evolution of the wealth of the young generation can be conditioned on two cases according to

$$w_{t|t-1}^A = \begin{cases} R_t^a w_{t-1}, & \text{if} \quad \rho_t > \underline{a} \\ (1-\delta_{t-1})R_t^a w_{t-1} + \delta_{t-1}R_t\underline{a}w_{t-1}, & \text{if} \quad \rho_t = \underline{a} \quad \text{and} \quad \delta_t \in (0,1]. \end{cases}$$

However, when $\rho_t = \underline{a}$ it means that $R_t\underline{a} = R_t^a$ by definition, and relationship (2.11) follows.

[11]For simplicity we assume that the exposure φ is time-invariant.

$A_{it+1} \geq \rho_{t+1}$ the agent borrows to the limit and invests all the funds in the capital technology. The investing adult agents also receive a random new supply of non-fundamental asset equal to b_{it}^A that she can immediately sell to the intermediary.[12] For simplicity we assume that all the adult investing agents receive the same non-fundamental asset shock, so that $b_{it}^A = b_t^A$. The capital produced by the investing adult i is

$$k_{it+1} = A_{it+1} \frac{w_t^A + b_t^A}{1 - \theta \frac{A_{it+1}}{\rho_{t+1}}}. \tag{2.12}$$

and her borrowing from the intermediary is $a_{it+1} = -\theta k_{it+1}/\rho_{t+1}$. To facilitate the representation of the aggregate behavior of the investing adult generation it is convenient to define an aggregate "leverage" function U as

$$U_\theta(\rho) \equiv \int_{A>\rho} \frac{A}{\rho} \frac{1}{1 - \theta \frac{A}{\rho}} dG \tag{2.13}$$

where G is the cumulative density function of the distribution for the productivity of capital investment. The leverage function is decreasing in ρ: the higher the relative cost of borrowing, the lower the total borrowing that can be done against the existing internal funds. On the other hand, U is increasing in θ: the more the fraction of future income that can be pledged, the higher the borrowing that can be done against the existing internal funds. A more subtle, but nonetheless crucial, property of U is that when θ is increased the leverage of the more productive agents is increased relatively more compared to that of the less productive ones. This is a consequence of the argument of the integral in (2.13) being non-linearly increasing in $A\theta$. The relevance of this property will become clear in our equilibrium analysis.

Aggregating across all investing adults at time t the total capital produced is

$$k_{t+1}^A = \rho_{t+1} U_\theta(\rho_{t+1}) (w_t^A + b_t^A),$$

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[12]The unexpected new fundamental asset supply plays the role of a random relaxation of the borrowing constraint which does not result in an increase in the amount borrowed, but rather in internal funds available, courtesy of the depositors buying the new asset.

while the total borrowing and the total intermediate saving demands for adults at time t are

$$a_{t+1}^A = -\theta U_\theta(\rho_{t+1})(w_t^A + b_t^A) \quad \text{and} \quad G(\rho_{t+1})w_t^A.$$

The total wealth that the adult generation expects to bring into their adult age is finally given by

$$w_{t+1|t}^O = R_{t+1}^a \left[w_t^A G(\rho_{t+1}) + (1 - \theta)(w_t^A + b_t^A)U_\theta(\rho_{t+1}) \right].$$

The old agent at time $t + 1$ consumes all the wealth delivered by her investments at time t. As in the case of the young agent turning adult, the adult agent turning old is facing the uncertainty related to the value of the non-fundamental asset held by the intermediary, according to the measure $1 - \varphi$. Aggregate consumption at time $t + 1$ is then

$$c_{t+1} = w_{t+1|t}^O + (1 - \varphi)(b_{t+1} - \mathbb{E}_t(b_{t+1})).$$

FINANCIAL ASSETS DEMAND AND SUPPLY. The portfolio strategies just described provide a description of the financial assets demand and supply in the economy. The aggregate demand for intermediated savings is

$$d_t(\rho_{t+1}) = (1 - \delta_t)w_t + G(\rho_{t+1})w_t^A \tag{2.14}$$

Deposits demand is an increasing function of the relative return ρ_{t+1} and of the wealth available to young and adult agents. Note that w_t^A depends on the return on the non-fundamental asset if any is held by the intermediary, so an increase (decrease) in the value of b_t has a positive (negative) effect on deposits demand, everything else equal.

The aggregate supply of fundamental financial assets consists of the total borrowing of the investing young and adult agents at time t, namely

$$l_t(\rho_{t+1}) = \theta \left[\frac{\delta_t}{1 - \theta}w_t + U_\theta(\rho_{t+1})(w_t^A + b_t^A) \right]. \tag{2.15}$$

When $\theta = 0$ the supply of fundamental assets is zero. As θ is increased the supply becomes positive. Given the properties of U the increase in fundamental asset supply is non-linear in θ.

EQUILIBRIUM. Any equilibrium of the economy is a function of the non-negative stochastic process $\{b_t, b_t^A\}_{t=0}^{\infty}$. Let ω_t be a specific realization of the process at time t, and define $\omega^t = \{\omega_0, \omega_1, ..., \omega_t\}$ as the history of non-fundamental value shocks up to time t, with Ω^t being the set of all possible histories, so that $\omega^t \in \Omega^t$. The specific realization of the history ω^t combined with the optimization and market clearing conditions for the output good, capital, labor and financial assets implies that the equilibrium path of aggregate capital and the financial interest rate are a function of the history ω^t, more formally $k_t = k_t(\omega^t)$ and $R_t^a = R_t^a(\omega^t)$. An equilibrium for the closed economy is defined as follows.

Definition. *Given a non-negative stochastic process $\{b_t, b_t^N\}_{t=0}^{\infty}$ an equilibrium for the economy is a sequence for aggregate capital allocation $\{k_{t+1}\}_{t=0}^{\infty}$ and financial interest rate $\{R_{t+1}^a\}_{t=0}^{\infty}$ such that individual optimization is achieved, all markets clear and the non-fundamental asset remains affordable.*

When $\{b_t, b_t^A\} = 0$ we say that the economy is in a fundamental equilibrium at time t. When not in a fundamental equilibrium the economy is experiencing non-fundamental dynamics. Under some conditions, the fundamental equilibrium is the only possible equilibrium in the economy. In this case, the allocation of capital, output and asset prices will be deterministic. Under other conditions, the non-fundamental dynamics can be a possibility in equilibrium. In this case the allocation of capital, output and asset prices reflect the behavior of investors' sentiments and become subjected to random fluctuations and sudden changes.

NON-FUNDAMENTAL ASSET, LEVERAGE AND CROWDING IN/OUT OF CAPITAL. Before characterizing an equilibrium it is useful to describe the transfers of funds engineered by the non-fundamental asset and their effect on capital accumulation. Let us consider first the case of $\theta = 0$, so that the only saving options available are direct investment and the non-fundamental asset. In the market for b_t at time t the buyers are δ_t of the the young agents when $\rho_{t+1} = \underline{a}$ and all the young agents plus the less productive adult agents when $\rho_{t+1} > \underline{a}$. On the other hand, the sellers of the asset are all the adult agents when $\rho_{t+1} = \underline{a}$ and the most productive adult agents plus the less productive adult agents from the previous period that are now old if $\rho_t > \underline{a}$ in the previous period. Therefore b_t transfers funds from the least productive young and adults to the most productive adults and some old consumers that have not directly invested when adult. Both a crowding out and a crowding in effect are contemporaneously present in such transfers. When the buying of b_t draws funds from young and adult agents that do not directly invest it operates a crowding out effect on capital. But when the selling of b_t channels funds to

productive adult agents it operates a crowding in effect on capital. If the latter effect is large enough, a positive value for the non-fundamental asset can be rationally sustained in the economy.

Suppose now that $\theta > 0$. In this case the transfer of funds from the least productive young and adult to the most productive adult is already happening because of the market for the fundamental asset l_t. The average efficiency level at which funds are invested is a function of the internal funds available to productive adult agents. In presence of the non-fundamental asset b_t, productive adult agents selling the asset accumulate more internal funds that increase their borrowing potential. The efficiency enhancement due to the non-fundamental asset is higher compared to the $\theta = 0$ case, and, given the same b_t, the crowding in effect is stronger. There is, however, a limiting factor in the circulation of b_t when θ gets larger, which is represented by the supply of the fundamental asset. The increased borrowing capacity of productive adults creates a supply of fundamental assets that compete with b_t in capturing the savings of young and unproductive adult agents. This imposes an upper bound on the attainable value of b_t in equilibrium as θ is increased. The working of these effects in equilibrium are formalized in the next section.

3 Non-Fundamental Dynamics in the Closed Economy

The objective of this section is to study the conditions under which a non-fundamental equilibrium can emerge in a closed economy of the type described in Section 2. In particular, we want to understand how financial development, measured by θ, affects the possibility of non-fundamental dynamics. To facilitate the analysis it is convenient to express the dynamics of the economy recursively. We re-scale all the variables by the "size" of the economy, represented by the total net wealth in the economy at time t after production of output and consumption of the old agents took place, but the portfolio choice and the capital investment did not. Let the total realized net wealth be denoted by $W_t = w_t + w_t^A$, then its distribution across adult and young is denoted by $n_t \equiv \frac{w_t^A}{W_t}$ and $1 - n_t \equiv \frac{w_t}{W_t}$. Let $W_{t|t-1}$ denote the total wealth in the economy at t as expected at the end of period $t-1$. We define the value of the non-fundamental assets relative to such wealth as

$$z_t \equiv \frac{b_t}{W_{t|t-1}}, \quad \text{and} \quad z_t^A \equiv \frac{b_t^A}{W_{t|t-1}}.$$

The difference between the expected and realized wealth is due to the difference in the expected and realized value of the non-fundamental asset. Let $\sigma_t \equiv z_t - \mathbb{E}_{t-1}(z_t)$, then the relationship between expected and realized wealth in the economy is

$$W_t = (1 + \varphi \sigma_t) W_{t|t-1},$$

where φ is the fraction of the change in the asset value that is accrued or sustained by young depositors turning adults at time t. Let $e_{t+1|t}$ denote the expected wealth of the young at $t+1$ in terms of the net realized wealth at t, this can be written (see Appendix for details) as

$$e_{t+1|t} \equiv \frac{1-\alpha}{\alpha} \left[\frac{\delta_t}{1-\theta}(1 - n_t) + U_\theta(\rho_{t+1}) \left(n_t + \frac{z_t^A}{1 + \varphi \sigma_t} \right) \right],$$

The expected wealth of the young at time $t+1$ is equal to the wage payment they receive from supplying their unit of labor. The level of the wage is a function of the capital available at time $t+1$, which was determined by the portfolio allocation chosen at t by the then young and adult generations. The larger is the fraction of output that remunerates labor, $1 - \alpha$, the larger the wealth of the young at $t+1$ for any level of capital available. The expected wealth of the adult at $t+1$ in terms of wealth at t is always equal to the fraction of wealth held when young at t, $1 - n_t$ (see Appendix). The following proposition provides a recursive representation of the equilibrium in the closed economy.

Proposition 1. *The non-negative stochastic process $\{z_t, z_t^A\}_{t=0}^\infty$ and the sequence $\{\delta_t \in [0,1], n_t, \rho_{t+1}\}_{t=0}^\infty$, with $\rho_{t+1} = \underline{a}$ when $\delta_t < 1$, constitute an equilibrium of the closed economy if the following conditions are satisfied:*

(a) expected return of non-fundamental asset

$$\mathbb{E}_t(z_{t+1}) = \frac{z_t + z_t^A\big(1 - G(\rho_{t+1})\big)}{1 - n_t + e_{t+1|t}} \frac{1}{1 + \varphi \sigma_t}, \tag{3.1}$$

(b) asset market clearing

$$\theta \frac{\alpha}{1-\alpha} e_{t+1|t} + \big[z_t + z_t^A\big(1 - G(\rho_{t+1})\big)\big] \frac{1}{1 + \varphi \sigma_t} = (1 - n_t)(1 - \delta_t) + n_t G(\rho_{t+1}), \tag{3.2}$$

(c) intergenerational wealth distribution

$$n_{t+1}(1 + \varphi\sigma_{t+1}) = \frac{1 - n_t}{1 - n_t + e_{t+1|t}} + \varphi\sigma_{t+1}, \tag{3.3}$$

Proof. See Appendix.   □

Along the fundamental dynamics, described by Equations (3.2)-(3.3) with $z_t = z_t^A = 0$, funds are transferred from young agents to adult agents through loans l_t and the equilibrium relative return ρ_{t+1} ensures that the demand for intermediated savings is equal to the supply of loans. A fundamental equilibrium always exists.

The non-fundamental dynamics display by definition at least one strictly positive realization for z_t. In this case equation (3.1) has to be satisfied as well. In the non-fundamental equilibrium funds are transferred from the young to the adult through z_t as well as l_t, but for that to be possible z_t must offer a return that is competitive with that of the loans. Equation (3.1) provides the restriction on the non-fundamental asset dynamics for this to happen. The total value of the non-fundamental asset is equal to the sum of the current value of the existing asset, z_t, and that of the new non-fundamental asset issued by investing adult agents $z_t^A(1 - G(\rho_{t+1}))$. Since both z_t and z_t^A are channeling funds from unproductive to productive investors, the average capital investment efficiency is increased in the economy, which helps to keep the purchase of the non-fundamental asset affordable. However, the necessary return on z_t might be such that its value eventually exceeds the resources available to the young. At this point the relative return ρ_{t+1} increases above $\underline{a}$ in order to attract the least productive adult agents to purchase the asset. The available funds are no longer channeled only to productive adults, but they begin to be channeled also to old agents - those that were the least productive when adult - for consumption. As this happens, the original crowding-in effect is contrasted by a crowding-out effect due to the increasing size of the non-fundamental asset. Eventually, the crowding-out effect might slow down capital accumulation enough to make conditions (3.1)-(3.2) jointly unattainable at some point in the future. Note that $e_{t+1|t}$ measures the income that the next generation of young is expected to receive at time $t + 1$, which depends on the total capital accumulated at time t into $t + 1$, and the share of the income produced with that capital that goes to young agents, represented by $\frac{1-\alpha}{\alpha}$. As $e_{t+1|t}$ is reduced, condition (3.1) implies a higher expected change in z_t, which can eventually result in (3.2) being violated.

In summary, for any given level of z_t, factors that increase the crowding-in effect or mitigate the crowding-out effect makes conditions (3.1)-(3.2) easier to achieve for all t. To isolate the role of such factors we turn the attention to a particular resting point of the dynamic system above.

STATIONARY STOCHASTIC EQUILIBRIUM (SSE)   The equations in Proposition 1 entirely describe the dynamics of the economy given some initial conditions on n_0, ρ_1 (or δ_0 in case $\rho_1 = \underline{a}$) and $\mathbb{E}_0(z_1)$, and as such identify a set of stochastic processes for the non-fundamental asset valuation that can be part of an equilibrium. We are interested in analyzing the sufficient conditions for such set to be non-empty, and characterize some basic features of the elements belonging to this set, such as the upper bound on the non-fundamental asset value. To do this we follow Weil (1987) and Kocherlakota (2009) and study a *stationary stochastic* non-fundamental equilibrium (SSE) of the dynamic system (3.1)-(3.3). More precisely, we focus on the case of a non-fundamental valuation z^* which is believed to disappear each period with some probability p, but which instead remains at the same stationary level z^*. This is a useful benchmark because it provides an upper bound for the expected value of a candidate stochastic process, conditional on the realization z_t, to be an equilibrium.[13]

To be more specific about the stochastic structure of the non-fundamental valuations, suppose that the state of the world at t, ω_t, can take two values, F and NF. If $\omega_t = F$ then the non-fundamental asset has no value and $z_t = z_t^A = 0$, irrespectively of whether the expected value from $t-1$, $\mathbb{E}_{t-1}(z_t)$, was positive or equal to zero. If the expected value was positive at $t-1$, at t the non-fundamental equilibrium collapses if $\omega_t = F$. If $\omega_t = NF$ then the non-fundamental asset can take a positive value, so that $z_t > 0$ and/or $z_t^A > 0$. The transition probabilities from the two states are such that the process is Markovian and they are defined as follows,

$$Pr(\omega_t = NF | \omega_{t-1} = F) = r \quad \text{and} \quad Pr(\omega_t = F | \omega_{t-1} = NF) = p.$$

Therefore, if $\omega_t = NF$ and $\mathbb{E}_t(z_{t+1}) > 0$, the probability of remaining in the non-fundamental equilib-

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[13]An alternative approach would be to study the derivative of $E_t(z_{t+1})$ with respect to z_t and make sure that it is smaller than 1 for $z_t = 0$. By continuity then there exist a z^* that provides an upper bound on the stochastic process for z_t to be a non-fundamental equilibrium. This is the approach taken by Martin and Ventura (2012). It can be showed that in our setting the conditions on the structural parameters for the derivative of $E_t(z_{t+1})$ being smaller than 1 is equivalent to a $z^* > 0$ existing. We choose the SSE approach because it allows a cleaner analysis of the effect of θ on equilibrium existence.

rium in $t + 1$ is equal to $1 - p$, which gives

$$z_{t+1} = \frac{\mathbb{E}_t(z_{t+1})}{1 - p}.$$

where $\mathbb{E}_t(z_{t+1})$ is restricted by the equilibrium return condition (3.1), and therefore a function of $\mathbb{E}_{t-1}(z_t)$. The SSE z^* is then the solution of the fixed point $\mathbb{E}_t(z_{t+1}) = \Phi(\mathbb{E}_{t-1}(z_t))$, where Φ is the mapping implied by the equilibrium conditions of Proposition 1.

We assume no continuous creation of new assets in steady state, so that eventually $z_t^A = 0$ for every t.[14] The SSE is then characterized by a vector $(z^*, \rho^*(\delta^* \text{ if } \rho^* = \underline{a}), n^*)$ from which all the other relevant variables can be derived. The following corollary provides the description of the SSE.

Corollary 1. *The Stationary Stochastic Equilibrium (SSE) of the closed economy is the solution to*

$$\frac{\delta^*}{1 - \theta} + (1 - p)U_\theta(\rho^*) = \frac{\alpha}{1 - \alpha}\frac{1}{1 - p}, \tag{3.4}$$

and

$$z^* = \left(\frac{1 - p}{2 - p}\right)\left[(1 - \delta^*) + (1 - p)G(\rho^*) - \theta\frac{\alpha}{1 - \alpha}\frac{1}{1 - p}\right] > 0, \tag{3.5}$$

with $\rho^* = \underline{a}$ when $\delta^* \in (0, 1]$.

For a SSE equilibrium to exist two conditions must be met. First, the non-fundamental asset must eventually grow at a rate that in expectations is the same rate at which wealth grows, and, for this to happen while providing a competitive return with other assets, equation (3.4) has to be satisfied. The left hand side of (3.4) is decreasing in ρ^* (and increasing in δ^*) so a SSE would fail to exist when

$$\frac{1}{1 - \theta} + (1 - p)U_\theta(\underline{a}) < \frac{\alpha}{1 - \alpha}\frac{1}{1 - p}. \tag{3.6}$$

Equation (3.6) suggests three reasons for failure of existence. First, the fraction of income that is appropriated by the new young generation might be too small, α too high, to allow them to keep purchasing the non-fundamental asset. Second, the probability of the non-fundamental valuation collapsing to zero, p, might be too high, which means that z has to grow faster to compensate for the event of a total loss

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[14]Appendix B studies the conditions under which $z_t^A > 0$ facilitates the existence of a non-fundamental equilibrium.

of value. Third, the "leverage potential" of the economy, as measured by θ, might be too low. We will return to the role of θ in the next section.

Even if condition (3.4) holds, a non-fundamental valuation equilibrium might still fail to exist if equation (3.5) is not satisfied. For a non-fundamental asset to have positive value there must be a saving demand in excess of loans assets. If loans as a share of the wealth in the economy, represented by $\theta \frac{\alpha}{1-\alpha} \frac{1}{1-p}$, are large enough, then there is no room in the financial market for the non-fundamental asset and a SSE will fail to exist.

Degree of Pledgeability θ and Existence of SSE

Corollary 1 suggests that the degree of pledgeability θ plays an ambiguous role in the existence of a non-fundamental valuation equilibrium. In this section we show that the existence of a SSE and the size of z^* have indeed a non-monotonic relationship with θ.[15]

For simplicity we assume that p is arbitrarily small (i.e. we set $p = 0$).[16] We first consider the case of θ being too small to allow a non-fundamental valuation equilibrium. Suppose that $\theta = 0$ and that condition (3.4) cannot be satisfied. This requires

$$1 + U_0(\underline{a}) < \frac{\alpha}{1 - \alpha}. \tag{3.7}$$

As θ is increased the left hand side of (3.7) is also increased since both young and adult agents can leverage off their internal funds. For a high enough θ the LHS can overcome the RHS, providing the first necessary condition for a SSE to exist. The intuition for this result lies on the role of θ for the crowding-in effect of $z^* > 0$ on capital accumulation. For any given $z^* > 0$, when $\theta = 0$ inefficient investment by the young generation is reduced and funds are transferred to the productive adult. When $\theta > 0$, the same $z^* > 0$ is enhancing the crowding-in effect. The reason for this is that adult agents with higher productivity are able to attract relatively more external funds for a given level of internal funds, a property that is a consequence of the non-linearity of U with respect to θA. In Section 2 we described the role of b_t as eliminating the inefficient investments and substituting them with efficient

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[15]Hirano and Yanagawa (2010) derive a similar result in a model with endogenous growth - output technology linear in capital - and infinitely lived agents. One key difference with their analysis is that while they focus on conditions for dynamic inefficiency in their economy, i.e. the relationship between the growth rate and the interest rate, we focus on the tension between the fundamental vs non-fundamental asset supply, which turns out to be a more general notion in our setting.

[16]This assumption is not crucial for the qualitative nature of the results since all the conditions are continuous in p at $p = 0$
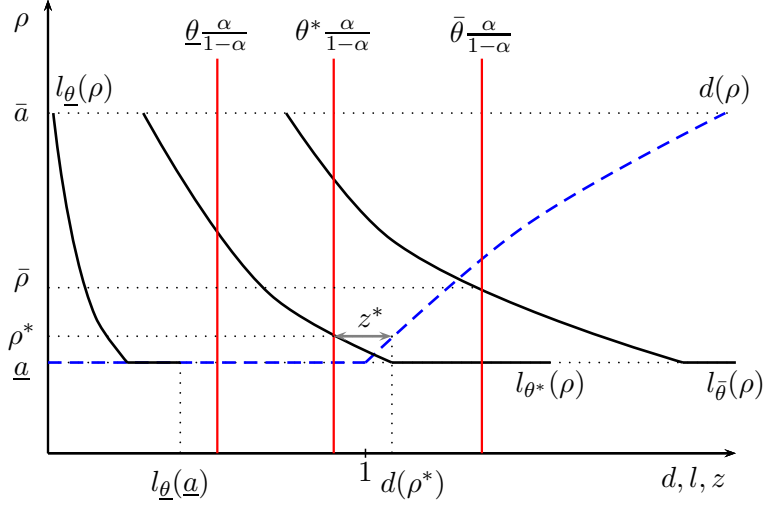
Figure 1: Metzler Diagram and Non-Fundamental Valuation

ones. An increase in the pledgeability parameter θ makes this selection process more effective, which means that for the same b_t a larger crowing-in effect is generated.

The increase in the efficiency by which z^* turns low productivity investments into high productivity ones, however, simultaneously generates an effect that works against the existence of a SSE. This is to be found on the *asset supply* side in equation (3.5). As θ is increased more loans assets are generated, which compete with the non-fundamental asset for the savings of the young and adult agents. Eventually, there will be enough fundamental asset supply that a $z^* > 0$ will no longer be possibly demanded by savers, and a non-fundamental equilibrium will cease to exist.

Figure 1 depicts the Metzler diagram for the closed economy that shows the case of three different increasing values for θ: $0 < \underline{\theta} < \theta^* < \bar{\theta}$. At $\underline{\theta}$ there is an excess savings demand that in principle could absorb the non-fundamental asset since the fundamental asset supply is at $l_{\underline{\theta}}(\underline{a}) < 1$, while savings demand can be anywhere between 0 and 1 as long as $\rho = \underline{a}$. However, any value for z will not be able to generate a crowding-in effect strong enough to remain affordable in the economy, which corresponds to the lack of an intersecting point for the fundamental supply curve $l_{\underline{\theta}}(\rho)$ and the minimal level of average investment efficiency necessary to sustain a non-fundamental asset valuation $\underline{\theta}\frac{\alpha}{1-\alpha}$ (recall that $l_{\theta}(\rho) = \theta\left(\frac{\delta}{1-\theta} + U_{\theta}(\rho)\right)$). When $\theta = \theta^*$, the crowding-in effect of the non-fundamental asset is larger

and the return on the non-fundamental asset can be sustained. As the graph shows, the fundamental supply curve $l_{\theta^*}(\rho)$ intersects $\theta^* \frac{\alpha}{1-\alpha}$ at $\rho^* > \underline{a}$. This is still not enough to guarantee the existence of an SSE, as the non-fundamental asset must find a demand from savers not satisfied by loans assets. From the diagram we see that at the relative return ρ^* there is an excess demand for deposits, $d(\rho^*) > \theta^* \frac{\alpha}{1-\alpha}$, and so a non-fundamental asset value $z^* > 0$ can indeed clear the financial market. Finally, when $\theta = \bar{\theta}$ the crowding-in effect is still strong enough for a non-fundamental equilibrium and the relative return is raised to $\rho = \bar{\rho}$. However, at that return the supply of fundamental assets is higher than the demand for deposits, $l_{\bar{\theta}}(\bar{\rho}) > d(\bar{\rho})$, which means that the financial market is already saturated by the fundamental supply and a SSE ceases to exist.

4   Equilibrium with Open Financial Markets

The global economy consists of two regions, "North" and "South", whose individual economies have the structure of the economy presented in Section 2. In what follows the variables with *tilde* refer to the South region. The two regions produce the same output good employing identical technologies. The only difference between the two economies is in their level of financial development measured by the degree of pledgeability: θ for the borrowers who operate the capital investment technology in the North and $\tilde{\theta}$ for those located in the South. We assume that $\theta > \tilde{\theta}$. We think of the degree of pledgeability as capturing the institutional environment in which loans are generated together with the ability of the financial intermediaries to evaluate investment projects. In this sense, a loan extended by an intermediary from a developed financial environment to a borrower located in a less developed one is subjected to some of the pledgeability restrictions of the institutions where the borrower is located since the recovery of the loan, in case of lack of repayment, has to take place in the location with weaker institutions. We capture this feature of the financial environment by assuming that for a financial intermediary from the North to extend a loan to an agent operating the investment technology in the South the degree of pledgeability is $\phi\theta$ with $\phi \in [0, 1]$. Thus, when financial markets are integrated, an investor from the South will be able to pledge the fraction $\theta_s = max\{\tilde{\theta}, \phi\theta\}$ of her future income from investment when borrowing from the financial intermediary.

Financial integration corresponds to the situation where the two regions can freely trade in financial assets and output. Once the investment in physical capital is made in a specific region, the capital

obtained can only be used in the output technology of that region, so capital goods are not directly tradable. The market for investment is nevertheless open, one unit of output good from the North can be directly invested in obtaining capital in the South, and viceversa. We assume that labor is not a mobile factor across regions. Therefore, under financial integration wages will not be equated across regions unless financial markets function smoothly and capital reaches the same level in the North and in the South.

Financial integration also means that the markets for the non-fundamental assets are integrated: intermediaries from both North and South can buy new non-fundamental assets from investing adults and can trade non-fundamental assets among them. We let the total value of the global non-fundamental asset at the beginning of period t be denoted by b_t^*.

Under financial integration the market clearing for the output good in the global equilibrium is

$$y_t + \tilde{y}_t = c_t + \tilde{c}_t + x_t + x_t^A + \tilde{x}_t + \tilde{x}_t^A$$

where $\tilde{y}_t$, $\tilde{c}_t$ denote output produced and consumption in the South, respectively, and $\tilde{x}_t$, $\tilde{x}_t^A$ is aggregate investment of young and adult agents in the South. Financial markets integration implies that intermediaries from the North and the South freely compete for deposits and loans. Free entry and the zero profit condition essentially mean that there is a representative global intermediary holding all the deposits and extending all the loans. The balance sheet of the global intermediary is

$$b_t^* + b_t^A + \tilde{b}_t^A + l_t + \tilde{l}_t = d_t + \tilde{d}_t, \tag{4.1}$$

where $\tilde{l}_t$ are the total loans extended to directly investing agents in the South and $\tilde{d}_t$ are deposits of saving agents from the South. The values of the specific components of the balance sheet of the global intermediary can be used to determine the flow of funds across regions, and, as a consequence, the current accounts of the two economies. Suppose that we are in a fundamental equilibrium so that all the non-fundamental assets have a value of zero. In the closed economy it must be $\tilde{l}_t = \tilde{d}_t$, so that all the domestic savings are channeled towards domestic investments. However, because of the financial frictions the asset supply from loans are limited and agents are forced to directly invest instead of depositing funds in intermediated savings. When financial markets are open, the saving demand of the South can be satisfied by the asset supply of the more financially developed North so that $\tilde{l}_t < \tilde{d}_t$, and

funds are now channeled from the South to the North. In the presence of a non-fundamental asset, the difference between the expected return and the realized return is assumed to be distributed across depositors from the North in fraction μ and from the South in fraction $1 - \mu$. Within each region, the difference is distributed across young and adult agents according to the fraction φ and $1 - \varphi$ in the North (resp. $\tilde{\varphi}$ and $1 - \tilde{\varphi}$ in the South).

The global equilibrium is characterized by a world financial interest rate R_{t+1}^* that clears the market for financial assets. Symmetrically to the closed economy analysis we define the relative financial returns for the two regions as $\rho_{t+1} = \frac{R_{t+1}^*}{R_{t+1}}$ and $\tilde{\rho}_{t+1} = \frac{R_{t+1}^*}{\tilde{R}_{t+1}}$. The presence of financial frictions in both regions, if severe enough, can prevent the return on capital from being equated across the economies. For example, if $\rho_{t+1} > \tilde{\rho}_{t+1}$, financial frictions are keeping funds from flowing from the North to the South, or, alternatively, they are channeling savings from the South to the North. We let q_{t+1} capture the tension between North and South in terms of net flow of funds, where

$$q_{t+1} \equiv \frac{\tilde{\rho}_{t+1}}{\rho_{t+1}} = \frac{R_{t+1}}{\tilde{R}_{t+1}}. \tag{4.2}$$

If a unit of output could be freely allocated across the regions, the value of q_{t+1} would provide the information necessary for an efficient allocation: when $q_{t+1} > 1$ the extra unit should be allocated to the North and when $q_{t+1} < 1$ to the South.

The definition of an equilibrium for the global economy is analogue to the definition of an equilibrium for the individual economy of Section 2, and so we omit its statement. To characterize the global equilibrium conditions recursively we define the fractions of wealth of each economy held by the adults as in Section 3 and denote them by n_t and $\tilde{n}_t$. In addition, we let v_t represent the relative size of net wealth of the South economy with respect to the North, where $v_t = \frac{\tilde{W}_t}{W_t}$. Next, we define the non-fundamental asset values in terms of the net wealth in the North at time t as expected at $t - 1$, so that $z_t^* \equiv \frac{b_t^*}{W_{t|t-1}}$, $z_t^A \equiv \frac{b_t^A}{W_{t|t-1}}$, $\tilde{z}_t^A \equiv \frac{\tilde{b}_t^A}{W_{t|t-1}}$. In presence of a positive value for the non-fundamental asset, the global economy is subjected to global stochastic fluctuations defined as $\sigma_t^* \equiv z_t^* - \mathbb{E}_{t-1}(z_t^*)$. The definition for $e_{t+1|t}$ is as in Section 3. Here we define the analogue variable for the South economy,

$$\tilde{e}_{t+1|t} \equiv \frac{1 - \alpha}{\alpha} \left[\frac{\tilde{\delta}_t}{1 - \theta_s} (1 - \tilde{n}_t) v_t + U_{\theta_s} (\rho_{t+1} q_{t+1}) \left(\tilde{n}_t v_t + \frac{\tilde{z}_t^A}{1 + \mu \varphi \sigma_t^*} \right) \right], \tag{4.3}$$

which represents the wealth of the young agents at time $t + 1$ as expected at time t in terms of the

North wealth at time t. The wealth of the adult agents at time $t+1$ as expected at time t in terms of the North wealth at time t is $v_t(1 - \tilde{n}_t)$. Finally, the expected relative size of wealth in the two regions is

$$v_{t+1|t} = \frac{v_t(1 - \tilde{n}_t) + \tilde{e}_{t+1|t}}{1 - n_t + e_{t+1|t}}.$$

The following proposition characterizes the dynamic equilibrium of the global economy.

Proposition 2. *The non-negative stochastic process $\{z_t^*, z_t^A, \tilde{z}_t^A\}_{t=0}^{\infty}$ and the sequence $\{n_t, \rho_t, \tilde{n}_t, q_t, v_t\}_{t=0}^{\infty}$ constitute an equilibrium of the global economy with integrated financial markets if the following conditions are satisfied:*

(a) expected return of non-fundamental asset

$$\mathbb{E}_t(z_{t+1}^*) = \frac{z_t^* + z_t^A(1 - G(\rho_{t+1})) + \tilde{z}_t^A(1 - G(\rho_{t+1}q_{t+1}))}{e_{t+1|t} + 1 - n_t} \frac{1}{1 + \mu\varphi\sigma_t^*}, \tag{4.4}$$

(b) asset market clearing

$$\frac{1}{1 + \mu\varphi\sigma_t^*}\left[z_t^* + z_t^A(1 - G(\rho_{t+1})) + \tilde{z}_t^A(1 - G(\rho_{t+1}q_{t+1}))\right] + \frac{\alpha}{1 - \alpha}\left[\theta e_{t+1|t} + \theta_s\tilde{e}_{t+1|t}\right] = \tag{4.5}$$

$$(1 - n_t)(1 - \delta_t) + n_t G(\rho_{t+1}) + v_t\left[(1 - \tilde{n}_t)(1 - \tilde{\delta}_t) + \tilde{n}_t\tilde{G}(\rho_{t+1}q_{t+1})\right], \tag{4.6}$$

(c) capital return inequality and size inequality

$$q_{t+1} = \left(\frac{\tilde{e}_{t+1|t}}{e_{t+1|t}}\right)^{\frac{1-\alpha}{\alpha}}, \qquad (1 + \mu\varphi\sigma_{t+1}^*)v_{t+1} = v_{t+1|t} + (1 - \mu)\tilde{\varphi}\sigma_{t+1}^* \tag{4.7}$$

(d) intergenerational realized wealth distribution

$$(1 + \mu\varphi\sigma_{t+1}^*)n_{t+1} = \frac{1 - n_t}{1 - n_t + e_{t+1|t}} + \mu\varphi\sigma_{t+1}^*, \tag{4.8}$$

$$\left(1 + \frac{1 - \mu}{v_{t+1|t}}\tilde{\varphi}\sigma_{t+1}^*\right)\tilde{n}_{t+1} = \frac{v_t(1 - \tilde{n}_t)}{\tilde{e}_{t+1|t} + v_t(1 - \tilde{n}_t)} + (1 - \mu)\tilde{\varphi}\sigma_{t+1}^*, \tag{4.9}$$

Proof. See Appendix. □

The conditions of Proposition 2 parallel those for the closed economy case. Aside from the evolution of q_{t+1} and v_{t+1}, which are essentially equilibrium accounting, the key difference lies in the asset market clearing condition, which now equates the global demand for savings with the global supply of both fundamental and non-fundamental assets.

5 Non-Fundamental Equilibrium in the Global Economy

In this Section we formally address the question raised in the introduction: can financial markets integration increase the subjectability of the global economy to the emergence of non-fundamental dynamics? To answer this question we proceed as we did in Section 3 and focus on the characterization of the stochastic stationary steady state of the global economy (GSSE), under the assumption that no new non-fundamental assets are created in steady state and that $p = 0$. To show that financial markets integration can open the door to non-fundamental dynamics in the global economy, we consider the case in which a non-fundamental steady state does not exist for the individual economies under autarky. We know from Section 3 that this is possible if the degree of pledgeability is high enough, e.g. in the North economy, or low enough, e.g. in the South economy. Next we show that under the same parameter values that ensure non-existence of non-fundamental steady states in autarky, a non-fundamental steady state is indeed possible when financial markets integrates.

The next corollary characterizes the GSSE for the global economy.

Corollary 2. *Suppose that $\phi\theta \geq \tilde{\theta}$. The GSSE of the global economy (z^*, ρ^*, q^*, v^*) is the solution to*

$$\frac{\delta^*}{1-\theta} + U_\theta(\rho^*) = \frac{\alpha}{1-\alpha} \tag{5.1}$$

where $\rho^* = \underline{a}$ when $\delta^* \in (0,1]$,

$$\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*) = \frac{\alpha}{1-\alpha} \tag{5.2}$$

where $q^*\rho^* = \underline{a}$ when $\tilde{\delta}^* \in (0,1]$, and

$$z^* = \frac{1}{2}\left[(1 - \delta^*) + G(\rho^*) + v^*\left[(1 - \tilde{\delta}^*) + G(q^*\rho^*)\right] - \theta\frac{\alpha}{1-\alpha}\left(1 + \phi v^*\right)\right] > 0, \qquad (5.3)$$

with $v^* = q^{*\frac{\alpha}{1-\alpha}}$.

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Conditions (5.1) and (5.3) parallel those of Corollary 1 for the closed economy. Condition (5.2) is, however, not immediately comparable to Corollary 1 since the dynamics of z_t^* are defined in terms of wealth in the North. The reason for equation (5.2) to be necessary in a GSSE is that we are imposing a constant ratio for the wealth of the two regions, v^*, in steady state. At z^*, for the non-fundamental asset to remain affordable and the relative size of the two economies to remain constant, (5.2) must necessarily hold.

In Proposition 2 we have assumed that the financial development of the North is strictly higher compared to the South. To simplify the analysis further without much loss of generality we assume that investing agents in the South cannot pledge any portion of their future income.

Assumption 1. $\tilde{\theta} = 0$.

An immediate consequence of Assumption 1 is that, in order for the South region to not allow non-fundamental valuation equilibria in autarky, the following condition is sufficient

Assumption 2. $1 + U_0(\underline{a}) < \frac{\alpha}{1-\alpha}$.

In the North economy $\theta > 0$ and we know from Section 3 that to ensure that a non-fundamental valuation equilibrium does not exist it is sufficient to assume that at the relative return that would ensure a sustainable non-fundamental asset value the supply of fundamental assets is already enough to satisfy the intermediated saving demand. Formally,

Assumption 3. $1 + G(\rho) \leq \theta\frac{\alpha}{1-\alpha} \quad for \quad \rho : \theta U_\theta(\rho) = \frac{\alpha}{1-\alpha}\theta$.

Can the conditions in Corollary 2 be satisfied with $z^* > 0$ under Assumptions 1-3? Condition (5.1) is implied by Assumption 3, and so it will hold for some $\rho^* \geq \underline{a}$. Condition (5.2) for q^* requires some additional elaboration. Consider first the case where $\phi = 0$, so that the degree of pledgeability of

investment returns from the South is not changed by financial integration. Then under Assumption 2, the condition in (5.2) cannot possibly hold, since by construction $q^* \rho^* \geq \underline{a}$. As $\phi > 0$ the left hand side of (5.2) is increased for any value of $q^* \rho^*$ and so for ϕ large enough condition (5.2) will eventually hold. Intuitively, for a positive non-fundamental asset value to remain sustainable in the global economy, there needs to be a minimal level of leverage potential for productive investors in the South region so to avoid the relative size of the region becoming arbitrarily small, i.e. $v^* \to 0$. For $\phi < 1$ conditions (5.1) and (5.2) imply that $q^* < 1$. In a GSSE of the global economy, as long as there is financial development heterogeneity, there will remain a difference in the marginal return on physical capital across regions. Note that this statement is conditional on a GSSE existing, which means that the financial development overall is already not enough to channel funds from savers to investors. The final requirement for a GSSE to exist for the global economy is that $z^* > 0$. Under Assumption 3 the total demand for savings of the North in the GSSE is entirely satisfied by the North supply of fundamental assets. For the non-fundamental asset to have a positive value then it is necessary that the financial integration generates a demand for intermediated savings from the South and that such demand is high enough that it absorbs the supply of fundamental assets in excess of the demand for savings of the North *and* the supply of fundamental assets due to the borrowing of the South, which was not possible in autarky.

Intuitively, for a non-fundamental equilibrium of the global economy to exist, the North region must be already close to a situation of asset supply "shortage". To make this as transparent as possible, suppose that the North economy is parameterized with θ, α and $\underline{a}$ such that $U_\theta(\underline{a}) = \frac{\alpha}{1-\alpha}$ and $\theta \frac{\alpha}{1-\alpha} = 1$. Then in autarky the North region would have a non-fundamental asset value exactly equal to zero in SSE since $\rho = \underline{a}$ and $\theta U_\theta(\underline{a}) = 1$. As financial integration happens, for ϕ high enough it will be possible that $\tilde{\delta}^* > 0$ such that (5.2) is satisfied. The higher is ϕ the higher will be the fraction of young agents in the South demanding intermediated savings, so that $\tilde{\delta}^*(\phi)$ defined by (5.2) is a decreasing function of ϕ. The condition for a GSSE in this specific case then boils down to

$$1 - \tilde{\delta}^*(\phi) > \phi. \tag{5.4}$$

Equation (5.4) captures the tension between asset demand and supply for the existence of a non-fundamental valuation equilibrium in the global economy. The left hand side of the inequality is the saving demand from the South, which is increasing in ϕ. The right hand side represents the fundamental
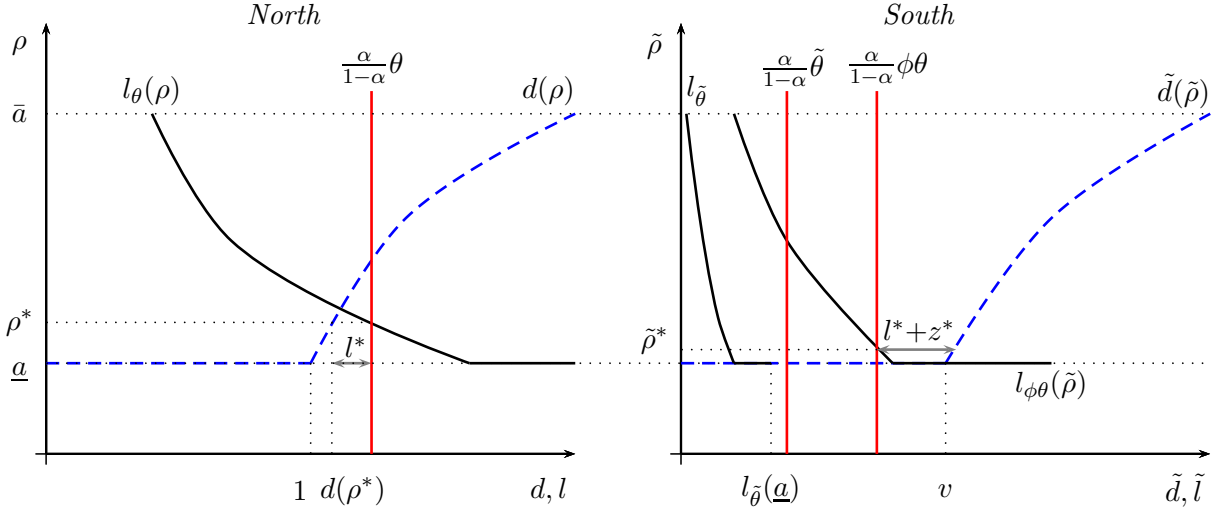
Figure 2: METZLER DIAGRAM AND GLOBAL ECONOMY SSE

asset supply generated by loans to the South, which is also increasing in ϕ. As long as the saving demand is larger than the supply, there is room for a non-fundamental asset. The inequality is not satisfied for $\phi = 0$ and $\phi = 1$, while it can hold for intermediate values. The same non-monotonic relationship between the degree of pledgeability and the existence of a non-fundamental equilibrium in the closed economy directly extends to the global economy, where it takes the form of non-monotonicity with respect to the degree of financial integration across regions ϕ. The following proposition summarizes our result.

Proposition 3. *Suppose that Assumptions 1-3 hold. Then, when in autarky both the North and the South economy do not allow for episodes of non-fundamental dynamics in financial assets. Episodes of non-fundamental dynamics become possible when financial markets are integrated if: (i) the North economy is relatively close to a condition of asset supply shortage in autarky; (ii) the degree of financial integration, measured in terms of the increase in pledgeability of investment income in the South, is intermediate.*

Figure 2 is the Metzler diagram for the global economy. In autarky a SSE is not possible in the North since at ρ^* the asset supply is bigger than the savings demand, while it is not possible in the South since $\tilde{l}_{\tilde{\theta}}(\underline{a}) < \frac{\alpha}{1-\alpha}\tilde{\theta}$. In a fundamental equilibrium of the integrated economy the Metzler diagram instructs one to look for the values of ρ and $\tilde{\rho}$ that ensure equality between the excessive supply of fundamental asset in the North economy, $l_{\theta}(\rho) - d(\rho) > 0$, and the shortage of fundamental assets in

the South economy, $l_{\phi\theta}(\tilde{\rho}) - \tilde{d}(\tilde{\rho}) < 0$. In presence of different financial developments, the fundamental equilibrium happens at $\rho > \tilde{\rho}$ since the marginal return on capital is higher in the South. In a GSSE with $z^* > 0$ the values for ρ^* and $\tilde{\rho}^*$ are determined by conditions (5.1) and (5.2), and they correspond to the points in the figure where the fundamental asset supply intersects $\frac{\alpha}{1-\alpha}\theta$ in the North and $\frac{\alpha}{1-\alpha}\phi\theta$ in the South. The Metzler diagram can then be used to check whether the excess fundamental asset supply in the North at ρ^*, given by $l^* = \frac{\alpha}{1-\alpha}\theta - d(\rho^*)$, is smaller than the excess savings demand from the South at $\tilde{\rho}^*$, given by $\tilde{d}(\tilde{\rho}^*) - \frac{\alpha}{1-\alpha}\phi\theta$. This is the case in the figure where a GSSE with $z^* > 0$ indeed exists.

We have focused on purpose on the starkest case of non-existence of non-fundamental dynamics in autarky and existence once the regions are financially integrated. A more general version of our result holds in terms of the maximum non-fundamental asset valuation that can be sustained in equilibrium in autarky as opposed to the case of financial integration. Suppose that both the North and the South allow for episodes of non-fundamental dynamics in autarky with upper bounds of $z_a^* \geq 0$ and $\tilde{z}_a^* \geq 0$ respectively, both measured in terms of steady state output of the North in autarky. It is then possible to show that under financial integration the maximum non-fundamental valuation can be higher than the sum of the two, namely $z^* > z_a^* + \tilde{z}_a^* \geq 0$.

6 GLOBAL EQUILIBRIUM DYNAMICS: NUMERICAL SIMULATION

In this section we study by numerical simulations the dynamics of asset values, output and interest rates in the global economy under financial integration. We are interested in understanding whether episodes of asset price fluctuations qualitatively similar to those reported in Figure 5 can be generated within the model we have presented once financial markets are integrated, even though the same episodes were not possible in autarky. When financial integration happens, the steady state capital levels of the individual regions in the global economy change as the incidence of financial frictions is different in the open economy. For the purpose of this section we do not focus on the adjustment path towards the new fundamental steady state, but we assume that as the global economy is integrated the new fundamental steady state is immediately achieved and non-fundamental asset prices valuations happen around the new steady state. We assume that the productivity distribution for adult agents is uniform between

$[\underline{a}, \bar{a}]$.[17] While in autarky, investors both in the North and in the South experience sentiment shocks that would allow them to sell a non-fundamental asset at a positive value. However, the structure of the economy is such that the non-fundamental value will never be rationally believed to be feasible in equilibrium and nobody will be willing to buy the asset. In the global economy the increased financial development of the South and the more stringent asset supply shortage provide the conditions for the sentiment shocks to affect asset prices, as they are now rationally believed to be sustainable. The sentiment shocks that were dormant in autarky can now actively affect the aggregates of the global economy.

We simulate the equilibrium dynamics of the global economy under the following parameterization: $\theta = .36$, $\alpha = .73$, $\phi = .6$, $\underline{a} = .5$, $\bar{a} = 1.5$. We assume that the probability of going from the fundamental state to the non-fundamental state in every period is $r = .05$, while the probability of going from the non-fundamental state back to the fundamental state is set at $p = .15$. We also assume that during a non-fundamental episode there is creation of new non-fundamental supply of $z_t^A = 0.002$ and $\tilde{z}_t^A = 0.001$ every period. This is not necessary for the existence of a non-fundamental equilibrium, but it generates some interesting properties of the dynamics of the key macroeconomic aggregates.

The choice of the functional forms and parameterization is not done with the intention to calibrate the numerical simulation to the actual economy. Our main interest is in understanding whether the model can generate the qualitative features outlined in the introduction.[18] Under the chosen parameterization both the North and the South regions in autarky would not allow for non-fundamental episodes, so $z_a^* = \tilde{z}_a^* = 0$. The financially integrated global economy would instead allow non-fundamental valuation episodes up to $z^* = .08$, or 8% of total wealth in the North region.[19]

Figure 6 shows the behavior of the simulated global equilibrium under a realization for the stochastic process of investors sentiments that is within the existence conditions of Proposition 2. The global

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[17]By setting $G(\rho) = \frac{\rho - \underline{a}}{\bar{a} - \underline{a}}$ the leverage function becomes

$$U_\theta(\rho) = -\frac{1}{(\bar{a} - \underline{a})\theta}\left[\bar{a} - \rho + \frac{\rho}{\theta}\log\left(\frac{1 - \bar{a}\theta/\rho}{1 - \theta}\right)\right].$$

These are also the functional forms used to draw the diagrams in Figures 1 and 2.

[18]For instance, both the assumption of uniform distribution of productivity for adult agents, and the symmetry of this distribution across regions are imposing unnecessary strong restrictions on the quantitative potential of our model.

[19]To provide a ballpark figure for the size of the non-fundamental value, households net worth in the United States in 2012 was estimated to be $66 Trillion (Federal Reserve Flow of Funds), making the North region correspond with the US Economy would give an upper bound of around $6 Trillion for the non-fundamental asset valuation. The values for z_a^* and $\tilde{z}_a^*$ have been calculated using the expressions in Corollary 1, the value for z^* has been computed using a version of the equations in Corollary 2 with $p > 0$.

economy is at the fundamental state for the first part of the simulation. At period $t = 33$ the state changes to non-fundamental and a new asset begins to be exchanged in the global financial markets. The increased total availability of assets draws funds from young and possibly adult agents in the South to investors in the North. Note that the non-fundamental asset is held by both North and South savers in measure μ and $1 - \mu$. For this simulation we set $\mu = .5$. The funds drawn into the intermediated savings market are channeled towards the North and so output and investment fall in the South. As expected, the global financial interest rate increases as the supply of assets is increased. The savers in both regions take advantage of the higher financial return and increase their consumption when old. The funds from the savers of the South are invested by productive adults in the North, where a boom in both investment and output takes place. The boom in output makes the increase in consumption in both regions feasible. The trade balance of the North is deteriorating during the non-fundamental episode because resources are being transferred from the South to the North. The flip side of the deficit in the North is that the South is running an increasing trade balance surplus. The relative return on capital q is smaller than 1 in the fundamental steady state, due to the lower financial development in the South. During the non-fundamental episode it falls even further, as funds are pulled from investment in the South and invested in capital in the North region.

When the non-fundamental episode ends, in period $t = 41$, there is a "forced" transfer of wealth from the holders of the non-fundamental asset to the next young generation, which cannot buy the asset anymore. The direction of the wealth transfer depends on who is holding the asset at the time of the collapse. In the numerical simulation it is assumed that the adjustment is equally sustained by North and South savers, which means a drop in consumption in both regions. The disappearance of the asset also drastically reduces the transfer of resources from the South to the North, which creates a sudden but transitory reversal in the trade balance. Both output and consumption fall sharply in the North at the time of the collapse and then begin a slow recovery towards the fundamental steady state level. Finally, output (and thus investment) in the South experiences a sudden upward reversal during a non-fundamental asset collapse since the funds of the young generation that would have been transferred to the North now have to be invested in the South. Accordingly, the global financial interest rate drops back to the fundamental steady state level and the capital return differential increases back to the initial value. In period $t = 63$ another non-fundamental episode starts and a patter similar to the first episode is once again observed.

7 CONCLUSION

We presented a stylized equilibrium model of the global economy and studied the conditions for shocks to investors sentiments to translate into non-fundamental assets valuations that can have real effects on output and consumption when financial markets integrate.

In the global equilibrium with integrated financial markets the non-fundamental asset is held by the intermediaries and owned, indirectly, by the savers of the North and of the South. Our model captures the allocation of the risk related to the sudden loss of value of the asset in very reduced-form through the parameter μ. We can think of μ as identifying the holders or the "location" of the non-fundamental asset. For example, when $\mu = 1$ any unexpected change in the value of the asset will affect the value of the deposits that North savers can claim on the intermediary, while the South savers are shielded by such direct risk. In this case we can say that the non-fundamental asset is located in the North region. Note that the location of the asset has nothing to do with where the asset originated in the first place. A new non-fundamental asset sold by a productive adult from the South is effectively located in the North if held by a saver in the North.

In our model a non-fundamental valuation is fueled by the demand of savers from the South, but in the end the savers in the North could end up holding the asset. Arguably, this is the sort of global financial imbalance that developed during the real estate bubble of the early 2000's in the global economy. The strong surge in demand for safe assets from emerging economies, such as China, provided the resources to make a non-fundamental asset affordable, but because such demand was mainly satisfied by purchase of fundamental assets from the US, primarily in the form of US Treasuries, the US savers ended up holding the non-fundamental asset, in the form of overvalued real estate. The version of the model presented in this paper abstracts from risk considerations, but it already contains the components to formally analyze episodes of global financial imbalances and non-fundamental valuations, such as the one just described, if some degree of risk aversion is introduced. We leave this to future work.

8 APPENDIX

8.1 DERIVATION OF RECURSIVE REPRESENTATION   Using the definitions in the text it is easy to see that

$$\frac{b_t}{W_t} = \frac{b_t}{W_{t|t-1}} \frac{W_{t|t-1}}{W_t} = \frac{z_t}{1 + \varphi \sigma_t}. \tag{8.1}$$

The evolution of total capital in the economy is

$$k_{t+1} = k_{t+1}^A + k_{t+1}^Y = w_t \underline{a} \frac{\delta_t}{1-\theta} + \rho_{t+1}(w_t^A + b_t^A) U_\theta(\rho_{t+1}), \tag{8.2}$$

where $\delta_t = 0$ when $\rho_{t+1} > \underline{a}$. Using the definition for n_t one has that

$$\frac{k_{t+1}}{W_t} \frac{1}{\rho_{t+1}} = (1 - n_t) \frac{\delta_t}{1-\theta} + \left(n_t + \frac{z_t^A}{1 + \varphi \sigma_t} \right) U_\theta(\rho_{t+1}). \tag{8.3}$$

Adult agents wealth at $t+1$ as expected at time t is $w_{t+1|t}^A = R_{t+1}^a w_t$ which results in

$$\frac{w_{t+1|t}^A}{R_{t+1}^a} \frac{1}{W_t} = 1 - n_t \tag{8.4}$$

The next period total wealth expected at time t can be written as

$$W_{t+1|t} = R_{t+1}^a \left(\frac{w_{t+1|t}^A}{R_{t+1}^a} + \frac{w_{t+1}}{R_{t+1}^a} \right) = R_{t+1}^a \left(\frac{w_{t+1|t}^A}{R_{t+1}^a} + \frac{w_{t+1}}{R_{t+1}} \frac{1}{\rho_{t+1}} \right) \tag{8.5}$$

Using the output technology under competitive returns

$$\frac{R_{t+1}}{w_{t+1}} = \frac{\alpha}{1-\alpha} \frac{1}{k_{t+1}}. \tag{8.6}$$

Combining expressions it follows that

$$\frac{W_{t+1|t}}{W_t} = R_{t+1}^a \left(1 - n_t + \frac{k_{t+1}}{W_t} \frac{1-\alpha}{\alpha} \frac{1}{\rho_{t+1}} \right) \tag{8.7}$$

This suggests the definition used in the main text for $e_{t+1|t}$,

$$e_{t+1|t} \equiv \frac{k_{t+1}}{W_t} \frac{1-\alpha}{\alpha} \frac{1}{\rho_{t+1}} = \frac{1-\alpha}{\alpha} \left[\frac{\delta_t}{1-\theta}(1 - n_t) + U_\theta(\rho_{t+1}) \left(n_t + \frac{z_t^A}{1 + \varphi \sigma_t} \right) \right]. \tag{8.8}$$

8.2 PROOF OF PROPOSITION 1   The return on the non-fundamental asset R_{t+1}^b has to be equal to the return on the fundamental asset R_{t+1}^a in a non-fundamental equilibrium, which means that

$$\mathbb{E}_t(b_{t+1}) = R_{t+1}^a \big(b_t + b_t^A(1 - G(\rho_{t+1})) \big). \tag{8.9}$$

Multiply both sides by $W_{t|t-1}W_t/W_{t+1|t}$ and using the expressions above one can immediately obtain (3.1). The asset market clearing condition is

$$l_t(\rho_{t+1}) + b_t + (1 - G(\rho_{t+1}))b_t^A = d_t(\rho_{t+1}). \tag{8.10}$$

Dividing both sides by W_t and using expressions (2.14) and (2.15) together with (8.1) expression (3.2) follows. Finally, the evolution of n_t is obtained using

$$n_{t+1} = \frac{w_{t+1}^A}{W_{t+1}} = \frac{w_{t+1|t}^A + \varphi(b_{t+1} - \mathbb{E}_t(b_{t+1}))}{W_{t+1|t} + \varphi(b_{t+1} - \mathbb{E}_t(b_{t+1}))} = \frac{\frac{w_{t+1|t}^A}{W_{t+1|t}} + \varphi(z_{z+1} - \mathbb{E}_t(z_{t+1}))}{1 + \varphi(z_{t+1} - \mathbb{E}_t(z_{t+1}))}, \tag{8.11}$$

and recognizing that

$$\frac{w_{t+1|t}^A}{W_{t+1|t}} = \frac{w_{t+1|t}^A}{W_t}\frac{W_t}{W_{t+1|t}} = \frac{1 - n_t}{1 - n_t + e_{t+1|t}} \tag{8.12}$$

Combining the expressions (3.3) follows.

8.3 PROOF OF COROLLARY 1   In SSE $\sigma_t = 0$ and $z_t^A = 0$ so the conditions of Proposition 1 under $z^* > 0$ become

$$(\rho^*):\qquad \frac{1-\alpha}{\alpha}\left[(1-n^*)\frac{\delta^*}{1-\theta} + n^*U_\theta(\rho^*)\right] + (1-n^*) = \frac{1}{1-p} \tag{8.13}$$

$$(z^*):\qquad \frac{z^*}{1-p} + \theta\left[(1-n^*)\frac{\delta^*}{1-\theta} + n^*U_\theta(\rho^*)\right] = (1-n^*)(1-\delta^*) + n^*G(\rho^*) \tag{8.14}$$

$$(n^*):\qquad \left[\frac{\delta^*}{1-\theta} + \frac{n^*}{1-n^*}U_\theta(\rho^*)\right] = \frac{\alpha}{1-\alpha}\frac{1-n^*}{n^*} \tag{8.15}$$

$$\tag{8.16}$$

where $\delta^* = 0$ when $\rho^* > \underline{a}$. Substituting (ρ^*) into (n^*) one obtains

$$\frac{1}{1-n^*}\left(\frac{1}{1-p} - (1-n^*)\right) = \frac{1-n^*}{n^*}. \tag{8.17}$$

Rearranging gives

$$\frac{n^*}{1-n^*} = 1-p \quad \text{and} \quad n^* = \frac{1-p}{2-p}. \tag{8.18}$$

Substituting n^* back into (ρ^*) expression (3.4) follows. Expression (3.5) is similarly obtained by substituting n^* and (8.18) into (z^*).

FUNDAMENTAL STEADY STATE   For completeness we report the characterization of a fundamental steady state. Setting $z^* = 0$ and disregarding (ρ^*), the fundamental steady state variables $\hat{n}$ and $\hat{\rho}$ are given by

$$(\hat{\rho}): \qquad \theta\left((1-\hat{n})\frac{\hat{\delta}}{1-\theta} + \hat{n}U_\theta(\hat{\rho})\right) = (1-\hat{n})(1-\hat{\delta}) + \hat{n}G(\hat{\rho}) \tag{8.19}$$

$$(\hat{n}): \qquad \frac{\hat{n}}{1-\hat{n}}\left(\frac{\hat{\delta}}{1-\theta} + \frac{\hat{n}}{1-\hat{n}}U_\theta(\hat{\rho})\right) = \frac{\alpha}{1-\alpha}, \tag{8.20}$$

where $\hat{\delta} = 0$ when $\hat{\rho} > \underline{a}$.

8.4 PROOF OF PROPOSITION 2   Under the definition given in the main text we have that

$$\frac{\tilde{b}_t^A}{\tilde{W}_t} = \frac{\tilde{z}_t^A}{v_t(1+\mu\varphi\sigma_t^*)}. \tag{8.21}$$

The following expressions can then be derived

$$\frac{\tilde{k}_{t+1}}{\tilde{W}_t}\frac{1}{\tilde{\rho}_{t+1}} = \left((1-\tilde{n}_t)\frac{\tilde{\delta}_t}{1-\theta_s} + \left(\tilde{n}_t + \frac{\tilde{z}_t^A}{v_t(1+\mu\varphi\sigma_t^*)}\right)U_{\theta_s}(\tilde{\rho}_{t+1})\right) \quad \text{and} \quad \frac{\tilde{w}_{t+1|t}^A}{R_{t+1}^*}\frac{1}{\tilde{W}_t} = (1-\tilde{n}_t), \tag{8.22}$$

where $\tilde{\delta}_t = 0$ when $\tilde{\rho}_{t+1} > \underline{a}$. The expression on the left can be used to define $\tilde{e}_{t+1|t}$ as we did for the closed economy case, so that (4.3) follows. For v_t one has

$$v_t \equiv \frac{\tilde{W}_t}{W_t} = \frac{\tilde{W}_{t|t-1} + (1-\mu)\tilde{\varphi}(b_t - \mathbb{E}_{t-1}(b_t))}{W_{t|t-1} + \mu\varphi(b_t - \mathbb{E}_{t-1}(b_t))} = \frac{\tilde{W}_{t|t-1}}{W_{t|t-1}}\left(\frac{1 + (1-\mu)\tilde{\varphi}\frac{W_{t|t-1}}{\tilde{W}_{t|t-1}}\sigma_t^*}{1+\mu\varphi\sigma_t^*}\right). \tag{8.23}$$

It is straightforward to show that

$$v_{t+1|t} \equiv \frac{\tilde{W}_{t+1|t}}{W_{t+1|t}} = \frac{\tilde{e}_{t+1|t} + v_t(1-\tilde{n}_t)}{e_{t+1|t} + 1 - n_t}, \tag{8.24}$$

and the equation for the evolution of v_{t+1} immediately follows. For q_{t+1}, given the function form of the output technology $q_{t+1} = \left(\frac{k_{t+1}}{\tilde{k}_{t+1}}\right)^{\alpha-1}$. Using the expressions for k_{t+1} and $\tilde{k}_{t+1}$ derived above and recalling that

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$q_{t+1}\rho_{t+1} = \tilde{\rho}_{t+1}$ one gets

$$q_{t+1} = \left(\frac{e_{t+1|t}}{q_{t+1}\tilde{e}_{t+1|t}}\right)^{\alpha-1}, \tag{8.25}$$

from which the left hand expression in (4.7) follows. The non-fundamental asset return and the market clearing condition are immediate consequences of the definition of the variables for the integrated economy and so we omit their derivation. The evolution of n_t and $\tilde{n}_t$ can be also derived by using the above expressions for $W_{t|t-1}$ and $\tilde{W}_{t|t-1}$ and so on, as was done for the closed economy case.

8.5 Proof of Corollary 2

Setting $z_t^A = \tilde{z}_t^A = \sigma_t^* = 0$ and $z_t = z^*$ in (4.4) one gets

$$\frac{1-\alpha}{\alpha}\left(\frac{\delta^*}{1-\theta} + U_\theta(\rho^*)\frac{n^*}{1-n^*}\right) = 1. \tag{8.26}$$

From the steady state version of the equation for v_{t+1} the above expression implies

$$\frac{1-\alpha}{\alpha}\left(\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*)\frac{\tilde{n}^*}{1-\tilde{n}^*}\right) = 1. \tag{8.27}$$

Using the steady state version of (4.8) gives $\frac{n^*}{1-n^*} = 1$, or $n^* = \frac{1}{2}$, and similarly $\frac{\tilde{n}^*}{1-\tilde{n}^*} = 1$. Substituting these relationships in (8.26) and (8.27), conditions (5.1) and (5.2) follow. Using these two conditions the steady state version of the equation for q_{t+1} immediately simplifies to $q^* = (v^*)^{\frac{1-\alpha}{\alpha}}$ which gives $v^* = (q^*)^{\frac{\alpha}{1-\alpha}}$. Finally, the asset market clearing condition in steady state is

$$z^* = \frac{1}{2}\left((1-\delta^*) + G(\rho^*) + v^*(1-\tilde{\delta}^* + G(q^*\rho^*))\right) - \frac{1}{2}\left[\theta\left(\frac{\delta^*}{1-\theta} + U_\theta(\rho^*)\right) + \phi\theta v^*\left(\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*)\right)\right]. \tag{8.28}$$

Substituting conditions (5.1) and (5.2), equation (5.3) is obtained.

Appendix B: New Asset Supply z_t^A and Non-Fundamental Dynamics

The issuance of a new non-fundamental asset z_t^A by investing adult agents has a direct crowding-in effect at time t as funds are channeled from low efficiency young agents to higher efficiency adult agents. However, the new non-fundamental asset will have to be sold by young agents in the future and so it will become part of the existing non-fundamental asset z_{t+1}, which requires a continuous demand from future savers. So a natural question to ask is whether the initial crowding in effect for a newly supplied non-fundamental asset can compensate the future crowding-out effect enough to allow for a non-fundamental equilibrium to exist. We address this question using the dynamic equations of Proposition (1) and following the method of Martin and Ventura (2012).[20] In order

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[20]The stationary steady state analysis for the case of the existence of a non-fundamental equilibrium and new asset

for z_t^A to relax the condition for an equilibrium it must shift the $E_t(z_{t+1})$ schedule downward. Assuming that $\rho_{t+1} = \underline{a}$ it is possible to show that downward shift happens if and only if

$$\frac{1 - n_t + n_t(1 - \alpha)U_0(\underline{a})}{(1 - \alpha)U_0(\underline{a})} < z_t < 1 - n_t. \tag{8.29}$$

This condition can be satisfied if the interval exists in the first place, which happens for n_t small enough compared to $1 - n_t$. This is intuitive, the smaller is the share of wealth available to the productive adult agents, the more effective would be a new non-fundamental asset to crowding-in funds towards efficient investments. Second, a crowding-in is possible only if an old non-fundamental asset has already a non-negligible size in the economy. For a non-fundamental equilibrium not to exists when $z_t^A = 0$ it has to be that $E_t(z_{t+1}) > z_t$ so that the a competitive return is not affordable. If condition (8.29) is satisfied, a new non-fundamental asset can create the condition for an equilibrium to exists if there is a $z_t^A > 0$ with $z_t + z_t^A \leq 1$ such that the inequality is reversed, i.e. $E_t(z_{t+1}) < z_t$. Suppose we want to give the best chance to the new asset by letting $z_t^A = 1 - n_t - z_t$, then together with (8.29) it must be that

$$z_t\left(1 - n_t + \frac{1 - \alpha}{\alpha}U_0(\underline{a})(1 - z_t)\right) < 1. \tag{8.30}$$

Characterizing the conditions for the case of $\rho_{t+1} > \underline{a}$ is more involved and we do not report it here, but similar conditions on z_t can be derived, together with less stringent conditions on the relationship between n_t and $1 - n_t$.

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creation is not appropriate since it would require to assume that a new asset is continuously created. Given the diminishing return to capital in the economy it is possible to show that such a steady state will not exist unless a non-fundamental equilibrium was already possible without the new asset being continuously created.

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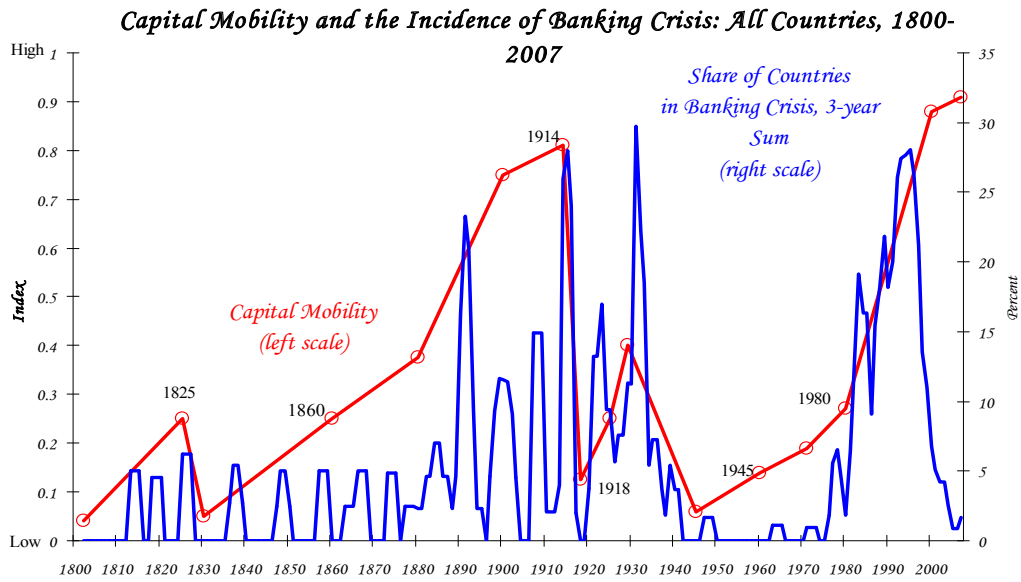
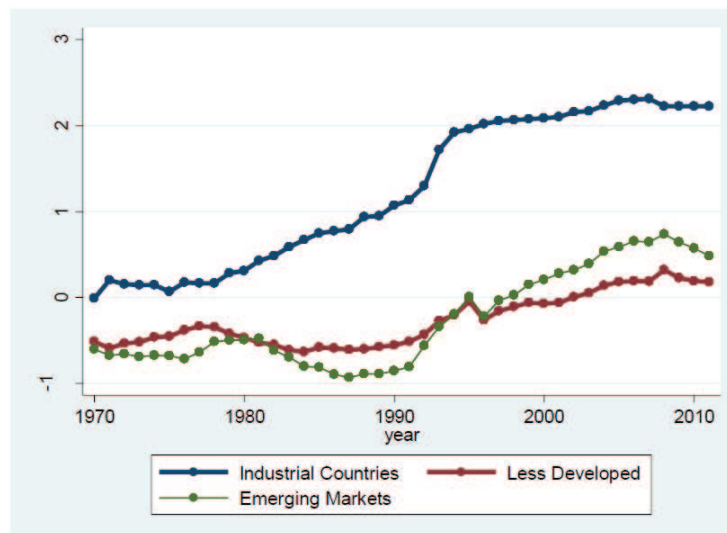Figure 3: FINANCIAL CRISES AND MARKETS INTEGRATION, REINHART AND ROGOFF (2009)

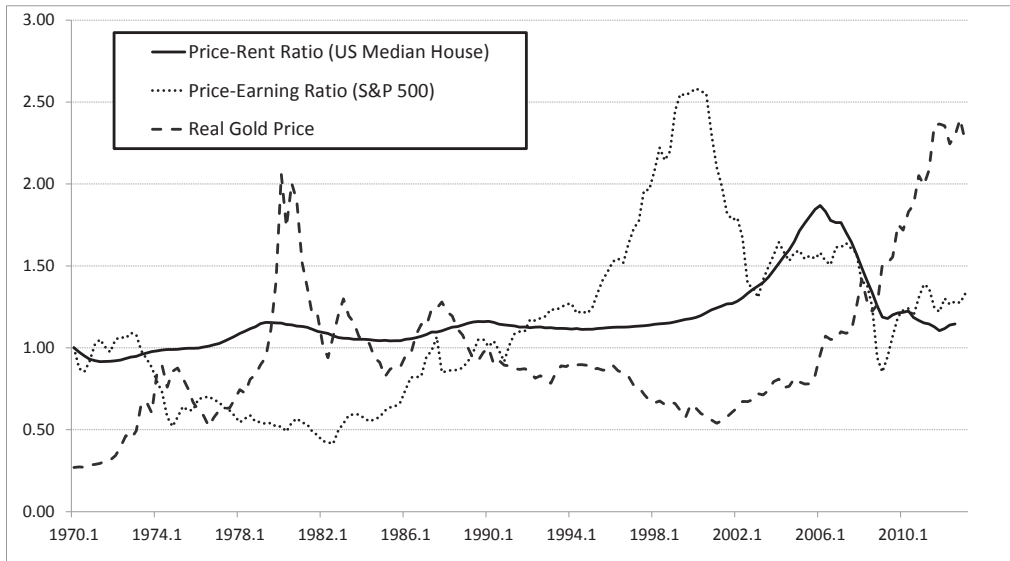Figure 4: GLOBAL FINANCIAL OPENNESS, CHINN AND ITO (2008)

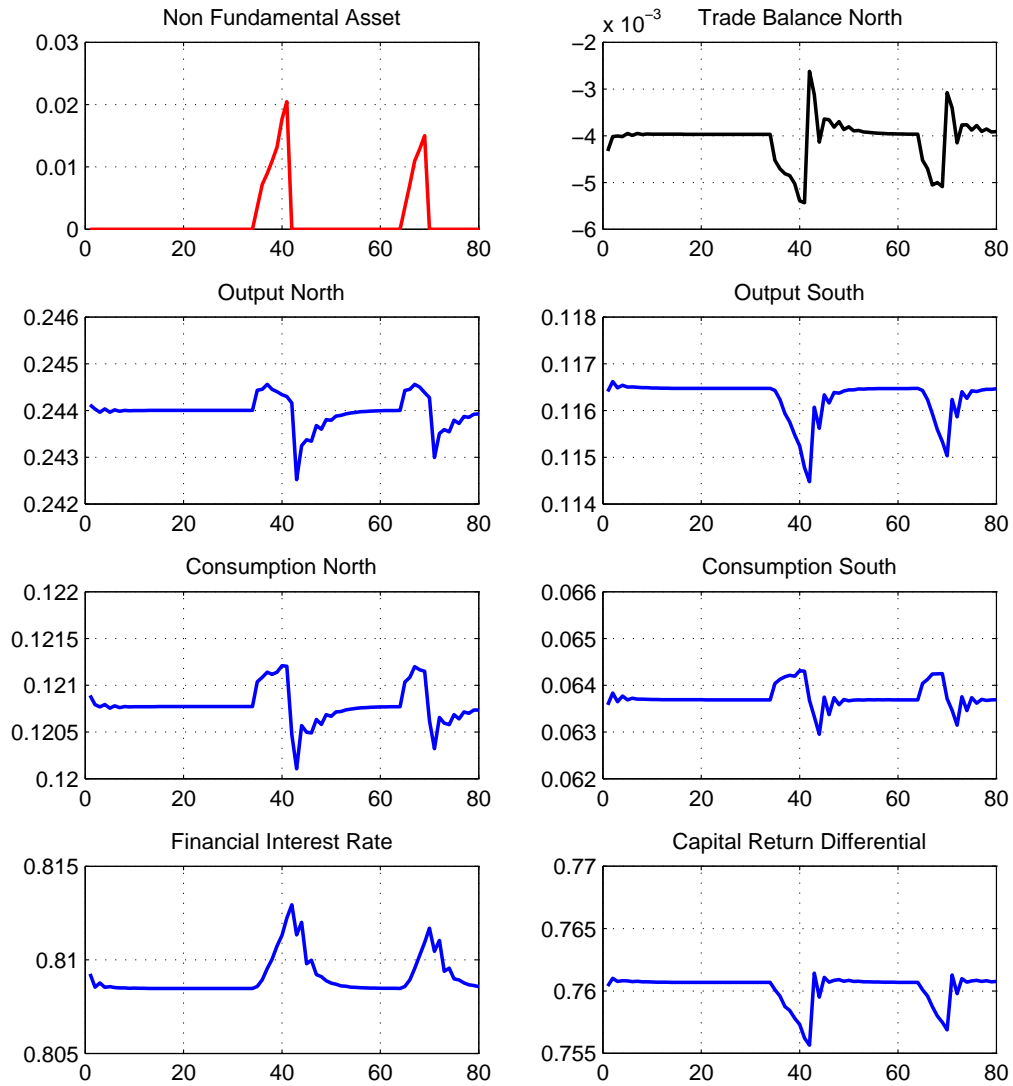Figure 5: Stock Market, Real Estate and Gold, 1970-2013

Figure 6: NON-FUNDAMENTAL DYNAMICS IN THE GLOBAL ECONOMY: NUMERICAL SIMULATION